Abstract

We study information aggregation in an election where agents can strategically choose when to vote, and can observe the votes of those who preceded them. We show that in a common value setting, such an election aggregates information efficiently regardless of the voting rule or the size of the electorate. This result stands in contrast to the extensive body of work examining simultaneous and sequential elections in the context of Condorcet’s Jury Theorem. If votes are cast one at a time, the result can be extended to the case where the number of voters is uncertain. The performance of the election in the presence of partisans and multidimensional signals is also analyzed.

1 Introduction

The idea that voting is a means to aggregate private information and ensure a better collective choice is one that dates back to Condorcet. An extensive literature in political economy has since examined the veracity of the so called Condorcet Jury Theorem, according to which majorities are more likely than any single individual to select the “better” of two alternatives when there exists uncertainty about which of the two alternatives is in fact preferred.

In this paper we analyze an election which permits agents to exploit the fact that not everyone in an electorate has the same quality of information. We consider an election where voters can choose when to cast their ballot and the intermediate tally of votes is

*We would like to thank Dilip Abreu and Wolfgang Pesendorfer for their valuable input and advice.
observable. By strategically choosing the time at which they cast their vote, early voters can now communicate their signals to those who will follow. We find that this added flexibility vastly improves information aggregation - agents endogenously order themselves with the better informed casting their ballots ahead of those less sure of themselves.

The voting environment described above is not uncommon. Voting in legislatures, corporate boardrooms and at the United Nations is often done by a show of hands and uncertain voters may delay casting their votes to gauge the level of consensus in the room. Perhaps the most notable instance of such an election is the way the Democratic National Party conducts the Iowa caucus. The caucus site is partitioned into sections according to candidate and for the first thirty minutes after the caucus begins voters can decide which "preference group" to join. At the end of this period, the number of voters in each section are counted and each candidate is awarded delegates to the county convention in proportion to the size of her preference group. The results in this paper should also be taken into consideration when examining the effect of exit polls on election outcomes. A recent survey (12) shows that laws pertaining to the broadcast of exit polls on election day vary considerably across countries. While most arguments in favor of exit polls are on the grounds of transparency and the constitutional freedom of speech, we believe our efficiency results may be viewed as a novel reason why exit polls should be instituted.

In a seminal contribution, Austen-Smith and Banks (2) show that in a common value setting, simultaneous elections aggregate information efficiently only under knife-edge conditions. In fact, they aggregate information efficiently under a simple majority rule if and only if all voters have the same quality of information. The inefficiency in these elections stems from the fact that in equilibrium, strategic voters who condition on being pivotal do not find it rational to vote according to their signal. In contrast, the election we consider is outcome equivalent to the first best (that is, the electorate chooses the same outcome as it would choose if all information were publicly available) regardless of the size of the electorate. Efficiency is obtained because under mild conditions, there exists an order, which voters follow in equilibrium, that (i) allows better-informed voters to communicate their signals without inadvertently deciding the election in favor of the wrong candidate and (ii) reveals all relevant information while leaving a sufficient number of voters to swing the election in either direction.

Information aggregation has also been studied in the context of sequential or roll-call elections where players vote according to a predetermined (exogenously specified) order. Fey

\footnote{For our main result, it suffices if the voters’ signals satisfy the maximum likelihood ratio property.}
Information Aggregation in Voting with Endogenous Timing

Wit and more recently Ali and Kartik (1) have examined the possibility of bandwagons (or informational cascades) developing in sequential elections. In another landmark paper, Dekel and Piccione\(^2\) prove that any symmetric responsive strategy profile is an equilibrium of the sequential election if and only if it is also an equilibrium of the simultaneous election. We identify two simple refinements - the first guarantees that no inefficient equilibrium of simultaneous elections can be an equilibrium of our election. The second ensures that bandwagons of the form discussed in the literature cannot occur in our election. All efficient equilibria survive both refinements.

Our work contributes to another strand of the literature on information aggregation that begins with the work of Fedderssen and Pesendorfer (8). Using pivotal voter arguments, (FP) argue that the unanimity rule, commonly used by juries to protect the innocent from being convicted, is in fact more likely to result in an incorrect judgement than the simple majority rule. In defense of the unanimity rule, Coughlan (4) shows that in a common value setting, adding a pre-play communication phase (such as a non-binding straw poll\(^3\)) allows all information to be aggregated prior to the jury's vote and renders all voting rules outcome-equivalent. This article presents an alternate solution. The election we described above achieves first best regardless of the voting rule. We find that information can be aggregated through the voting process itself, without having to resort to a cheap-talk phase. We believe this is an important finding for at least two reasons. First, pre-play communication is not always possible. Voters may be geographically separated. The electorate may be large. Communicating with every member of the jury can be costly\(^4\). Second, an experimental study conducted by Guarnaschelli and Palfrey (10) shows that voters do not use the information revealed in a straw poll correctly and election outcomes can vary with the voting rule. While we cannot convince the reader that our election design is more robust than the straw poll, we hope that in the light of Guarnaschelli et. al's results, the reader will recognise the merit in considering alternate voting mechanisms.

We examine several natural extensions. We find that our results generalize to scenarios where the voter is uncertain about the number of voters. To prove this generalization we resort to a slightly richer model of the election where time is continuous and the tally is

\(^2\)Our work was motivated in part by a quote from Dekel and Piccione, pg. 35 which reads “On the negative side, it [our result] completely demolishes any hope of obtaining strong conclusions about endogenous timing...”. While we don't believe Dekel and Piccione meant to use the phrase “endogenous timing” as we have in this paper, we would like to thank them for leading us to this project.

\(^3\)The Republican National Party holds a non-binding straw poll in Ames, IA before the primaries.

\(^4\)For a more extensive discussion of why relying on communication before voting is not always appropriate, we refer the reader to Nicola Persico.
broadcast every $\Delta$ seconds. Voters who were previously voting at discrete points in time, now choose an interval between two broadcasts and are randomly assigned a position in a queue with other voters who have chosen the same interval. Votes wait their turn in the queue to cast their vote. Our main results discussed above also extend to this modified game.

Next we provide some insights about this election when there is diversity in voter preferences. We have been unable to derive any strong results in this situation as we find that even the simplest model becomes intractable when we analyze small electorates. But we do provide two instances where the presence of partisans does not prevent the election from achieving efficiency. Finally, we provide an example showing that this election format can efficiently aggregate information even when it does not satisfy the maximum likelihood ratio property. A complete characterization is pending.

2 Model setup for Simultaneous Elections

Our model follows the lines of Austin-Smith and Banks (2). Let’s consider an odd number of voters $i = \{1, 2, ..., n\}$. There are two states of the world, Left and Right, so that the state of the world $\omega \in \{L, R\}$. The voters can collectively decide which of two alternatives they prefer, $l$ or $r$, so that each alternative $a \in \{l, r\}$. We will be assuming common values, and each voter will prefer to pick the alternative that corresponds to the present state of the world. Hence, the utility function for each and every voter will satisfy $U(L, l) = U(R, r) = 1$ and $U(R, l) = U(L, r) = 0$.

Without loss of generality, we can assume that the prior probability of the state of the world being $L$ satisfies $\pi_L \geq \frac{1}{2}$. Each voter receives a private signal $s_i \in \{0, 1\}$ that conveys information about the state of the world, with $p_R = P(s_i = 1|\omega = R) \geq \frac{1}{2}$ and $p_L = P(s_i = 0|\omega = L) \geq \frac{1}{2}$. Hence, getting a 0-signal is more likely in state $\omega = L$ and getting a 1-signal is more likely in state $\omega = R$.

A standard assumption that is often used is that individual signals dominate priors, so $\frac{\pi_L}{1-\pi_L} < \frac{p_R}{1-p_R}$, that is, voting is informative. While this assumption was used in (2), we won’t be making use of this assumption in our results. Another standard assumption is the monotone likelihood ratio property (MLRP). In our model, MLRP is equivalent to $\frac{pl}{1-p_L} \geq \frac{1-pR}{p_R}$, which follows immediately from $p_L, p_R \geq \frac{1}{2}$.

In this paper we will be studying conditions necessary for achieving efficient aggregation of information. We define the efficient outcome in an election as follows:
Definition 1 We say that the efficient outcome in an election is achieved if the outcome of the election is the same whether or not each voter knows the signal vector $s = (s_1, s_2, ..., s_n)$. That is, the efficient outcome is the outcome preferred by each and every voter when all $n$ signals were publicly available. Assuming that all $n$ signals are available, define $k_0$ as the number of 0-signals among the $n$ total signals received by the voters, so that $k_0 = \sum_{i=1}^{n} s_i$ for $s_i \in \{0, 1\}$, and define $k_0^*$ as the minimum number of 0-signals necessary for $l$ to be the efficient choice. $k_1$ and $k_1^*$ can be defined similarly for 1-signals. Consider the log-likelihood-ratio $\beta(k_0, \pi_L, n) = \frac{P(\omega=L|n_0=k_0)}{P(\omega=R|n_0=k_0)}$ or equivalently $\beta(k_0, \pi_L, n) = \frac{\pi_L^{k_0}(1-\pi_L)^{n-k_0}}{(1-\pi_L)p_L^{-k_0}(1-p_R)^{k_0}}$. Then $k_0^*(n, \pi_L, p_L, p_R)$ is the unique $k_0^*$ that satisfies $\beta(k_0^* - 1, \pi_L, n) < 1 \leq \beta(k_0^*, \pi_L, n)$. The existence of this $k_0^*$ has been proved on (2) and follows immediately from the MLRP condition.

The next step is to define the voting rule. We define a q-rule in which alternative $l$ is chosen if it receives $m$ or more votes, where $m = \lceil nq \rceil$, with $q \geq \frac{1}{2}$. We will first study a simultaneous election under this setting. Rational voters always condition on the pivotal scenario, that is, they vote under the assumption that everyone else has voted in such a way so as to make them pivotal. On the other hand, for votes to be informative, they need to satisfy $v(0) = l$ and $v(1) = r$. Will a rational voter that conditions on the pivotal scenario choose to vote informatively?

Looking at figure 2 we can see that informative voting will not always lead to the efficient result. For example, under informative voting, if the number of voters that received a 0-signal is greater than $k_0^*$ but lower than $m$, then $l$ is the efficient choice, but $r$ is the alternative chosen, as there are not enough votes for $l$. It is easy to see that this problem goes away in the special case that $k_0^* = m$. Austen-Smith and Banks (2) have proved the following result:
Figure 2: Informative voting

**Proposition 1** Informative voting in a simultaneous election is rational if and only if the aggregation rule is such that $k_0^* = m$. That is, the efficient outcome can always be achieved in a simultaneous election with $k_0^* = m$. (Austin-Smith and Banks (2))

In our paper we will try to overcome this issue and show that, under sequential voting with endogenous timing, informative voting can be rational for any aggregation rule.

### 3 Model Setup for Sequential Elections

We will now allow for elections to take place over a number of periods $t = 1, 2, 3\ldots$ Voters can choose at which period to cast their vote, and all the votes casted are revealed at the end of each period. We will assume that the election will end if no votes are cast for $T$ consecutive periods. There is no cost of waiting and voters are permitted to abstain and choose to vote at a later time. The rest of the setting will be as in the previous section.

The main idea now is that early voters can communicate their signal to later voters through their vote. Consider an extreme example in which $p_L = \frac{1}{2}$ and $p_R = 1$, so that if any of the voters receives a 0-signal, then we can be certain that the state of the world is $L$. This scenario is depicted in the following diagram.

In this scenario, the presence of a single voter with a 0-signal is enough to determine with certainty the state of the world. However, in a simultaneous election, that single voter has no way of communicating his very valuable signal to the rest of the voters. But if we were to allow for sequential voting, in which voters with a 0-signal choose to vote for $l$ in the first period, and all others vote in the second period, then, after the first period is over, all information is revealed. Each voter knows exactly how many voters received each signal, and the efficient result can always be achieved. There is never a risk that voters in the first period will win the election inefficiently, as the presence of even one voter in the first period signifies that we are in state $L$. 
This result can be extended for more general $p_L$, $p_R$. Assume general $p_L$, $p_R$ and that $k_0^* < m$. We will show that we can always achieve the efficient outcome in a sequential setting in which the voters that receive a 0-signal vote first. If the election is already determined after the first period, then it has been determined efficiently. If on the other hand the election has not been determined, than the efficient outcome has been fully revealed and the voters of the second period can make sure that the election achieves the efficient outcome. The following diagram explains the intuition of our result.

![Diagram](image)

Figure 3: $p_L = 0.5$ and $p_R = 1$

Notice here that the order of voting actually matters for efficient aggregation of information. If the voters that receive a 1-signal had voted first, then they may have been enough of them to determine the election but not enough to ensure that the efficient result was the one voted upon. This issue occurs when $n - m + 1 \leq k_1 < k_1^*$. Figure 5 illustrates how the order of voting matters.

We will prove that an efficient voting order always exists for all information structures.
Who votes first depends on the structure of the signal and on the voting rule. The key insight of the proof is that one of either $k_0^* \leq m$ or $k_1^* = n - k_0^* + 1 < n - m + 1$ always has to be true, effectively determining the optimal order of voting.

**Proposition 2** The following equilibrium strategies achieve efficiency in a two-period election with two signals 0 and 1, $k_0^* \neq m$ and in which all voters have common values:

- If $k_0^* < m$, then 0-signal voters vote in the first period for $l$.
- If $k_0^* > m$, then 1-signal voters vote in the first period for $r$.

In both cases, the voters that didn’t vote in the first period are perfectly informed about $k_0$ and can vote in the second period, achieving the efficient outcome.

**Proof.** We will investigate what happens after the first period, when the number of 0-signal and 1-signal voters $k_0$ and $k_1$ respectively have been observed.

- If $k_0^* < m$ and $k_0 \geq m$ then $l$ has won the election. But $k_0 \geq m > k_0^*$, so the efficient outcome was chosen.
- If $k_0^* < m$ and $k_0 < m$ then the election has not been determined yet. However $k_0$ has been revealed and all voters in the second period will vote for $l$ if $k_0 \geq k_0^*$ and for $r$ if $k_0 < k_0^*$, so the efficient outcome will be achieved.
- If $k_0^* > m$ and $k_1 > n - m$, then $r$ has won the election and $k_0 = n - k_1 < m < k_0^*$, so the efficient outcome was achieved.
- If $k_0^* > m$ and $k_1 \leq n - m$, then $k_0 = n - k_1 \geq m$. The efficient choice is now common knowledge since $k_0$ is perfectly revealed. Suppose $k_0 \geq k_0^*$ so that $l$ is the efficient choice. In the worse case scenario, the maximum number of voters who chose $r$ in the first period period is $k_1 = n - m$. Therefore, there are at least $n - n + m = m$ voters remaining, which is enough for $l$ to win the election. Alternatively, if $k_0 < k_0^*$, then $r$ is the preferred alternative and all voters choose $r$ in the second period as well thereby ensuring efficiency. ■
4 Generalization for more than two signals

The previous result can be generalized to more than two signals if we allow for a larger number of voting periods. Assume that each voter $i$ receives a private signal $s_i \in \{0, 1, ..., d\}$. Moreover, assume that signals satisfy the monotone likelihood ratio property. That is, if $s > s'$, then $\frac{P(s|\omega=L)}{P(s'|\omega=L)} \geq \frac{P(s|\omega=R)}{P(s'|\omega=R)}$. Let $\beta(k_0, k_1, ..., k_d, \pi_L, n)$ be the log-likelihood-ratio, with $\sum_{i=0}^{d} k_i = n$. Similarly to the case with two signals, we define $k^*_0$ as the $k^*_0$ that satisfies $\beta(k^*_0 - 1, 0, 0, ..., n - k^*_0 + 1, \pi_L, n) < 1 \leq \beta(k^*_0, 0, 0, ..., n - k^*_0, \pi_L, n)$. It is easy to see that this definition has also implicitly defined $k^*_d$ as $k^*_d = n - k^*_0 + 1$.

We have:

**Proposition 3** There exists an equilibrium strategy that achieves the efficient outcome in an election in which voters receive one of $d+1 > 2$ possible signals each and in which voting takes place in $t = d + 1$ periods and voters choose in which period to vote.

**Proof.** We know that $k^*_0$ is the minimum number of 0-signals to ensure that $l$ is the efficient choice, regardless of what the rest $n - k^*_0$ signals are. Similarly, $k^*_d$ is the minimum number of d-signals that ensure that $r$ is the efficient choice, regardless of what the rest $n - k^*_d$ signals are. For example, if we have $k^*_d$ d-signals and $n - k^*_d$ 0-signals, then $r$ is the efficient choice, but if we have $k^*_d - 1$ d-signals and $n - k^*_d + 1$ 0-signals, then $l$ is the efficient choice. But since the 0-signals and d-signals are the strongest signals for $l$ and $r$ respectively, using the monotone likelihood property, it is easy to see that the relationship $k^*_0 + k^*_d = n + 1$ must hold. Now either $k^*_d \leq m$ or else $k^*_d \geq m + 1$.

- If $k^*_d \leq m$ then the voters will follow the following strategy when voting in the first period: Voters that got a d-signal will vote for $r$ and all the rest will just wait and vote in the following periods. Let $k_d$ the number of voters who got a d-signal and hence voted in the first period.
  We have $n - k_d$ voters left. If $k_d \geq k^*_d$ then the efficient choice has been revealed - it is $r$. All voters in the first period have voted for $r$ already, all voters in the following periods observe $k_d$ and realize that the efficient choice is $r$ and vote for $r$ as well in the next period. The election is over and the efficient outcome is achieved. If, on the other hand, $k_d < k^*_d$, then the number of voters that got a d-signal is perfectly revealed to the rest. The rest of the voters all received signals $s_i \in \{0, 1, ..., d - 1\}$ and now they have a new problem to solve in which there are $\hat{n} = n - k_d$ voters, a status-quo $l$ that requires $\hat{m} = m - k_d > m - k^*_d > 0$ votes to be overturned and $d$ types of different signals. Hence, the problem that voters face when they receive one of $d + 1$ possible signals can be reduced to an equivalent problem with
voters receiving one of \( d \) possible signals. But we have already proved the result for the case \( d = 1 \). So, by induction, the result has to hold for any integer \( d > 1 \).

- If \( k^*_d \geq m + 1 \), then \( n + 1 - k^*_d \geq m + 1 \) or equivalently \( k^*_0 \leq n - m \). Voters will follow the following strategy when voting in the first period: Voters that got a 0-signal will vote for \( l \) and all the rest will just wait and vote in the following periods. Let \( k_0 \) the number of voters who got a signal of type 0 and, hence, voted in the first period. If \( k_0 \geq k^*_0 \) then the efficient choice has been revealed - it is \( l \). All voters in the first period have voted for \( l \) already, all voters in the following periods observe \( k_0 \) and realize that the efficient choice is \( l \) and vote for \( l \) as well in the next period. The election is over and the efficient outcome is achieved.

If, on the other hand, \( k_0 < k^*_0 \leq n - m \), then the number of voters who got a 0-signal is perfectly revealed to the rest. The rest of the voters all received signals \( s \in \{1, \ldots, d\} \) and now they have a new problem to solve in which there are \( \hat{n} = n - k_0 \) voters, a status-quo \( l \) that requires \( \hat{m} = m \) votes to be overturned and \( d \) types of different signals, with \( \hat{m} < \hat{n} \).

Hence, the problem that voters face when they receive one of \( d + 1 \) possible signals can be reduced to an equivalent problem with voters receiving one of \( d \) possible signals. But we have already proved the result for the case \( d = 1 \). So, by induction, the result has to hold for any integer \( d > 1 \). \( \blacksquare \)

In the above proposition, we have proved that we can get an efficient outcome regardless of the voting rule or the size of the electorate. The election need not end after \( t = d + 1 \) periods, but regardless of when it will end, the outcome will be efficient. Note here that the efficient order of voting is not unique. There are instances in which there can be multiple different orderings, all of which are efficient. An interesting question to ask is if all Nash equilibria in our game lead to efficient outcomes, and if not, if there are any refinements under which efficient equilibria are the only equilibria of our game.

5 Extension of our result for special cases of partisans

Introducing partisans whose incentives don’t agree with the common value framework of our model is a next natural step. In the most general case, we have an unknown number of partisans, some of which always vote in a way so as to maximize the chances that alternative \( l \) is selected (l-partisans) and some of which act in a way so as to maximize the chances that alternative \( r \) is selected (r-partisans). Extending our results to ensure that the efficient outcome is achieved in this most general case seems very difficult. Those partisans will always try to do their best to confuse the other voters and mimic the most informed voters,
making it impossible for all the rest to distinguish partisans from informed voters.

However, our results still hold for some special cases. If the number of l-partisans and the number of r-partisans are known, then the efficient outcome can still be achieved. The intuition for this is clear: the partisans will mimic the most informed voters (the ones with the most extreme signals 0 or \(d\)), and, as everyone else knows exactly how many partisans there are, everyone else will discount the partisans from the informed voters. The partisans can hardly do anything to confuse the other voters further - they can choose not to mimic the most informed voters but this will hardly help them, as everyone else will discount the number of informed voters under the assumption that partisan were also in that group - so if the partisans don’t do as expected they will only make things worse for themselves.

Another special case in which the efficient outcome can be achieved is the case of overconfident voters. Overconfident voters are voters that received the most extreme signals (0 or \(d\)) but instead of believing in the common value framework under which all voters have an impact in which outcome is the preferable, they believe that their signal is right and choose to ignore everyone else’s signal. Here, overconfident voters also have a useful signal to contribute to common value voters, unlike partisans which are usually assumed to carry no informative signals. However, overconfident voters will choose to act just like informed voters - once again, there is nothing they can do to further their own incentives more other than to act as common value voters want them to act - as voters that received a very informative signal.

6 Extension of our result when the total number of voters is unknown

In large elections the precise number of voters is often unknown to the other voters. In this section we will assume that the number of voters \(n\) is unknown and comes from a probability distribution. As before, alternative \(l\) is chosen if it receives \(m(n)\) or more votes. We will restrict our results to simple majority rule, where \(m(n) = \lceil \frac{n}{2} \rceil\), but there is no reason why our results won’t hold for any general q-rule. Voters now don’t know \(m(n)\) as they don’t know \(n\). We will assume \(s_i \in \{0, 1\}\) and that \(k_0(n)\) is the minimum number of 0-signals necessary for \(l\) to be the efficient choice, given that there are \(n\) voters. \(k_1(n)\) is similarly defined.

To simplify the calculations, we will assume that \(n = 2\lambda + 1\), where \(\lambda\) is a random variable
that comes from a probability distribution \( f(\lambda) \), with \( \lambda \in [1, \infty) \bigcap \mathbb{Z} \). This setting has the convenient property that \( m(n) = \lceil \frac{n}{2} \rceil = \lambda + 1 \) for all the values of the random variable \( \lambda \).

For our results to work, we will modify our model and assume that time is continuous, but votes are declared publicly at discreet intervals. That is, a voter can vote at any time \( t \in [1, \infty) \), but information about the votes so far is only revealed at \( t = 1, 2, 3, \ldots \). A voter can still observe how many other voters have voted before him, but only knows the votes of voters up until the last time interval. Moreover, a voter can choose in which period to vote, but cannot choose the exact time - the exact time of his vote is determined randomly. No two voters vote at the same time.

Our main result will be making use of the following lemmas:

**Lemma 1** If \( \frac{1}{2} \leq p_L \leq p_R \) then for any \( n \), \( k_0^*(n + 2) - k_0^*(n) \leq 1 \). Similarly, if \( \frac{1}{2} \leq p_R \leq p_L \) then for any \( n \), \( k_1^*(n + 2) - k_1^*(n) \leq 1 \).

**Proof.** Let \( \frac{1}{2} \leq p_L \leq p_R \), so \( \frac{p_L}{1-p_R} \geq \frac{p_R}{1-p_L} \geq 1 \). For fixed \( n \), let \( k_0^*(n + 2) = k_0^*(n) + \mu \) and \( k_0^*(n) = k_0^* \). From the definition of \( k_0^*(n) \), we have \( \frac{p_{L}^{k_0^* - 1} (1-p_L)^{n-k_0^* + 1}}{p_{R}^{k_0^*} (1-p_R)^{k_0^*}} < \frac{1-\pi_L}{\pi_L} \leq \frac{p_{L}^{k_0^*} (1-p_L)^{n-k_0^*}}{p_{R}^{k_0^*} (1-p_R)^{k_0^*}} \) (1). From the definition of \( k_0^*(n + 2) \), we have \( \frac{p_{L}^{k_0^* + \mu - 1} (1-p_L)^{n-k_0^* + 3-\mu}}{p_{R}^{k_0^*} (1-p_R)^{k_0^*}} < \frac{1-\pi_L}{\pi_L} \leq \frac{p_{L}^{k_0^* + \mu} (1-p_L)^{n-k_0^* + 2-\mu}}{p_{R}^{k_0^*} (1-p_R)^{k_0^*}} \) or equivalently \( \frac{p_{L}^{\mu-1} (1-p_L)^{3-\mu}}{p_{R}^{\mu} (1-p_R)^{\mu-1}} < \frac{1-\pi_L}{\pi_L} \leq \frac{p_{L}^{\mu} (1-p_L)^{3-\mu}}{p_{R}^{\mu} (1-p_R)^{\mu-1}} \) (2). If \( \mu \geq 2 \) then \( \frac{p_{L}^{\mu-1} (1-p_L)^{3-\mu}}{p_{R}^{\mu} (1-p_R)^{\mu-1}} \geq \frac{1-\pi_L}{\pi_L} \geq 1 \) and (1), (2) give us \( \frac{1-\pi_L}{\pi_L} \leq \frac{p_{L}^{k_0^*} (1-p_L)^{n-k_0^*}}{p_{R}^{k_0^*} (1-p_R)^{k_0^*}} \), which is a contradiction. Hence, \( \mu \leq 1 \) and \( k_0^*(n + 2) - k_0^*(n) \leq 1 \) for any \( n \). The proof for \( \frac{1}{2} \leq p_R \leq p_L \) is similar.

**Lemma 2** If \( \frac{1}{2} \leq p_L \leq p_R \), then for all integers \( n \) of the form \( n = 2\lambda + 1 \) with \( \lambda \in [1, \infty) \bigcap \mathbb{Z} \) we have \( m(n) \geq k_0^*(n) \). Similarly, if \( \frac{1}{2} \leq p_R \leq p_L \), then for all integers \( n \) of the form \( n = 2\lambda + 1 \) with \( \lambda \in [1, \infty) \bigcap \mathbb{Z} \) we have \( m(n) \geq k_1^*(n) \).

**Proof.** Let \( \frac{1}{2} \leq p_L \leq p_R \). Then \( m(n) = m(2\lambda + 1) = \lambda + 1 \) and, from lemma 1, \( k_0^*(n) = k_0^*(2\lambda + 1) \leq \lambda + 1 \), so \( k_0^*(n) \leq m(n) \). The proof is similar for \( \frac{1}{2} \leq p_R \leq p_L \).
Proposition 4 There exists an equilibrium strategy that achieves the efficient outcome in an election with an unknown number of voters and with continuous time.

Proof. Without loss of generality, let $\frac{1}{2} \leq p_L \leq p_R$. Assume that all 0-signal voters vote in the first period for $l$, and that there are $k_0$ of them. (If $\frac{1}{2} \leq p_R \leq p_L$ then 1-signal voters will vote in the first period for $r$.) If $k_0 \geq m(n)$ then the election is over and the efficient outcome has been achieved, since $m(n) \geq k_0^*(n)$. (lemma 2) Suppose $k_0 < m(n)$. After the first period, all the remaining voters observe $k_0$, but they don’t know $n$. Let $\tilde{n}$ be defined as the smallest odd $\tilde{n} = 2\bar{\lambda} + 1$ with $k_0^*(\tilde{n}) > k_0$. From lemma 1, we know that $k_0 < k_0^*(2\bar{\lambda} + 1) \leq \bar{\lambda} + 1$. Notice that $\tilde{n}$ is known by all the remaining voters after the first period. Now, if the remaining $n - k_0$ voters knew $n$, then they would like to all vote for $r$ if $n \geq \tilde{n}$ and for $l$ if $n < \tilde{n}$. In our equilibrium we will achieve the same result without requiring from the voters to know $n$. The idea is as follows: the first $\bar{\lambda} - k_0 \geq 0$ remaining voters vote for $l$ and the ones remaining after that (if any) vote for $r$. Then the efficient result will always be achieved, regardless of the value of $n$, as $r$ will be the outcome of the election if and only if $n \geq 2\bar{\lambda} + 1 = \tilde{n}$. This outcome can always be implemented as voters can always see how many others voted before them and vote or postpone voting accordingly.

7 Extension of our result for a special case of multidimensional signals

In this section we will provide an example to show that our method can still work in certain cases even when signals don’t satisfy the maximum likelihood property. Suppose that each of the $n$ voters receives a multidimensional signal $s_i = (s_i^1, s_i^2) \in \{0, 1\}^2$. Notice that we can always construct a mapping from a multidimensional signal to a signal in one dimension, and then use proposition 3 to efficiently aggregate the signals that each voter receives through a sequential election. However, proposition 3 requires the maximum likelihood property, and a multidimensional signal cannot always be mapped to a signal in one dimension that satisfies the maximum likelihood property. In this example, we will use a two dimensional signal that satisfies the maximum likelihood property in each dimension only. Suppose that we have an efficient aggregation rule $E : \{s_1, s_2, ..., s_n\} \rightarrow \{l, r\}$ as follows, for given $S_1(n), S_2(n)$:
\[ E(s_1, s_2, \ldots, s_n) = \begin{cases} 
  l, & \text{if } \sum_{i=1}^n s_i^1 < S_1 \text{ and } s^2 = \sum_{i=1}^n s_i^2 < S_2 \\
  r, & \text{otherwise} 
\end{cases} \]

This aggregation rule has an intuitive interpretation - it represents a scenario in which we want to figure out which of two candidates is best in each of two dimensions and candidate \( r \) is preferred only when she is best in both dimensions. In the following figure, the aggregation rule is represented by the blue line that divides the graph into two areas, one in which \( l \) and one in which \( r \) is the efficient outcome.

![Figure 6: Efficient information aggregation with multidimensional signals](image)

Notice that this aggregation rule has no equivalent in one dimensional signals satisfying the maximum likelihood property. To see this, let \( s_i = (0, 1) \) and \( s_j = (1, 0) \). It is impossible to order \( s_i \) and \( s_j \) and tell which one of the two brings the efficient outcome closer to \( l \) or \( r \) - to order \( s_i \) and \( s_j \) we will need to know all the other signals of all the other voters. For example, an \( s_i \) can bring the outcome closer to \( r \) when we are close to the horizontal part of the blue line, whereas an \( s_j \) can bring the outcome closer to \( r \) when we are close to the vertical part of the blue line in the above figure.

Nevertheless, we will show that sequential voting with endogenous timing can again achieve the efficient outcome.
Proposition 5  The efficient outcome can be achieved in a sequential election with endogenous timing in which each voter receives a two-dimensional signal $s_i = (s^1_i, s^2_i) \in \{0,1\}^2$ and in which the efficient aggregation rule $E$ is as described above.

Proof. Assume that $S_2 < m$ as in figure 6. The case in which $S_2 \geq m$ can be proved similarly by substituting 0-signals with 1-signals and vice-versa. We will allow the voters with signal $s_i = (1,1)$ to vote in the first period for $r$. Suppose there are $k_{(1,1)}$ such voters.

- If $k_{(1,1)} \geq m$, then the election is determined, but since $k_{(1,1)} \geq m > S_2$ the efficient outcome $r$ has been achieved.

- If $k_{(1,1)} < m$, then we will allow voters with signal $s_i = (1,0)$ to vote in the second period for $r$. Suppose there are $k_{(1,0)}$ such voters. If $k_{(1,1)} + k_{(1,0)} > S_2$, then the efficient outcome has been revealed, it is $r$, and voters in the next period can also vote for $r$ and ensure that $r$ is achieved. If, on the other hand, $k_{(1,1)} + k_{(1,0)} \leq S_2$ then we have only voters of type $(0,0)$ and $(0,1)$ remaining and the election has not yet been determined, since $k_{(1,1)} + k_{(1,0)} \leq S_2 < m$. But this is just equivalent to the single dimensional problem solved in proposition 2 earlier in this paper. ■

8 Conclusion

We have shown that allowing the voters to choose when to cast their vote can lead to substantial improvements in the aggregation of information. We have examined a number of different scenarios and extensions of our results in various directions. However, all of our results hold only under the assumption of common values. An interesting extension would be to see if endogenous timing can lead to improvements in the aggregation of information even without common values, for example when each voter has both a common value component in her utility when choosing an alternative as well as a private component.

References


