Can Fuzzy Decision-Making Explain Growing Consumption During Eras of Negative Real Interest Rates?

Aram Balagyan* and Christos Giannikos†

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Abstract

Treatment of fuzziness as a distinct source of uncertainty in intertemporal consumption models allows for explicit modeling of the effect of consumers’ optimism-pessimism on optimal consumption decisions. By exploring the usefulness of this particular feature in uncovering the ‘excessive consumption growth’ puzzle we demonstrate in simple two-period life-cycle settings that there are some reasonable ranges of individuals’ optimism-pessimism and risk-aversion for which the optimal consumption is expected to grow ‘excessively’ despite negative real interest rate.

JEL Classifications: E21, D11, D91

1 Introduction

One of the manifests offered by a large family of classical life-cycle representative-agent models is that personal consumption is expected to decline whenever individuals’ time preference exceeds the real rate of interest. Deaton (Deaton 1986), however, reports some puzzling evidence that there were prolonged periods in the U.S. post-war history when aggregate consumption continued growing despite prevailing negative real rates of interest. Responding to this conundrum many authors¹ show that

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*The Graduate Center, City University of New York, abalagyozyan@gc.cuny.edu
†Columbia University and Zicklin School of Business, Baruch College, City University of New York, cig3@columbia.edu
¹See for example (Caballero 1990)
the existence of precautionary saving motives can, to a large degree, be accountable for this ‘excessive growth’ puzzle. As it is noted by Deaton (Deaton 1992) explicit treatment of precautionary saving in a representative agent framework, nevertheless, does not entirely uncover this discrepancy between the average consumption growth and the negative real rate of return. Given the average rate of interest and variance of consumption growth that were historically observed in the U.S. the coefficient of relative risk aversion is required to be unreasonably large to account for positive consumption growth. This paper addresses the ‘excessive growth’ puzzle by explicitly treating fuzziness of information as a possible source of uncertainty in a simple, two-period life-cycle consumption model. Here we draw a strict distinction between fuzzy and stochastic uncertainty. An event is said to be uncertain in a stochastic sense if its occurrence is a matter of chance, hence there exist a certain probability distribution strictly less than one that describes the likelihood of occurrence of every element in the sample space. When an event is fuzzy, on the other hand, its occurrence is a matter of degree, rather than likelihood, and there exist a membership function less than or equal to one that describes the degree to which every element in the sample space belong to the fuzzy outcome set. It may be possible that some events are uncertain in a fuzzy sense only even though they are not stochastic at all. When asked what is their forthcoming income many people are absolutely sure that is going to be around some round number. For instance an individual may be absolutely certain that her forthcoming disposable income is going to be around hundred thousand dollars. Clearly, what constitutes uncertainty here is the word "around" rather than the stochastic nature of the outcome and may be due, for instance, to the complexity of the tax system. A fundamental difference between stochastic and fuzzy uncertainty, is that people usually attempt to forecast stochastically uncertain events that are related to their current choice. When it comes to fuzziness, on the other hand, people rarely attempt to forecast it. In reality, when it comes to decision making, precision is rarely a descriptive attribute. Naturally, the usage of fuzziness and, hence, of the elements of fuzzy set theory, rather than more traditional elements of probability theory should be further justified when it comes to the puzzle of excessive consumption growth. One candidate explanation of the excessive consumption growth phenomenon in post-war years would be widespread pessimism among U.S. population (insert a citation of Acemoglu here) Even so, it is not clear why would people remain pessimistic for such a prolonged period of time if the only uncertainty was stochastic (such as?). For example when one faces a gamble that consists of a flip of a coin, then sooner or later the gambler should learn the true probability structure of the gamble and his optimism-pessimism should no longer affect his gambling decisions. and the price she wants to pay for this gamble. The same may not, however, be true if the relevant information is fuzzy. In city- or state-wide lotteries, for example, people know that the exact take-home amount
of the win is "large" and that the probability of win is "small". Even though, this
type of basic information about any lottery is fairly trivial to obtain, lottery players
rarely go through the trouble of obtaining it and as a consequence their decision to
play lottery or not is largely driven by their optimism or pessimism.

By extending the methodology first proposed by Hauenschild and Stahlecker
(Hauenschild & Stahlecker 2001) to a constant relative risk aversion felicity func-
tion we show that when expectations about future income are only uncertain in a
fuzzy sense then there can be some very reasonable ranges of consumers’ pessimism
optimism and risk-aversion for which the growth rate of aggregate consumption is
expected to be positive even with prevailing negative real rates of interest and a
comparatively large individual discount factor. The resulting Euler’s equation can
nest a family of non-fuzzy alternatives and, in principle, can easily be tested.

2 The Model

A representative agent maximizes the following two-period intertemporal separable
utility function
\[ U(c_1, c_2) = u(c_1) + \frac{1}{1 + \delta} u(c_2) \]

Subject to constraint:
\[ A_2 = y_2 + (1 + r)(y_1 - c_1) \]

and transversality condition:
\[ c_2 = A_2 \]

Here \( c_1 \) and \( c_2 \) are the present and future consumptions, \( y_1 \) and \( y_2 \) are the present and
future incomes, \( r \) is the real rate of interest, and \( \delta \) is the rate of time preference. The
sole source of uncertainty in this model is the second period’s wealth, \( A_2 \), that is only
uncertain in a fuzzy sense. The consumer knows for certain that her future wealth
is going to be around some known value \( A_2^* \). What constitutes uncertainty in these
settings is the word "around", which may arise due, for example, to complexity of the
tax system that the consumer does not attempt to predict. The degree of membership
of the future wealth \( A_2 \) to the fuzzy set described by the sentence "wealth around
\( A_2^* \)" is modeled by a commonly used Gaussian membership function
\[ \mu(A_2) = e^{-\left(\frac{(A_2 - A_2^*)^2}{2(zA_2^*)^2}\right)} \]

where \( A_2 \) is the actual wealth in the second period. Changes in the parameter \( z \)
alter the degree of fuzziness carried by the membership: fuzziness vanishes as \( z \)
approaches to zero. Hence the conventional, non-fuzzy alternative of the model (1)-(4) can be modeled as a limiting case of the fuzzy representation here. The $\alpha$-cut of the membership function (4) is given by:

$$A_2(\alpha^*) = A_2^* \pm A_2^* z \sqrt{-2 \ln(\alpha)}$$  \hfill (5)

Where, $\alpha \in (0, 1]$. Expression $z \sqrt{-2 \ln(\alpha)}$ in (5) represents some relative or percent measure of fuzziness that distorts the benchmark value of the second-period wealth $A_2^*$ from its actual value $A_2$. Given an $\alpha$-cut at $\alpha = \alpha^*$ an extremely optimistic consumer knows that her wealth at period two is going to be:

$$A_2(\alpha^*) = A_2^* + A_2^* z \sqrt{-2 \ln(\alpha^*)}$$  \hfill (6)

Substituting (6) into (3) and then substituting the resulting expression into (1) we rewrite the utility function of an optimistic consumer as:

$$U(c_1) = u(c_1) + \frac{1}{1 + \delta} u(A_2^* + A_2^* z \sqrt{-2 \ln(\alpha^*)})$$  \hfill (7)

Similarly, given an $\alpha$-cut at $\alpha = \alpha^*$, we rewrite the utility of a pessimistic individual

$$U(c_1) = u(c_1) + \frac{1}{1 + \delta} u(A_2^* - A_2^* z \sqrt{-2 \ln(\alpha^*)})$$  \hfill (8)

Utility of an individual whose optimism falls between these two extremes is

$$U(c_1) = u(c_1) + \frac{1}{1 + \delta} \{q u((1 - z \sqrt{-2 \ln(\alpha^*)})A_2^*) + (1 - q) u((1 + z \sqrt{-2 \ln(\alpha^*)})A_2^*)\},$$  \hfill (9)

where $q$ is Hurwicz optimism-pessimism index that takes values between 0 and 1 inclusively and represents extreme optimism and extreme pessimism respectively at those extremes. In order to ensure that the arguments inside of the felicity functions in (9) are non-negative we further restrict parameter $\alpha \in [e^{-2z^2}, 1]$ Aggregating (9) over all possible $\alpha$-cuts we get a fuzzy version of the utility function in (1):

$$U(c_1) = u(c_1) + \frac{1}{1 + \delta} \left\{ q \int_{e^{-2z^2}}^{1} u((1 - z \sqrt{-2 \ln(\alpha)})A_2^*) \, d\alpha + \right.$$

$$\left. (1 - q) \int_{e^{-2z^2}}^{1} u((1 + z \sqrt{-2 \ln(\alpha)})A_2^*) \, d\alpha \right\}$$  \hfill (10)
The optimal first-period consumption is subject to a first order necessary condition that can be obtained by substituting transition equation (2) into (10) and differentiating the resulting expression with respect to first-period consumption.

\[
  u'(c_1) = \frac{1 + r}{1 + \delta} \{ q \int_1^{1 - z} (1 - z \sqrt{-2 \ln(\alpha)}) u'((1 - z \sqrt{-2 \ln(\alpha)})A^*_2) \, d\alpha + (1 - q) \int_1^{1 + z} (1 + z \sqrt{-2 \ln(\alpha)}) u'((1 + z \sqrt{-2 \ln(\alpha)})A^*_2) \, d\alpha \}
\]

We modify (11) by assuming that consumer's preferences can be described by a conventional constant relative risk aversion utility function of the form \( u(c) = \frac{c^{1-\lambda}}{1-\lambda} \), so that \( u'(c) = c^{-\lambda} \), where \( \lambda > 0 \) is the coefficient of risk aversion:

\[
  c^{-\lambda}_1 = \frac{1 + r}{1 + \delta} \{ q \int_1^{1 - z} (1 - z \sqrt{-2 \ln(\alpha)})^{1-\lambda} \, d\alpha + (1 - q) \int_1^{1 + z} (1 + z \sqrt{-2 \ln(\alpha)})^{1-\lambda} \, d\alpha \}
\]

where \( c_2 \) is what the second-period consumption would have been had the consumer have a non-fuzzy perfect foresight that her future wealth was going to be equal to the benchmark amount \( A^*_2 \). We rewrite expression (12) in the following more convinient format:

\[
  \ln\left( \frac{c_2}{c_1} \right) = \frac{1}{\lambda} (r - \delta) + \frac{1}{\lambda} \ln(qg^-(\lambda, z) + (1 - q)g^+(\lambda, z))
\]

Where \( g^\pm(\lambda, z) = \int_1^{1 \pm z} (1 \pm z \sqrt{-2 \ln(\alpha)})^{1-\lambda} \, d\alpha \) and an assumption was made that the interest rate and the discount factor are reasonably small to allow for this ap-
proximation. Unfortunately, the integrals on the right hand side of (12) can not be evaluated analytically and in consequent analysis we resort to numerical methods to evaluate them. It is easy to show (see Balagyan & Giannikos 2006)) that Euler’s equation (12) carries a few very interesting features. It can be shown, for example, that when future labor income is uncertain in a fuzzy sense, the difference in consumption patterns of two identically optimistic or pessimistic but differently risk-averse consumers are the largest when both are neither excessively optimistic nor pessimistic about future labor income. Hence as excessive optimism or pessimism becomes widespread the differences in consumption patterns between more risk-averse and less risk-averse individuals disappear. It also can be readily noted from (12) that as $z$ converges to zero, thus as fuzzy uncertainty disappears, fuzzy Euler’s equation (12) converges to a conventional intertemporal Euler’s equation of the form:

$$\left(c_1\right)^{-\lambda} = \frac{1 + r}{1 + \delta} \left(c_2\right)^{-\lambda}$$

(14)

Hence a more traditional Euler’s equation such as (14) is nested in (12) and, therefore, two alternatives can easily be tested against each other.

### 3 Results

The question that we address in this paper is whether there are any reasonable values of risk aversion, optimism-pessimism, relative fuzziness, and rates of time preference such that the Euler’s equation (13) holds positive consumption growth that is commensurate to the one observed historically in after-world-war II years despite a negative real rate of interest. We use historical data statistics reported by Blinder and Deaton (Blinder & Deaton 1985) and Deaton (Deaton 1992). These authors report that between the third quarter of 1953 and the fourth quarter of 1984 the growth rate of per capita quarterly consumption has averaged to 0.5%. The real rate of interest in that period, on the other hand, was on average -0.06%. Substituting the former number into the right hand side of (13) and the later one into the right hand side of (13) we search for a space of optimism-pessimism index, $q$, coefficient of risk-aversion,$\lambda$, and relative fuzziness, $z$, that solve the Euler’s equation in (13). Without making any guesses about the coefficient of time preference, $\delta$, we solve (13) by allowing $\delta$ to take four discreet values: $\delta = 0.03\%$, $\delta = 0.1\%$, $\delta = 0.5\%$, and $\delta = 3\%$ per quarter. The graphical results that correspond to each of these cases are presented in Figures 1-4. The solution curves characterize all the combinations of optimism-pessimism and risk-aversion that given historical consumption growth, ex-post real rate of interest,
relative fuzziness, and the rate of time preference conform with (13). It can easily be spotted from all the graphs presented here that there are some degrees of optimism-pessimism and coefficients of risk aversion less than ten such that despite negative real rate of quarterly interest of -0.06% per capita consumption grew at positive 0.5% quarterly rate. For instance, if optimal consumption patterns are governed by (12) and, therefore, (13) and consumers discount future consumption relatively heavily at a quarterly rate of $\delta = 3\%$, (Figure ??), and the relative fuzziness $z = 6\%$, then the coefficient of relative risk aversion $\lambda = 2$ coupled with an optimism-pessimism index $q = 0.63$ (moderately pessimistic) would result in 0.5% quarterly consumption growth while the quarterly real rate of interest is -0.06%. It is worth noting that the solution curves in all the figures can be regarded as ‘indifference curves’. There is a trade-off between risk-aversion and optimism-pessimism. Increase in consumers’ optimism-pessimism can be compensated by increase in risk aversion to maintain the previous consumption pattern. When those curves are relatively flat (steep) risk aversion (optimism-pessimism) compared to optimism pessimism (risk aversion) is unimportant and does not dramatically influence consumer’s decision under fuzzy uncertainty. The prevalence of relative flatness of these indifference curves signifies that consumer’s optimism-pessimism plays an important role in their consumption decision processes. Empirical findings of Acemoglu and Scott (Acemoglu & Scott 1994) imply that consumer confidence, hence consumers’ optimism-pessimism, is a strong predictor of consumption growth, a result that is consistent with our analysis.

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4Estimates of $\lambda$ vary widely in empirical literature. Altug (Altug 1983) find it to be near zero. Mankiw, Rotemberg, and Summer (Mankiw & Summers 1985) establish that it is near 0.5. Kehoe (Kehoe 1983), and Hansen and Singleton (Hansen & Singleton 1983) estimate $\lambda$ to be around 1. Tobin and Dolde (Tobin & Dolde 1971) 1.5. Friend and Blume (Friend & Blume 1975) found it to be around 2. Zeldes’s (Zeldes 1989) estimates of $\lambda$ are in the region of 2.3. Mankiw (Mankiw 1981), (Mankiw 1985) 4, and 3 respectively.
Figure 1: Optimism-pessimism index, $q$, vs. positive coefficients of risk aversion, $\lambda$, that turn quarterly consumption growth equal to its historical after-world-war II average of 0.5% per Quarter $\delta=1\%$, $r=-0.06\%$.

Figure 1: Optimism-pessimism index, $q$, vs. positive coefficients of risk aversion, $\lambda$, that turn quarterly consumption growth equal to its historical after-world-war II average of 0.5% per Quarter $\delta=1\%$, $r=-0.06\%$.
Figure 2: Optimism-pessimism index, $q$, vs. positive coefficients of risk aversion, $\lambda$, that turn quarterly consumption growth equal to its historical after-world-war II average of 0.5% under conditions of negative quarterly real interest rate -0.06% and positive quarterly discount factor of 0.1%
Figure 3: Optimism-pessimism index, \( q \), vs. positive coefficients of risk aversion, \( \lambda \), that turn quarterly consumption growth equal to its historical after-world-war II average of 0.5% per Quarter \( \delta = 5\% \), \( r = -0.06\% \).
Figure 4: Optimism-pessimism index, $q$, vs. positive coefficients of risk aversion, $\lambda$, that turn quarterly consumption growth equal to its historical after-world-war II average of 0.5% per Quarter $\delta=10\%$, $r=-0.06\%$.
References


