An Econometric Workbench for Comparing the Substantive and Dominance Tests in Horizontal Merger Analysis

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Abstract  
This study investigates for the relative accuracy of the simulation approach with respect to the concentration approach for clearing a pro (rejecting an anti) competitive merger. We propose a methodology to compare the predictive and screening power of both tests by implementing them in a hypothetical economy. We generate this economy within a context of multi-product firms competing in differentiated products markets and supplying to consumers purchasing in function of the products’ characteristics and their idiosyncratic tastes. Within this “true” economy an “approximate” economy that fits the pre-merger equilibrium is estimated. We then compare the post-merger predicted unilateral effects as well as the HHI levels and deltas in both the “true” and the “approximate” economies. We find that when a structural econometric approach (for computing unilateral effects) is missed, the concentration approach HHI (the dominance test) is biased upwards, implying a higher risk of incurring on error type I. Moreover, we find that the evaluation of unilateral effects by means of simulation is very much influenced by the market size under scrutiny. In turn, when the share of the outside good is large enough a decision based on the change of HHI would have nothing to do with a decision based on unilateral effects.

Keywords: mergers, antitrust, differentiated product markets, simulation.

JEL codes: L10, L41, K21, D78, C60.
1. Introduction

The identification and quantification of horizontal merger effects is one of the main concerns of antitrust authorities. The reason is that horizontal mergers may enhance and/or strengthen market power which in turn could facilitate firms to increase prices, reduce output choice or quality and deter innovation at the expense of consumers.¹ Since competition authorities generally delineate welfare as consumer surplus, the previous merger effects are perceived as welfare detrimental. A major concern to policy makers is then the availability of effective analytical and quantitative tools that make possible to identify whether and to what extent a merger will induce market power.

Responding to these concerns, a growing body of models of competition that study horizontal merger effects have been proposed in the literature of industrial economics. On its side, the literature of applied industrial economics has also considerably advanced in providing quantitative methods that answer the analytical complexity of the theoretical models. In turn, competition agencies have increased reliance on such academic contributions to merger analysis and updated their modes of investigation and implementation. For instance, in 2004, the European Community Merger Regulation (ECMR), has undergone into a reform that modifies the (previously more restraint) scope of investigation for merger cases. In particular, the criterion of market dominance has been enlarged to include any form of dominance (not only collective dominance) as well as cases that do not involve dominance but still entail anticompetitive concerns. This restructuring of merger regulation takes concrete form in the accompanying Horizontal Merger Guidelines (HMG) which write a list of factors that have to be analyzed in merger cases. Among them, two major issues are: the degree of market concentration and the possible anticompetitive effects of the merger.²

For the analysis of market concentration the HMG stipulate applying the *Herfindahl Hirschman Index* (HHI) and its post-merger change besides the combined market share levels of the undertaking firms. For the assessment of a possible anticompetitive harm, the HMG do not stipulate specific tools but state that a “but for” analysis is required. Such analysis is actually interpreted as a quantitative projection of the (short-term) price change as a result of

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² In addition, countervailing factors to the increase in market power like buyer power and, more importantly, efficiencies gains of the merger are to be analyzed too. Another factor applied to a less extent is the *failing firm defence*. 
the merger. Thus, *merger simulation models* are implicitly called and indeed put into practice more and more frequently.\(^3\)\(^4\) More specifically, whereas the HHI aims at determining market power in terms of concentration, merger simulation aims at predicting market power through the so-called *unilateral effects*, i.e., the increase in prices due to the merger.

In practice, thresholds for market shares and the HHI are systematically applied to assess horizontal merger cases whereas the merger simulation model is mostly implicitly called during the investigation process.\(^5\) The reason for this preference towards the HHI is that, in terms of data, the HHI only requires the firms’ market shares of the market under analysis. The simulation model, in turn, requires further information about the market’s demand, and not necessarily but preferable, about the undertakings’ cost structure. In terms of implementation, the HHI entails a simple calculation whereas the simulation analysis requires structural econometric estimation and therefore further time and technical abilities. As a result, the European competition authorities still hesitate in going forward with the more structural economic approach of the simulation model and mostly rely on the HHI criteria to close horizontal merger cases.

Nevertheless, the simplicity of the HHI does not come without a cost. It can accurately predict the competitive effects of a horizontal merger only if the latter takes place in a market of homogeneous products. In fact, within such a market structure the predictions of the two tests coincide because the measure of market power, the mark-up, resulted from the structural analysis is proportional to the HHI. Yet, how much the two methods converge or diverge according to changes in different factors of more general merger models remains to be systematically investigated. In particular, in markets of differentiated products, the focus on market shares and concentration is problematic and merger simulation is predominantly useful. When the analysis deals with such markets, there is a tradeoff between the simplicity and accuracy of these two market power tests. Indeed, a still open question to analysts and practitioners is whether the two types of measures should be substitutes or complements.

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3 Paragraph 9 of the EC Guidelines for horizontal mergers reads: “In assessing the competitive effects of a merger, the Commission compares the competitive conditions that would result from the notified merger with the conditions that would have prevailed without the merger”. Other methodologies can also be applied for predicting the effects of the merger, for instance, reduced-form regressions or price correlations among competitors attempt to disentangle if “parallelism” in prices is due to competition or to collusion. See Baker and Rubinfeld (1999) for a broad discussion of these alternative methods.

4 The 2001 Volvo/Scania case (COMP/M.1672) is the earliest example of the application of the simulation model; it was performed by Ivaldi and Verboven.

5 This is understood in the HMG recommendation of elaborating a hypothetical post-merger situation to compare it to the pre-merger situation.
In this study, we intend to answer under which circumstances one test should be preferred to the other. That is, we aim at determining under which conditions the various strands of the merger simulation approach, may bring value added, in light of the market power test in markets of differentiated products, with respect to the traditional HHI. In particular, we want to answer to questions like: How accurately can an impediment to competition be estimated by the HHI and simulation quantitative tools? Is simulation analysis a more appropriate tool for accurately predicting anticompetitive merger effects? Does the new substantive test effectively reduces the probability of incurring on errors type I and II (prohibiting a pro-competitive and clearing an anti-competitive merger, respectively) compared to the previous dominance test?

To address these questions, we propose a generic methodology that evaluates the performance of both market power tests and allows to critically asses the advantages and drawbacks of the simulation analysis relative to the concentration test. Our methodology consists in constructing and econometric workbench in which we implement the two tests and subsequently compare their results. We do so, by generating market-level data of a differentiated products’ industry that supplies to heterogeneous consumers. In this true economy a merger takes place and its effects in terms of HHI and price increases (unilateral effects) are measured. Then, with the generated data of the true economy, an approximate economy is estimated to recover the actual equilibrium of the market (as it is usually done in the case-by-case analysis). In this approximate economy, we estimate the unilateral effects with the merger simulation model. Then, by correlation analysis we compare the relationship between the resulted predictions of the two market power tests. Our results confirm that the HHI test tends to be upwards biased compared to the test of unilateral effects. These results are sensitive to the choice of the size of the market, or more precisely to the size of the market share of the outside good (composed of the goods that do not belong to the actual market under scrutiny). In particular, when the size of the outside good is small, a prediction based on the HHI test is roughly in accordance with a prediction based on the unilateral effects test. On the other side, when the size of the outside good is large, a prediction based on the HHI test is different from that based on the unilateral effects test. That is, in the latter case, the increase in prices and in HHI, induced by the merger, are not related to each other. Consequently, a decision based on the dominance test, HHI would have nothing in common with a decision based on the substantive test of unilateral effects.
These results have important implications for merger control. In our example, they suggest that when uniquely relying on the HHI test, authorities risk incurring on type I error, i.e., prohibiting a merger although there is no serious competition concern.

In Section 4.2 we describe the two market power tests, namely the HHI dominance test and the test of unilateral effects through merger simulation. We also describe the required data for the tests to be implemented. In Section 4.3 we expose our methodology and describe the generation of the true economy and estimation of the approximate economy. Section 4.4 summarizes the results of our comparative experiments and Section 4.5 concludes.

2. Market Power Tests in Horizontal Merger Analysis

In analyzing horizontal merger effects, the main question is whether the merger will enhance or strengthen market power, or more specifically, whether it will lead to substantial price increases.\(^6\) In oligopoly theory, the standard measure of a firm’s market power is based on the difference of the price the firm’s charges and its marginal costs, that is, the mark-up. In turn, this price-cost margin is inversely proportional to the elasticity of demand. So, in reality, the analysis of market power largely relies on the estimation of demand elasticities. However, even when elasticities are an important driver to exert market power, oligopoly theory does not suggest ignoring market shares or concentration in merger analysis. On the contrary, market concentration remains important in the competitive effects analysis of mergers. The reason is that market power may be induced by a merger through at least two distinct mechanisms: collusion (or coordinated effects) between otherwise competing firms or, independent action of the merging firms (non-coordinated effects). Within the ECMR the first mechanism is analyzed under the collective dominance concept, while the second refers to the more recently introduced concept of unilateral effects whose analysis does not require dominance. Coordinated effects would result from the merger if the reduction of competitors in the market enables the remaining players to (implicitly) coordinate their behavior to set higher than competitive prices. Unilateral effects would result if the merging firms, facing inelastic demands, can capture ongoing profits from a price increase of the merging substitute product (irrespective of the pricing decisions or actions of their competitors).

\(^6\) Here, we summarize enhancement of market power as post-merger price increases but, the broader concept includes reducing output or quality of goods and services, deterring innovation or influencing other parameters of competition.
Previous to the 2004 ECMR’s reform, merger investigation was focused on the coordinated effects of the merger and tended to rely on the market shares of the undertaking firms. The prohibition criteria were predetermined according to whether the merger would create or strengthen “a dominant position as a result of which effective competition would be significantly impeded”. After having recognized that focusing on coordinated effects, other anticompetitive mergers (below the dominance criteria) could not be covered, the new prohibition criterion of the ECMR now reads that a merger must be blocked if it would “significantly impede effective competition, in particular as a result of the creation or strengthening of a dominant position”. This is referred as the SIEC or substantive test. That is, the SIEC test has been adopted to include not only coordinated effects but also unilateral effects of mergers.

More specifically, the introduction of unilateral effects in the ECMR’s SIEC test replaces the traditional analysis of horizontal mergers systematically leading to coordinated effects. This previous criteria was based on the so-called SCP (structure-conduct-performance) paradigm. This theory states that the structure of the market (i.e., the number of firms) determines the conduct of firms (treated as a black box) that results in performance (prices or profits). The SCP concept was however undermined by both empirical and theoretical research. On the empirical side, cross-sectional studies linking market concentration to prices and margins were seriously challenged in grounds of identification, causality and measurement error concerns. On the theoretical side, the analysis was replaced by the modern theory of industrial economics which does not suggest ignoring market concentration as a factor facilitating market power but which also shows that post-merger market dominance (or relative large market shares) is not a necessary condition to enhance market power.

In practice, when implementing the SIEC, the first step of the analysis involves the definition of the relevant market under scrutiny. A detailed discussion of the advantages and drawbacks of this step of the analysis are out of the scope of this study. Once the relevant market defined, explicit distinction between coordinated and unilateral effects is established. The reason is that competition authorities generally consider that coordinated effects are more likely to result from the merger in industries of homogeneous products and that unilateral effects are more likely to take place in mergers of differentiated product industries. Since it is

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7 Also, the use of accounting profits (usually employed as the dependent variable in the regressions) was seen as a poor indicator of the firm’s performance resulting from the exercise of market power.
8 In some cases, both such practices may be considered in a merger case, which substantially increases the degree of complication of the analysis.
also broadly recognized that horizontal mergers mostly occur in differentiated product markets, a great deal of importance is switched nowadays to the analysis of merger unilateral effects.

More specifically, in implementing the SIEC test, the HMG establish specific thresholds of market shares and HHI when analyzing the concentration factor. In what respects to the potential anticompetitive harm factor: on the one hand, if the concern is coordinated effects, an analysis of the facility of coordination and monitoring among firms besides concentration is required. There are however, no specific empirical tools that could straightforwardly include the large list of factors facilitating collusion. On the other hand, if the concern is unilateral effects it is recommended to realize a quantitative projection of the (short-term) price change as a result of the merger. As explained before, for this analysis, merger simulation is implicitly call and indeed put into practice more and more frequently.

At this point, it is important to remark that the HHI test of dominance is developed on a concept in which coordinated effects are the issue and it can be a poor predictor of harm to competition when the case involves unilateral effects and thus competitors of differentiated products. One reason is the difficulty in defining markets when products are differentiated. Another reason is that oligopoly theory predicts that market concentration does not necessarily determine market power in these markets. In particular, the two approaches may predict different effects and direct towards different decisions (allowing or blocking a particular merger). Thus, investigating whether the two different tests of market power lead to different positions towards the merger and examining their accuracy in the post-merger situation is crucial in a policy perspective. Such investigations are however inherently difficult. They require a comparison between the actual effects induced by the merger decision with the effect that would have prevailed if the contrary decision had been taken. At least two approaches have been proposed in the economic literature to address these questions.

One is to gather historical data on past mergers and to proceed to an ex post evaluation of decisions taken by competition authorities but this is very data demanding. First of all, it is fairly difficult to collect data from past merger cases that would permit to estimate unilateral effects when it has not been done during the case. Second, it is also very difficult to collect data on the industry and merged firms post-merger, but necessary to check for the accuracy of

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9 Those factors are among others: entry barriers, frequent interaction among firms, market transparency, demand growth, business cycles and demand fluctuations, innovation and cost asymmetries among firms. See Ivaldi et al. (2003a) for a review of the factors facilitating collusion.
the different tests. So, if this approach can be applied in a few cases in the best conditions, it cannot be used to draw general conclusions. To our knowledge, analyzing the accuracy of merger simulation in merger decision has been undertaken only twice. Hausman and Leonard (2002) compare the observed change in prices of the toilet paper industry in the US after a merger with the predicted ones by the simulation model. They find that both price increases (observed and predicted by simulation) were reasonably close. Peters (2006) generates predicted post-merger price increases and compares them with the observed price changes of five mergers in the Airline industry of the U.S. The author finds that the observed prices are greater than the predicted by the simulation model and affirms that the difference relies in that the simulation model does not consider the possibility of coordinated effects. To our knowledge, no equivalent study has been undertaken for an EC case.

Another solution is to apply the event-study methodology on merger decisions occurred in the past. Neven and Roller (2002) employed this technique to identify mergers that were considered as pro or anti-competitive by the market but prohibited or cleared by the EC antitrust authority based on the dominance test. In a sample of 100 mergers reviewed from 1990 to 2000 they find that the Commission incurred in error type I (blocking a pro-competitive merger) in 25% of the cases and error type II (allowing an anti-competitive merger) in 46% of the cases. The authors emphasize that (at that time) the scope of the dominance concept and the lack of the efficiency defence in the evaluation of the decision were the sources of such discrepancies. In a continuation of the study Duso, Neven and Roller (2007) enlarge the period (until 2002) and thus the sample of investigation to 167 mergers. They find that the frequency of error type I slightly decreases to 21% and type II error is found in 23% of the cases. The authors conclude that the source of the error relies on the market definition feature of the case and the length of the investigation phases among other institutional and political factors. As the authors recognized however, a strong assumption is needed to perform this test, namely economic profits of the merging (and non-merging firms) firms are perfectly reflected in a one-to-one relationship on the firms’ stock prices. In other

10 The event-study technique consists in computing the abnormal returns due to the merger announcement. It is based on the market model. That is, it runs a regression of the merging firms’ stock returns on a constant and the market returns (or industry index) i.e., \( R_i = \alpha_i + \beta R_m + \epsilon_i \), where \( R_i \) are the actual returns to firm \( i \) at time \( t \) and \( R_m \) are the actual returns to a market portfolio for firm \( i \) at time \( t \). Thus, abnormal returns are obtained by subtracting the predicted merging firms’ returns, obtained from estimations for a period prior to the merger, to the ones observed for a period after the merger, that is, by computing: \( AR_i = R_i - (\hat{\alpha}_i + \hat{\beta}_m R_m) \). The abnormal returns are then summed-up over the length of the selected post-merger period to finally obtain cumulated abnormal returns.
terms, the whole procedure is based on the assumption that financial markets are efficient and that insiders and outsiders (with respect to the merger process) have the same information. However, movements in stock prices are due to a broad list of events that could happen in the stock market. Thus the true merger effect and the effect of the competition authority’s decision might be difficult to isolate.

Moreover, this evidence is not precisely answering our question about the relative accuracy of the two market power tests competition authorities employed in deciding for clearing of prohibiting mergers. The first group of studies evaluates the predictive power of the merger simulation model and the second group of studies evaluates the predictive power of the dominance test. But they do not compare the relative accuracy of the two tests. Thus, this evidence is not answering if the two different tests lead to different positions towards the merger. We consider that, in order to compare the HHI and the merger simulation approaches in terms of predictive and screening power, a specific methodology is needed. This methodology has to be general enough to avoid the drawbacks of the approaches just discussed above. We propose an approach based on the econometrics of differentiated products markets that allows implementing, at the time, both such tests to evaluate their predictive accuracy in detecting market power enhancement due to the merger.

We turn next to describe both market power tests, highlighting their pros and cons, to subsequently implement them in our workbench.

2.1. Market concentration and HHI

The European HMG thresholds for market shares, HHI and its changes in the relevant market are considered as first indicators of the change in competitive conditions due to the merger. In order to applied the thresholds it is necessary to first define the relevant market under scrutiny. By doing so, the boundaries of competition between firms are supposed to be identified and delineated. The HMG recommend delineating the relevant market in terms of demand substitability.\(^\text{11}\) Once the relevant market is defined, market shares for each supplier can be calculated on the basis of their individual product sales or revenues.\(^\text{12}\) If a combined post-merger market share of the merging firms is lower than 25% there is no risk of effective competition whereas one higher than 50% may be evidence of the existence of a dominant

\(^{11}\) See the Commission notice on the definition of the relevant market available at http://eu-ex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX:31997Y1209(01):EN.

\(^{12}\) It has also been proposed to use the profit shares in the market instead of the output shares. See Mariuzzo, Walch and Whelan (2004).
The HHI is calculated as the sum of squares of the individual market shares of all firms in the market. The index involves therefore the number of firms and their dispersion on the market. It ranges from close to zero (in an atomistic market) to 10000 (in a monopolistic market) and it gives greater weight to larger players. That is, if \( J \) is the number of total single-product firms competing in the market under scrutiny and \( s_j \), is the market share of firm (or product) \( j \), then, the HHI is obtained by \( HHI = \sum_{j=1}^{J} s_j^2 \). In turn, market shares \( s_j \), are defined as \( q_j/Q \), where \( q_j \) is the quantity of product \( j \) sold in market \( Q \) and \( \sum_{j=1}^{J} q_j = Q \).

So, the concern in computing the HHI is to define \( Q \), the relevant market. In turn, the HHI criteria state that the merger is unlikely to raise competitive concerns if it takes place in markets with:

- post-merger HHI below 1000.
- post-merger HHI between 1000 and 2000 and \( \Delta HHI \) below 250.
- post-merger HHI above 2000 and a \( \Delta HHI \) of less than 150.\(^\text{14}\)

Two pitfalls are worth mentioning about the merger effects predicting power of the HHI test in the context of differentiated product industries. First, the HHI is constructed under the assumption of homogeneous goods. Indeed, the unique case in which the HHI yields accurate predictions of market power, is the one in which firms compete in quantities with homogeneous products, constant elasticity of demand and symmetric constant marginal costs.\(^\text{15}\) Under markets of differentiated products the HHI tells little about the underlying competitive structure of the market. In these industries, the competitive constraint of firms is determined by the degree of substitutability between the various differentiated competing products. In particular, firms with small market shares may well be able to extract high mark-ups if their product is highly differentiated and firms with large market shares may well be

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\(^\text{13}\) See http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX:52004XC0205(02):EN:NOT

\(^\text{14}\) There are exceptions to these rules. They are:
- a) If a merger involves a potential entrant or a recent entrant with a small market share.
- b) If one or both parties are important innovators in ways not reflected in their market shares.
- c) If there are significant cross-shareholdings among the market participants.
- d) If one of the merging firms is a maverick with a high likelihood of disrupting collusive conduct.
- e) If indications of collusive behaviour are present.
- f) If one of the pre-merger parties has a market share of 50% or more.

\(^\text{15}\) That is, the HHI test is derived from the Cournot model of quantity competition with homogeneous goods. To illustrate this, recall that in this model the mark-up of product \( j \) is expressed as \( p - c_j/p = s_j/\eta \), where \( c_j \) is the constant marginal cost of product \( j \) and \( \eta \) is the price elasticity of aggregate demand. It turns out that if marginal costs are all equal and constant, then the average unit cost is \( c = \sum_{j=1}^{J} s_j c_j \). By multiplying both sides of the equation by \( \sum_{j=1}^{J} s_j \) and summing over all firms, we get \( p - c/p = \sum_{j=1}^{J} s_j^2 / \eta = HHI/\eta \). So, for a fixed elasticity of demand, the mark-up or market power measure is proportional to the HHI.
following intensive competition if consumers perceive their products as close substitutes. In differentiated products industries the higher the degree of substitutability between the merging firms' products, the more likely it is that the merging firms will rise prices significantly. Moreover, market shares are not robust to inappropriate market delineation and thus if wrongly defined they may not reflect the degree of competition. As a result, the HHI is a poor indicator of market power in markets of differentiated products. Here, the essential instrument to analyze market power is the ability to set high prices and this is correctly done only if computing price elasticities. Nevertheless, sometimes the difficulty of obtaining price elasticities returns the analysis to the definition of market shares as indirect evidence of their value but still market shares are not the issue.

The second pitfall of the HHI has to be with its post-merger computation and thus with its corresponding change. In order to compute the post-merger HHI, it is assumed that the post-merger market shares remain the same and are not recalculated according to the post-merger equilibrium (they are summed up in their pre-merger values). That is, the computation of the change in HHI does not take into account the oligopolistic interaction of the market. It ignores that horizontal merger models predict that the merging firms’ market shares will tend to overestimate (underestimate) the unilateral effects of the merger if the firms’ merging products are especially distant (close) substitutes within the relevant markets.

2.2. Merger Simulation of Unilateral Effects

2.2.1. Unilateral effects

The pioneer work of Deneckere and Davidson (1985) show that in markets of differentiated products, and in the context of price competition, a merger always results in higher prices, unless strong synergies are a consequence of the merger. In the unilateral effects models higher prices result even if the industry members are not coordinating their actions, that is, even when collusion is not an option. The main element driving the results of these models is product differentiation. When products are differentiated they are not perfect substitutes and consumers may (not) easily switch to a non-perfect substitute product when facing a price increase. Merging firms producing substitute goods will benefit from raising prices to some degree, because they will recapture some of the customers who switch in favor of what previously was a competing product. In turn, responding to merging firms price increases, outsiders will also find profitable to unilaterally raise prices (since in price competition, prices are strategic complements). The term unilateral arises from the fact that merging and non-merging firms act in their unilateral self interest (without coordinating
among them). As explained above, in the differentiated products industries, a post-merger price increase does not depend on the merged firm being the dominant player in the market, but instead, on the substitutability of the products in question. Since products substitutability is measured through price elasticities, the analysis of unilateral effects of mergers strongly relies on the computation of the own and cross elasticities of the products of the market under scrutiny.

An important point to mention here is that within the analysis of unilateral effects, market definition is not an issue. The economic analysis is the same regardless of whether the case is generating high concentration within a narrow market or concentration will stay low within a broader market. The reason is that, the profitability of a price increase in the unilateral effects model depends on the demand system and factors determining the proximity between the merging products’ firms, and not on drawing lines between products that are inside or outside the relevant market. In contrast, market definition is an issue within the HHI analysis. The index will be much higher if high market shares are involved in a tiny market than if they are low in a broader market.

2.2.2. Merger simulation model

Merger simulation is the formal use of the unilateral effects model to make quantitative predictions of price increases due to the merger. It ought to match the closest possible the critical features of the market under analysis. Merger simulation of unilateral effects proceeds in three main steps. The first step is the specification of the two building blocks of the merger model, namely, demand and supply. The second step consists of the estimation or calibration of the parameters of such functions. The third step uses the estimated parameters (from the second step) to predict the post-merger prices and market shares and compare them to the pre-merger ones. At this step, efficiency gains (i.e., fixed decreases in marginal costs) can be implemented in the simulation exercise. From this comparison, unilateral effects are finally measured. That is, the merger simulation model of unilateral effects computes the difference between the pre-merger and the post-merger price equilibrium.

2.2.2.1. Demand

The challenge of the first block of merger simulation, namely demand, is to deal with markets of differentiated products. Two types of demand models have been proposed in the literature. The one defining consumer preferences on product space or simply demand system,
and the one defining consumer preferences in *characteristic space or discrete choice* demand. A demand based on product space has the inconvenient that the number of parameters grows exponentially with the number of products under analysis. The reason is that the number of these parameters to be estimated is directly linked to the own and cross-price elasticities. For example, if the market is composed of *J* products, a demand system specified in product space requires estimating *J*² parameters: for each product *j*, one own-price elasticity plus *(J − 1)* cross-price elasticities. Some methods have been proposed to limit this number of parameters. Hausman, Leonard and Zona (1994) suggest a multistage budgeting procedure under which products are ranked into distinct levels of expenditure. The procedure consists in assuming that a representative consumer chronologically allocates his income in an *upper, lower and bottom* level of demand. This sequential expenditure of consumers is supposed to capture the differentiation of products. Then, at the bottom level of demand an AIDS (Almost Ideal Demand System) is estimated to obtain cross-price elasticities. The advantage of the method is that it has desirable flexibility properties of elasticities. The disadvantage is however, that the greater the restrictions built, in terms of the chronological allocation of income, the greater the sensitivity of the resulting price predictions. Moreover, the system still remains intractable when the number of products at the bottom level is large.¹⁶

This drawback is avoided by employing the second category of demand, the discrete choice one, which includes the various kinds of logit models. In these models, product differentiation is captured by defining products as bundles of characteristics.¹⁷ For example, the pioneer work of Berry, Levinsohn and Pakes (1995) defines a car by its size, horsepower, number of miles the car can drive per gallon of gasoline, quality of the service network, etc. Nevo (2000) defines the ready-to-eat cereal by its contents of calories, sodium and fiber. Furthermore, the technique enables to introduce in the analysis not only the products’ observed characteristics but also the unobserved characteristics to the analyst. This is important because unobserved characteristics represent factors that are observed by the firms that produce them and perceived by consumers but are not quantifiable by the analyst. Examples of such characteristics are the product’s quality or the effect of advertising on the product. It is clear that such characteristics are a decisive factor for buyers’ choice and firms’

¹⁶Within this demand system context the most flexible specification is the metric approach proposed by Slade and Pinkse (2004). It only constrains demand elasticities to be in function of the “closeness” (or distance) of its competitors in product characteristic space. That is, this specification does not impose any a priori substitution pattern except the assumption that brands that have similar characteristics might be closer substitutes. However, this approach has also the disadvantage of estimating (too) many parameters.

The methodology consists in modeling consumer preferences as a function of products’ characteristics instead of products themselves. This technique gives rise to a model that is easily tractable because the number of parameters to be estimated will be linked to the number of characteristics the product is composed of. The most straightforward version of these models is the multinomial logit. Its simplicity relies on the assumption of the *independence of irrelevant alternatives* (IIA). This property implies that any pair of products, say \((j,l)\), will have the same cross price elasticity with respect to any third product in the market, say \(h\), i.e., \(\eta_{jh} = \eta_{lh}\), where \(\eta_{jh}\) and \(\eta_{lh}\) are the cross price elasticities of products \(j,l\), respectively, with respect to product \(h\). The nested logit model, partially relaxes the IIA property by segmenting consumers’ decision into nests (groups of similar products). It offers more flexible substitution patterns because the nesting procedure implies greater substitution among products *within* nests than *between* (across) nests. The mixed-logit, or random coefficient model, offers the largest flexibility in substitution patterns by introducing the interaction between products’ characteristics and consumers’ characteristics (the random component of the utility). In doing so, substitution patterns entirely depend on the products characteristics and not on products belonging to the same group. That is, consumers will substitute to products of similar characteristics and consumers with similar characteristics will have similar preferences for the differentiated products.

Undeniably, the main advantage of the discrete choice models is their parsimony in the number of parameters to be estimated. This number is independent of the number of products in the market under scrutiny. It is substantially reduced from \(J^2\) to the number of characteristics the product is composed of.\(^{18}\) Thus, one can typically include many more products within the discrete choice approach and still obtain fairly accurate parameter estimates.

Crooke *et al.* (1999) evaluate how different demand functions affect the predictions of the merger simulation model of unilateral effects. They found that merger simulation predicts relatively lower price increases with the logit demand function than with the AIDS function. The reason is that in the logit demand, own price elasticities increase relatively rapidly as prices rise whereas in the AIDS own price elasticities increase relatively slow as prices rise.

\(^{18}\) In a random coefficients model it is twice the number of characteristics indeed, one time for the mean of such characteristic and a second time for the variance.
For its advantages, our study employs the discrete choice logit demand. As we will see below, we model a random coefficients utility function to generate consumer choices in the true economy to ensure larger flexibility on consumer substitution patterns, and we model a multinomial logit demand in our approximated economy for its relative advantages in terms of tractability.

### 2.2.2.2 Supply

The other building block of merger simulation is supply. For the supply side of the market two features are required, an assumption about the nature of competition among firms and an assumption about their cost function. Concerning the nature of competition, the Bertrand model is the most convenient within the differentiated products framework. Indeed, some merger simulation studies have tested the fit of this assumption. For instance, Nevo (2000) and Pinkse and Slade (2004) performed a statistical test on observed and estimated average margins and found that the Bertrand hypothesis could not be rejected. In addition, modeling firms as multiproduct producers is required to ensure realism. This is important in the analysis of merger effects because when a multiproduct firm considers increasing the price of one of its products, it takes into account how much of the lost demand will go to its other products. In fact, not considering the multiproduct aspect (and modeling firms as single-product entities) would lead to a downward bias in the estimate of the firm’s mark-up. The reason is that in the single product context, firms’ mark-ups are only in function of their corresponding own price elasticity. This ignores the fact that a price increase in brand \( j \) will lead to an increase in demand for other goods produced by the same firm. Hausman, Leonard and Zona (1994) found that when assuming single product firms, the mark-up of product \( j \) is 17% lower than the mark-up of the same product if assuming multiproduct firms instead.

Concerning the cost structure, it is common practice to rely on the assumption of constant marginal costs.\(^\text{19}\) The reason for opting to a simple marginal cost function is that in merger analysis, careful attention to the supply side becomes important only when the concern shifts to whether changes on supply (due to efficiencies or remedies) would offset the exercise of market power.

At this point, it is important to insist in that the simulation model undertaken with the discrete choice demand addresses the question of defining the potential size of the market and not the relevant market. In merger simulation, the observation of product’s sales is not enough to compute market shares and it is necessary to introduce the concept of the potential market

\(^{19}\) More flexible cost structures can also be modeled. For an example see Ivaldi and McCullough 2005.
size as distinct from the observed market size. Indeed, the potential market size can be assumed or estimated from the available market-level data. The reason is that the discrete choice demand takes into account that consumers may choose the outside alternative, the composed good which gathers all the products that are not of direct interest for the merger under investigation but that could act as substitutes. Thus, in the Bertrand industry the issue turns into what differentiated products one should include in the model. This is not problematic since the model implies that the prices of the excluded goods remain constant. As the prices of all substitutes for the merging products actually increase as a result of the merger, any exclusion of substitutes biases downwards the price increase effect of the merger. Still, since the increase in prices of most non-merging goods is small, their exclusion does not imply a substantial bias of the estimated unilateral effect of the merger. In contrast, when implementing the HHI test one has to carefully define the relevant market because the predictions of anticompetitive effects strongly rely on such definition.

In what follows we explain how we develop our methodology of comparison of these two predictive merger effects tests, the HHI and the merger simulation model.

3. A Methodology of Comparison

The procedure we implement consists in the generation of a hypothetical economy which serves as a benchmark to simulate horizontal mergers. The unilateral effects and the concentration index can be measured and compared in these hypothetical situations. While this method is very flexible because it allows for a large set of experiments, the conditions under which the hypothetical economy is built are crucial for the accuracy of the approach. That is, the robustness of results depends on the features of the setup itself. In this context, it is obvious that the approach provides an approximation of reality but given that the assumptions we use to generate the hypothetical economy are fairly general, our examples provide accurate comparisons of the market power tests in a differentiated products context.

More specifically, to develop our experiments we construct a workbench consisting on a dataset about a market composed of heterogeneous consumers and multiproduct firms producing differentiated products. We generate this economy by random draws. It represents the true economy in our analysis. Within this true economy we observe the actual market equilibrium in terms of prices and market shares as well as consumer preferences. Then, a merger takes place in this economy. We thus observe post-merger equilibrium prices and market shares. Since we observe pre and post-merger market shares in this true economy, we
can compute the true post-merger $\text{HHI}$, and its corresponding true delta. That is, we consider post-merger observed market shares to accurately calculate the concentration index. We also compute the HHI as it is usually done by the antitrust authorities, i.e., by summing up the pre-merger market shares of the merging firms, we call this index the ex-ante post-merger $\text{HHI}$. To sum up, in this true economy we observe post-merger true HHI and unilateral effects i.e., true increase in prices.

Next, with the data available in the true economy we estimate an approximated economy as it is usually done in merger cases. That is, we assume that we observe product characteristics and cost factors to estimate demand and the pricing decisions of the merging firms. In this approximated economy, we implement the merger simulation model to estimate the post-merger equilibrium in prices and market shares. That is, we estimate the unilateral effects of the merger.

Hence, with the available data we can evaluate the predicting power of the two market power tests, the HHI and the unilateral effects through simulation. We do so, by comparing the true post-merger $\text{HHI}$ and its corresponding change with the wrongly computed ex-ante post-merger $\text{HHI}$, in the true economy. Then, we evaluate the accuracy of the merger simulation model. We do so, by comparing the true and estimated unilateral effects, that is, the true and estimated increase in prices due to the merger. Finally, we evaluate the relative performance of both such tests, the HHI and the merger simulation model of unilateral effects by contrasting their corresponding predictions in terms of market power effects. This procedure is summarized and depicted in Diagram 1 and in the following paragraphs:

(a) A true economy is generated and the initial equilibrium in prices and market shares (or quantities) is obtained. These are the pre-merger data that the competition authority usually observes (in diagram 1 the vector of initial prices of all products is denoted $p_{0}^{t}$). A merger is then proposed and the initial market shares are used to compute the initial $\text{HHI}$ denoted as $\text{HHI}_{0}^{t}$.

(b) In the true economy, we allow the proposed merger to occur. Using the pre-merger market shares the ext-ante post-merger $\text{HHI}$ and its corresponding delta are computed and denoted $\text{HHI}_{0,1}^{t}$ and $\Delta \text{HHI}_{0,1}^{t}$, i.e., $\Delta \text{HHI}_{0,1}^{t} = \text{HHI}_{0}^{t} - \text{HHI}_{0,1}^{t}$. These are the index’s level and change as usually computed by the competition authority.

(c) Still, in the true economy, the post-merger equilibrium in true prices and true market shares is computed. This generates the post-merger data that are not available on real cases (in diagram 1 the vector of post-merger prices is denoted $p_{1}^{t}$).
The true post-merger HHI and its corresponding delta are computed and denoted by $\text{HHI}_t$ and $\Delta\text{HHI}_t$, i.e., $\Delta\text{HHI}_t = \text{HHI}_0 - \text{HHI}_t$. That is, this delta is the true post-merger measure of change in concentration. Note that this change is not observable on real cases.

The unilateral effects in the true economy are observed, i.e., the difference in pre and post-merger prices are computed and denoted $\Delta p^t$.

Using the generated pre-merger data of this economy, the approximate economy is estimated by fitting a demand and supply system. In other words, demand and supply conditions are estimated to fit the observed pre-merger equilibrium.

In this approximated economy the merger is simulated; post-merger prices are obtained and denoted $p^o_t$ and thus estimated unilateral effects, denoted $\Delta p^o_t$. That is, steps (d) and (e) are the usual tasks that the analyst performs by applying the simulation approach to a real case.

Tests comparison: The predictions of the HHI test and the unilateral test are confronted.

Tests accuracy: Observed changes in true prices and true market shares are contrasted with the results predicted by the HHI test and the unilateral effects test.

We now turn into the detailed description of the methodology.

### 3.1. Generation of the true economy

For the implementation of the market power tests we require data about products, firms, and consumers. These data are generated from underlying assumptions about products characteristics, consumer preferences, firms’ cost structure and behavior. Hence, we first generate a set of differentiated products composed of a bundle of characteristics.

Second, we generate a population of consumers whose utility is modeled such that it depends on common and own tastes for the characteristics of the products and on income. That is, we model the most flexible discrete choice demand system, namely the random-coefficients logit, to ensure realistic and accurate substitution patterns. Then, individuals are assumed to make a discrete choice i.e., to select the product among the whole variety of existing products that gives them the greatest utility. For this, a maximization program of utility is built up to derive consumer choice. By summing up consumers’ choices demand in terms of market shares is determined.
Third, firms are modeled as profit-maximizing multiproduct entities that set prices for each of their differentiated products taking into account that their competitors do the same. They are assumed to produce with constant marginal costs which in turn depend on the characteristics of the products. In order to set their maximization program that determines prices, we assume that even if they do not perfectly know consumers tastes, they know the components entering their utility up to a distribution function. So that firms are able to compute their expected demand according to this distribution and to finally decide about prices.

Jointly estimating the solutions of these two maximization programs we obtain the equilibrium of the market in term of prices and quantities (or market shares). In what follows we further describe the generation process of each element of our true economy.

3.1.1. Generation of products

On the supply side of the economy we generate a market composed of five firms indexed by \( f \), where \( f = 1, \ldots, F \) and \( F = 5 \). These firms are symmetric in size and in cost structure. Each firm \( f \) produces a set of 100 differentiated products denoted by \( \Omega_f \). We generate then 500 differentiated products indexed by \( j = 1, \ldots, J \) and \( J = 500 \). Their differentiation relies on the characteristics they are composed of. We assume that some of the product characteristics are observed and some are unobserved (observed by producers and consumers but unobserved by the analyst). The former can be physical characteristics like the horsepower, size and air conditioning of a car, or the calories, sodium and fiber content of the ready-to-eat cereal. The latter can account for the quality of the product, promotional activity or other unquantifiable factors.\(^{20}\) We also create market segmentation by positioning each product \( j \) in one of four categories \( g = 1, \ldots, G \), and \( G = 4 \). Categories are clusters of products of similar characteristics, for example, large and small size would be two distinct categories of cars and high or low content of fiber would be two distinct categories of the ready-to-eat cereal. Each product \( j \) is thus generated as a set of six observed characteristics grouped in vector \( x_j \) where \( x_j \) is composed of \( K = 5 \) observed characteristic and a category \( g \), that is, \( x_j = x_{jk} + x_{jg} \), the sixth characteristic is the unobserved one denoted by \( \xi_j \).

\(^{20}\) The unobserved characteristic is anything observed by consumers and producers but that is not quantifiable by the analyst. Demand shocks could also be represented by unobserved characteristics.
Two of the product characteristics, $x_i$ and $x_4$, are generated as continuous random variables with a normal distribution. The remaining three observed characteristics are generated as discrete random variables with a binomial distribution. That is, $x_k$ is generated such that $x_k \sim N\left(0, \sigma_k^2\right)$ for $k=1,2$ and $x_k \sim U\left(0, \sigma_k^2\right)$ for $k=3,4,5$. The category variable, $x_{jg}$, is generated with a uniform distribution by giving a fraction of the interval $[0,1]$ to each group $g$, i.e., $x_{jg} \sim U\left(0, \sigma_g^2\right)$. Each random variable has either an associated probability function (in the discrete case) or a probability density function (in the continuous case). Finally, the unobserved characteristic, $\xi_j$, is generated as a continuous variable with a normal distribution such that, $\xi_j \sim U\left(0, \sigma_{\xi}^2\right)$. All the variables following a normal distribution are assumed to have a zero mean and their variance is generated by a randomization routine.

### 3.1.2. Generation of individuals utility and choices

For the demand side of the market we generate a population of $M$ heterogeneous individuals and potential consumers indexed by $i$, where $i=1,\ldots,M$ and $M=500,000$. The population $M$ represents therefore the potential size of the market. Individuals are endowed with an income and assumed to have common and idiosyncratic tastes for the attributes of the product $j$. In real databases, examples of individual characteristics that may reveal idiosyncratic tastes are related to demographics, i.e., age, family size, race or education among others. We represent individual $i$’s income by $y_i$ and we cluster his idiosyncratic tastes in the variable $v_i$. Then, the indirect utility of consumer $i$ for purchasing product $j$, denoted hereinafter by $U_{ij}$ is assumed to be a function of the observed and unobserved product $j$’s characteristics $x_j$ and $\xi_j$, the price of the product, $p_j$, individual $i$’s income, $y_i$ and idiosyncratic characteristics $v_i$. That is,

\[
U_{ij}\left(y_i, v_i, x_j, p_j, \xi_j\right),
\]

More specifically consumers’ utility is modeled as a random coefficients function such as:
\[ U_{ij} = \alpha_i \left( y_i - p_j \right) + \sum_k x_{jk} \beta_{ik} + \xi_j + \epsilon_{ij}, \]  

where \( \alpha_i \) is consumers \( i \)'s marginal utility of income and marginal disutility of price, \( \beta_{ik} \) is a \( K \)-dimension vector of individual-specific taste coefficients and \( \epsilon_{ij} \) is a mean-zero stochastic term, specific to both, individual \( i \) and product \( j \) that reflects differences in tastes and/or other non measured factors.

Within the discrete choice approach, the specification of the demand side of the market is completed with the introduction of the outside good. This is a set that encloses all the other alternatives to which the consumer could allocate income. If the outside good is not included in the analysis, market shares cannot be computed with the single observation of the inside goods. That is, the introduction of the outside good is necessary because without it, a price increase of the merging firms’ products would not change the quantities purchased. The reason is that in the absence of the outside good consumers would be forced to choose one of the inside goods and demand would only depend on differences in prices. This would imply that a general price increase would not decrease demands for the inside products and then computing demand elasticities to infer market power would have no sense. Since the outside good represents a composite good, its price and characteristics are not defined, therefore the utility of choosing the outside good resumes to: 

\[ U_{i0} = \alpha_i y_i + \epsilon_{i0} \]  

As \( \alpha_i y_i \) will eventually vanish because it is common to all products, it is natural to normalize the mean utility of the outside good equal to zero, i.e., 

\[ U_{i0} = 0. \]

Individual heterogeneity, \( v_i \), is introduced through the marginal utility of income, \( \alpha_i \), and by decomposing the utility of each of the characteristics of the product, \( \beta_{ik} \), into an idiosyncratic and a common taste parameter. That is:

\[ \beta_{ik} = \beta_k + \sigma_k \tilde{\beta}_{ik}, \]

\[ \alpha_i = \alpha + \bar{\alpha}_i. \]

Thus, the expression that generates individuals’ choices translates into:
This utility function can be rearranged as:

\[ U_{ij} = (\alpha + \tilde{\alpha}_i) y_i + \sum_k x_{jk} \left( \beta_k + \sigma_k \tilde{\beta}_k \right) + \xi_j + \epsilon_{ij}, \]

\[ U_{i0} = (\alpha + \tilde{\alpha}_i) y_i + \epsilon_{i0}. \]  \((4)\)

That is, the utility function can be expressed as the sum of four elements.\(^{21}\) A marginal utility of income, \((\alpha + \tilde{\alpha}_i) y_i\), which will further vanish since it is common to all products. A mean utility, \(\delta_j\), common to all consumers that contains the vector of parameters to be estimated denoted \(\theta = (\beta_k, \alpha)\). A term \(\mu_{ij}\) that accounts for the individual tastes for the products’ characteristics, \(v_i = (\tilde{\beta}_k, \tilde{\alpha}_i)\), that is, \(\mu_{ij}\) is the fraction of heteroskedastic deviations from the mean utility that captures the effects of the random coefficients model. And finally, an error term \(\epsilon_{ij}\), which is assumed to be identically and independently distributed across products and consumers and to follow an extreme value distribution such that its cumulative distribution is \(F(\epsilon) = \exp(-\exp(-\epsilon))\).

This structure of consumers’ characteristics ensures flexible and reliable substitution patterns because it allows for interactions between consumer and product characteristics. This means that it takes into account consumers’ idiosyncratic tastes which in turn implies that price elasticities will depend on consumer real preferences for the products’ attributes and not only on the products’ market shares. For example, this specification would that imply larger families prefer larger cars or richer families are less responsive to price increases.\(^{22}\)

\(^{21}\) Note that if we do not add idiosyncratic tastes for the product characteristics, consumers’ utility is only a function of the common parameters \(\alpha\) and \(\beta_k\) such as: \(U_{ij} = \alpha y_i + \sum_k x_{ij} \beta_k - \alpha p_j + \xi_j + \epsilon_{ij}\).

\(^{22}\) Introducing the interaction of price and the unmeasured variable \(\tilde{\alpha}_i\) may also help to capture sources of wealth that the analyst cannot measure and different degrees of price sensitivity among individuals with the same wealth.
In order to generate consumers’ utility expressed in equation (4.5) we then draw a vector of individuals’ income following a log-normal distribution such that \( \log(y_i) \rightarrow N(\bar{y}, \sigma^2_y) \) with predetermined mean and variance \( \bar{y} \) and \( \sigma^2_y \). The distributions of consumers idiosyncratic tastes, \( v_i = (\tilde{\beta}_i, \tilde{\alpha}_i) \) are drawn as random variables that follow a normal distributions such that \( \tilde{\beta}_i \rightarrow N(0, \sigma^2_k) \) and \( \tilde{\alpha}_i \rightarrow N(0, \sigma^2_a) \) for \( k=1, \ldots, K \), where \( \sigma^2_k \) is the variance of the individual taste for product characteristic \( k \) and \( \sigma^2_a \) is the variance of the random coefficient that accounts for individual wealth and price effects. Accordantly, we construct a corresponding identity covariance matrix such that it is independent of the level of income \( y_i \).

To generate a vector of choices, consumers are assumed to purchase (one unit of) the good that gives them the highest utility. For this, we build up a maximization program for equation (5). Then, for a given vector of prices, we provide initial values to the common parameters \( \theta = (\beta_k, \alpha) \), and to the individual parameters \( \sigma_k \) and \( \sigma_a \) of the random variables involved. Recall that the vector of observed and unobserved characteristics \( x_j \) and \( \xi_j \) was previously generated. That is, given the exogenous variables and the arbitrary vector of prices, consumer \( i \) is assumed to choose product \( j \) if:

\[
U(v_i, p_j, x_j, \xi_j, \epsilon_i; \theta) \geq U(v_l, p_l, x_l, \xi_l, \epsilon_l; \theta),
\]

where \( l \neq j \). Note that since each individual purchases only one good, the number of consumers choosing good \( j \) divided by the size of the market \( M \) gives the market share of good \( j, s_j \).

**3.1.3. Generation of supply**

Firms are assumed to produce differentiated goods at a constant marginal cost (independent of output). We then generate a log-linear constant marginal cost function for each product \( j \) that depends on three distinct components: the observed characteristics of the product, \( x_j \), an exogenous variable that is assumed to affect the cost structure but not demand, \( z_j \), and an unobserved term denoted by \( \omega_j \). The random variables, \( \omega_j \) and \( z_j \), are generated
with a normal distribution such that \( \omega_j \sim N\left(0, \sigma^2_{\omega} \right) \) and \( z_j \sim N\left(0, \sigma^2_z \right) \). Then, marginal costs of producing the differentiated product \( j \), \( mc_j \) is expressed as:

\[
mc_j = \exp\left( \sum_k \gamma_k x_{jk} + \gamma_z z_j + \omega_j \right),
\]

and generated by giving initial values to the parameters of the exogenous variables, \( \gamma_k \) and \( \gamma_z \). Once marginal costs generated, we assume that the \( F \) multiproduct firms compete in prices. In maximizing their profits with respect to their own products’ prices, they take into account their competitors’ strategies as well as their potential demand. Then, the profit function of each firm \( f = 1,...,F \) that produces the set of differentiated products, \( \Omega_f \), at a marginal cost, \( mc_j \), is expressed as:

\[
\Pi_f = \sum_{j \in \Omega_f} \left( p_j - mc_j \right) M s_j \left( p, x, \xi; \theta \right).
\]

Firms are assumed to perfectly know their own marginal cost of product \( j \), \( mc_j \), the potential size of the market \( M \) and an approximation of the demand for their product \( j \). That is, as in Berry Levinsohn and Pakes (1995) we assume that, in order to determine prices, producers have information about the population’s preferences and are able to estimate an expected demand for their product. In particular, we assume that firms perfectly know the components entering consumers’ utility function including their heterogeneous characteristics, i.e., idiosyncratic tastes \( \nu_i \), income \( \gamma_i \) and \( \epsilon_{ij} \), up to their distribution function. Denote the distribution function of consumers characteristics as \( P(\nu, \gamma, \epsilon) \). Then, knowing that there exist a set of consumers with values \( \nu \) that induce the choice of product \( j \), defined as \( A_j \), such that:

\[
A_j = \left\{ (\nu, \gamma, \epsilon) : U \left( \nu, \gamma, x_j, p_j, \xi, \epsilon_j; \theta \right) \geq U \left( \nu, \gamma, x_l, p_l, \xi, \epsilon_l; \theta \right), \; l \neq j \right\},
\]

firm producing good \( j \) will compute its expected demand, by capturing the heterogeneity in the population through \( P(\cdot) \), which in the discrete choice context is the probability of product
\( j \) being chosen and thus the expected market share for product \( j \). Denote this expected market share for product \( j \) as, \( \tilde{s}_j \). It is computed (by firms) as:

\[
\tilde{s}_j = \Pr(\text{consumer } i \text{ choosing product } j | v, x, \xi, p; \theta) = \tilde{s}_j(p, x, \xi; \theta) = \int_{(v, y, \varepsilon) \in A_j} P(dv dy d\varepsilon).
\]

This market share is a function of the characteristics and prices of all the goods competing in the market. According to the specification of the logit preferences, (10) translates into:

\[
\tilde{s}_j(p, x, \xi; \theta) = \int_{\tilde{\beta}_k, \tilde{\alpha}} \frac{\exp \left[ \sum_j x_{kj} (\beta_k + \tilde{\beta}_k) - (\alpha + \tilde{\alpha}) p_j + \xi_j \right]}{1 + \sum_j \exp \left[ \sum_k x_{kj} (\beta_k + \tilde{\beta}_k) - (\alpha + \tilde{\alpha}) p_j + \xi_j \right]} dP(\tilde{\beta}_k, \tilde{\alpha}).
\]

That is, firms integrate the logit probabilities over the random components \( (\tilde{\beta}_k, \tilde{\alpha}) \) which correspond to the idiosyncratic individuals’ characteristics \( (v, y, \varepsilon) \). In order to solve for the multidimensionality of the integral (4.11) we undertake its computation with a simulation technique. We obtain an estimation of \( P \) by carrying out \( ns=100 \) random draws from the distribution of \( P(v, y, \varepsilon) \) and then replace \( P \) by \( P_{ns} \) (the simulated estimator of \( P \)). That is, giving initial values of \( \theta \), the vector of simulated market shares that firms integrate in their maximization problem is:

\[
\bar{s}_j(v, \delta, p, x, \theta, P_{ns}) = \frac{1}{ns} \sum_{i=1}^{ns} f_j(v, \delta, p, x, \theta) .
\]

Since the demand of product \( j \) firm \( f \) faces is the probability of choosing good \( j \) times the number of consumers in the economy, that is, \( Ms_j(p, x, \xi; \theta) \), the profit function of firm \( j \) translates into:
\[ \Pi_f = \sum_{j \in \Omega_f} \left( p_j - mc_j \right) M \tilde{s}_j \left( p, x, \xi; \theta \right). \] (13)

Firms chose then prices of products so as to maximize profits knowing that their competitors do the same. In other words, assuming firms enroll in Bertrand competition the first order conditions are:

\[ \tilde{s}_j \left( p, x, \xi; \theta \right) - \sum_{i \in \Omega_j} \left( p_i - mc_i \right) \frac{\partial \tilde{s}_i \left( p, x, \xi; \theta \right)}{\partial p_j} = 0, \quad l = 1, \ldots, J \] (14)

If \( \Delta \) is a \( J \) by \( J \) matrix whose element \( \left( j, l \right) \) is given by

\[ \Delta_{jl} = \begin{cases} -\frac{\partial \tilde{s}_l}{\partial p_j} & \text{if } j \text{ and } l \text{ are produced by the same firm,} \\ 0 & \text{otherwise.} \end{cases} \] (15)

then, in matrix notation the first order conditions are expressed as:

\[ s \left( p, x, \xi; \theta \right) - \Delta \left( p, x, \xi; \theta \right) \left[ p - mc \right] = 0, \] (16)

or

\[ p = mc + \Delta \left( p, x, \xi; \theta \right)^{-1} s \left( p, x, \xi; \theta \right). \] (17)

With this firms’ optimization program we finally generate the pre-merger prices of all products \( J \). Equation (17) indicates that the price of product \( j \) is equal to the marginal cost of product \( j \) plus a mark-up term which is in turn in function of product’s \( j \) price elasticity (the derivative of the market share of product \( j \) with respect to price). In other words the price-cost margin for each product \( j \) is in function the willingness to pay of consumers for product \( j \). Firms are then confronted to compute the price elasticities of the market shares defined by:
\[ \frac{\delta \tilde{\delta}_j p_l}{\delta p_j \tilde{s}_i}(p, x, \tilde{\xi}; \theta) = \begin{cases} \frac{P_l}{\tilde{s}_j} \int_{\beta_k, \tilde{\alpha}} \tilde{s}_j (1 - \tilde{s}_j) dP(\tilde{\beta}_k, \tilde{\alpha}) & \text{if } j = l \\ \frac{P_l}{\tilde{s}_j} \int_{\beta_k, \tilde{\alpha}} \tilde{s}_j dP(\tilde{\beta}_k, \tilde{\alpha}) & \text{otherwise} \end{cases} \] (18)

The elasticities here refer to the percentage change in the market share of product \( j \) in response to a change in the price of product \( l \), where \( l \in \Omega_f \).

Summing-up the equilibrium of the true economy in terms of market shares and prices is fully characterized by equations (6) and (17).

### 3.1.4. Merger simulation

We assume that, before and after the merger, the merging firms set prices independently. That is, Bertrand competition remains after merger which eliminates the possibility of cooperative behavior. Market size, consumer characteristics, the total number of products \( J \), the characteristics of the products \( x_{jk}, x_{lg}, jk, x_{jg}, jk, \) and marginal costs \( mc_j \) are assumed to remain the same after merger. Then, using the estimated pre-merger marginal cost and demand parameters we predict the post-merger equilibrium prices. We do so by changing firm’s \( f \) ownership set of products, i.e., its \( \Omega_f \). For example, if firm 1 and 2 merge, what previously was the set of production \( \Omega_1 \) and \( \Omega_2 \) post-merger becomes \( \Omega_{1,2} \) for which we solve the new equilibrium post-merger conditions. That is, we take into account that after merger firms internalize the fact of jointly producing substitute products and that the new merged firm modifies its maximization problem internalizing a new joint set of products. The reason is that when increasing the price of good 1, the merger firm anticipates that it can compensate the loss of consumers buying product 1 by a gain of consumers who will switch to product 2. Since products are substitutes, these gains (higher price of 1 and higher demand for 2) outweigh the loss of the lower demand for good 1. The same effect derives for good 2, that is, when increasing the price of product 2, the firm will outweigh the loss of forgoing demand of good 2 by an increase in demand for good 1. Moreover, since in price competition prices are strategic complements, whenever the price of a competitor increases, the other competitors’ response will be to increase prices too. As a result, in the post-merger equilibrium, merging and non-merging firms’ prices are higher.
3.1.5. Data from the true economy

With the previous generated data we observe an economy with available information about pre-merger prices, quantities, market shares, products’ and individuals’ characteristics. Prices are obtained from the maximization program of firms’ profits, equation (17). Market shares are obtained from the maximization program of individual’s utility, equation (5) provided (6) is satisfied. Products’ characteristics and consumers’ idiosyncratic tastes are obtained from random routines. We also observe the size of the potential market and of the actual market (the quantities bought for the 5 firms’ products) for which market shares and initial index of concentration HHI are computed.

We denote the pre-merger data of the true economy as $p_0^t$, $s_0^t$ and $HHI_0^t$, for the vector of pre-merger equilibrium prices and market shares of the $J$ products and the initial HHI, respectively. The subscript $0$ denotes the period prior merger and the superscript $t$ denotes true economy. After the merger simulation of firms 1 and 2 we obtain the equivalent information, i.e., prices, quantities, market shares, HHI and its corresponding delta induced by the merger. We denote these post-merger data by $p_1^t$ and $s_1^t$, for the vector of post-merger equilibrium prices and market shares where the subscript $1$ accounts for the period after merger. In other words, we observe in the true economy, the true unilateral effects of the merger, i.e., the increase in prices, denoted by $\Delta p^t$.

Moreover, by summing up the pre-merger market shares of the merging firms 1 and 2, say, $s_{01}$ and $s_{02}$, we compute the ex-ante post-merger HHI, the $HHI_{0,1}^t$. This corresponds to the index that competition authorities actually compute when facing a merger case. We also compute the true post-merger HHI, the $HHI_1^t$ which takes into account the firms’ post-merger market shares resulting from merging and non-merging firms’ reaction to price competition, i.e., from the unilateral effects.

To summarize, in our generated market, we observed supply side data that include pre and post-merger: product characteristics, prices and market shares of the $J$ products as well as the corresponding HHIs. The consumers (demand) side data include: the potential market size $M$, the actual market size of the market, $\sum_j s_j$, consumers’ income and information on the distributions of variables that account for the idiosyncratic tastes of consumers. Note that

\[ M = \sum_j s_j + s_o, \] where $s_o$ is the market share of the outside good.
whereas prices and market shares will change after merger, product and consumers characteristics are assumed to remain the same post-merger.

At this stage, we have all the necessary data from the true economy to perform a structural econometric analysis of merger unilateral effects through simulation.

3.2. Estimation of the approximate economy

In order to estimate the unilateral effects of the merger taking place in the true economy, we use the required and available data: products’ characteristics, prices and market shares. To estimate demand we employ the discrete choice multilogit model. To derive the pricing equations, we keep the assumption of price competition and constant marginal costs. We then simultaneously estimate our functional forms of demand and supply to obtain estimated pre-merger equilibrium prices and market shares. We next simulate a merger to predict the post-merger equilibrium prices and market shares. That is, here too we use the parameters estimated based on pre-merger conditions to solve for the post-merger conditions.

3.2.1. Demand, supply, pre and post-merger equilibrium

We consider the multinomial logit specification of demand as the suitable benchmark for our approximate economy because it is the most conservative in terms of price increase predictions induced by the merge. The multinomial logit model assumes that each individual $i$’s indirect utility from consuming good $j$ is linear in characteristics and expressed as:

$$U_{ij} = x_j \beta - \alpha p_j + \xi_j + \epsilon_{ij}, \tag{19}$$

where $x_j$ is a vector of observed characteristics, $p_j$ is the price and $\xi_j$ is an unobserved (by the econometrician) characteristic of product $j$. Here, product characteristics are treated as exogenous, although product prices are determined within the model. $\epsilon_{ij}$ represents the distribution of consumer preferences around the mean utility and it is assumed to be identically and independently distributed across both consumers and products, which means

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24 This property has been shown by Crooke et al. (1999). They use Monte Carlo experiments for four demand systems (linear, log-linear, logit and AIDS) and show that the predicted price increase of a merger is, ceteris paribus, highest with the log-linear demand (i.e. constant elasticity) followed by the AIDS and then by the logit demand (the linear having the lowest price increase, however its disadvantages –notably having negative prices– make it unappealing).
that their choices are ruled by the same probability distribution (in this context it accounts for the individual specific deviation from the mean).

Note that with this specification $\alpha$, which represents the marginal disutility of price, and $\beta$, the vector of taste parameters, are assumed to be invariant across consumers. That is, in this model both, observed and unobserved characteristics are assumed to be the same across individuals and then consumers’ heterogeneity enters only through the error term $\varepsilon_j$. The specification can be re-arranged as:

$$ U_{ij} = \delta_j + \varepsilon_{ij}, \quad (20) $$

where $\delta_j = x_j \beta - \alpha p_j + \xi_j$ is the element accounting for the mean utility of product $j$. Then, if $\varepsilon_{ij}$ follows the extreme value distribution such that $\exp(-\exp(-\epsilon))$, in the multinomial logit model, the discrete choice market share function, $s_j$ is derived from the principle that consumer $i$ will purchase the good that gives him the highest utility. Since each individual is defined by a vector of random choices, $\bar{x}_j = [\varepsilon_{i1}, \varepsilon_{i2}, ..., \varepsilon_{iJ}]$, the set of individuals choosing good $j$ will be

$$ I_j = (\bar{x}, \bar{p}, \bar{\xi}, \alpha, \beta) = (\varepsilon_j | U_{ij} \geq U_{il} \forall l \neq j) $$

where $x = (x_1, x_2, ..., x_J)$, $\bar{p} = (p_1, p_2, ..., p_J)$ and $\bar{\xi} = (\xi_1, \xi_2, ..., \xi_J)$. In other words, product $j$ will be chosen if an only if for all $l \geq 0$ and for $l \neq j$, $\Pr(U_{ij} \geq U_{il}) = \Pr(\varepsilon_j - \varepsilon_l \leq \delta_j - \delta_l) = \int dP\varepsilon$. According to the distribution of $\varepsilon_{ij}$ this probability expresses the choice probability of product $j$ as:

$$ s_j = \frac{\exp(x_j \beta - \alpha p_j + \xi_j)}{\sum_{l=1}^{J} \exp(x_l \beta - \alpha p_l + \xi_l)}, \quad \text{for any good } j, \quad (21) $$

and

$$ s_0 = \frac{1}{\sum_{l=1}^{J} \exp(x_l \beta - \alpha p_l + \xi_l)}, \quad \text{for the outside good.} \quad (22) $$
These choice probabilities are measures of the market shares. Applying Berry, Levinhson and Pakes (1995) methodology here too, the ratio of the logarithm of the two market shares gives us the linear demand expression for product $j$ to be estimated as:

$$\ln s_j - \ln s_0 = x_j \beta - \alpha p_j + \xi_j,$$  

(23)

Recall that from the generated market data of the true economy we observe the market shares, $s_j$, $s_0$, the product characteristics, $x_j$ and the price of product $p_j$. Here, the term $\xi_j$ represents the unobserved element of demand. With this demand function own and cross price elasticities are:

$$\eta_{jl} = \frac{\partial s_j p_l}{\partial p_j s_j} = \begin{cases} -\alpha p_j (1-s_j) & \text{if } j = l, \\ \alpha p_j s_l & \text{otherwise.} \end{cases}$$  

(24)

This last expression of elasticities indicates that the ratio between the choice probabilities of $j$ and $l$ is independent of the rest of the choices included in set of products $J$. This is the so called Independence of Irrelevant Alternatives property (IIA). It implies that, conditional on market shares, substitution patterns do not depend on the mean utility generated from the product $\delta_j \equiv x_j \beta - \alpha p_j + \xi_j$, but instead on the market shares. That is, in the setting of differentiated products, the IIA means that the substitution patterns between one product and all alternative products are proportional to their respective market shares. For example, any two goods $j$ and $k$ with the equal market shares, $s_j = s_k$ are constrained to have equal own price elasticity, $\eta_{jj} = \eta_{kk}$ and cross price elasticity with any third good, $\eta_{jl} = \eta_{kl}$, regardless of whether both, $j$ and $k$, are highly differentiated.\textsuperscript{25} This implies that the anticompetitive effects of mergers will indeed depend on the market shares of the firms. Still, the issue is not the definition of the relevant market to compute market shares, as it is for the computation of the HHI. Despite these restrictions, the multi-logit model still provides the most natural default assumption for substitution patterns. In fact, assuming substitutability as a function of the share of the good is generally seen as an appealing assumption even if the IIA property does not define what it means for all goods in the choice set to be equally good.

\textsuperscript{25} This is a consequence of assuming an independently and identically distributed additive error and not of any specific distributional assumption on the errors.
substitutes for each other. Imposing the IIA property assures that all estimated elasticities make sense, i.e., that products known to be substitutes actually have positive estimated cross elasticities of demand.

For the estimation of the approximated economy’s supply side, as previously, we assume that symmetric multiproduct firms compete in prices and that their marginal cost for producing each differentiated product \( j \), \( mc_j \), is linear in product’s characteristics. Then marginal costs are estimated by:

\[
\ln(mc_j) = w_j \gamma + \omega_j,
\]

where \( w_j \) is a vector of product \( j \)'s characteristics, \( \gamma \) is the vector of parameters to be estimated and \( \omega_j \) is the error term that accounts for unobserved costs. Firms are assumed to maximize their profits with respect to prices, taking into account that their competitors do the same, the profit equation is:

\[
\Pi_j = \sum_{j=\Omega_j} \left( p_j - mc_j \right) Ms_j(p),
\]

from which the first order condition results in:

\[
p_j = mc_j + \frac{s_j}{\partial s_j / \partial p_j}.
\]

Again, the pricing equation for product \( j \) is equal to its marginal cost plus a term that is in function of its inverse elasticity of demand, \( \eta_{j\beta} \). Note that \( p_j \) and \( s_j \) are known and \( \partial s_j / \partial \delta_j \) is obtained from the estimation of demand. Then, for the joint estimation of supply and demand, we substitute the term \( \partial s_j / \partial \delta_j \), by \( s_j \left(1-s_j \right) \) from (4.24) to finally obtain a pricing (mark-up) equation expressed as:

\[
p_j = w_j \gamma + \frac{1}{\alpha \left[1 - \frac{1}{1-s_j} \right]} + \omega_j,
\]
where \( w_j \) is observed and \( \alpha \) is estimated from the demand equation.

We account for the endogeneity problem of prices and market shares by estimating the system of demand and pricing equations with the non-linear three-stage least squares estimator (NL3SLS) using instruments previously generated from the data of the true economy. As conventionally (See Berry, 1994), these instruments are created as: the number of products per category, the number of products per firm and per category, the number of products per firm and per each of the discrete variables.

Again, the pre-merger equilibrium in terms of prices and market shares, of the approximate economy, is fully characterized by equations (23) and (28). We denote these pre-merger prices and market shares vectors by \( p_0^a \) and \( s_0^a \), respectively, where the suffix 0 accounts for the pre-merger period and \( a \) for the approximated economy.

### 3.2.2. Merger simulation in the approximated economy

To simulate the merger, we proceed as before. That is, using the estimated pre-merger marginal costs’ and the demand’s parameters we predict the post-merger equilibrium prices and market shares. We denote this information by \( p_1^a \), and \( s_1^a \) where the suffix 1 accounts for the post-merger period and \( a \) for the approximated economy. From these data we can then compute the unilateral effects of the merger in terms of the increase in prices, that is, \( \Delta p^a \).

### 3.3. Evaluation and Comparison of the HHI and Unilateral Effects Tests

We can now precede to the evaluation and comparison of the two market power tests, namely, the dominance test of HHI and the unilateral effects test. For this, we generate the true economy and estimate the approximated economy for two distinct cases: large and small market share of the outside good. Thus, we run our experiments 210 (150) times for cases of the market share of the outside good being large (small) to obtain confidence intervals. Accordingly, the statistics resulted from our experiments are averaged figures.

In each of the experiments we assess the performance of the dominance test by evaluating the difference of the \textit{ex-ante} post-merger HHI and its corresponding change, that is, the \( HHI_{0,1}^i \) and \( \Delta HHI_{0,1}^i \), respectively, with the \textit{true} post-merger HHI, and its corresponding delta, the \( HHI_{1}^i \) and \( \Delta HHI_{1}^i \), respectively. That is, we compare the HHI as
actually computed by the competition authorities with the value it should have been obtained if correctly computed.

Second, we evaluate the predicting power of the substantive test by comparing the estimated merger unilateral effects of the approximated economy, $\Delta p^a$, with the true unilateral effects, i.e., true increase in prices, $\Delta p^i$.

Third, we assess the relative performance of both such tests of merger effects by comparing their predicting results to each other. That is, we analyze if the unilateral effects test, $\Delta p^a$, and the dominance test, $HHI^a_{0,1}$ and $\Delta HHI^a_{0,1}$ are in accordance with each other and with the true merger effects, namely $\Delta p^a$ and $\Delta HHI^i_1$. See Diagram 1 for a recapitulative of the procedure of the evaluation and comparison of the two market power tests.

4. Results of Comparisons

Four main results can be drawn from our statistical analysis. They are gathered in Tables 1 to 4 below. The first one concerns the accuracy of the HHI as a predictive market power test of merger effects. The second one concerns the accuracy of the merger simulation approach to assess the merger unilateral effects. The third and fourth one concern the relative performance of both tests.

As expected the first result reveals that the levels of the post-merger HHI are biased upwards when they are computed ex-ante, that is, before the new post-merger equilibrium is obtained. See Tables 1 and 2. In our experiments, the average pre-merger value of the HHI in the true economy, $HHI^i_0$, is of 2016 (2018) for the case of small (large) market share of the outside good. Its average ex-ante post-merger value, $HHI^i_{0,1}$, is 2809 (2808) for small (large) market share of the outside good. The average true post-merger HHI, that is, the average value of the HHI computed with the post-merger equilibrium market shares, $HHI^i_1$, is 2698 (2726) for small (large) market share of the outside good. The upward bias of the HHI levels is not large but it is significant, it is of 4.14 percent (3.06 percent) for small (large) market share of the outside good. The upwards bias is also revealed in the change of the HHI. In our experiments, the ex-ante average change, $\Delta HHI^a_{0,1}$, is of 793 (789) under small (large) market share for the outside good. In the same order, the average true change, $\Delta HHI^i_1$, is of 682
In both cases, large and small market share of the outside good, the true change in the HHI is lower than the ex-ante post-merger change, that is, \( \Delta HHI'_1 < \Delta HHI'_{0,1} \). This upwards bias is a result of the HHI not taking into account that when products are substitutes and firms do not benefit from any efficiency gains after merger (revealed in lower marginal costs), the merging firms find profitable to increase their prices. In turn, the non-merging firms also raise their prices, but to a smaller extent. It follows that the share of the merging entity is smaller when one takes into account the unilateral effects. Thus, the post-merger HHI computed ex-post is smaller than the post-merger HHI computed ex-ante.

This first result implies that in a market structure like the one of our true economy, namely, five symmetric firms producing equal number of differentiated products, with similar market shares among them, the HHI ex-ante post-merger level and its respective delta would point towards blocking the merger. Hence, when one is not able to compute the post-merger equilibrium (the merger unilateral effects), computations of post merger HHI and change in HHI are biased upward, and this would increase the risk of type I error, that is to say, the risk to prohibit a merger although there are no serious competition concerns.

The second result concerns the accuracy of the unilateral effects test through the merger simulation model. This result is very much related to the size of the market, or more precisely to the size of the market share of the outside good. In the approximated economy, the average estimated change in price, \( \Delta p'_i^e \), is 10.52 percent (2.79 percent) if the market share of the outside good is small (large). In both cases, these figures are higher than the true increase in price, \( \Delta p'_i^t \) which are 1.99 percent (0.80 percent) when the market share for the outside good is small (large). These numbers indicate that the simulation model overestimates the price effect of the merger and that such overestimation is higher when the share of the outside good is defined as small. In particular, in case of a small market share of the outside good the test would suggest to prohibit the merger because of large unilateral effects. In case of a large market share of the outside good, the unilateral merger effects test would take the opposite decision. In other words, the evaluation of unilateral effects by means of the simulation tool is very much influenced by the market size. Not taking into account for the fact that the market could be much larger might strongly bias upwards the measure of unilateral effects. It is important to mention that it would be useful to evaluate to what extent this result depends on the specification of the simulation tool.\(^{26}\) Also, if we would have

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\(^{26}\) For example, the restrictions on substitution patterns that impose the logit model results in higher price increases compared to a more flexible demand system like the nested logit.
accounted for efficiency gains, the results of the merger simulation exercise would have been different, in particular, we would have obtained a lower post-merger price increase, $\Delta p_1^a$. We did not introduce any possible efficiency gains in our merger simulation model because our objective is to compare it with the dominance test. That is, given that in the analysis of the HHI there is no explicit way to trade-off cost reductions we found useless to make use of them in the simulation model.

**TABLE 1**

<table>
<thead>
<tr>
<th>Statistics of the Experiments (Small $s_0$)</th>
<th>Mean</th>
<th>S.D.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1’s market share</td>
<td>$s_1$</td>
<td>19.87</td>
<td>1.69</td>
<td>15.18</td>
</tr>
<tr>
<td>Firm 2’s market share</td>
<td>$s_2$</td>
<td>20.01</td>
<td>1.93</td>
<td>14.42</td>
</tr>
<tr>
<td>Merging entity’s market share</td>
<td>$s_{1,2}$</td>
<td>39.87</td>
<td>2.23</td>
<td>34.51</td>
</tr>
<tr>
<td>Outside option’s market share</td>
<td>$s_0$</td>
<td>0.03</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>Aggregate elasticity</td>
<td>$\eta$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Estimated average change in prices</td>
<td>$\Delta p_1^a$</td>
<td>10.52</td>
<td>1.83</td>
<td>6.42</td>
</tr>
<tr>
<td>True average change in prices</td>
<td>$\Delta p_1^t$</td>
<td>1.99</td>
<td>0.74</td>
<td>0.41</td>
</tr>
<tr>
<td>Bias on change in prices</td>
<td></td>
<td>494.74</td>
<td>236.14</td>
<td>210.10</td>
</tr>
<tr>
<td>Average pre-merger</td>
<td>$HHI_0'$</td>
<td>2016.20</td>
<td>11.33</td>
<td>2000.79</td>
</tr>
<tr>
<td>Average ex-ante post-merger</td>
<td>$HHI_{0,1}'$</td>
<td>2809.56</td>
<td>89.00</td>
<td>2620.73</td>
</tr>
<tr>
<td>Average true post-merger</td>
<td>$HHI_1'$</td>
<td>2698.77</td>
<td>65.67</td>
<td>2579.44</td>
</tr>
<tr>
<td>Bias on HHI levels</td>
<td></td>
<td>4.14</td>
<td>3.58</td>
<td>-5.85</td>
</tr>
<tr>
<td>Average ex-ante change in HHI</td>
<td>$\Delta HHI_{0,1}'$</td>
<td>793.36</td>
<td>89.49</td>
<td>584.50</td>
</tr>
<tr>
<td>Average true change in HHI</td>
<td>$\Delta HHI_1'$</td>
<td>682.57</td>
<td>66.56</td>
<td>549.30</td>
</tr>
<tr>
<td>Bias on change in HHI</td>
<td></td>
<td>16.95</td>
<td>14.74</td>
<td>-19.73</td>
</tr>
</tbody>
</table>

$^{27}$ In this context, the simulation model assumes that reductions in marginal costs are, to some extent, passed through consumers in the form of lower prices.
### Table 2

Statistics of the Experiments (Large $s_0$)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mean</th>
<th>S.D.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1’s market share</td>
<td>$s_1$</td>
<td>19.85</td>
<td>1.84</td>
<td>14.91</td>
</tr>
<tr>
<td>Firm 2’s market share</td>
<td>$s_2$</td>
<td>19.93</td>
<td>1.85</td>
<td>14.60</td>
</tr>
<tr>
<td>Merging entity’s market share</td>
<td>$s_{1,2}$</td>
<td>39.79</td>
<td>2.29</td>
<td>33.41</td>
</tr>
<tr>
<td>Outside option's market share</td>
<td>$s_0$</td>
<td>68.29</td>
<td>1.20</td>
<td>64.65</td>
</tr>
<tr>
<td>Aggregate elasticity</td>
<td>$\eta$</td>
<td>1.54</td>
<td>0.05</td>
<td>1.38</td>
</tr>
<tr>
<td>Estimated average change in prices</td>
<td>$\Delta p^e_1$</td>
<td>2.79</td>
<td>0.48</td>
<td>1.76</td>
</tr>
<tr>
<td>True average change in prices</td>
<td>$\Delta p^t_1$</td>
<td>0.80</td>
<td>0.51</td>
<td>0.01</td>
</tr>
<tr>
<td>Bias on change in prices</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average pre-merger HHI</td>
<td>$HHI^t_0$</td>
<td>2018.59</td>
<td>13.27</td>
<td>2000.75</td>
</tr>
<tr>
<td>Average ex-ante post-merger HHI</td>
<td>$HHI^t_{0,1}$</td>
<td>2808.56</td>
<td>89.54</td>
<td>2608.67</td>
</tr>
<tr>
<td>Average true post-merger HHI</td>
<td>$HHI^t_1$</td>
<td>2726.68</td>
<td>79.42</td>
<td>2574.98</td>
</tr>
<tr>
<td>Bias on HHI levels</td>
<td></td>
<td>3.06</td>
<td>3.77</td>
<td>-4.72</td>
</tr>
<tr>
<td>Average ex-ante change in HHI</td>
<td>$\Delta HHI^t_{0,1}$</td>
<td>789.97</td>
<td>91.35</td>
<td>557.11</td>
</tr>
<tr>
<td>Average true change in HHI</td>
<td>$\Delta HHI^t_1$</td>
<td>708.09</td>
<td>80.86</td>
<td>548.49</td>
</tr>
<tr>
<td>Bias on change in HHI</td>
<td></td>
<td>12.53</td>
<td>15.61</td>
<td>-17.31</td>
</tr>
</tbody>
</table>

As explained before, the HHI is based only on the inside product market shares. When the outside good matters and its size is imperfectly known, the HHI cannot discriminate between situations corresponding to different levels of the outside good’s market share. This has a critical implication: the HHI cannot be a good proxy for measuring market power, i.e., for measuring the ability of firms to raise prices above competitive levels. For instance, when the market share of the outside good is large, meaning that customers are not very captive, the post-merger HHI can be large even though the merging firm has little market power over consumers. This is supported by our third main result which bears on the correlation between the true change in prices, $\Delta p^t_1$, and the estimated change in HHI, $\Delta HHI^t_{0,1}$. In Tables 3 and 4, one observes that there is almost no correlation between these two figures when the market size of the outside good is large. In turn, they are correlated at a thirty-five percent level when the size of the market share of the outside good is small. In other words, when there is a reason to account for an outside good, i.e., when its market share is not negligible, the change in prices and the changes in HHI due to a merger are independent. This last implies that, when the market share of the outside good is important, a decision based on the change in HHI is not related to a decision based on the unilateral effects test.
Finally, our fourth result is related to the relative accuracy of the HHI and the unilateral effects test. The correlation between the true change in prices $\Delta p^t_i$ and the estimated change in price $\Delta p^e_i$ is substantially higher, in both cases (small and large size of the market share of the outside good), than the correlation of the former with the change in concentration, $\Delta HHI_{0,1}^t$. When the market share of the outside good is small (large) the correlation of the true price increase with the estimated one is 0.54 (0.14). This number is much larger than the correlation of the true price increase with the change in the ext-ante HHI which is of 0.35(0.06) when the market share of the outside good is small (large). This last result indicates that the substantive test of anticompetitive harm implemented with the simulation model performs better in capturing the true situation of the market compared to the dominance test of concentration.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta HHI_{0,1}^t$</th>
<th>$\Delta HHI_1^t$</th>
<th>$\Delta p^e_i$</th>
<th>$\Delta p^t_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta HHI_{0,1}^t$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta HHI_1^t$</td>
<td>0.99 (0.00)</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta p^e_i$</td>
<td>0.17 (0.01)</td>
<td>0.15 (0.03)</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$\Delta p^t_i$</td>
<td>0.35 (0.00)</td>
<td>0.34 (0.00)</td>
<td>0.54 (0.00)</td>
<td>1.00</td>
</tr>
</tbody>
</table>

$\Delta HHI_{0,1}^t$, is the average ex-ante change in HHI, $\Delta HHI_1^t$ is the average ex-ante post-merger HHI, $\Delta p^e_i$ and $\Delta p^t_i$ are the estimated and the true average change in prices, respectively.
### Table 4

**Correlation Study on the Experiments (Large $s_0$)**

<table>
<thead>
<tr>
<th></th>
<th>$\Delta\text{HHI}_{0,1}$</th>
<th>$\Delta\text{HHI}_i$</th>
<th>$\Delta\rho_i^*$</th>
<th>$\Delta\rho_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta\text{HHI}_{0,1}$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta\text{HHI}_i$</td>
<td>0.98 (0.00)</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta\rho_i^*$</td>
<td>0.10 (0.14)</td>
<td>0.09 (0.18)</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$\Delta\rho_i$</td>
<td>0.06 (0.39)</td>
<td>0.05 (0.44)</td>
<td>0.14 (0.04)</td>
<td>1.00</td>
</tr>
</tbody>
</table>

$\Delta\text{HHI}_{0,1}$ is the average ex-ante change in HHI, $\Delta\text{HHI}_i$ is the average ex-ante post-merger HHI, $\Delta\rho_i^*$ and $\Delta\rho_i$ are the estimated and the true average change in prices, respectively.

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### 5. Conclusion

The objective of this to compare the accuracy of two different tests of horizontal mergers effects in terms of market power in a context of differentiated products. The first test consists in estimating the *unilateral effects* of a merger through the merger simulation model. That is, it consists in computing the increase in prices facilitated by the higher market power due to the merger. The second test is based on the idea that market power is closely related to *market concentration* and relies on the increase of this market concentration to conjecture the scope of market power. Market concentration is measured by the Herfindahl-Hirschman Index (HHI) which is in turn computed as the sum of the squared market shares of all firms in the market. In order to compare the absolute and relative performance of these two market power tests we put them into practice in a *merger case* and asses the relation of their respective predictions. To do so, we construct and econometric workbench based on a differentiated products industry that supplies to heterogeneous consumers. More specifically, our procedure consists in the generation of a hypothetical economy which serves as a benchmark to simulate hypothetical mergers in which unilateral effects and concentration index are measured and compared.

We find that the HHI test tends to be upwards biased compared to the test of unilateral effects. In addition, our experiments show that the results of the merger simulation model are
sensitive to the choice of the size of the market, or more precisely to the size of the market share of the outside good (composed of the goods that do not belong to the actual market under scrutiny). On the one side, when the size of the outside good is small, a prediction based on the HHI test is roughly in accordance with a prediction based on the unilateral effects test. On the other side, when the size of the outside good is large, a prediction based on the HHI test is different from that based on the unilateral effects test. Consequently, a decision based on the dominance test, the HHI is not related with a decision based on the substantive test of unilateral effects. In our correlation analysis that compares the relationship between the resulted predictions of the two market power tests with the true post-merger effects, we found that while the dominance test is only slightly correlated with the true increase in prices, the unilateral effects test in substantially higher correlated. Thus, the unilateral effects test performs better in predicting merger effects than the index concentration test.

From our results, our conclusions are twofold. On the one side, in terms of implementation and taking into account technical and time constraints, we conclude that the simulation model is a very useful tool to complement the HHI analysis but it is not a substitute for it. The reason is that the higher cost in terms of time, data and technical abilities that the simulation model require prevent it from being an easy substitute of the dominance test. On the other side, in terms of predictive accuracy, we conclude that market concentration remains important in analyzing merger effects since, all things equal; a higher concentration makes unilateral effects more likely. However, the advantage of the merger simulation approach compared to the market concentration approach is that the former can indeed offer an estimate of the price effect induced by the merger. The structural simulation model tells what matters, why, and by how much i.e., it identifies where more evidence is needed and it does not necessarily requires market definition. In our view, the simulation approach is more complete than the HHI approach since the former looks at both demand and supply whereas the latter regards only at the supply side of the market.

Finally, we recognize that further research is still required to improve our workbench of comparisons, for instance, building up the hypothetical economy with: asymmetric firms, a more flexible cost function and estimating the approximate economy with a more flexible demand function like the nested logit one.
References

Appendix

A.1 Presentation of the testing procedure

**Diagram 1: Presentation of the testing procedure**

- Generated data about consumer and product characteristics
- True pre-merger equilibrium
  - Prices and market shares
- Estimated pre-merger equilibrium
  - Prices and market shares
- Merger Simulation
- Evaluation of Unilateral Effects Test
  - \( \Delta \text{HHI}_0 \)
  - \( \Delta \text{HHI}_1 \)
- Evaluation of Dominance Test
  - \( \Delta \text{HHI}_0 \)
  - \( \Delta \text{HHI}_1 \)

Comparisons among the two Tests

- \( \text{HHI}_0 \)
- \( \text{HHI}_1 \)
- \( \rho^a_0 \)
- \( \rho^a_1 \)
A.2 Histograms of Average Changes of HHI and Prices

Graph 1: Histogram of levels of HHI (Small market size for the Outside Good)

Graph 2: Histogram of levels of HHI (Large market size for the Outside Good)
Graph 5: Histogram of Changes in Prices (Small Market Size for the Outside Good)

Graph 6: Histogram of Changes in Prices (Large Market Size for the Outside Good)