Public Capital Maintenance and Congestion:
Long-Run Growth and Fiscal Policies

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Abstract

In this paper we study the equilibrium properties of an endogenous growth model, in which public main-
tenance expenditures affect the depreciation rate of public capital and the latter is subject to congestion. A
rise in ‘new’ public investment and output also raises the public capital depreciation rate and reduces public
capital accumulation due to increased public capital usage. We then point out a mechanism under which
thresholds of policy variables for economic growth arise and we find that economies with low congestion in
public infrastructure, in which the rise in output is high, will require a threshold level of public capital main-
tenance for ongoing growth. We also examine the fiscal implications of public capital maintenance policies
and we find that the composition of public capital expenditures under congestion is a crucial determinant of
optimal and growth-maximizing fiscal policies. The government can affect the return of public capital by re-
allocating public expenditures between ‘new’ public investment and maintenance and hence avoid excessive
taxation that is required under increasing congestion.

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1. Introduction

Public capital maintenance expenditures are important for infrastructure services, which in turn are a key instrument for long-run growth.\(^1\) Several country reports and case studies from developing countries demonstrate that the lack of sufficient public capital maintenance has been a critical factor for the observed growth stagnation, since in most cases infrastructure building is given a higher priority than maintenance. According to the World Bank (1994), lack of maintenance in key infrastructure sectors (like roads, railways, power, and water) in developing countries caused losses equivalent to a quarter of their annual investment in infrastructure in the early 1990s, whereas a similar situation was faced in most South Eastern European countries during the last decade (World Bank, 2000). A large fraction of public capital maintenance involves spending on roads, for which maintenance expenditures typically account between 30 to 60% of total expenditures and as much as 0.5% of GNP (Gwilliam and Shalizi, 1999). Harral and Faiz (1988) estimated that the maintenance level required to prevent road deterioration amounted to 0.2% of GDP for East Asia and Pacific countries and to 1% for West African countries for 1986–1990; in turn, the backlog of maintenance work varied from 1.6% of GDP in East Asia and the Pacific to 3.5% in South Asia.\(^2\) The influence that public capital maintenance may have on the growth process should therefore make fiscal policies related to capital expenditures and their allocation an important factor in models that attempt to understand the behavior of the macroeconomy.

Despite the intuitive consensus on the crucial impact of maintenance expenditures in public cap-

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\(^1\)A major obstacle in assessing the role of capital maintenance is that there is no systematic recording of maintenance expenditures, because they are treated as a current expenditure and are not assigned a separate category in the national accounts or any other macroeconomic data source. However, some evidence suggests that public capital maintenance comprises a non-negligible share of output. Globally, the only source of long-run data on capital spending in newly purchased assets and maintenance is the Canadian survey on ‘Capital and Repair Expenditures’, which shows that total public capital maintenance and repair expenditures in Canada amounted on average to 1.5% of GDP for the period 1956-93 and comprised 21% of total public capital spending; see McGrattan and Schmitz (1999) and Kalaitzidakis and Kalyvitis (2005) for a more detailed presentation of this dataset. Yepes (2004) estimates that infrastructure maintenance in East Asian countries amounted to 2.2% of GDP over the period 1996-2005 and covered roughly 30% of total capital expenditures.

\(^2\)These estimates concur with anecdotal evidence from various studies. The World Development Report (World Bank, 1994) stated that an additional $12 billion spent on timely road maintenance in Africa could have saved $45 billion spent in reconstruction. Also, the allocation of recurrent expenditures in the 1993/4 budget for Ethiopia was estimated to be less than half of what would be needed for regular road maintenance (International Monetary Fund, 1995); see also Table 1 in Gwilliam and Shalizi (1999) for additional anecdotal evidence. The study by Heggie and Vickers (1998) corroborates these assessments by stating that the return for road maintenance projects between 1961 and 1988 was 38.6 percent compared to 26 percent for all transport projects and 21 percent for all World Bank investment projects.
ital formation, there have been only recently some systematic attempts to investigate their growth impact. The author shows that the optimal maintenance level (as a share of GDP) depends upon various parameters and presents calibration results from Latin American countries that confirm the importance of maintenance for the pattern of growth in these countries. Kalaitzidakis and Kalyvitis (2004) have extended Rioja’s (2003) model by concentrating on the growth implications of public capital maintenance expenditures. In their model, both types of expenditures are financed by a tax on output as in Barro (1990); by altering their allocation the government can use the share of maintenance as a policy instrument to raise the shadow value of private capital and the growth rate of the economy. An appealing implication is that the growth-maximizing tax rate is higher than the elasticity of public capital in the production function (which reverses a standard argument put forward by, among others, Barro (1990), Glomm and Ravikumar (1994), Devarajan et al. (1998)), as the additional positive effect of maintenance expenditure on the accumulation of public capital raises the benefits of taxation compared to the standard models.

A natural extension of this framework envisages the role of public capital as a rival and non-excludable good that is subject to congestion; as argued by Barro and Sala-I-Martin (1992), virtually all services provided by the public sector, like transport and energy, are subject to congestion. In turn, models of productive government services, such as those developed by Barro and Sala-I-Martin (1992), Turnovsky (1997a, 1997b), and Fisher and Turnovsky (1998), have assumed that the input of public capital to private production is subject to congestion and have established that

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3The assessment of the impact of public capital maintenance is usually performed in the context of cost-benefit analysis and examines primarily the issue of road damage and optimal user charges, which rely on required repairs and their timing (see Newbery, 1988).

4In an empirical context, Kalaitzidakis and Kalyvitis (2005) provide evidence that the Canadian economy would benefit in terms of growth from a reallocation between ‘new’ public investment and maintenance expenditures.

5Although the concept of congestion appears theoretically sound and rising congestion levels in public services are typically observed in most countries and regions, it does not translate easily into operational guidelines regarding its growth effects. Data on congestion are not systematically collected on a cross-country basis and the existing evidence is based on local or national studies involving mainly the transportation sector. For instance, Schrank and Lomax (2005) report that the cost of congestion in the US transportation sector, which accounts for more than 30% of total infrastructure, has increased from 12.5 billion Dollars in 1982 to 63.1 in 2003 (in constant 2003 prices). Some cross-country estimates indicate that the ensuing costs of congestion are found to exceed 3.5 per cent of GDP (United Nations, 2002).
the degree of congestion is important in assessing the linkages between growth, public and private capital, and represents a key determinant of tax policies.

The task of integrating public capital maintenance in a growth model with congestion in public capital appears therefore to be a stimulating challenge. The purpose of the present paper is to provide a unified framework for analyzing the macroeconomic implications of these issues by formulating a growth model with endogenous public capital depreciation and congestion in public capital services, in order to explore the steady-state properties of the economy and the associated first-best and growth-maximizing fiscal policies. To this end, we follow the mainstream class of infrastructure-led endogenous growth models developed by Barro (1990) and Barro and Sala-I-Martin (1992), which have stressed the role of government productive activities as key determinants of long-run growth, by assuming that private sector productivity is affected by government productive services in a balanced budget framework. In our approach the production side of the economy follows Turnovsky (1997a, 1997b) and Fisher and Turnovsky (1998) by adopting the view that the public capital stock rather than the flow of government expenditures is crucial for economic activity. In this setup, where the public capital stock is subject to congestion, the government can decide on the public capital accumulation rate by investing either in ‘new’ public capital, or by retarding the rate of decay of installed capital through public maintenance expenditures.

Our major findings can be summarized as follows. We find that a threshold level of expenditures for public capital maintenance, which depends qualitatively on the level of congestion in the economy, is necessary for the existence of the balanced growth path. The key mechanism is that a rise in ‘new’ public investment and, consequently, output also raises the public capital depreciation rate and reduces public capital accumulation due to increased public capital usage. The magnitude of the rise in public capital usage depends upon congestion and hence economies with low (high) congestion in public infrastructure, in which the rise in output is high (low), will require a threshold level of public capital maintenance (‘new’ public investment) for ongoing growth.

We also establish that, in the presence of congestion, the share of maintenance expenditures in total public capital expenditures is a critical determinant of the optimal and the growth-maximizing

Interestingly, several reports from developed and developing countries often associate the rising costs of congestion with insufficient infrastructure maintenance; see United Nations (2006).
government size. Consequently, if the level of congestion in the economy changes the government can use the two fiscal policy instruments, namely the tax rate and the composition of public capital expenditures, for the design of fiscal policies to improve welfare and growth. In particular, the optimal tax rate is positively related to the degree of congestion because of the negative externality leading to an over-accumulation of private capital. The optimal tax rate will be even higher in the presence of public capital maintenance, because agents do not take into account the positive effect on aggregate activity generated by their over-investment in private capital. However, the implied rise in aggregate public capital expenditures reduces the depreciation rate of public capital and, as a result, the additional tax revenues can finance ‘new’ public investment at a greater proportion. Hence, the government also benefits from the re-allocation between ‘new’ public investment and maintenance to improve welfare.

These results have some important policy implications. First, the requirement of a minimum level for public capital maintenance (and also for ‘new’ public investment) in order for the economy to attain sustainable growth conforms with the common view that stagnant growth is often associated with the lack of a critical level of public capital maintenance required to boost growth. Second, our results underline the importance of the efficient policy mix between public capital maintenance and ‘new’ public investment for the design of fiscal policies. In particular, our findings imply that when congestion rises (falls), the government can benefit by steering public capital expenditures away from (towards) maintenance and towards (away from) ‘new’ public investment without necessarily altering the size of public expenditures. In the same vein, a government that faces crowded roads and empty ports can steer the mix of the existing public capital expenditures towards building new roads and maintaining the existing ports, in order to improve public sector efficiency.

The rest of the paper is structured as follows. Section 2 solves the decentralized equilibrium problem and section 3 studies the steady state and the dynamic properties of the model. Section 4 presents the social planner problem and derives the first-best fiscal policies. Section 5 investigates the growth-maximizing fiscal policies and, finally, section 6 discusses the results and concludes the paper.
2. The model

This section presents a representative agent model with congestion in public capital, in which maintenance expenditures by the government affect the depreciation of the public capital stock. The main features of the model are as follows: (a) the production function of the firm depends upon the private and public capital stocks, with the latter providing a positive production externality to private firms, (b) the production function exhibits a congestion effect in the acquirement of public capital services by the private sector, (c) public maintenance expenditures affect negatively the decay of public capital, and (d) public capital services in the form of expenditures on ‘new’ public investment and public capital maintenance are financed by a tax on output. Lower- and upper-case variables denote individual and aggregate quantities, respectively.

2.1. The representative agent

Consider an economy populated by $N$ homogenous agents with no population growth. The representative consumer-producer in this economy consumes, $c$, of the production good, in order to maximize the following intertemporal logarithmic utility function:

$$\max \int_0^\infty \log(c)e^{-\rho t}dt$$

(1)

The representative agent produces a single traded good, $y$, by facing the following Cobb-Douglas production function:

$$y = f(k, K^s_g) = y = (k)^a \left( \frac{K^s_g}{N^{1-\sigma}} \right)^{1-a}, 0 < a < 1$$

(2)

where $k$ denotes the individual private capital stock and $K^s_g$ denotes the public capital services. We assume that public capital services are subject to congestion and parameter $\sigma \in (0, 1)$ measures the degree of congestion. To eliminate any scale effects we assume that the productivity of individual private capital depends on the average level of public capital services, which is in turn determined by the number of individual that uses them and the degree of congestion. For instance, under no congestion ($\sigma = 0$) all individuals make efficient use of the public services and the productivity
of private capital stock depends on the average public capital services given by \( \frac{K^s}{N} \). The services derived by the agent from the public capital stock are then given by:

\[
K^*_g = K_g \left[ \frac{k}{K} \right]^{\sigma}
\]  

(3)

where \( K \) denotes the aggregate private capital stock. Specification (3) follows a concept put forward by Edwards (1990) and formalized by, among others, Turnovsky (1997a, 1997b), Fisher and Turnovsky (1998), and Gomez (2004), and implies that public capital is congested by the use of private capital. This specification embodies ‘relative’ congestion where the level of services derived by the agent from the provision of a public good is in terms of the usage of the individual capital stock relative to the aggregate capital stock (as, for instance, in the transport sector).\(^7\) Within this context, \( \sigma = 0 \) corresponds to the no-congestion case, whereas \( \sigma = 1 \) implies full congestion and the private capital has to rise in direct proportion to the aggregate private capital stock to maintain a fixed level of public services available to the firm.

The representative agent is endowed with an initial capital stock \( k(0) > 0 \) and accumulates private capital by spending on investment, \( i \). The private capital accumulation constraint is then given by the following law of motion:

\[
\dot{k} = i - \delta_k k
\]  

(4)

where \( \delta_k \) denotes the constant private capital depreciation rate.\(^8\)

2.2. Government Sector

The government invests in ‘new’ public capital, \( I_g \), and maintains the public capital stock, \( M_g \),

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\(^7\)Relative congestion is opposed to ‘absolute’ congestion where the public capital stock is congested by the aggregate private capital stock (like, for instance, in the case of police protection); see Eicher and Turnovsky (2000) for an extensive discussion on these two concepts of congestion. We opted for the specification with relative congestion for two reasons. First, relative congestion is associated with sectors of economic activity that are closer to infrastructural capital. Second, ‘absolute’ congestion in the production function yields a scale effect that is empirically implausible. Notice that the results derived later on are not qualitatively affected if the specification with ‘absolute’ congestion is adopted.

\(^8\)Assuming that private capital depreciation is endogenously determined would not affect qualitatively the results derived later on, provided that private maintenance enters as a ratio of the private capital stock, rather than output, in the depreciation function. This would be a plausible assumption since it implies that private capital depreciation is affected by the usage of the private capital stock, in contrast to public capital depreciation that depends upon its aggregate usage given here by output; see the next subsection.
to affect its decay rate. We assume that the law of motion for public capital accumulation is given by:

\[ \dot{K}_g = I_g - \delta_g \left( \frac{M_g}{Y} \right) K_g \]  

(5)

where the depreciation function \( \delta_g(\cdot) \) is continuous and has the following properties: \( \delta'_g(\cdot) < 0 \), \( \delta''_g(\cdot) > 0 \), \( \lim_{\frac{M_g}{Y} \to 0} \delta_g(\cdot) = 1 \) and \( \lim_{\frac{M_g}{Y} \to \infty} \delta_g(\cdot) = \bar{\delta} \) where \( \bar{\delta} \in (0, 1) \). This setup covers the general definition of capital maintenance as the “...deliberate utilization of all resources that preserve the operative state of capital goods” (Bitros, 1976), where as far as public capital is concerned it encompasses the “...wide range of activities aiming at keeping infrastructure at a serviceable condition” (Heller, 1991). We assume that the depreciation rate cannot be diminished below a threshold, \( \bar{\delta}_g \), determined by the ‘physical’ depreciation due to capital ageing and other technical factors. We also assume that the public capital depreciation rate is a negative function of public maintenance expenditures as a ratio of aggregate economic activity, \( Y \). Hence, public capital depreciation is a positive function of the usage of public capital stock by all agents of the economy (including the public sector) given by total economic activity. This assumption implies, for instance, that public roads will deteriorate faster if the aggregate production activity by both the private and public sectors in the economy is high. In turn, the government can choose the level of maintenance expenditures as a share of output to determine the depreciation rate of its capital stock.

The government finances expenditures by levying a flat tax rate on private output, \( \tau \in (0, 1) \). Assuming a balanced budget the government budget constraint is given by:

\[ \tau Y = I_g + M_g \]  

(6)

To ease exposition, we parameterize public maintenance \( M_g \) as a share on tax revenues by \( \mu_g \in (0, 1) \) and thus the corresponding share for ‘new’ public investment is given by \( (1 - \mu_g) \). The internal allocation of government expenditures can then be written as:

\[ I_g = (1 - \mu_g)\tau Y \]  

(7)
\[ M_g = \mu_g \tau Y \quad (8) \]

### 2.3. The representative agent problem

The representative agent in the economy receives income from after-tax production (income) that is allocated to investment and consumption:

\[ (1 - \tau)y = i + c \quad (9) \]

The aggregate production of the economy is allocated to consumption, private and public investment, and public capital maintenance. The economy-wide resource constraint is given by:

\[ Y = C + I_k + I_g + M_g \quad (10) \]

By summarizing the above constraints and using (4) in (9) we get that the flow budget constraint is given by:

\[ \dot{k} = (1 - \tau)y - c - \delta_k k \quad (11) \]

given the initial capital endowment \( k(0) > 0 \).

The intertemporal problem of the representative agent is to maximize lifetime utility (1) subject to (11). The current-value Hamiltonian of the problem is given by:

\[ H = \log(c) + \lambda((1 - \tau)y - c - \delta_k k) \]

The first-order conditions are given by:

\[ \frac{1}{c} = \lambda \quad (12) \]

\[ (1 - \tau) \left[ (a + (1 - a)\sigma)k^{-(1-a)(1-\sigma)} \left( \frac{1}{K} \right)^{(1-a)} (K_g)^{1-a} N^{-(1-a)(1-\sigma)} \right] = \rho + \delta_k - \frac{\lambda}{\lambda} \quad (13) \]

\[ \lim_{t \to \infty} \lambda(t)k(t)e^{-\rho t} = 0 \quad (14) \]

Equation (12) is the standard condition stating that at the optimum the marginal utility of
consumption equals the shadow price of wealth. Equation (13) states that at the optimum the
after-tax marginal product of capital equals to the opportunity cost of investing in capital, which
in turn equals the depreciation of physical capital, and the rate of time preference minus the capital
gain. The after-tax marginal product of private capital is augmented by the level of congestion
because investment in private capital also raises the marginal benefits that the individual derives
from the public capital stock given the aggregate private capital stock. Congestion operates then as
an externality of the public capital stock on private decisions; see also Turnovsky (1997a, 1997b).

3. Balanced growth and equilibrium dynamics in the decentralized economy

The Balanced Growth Path (BGP) is defined a state where the variables of the economy grow at
a constant rate. Since all agents in the economy are identical, the equilibrium relationship \( K = Nk \)
links the aggregate and individual capital stocks.

**Definition 1.** The competitive equilibrium of the economy is defined for the exogenous policy
instruments \( \tau, \mu_g \), and aggregate allocations \( I_k, I_g, M_g, C, K, K_g \) such that the individuals solve their
intertemporal utility maximization problem by choosing \( c \) and \( i \) given \( \tau \) and \( \mu_g \).

Taking the time derivative of (12) and substituting in (13), we get after aggregating that the
equilibrium growth rate of aggregate consumption is given by:

\[
\frac{\dot{C}}{C} = (1 - \tau)(1 - (1 - a)(1 - \sigma)) \left( \frac{K_g}{K} \right)^{(1-a)} - \rho - \delta_k
\]

(15)

Also, after aggregating the feasibility constraint of our economy, (11), we get that the aggregate
private capital stock evolves as:

\[
\frac{\dot{K}}{K} = (1 - \tau) \left( \frac{K_g}{K} \right)^{(1-a)} - \frac{C}{K} - \delta_k
\]

(16)

Using (7) and (8) in (5) the equilibrium growth rate of public capital is given by:

\[
\frac{\dot{K}_g}{K_g} = (1 - \mu_g) \tau \left( \frac{K_g}{K} \right)^{-\sigma} - \delta_g (\mu_g \tau)
\]

(17)
and the transversality condition under (12) is now modified to:

\[ \lim_{t \to \infty} \frac{k(t)}{c(t)} e^{-\rho t} = \infty \]  

(18)

At the BGP the growth rates of consumption, public and private capital have to grow necessarily at the same rate, i.e. \( \frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{K}_g}{K_g} = g \). This result is easily obtained by investigating the equilibrium growth rates of these variables separately. In particular, for the consumption growth rate, given by (15), to be constant in the steady-state we see that both \( K \) and \( K_g \) have to grow at the same constant rate, say \( g_k = g_{k_g} = g \). Since the steady-state ratio of public to private capital will be constant, the equilibrium growth rate of consumption, \( g_c \), will be constant too. Then, by inspection of (16), in order for the growth rate of private capital to be constant we need that \( g_k = g_{k_g} \) and \( g_c = g_k \) (for \( \frac{K}{K_g} \) and \( \frac{C}{K} \) to be constant respectively). Taken together with the previous condition on the consumption growth rate to be constant these conditions imply that \( g_c = g_k = g_{k_g} = g \). Since this condition also satisfies (17), as well as the transversality condition, (18), the necessary condition for the existence of a BGP in this economy is that all variables grow at the same rate, \( g \).

Given the above necessary condition for a BGP we can now derive the equilibrium growth rate of the economy. We first define the following auxiliary stationary variables, namely \( \omega \equiv \frac{\dot{C}}{C} \Rightarrow \frac{\dot{\omega}}{\dot{C}} \equiv \frac{\frac{\dot{C}}{C} - \frac{\dot{K}}{K}}{\frac{\dot{K}}{K}} \equiv \frac{\dot{\omega}}{\dot{C}} \equiv \frac{\dot{K}_g}{K_g} - \frac{\dot{\omega}}{\dot{K}} \). By solving (15) for \( z \) we obtain that at the BGP:

\[ \ddot{z} = \left( \frac{\bar{g} + \rho + \delta_k}{(1 - \tau)(1 - (1 - a)(1 - \sigma))} \right)^{\frac{1}{1 - \sigma}} \]  

(19)

Substituting then (19) in (17) we obtain the equilibrium growth rate of the economy implicitly as:

\[ \Phi(\bar{g}) \equiv \bar{g} - (1 - \mu_g)\tau \left( \frac{\bar{g} + \rho + \delta_k}{(1 - \tau)(1 - (1 - a)(1 - \sigma))} \right)^{\frac{-a}{1 - a}} + \delta_g \left( \mu_g \tau \right) = 0 \]  

(20)

where the solution of this continuous function for \( \bar{g} > 0 \), such that \( \Phi(\bar{g}) = 0 \), determines the existence and the properties of the equilibrium growth rate.

**Proposition 1.** A unique positive equilibrium growth rate, \( \bar{g} \), exists in our economy and
is given by (20) iff \( \delta_g(\mu_g) \frac{\mu_g}{1-\mu_g} (1 - \tau) \frac{\rho}{1-a} < \left( \frac{1-(1-a)(1-\sigma)}{\rho + \delta_k} \right) \frac{a}{1-a} \) holds for given parameter values and exogenous policy instruments.

**Proof.** See Appendix 1.■

Notice that our economy will exhibit a zero equilibrium growth rate if the necessary and sufficient parametric condition holds with equality. A crucial remark is that the two fiscal instruments, \( \tau \) and \( \mu_g \), are critical not only for the quantitative determination of the equilibrium growth rate, but also for the existence of a non-negative BGP. In fact, a direct consequence of Proposition 1 is that there exists a subset in the domain of the policy instruments that forms a set of sufficient values for sustainable growth. The following Corollary formalizes this point.

**Corollary to Proposition 1.** Given the parametric characteristics of the economy, there exists a range \((\hat{\mu}_g, \hat{\mu}_g) \) of public maintenance expenditures as a share of public capital expenditures that has to be implemented in order for Proposition 1 to hold. The boundaries of the public maintenance expenditure share are given by the following conditions.

(I) Lower bound of maintenance share: The share of maintenance expenditures in total public capital expenditures has to exceed a minimum value, \( \hat{\mu}_g > 0 \), where \( \frac{\delta_g(\hat{\mu}_g)}{(1-\mu_g)^\tau} (1 - \tau) \frac{\rho}{1-a} = \left( \frac{1-(1-a)(1-\sigma)}{\rho + \delta_k} \right) \frac{a}{1-a} \); in order for the economy to attain the BGP for any value of \( \tau \in (0, 1) \). This share exists iff \( 1 > \tau \left( \frac{(1-\tau)(1-(1-a)(1-\sigma))}{\rho + \delta_k} \right) \frac{a}{1-a} \). A sufficient parametric condition for a non-zero lower bound for maintenance expenditures to exist is given by \( \sigma < \frac{\rho + \delta_k - a}{1-a} \).

(II) Upper bound of maintenance expenditures: There exists an upper bound for the share of maintenance expenditures in public capital expenditures, \( \hat{\mu}_g < \hat{\mu}_g < 1 \), where \( \frac{\delta_g(\hat{\mu}_g)}{(1-\mu_g)^\tau} (1 - \tau) \frac{\rho}{1-a} = \left( \frac{1-(1-a)(1-\sigma)}{\rho + \delta_k} \right) \frac{a}{1-a} \) for any value of \( \tau \in (0, 1) \), in order for the economy to attain the BGP.

**Proof.** See Appendix 1.■

Proposition 1 in conjunction with cases (I) and (II) of the Corollary show that there exists a range \( \hat{\mu}_g < \mu_g < \hat{\mu}_g \) that is necessary for the existence of a positive long-run growth rate in the economy for any value of \( \tau \in (0, 1) \). In particular, economies that are described by (I) will face non-positive steady-state growth regardless of their government size, unless adequate resources on public capital maintenance are spent. For sufficiently high levels of public investment an additional
unit of ‘new’ public capital boosts aggregate economic activity and hence the usage of the existing public capital stock, which depends upon the degree of congestion, rises. This generates the need for additional public capital maintenance because the depreciation rate of public capital is endogenously determined by the share of maintenance expenditures in output, $\mu_g \tau$. Due to the convexity of the depreciation function, at sufficiently low levels of maintenance activity the positive effect of public investment on public capital accumulation is outweighed by the negative effect generated through the rise on the usage of public capital through its depreciation rate. As a result, a minimum level of maintenance is necessary to sustain a positive growth rate of public capital in the long run. The inverse happens for sufficiently high levels of maintenance activities: the benefits on the depreciation rate are diminishing and, moreover, are bounded by the natural depreciation rate. A threshold level of ‘new’ public investment is therefore necessary as well for attaining long-run growth.

These effects depend on the level of congestion in the economy through the degree of public capital usage determined by economic activity, $Y$. As indicated by Part I of Corollary to Proposition 1, when congestion is sufficiently low the usage of the public capital stock will be high. This raises the depreciation of the existing capital stock and in order for the growth rate of public capital to follow a balanced growth path a minimum level of maintenance is required to preserve the existing capital stock at the higher usage.\(^9\)

Hence, when the minimum level of public capital maintenance is not reached, ‘new’ public investment will not be adequate to attain long-run growth due to the high depreciation of public capital. This result extends the point by Rioja (2003) and Kalyvitis and Kalaitzidakis (2004) on the importance of these expenditures for growth maximization by showing that public capital maintenance expenditures are necessary for the existence of sustainable growth. On the flipside, the condition on the upper bound for maintenance expenditures states that a country has to set

\(^9\)Notice that the other structural characteristics of the economy are important for the BGP as well. In particular, economies characterized by a high elasticity of public capital in the production function, a high depreciation rate of private capital, a high rate of time preference, and low congestion affect ceteris paribus the threshold level of public maintenance expenditures to attain positive growth. For instance, a high rate of time preference (low propensity to save) and a high depreciation rate of private capital affect negatively the growth rates of consumption growth and private capital. In these cases the private agents cannot benefit adequately from public capital services and the high usage of the public capital stock requires a threshold level of maintenance expenditures as a share of output in order for capital accumulation to generate positive growth.
public capital maintenance expenditures at a level that will ensure a minimum level of investment in ‘new’ public capital.\textsuperscript{10}

Finally, we can examine the local stability of the economy, which is determined by the two-dimensional system:

\[
\begin{align*}
\dot{\omega} &= -(1 - \tau)(1 - \sigma)(1 - a)z^{1-a} + \omega - \rho \\
\dot{z} &= (1 - \mu_g)\tau z^{-a} - \delta_g (\mu_g \tau) - (1 - \tau)z^{1-a} + \omega + \delta_k
\end{align*}
\]  

where a dot above a variable denotes the corresponding derivatives with respect to time. In matrix notation we can write:

\[
\begin{bmatrix}
\omega \\
z
\end{bmatrix} = \begin{bmatrix}
\bar{\omega} & -((1 - \tau)(1 - \sigma)(1 - a)z^{-a}\bar{\omega} \\
\bar{z} & (1 - \mu_g)\tau (a)(\bar{z})^{-1-a} - (1 - \tau)(1 - a)(\bar{z})^{-a}
\end{bmatrix} \begin{bmatrix}
\omega - \bar{\omega} \\
z - \bar{z}
\end{bmatrix}
\]

The determinant of the above system is given by:

\[
J = \bar{\omega} \bar{z} \left[-(1 - \mu_g)\tau a(\bar{z})^{-1-a} - (1 - (1 - \sigma)(1 - a)) (1 - \tau)(1 - a)(\bar{z})^{-a}\right]
\]

with \(J < 0\), since \((1 - \mu_g)\tau a(\bar{z})^{-1-a} + (1 - (1 - \sigma)(1 - a)) (1 - \tau)(1 - a)(\bar{z})^{-a} > 0\) for any positive value of \(\bar{z}\) (which can be easily shown to exist). Thus the system comprised by (21) and (22) is locally saddle-path stable.

\section*{4. Optimal policies}

In this section we derive the social planner equilibrium in order to determine the optimal fiscal policy. To this end, we will derive the optimality conditions for the social planner economy and we will then determine the optimal government size (tax rate) with the shares of its components, ‘new’ public investment and public capital maintenance, chosen optimally in order to replicate the first-best environment.

\textsuperscript{10}Notice that, as in Barro (1990), a similar reasoning applies for the lower and upper bound of the government size required to ensure positive growth. It is straightforward to show that Proposition 1 imposes boundaries for the tax rate since it does not hold for \(\tau = 0\) or \(\tau = 1\).
4.1. The Social Planner problem

The objective of the social planner is to maximize social welfare given by:

$$\max \int_0^\infty \log(C)e^{-\rho t} dt$$

subject to the economy’s aggregate resource constraint:

$$Y = K^a K_g^{1-a} = C + I_k + I_g + M_g$$  \hspace{1cm} (23)

and the law of motion for the aggregate public and private capital stocks:

$$\dot{K} = I - \delta_k K$$  \hspace{1cm} (24)

$$\dot{K}_g = I_g - \delta_g \frac{M_g}{Y} K_g$$  \hspace{1cm} (25)

given the initial public and private capital stocks, $K(0)$ and $K_g(0)$. Equation (23) denotes the aggregate resource constraint in our economy. In contrast to the decentralized equilibrium, the production technology in the social planner problem internalizes the congestion externality, since the social planner solves for the aggregate quantities.

The current value Hamiltonian for the above problem is given by:

$$H = \log(C) + \zeta(K^a K_g^{1-a} - C - \delta_k K - I_g - M_g) + \xi(I_g - \delta_g \frac{M_g}{Y} K_g)$$

where the choice variables are $C$, $I_g$, $M_g$, and the state variables of the economy are $K$, $K_g$. The first-order conditions are given by:

$$\frac{1}{C^*} = \zeta$$  \hspace{1cm} (26)

$$\zeta = \xi$$  \hspace{1cm} (27)

$$-\zeta \delta'_g \left( \frac{M_g}{Y^*} \right) \frac{K_g^*}{Y^*} = \xi$$  \hspace{1cm} (28)
Equations (26) to (30) yield the optimal allocations in the social planner economy given the initial conditions and the associated transversality conditions. Specifically, equation (26) shows that at the optimum the marginal utility of consumption equals the marginal change of wealth. Equation (27) implies that the benefit of increasing public investment measured by the shadow price of public capital equals the cost in wealth by allocating expenditures of consumption and maintenance on investment. As opposed to the decentralized equilibrium, the social planner decides for the optimal allocation of public investment and maintenance expenditures with the latter given by (28). The government faces then a trade-off in allocating resources to maintenance and investment as both expenditures affect positively the accumulation rate of public capital and are related through the government resource constraint.

Substituting (26), (28), and (27) in (29) we can obtain the consumption growth rate in the social planner economy, $g_{sp}^c$:

$$
g_{sp}^c = \frac{\dot{C}^*}{C^*} = a \left( \frac{K_g^*}{K^*} \right)^{1-a} \left[ 1 - M_g^* \frac{Y_g}{Y^*} \right] - \rho - \delta_k \quad (31)$$

Also, by the feasibility constraint and the private and public capital accumulation rates, and by (7) and (8), we obtain that:

$$
g_{sp}^p = \frac{\dot{K}^*}{K^*} = (1 - \tau) \left( \frac{K_g^*}{K^*} \right)^{1-a} - \frac{C^*}{K^*} - \delta_k \quad (32)$$

$$
g_{sp}^g = \frac{\dot{K}_g^*}{K_g^*} = (1 - \mu_g)\tau \left( \frac{K_g^*}{K^*} \right)^{-a} - \delta_g(M_g^* \frac{Y_g}{Y^*}) \quad (33)$$

where $g_{sp}^p$ and $g_{sp}^g$ denote the equilibrium growth rates of public and private capital in the social planner economy. In the presence of congestion the share of public expenditures for capital maintenance affects the growth rates of consumption and public capital in the social planner economy. This occurs because of the required taxation imposed on private agents, since the return to private
investment exceeds the social one. To internalize the distortion, the individual producer who accumulates $k$ (and thus contributes positively to aggregate usage given by $Y$) has to provide additional resources to maintain the public services available to other agents, where the required compensation should equal $M/Y$ times the addition to $Y$. In turn, by substituting (27) in (28) we can obtain the optimal allocation of maintenance expenditures to output by the following condition:

$$-\delta'_g \left( \frac{M^*_g}{Y^*} \right) = \frac{Y^*}{K^*_g} \equiv (z^*)^{-a}$$

Equation (34) determines the optimal public to private capital ratio and states that it is optimal for the government to increase maintenance expenditures at a level where the change in the depreciation rate of public generated by public capital maintenance relative to its usage equals the average product of public capital. Furthermore, (29) and (30) represent the optimal conditions with respect to the private and public capital stocks respectively. Using (27) and (28) in (29) and (30), and by virtue of (8), we can obtain that:

$$a(1 - \mu_g \tau)(z^*)^{1-a} - \delta_k = (1 - a)(1 - \mu_g \tau)(z^*)^{-a} - \delta_g \left( \mu_g \tau \right)$$

Equation (35) yields the ratio of public to private capital in the social planner economy as a function of the parameters of the economy and the policy instruments and shows that the net returns of private and public capital are equalized at equilibrium. The marginal products of private and public capital are now reduced as the output share of public capital maintenance increases. The marginal benefit of spending one unit in public capital maintenance, $M^*_g$, is given by the fall in $\delta_g \left( \mu_g \tau \right)$ and the rise in the net return on $K^*_g$. At the same time, there are less available resources (output units) for $C + I_k + I_g$ and as $M^*_g$ rises the marginal products of the capital stocks are reduced by a fraction $\left( \frac{C + I_k + I_g}{Y} \right)$, as the last unit of output spent on maintenance has to have the same impact on welfare with the corresponding one spent on the other components of output.

Notice that if the exogenous depreciation rate of private capital happens to coincide with the endogenous depreciation rate of public capital evaluated at the optimum, then the ratio of public to private capital is given by the familiar condition $z^* = \frac{a}{1-a}$, which states that it depends upon the
ratio of the corresponding capital stock elasticities in the production function. This condition however is only a special case here, as the public to private capital ratio will depend on other variables and parameters of the economy as well, such as the maintenance to output ratio, the endogenous depreciation rate of public capital and the private depreciation rate. It is straightforward to show that a well-defined \( z^* > 0 \) exists for any parameter value of our economy in the assumed domain and that it is a function of the structural characteristics of our economy.\(^{11}\)

4.2. Optimal fiscal policies

Given the solution of the social planner and the decentralized equilibrium, the government aims at choosing that tax rate and the composition of public capital expenditures that enable the competitive equilibrium to achieve a first-best allocation of resources. Using (8) the consumption growth rate in the social planner economy can be written as:

\[
g_{cp}^{sp} = a \left( 1 - \mu_g \tau^* \right) \left( z^* \right)^{1-a} - \rho - \delta_k
\]

Since, by (36), (32), and (33), at the BGP consumption, public capital, and private capital have to grow at the same rate, in order to enable the decentralized equilibrium to replicate the first-best outcome the government has to set the tax rate at the level that equates the growth rate of consumption of the decentralized economy with the one attained by the social planner.\(^{12}\) From (15) and (36) we obtain the optimal tax rate with public capital maintenance under congestion:

\[
\tau^* = \frac{(1-a)\sigma}{(1-a)\sigma + a(1-\mu_g)}
\]

\(^{11}\)Equation (35) can be written as \( \phi(z^*) = a(1-\mu_g)z^* - (1-a)(1-\mu_g)z^{-a} + \delta (\mu_g) = 0 \) and has the following properties:

1. \( \phi(z^*) \) is continuous function for \( z^* > 0 \) since is the addition of continuous functions.
2. \( \frac{\partial \phi(z^*)}{\partial \tau} = a(1-a)(1-\mu_g)z^* - (1+(z^*)^{-1}) > 0 \)
3. \( \lim_{z^* \to \infty} \phi(z^*) = +\infty \)
4. \( \lim_{z^* \to 0} \phi(z^*) = -\infty \)

From 1-4 it follows that there exists \( z^* > 0 \) that solves \( \phi(z^*) \) such that \( \phi(z^*) = 0 \) and it is unique.

\(^{12}\)Notice that the growth rates of public and private capital are quantitatively different than the ones obtained in the decentralized economy because the corresponding consumption growth rates are different. By equating the consumption growth rate of the social planner with the competitive equilibrium through the optimal tax rate, the decentralized economy will achieve the outcome of the social planner in terms of the growth rates of public and private capital as well.
The optimal tax rate is clearly feasible for any value of congestion and maintenance in their domain. Under public capital maintenance and congestion, the optimal tax rate differs from that derived by Barro (1990) and is a more general case of the policy rule derived by Kalaitzidakis and Kalyvitis (2004). To highlight equation (37), we first consider some limit cases. If there is no congestion in the economy \((\sigma = 0)\), the optimal tax rate is equal to zero since there is no distortion on individual decisions; this is a manifestation of the well-know Chamley (1986) result that capital should be untaxed at the optimum. On the opposite, under full congestion the optimal tax rate is given by \(\tau^*_{\sigma=1} = \frac{1-a}{1-\alpha} \), which states that the optimal tax rate depends positively on the public maintenance expenditures to output ratio.\(^{13}\) As has been pointed out in Kalyvitis and Kalaitzidakis (2004), the endogeneity of public capital depreciation renders the Barro (1990) optimal taxation rule suboptimal, as maintenance expenditures increase the marginal cost of public funds and the government size has to exceed the elasticity of public capital in the production function to equal the corresponding marginal benefit. In the case of partial congestion \((0 < \sigma < 1)\), the tax rate is lower and we can establish that it is a positive function of the level of congestion, \(\sigma\), for a given share of public capital maintenance:

\[
\frac{\partial \tau^*}{\partial \sigma} = \frac{a(1-a)(1-\mu_g^*)}{(1-(1-\sigma)(1-a)-\mu_g^*a)^2} > 0
\]  

Equations (37) and (38) extend the optimal taxation rule derived by Gomez (2004), which also states that the government has to implement a positive time-invariant tax rate in order to drive the decentralized economy to the first best outcome. The optimal tax rate is positively related to the degree of congestion in the economy as a result of the implied intensity of the negative externality: the higher return to private investment and the over-accumulation of private capital triggered by congestion have to be diminished by imposing a higher tax on output in order to attain the social optimum. Moreover, the optimal tax rate is higher in the presence of public capital maintenance because agents do not take into account the positive effect on aggregate activity generated by their over-investment in private capital. Hence, the optimal tax rate will be positively related to

\(^{13}\)Note that in the case of full congestion the optimal taxation rule coincides with the growth-maximizing rule derived later on, because the economy has to grow at the maximum rate, in order for the decentralized economy to eliminate the externality and replicate the first-best environment.
maintenance expenditures, which have to be raised to sustain the high usage of public capital.

However, the effect of congestion on the optimal tax rate is now mitigated as the government can benefit from a re-allocation between ‘new’ public investment and public capital maintenance to achieve the first-best outcome. In particular, we can investigate the long-run impact of congestion by using equations (34), (35) and (37), which determine jointly the three endogenous variables of the model, \( \tau^*, z^*, \mu_g^* \), in terms of the exogenous parameters. The total effects of congestion on the optimal tax rate and the optimal allocation of government revenues to maintenance are given by:

\[
\frac{d\tau^*}{d\sigma}\bigg|_{\text{total}} = \frac{A\tau^*}{B\mu_g^* + \tau^*} > 0
\]

\[
\frac{d\mu_g^*}{d\sigma} = -\frac{A\mu_g^*}{B\mu_g^* + \tau^*} < 0
\]

where \( A \equiv \frac{\partial\tau^*}{\partial\sigma} > 0 \) is the partial derivative of the optimal tax rate w.r.t. \( \sigma \) given by (38) and \( B \equiv \frac{\partial\tau^*}{\partial\mu_g^*} = \frac{a\sigma(1-a)}{(1-(1-\sigma)(1-a)-\mu_g^* a)} > 0 \) is the partial derivative of the optimal tax rate w.r.t. \( \mu_g \) determined by (37). Taking the total derivative of the optimal tax rate given in (37), we obtain equation (39) alternatively as:

\[
\frac{d\tau^*}{d\sigma}\bigg|_{\text{total}} = A + B \frac{d\mu_g^*}{d\sigma}
\]

Here, \( A \) represents the positive direct effect of congestion on the optimal tax rate (‘taxation’ effect) and \( B \frac{d\mu_g^*}{d\sigma} \) represents the negative indirect effect triggered by congestion through the change in the optimal share of maintenance expenditures (‘reallocation’ effect). The ‘taxation’ effect clearly dominates the ‘reallocation’ effect, since the total effect of congestion on the optimal tax rate is positive, as shown by (39). These results are summarized in the following Proposition.

**Proposition 2.** The ‘taxation’ effect of congestion on the optimal tax rate is positive. The ‘reallocation’ effect of congestion on the optimal share of public capital maintenance in total public capital expenditures is negative. The total effect of congestion on the optimal tax rate is positive.

*Proof. See Appendix 2.*

Proposition 2 generalizes the impact of congestion on optimal fiscal policy. The policy change in the optimal allocation of resources brought about by congestion, which is captured by the term
(B \frac{dp_{g}^{*}}{dx})$, mitigates the required increase in the optimal tax rate, because the government can now benefit from the optimal re-allocation of public capital expenditures between ‘new’ investment and public capital maintenance. Intuitively, a rise in congestion reduces the amount of public services extracted by the private agent and increases their marginal value, thus increasing the optimal tax rate (‘taxation’ effect). At the same time, the rise in aggregate public capital expenditures reduces the depreciation rate of public capital. With a constant differential between the marginal products of private and public capital, equation (35) implies that the share of public capital maintenance in output has to fall to equalize the net returns of the capital stocks. As a result, the additional tax revenues finance ‘new’ public investment at a greater proportion (‘reallocation’ effect).

We can also investigate the impact of congestion on the allocation of public capital expenditures when the government aims at holding the tax rate constant. In such a case, the government can use the internal allocation of resources as an alternative policy instrument, whereas the tax rate remains unaffected by congestion. This effect is given by:

$$
\frac{d\mu_{g}^{*}}{d\sigma} \bigg|_{dx^{*}=0} = -\frac{1}{\sigma} < 0
$$

Equation (42) gives the optimal response of government expenditures in maintenance as a share of total public capital expenditures following a marginal change in the level of congestion for a fixed government size. In particular, following a rise (fall) in congestion the government should reduce (increase) the level of public capital maintenance. Therefore, following a rise in congestion the government can improve efficiency by altering the composition of public capital expenditures between maintenance and ‘new’ investment instead of imposing a higher tax on production to raise public services, as Proposition 2 would imply. A direct consequence of (42) is that countries or regions facing fiscal limitations can re-allocate the existing public resources by steering expenditures towards (away from) ‘new’ public investment and away (towards) public capital maintenance in response to higher (lower) congestion. Therefore, in the presence of increasing congestion in public capital (and assuming that maintenance expenditures and ‘new’ investment exceed their threshold values dictated by the Corollary to Proposition 1), the government is equipped with an extra policy instrument for the conduct of optimal fiscal policy.
5. Growth-maximizing policies

In this section we analyze growth-maximizing fiscal policy rules. Modern growth theory has shown particular interest in growth-enhancing policies, as the understanding of the forces of economic growth is crucial in order to identify the relative merits and synergies of government interventions in areas like the formation and allocation of public capital. Moreover, the growth rate is usually the main measurable objective of the government and hence it is useful to assess the contribution of the components of public capital expenditures aiming at long-run growth.

To analyze the fiscal policies that aim at growth-maximization in the context of the present model, we extend the approach adopted by Kalyvitis and Kalaitzidakis (2004) by taking into account the impact of congestion on the effects on the growth-maximizing government size and the share of maintenance expenditures as follows.

**Definition 2.** *Growth-maximizing policies in the competitive equilibrium of the economy are given under Definition 1 when the government chooses $\tilde{\tau}$ and $\tilde{\mu}_g$ in order to maximize the long-run growth rate of the economy by taking into account the aggregate maximizing behavior of individuals, and the government budget constraints and the feasibility and technological conditions are met.*

The problem of long-run growth-maximizing policies can then be formulated as:

$$\max \tilde{g}(\tau, \mu_g, z) = (1 - \tau)(1 - (1 - a)(1 - \sigma))\tilde{z}^{1-a} - \rho - \delta_k$$

subject to the decentralized equilibrium response by the private agent given by the system (21) and (22). Taking the first-order conditions, we can obtain after some algebra the following growth-maximizing conditions:

$$\tilde{\tau} = \frac{1 - a}{1 - a\tilde{\mu}_g}$$  \hspace{1cm} (43)

$$-\delta'_g (\tilde{\mu}_g \tilde{\tau}) = \tilde{z}^{-a}$$ \hspace{1cm} (44)

$$(1 - \tilde{\mu}_g)\tilde{z}^{-a} \tilde{\tau} - \delta_g (\tilde{\mu}_g \tilde{\tau}) - (1 - \tilde{\tau})(1 - (1 - \sigma)(1 - a))\tilde{z}^{1-a} + \delta_k + \rho = 0$$ \hspace{1cm} (45)

Equations (43), (44), and (45) express the three endogenous variables, $\tilde{\tau}$, $\tilde{\mu}_g$, $\tilde{z}$, in terms of the
model parameters and determine the rules that the government has to satisfy to attain growth-maximization. Equation (43) coincides with the growth-maximizing tax rate obtained by Kalaitzidakis and Kalyvitis (2004) and states that public maintenance expenditures affect positively the growth-maximizing size. The size of these expenditures as a share of total public capital expenditures is then determined by the curvature of the public capital depreciation function, determined in (44) in terms of the average product of public capital determined. As in Kalaitzidakis and Kalyvitis (2004), the Barro (1990) growth-maximizing tax rate is suboptimal and depends upon public maintenance expenditures. However, the growth-maximizing tax rate does not depend here solely on the growth-maximizing share of expenditures in public capital maintenance. Taking the total derivatives in the above system we can assess the response of the policy variables with respect to congestion, which are summarized in the following Proposition.

Proposition 3. The effects of congestion on the growth-maximizing tax rate and the growth-maximizing share of public capital maintenance in total public capital expenditures are ambiguous and depend upon the magnitude of the elasticity of the public capital depreciation rate with respect to the share of public capital maintenance expenditures in total public expenditures.

Proof. See Appendix 3.

Congestion impacts on the growth-maximizing government size through the change in the growth-maximizing allocation of government expenditures on maintenance. However, the outcome differs from the first-best solution where the planner takes the public to private capital ratio, $z$, as given by (34). The tax rate depends now on the response of the public capital depreciation rate (affected by the equilibrium level of maintenance to output ratio) and the associated average product of public capital. For instance, a rise in congestion increases initially the average product of public capital as private agents over-accumulate private capital. This lowers the public to private capital ratio and raises output directly through the production function. As output rises the usage of public capital rises too, which in turn raises the public capital depreciation rate and reduces the public capital stock. If the elasticity of the depreciation function is sufficiently low the positive change in the marginal benefit of public funds through congestion is outweighed by the fall in the
marginal benefit of public funds and maintenance expenditures, and the government size has to fall. On the flip side, a fall in congestion will raise the marginal benefit of maintenance expenditures and the average product of public capital if the response of the depreciation rate w.r.t. public capital maintenance (measured by the corresponding elasticity) is large enough. This will generate the need for additional expenditures on public capital maintenance, which in turn requires a higher tax rate to finance these outlays.14

6. Discussion and conclusions

The aim of the paper was to explore the steady state and fiscal policy implications of public capital maintenance in an endogenous growth model under the presence of congestion in public capital services. We showed that a minimum amount of capital expenditures has to be devoted in public capital maintenance and ‘new’ public investment in order for the economy to attain steady-state growth. This result extends the set of mechanisms that underline the importance of threshold levels for fiscal variables in order to attain long-run growth. These thresholds stem now from the composition of public productive expenditures in two separate categories with differential impacts on public capital accumulation and growth, and as in the case of the tax rate in the Barro (1990) model, the threshold levels arise in the form of sine qua non conditions (rather than as a priori assumptions) for long-run growth.

In practice, however, the threshold level for maintenance expenditures can be a more difficult policy target to achieve: public capital maintenance often has a low priority in government budgets, as this form of outlays are politically less appealing and visible than ‘new’ investment projects, and hardly affect the condition of the public capital stock for some time. This has often led to ‘maintenance deferral’, as the consequence of deferring many types of public capital are not visible in the short or medium-run to myopic voters and further hardens the provision of adequate resources for public capital maintenance.15

We also found that in the presence of congestion, maintenance expenditures are important for

14Substituting (43) and (45) in (44), it is straightforward to show that the total effect of congestion on growth-maximizing fiscal policies under congestion depends on the initial level of congestion and the structural parameters of the economy. For instance, if the initial congestion level and the intensity of public capital in the production function are low, the effect of congestion on the growth-maximizing share of maintenance is more likely to be positive.

15Hulten and Peterson (1984) discuss in more detail the issue of ‘maintenance deferral’ in the public sector.
the first-best taxation rule. The share of maintenance expenditures affects positively the optimal government size due to the change on the marginal cost of public funds. The government can also benefit from the re-allocation between ‘new’ public investment and maintenance to improve efficiency at the existing tax rate in the presence of increasing (decreasing) congestion by shifting capital expenditures away from (towards) public capital maintenance. Thus, following the spirit of Devarajan et al. (1996) our findings extend the set of mechanisms that highlight the importance of the composition of public productive expenditures for growth.\(^{16}\) Given that most countries have a dual budget structure comprised by the recurrent budget (presenting spending on salaries and operations-maintenance) and the investment budget (involving one-off capital expenditures on projects), it is always important for policymakers to achieve efficient resource allocation.\(^{17}\)

Efficient composition of public capital expenditures becomes more important in the presence of binding financial constraints, such as those posed by the International Monetary Fund and the World Bank to less developed economies or faced by local authorities under fiscal distress, which limit the provision of expenditures for public services.\(^{18}\) According to the optimal fiscal policy rules presented here, public maintenance expenditures are necessary to ensure adequate replacement of obsolete capital and reduce the need for future investment to rehabilitate deteriorated assets, and should therefore be treated as an integral part of productive outlays, particularly when fiscal targets distinguish between current and capital expenditures.

We close the paper by noting that another important determinant of the depreciation rate that merits further investigation involves the utilization rate of public capital. Although casual empiricism suggests that the decay of public services varies along with their utilization rate, the latter is typically introduced only in the context of the endogenous determination of private capital.

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\(^{16}\)The existing growth literature on the optimal composition of public spending typically distinguishes between productive expenditures, which affect private sector productivity, and unproductive (or consumption) expenditures, which affect intertemporal utility, and identifies their impacts on growth and welfare. See, for instance, Lee (1992) and Turnovsky and Fisher (1995).

\(^{17}\)The problem may be more acute in developing countries where the total budget often includes various donor-paid expenditures on recurrent items that generate a mixture of recurrent and capital budget items; see also World Bank (1998). An additional reason for policymakers to favour ‘new’ investment projects when forming the budget is that maintenance activities may be more difficult to monitor in practice.

\(^{18}\)Bumgarner et al. (1991) provide empirical evidence that support the hypothesis that deficiencies in public capital maintenance are to a large extent caused by fiscal distress. See Blanchard and Giavazzi (2004) and Estache (2004) for a more general discussion on the importance of public capital maintenance expenditures in the presence of fiscal rules.
depreciation. A promising route for further research might therefore deal with the analysis of a growth model involving public capital maintenance and utilization, along with their implications for fiscal policies.

\[\text{\textsuperscript{19}}\] See the early contributions by Nickell (1978, chapter 7), Schworm (1979) and the more recent ones by, among others, Boucekkine and Ruiz-Tamarit (2003) at the firm level, in Licandro et al. (2001) and Aznar-Marquez and Ruiz-Tamarit (2004) in growth models.
Appendix 1. Proof of Proposition 1 and Corollary.

Proof of Proposition 1. We can establish the following properties of $\Phi(g)$ for $g > 0$:

1. $\Phi(0) = \delta_g (\mu_g \tau) - (1 - \mu_g) \tau \left( \frac{(1-\tau)(1-(1-a)(1-\sigma))}{\rho + \delta_k} \right)^{\frac{a}{1-a}}$

2. $\frac{\partial \Phi(g)}{\partial g} = 1 + (1 - \mu_g) \frac{a}{1-a} \tau \left( \frac{(1-\tau)(1-(1-a)(1-\sigma))}{g + \rho + \delta_k} \right)^{\frac{a}{1-a}} \left( \frac{1}{(1-\tau)(1-(1-a)(1-\sigma))} \right)^{\frac{1}{1-a}} > 0$

3. $\frac{\partial^2 \Phi(g)}{\partial g^2} = -(1 - \mu_g) \frac{a}{1-a} \tau \left( \frac{1}{(1-\tau)(1-(1-a)(1-\sigma))} \right)^{\frac{1}{1-a}} \left( \frac{1}{g + \rho + \delta_k} \right)^{\frac{1}{1-a}} < 0$

4. $\lim_{g \to -\infty} \Phi(g) = +\infty$

5. $\lim_{g \to +\infty} \frac{\partial \Phi(g)}{\partial g} = 1$

6. $\Phi(g)$ is continuous in $g$.

Under the necessary and sufficient condition established in Proposition 1 and from the properties of the continuous function $\Phi(g)$ it follows that $g^*$ exists and is unique. Under the continuous and strictly increasing $\Phi(g)$, and under its limit properties for an increasing and positive $g$, it follows straightforward that, if $\Phi(g)$ starts from a non-positive value as implied by condition $\frac{\delta_g (\mu_g \tau)}{(1-\mu_g) \tau} (1 - \tau) \frac{1}{1-a} \left( \frac{(1-(1-a)(1-\sigma))}{\rho + \delta_k} \right)^{\frac{a}{1-a}}$, then it crosses the horizontal locus of the $(g, \Phi(g))$ space. Then, $\Phi(g)$ has a fixed point $\bar{g} > 0$ such that $\Phi(\bar{g}) = 0$. □

Proof of Corollary to Proposition 1.

Proof of (I). We will show that $\mu_g = 0$ implies a non-positive growth rate for the economy according to Proposition 1. Assuming $\mu_g = 0$, Proposition 1 implies that $1 < \tau (1 - \tau) \frac{(1-(1-a)(1-\sigma))}{\rho + \delta_k} \frac{1}{1-a}$. Since $\tau(1-\tau) \frac{1}{1-a} < 1$, a sufficient parametric condition for $1 < \tau (1 - \tau) \frac{(1-(1-a)(1-\sigma))}{\rho + \delta_k} \frac{1}{1-a}$ not to hold is $\left[ \frac{(1-(1-a)(1-\sigma))}{\rho + \delta_k} \frac{1}{1-a} \right]^{-\frac{a}{1-a}} < 1$. It follows that $\sigma < \frac{\rho + \delta_k - a}{1-a}$ is a sufficient parametric condition under which Proposition 1 does not hold for any $\tau \in (0, 1)$ and $\mu_g = 0$ generates non-positive growth. Thus, the government has to implement a level $\mu_g > \bar{\mu}_g > 0$, where $\frac{\delta_g (\bar{\mu}_g \tau)}{(1-\mu_g) \tau} (1 - \tau) \frac{1}{1-a} = \frac{(1-(1-a)(1-\sigma))}{\rho + \delta_k} \frac{1}{1-a}$ implies $g = 0$. This level $\bar{\mu}_g$ exists and is unique, since $\delta_g (\mu_g \tau)$ is a convex function with respect to $\mu_g$ and $(1 - \mu_g) \tau \left( \frac{(1-(1-a)(1-\sigma))}{\rho + \delta_k} \frac{1}{1-a} \right) \frac{1}{1-a}$ is linear and strictly decreasing in $\mu_g$. It follows that Proposition 1 holds for values of $\mu_g > \bar{\mu}_g$.

Proof of (II). This follows straightforward from Proposition 1. If $\mu_g = 1$ then from Proposition 1 we get that $\delta_g (\tau) < 0$, which is a contradiction for any $\tau \in (0, 1)$ by the properties of the depreciation rate function for public capital. Since $\delta_g (\mu_g \tau)$ is a convex function with respect to $\mu_g$
while \((1 - \mu_g)\tau \left( \frac{\rho + \delta_k}{1 - \rho} \right)^{\frac{\sigma}{1 - a}} \) is linear and strictly decreasing, there exists \( \mu_g < \hat{\mu}_g < 1 \), where

\[
\frac{\delta_g(\hat{\mu}_g \tau)}{(1 - \mu_g)\tau} (1 - \tau)^{\frac{\sigma}{1 - a}} = \left( \frac{\rho + \delta_k}{1 - (1-a)(1-\sigma)} \right)^{\frac{\sigma}{1 - a}},
\]

for which Proposition 1 holds. \(\blacksquare\)

Appendix 2. Proof of Proposition 2.

Equations (34), (35) and (37) give the solution for the optimal values of the maintenance to output ratio, the tax rate and the public to private capital ratio. Taking the total derivatives of the above system for \( \mu_g^*, \tau^*, z^* \) w.r.t. \( \sigma \) we can study the effect of congestion on optimal tax rate and the optimal maintenance share of government revenues. After some algebra the corresponding matrix for the above system of equations is given by:

\[
\begin{bmatrix}
\gamma_{11} & \gamma_{12} & \gamma_{13} \\
\gamma_{21} & \gamma_{22} & \gamma_{23} \\
0 & 1 & -B
\end{bmatrix}
\begin{bmatrix}
dz^* \\
d\tau^* \\
d\mu_g^*
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
A
\end{bmatrix}
\]

where \( \gamma_{11} = a(z^*)^{-1-a}, \gamma_{12} = -\delta^\prime \prime(\mu_g^* \tau^*) \mu_g^*, \gamma_{13} = -\delta^\prime \prime(\mu_g^* \tau^*) \tau^*, \gamma_{21} = a(1 - \mu_g^* \tau^*)(1 - a)(z^*)^{-a} + a(1 - a)(1 - \mu_g^* \tau^*)(z^*)^{-1-a}, \gamma_{22} = -a \mu_g^*((z^*)^{-1-a} + (z^*)^{-a}), \gamma_{23} = -a \tau((z^*)^{1-a} + (z^*)^{-a}), \) and \( B > 0, A < 0 \) are given in the text. The determinant of the matrix is given by:

\[
D_\gamma = \xi \left[ a(z^*)^{-a} - \delta^\prime \prime(\mu_g^* \tau^*)(1 - a)(1 - \mu_g^* \tau^*) \right]
\]

where \( \xi = ((z^*)^{-a} + (z^*)^{-a-1})(Ba \mu_g^* + a \tau^*) > 0 \). The sign of the determinant is ambiguous and depends on the curvature of the depreciation function, on the technology parameters of public and private capital in the production function, and on the optimal levels of the tax rate and the ratio of public capital maintenance to total public capital expenditures. We shall henceforth assume that \( [a(z^*)^{-a} - \delta^\prime \prime(\mu_g^* \tau^*)(1 - a)(1 - \mu_g^* \tau^*)] \neq 0 \) holds in order for the system to have a solution. Applying Cramer’s rule on the above system of equations we can show that:

\[
\frac{d\tau^*}{d\sigma} = \frac{Aa \tau^*((z^*)^a + (z^*)^{-1-a})}{\xi} > 0
\]

\[
\frac{d\mu_g^*}{d\sigma} = -\frac{Aa \mu_g^*((z^*)^a + (z^*)^{-1-a})}{\xi} < 0
\]

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which correspond to equations (39) and (40) in the text. The last part of Proposition 2 follows
then directly from (41).

Appendix 3. Proof of Proposition 3.

Equations (43), (44), and (45) characterize the solution of the system for the three unknowns,
namely \( \tilde{\mu}_g, \tilde{\tau}, \tilde{z} \). The system of total derivatives is given in matrix notation as follows:

\[
\begin{bmatrix}
v_{11} & v_{12} & v_{13} \\
v_{21} & v_{22} & 0 \\
0 & 1 & -W
\end{bmatrix}
\begin{bmatrix}
d\tilde{z} \\
d\tilde{\tau} \\
d\tilde{\phi}_g
\end{bmatrix} =
\begin{bmatrix}
(1 - \tilde{\tau})(1 - a)\tilde{z}^{1-a} \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
d\sigma \\
-1 \\
0
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
d\rho
\end{bmatrix}
\]

where \( v_{11} = a\tilde{z}^{-1-a} \), \( v_{12} = -\delta''(\tilde{\mu}_g \tilde{\tau})\tilde{\mu}_g \), \( v_{13} = -\delta''(\tilde{\mu}_g \tilde{\tau})\tilde{\tau} \), \( v_{21} = -a\tilde{\tau}(1 - \tilde{\mu}_g)\tilde{z}^{-1-a} - (1 - a)(1 - \tilde{\tau})(1 - (1 - \sigma)(1 - a))\tilde{z}^{-a} \), \( v_{22} = \tilde{z}^{-a} + (1 - (1 - \sigma)(1 - a))\tilde{z}^{1-a} \), and \( W \) is a positive constant. The determinant of the matrix is given by:

\[
D_v = -\left[ \tilde{z}^{-1-a} + \tilde{z}^{-a}(1 - (1 - \sigma)(1 - a)) \right] \left[ \tilde{z}^{-a} - \delta''(\tilde{\mu}_g \tilde{\tau})(1 - \tilde{\mu}_g)\tilde{\tau} \left( \frac{\tau}{W} + \tilde{\mu}_g \right) \right] \tilde{\phi}_g \tilde{z} \tilde{\phi}_g > 0
\]

and has an ambiguous sign. From (44) and defining the elasticity of the derivative of the depreciation
function with respect to maintenance to output ratio as \( \varepsilon_{\tilde{\mu}_g \tilde{\tau}} \equiv \frac{-\delta''(\tilde{\mu}_g \tilde{\tau})}{\delta''(\tilde{\mu}_g \tilde{\tau})} \tilde{\mu}_g \tilde{z} > 0 \), we can set the following conditions:

\( \Xi.1 \): The depreciation function is linear, \( \delta''(\tilde{\mu}_g \tilde{\tau}) = 0 \), or the elasticity of the derpeciation
function with respect to the maintenance to output ratio is sufficiently low, such that \( \varepsilon_{\tilde{\mu}_g \tilde{\tau}} < \frac{\tilde{\mu}_g}{(1 - \tilde{\mu}_g)(\frac{\tilde{\phi}_g}{\tilde{\phi}_g})} \tilde{\mu}_g \tilde{z} \), then \( D_v < 0 \).

\( \Xi.2 \): The convexity of the depreciation function measured by the elasticity of the depreciation
function is high, such that \( \varepsilon_{\tilde{\mu}_g \tilde{\tau}} > \frac{\tilde{\mu}_g}{(1 - \tilde{\mu}_g)(\frac{\tilde{\phi}_g}{\tilde{\phi}_g})} \tilde{\mu}_g \tilde{z} \), then \( D_v > 0 \).

Thus, when the depreciation function curvature is sufficiently low (or, equivalently, the growth-
maximizing maintenance to output ratio is sufficiently high) we have \( D_v < 0 \) for any parameter
value. Applying Cramer’s rule on the above system we can characterize the effect of congestion on
the growth-maximizing policy variables, $\tilde{\tau}$ and $\tilde{\mu}_g$. The effect of congestion on $\tilde{\tau}$ is given by:

$$
\frac{d\tilde{\tau}}{d\sigma} = -\frac{a\tilde{z}^{1-a}(1 - \tilde{\tau})(1 - a)\tilde{z}^{1-a}W}{D_v}
$$

The effect of congestion on the growth-maximizing government size depends the sign of the determinant and it is positive for $\Xi,1$ and negative for $\Xi,2$. Increasing congestion increases the growth-maximizing government size. This is an indirect effect that comes from the effect of congestion on the growth-maximizing allocation on maintenance expenditures where, by (43), maintenance affects the growth-maximizing government size (you will see later on all relations together). The effect of congestion on the allocation of government revenues to maintenance is then given by:

$$
\frac{d\tilde{\mu}_g}{d\sigma} = -\frac{a\tilde{z}^{1-a}(1 - \tilde{\tau})(1 - a)\tilde{z}^{1-a}}{D_v}
$$

This multiplier shows that under $\Xi,1$ congestion affects positively the growth-maximizing allocation of maintenance expenditures in government revenues, whereas the effect is negative under $\Xi,2$; again, the sign depends on the response of the depreciation function. By a closer inspection of (44) the degree that congestion affects the internal allocation of expenditures depends on the public to private capital ratio and, in turn, on its effect on the average product of public capital. This effect is given by:

$$
\frac{d\tilde{z}}{d\sigma} = -\frac{\sigma''(\tilde{\mu}_g\tilde{\tau})(1 - \tilde{\tau})(1 - a)\tilde{z}^{1-a}(\tilde{\mu}_gW + \tilde{\tau})}{D_v}
$$

which is positive under $\Xi,1$ and negative under $\Xi,2$. Taking into account the effect of $\sigma$ on $\tilde{z}$ and using the conditions for $\frac{d\tilde{z}}{d\sigma}$ and $\frac{d\tilde{\mu}_g}{d\sigma}$, we get that:

$$
\frac{d\tilde{\tau}}{d\sigma}_{total} = W\frac{d\tilde{\mu}_g}{d\sigma} \geq 0
$$

$$
\frac{d\tilde{\mu}_g}{d\sigma}_{total} = \frac{a\tilde{z}^{1-a}}{\sigma''(\tilde{\mu}_g\tilde{\tau})(\tilde{\mu}_gW + \tilde{\tau})} \frac{d\tilde{z}}{d\sigma} \geq 0
$$

from which Proposition 3 follows directly.
References


Newbery D., 1988, ‘Road damage externalities and road user charges’, *Econometrica*, 56, 295-316.


