

# Financial Market Segmentation, Stock Market Volatility and the Role of Monetary Policy

Anastasia S. Zervou \*

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## Abstract

This paper explores the role of monetary policy in a segmented stock market model. Previous research (e.g Mankiw and Zeldes (1991), Vissing-Jørgensen (2002)) reports that only a fraction of the households participates in the stock market. In this paper participating households have stochastic dividend as part of their income and are, therefore, subject to financial market risk. Also, only participants receive monetary transfers. In such a setting, optimal monetary policy has the new role of perfectly sharing the financial market risk among all agents in the economy. This policy might not minimize neither stock price volatility nor inflation volatility. Furthermore, optimal monetary policy does not depend on the degree of participation and increased participation does not necessarily decrease stock prices volatility.

## 1 Introduction

We often witness the media's concerns about the Fed's reaction to asset markets' changes, reflecting possibly the concerns of the increasing number of investors in these markets. Especially after Alan Greenspan's talk on December 1996, it became clear that the interplay between monetary policy and the financial markets is far from explicit and more research has to be done in the area. A new stream of research developed with focus on examining both the effects of monetary policy on stock price volatility and the influence stock market advances should have on monetary policy. Bernanke and Gertler (2001) suggest that if monetary authorities follow a type of inflation targeting then low stock prices volatility is ensured, coming either from fundamentals or bubbles. Rigobon and Sack (2003) examine the effects of stock prices on monetary policy and find that

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\*Washington University in St. Louis, Campus Box 1208, St. Louis, MO 63130, azervou@artsci.wustl.edu, <http://artsci.wustl.edu/azervou>. I am very grateful to my advisor Stephen Williamson and to Costas Azariadis and James Bullard for their constructive comments and support. I also thank Jacek Suda, the entire Macro Reading Group at Washington University and participants of the 2008 Missouri Economics Conference for helpful comments. Finally, I thank the Olin Business School's Center for Research in Economics and Strategy (CRES) for funding.

monetary authorities do react to stock prices, but only to offset their effects on the economy. On the other side of the interplay, Bernanke and Kuttner (2005) don't find strong effects of monetary policy on stock prices while Rigobon and Sack (2004) do.

This paper attempts to shed light on some of the interactions between the stock market and monetary policy, through studying a simple theoretical model where only a fraction of the population participates in financial markets. The fact that only a fraction of the population holds financial assets is well documented by empirical research like Vissing-Jorgensen (2002) who reports that in her dataset approximately 22% of the households hold stocks and Mankiw and Zeldes (1991) who describe that even among high liquidity households less than half of them hold stocks.

Former theoretical research employed the limited participation insight in order to capture the liquidity effect, as in Alvarez, Lucas and Weber (2001) and forms of non-neutrality of money as in Williamson (2005) and Williamson (2006), or to study the asset premium puzzle, as in Guo (2000), or stock price volatility as in Allen and Gale (1994). In addition, the limited participation assumption is recently incorporated in the New Keynesian literature like Bilbiie (2005) who attempts to investigate whether monetary authorities react differently due to changes in the financial market participation rate.

This paper builds on Alvarez, Lucas and Weber (2001) limited participation model, by adding stock market and abstracting from variable velocity in order to examine aspects of the interplay between monetary policy and the stock market. Specifically, we ask the question of how monetary authorities acting optimally, react to the fact that a fraction of the population is subject to stock market risk. Furthermore, we analyze stock price volatility as Allen and Gale (1994) do, but here we can also investigate whether optimal monetary policy decreases this volatility and compare across various policy assumptions. Additionally, we explore how monetary policy is affected by different participation rates as Bilbiie (2005) does and appraise the effects on the model economy that the increased participation in financial markets has, a manifest phenomenon in the United States after the early 90's.

More precisely, this work assumes that there are two groups of agents residing in the model economy, one of them only participating in financial markets. The participating fraction of the population receives, except from its deterministic income, a share of the risky total dividend income. This total dividend income is random, resembling Lucas (1978) tree income, and is shared among the financial markets' participants analogously to the amount of shares each of them holds.

In addition, as usually assumed in the limited participation literature, financial market participants are the first to absorb monetary policy changes, while non participants are affected only indirectly through price adjustments. Specifically, participants receive a positive transfer every time monetary authorities follow expansionary policy, while they get taxed whenever policy is contractionary.

In such an environment, monetary policy valuing equally all agents can reduce the participants' risk by sharing it among participants and non-participants, increasing in this way total welfare. In particular, optimal monetary policy

becomes expansionary whenever dividend income is lower than expected, subsidizing participants with a positive income transfer. Such a policy increases goods prices, dismaying non-participants whose consumption decreases. On the other hand, whenever dividend income is higher than expected, monetary policy contracts, taxing participants, taking away part of their increased income. The good becomes more affordable and non-participants' consumption increases. We find a new role assigned to monetary policy and a different reason for monetary policy intervention in financial markets, which is to share the dividend risk participants face, with the non-participating fraction of the population.

While monetary policy can have a welfare maximizing effect only as long as there is a fraction of the population incurring financial risk, we find that contrary to Bilbiie (2005), the decisions that monetary authorities take do not depend on the participation rate. As soon as there are some agents of both groups in the economy, monetary policy that values them both equally, can increase total welfare by sharing the risk among the two types of agents, irrespectively of the participation rate. This risk sharing property is not sensitive to standard utility specifications and implies that both groups share equally the economy's total output and dividend risk. In this way, despite the fact that we do not consider endogenous participation choice, optimal monetary policy makes agents indifferent between participating or not in the financial markets. Optimal monetary policy corrects the market's imperfection of limited participation.

Furthermore, we examine the implications that optimal risk sharing policy has on stock price volatility. We find that optimal monetary policy does not necessarily involve low stock price volatility. We compute specific examples of optimal monetary policy implying higher stock price variability compared to a constant money growth policy. Additionally, converse to what Allen and Gale suggest (1994), there are parameters' values for which increased participation does not necessarily entail lower stock market volatility. Concluding, optimal monetary policy maximizes welfare, shares the dividend risk between participants and non-participants, corrects the limited participation imperfection and does not necessarily imply low stock prices volatility.

As a last exercise we examine how inflation variation behaves under the optimal monetary policy assumption and compare across other policy specifications.

## 2 The Model Economy

### 2.1 Set-up

The model economy consists of a good market and three asset markets: nominal bond, stock and money market. Bond and stock markets are segmented, so that from a continuum of infinitely lived households of measure one, only  $\lambda \in (0, 1)$  participates in these markets while  $1 - \lambda$  doesn't. The stock market is introduced similarly to Lucas (1978) model. Participating agents receive a share of the stochastic dividend tree according to the amount of stocks they

hold. Decisions about bonds do not affect the agents behavior and they are introduced for examining the asset pricing of the model.

All agents have identical preferences and seek to maximize their lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad (1)$$

where  $0 < \beta < 1$

Endowments are defined in terms of a unique, non-storable good. The  $1 - \lambda$  fraction of the population which doesn't participate in financial markets, or the non-traders, receive every period a fixed real endowment  $y^N$ . The  $\lambda$  fraction of the population which does participate in financial markets, or the traders, receive every period a fixed real endowment  $y^T$  and a share of the stochastic real endowment  $\varepsilon_t$ . Specifically, at period  $t$  each trader buys  $z_{t+1}$  share of the stochastic part of output so  $z_{t+1}\varepsilon_t$  is interpreted as the real dividend the traders receive that period. Consequently, traders have a risky component in their income, while non-traders get only the fixed endowment  $y^N$ . To make the analysis more interesting and see various aspects of monetary policy, it is assumed that the mean income of traders equals that of non-traders i.e.  $y^T + \frac{\bar{\varepsilon}}{\lambda} = y^N = \bar{y}$  where  $\bar{\varepsilon} = (\bar{y} - y^T)\lambda$  is the mean of the dividend shock  $\varepsilon_t$  and is shared among the  $\lambda$  traders. Note that  $\bar{\varepsilon}$  depends on  $\lambda$ , so the mean per trader dividend does not.

This is a cash-in-advance model, with the following timing: At the beginning of period  $t$ , agents are promised their endowments  $y^T$  and  $y^N$  which they only receive at the end of period  $t$ , after they make their saving and consumption choices. Traders realize  $\varepsilon_t$ , which is received at the end of the period too. Traders and non-traders start period  $t$  with available money holdings  $m_t^T$  and  $m_t^N$  respectively. Traders also receive a monetary transfer  $\tau_t$  from the monetary authorities. This assumption captures a direct effect that monetary policy has on the financial markets participants. Agents can only use cash when entering the financial and goods markets. Credit is assumed away, introducing cash-in-advance constraints. The financial markets open first, where at period  $t$  traders can sell the  $b_t$  amount of bonds and  $z_t$  amount of stocks they bought at  $t - 1$ . Note that  $z_t$  is defined in terms of the number of titles each trader holds and can be sold at the price  $q_t$ , so traders receive  $q_t z_t$  dollars for holding  $z_t$  stocks titles for a period. On the other hand, bonds are bought at period  $t - 1$  at the price  $s_{t-1} < 1$  and pay back one unit of money at period  $t$ . Using their money holdings  $m_t^T$ , the money for selling their  $z_t$  stocks, the returns from holding  $b_t$  and the money transfer  $\tau_t$ , traders can decide how many new bonds and stocks titles to buy and use the rest of their resources for buying consumption good.

Non traders cannot participate at the financial markets and do not receive monetary transfers. They only decide how much of their money holdings  $m_t^N$  to spend in buying consumption good.

The described cash-in-advance constraints are given bellow:

For participants:

$$m_t^T + q_t z_t + b_t + \tau_t \geq p_t c_t^T + q_t z_{t+1} + s_t b_{t+1} \quad (2)$$

For non-participants:

$$m_t^N \geq p_t c_t^N, \quad (3)$$

where  $p_t$  is the price of the consumption good and  $q_t$  is the price of the share.  $s_t$  is the price of the nominal bond which pays one unit of money next period.

After new shares, bonds and consumption good are bought, the agents receive their endowments and dividends. The budget constraints are given as follows:

For participants:

$$m_t^T + q_t z_t + b_t + \tau_t + p_t z_{t+1} \varepsilon_t + p_t y^T \geq m_{t+1}^T + p_t c_t^T + q_t z_{t+1} + s_t b_{t+1} \quad (4)$$

where  $d_t = z_{t+1} \varepsilon_t$  are the real dividend payments distributed at period  $t$  (but available to use on  $t + 1$ ).

For non-participants:

$$m_t^N + p_t y^N \geq m_{t+1}^N + p_t c_t^N \quad (5)$$

Because assets markets operate before goods markets open, holding money after the financial markets close, bears positive opportunity cost when we assume positive return for bonds or stocks. Only the amount of money required for purchasing the desired amount of consumption good is held and the cash-in-advance constraints is assumed to bind. The implications for the budget constraints are:

For participants:

$$p_t z_{t+1} \varepsilon_t + p_t y^T = m_{t+1}^T \quad (6)$$

For non-participants:

$$p_t y^N = m_{t+1}^N \quad (7)$$

The above equations reveal that the cash balances with which agents begin period  $t + 1$  match the fraction of their wealth that the cash-in-advance constraints prevented them from using at period  $t$ . These are, the proceeds from selling the real endowments at the goods market and for the case of traders, cashing out the real dividends distributed at period  $t$ .

## 2.2 Competitive Equilibrium and Asset Pricing

The analysis below assumes that the cash-in-advance constraints bind, while later the conditions under which this assumption holds are discussed.

The four market clearing conditions in this economy are as follows:

For the goods' market to clear, total real endowment is completely consumed by traders and non-traders at every period, as this is a non-storable good.

$$Y_t = \varepsilon_t + \lambda y^T + (1 - \lambda)y^N = \lambda c_t^T + (1 - \lambda)c_t^N$$

Using the assumption that the mean income of the two groups is the same, it turns out that

$$Y_t = \bar{y} + \varepsilon_t - \bar{\varepsilon} = \lambda c_t^T + (1 - \lambda)c_t^N \quad (8)$$

For the stock market to clear, the sum of all shares each trader holds equals the total share of the stochastic output distributed as shares. We assumed that all the stochastic output is distributed, thus:

$$\lambda z_{t+1} = 1 \Rightarrow z_{t+1} = \frac{1}{\lambda} \quad (9)$$

For the bonds market to clear, the sum of all real bonds each trader holds equals the total supply of them, which is zero.

$$\lambda b_t = 0 \quad (10)$$

For the money market to clear, the total money holdings of traders and non-traders should equal the total amount of money supplied in the economy,  $\bar{M}_t$ . Then:

$$\lambda m_{t+1}^T + (1 - \lambda)m_{t+1}^N = \bar{M}_t = \bar{M}_{t-1}(1 + \mu_t) = \lambda \tau_t + \bar{M}_{t-1} \quad (11)$$

where  $\mu_t$  denotes money growth from time  $t - 1$  to time  $t$ . The extra money supplied at time  $t$  are distributed as transfers to the  $\lambda$  traders.

Applying the bond, stock and money market clearing conditions to the cash-in-advance equations (2) and (3) we get the straight forward cash-in-advance restrictions describing that consumption expenditure cannot exceed the monetary resources agents began the period plus, for the case of participating agents, the monetary transfer. This is true because agents do not receive their endowments and dividend incomes until the good and asset markets have closed.

For participants:

$$m_t^T + \tau_t \geq p_t c_t^T$$

For non-participants:

$$m_t^N \geq p_t c_t^N,$$

Furthermore, from the goods market clearing condition (8) and the cash-in-advance constraints (2) and (3) holding with equality, it turns out that:

$$p_t Y_t = \lambda q_t (z_t - z_{t+1}) + \bar{M}_t + \lambda (b_t - s_t b_{t+1})$$

Applying the bond, stock and money market clearing conditions (10), (11) and (12) we get a version of the quantity equation where total output is the sum of a deterministic and a stochastic part, and velocity equals one:

$$p_t = \frac{\bar{M}_t}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})} \quad (12)$$

While in Alvarez, Lucas and Weber (2001) the instability between prices and money stems from velocity shocks, in this model important role plays the stochastic part of output distributed as dividends to the stock market participants. In particular, an increase in the dividend real income shock the participants receive puts downward pressure in prices and vice versa.

As in Alvarez, Lucas and Weber (2001), consumption can be calculated assuming binding cash-in-advance constraints. There is no need for solving the maximization problem in this point and no utility assumptions are made.

In particular, combining equations (3) with equality and equation (7) we find that:

$$p_{t-1} y^N = p_t c_t^N$$

Substituting for the good price from equation (12), it turns out that the consumption of non-traders is given by:

$$c_t^N = \bar{y} \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \frac{1}{1 + \mu_t} \quad (13)$$

Using the above equation and the market clearing condition for the good market given by (8) it turns out that the traders' consumption is represented by the following equation:

$$c_t^T = \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\lambda} \frac{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))(1 + \mu_t)} \quad (14)$$

Here we see that an increase in the current dividends distributed,  $\varepsilon_t$  translates in lower good price at period  $t$ , increasing consumption for both traders and non-traders. On the other hand, an increase in the real dividends distributed last period  $\varepsilon_{t-1}$ , decreases the price of the good at period  $t-1$  and thus the value of the good carried in the form of money balances from period  $t-1$  to period  $t$ . Assuming that monetary policy does not react to such a shock and that dividend shocks are independent across time, consumption at period  $t$  decreases for non-participants while for participants the change is positive:

$$\frac{\partial c_t^N}{\partial \varepsilon_{t-1}} = - \frac{(\bar{y} + (\varepsilon_t - \bar{\varepsilon}))\bar{y}}{(1 + \mu_t)(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))^2}$$

and

$$\frac{\partial c_t^T}{\partial \varepsilon_{t-1}} = \frac{(\bar{y} + (\varepsilon_t - \bar{\varepsilon}))\bar{y}(1 - \lambda)}{\lambda(1 + \mu_t)(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))^2}$$

This is because except from the indirect effect affecting negatively all agents, participants are also affected positively by earning part of the  $\varepsilon_{t-1}$  shock as dividend, and this effect is stronger. The cash-in-advance constraints allow  $\varepsilon_{t-1}$  to be observed at period  $t-1$  but is available for the participants consumption only at period  $t$ . The increase in dividends is higher than the decrease in money balances and thus participants are better off. Also note that the increase in traders consumption is  $\frac{1-\lambda}{\lambda}$  higher than the decrease in non-traders consumption. This is because the increase on traders consumption does depend on the participation rate, as the more participating agents, the less share of the  $\varepsilon_{t-1}$  shock each of them receive. Then the increase in traders consumption is negatively affected by the participation rate.

While the above analysis was not requiring solving the maximization problem, the analysis of the asset prices requires such a procedure. In particular the traders' utility maximization problem needs to be solved, subject to the cash-in-advance and budget constraints.

It turns out that the bonds prices in terms of currency are determined by:

$$\beta E_t \frac{u'(c_{t+1}^T)}{p_{t+1}} = \frac{u'(c_t^T)}{p_t} s_t, \quad (15)$$

Equation (15) describes the pricing of the nominal bond: the utility increase traders expect to receive at period  $t+1$ , when the bond matures and pays back, equals the foregone utility they suffer from buying the nominal bond at period  $t$ . In addition, this equation reveals a Fisher effect. Defining the nominal rate as  $r_t^n \equiv \frac{1}{s_t} - 1$ , the real rate as  $r_t^r \equiv \frac{p_t}{s_t p_{t+1}} - 1$  and inflation as  $\pi_t \equiv \frac{p_{t+1}}{p_t} - 1$ , we have that  $r_t^n = r_t^r \frac{p_{t+1}}{p_t} + \pi_{t+1}$  which gives approximately the Fisher effect.

Note also that for the binding cash-in-advance assumption to hold, the multiplier of the bonds first order condition should be strictly positive, implying that  $s_t < 1$  and then the nominal rate is strictly positive.

In addition, the stock market first order condition implies that:

$$\beta E_t \frac{u'(c_{t+1}^T)}{p_{t+1}} (q_{t+1} + p_t \varepsilon_t) = \frac{u'(c_t^T)}{p_t} q_t, \quad (16)$$

which evaluates that the additional utility expected at period  $t+1$ , when the dividends are paid and the stock can be re-traded, equals the foregone utility at time  $t$  incurred for buying the stock.

### 3 Optimal Monetary Policy

In this section we study the implications of optimal monetary policy. The assumption adopted is that monetary authorities set the money supply growth attempting to maximize total welfare.

The monetary authority is assumed to assign equal weight to each agent, so there is  $\lambda$  weight assigned to the group of traders and  $1 - \lambda$  to the group of non-traders. The maximization problem is as follows:

$$Max_{\mu_t} V_t = Max_{\mu_t} \sum_{t=0}^{\infty} \beta^t (\lambda u(c_t^T) + (1 - \lambda) u(c_t^N)) \quad (17)$$

The first order conditions imply that:

$$\lambda \frac{\partial u(c_t^T)}{\partial c_t^T} \frac{\partial c_t^T}{\partial \mu_t} + (1 - \lambda) \frac{\partial u(c_t^N)}{\partial c_t^N} \frac{\partial c_t^N}{\partial \mu_t} = 0 \quad (18)$$

From equations (13) and (14) which determine consumption in equilibrium, we can calculate the derivative of consumption with respect to money growth:

$$\frac{\partial c_t^N}{\partial \mu_t} = - \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \frac{\bar{y}}{(1 + \mu_t)^2}$$

and

$$\frac{\partial c_t^T}{\partial \mu_t} = \frac{1 - \lambda}{\lambda} \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \frac{\bar{y}}{(1 + \mu_t)^2}$$

Note that the consumption of non-traders decreases whenever monetary policy expands while traders consumption increases. This is because expansionary monetary policy increases current prices, affecting negatively all agents. The traders however, receive the monetary transfer and it seems that the positive effect is stronger. Also note, as for the case of a change in  $\varepsilon_{t-1}$ , the increase in traders consumption is  $\frac{1-\lambda}{\lambda}$  higher than the decrease in non-traders consumption. This is because the increase on traders consumption does again depend on the participation rate. The higher the participation rate, the less share of the monetary transfer,  $\tau$  each of the traders receive. An increase in traders consumption is negatively affected by the participation rate.

Substituting the above equations into equation (18) we find that:

$$\frac{\frac{\partial u(c_t^N)}{\partial c_t^N}}{\frac{\partial u(c_t^T)}{\partial c_t^T}} = 1 \quad (19)$$

Optimal monetary policy will attempt to equate the marginal utility of consumption for the two types of agents. Then for any concave utility specification it turns out that optimal monetary policy equates consumption for the two groups:

$$\frac{\frac{1}{c_t^N}}{\frac{1}{c_t^T}} = 1$$

This implies that:

$$\mu_t = \frac{-(\varepsilon_{t-1} - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})}$$

or,

$$1 + \mu_t = \frac{\bar{y}}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \quad (20)$$

For real dividend lower than the mean dividend  $\bar{\varepsilon}$ , optimal monetary policy is expansionary, increasing money supply by the fraction of extra dividends to total output. This is because whenever traders receive low dividend payments,

their consumption decreases. Optimal monetary policy expands and increases traders consumption by distributing them higher transfers. As monetary policy expands, current prices increase, decreasing non-traders consumption. On the other hand, whenever dividends are higher than the mean dividend, offering high consumption to traders, optimal monetary policy contracts, taxing traders and decreasing prices, so traders consumption decreases and non-traders consumption increases.

Following optimal monetary policy, consumption of traders and non traders each period is given bellow:

$$c_t^{N*} = c_t^{T*} = Y_t = \bar{y} + (\varepsilon_t - \bar{\varepsilon})$$

We see that optimal monetary policy shares the risk perfectly between the two groups.

Also note that optimal monetary policy does not depend on the participation rate. This is because the increase in traders consumption is as we saw before,  $\frac{1-\lambda}{\lambda}$  times higher than the decrease in non-traders consumption whenever  $\varepsilon_{t-1}$  increases. Optimal monetary policy reacts on that by decreasing money supply. But this change decreases traders consumption by  $\frac{1-\lambda}{\lambda}$  more than the increase in non-traders consumption. These changes cancel out when optimal monetary policy wishes to equate the consumption of the two groups, so the optimal money growth does not depend on participation rate.

## 4 Stock Price and Inflation Volatility

### 4.1 Stock Price Volatility

In this section we compute the stock price volatility for two cases of policy: optimal monetary policy and constant money supply. We ask the question whether optimal monetary policy in this model takes care of stock price volatility and we find that this is not necessarily true.

From equation (16) we calculate the real stock price,  $\hat{q}_t$ . The recursive solution assuming that the transversality condition holds and log utility function is:

$$\hat{q}_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{c_t^T}{c_{t+j}^T} \frac{p_t}{p_{t+j}} \varepsilon_{t+j-1}, \quad (21)$$

and by substituting prices and trader's consumption given by equations (12) and (14) we find that:

$$\hat{q}_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))(1 + \mu_t)} \frac{(\bar{y} + (\varepsilon_{t+j-1} - \bar{\varepsilon}))(1 + \mu_{t+j})}{(\varepsilon_{t+j-1} - \bar{\varepsilon})(1 + \mu_{t+j}) + \bar{y}(\lambda + \mu_{t+j})} \frac{\varepsilon_{t+j-1}}{\prod_{s=1}^j (1 + \mu_{t+s})}$$

We first compute stock prices under the assumption that monetary policy acts optimally every period. Stock price is given then by the following formula:

$$\hat{q}_t^{opt} = E_t \sum_{j=1}^{\infty} \beta^j \frac{\varepsilon_{t+j-1}}{\bar{y}} \prod_{s=1}^j (\bar{y} + (\varepsilon_{t+s-1} - \bar{\varepsilon}))$$

Assuming that the dividend shocks are iid and by linearizing around their mean value  $\bar{\varepsilon}$  we find that the unconditional variance of the stock price when optimal monetary policy is followed, is given by equation (23):

$$Var(\hat{q}_t^{opt}) = \frac{\beta^2 \sigma_{\varepsilon}^2}{\bar{y}^2 (1 - \beta)^2} [\bar{y}(1 - \beta) + \lambda(\bar{y} - y^T)]^2 \quad (22)$$

where  $\bar{\varepsilon} = (\bar{y} - y^T)\lambda$  as discussed earlier and  $\sigma_{\varepsilon}^2$  is the variance of the dividend shock.

We now compute the stock price and variance for the case of zero money growth, assuming again iid shocks and linearizing around their mean:

$$\hat{q}_t^{\mu=0} = E_t \sum_{j=1}^{\infty} \beta^j \frac{\lambda \bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \frac{\bar{y} + (\varepsilon_{t+j-1} - \bar{\varepsilon})}{\lambda \bar{y} + (\varepsilon_{t+j-1} - \bar{\varepsilon})} \varepsilon_{t+j-1}$$

$$Var(\hat{q}_t^{\mu=0}) = \frac{\beta^2 \sigma_{\varepsilon}^2}{\bar{y}^2 (1 - \beta)^2} [(1 - \lambda)^2 (\bar{y} - y^T)^2 + ((\bar{y} - y^T) + \frac{y^T}{\lambda})^2 (1 - \beta)^2] \quad (23)$$

Comparing equations (23) and (24) we see that it is not obvious which policy produces higher stock price volatility. The result would depend on the parameters' values. For example, leaving the participation rate  $\lambda$  free and for  $\bar{y} = 1$ ,  $y^T = 0.9$ ,  $\beta = 0.9$  and  $\sigma_{\varepsilon} = 0.06$  we see at figure (1) that there is a critical value for the participation rate in the horizontal axes, below which optimal monetary policy produces less volatility of asset prices than the constant money supply policy and above which, optimal monetary policy generates higher volatility.

The analysis implies that optimal monetary policy does not necessarily involve low stock price volatility. In addition, as we see at figure (1), increased participation does not necessarily imply lower stock price volatility either. This observation is contrary to Allen and Gale (1994) who argue that high variability in stock prices is encouraged by low stock market participation.

## 5 Conclusions

In a limited participation model, a new role for monetary policy is explored, arising from the fact that only a part of the population participates in the stock market, being subject to dividend income risk. In such a setting, optimal monetary policy can share the risk between the stock market participants and non-participants, maximizing in this way total welfare. Whenever dividend income is low, monetary authorities acting optimally increase the money transfers they distribute to the financial market participants. Such a response increases

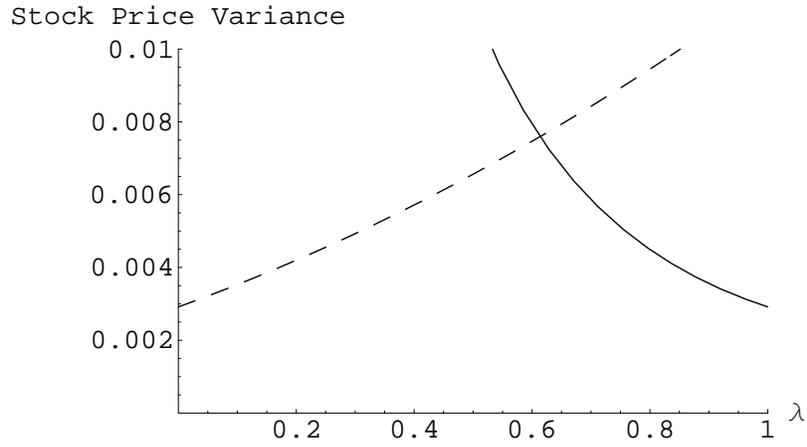


Figure 1: Stock price volatility of optimal monetary policy given by the dashed line and of constant money supply by the solid line.

prices and hurts the non participants. On the other hand, whenever dividend income is high, monetary policy contracts, taxes participants, prices decrease and non participants are benefited. Such a policy though does not necessarily lowers stock price volatility. We provide an example where constant money supply policy produces less stock price volatility than what optimal monetary policy does.

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