PUBLIC GOODS AND TAX COMPETITION IN A TWO-SIDED MARKET

Christos Kotsogiannis† and Konstantinos Serfes‡

This version: May 23, 2008

Abstract: A neglected issue in the tax competition literature is the dependence of equilibrium outcomes on the presence of firms and shoppers (two-sided markets). Making use of a model of vertical and horizontal differentiation within which jurisdictions compete by providing public goods and levying taxes to attract firms and shoppers, this paper characterizes the non-cooperative equilibrium. It also evaluates the welfare implications for the jurisdictions of two popular policies of tax coordination, the imposition of a minimum tax and tax harmonization. It is shown that the interaction of the two markets affects the intensity of tax competition and the degree of optimal vertical differentiation chosen by the competing jurisdictions. Though the non-cooperative equilibrium is—as is typically the case—inefficient, such inefficiency is mitigated by the strength of the interaction in the two markets. A minimum tax policy is shown to be effective when the strength of the interaction is weak and ineffective when it is strong. A tax harmonization policy, however, cannot make the jurisdictions better off.

Keywords: Public goods; Tax competition; Two-Sided Market; Vertical Differentiation.

JEL classification: D72, H77.

Preliminary and incomplete Comments welcome

Acknowledgments: We thank Hideo Konishi and Benjamin Zissimos for comments and suggestions. We are responsible for any errors and omissions.

†Department of Economics, School of Business and Economics, University of Exeter, Streatham Court, Rennes Drive, Exeter EX4 4PU, England, UK. Tel: +44 (0)1392 264500. E-mail: c.kotsogiannis@exeter.ac.uk

‡Department of Economics and International Business, Bennett S. LeBow College of Business, Drexel University, Matheson Hall, 32nd and Market Streets, Philadelphia, PA 19104, USA. E-mail: ks346@drexel.edu. Phone: (215) 895-6816. Fax: (215) 895-6975.
1 Introduction

Moorestown and Haddonfield are two U.S. towns that are in many respects similar: They are located near each other in the greater Philadelphia metropolitan area (south New Jersey), are affluent with very good school districts, and are predominantly residential. But they also differ in one important aspect: Haddonfield’s main street offers a better selection of restaurants and shopping areas, than Moorestown’s main street does. This better selection of shopping areas in Haddonfield has two, and closely related, effects on the economic activity there. Firstly, given that shoppers value, in general, variation in shopping areas, Haddonfield is attracting shoppers from many neighboring towns (including Moorestown). But, secondly, and perhaps more importantly, the higher shopper traffic into Haddonfield results in even more (and higher quality) businesses being attracted by this town, which, in turn, attracts even more shoppers in the Haddonfield area. An explanation for the lack of shopping variety in the Moorestown area that is frequently given is that Moorestown is expensive relative to Haddonfield, in terms of all the fees/taxes, to start-up business (and relative to the public infrastructure being offered in the respective towns).

Though the previous example originates from U.S towns, it is not difficult for one to be convinced that a similar tendency appears when one compares levels of taxation and public good provision across other jurisdictional units such as localities, states, and countries. As another example, that nicely illustrates this point, take the recent—and for most policy observers unfavorable for businesses—change in the corporate income taxation in the UK. Following the announcement of the new UK corporate tax structure a number of multinationals announced their decision to relocate to lower-tax regimes. But, it seems that, an exodus of businesses is unlikely to happen. The reason for this, and arguably a convincing one, is that the benefits of relocation for tax purposes are not evenly spread across—once one accounts for requirements of public goods and market access of—different types of businesses. And these benefits matter when governments decide on the taxation of fiscally footloose firms.

The two previous examples emphasize that, firstly, there is an important interaction between footloose firms and the number of shoppers (consumers) a given jurisdiction is able to attract and, secondly, that such interaction should impinge upon how jurisdictions behave in a fiscal competition game. It is these issues that this paper is concerned with. More specifically, the objective of this paper is to develop a model that is rich enough to capture some of the central features of the incentives of jurisdictions to set taxes and provide public goods but also simple enough to yield sharp insights into the issues.

Though the literature on tax competition is fairly sizeable, it has neglected such link, analyzing the problem either from the firm or the consumer side (see Wilson (1999) for a review). Closer to the issues addressed in this paper is the contribution by Zissimos and Wooders (2008) who, too, analyze how vertical differentiation affects tax competition. Though their analysis provides

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1 This view is nicely expressed in ‘Moorestown works to make business boom,’ (December 1, 2006, Moorestown-NEWSWeekly.com).
a number of insights, it focuses on the firm side of the economy only, abstracting, in particular, from the consumer side.

Incorporating the consumer side in a formal analysis has an interesting implication for equilibrium outcomes. More specifically, the interaction of the two markets in the tax competition game will be shown to give rise to a cross-group externality, with important implications for the equilibrium outcomes. One such implication (as shown in Proposition 1) is that the equilibrium of the fiscal competition between jurisdictions results in asymmetric shares (firm and shopper/consumer) between the jurisdictions. Jurisdictions also choose different public goods investments and so there is vertical differentiation (Proposition 2). The degree of vertical differentiation is also affected by the intensity of interaction (intensity of cross-group externality). For low levels of the cross-group externality the firm side is shared between the two jurisdictions and an increase in the magnitude of the externality intensifies tax competition. On the other hand, when the cross-group externality is strong all firms locate in the high public good jurisdiction, and further increases in the cross-group externality will lead to a higher tax rate levied by the high public good jurisdiction (Proposition 1). The high public good jurisdiction over-invests, relative to the social optimum, when the cross-group externality is strong and under-invests when the cross-group externality is weak. The first best optimal policy always involves all firms locating in the high public good jurisdiction. In that sense, the inefficiency of the non-cooperative outcome is mitigated as the cross-group externality increases and all firms locate in the high public good jurisdiction (Proposition 3). A minimum tax is effective when the cross-group externality is weak and ineffective when the cross-group externality is strong (Proposition 4). Tax harmonization cannot make the jurisdictions better off (Proposition 5).

The rest of the paper is organized as follows. The model is presented in Section 2. Section 3 presents the main analysis. The social planner’s problem is solved in Section 4, whereas Section 5 evaluates two popular policy proposals: the imposition of a minimum tax and tax harmonization across the two jurisdictions. Section 6 compares, briefly, the equilibrium outcomes of the present analysis to those of the one-sided model. Finally, Section 7 concludes.

2 The structure of the model

The model is that of Zissimos and Wooders (2008) but appropriately modified to incorporate the demand side of the economy. It features two jurisdictions $A$ and $B$, indexed by $k$, and two distinct groups of agents: Firms, denoted by $f$, and shoppers, denoted by $s$. Each jurisdiction provides a public good, denoted by $x_k$, and levies taxes to the firms, denoted by $\tau_{fk}$.

**Firms.** Firms are perfectly mobile across jurisdictions and, taking the level of the public good $x_k$ and the tax rate $\tau_{fk}$ in jurisdiction $k = A, B$ as given, make a decision upon which jurisdiction to locate. Each firm can locate in at most one jurisdiction. A shopper that travels in jurisdiction

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2 This paper also relates to a fast-growing literature on two-sided markets with applications that mainly fall into the industrial organization area (see, for example, Caillaud and Julien (2003), Armstrong (2006), Rochet and Tirole (2003 and 2006), Choi (2006), and Liu and Serfes (2007)). Two-sided markets have also been recently the subject of analysis in Public Finance. See Kind, Kothenbuerger and Schjelderup (2008).
k buys one unit of good from each firm that is located in that jurisdiction. The implication of this, as will be discussed in more details shortly, is that an additional firm in a given jurisdiction k exerts a positive externality on all other firms that have located in that jurisdiction.\footnote{This will be, for instance, the case if there is sufficient complementarity between the goods being produced.}

Firms, on the production side, are heterogeneous and characterized by a parameter w which is uniformly distributed on the [0, 1] interval. The mass of firms is one. A firm with parameter w has cost function \( F - wx_k \). The component \( F > 0 \) is independent of the jurisdiction the firm operates in, whereas \( wx_k \) depends on the level of public good provided \( x_k \) by the jurisdiction \( k = A, B \). The interpretation of this latter component is that a firm with a higher \( w \) receives a higher benefit from the public good provided by jurisdiction \( k \). The cost of the public good incurred by jurisdiction \( k \) is increasing and convex in the level of the public good and given by \( x_k^2/2 \). Each firm is able to sell each unit of output at, the exogenously set, price \( p = 1 \).\footnote{One may wonder about the lack, from the specification of the cost function, of a component that captures the cost of producing the demanded level of output. This omission is, however, without loss of generality as long as the marginal cost of production is independent of the level of demand.}

In deciding in which jurisdiction to locate, firms care about the number of shoppers in the jurisdictions, the level of taxes levied, as well as the level of the public good provided by both jurisdictions. Denoting by \( n_{sk}^w \) the number of shoppers firms expect to make their purchases in jurisdiction \( k \), the level of profits for firm \( w \) who locates in jurisdiction \( k \) is given by\footnote{This is not an uncommon assumption in the literature. See, also Kanbur and Keen (1993), and Zissimos and Wooders (2008).}

\[
\pi_{wk} = n_{sk}^w - (F - wx_k) - \tau_f k . 
\]

**Shoppers.** Shoppers in jurisdiction \( k \) take the level of the public good \( x_k \) parametrically. Shoppers, the mass of which is also one, are uniformly distributed on the [0, 1] interval with jurisdiction A located at 0 and jurisdiction B located at 1. Shoppers, as is typically the case (and unlike firms), will be taken to be attached to a particular jurisdiction. Shoppers are located uniformly on the [0, 1] and they incur a disutility if they do not shop in their ideal location, denoted by \( z \). The (strictly positive) per-unit travel cost is denoted by \( t \). The price shoppers pay, for one unit of a good they purchase, is the same across both jurisdictions. Shoppers value product variety and, thus, a shopper that has traveled to a given jurisdiction makes a purchase from all firms located in that jurisdiction. The utility gain for a typical shopper of jurisdiction \( k \) of an additional firm located in that jurisdiction is captured parametrically by \( \alpha \geq 1 \). The implication of this is that if a jurisdiction attracts an additional firm this will lead to more shoppers traveling to that jurisdiction, which in turn will attract even more firms as so on. For wanting a convenient label, \( \alpha \) will be called ‘cross-group externality’.\footnote{Notice that by setting \( p = 1 \) there is no guarantee that all firms’ profits are nonnegative. This, however, is not problematic given that \( p \), being exogenous, can be scaled up so to ensure nonnegativity of profits. Equally, the cost \( F - wx_k \) \( k = A, B \) maybe negative if the public good investment is high enough. The model, however, does not place any constraints on the sign of the fixed cost. It it conceivable, for instance, that the cost is negative and so jurisdictions subsidize the firms (a possibility that can be excluded by scaling up the fixed cost \( F \)).}

\footnote{Notice also that in the limiting case in which \( \alpha = 1 \) the cross-group externality disappears and the model reduces to the benchmark one-sided vertical differentiation model explored by Gabszewicz and Thisse (1979), Shaked and Sutton (1982), and Zissimos and Wooders (2008). We return to this in Section 6.}
Denoting by $n_{fk}$ the number of firms shoppers expect to travel to jurisdiction $k$, a shopper with location characteristic $z$ receives utility

$$U_z = \begin{cases} 
V + (\alpha - 1) n_{fA} - tz, & \text{if she shops in jurisdiction } A \\
V + (\alpha - 1) n_{fB} - t(1 - z), & \text{if she shops in jurisdiction } B .
\end{cases}$$  \hspace{1cm} (2)$$

It is assumed that $V$ is sufficiently high so that all shoppers shop and so the market is covered.

Alternatively, we can assume that each shopper does not buy from all firms, but more firms are valued by consumers because the probability of a better match increases (see also Konishi (2005)). Although we believe that this modeling approach will not change our main insights qualitatively, it will make the analysis considerably more difficult. Our modeling approach captures consumer preference for variety and yet it is simple enough to yield clear and sharp predictions.

The sequence of events in the game is as follows. In stage 1, jurisdictions choose the level of the public good. The strategic interaction between the jurisdictions can take a number of forms. In many contexts, it is natural to conceive of a dominant jurisdiction having a first-mover advantage relative to the other jurisdiction. In others, however, it may be more appealing to conceive of all governments moving simultaneously (as in Zissimos and Wooders (2008)). Interestingly, however, if, in the present two-sided-market context, both jurisdictions move simultaneously a pure strategy equilibrium may not exist.\(^8\) Existence is, however, restored if jurisdictions move sequentially in the public goods provision game. This is the case that attention is confined to here.\(^9\) In stage 2, and given the level of the public good $x_k$, jurisdiction $k = A, B$ chooses taxes $\tau_{fk}$. In stage 3, firms and shoppers, after they observe the strategic choices made by the jurisdictions, choose in which jurisdiction to locate and in which jurisdiction to make their purchases, respectively. The solution concept used in the solution of this game is that of a subgame perfect equilibrium in pure strategies. We turn now to the analysis, starting from the last stage, of the game.

3 Solving for the subgame perfect equilibrium

3.1 Stage 3: Firms’ and shoppers’ location decision

Suppose that $x_A \geq x_B$, and thus the difference in public good provision between the two jurisdictions denoted by $\Delta$, is non-negative that is, $\Delta \equiv x_A - x_B \geq 0$.\(^{10}\)

\(^8\)The proof of this is available upon request. Roger (2007), in a two-sided model where both sides are vertically differentiated, obtains a similar non-existence result.

\(^9\)An alternative to the sequential moves in the public good game is to look for a mixed strategy equilibrium. It is difficult, however, for one to find convincing arguments in favor of mixed strategies in the context of public good investments (or investments in general), as these investments take place very infrequently, require long planning and are primarily irreversible. It seems, thus, more appropriate to assume that jurisdictions move sequentially. Since the jurisdiction that moves first makes (weakly) higher net revenues than the jurisdiction that follows, both jurisdictions will prefer to have the first mover advantage. Though this preemption game is not formally modeled, it is used implicitly as a justification for the approach taken by the present analysis. Tirole (1988, p.297) offers a discussion on this issue.

\(^{10}\)That $\Delta \geq 0$, at this stage, is without loss of generality. Of course, as it will be shown shortly, in the first of the game, it might be the case that $\Delta \leq 0$. We turn to this shortly below.
Firms can freely, but not costlessly, locate in either jurisdiction. Following (1), the marginal firm makes the same profits by locating either in jurisdiction A or jurisdiction B, that is

\[ n_{sA}^e - (F - wx_A) - \tau_{fA} = n_{sB}^e - (F - wx_B) - \tau_{fB}, \]  

which, upon solving for \( w \), identifies the location of the firm, denoted by \( \hat{w} \), and given by

\[ \hat{w} = \frac{(\tau_{fA} - \tau_{fB}) - \left(n_{sA}^e - n_{sB}^e\right)}{\Delta}, \]  

that is indifferent between locating in jurisdiction A or B. Firms with \( w \geq \hat{w} \) locate in jurisdiction A and firms with \( w \leq \hat{w} \) locate in jurisdiction B. A word of clarification is in order here. Since \( \hat{w} \in [0,1] \), it is possible that the public good offered by the two jurisdictions is homogeneous in the sense that \( \Delta \equiv x_A - x_B = 0 \). Firms, in this case, respond to a Bertrand-type game by locating in the jurisdiction that offers them the lowest fiscal burden (relative to the difference in the size of shoppers in the two jurisdictions).\(^{11}\) Thus, when \( \Delta = 0 \), equation (4) takes the value of\(^{12}\)

\[ \hat{w} = \begin{cases} 1, & \text{if } (\tau_{fA} - \tau_{fB}) > \left(n_{sA}^e - n_{sB}^e\right) \\ 0, & \text{if } (\tau_{fA} - \tau_{fB}) < \left(n_{sA}^e - n_{sB}^e\right) \\ 0, & \text{if } (\tau_{fA} - \tau_{fB}) = \left(n_{sA}^e - n_{sB}^e\right). \end{cases} \]  

With \( \hat{w} \) denoting the marginal firm, the fraction of firms that locates in jurisdiction A and in jurisdiction B are given, respectively, by

\[ n_{fA} = 1 - \hat{w} \text{ and } n_{fB} = \hat{w}. \]  

Consumers can shop in either jurisdiction. Following the utility function in (2), the marginal shopper derives the same utility by shopping in either jurisdiction A or in jurisdiction B implying that

\[ V + (\alpha - 1) n_{fA}^e - tz = V + (\alpha - 1) n_{fB}^e - t (1 - z). \]  

Solving equation (7) for \( z \), the location of the marginal shopper, denoted by \( \hat{z} \), is given by

\[ \hat{z} = \frac{(\alpha - 1) \left(n_{fA}^e - n_{fB}^e\right) + t}{2t}. \]  

It is, thus, the case that shoppers with \( z \geq \hat{z} \) locate in jurisdiction B and shoppers with \( z \leq \hat{z} \) locate in jurisdiction A. With \( \hat{z} \in [0,1] \), the fraction of shoppers that locates in jurisdiction A and in jurisdiction B are given, respectively, by

\[ n_{sA} = \hat{z} \text{ and } n_{sB} = 1 - \hat{z}. \]  

\(^{11}\)In the one-sided market, in which \( n_{sA}^e = n_{sB}^e = 0 \), the share of firms in the two jurisdictions is driven by a comparison between \( \tau_{fA} - \tau_{fB} \). In the two-sided market the share of firms is more complex because of the interconnection between shoppers, who value variety (and so a large pool of firms in their locality), and firms, who value a large pool of shoppers (in the place of location).

\(^{12}\)When the difference in taxes is equal to the difference in the number of shoppers, to ensure the existence of an equilibrium, it is assumed that all firms locate in jurisdiction A. Alternatively, we could assume that, in this case, fifty percent of the firms locate in jurisdiction A. To see the possibility of this, suppose that eighty percent of the firms locate in jurisdiction A. This will attract more shoppers to jurisdiction A, which implies that \( n_{sA}^e - n_{sB}^e > 0 \). This implies that in equilibrium it must be \( \tau_{fA} - \tau_{fB} > 0 \). But then jurisdiction A has an incentive to lower its tax infinitesimally in order to attract the remaining twenty percent of the firms.
In equilibrium it must be the case that \( n_{fA} = n_{fA}^e \), \( n_{fB} = n_{fB}^e \), \( n_{sA} = n_{sA}^e \) and \( n_{sB} = n_{sB}^e \).

One of the interesting features of the model, as will be shown shortly below, is that the tax-subgame equilibrium can be an interior, and so both jurisdictions set strictly positive taxes, or a corner one and so one jurisdiction sets a strictly positive tax whereas the other jurisdiction sets a zero one. In the latter equilibrium, as will be shown shortly, the firm market ‘tips’ in the sense that all firms locates in the jurisdiction that sets a strictly positive tax rate.

Notice, for later use, that at an interior equilibrium—solving simultaneously the system of equations (6) and (9)—the firm shares in the two jurisdictions are strictly positive and given by,

\[
\begin{align*}
n_{fA} &= \frac{\Delta t + t(\tau_{fB} - \tau_{fA}) - (\alpha - 1)}{\Delta t - 2(\alpha - 1)} \in (0, 1) \quad \text{and} \quad n_{fB} = \frac{t(\tau_{fA} - \tau_{fB}) - (\alpha - 1)}{\Delta t - 2(\alpha - 1)} \in (0, 1),
\end{align*}
\]

whereas the jurisdictions’ shares for the shoppers are given by

\[
\begin{align*}
n_{sA} &= \frac{2(\alpha - 1)(\tau_{fB} - \tau_{fA} - 1) + \Delta (-1 + t + \alpha)}{2[\Delta t - 2(\alpha - 1)]} \in (0, 1), \quad \text{and} \\
n_{sB} &= \frac{2(\alpha - 1)(\tau_{fA} - \tau_{fB} - 1) + \Delta (1 + t - \alpha)}{2[\Delta t - 2(\alpha - 1)]} \in (0, 1).
\end{align*}
\]

When tipping occurs the derivation of the limits of these shares will be, of course, different (and will be provided in the proofs of the corresponding Propositions).

The analysis now turns to the tax competition stage of the game.

### 3.2 Stage 2: Competition in taxes

The revenue function of jurisdiction \( k = A, B \) is given by\(^{13}\)

\[
R_k(\tau_{fA}, \tau_{fB}) = n_{fA}(\tau_{fA}, \tau_{fB})\tau_{fk}.
\]

Each jurisdiction \( k = A, B \), taking the cost of the public goods in both jurisdictions as given, chooses own destination-based tax \( \tau_{fk} \) to maximize revenues given by (12) holding Nash conjectures against the other jurisdiction. The following Proposition summarizes, for any \( \Delta \equiv x_A - x_B \geq 0, \alpha > 1 \) and \( t > \alpha - 1,^{14} \) the Nash equilibrium of the tax competition subgame.

**Proposition 1 (Tax competition).** The tax subgame equilibrium is characterized by:

i. **Firms taxes**

\[
\begin{align*}
\tau_{fA}^*(x_A, x_B) &= \begin{cases} 
\frac{2\Delta}{3} - \frac{(1 - \alpha)}{t} & \text{if } \Delta \geq \frac{3(\alpha - 1)}{t} \\
\frac{\alpha - 1}{t} & \text{if } \Delta \leq \frac{3(\alpha - 1)}{t}
\end{cases} \\
\tau_{fB}^*(x_A, x_B) &= \begin{cases} 
\frac{\Delta}{3} - \frac{1 - \alpha}{t} & \text{if } \Delta \geq \frac{3(\alpha - 1)}{t} \\
0 & \text{if } \Delta \leq \frac{3(\alpha - 1)}{t}
\end{cases}
\end{align*}
\]

\(^{13}\)For expository convenience, throughout, the dependence of functions on parameters are suppressed.

\(^{14}\)The assumption that \( t > \alpha - 1 \) is common in two-sided market models. Armstrong (2006), for example, in a model where both sides are horizontally differentiated, introduces this assumption to ensure that both sides of the market do not tip in favor of one ‘platform’ (choice of policy). In the model analyzed here, this assumption prevents the shopper side from tipping. We do not expect the results to change much qualitatively if we also allow the shopper side to tip. We will only have to examine more cases.
ii. Jurisdiction firm- and shopper-shares

\[ n^*_{fA}(\cdot) = \begin{cases} 
\frac{2\Delta t - 3(\alpha - 1)}{9|\Delta t - 2(\alpha - 1)|} & \text{if } \Delta \geq \frac{3(\alpha - 1)}{t} \\
1, & \text{if } \Delta \leq \frac{3(\alpha - 1)}{t} 
\end{cases} \quad n^*_{fB}(\cdot) = \begin{cases} 
\frac{\Delta t - 3(\alpha - 1)}{3|\Delta t - 2(\alpha - 1)|} & \text{if } \Delta \geq \frac{3(\alpha - 1)}{t} \\
0, & \text{if } \Delta \leq \frac{3(\alpha - 1)}{t} 
\end{cases} \quad (14) \]

\[ n^*_{sA}(\cdot) = \begin{cases} 
\frac{1 + \frac{\Delta (\alpha - 1)}{6|\Delta t - 2(\alpha - 1)|}}{2t} & \text{if } \Delta \geq \frac{3(\alpha - 1)}{t} \\
\frac{1 + \frac{\alpha - 1}{2t}}{2} & \text{if } \Delta \leq \frac{3(\alpha - 1)}{t} 
\end{cases} \quad n^*_{sB}(\cdot) = \begin{cases} 
\frac{1 - \frac{\Delta (\alpha - 1)}{6|\Delta t - 2(\alpha - 1)|}}{2t} & \text{if } \Delta \geq \frac{3(\alpha - 1)}{t} \\
\frac{1 - \frac{\alpha - 1}{2t}}{2} & \text{if } \Delta \leq \frac{3(\alpha - 1)}{t} 
\end{cases} \quad (15) \]

iii. Revenue functions (excluding the cost of the public good)

\[ R_A(\cdot) = \begin{cases} 
\frac{[2\Delta t - 3(\alpha - 1)]^2}{9|\Delta t - 2(\alpha - 1)|}, & \text{if } \Delta \geq \frac{3(\alpha - 1)}{t} \\
\frac{\alpha - 1}{t}, & \text{if } \Delta \leq \frac{3(\alpha - 1)}{t} \end{cases} \quad R_B(\cdot) = \begin{cases} 
\frac{(\Delta t - 3(\alpha - 1))^2}{9|\Delta t - 2(\alpha - 1)|}, & \text{if } \Delta \geq \frac{3(\alpha - 1)}{t} \\
0, & \text{if } \Delta \leq \frac{3(\alpha - 1)}{t} \end{cases} \quad (16) \]

Proposition 1 reveals that the tax competition equilibrium is affected by the extent of vertical differentiation across the two jurisdictions—given by \( \Delta \equiv x_A - x_B \geq 0 \)—measured relative to the threshold \( 3(\alpha - 1)/t \). In particular, if the degree of vertical differentiation is high, in the sense that \( \Delta \geq 3(\alpha - 1)/t \), then the solution is interior, whereas if the degree of vertical differentiation is low, in the sense that \( \Delta \leq 3(\alpha - 1)/t \), then the solution is a corner one. Notice that, though the precise value of the threshold is, arguably, model specific—and is not of central concern here—it is worth emphasizing, for later use, that it critically depends on the cross-group externality \( \alpha \) and the transportation cost \( t \).

It helps the exposition if the circumstances under which these two equilibria arise are discussed. We turn to this, starting with the interior solution, next.

### 3.2.1 Interior equilibrium of the tax competition subgame (\( \Delta \geq 3(\alpha - 1)/t \))

In an interior equilibrium the high public good jurisdiction, following (13), levies a higher tax, attracts more firms and more shoppers—following (14) and (15)—respectively, and, enjoys more revenues, following (16), than the low public good jurisdiction.

It also helps intuition, at this stage, to consider the comparative statics properties of the tax competition subgame of Proposition 1. Routinely differentiating (the appropriate parts of) (13) and (14) one obtains

\[ \frac{\partial f^*_{fA}(x_A, x_B)}{\partial \Delta} = \frac{2}{3} > 0, \quad \frac{\partial f^*_{fB}(x_A, x_B)}{\partial \Delta} = \frac{1}{3} > 0, \quad (17) \]

\[ \frac{\partial f^*_{fA}(x_A, x_B)}{\partial \alpha} = -\frac{t(\alpha - 1)}{3 [\Delta t - 2(\alpha - 1)]^2} < 0, \quad \frac{\partial f^*_{fB}(x_A, x_B)}{\partial \alpha} = \frac{\partial f^*_{fA}(x_A, x_B)}{\partial \Delta} > 0, \quad (18) \]

\[ \frac{\partial n^*_{fA}(x_A, x_B)}{\partial \Delta} = \frac{\Delta t}{3 [\Delta t - 2(\alpha - 1)]^2} > 0, \quad \frac{\partial n^*_{fB}(x_A, x_B)}{\partial \Delta} = -\frac{\partial n^*_{fA}(x_A, x_B)}{\partial \alpha} < 0. \quad (19) \]

\[ \frac{\partial n^*_{fA}(x_A, x_B)}{\partial \alpha} = \frac{\Delta t}{3 [\Delta t - 2(\alpha - 1)]^2} > 0, \quad \frac{\partial n^*_{fB}(x_A, x_B)}{\partial \alpha} = -\frac{\partial n^*_{fA}(x_A, x_B)}{\partial \alpha} < 0. \quad (20) \]

\[ \text{What this threshold also reveals is that it depends inversely on the cross-group externality } \alpha \text{ and positively on the extent of horizontal differentiation } t. \]
That, following (17), an increase in vertical differentiation makes tax competition less intense, is not new but has previously explored, in this context, by Zissimos and Wooders (2008). What is new, however, here, and perhaps somewhat surprising, is that the equilibrium tax rate of both jurisdictions decrease, following (18), as the indirect externality $\alpha$ increases, for given level of vertical differentiation $\Delta$. It is, thus, the case that tax competition becomes more intense with an increase in the indirect externality. As one would expect, the intuition of such a comparative statics outcome involves both the precise strategic relationship of taxes and their response to a change in the indirect externality. With respect to the former one can straightforwardly verify that taxes are strategic complements (and so both best response functions are upward sloping). With respect to the latter, one can also verify that as the level of the indirect externality increases, for given tax rates, two effects occur. Firstly, following (19), the tax base of the high public good jurisdiction $A$ increases while that of $B$ decreases. The consequence of this is that the firm shares across the two jurisdictions become more asymmetric. This makes tax cuts more costly for the high public good jurisdiction, because its tax base has increased and less costly for the low public good jurisdiction. Secondly, from (10) and (11), as $\alpha$ increases a tax cut attracts more marginal firms (the feedback loop gets stronger). Both effects make tax cuts more profitable for the low public good jurisdiction as the externality increases, but they are opposing for the high public good jurisdiction. It turns out that the second effect is stronger and the high public good jurisdiction also becomes more aggressive in setting its taxes as the externality intensifies. Hence, with a higher $\alpha$ each jurisdiction’s best response function shifts so that it sets a lower tax for any tax levied by the rival. This in turn implies lower equilibrium taxes. (The intuition behind an increase in the cost of traveling $t$ follows similar reasoning and it is, therefore, omitted.)

### 3.2.2 Corner equilibrium of the tax competition subgame ($\Delta \leq 3(\alpha - 1)/t$)

Proposition 1 also emphasizes the possibility of a corner solution (or ‘tipping’), that arises if the degree of vertical differentiation is sufficiently low in the sense that $\Delta \leq 3(\alpha - 1)/t$. To see why this happens suppose that the degree of vertical differentiation $\Delta$ decreases by one unit (keeping $\alpha$ and $t$ fixed). The consequence of such a reduction is that the tax differential between the two jurisdictions $\tau^*_fA - \tau^*_fB$, following from (17), falls by a fraction of one unit (and, in particular, $1/3$). How do firms evaluate this simultaneous reduction in vertical differentiation and tax differential? The benefit of being a high public good jurisdiction is multiplied by the presence of the cross-group externality, implying that the high public good jurisdiction attracts more firms than when the externality is absent ($\alpha = 1$). This can be readily seen from (4) and given that $n_{sA} - n_{sB} > 0$. Consequently, the marginal firm does not value public good provision much, in the sense that its location, $\hat{w}$, is low (in particular, it is less than $1/3$, the location of the marginal firm in a model with no consumer side (one-sided model)). On the other hand, all firms value the reduction in tax differential the same. Therefore, following a reduction in $\Delta$ and

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16 This can be straightforwardly seen from the first order conditions given in Appendix A.1.

17 It can be verified that, for given equilibrium taxes, $\partial n_{fA}(x_A, x_B)/\partial \alpha = \Delta t/[\Delta t - 2(\alpha - 1)]^2 > 0$. Since $n_{fA} + n_{fB} = 1$ it is the case that $\partial n_{fB}(x_A, x_B)/\partial \alpha = -\partial n_{fA}(x_A, x_B)/\partial \alpha < 0$.

18 This is a consequence of the fact that the high public good jurisdiction $A$ responds by reducing its tax by more than jurisdiction $B$. 

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τ_fA − τ_fB, the marginal firm will prefer to switch to the high public good jurisdiction. Following this logic, as the degree of vertical differentiation Δ decreases, jurisdiction B looses more and more firms. What Proposition 1 emphasizes is that when Δ falls below a threshold all firms locate into jurisdiction A. Not all shoppers, however, would locate into jurisdiction A. The reason for this is that for some consumers of jurisdiction B, jurisdiction A is ‘very far away’ and so locating there is costly relative to the benefit they would derive from the existence of more varieties there. Since jurisdiction B has no tax base, it also sets a zero tax rate. Jurisdiction A, on the other hand, having the market tipped in its favor, can afford to set a strictly positive tax.

Tipping in the firm market is also likely to occur with high cross-group externality α. The reason for the dependence of tipping on these measures is intuitive. A high cross-group externality α means that shoppers care more about the number of firms in a given jurisdiction since they offer more product variety. But more shoppers in a given jurisdiction induce more firms wanting to locate to that jurisdiction. If the market has tipped, an increase in α allows jurisdiction A to further increase its tax.

The discussion in the last two subsections shows that, overall, the effect of the cross-group externality α on the tax of the high public good jurisdiction is non-monotonic (and in particular U-shaped): Decreasing when the firm side is shared (in which case α is low) and increasing when tipping has occurred (in which case α is high). The same effect is monotonic for the low public good jurisdiction (decreasing).

We turn now to the first stage of the game in which jurisdictions compete in the provision of the public goods.

3.3 Stage 1: Competition in public goods provision

The government of jurisdiction k = A, B maximizes net revenues. Combining (16) with the cost of the public good one obtains the net revenue function of jurisdiction A and B given, respectively, by

\[ R_A (x_A; x_B) = \begin{cases} \frac{[2\Delta - 3(\alpha - 1)]^2}{9\Delta - 2} - \frac{x_A^2}{2} & \text{if } \Delta \geq \frac{3(\alpha - 1)}{t} \\ \frac{\alpha - 1}{t} - \frac{x_A^2}{2} & \text{if } \Delta \leq \frac{3(\alpha - 1)}{t} \end{cases} \]

\[ R_B (x_B; x_A) = \begin{cases} \frac{[\Delta - 3(\alpha - 1)]^2}{9\Delta - 2} - \frac{x_B^2}{2} & \text{if } \Delta \geq \frac{3(\alpha - 1)}{t} \\ 0 - \frac{x_B^2}{2} & \text{if } \Delta \leq \frac{3(\alpha - 1)}{t} \end{cases} \]

Close inspection of the net revenue functions in (21) and (22) reveals that they are discontinuous at the point where vertical differentiation is zero, Δ = 0. To see this recall from Proposition 1 that with zero vertical differentiation, and as long as the cross-group externality exists (in the sense that α > 1), jurisdiction B sets a zero tax whereas jurisdiction A is able to sustain a strictly

\[19\text{We assume that jurisdiction A is the high public good jurisdiction. If the roles get reversed then we can simply re-label the net revenue functions.}\]
positive one. In this case the market is tipped in favor of jurisdiction A, the jurisdiction with the largest pool of shoppers and, consequently, firms. The presence of the shopper side introduces a second source of differentiation on top of the differentiation in public goods provision. Therefore, even when $\Delta = 0$, jurisdiction A can sustain a strictly positive tax because it attracts more shoppers. With the firm market tipped in favor of jurisdiction A, jurisdiction B’s net revenues, following (22), are given by $R_B(x_A; x_B) = -x_B^2/2$. If now jurisdiction B changes its public good provision marginally by say $dx_B > 0$, then, with $x_A = x_B$ and $\Delta = -dx_B < 0$, jurisdiction B becomes the high public good jurisdiction and the firm side tips in its favor. The net revenue function for jurisdiction B, in this case, jumps to $R_B(x_A; x_B) = (\alpha - 1)/t - (dx_B)^2/2$ and, interestingly, its magnitude depends positively on the level of the cross-group externality $\alpha$ (and the cost of traveling $t$). This increase in revenues reflects the fact that, as noted earlier, the jurisdiction who has attracted all firms increases its tax rates as $\alpha$ increases. Such benefit of course disappears (and so does the discontinuity) if the cross-group externality is not present, $\alpha = 1$. With this in mind we turn now the analysis to the public good provision stage.

It is clear from Proposition 1 that for the first stage public good equilibrium there are 4 possible configurations: Jurisdiction A is the high (low) public good jurisdiction and the market is shared (no tipping), and jurisdiction A is the high (low) public good low jurisdiction and the market is tipped in favor of this jurisdiction. It is assumed, without loss of generality, that jurisdiction A is the Stackelberg leader in the public goods game. The strategy now is to describe the revenue function for jurisdiction B under the four different configurations. There are four different regions (see also figure 1).

**Region 1** ($0 \leq x_B \leq 3(\alpha - 1)/t$): Jurisdiction B is the low public good jurisdiction and the firm side is shared (no tipping). In this case, the net revenue function is decreasing. The reason is that a reduction in $x_B$, given that $\Delta \geq 0$, has three (positive) effects on the revenues of this jurisdiction.

- Firstly, a higher degree of vertical differentiation implies that, following (17), tax competition is less intense and, consequently, jurisdiction B can levy a higher tax (tax competition effect).

- Secondly, the tax base of jurisdiction B, following (19), expands as vertical differentiation increases (market share effect). Despite the fact that jurisdiction B provides a lower level of the public good and its tax goes up, it is still able to attract a higher share of firms. This is because, following (17), the rival jurisdiction raises its tax faster (than jurisdiction B) as the degree of differentiation increases, which, in the presence of cross-group externalities, results in some firms leaving jurisdiction A and locating in the low public good jurisdiction B.

- Thirdly, a reduction in $x_B$ reduces the total cost of the public good (public good cost effect).

**Region 2** ($3(\alpha - 1)/t \leq x_B \leq x_A$): Jurisdiction B is the low public good jurisdiction and the firm side has tipped in favor of jurisdiction A. In this case jurisdiction B has no tax revenues and its net revenue function, which is decreasing in $x_B$, is given by $R_B(x_A, 0) = -x_B^2/2$. 

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Things are, however, a bit more complicated if jurisdiction $B$ is the high public good jurisdiction. Though the precise details of this are not of central concern here (Appendix A.3 formally discusses this), it suffices to say that the shape of the revenue function for jurisdiction $B$ is discontinuous (with its extent being dependent on the level of the externality $\alpha$) and may not be quasi-concave, giving rise to the following two regions.

**Region 3** ($x_A < x_B \leq x_A + 3(\alpha - 1)/t$): Jurisdiction $B$ is the high public good jurisdiction, and so $\Delta \equiv x_A - x_B < 0$, and the firm side has tipped in its favor. In this case the net revenue function jumps to $R_B(x_A, x_B) = (\alpha - 1)/t - x_B^2/2$. After the jump the revenue function decreases up to $x_B = x_A + 3(\alpha - 1)/t$.

**Region 4** ($x_B \geq x_A + 3(\alpha - 1)/t$): Jurisdiction $B$ is the high public good jurisdiction and the firm side is shared (no tipping). In this case the revenue function of jurisdiction $B$ is given by $R_B(x_A, x_B) = [2\Delta t - 3(\alpha - 1)]^2 /[9t [\Delta t - 2(\alpha - 1)]]$ with $\Delta \equiv x_A - x_B < 0$. If $x_A \leq 3(\alpha - 1)/t$, then region 1 vanishes, otherwise all four regions exist.

It is, thus, the case that, for any level of public good of jurisdiction $A$, jurisdiction $B$ (the follower)
maximizes

\[ R_B (x_B; x_A) = \begin{cases} 
\frac{[\Delta t - 3(\alpha - 1)]^2}{9[\Delta t - 2(\alpha - 1)]} - \frac{x_B^2}{2} & \text{if } \Delta \geq \frac{3(\alpha - 1)}{t} \\
\frac{x_B^2}{2} & \text{if } \Delta \leq \frac{3(\alpha - 1)}{t} \\
\frac{\alpha - 1}{t} - \frac{x_B^2}{2} & \text{if } -\Delta \leq \frac{3(\alpha - 1)}{t} \\
\frac{-2\Delta t - 3(\alpha - 1)]^2}{9[\Delta t - 2(\alpha - 1)]} - \frac{x_A^2}{2} & \text{if } -\Delta \geq \frac{3(\alpha - 1)}{t} 
\end{cases} \]  

(23)

by the choice of \( x_B \). Jurisdiction \( A \), in turn, maximizes net revenues, given by (21), choosing \( x_A \) and anticipating the behavior of jurisdiction \( B \) as defined by the maximization problem of jurisdiction \( A \). The next Proposition summarizes the subgame perfect Nash equilibrium (SPNE).

**Proposition 2** The subgame perfect Nash equilibrium is described as follows:

**i. High cross-group externality:** Jurisdiction \( A \) (the first-mover) chooses \( x_A^* = \sqrt{\frac{2(\alpha - 1)}{\Delta}} \) and jurisdiction \( B \) (the follower) chooses \( x_B^* = 0 \). The firm side tips in favor of the jurisdiction \( A \). Both jurisdictions earn zero net revenue. As the cross-group externality decreases, jurisdiction \( A \) lowers its public good investment.

**ii. Medium cross-group externality:** Jurisdiction \( A \) (the first-mover) chooses \( x_A^* \in \left[ \frac{3(\alpha - 1)}{t}, \sqrt{\frac{2(\alpha - 1)}{\Delta}} \right] \) and jurisdiction \( B \) chooses \( x_B^* = 0 \). The firm side is shared between the two jurisdictions (no tipping). Both jurisdictions earn strictly positive net revenue, with the jurisdiction \( A \)'s net revenue being higher than \( B \)'s. As the cross-group externality decreases, the leader lowers its public good investment.

**iii. Low cross-group externality:** Jurisdiction \( A \) (the first-mover) chooses \( x_A^* = \max \{ x_A^*, \bar{x} \} \), where \( x_A^* \in \left[ \frac{3(\alpha - 1)}{t}, \infty \right) \) and \( \bar{x} \in \left( \frac{3(\alpha - 1)}{t}, \sqrt{\frac{2(\alpha - 1)}{\Delta}} \right) \), and the follower chooses \( x_B^* = 0 \). The firm side is shared between the two jurisdictions (no tipping). Both jurisdictions earn strictly positive net revenue, with jurisdiction \( A \)'s net revenue being higher than \( B \)'s.

Proposition 2 shows that in the subgame perfect Nash equilibrium, irrespective of the significance of the cross-group externality, the follower always chooses zero public good provision. This results in a high degree of vertical differentiation. Jurisdiction \( A \), surprisingly, having a first mover advantage does not have any strict advantage over the follower, when the externality is sufficiently high. Both jurisdictions, in this case, earn zero (net) revenues.\(^{21}\) Having described the sub-game perfect equilibrium of the game, naturally, one might wonder how (in)efficient the equilibrium in the public good stage of the game is. To facilitate such comparison one needs to solve for the social planner’s problem and compare the solution with the non-cooperative outcome of Proposition 2. We turn to this next.

\(^{20}\) We still use \( \Delta \equiv x_A - x_B \), but now we allow \( \Delta \) to be negative, that is \( x_B \geq x_A \).

\(^{21}\) It can also be straightforwardly verified that the endogenously chosen degree of vertical differentiation is non-monotonic (U-shaped) in the cross-group externality \( \alpha \).
4 Welfare analysis (first-best)

A social planner maximizes social surplus (denoted by $\omega$ and defined to be equal to the sum of aggregate profits and aggregate utility) given by

$$\omega(x_A, x_B, z, w) =$$

$$\int_0^z [V + (\alpha - 1)(1-w) - tl] \, dl + \int_z^1 [V + (\alpha - 1)w - t(1-\ell)] \, d\ell$$

$$+ \int_0^w [(1-z) - (F - \ell x_B)] \, d\ell - \frac{x_A^2}{2} + \int_0^1 [z - (F - \ell x_A)] \, d\ell - \frac{x_B^2}{2},$$

(24)

by choosing public goods investments $x_A$ and $x_B$, the location of the marginal shopper $z$, and the location of the marginal firm $w$.\(^{22}\) It is easy to verify that (24) reduces to

$$\omega(x_A, x_B, z, w) = V - F + \frac{x_A(1-w^2)}{2} + \frac{x_Bw^2}{2} + \alpha w(1-z) + \alpha z(1-w)$$

$$- \left( \frac{t}{2} + tz^2 - tz \right) - \left( \frac{x_A^2}{2} + \frac{x_B^2}{2} \right),$$

(25)

and so it is the components of (25) that the social planner seeks to maximize. Leaving aside the term $V - F$ which is fixed, the terms $x_A(1-w^2)/2 + x_Bw^2/2$ give the total cost reduction for the firms in both jurisdictions. Such terms are maximized at an asymmetric solution, where only one jurisdiction invests and attracts all the firms, due to the complementarity between public good investment and the number of firms in a given jurisdiction. The terms in $\alpha w(1-z) + \alpha z(1-w)$ give the total cross-group externality which is maximized at an asymmetric solution where one jurisdiction attracts all firms and shoppers and, thus, requires either $w = 0$ and $z = 1$, or $w = 1$ and $z = 0$. This is due to the complementarity between the number of firms and shoppers in a given jurisdiction. The terms $t/2 + tz^2 - tz$ give the total travel cost which, as it is usual in Hotelling-type models, is minimized at a symmetric solution, that is when $z = 1/2$. Finally, the terms in $x_A^2/2 + x_B^2/2$ give the the cost of the public goods, minimized at an asymmetric solution. This is because the cost functions are convex and the social planner would like to spread the investment cost across the two jurisdictions (as long as a jurisdiction has attracted some firms). The first-best solution will balance optimally the above trade-offs.

Maximization of (25) with respect to $x_A$, $x_B$, $z$ and $w$ gives

$$\frac{\partial \omega}{\partial x_A} = \frac{1-w^2}{2} - x_A,$$

(26)

$$\frac{\partial \omega}{\partial x_B} = \frac{w^2}{2} - x_B,$$

(27)

$$\frac{\partial \omega}{\partial z} = \alpha (1-2w) + t - 2tz,$$

(28)

$$\frac{\partial \omega}{\partial w} = \alpha (1-2z) - w(x_A - x_B).$$

(29)

\(^{22}\)Notice that taxes do not enter the specification in (24) since they are simply transfers from firms to jurisdictions.
If an interior maximum exists then it will satisfy (26)-(29) with equality. It can be shown that such solution does not exist.\textsuperscript{23} Denoting optimality by $sp$, the optimal solution is given by $x_{A}^{sp} = 1/2$, $x_{B}^{sp} = 0$, $w^{sp} = 0$, $z^{sp} = 1/2 + \alpha/(2t)$.

It is thus the case that, since $w^{sp} = 0$, the social planner always wants the firm side to be ‘tipped’ in favor of the high public good jurisdiction. Summarizing:

**Proposition 3 (Efficiency).** The social planner’s solution is described as follows:

i. Jurisdiction A is the high public good jurisdiction, with $x_{A}^{sp} = 1/2$, and jurisdiction B makes the minimum public good investment that is, $x_{B}^{sp} = 0$.

ii. All firms are located in the high public good jurisdiction, jurisdiction A, that is, $n_{jA}^{sp} = 0$.

iii. Jurisdiction A (the high public good jurisdiction) attracts more shoppers ($z^{sp} = 1/2 + \alpha/(2t)$) than jurisdiction B, the low public good jurisdiction.

Proposition 2 has emphasized that the low public good jurisdiction always chooses the first-best level of investment. But it is only if the cross-group externality is high enough the market is ‘tipped’ as in the efficient level (Proposition 2i). Even in this case, however, the high public good jurisdiction over-invests as $x_{A}^{*} > x_{A}^{sp} = 1/2$. If the cross-group externality is medium (in the sense of Proposition 2ii) then the market in not tipped, and the low public good jurisdiction attracts some firms. In this case, the high public good jurisdiction selects a level of investment that is above the first-best level from a social point of view, when the externality is high, and under-invests (in the sense that $x_{A}^{*} < x_{A}^{sp} = 1/2$) when the externality is low. The low public good jurisdiction, however, makes the efficient level of investment in the non-cooperative equilibrium. To emphasize:

**Corollary 1** At the non-cooperative equilibrium, the low public good jurisdiction (jurisdiction B) makes the efficient level of investment. The level of the high public good jurisdiction (jurisdiction A) is inefficient. Depending on the level of the cross-group externality, this investment can be above or below the efficient level.

Holding public goods investments fixed, the presence of externalities makes the non-cooperative outcome more efficient. The first-best always involves tipping and a strong externality moves the non-cooperative outcome closer to that outcome.

The proceeding discussion has pointed out that the non-cooperative outcome may be inefficient. The issue that arises, then, is whether tax coordination policies can improve upon this inefficiency. In the next section we look at two policy coordination proposals: The minimum tax and tax harmonization. We start the analysis with the latter policy.

\textsuperscript{23}Solving (26)-(28) and substituting into (29) yields a third degree polynomial in $w$, given by $\partial SS/\partial w = [2tw^{3} - (t - 4\alpha^{2}) w - 2\alpha^{2}]/(2t) = 0$. To solve for the $w$ we calculate the slope of $w$ at the two boundaries. It can be easily shown that $\partial SS(w = 0)/\partial w = -2\alpha^{2}/(2t) < 0$ and $\partial SS(w = 1)/\partial w = (t + 2\alpha^{2})/(2t) > 0$. If there is an interior local maximum in $w$, it must be that there also exists an interior local minimum, since the first order condition is strictly negative at the one boundary and strictly positive at the other. This implies that the polynomial of third degree must have at least two real positive roots. Using the Descartes’ rule of signs it can be shown that the polynomial has only one real positive root (because it changes sign only once). This root must correspond to a local minimum. Based on these arguments, an interior solution does not exist.
5 Tax policies coordination

5.1 A minimum tax

Suppose now that the jurisdictions—for levels of the public goods investments given in Proposition 3—negotiate in order to reach an agreement on a minimum tax. For a minimum tax to influence policy, and so to be binding, it must be that it is strictly higher than the equilibrium tax of jurisdiction $B$ (as it is given in Proposition 1). For an agreement to be feasible the minimum tax, denoted by $\mu$, must be

1. constraint Pareto efficient, in the sense that no jurisdiction $k = A, B$ can increase its tax revenues by deviating from this minimum tax without reducing the tax revenues of the other jurisdiction$^{24}$ and
2. individual rational, in the sense that both jurisdictions earn strictly higher revenues than before the imposition of the minimum tax (strict individual rationality). We so search for the set of all feasible minimum taxes, starting from the tax rates of Proposition 1. To this end, let $M = \{\mu \in \mathbb{R}_+: \text{conditions i) and ii) are satisfied}\}$. There are two cases to consider: The case in which the market is shared and the case in which the market is tipped.

Recall that in the case in which the market is shared (which occurs if $\Delta > 3(\alpha - 1)/t$) the equilibrium taxes, following Proposition 1, are given by $\tau_{fA}^* = 2\Delta/3 - (\alpha - 1)/t$ and $\tau_{fB}^* = \Delta/3 - (\alpha - 1)/t$. Since $\tau_{fA}^* > \tau_{fB}^*$, the minimum tax, denoted by $\tau_{\mu}^\text{min}$, should bind jurisdiction $B$ and should take the form

$$\tau_{\mu}^\text{min}_B = \tau_{fB}^* + \mu = \frac{\Delta}{3} - \frac{(\alpha - 1)}{t} + \mu. \quad (30)$$

Given $\tau_{\mu}^\text{min}_B$, jurisdiction $A$’s best response is to set,$^{25}$

$$\tau_{\mu}^\text{min}_A = 2\Delta/3 - \frac{(\alpha - 1)}{t} + \frac{\mu}{2}. \quad (31)$$

At these taxes, the revenue functions of the two jurisdiction $A$ and $B$—making use of (10) into (12)—are given, respectively, by

$$R_A(\mu) = \frac{[4\Delta t - 6(\alpha - 1) + 3\mu t]^2}{36t[\Delta t - 2(\alpha - 1)]}, \quad (32)$$

$$R_B(\mu) = \frac{[\Delta t - 3(\alpha - 1) + 3\mu t][2\Delta t - 6(\alpha - 1) - 3\mu t]}{18t[\Delta t - 2(\alpha - 1)]}. \quad (33)$$

It can be verified that the revenue function of jurisdiction $A$ in (32) is increasing in $\mu$, whereas the revenue function of jurisdiction $B$ in (33) is concave in $\mu$ and attains a maximum at

$$\mu^* = \frac{\Delta}{6} - \frac{\alpha - 1}{2t}. \quad (34)$$

It is intuitive that any minimum tax that is Pareto constrained cannot be less than the $\mu^*$ defined by (34). For if it is, following the argument regarding the shape of the revenue functions of the two jurisdictions, both jurisdictions can increase their revenues. It, then, follows that if a set of

$^{24}$This implies that that a minimum tax agreement is on the constrained Pareto frontier. It is constrained because after the imposition of the minimum tax the jurisdictions act non-cooperatively.

$^{25}$This follows from the first order condition of jurisdiction $A$, given in Appendix A.
minimum taxes \( M \) exists then it must contain minimum taxes with the property that \( \mu > \mu^* \). To state this differently: Any agreed upon minimum tax must involve \( \mu \geq \mu^* \) as is defined by (34) as this ensures that a minimum tax agreement is on the constrained Pareto frontier.

The minimum tax agreement must also be individually rational for both jurisdictions. For jurisdiction \( A \) things are simple: As its revenues are increasing in \( \mu > 0 \) it is individually rational to accept any minimum tax. This is not, however, true—given the concavity of the revenue function under a minimum tax—for jurisdiction \( B \). Individual rationality for this jurisdiction dictates that the minimum tax agreeable should not yield lower revenues than in the no minimum tax equilibrium of Proposition 1. A simple comparison of the revenue function of jurisdiction \( B \) under no minimum tax, given by Proposition 1, and the revenue function in (33) reveals that this is the case if and only if

\[
\mu < \frac{\Delta}{3} - \frac{\alpha - 1}{t}. \tag{35}
\]

From (34) and (35) it, then, follows that any

\[
\mu \in \left( \frac{\Delta}{6} - \frac{\alpha - 1}{2t}, \frac{\Delta}{3} - \frac{\alpha - 1}{t} \right), \tag{36}
\]

is (strictly) individually rational and lies on the constrained Pareto frontier.

It is thus the case that coordination, in the form of a minimum tax, is beneficial for the governments of the jurisdiction. This is an implication of the fact that minimum tax coordination improves efficiency in the sense that more firms now locate in the high public good jurisdiction \( A \), relative to the no minimum coordination case, thereby resulting in a more asymmetric outcome in terms of firm shares across the two jurisdictions.\(^\text{26}\) This, as emphasized in Proposition 3, improves efficiency as the social planner’s solution involves tipping in favor of the high public good jurisdiction.\(^\text{27}\)

Interestingly, as the cross-group externality increases, a minimum tax agreement is less likely to exist. The reason for this is that—as demonstrated in the previous section—as \( \alpha \) increases the non-cooperative equilibrium, holding the levels of the public goods fixed, becomes more efficient. The implication of this is that there is less room for a Pareto improvement.

Suppose now that the firm side has tipped that is, \( \Delta \leq 3(\alpha - 1)/t \). In this case the equilibrium taxes, following Proposition 1, are given by, \( \tau^*_fA = (\alpha - 1)/t \) and \( \tau^*_fB = 0 \). The minimum tax, in this case, is

\[
\tau^\text{min}B = \tau^*_fB + \mu = \mu. \tag{37}
\]

Given \( \tau^\text{min}B \), jurisdiction \( A \)'s best response is to set,

\[
\tau^\text{min}A = \frac{\alpha - 1}{t} + \mu. \tag{38}
\]

\(^\text{26}\)To see this first notice that, following (30) and (31), minimum tax coordination, for given vertical differentiation \( \Delta > 0 \), implies \( (\tau^\text{min}_A - \tau^\text{min}_B) = \Delta/3 - \mu/2 \) and so a reduction in the tax differential relative to the no minimum tax coordination case. Following (14), then more firms locate in the high public good jurisdiction \( A \).

\(^\text{27}\)This is in the spirit of Kanbur and Keen (1993), and Zissimos and Wooders (2008).
In this case, the high public good jurisdiction responds by increasing its tax by the same amount the low public good jurisdiction increased its tax. There will be no effect on the firm shares and the revenue of the low public good jurisdiction will not increase as a result of a minimum tax imposition. Therefore, there does not exist a (strictly) individually rational agreement that is different from the equilibrium under no minimum tax.

Summarizing the preceding discussion:

**Proposition 4** (Minimum tax). Fix \( \Delta \equiv x_A - x_B \) as it is given in Proposition 3.

i. Firm side is shared and so \( \Delta > 3(\alpha - 1)/t \). Any minimum tax \( \mu > 0 \) in the set \( M = (\Delta/6 - (\alpha - 1)/(2t), \Delta/3 - (\alpha - 1)/(t)) \) is strictly individually rational for both jurisdictions and lies on the constrained Pareto frontier.

ii. Firm side has tipped and so \( \Delta \leq 3(\alpha - 1)/t \). There does not exist a minimum tax that makes both jurisdictions strictly better off and lies on the constrained Pareto frontier, that is \( M = \emptyset \).

### 5.2 Tax harmonization

Suppose now that the two jurisdictions decide to harmonize their taxes in the sense that—starting from the non-cooperative level of taxes of Proposition 1—the high public good jurisdiction, jurisdiction \( A \), will reduces it tax by \( \mu_A \), while jurisdiction \( B \) (the low tax jurisdiction) will raise its tax by \( \mu_B \). The question then is whether such coordination exists.

Suppose that firm market is shared and, thus, taxes, following Proposition 1, take the form

\[
\tau^h_A = \frac{2\Delta}{3} - \frac{(\alpha - 1)}{t} - \mu_A \quad \text{and} \quad \tau^h_B = \frac{\Delta}{3} - \frac{(\alpha - 1)}{t} + \mu_B.
\]

(39)

We have shown that \( R_B \) is decreasing in \( \mu_B \), for any \( \mu_A \) and \( R_A \) is decreasing in \( \mu_A \), for any \( \mu_B \). Therefore, there does not exist \( \mu_A > 0 \) and \( \mu_B > 0 \) that make both jurisdictions strictly better off. The same is true when the firm side has tipped in favor of jurisdiction \( A \). The next Proposition summarizes this result.

**Proposition 5** (Tax harmonization). Starting from the non-cooperative equilibrium of Proposition 1, tax harmonization cannot make the jurisdictions better off.

A recurring feature of the analysis is the importance of the cross-group externality. One, then, might wonder how the equilibrium of the fiscal competition game in a two-sided market compare to equilibrium in the one-sided market? We turn to this next.

### 6 Two-sided versus one-sided market

Notice that if the cross-group externality is equal to one then Proposition 1 reduces to the equilibrium of the one-sided market analyzed by Zissimos and Wooders (2008). In such an equilibrium, the equilibrium is always interior and the equilibrium taxes are given by,

\[
\tau^*_fA(x_A, x_B, \alpha = 1) = 2\Delta/3 \quad \text{and} \quad \tau^*_fB(x_A, x_B, \alpha = 1) = \Delta/3.
\]

(40)
In the non-cooperative equilibrium analyzed by Zissimos and Wooders (2008) the equilibrium levels of the public good are given by \( x_A^* = 4/9 \) and \( x_B^* = 0 \),

\[ (41) \]

whereas the market shares are given by

\[ n_{fA}^* = 2/3 \quad \text{and} \quad n_{fB}^* = 1/3. \]

\[ (42) \]

In such a one-sided market, efficiency, denoted by \( s \), dictates that \( x_A^s = 1/2 \) and \( x_B^s = 0 \),

\[ (43) \]

and

\[ n_{fA}^s = 1 \quad \text{and} \quad n_{fB}^s = 0. \]

\[ (44) \]

It is, thus, the case that in a one-sided model, the high public good jurisdiction always under-invests relative to the social optimum since, following from (41) and (43), \( x_A^* = 4/9 < x_A^s = 1/2 \).

It is also the case that in such a market, following (42) and (44), there is always a divergence between the social optimum and the non-cooperative outcome in terms of firm shares. The (one-sided market) first-best always entails one jurisdiction (the high public good one) attracting all firms, while in the non-cooperative solution both jurisdictions have strictly positive firm shares.

While the social optimum in the two-sided market (of Proposition 3) is identical to the social optimum of the one-sided market,\(^{29}\) the non-cooperative equilibrium explored here distinctively differs in terms of \( i \) the type of equilibrium (interior or corner), \( ii \) the extend of inefficiency of public good supply (for the high public good jurisdiction) relative to the social optimum, and \( iii \) the level of taxation.

In a two-sided market there is the possibility of over-investment. To see this take, for instance, the case of Proposition 3\( i \). In this case the equilibrium choice of jurisdiction \( A \) is given by \( x_A^* = \sqrt{2(\alpha - 1)} / \sqrt{t} \) which is strictly greater than the equilibrium level of public good in the one-sided market. One can easily verify that this is also the case even if the externality is sufficiently high (as \( x_A^* \in \left( 3(\alpha - 1) / t, \sqrt{2(\alpha - 1)} / \sqrt{t} \right) \) is strictly greater than \( x_A^* = 4/9 \)). Some ambiguity regarding the sign arises if the externality is relatively low (discuss this).

When tipping occurs, equilibrium taxes are more asymmetric across the two jurisdictions than in a one-sided model. The high public good jurisdiction levies a higher tax (when \( \alpha \) is high) than the one-sided tax, \( \tau_{fA}^* = (\alpha - 1) / t > 2\Delta / 3 = 8/27 \) (since \( \Delta = 4/9 \)). The low public good jurisdiction levies a lower tax than the one-sided tax, \( \tau_{fB}^* = 0 < \Delta / 3 = 4/27 \). When the firm side is shared, equilibrium taxes for both jurisdictions are lower than in the benchmark one-sided model.

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\(^{28}\)Note that even if jurisdictions move sequentially when they choose public goods investments, in the one-sided model, the equilibrium does not change. Proof is available upon request.

\(^{29}\)With the additional, of course, characterization for the shoppers’ side.
7 Concluding remarks

A neglected issue in the tax competition literature is the dependence of equilibrium outcomes on the presence of firms and shoppers (two-sided markets). Making use of a model of vertical and horizontal differentiation within which jurisdictions compete, by providing public goods and levying taxes to attract firms and shoppers, the paper has explored such dependence. An attractive feature of the model is that it is flexible enough to yield firm and shopper shares that are very asymmetric across jurisdictions, depending on the degree of the cross-group externality.

The interaction of the two-sides in the tax competition game has been shown to give rise to a cross-group externality, which has profound implications for the equilibrium outcomes. For instance, as shown in Proposition 1, this cross-group externality intensifies the asymmetry of the shares (firm and shopper/consumer) between the jurisdictions (an observation that is absent by the one-sided model). Within such framework it has been shown that jurisdictions choose different public goods investments (vertical differentiation). The low public good jurisdiction chooses the minimum possible (zero) level and the high public good jurisdiction chooses a strictly positive investment. The degree of vertical differentiation relative to that in the one-sided model is: i) higher if the externality is strong and ii) lower if the externality is weak (Proposition 2). For low levels of the cross-group externality the firm side is shared between the two jurisdictions and an increase in the magnitude of the externality intensifies tax competition. On the other hand, when the externality is strong the firm side tips in favor of the high public good jurisdiction and further externality increases will lead to a higher tax rate levied by the high public good jurisdiction (Proposition 1). The high public good jurisdiction over-invests, relative to the social optimum, when the externality is strong and under-invests when the externality is weak. The first best always involves firm tipping. In that sense, the inefficiency of the non-cooperative outcome is mitigated as the cross-group externality increases and the firm side tips (Proposition 3). A minimum tax is effective when the externality is weak and ineffective when the externality is strong (Proposition 4). Tax harmonization cannot make the jurisdictions better off (Proposition 5).

The limitation of this paper suggests avenue for future research. Firms have been assumed to be footloose. It would be interesting to investigate the role of the cross-group externality on public goods investments and tax competition when firms have a certain degree of attachment to a particular jurisdiction.

Public goods also confer utility to consumers. As an extension to the model analyzed here, one could also consider a situation where the public goods investments benefit both sides of the market and not only firms. Investments in infrastructure is an example that fits well here. In such an environment, is it still the case that the principle of maximum differentiation holds? Intuition, however, suggests that it might not.
Appendices

A.1 Proof of Proposition 1.

The first order conditions of the revenue functions for jurisdiction $A$ and $B$ with respect to taxes are given, respectively, by

$$\frac{\partial R_A}{\partial \tau_{fA}} = \frac{\Delta t - (\alpha - 1) + t\tau_{fB} - 2t\tau_{fA}}{\Delta t - 2(\alpha - 1)} = 0$$

and

$$\frac{\partial R_B}{\partial \tau_{fB}} = \frac{-(\alpha - 1) + t\tau_{fA} - 2t\tau_{fB}}{\Delta t - 2(\alpha - 1)} = 0. \quad (A.1.1)$$

Sufficiency requires that

$$\frac{\partial^2 R_k}{\partial \tau_{f_k}^2} = -\frac{2t}{\Delta t - 2(\alpha - 1)} < 0, \quad k = A, B,$$

which is satisfied if and only if

$$\Delta > \frac{2(\alpha - 1)}{t}. \quad (A.1.2)$$

The proof now explores the dependency of optimality on the values of $\Delta$. There are two cases to consider.

**Case 1: Interior equilibrium.** Suppose that $\Delta \geq \frac{3(\alpha - 1)}{t}$ and so, following from (A.1.2), the revenue functions are strictly concave. It is then clear that, in this case, no jurisdiction has an incentive to unilaterally deviate from the solution to the system of the first order conditions in (A.1.1) given by $\tau_{fA} = \frac{2\Delta}{3} - \frac{(\alpha - 1)}{t} > 0$ and $\tau_{fB} = \Delta - \frac{(\alpha - 1)}{t} > 0$ (the inequality follows upon the restriction on $\Delta$).

At this interior solution, the location of the marginal firm and the marginal shopper—after we substitute equation (13) into (4) and (8)—are given, respectively, by

$$\hat{w} = \frac{\Delta t - 3(\alpha - 1)}{3[\Delta t - 2(\alpha - 1)]} \quad \text{and} \quad \hat{z} = \frac{1}{2} + \frac{\Delta (\alpha - 1)}{6[\Delta t - 2(\alpha - 1)]}. \quad (A.1.3)$$

The denominators of both $\hat{w}$ and $\hat{z}$ in (A.1.3) are strictly positive if and only if $\Delta > \frac{2(\alpha - 1)}{t}$, a condition that is satisfied from (A.1.2). Moreover, $\hat{w} \geq 0$ if and only if $\Delta \geq \frac{3(\alpha - 1)}{2t}$, $\hat{w} \leq 1$ if and only if $\Delta \geq \frac{3(\alpha - 1)}{2t}$ and $\hat{z} \leq 1$ if and only if $\Delta \geq \frac{6(\alpha - 1)}{3\Delta - (\alpha - 1)}$. A simple comparison between $\Delta \geq \frac{3(\alpha - 1)}{t}, \Delta \geq \frac{3(\alpha - 1)}{2t}$ and $\Delta \geq \frac{6(\alpha - 1)}{3\Delta - (\alpha - 1)}$ reveals that the critical $\Delta$ that satisfies $\hat{w}, \hat{z} \in [0,1]$ (under the assumption that $t > \alpha - 1$) is given by $\Delta = \frac{3(\alpha - 1)}{t}$. It is thus the case that the tax rates, given in (13), $\tau_{fA}^* = \frac{2\Delta}{3} - \frac{(\alpha - 1)}{t}$ and $\tau_{fB}^* = \Delta - \frac{(\alpha - 1)}{t}$, are the equilibrium ones. In such an equilibrium, the revenue functions, excluding the cost of the public good, are given by

$$R_A = \frac{[2\Delta t - 3(\alpha - 1)]^2}{9t[\Delta t - 2(\alpha - 1)]} \quad \text{and} \quad R_B = \frac{[\Delta t - 3(\alpha - 1)]^2}{9t[\Delta t - 2(\alpha - 1)]}. \quad (A.1.4)$$

**Case 2: Firm side tips in favor of jurisdiction $A$.** Consider now the case $0 \leq \Delta \leq \frac{3(\alpha - 1)}{t}$. In this case there is an equilibrium under which

$$\tau_{fA}^* = \frac{\alpha - 1}{t} \quad \text{and} \quad \tau_{fB}^* = 0. \quad (A.1.5)$$
To show that this is an equilibrium we proceed as follows. Suppose that $\tau_{fB}^*$ is zero. Then, as jurisdiction $B$ does not have any revenues, $x_B = 0$ and so, for any $\Delta > 0$, all firms have an incentive to move to jurisdiction $A$. The level of jurisdiction $A$’s tax rate under which this happens is the one that sets $n_{fA} - n_{fB} = 1$ and is given by $\tau_{fA}^* = \frac{\alpha - 1}{t}$. In this equilibrium, following from (8), $n_{sA} - n_{sB} = \frac{\alpha - 1}{t} < 1$ (given the assumption $t > (\alpha - 1)$), implying that jurisdiction $B$’s shopper share will never be zero. Now, for (A.1.5) to be an equilibrium, jurisdiction $B$ must have no incentive to deviate from $\tau_{fB}^* = 0$. Indeed this is the case since the share of firms located in $B$ is zero. Jurisdiction $A$, too, has no incentive to deviate from $\tau_{fA}^* = \frac{\alpha - 1}{t}$ by lowering the level of the tax, since by doing so it cannot increase the number of firms located there. What remains to be checked is whether jurisdiction $A$ has the incentive to raise its tax.

To show that it does not, suppose, first, that $\Delta \in \left(\frac{2(\alpha - 1)}{t}, \frac{3(\alpha - 1)}{t}\right]$. Under this range of $\Delta$ jurisdiction $A$’s tax rate response will be continuous and given by the derivatives of $n_{fA}$ and $n_{sA}$ from (10) and (11) with respect to $\tau_{fA}$. Following from (A.1.1), the first order condition of jurisdiction $A$ evaluated at $\tau_{fA}^* = \frac{\alpha - 1}{t}$ and assuming that $\tau_{fB} = 0$, gives

$$\frac{\partial R_A}{\partial \tau_{fA}}|_{\tau_{fA}^* = \frac{\alpha - 1}{t} \text{ and } \tau_{fB} = 0} = \frac{\Delta t - 3(\alpha - 1)}{\Delta t - 2(\alpha - 1)}, \quad (A.1.6)$$

which is strictly negative when $\Delta \in \left(\frac{2(\alpha - 1)}{t}, \frac{3(\alpha - 1)}{t}\right]$. Since under this $\Delta$ the revenue function is concave, jurisdiction $A$ cannot profit by increasing the level of the tax.

Suppose now that $\Delta \leq \frac{2(\alpha - 1)}{t}$. In this case a small increase in $\tau_{fA}$ will result in jurisdiction $A$ loosing all firms. The reason behind this is the following. At the candidate equilibrium (A.1.5), we have $\dot{w} = 0$ (all firms locate in jurisdiction $A$). Suppose $\tau_{fA}$ increases. From (4), jurisdiction $A$ will, initially, lose $\frac{1}{A}$ firms to jurisdiction $B$. Given this, the firm difference between the two jurisdictions increases by $\frac{2}{A}$. This, in turn, implies, following from (8), that jurisdiction $A$ will lose $\frac{(\alpha - 1)}{2t} \frac{2}{A}$ shoppers to jurisdiction $B$. This implies that the shopper difference between the two jurisdictions will increase by $\frac{(\alpha - 1)}{2t} \frac{2}{A}$. As a consequence, in addition to the initial $\frac{1}{A}$, $\frac{2(\alpha - 1)}{2t}$ more firms will be lost. It is straightforward to show that his process continues to infinity and is described by the series

$$\sum_{k=1}^{\infty} \frac{1}{\Delta^k} \left(\frac{2(\alpha - 1)}{t}\right)^{k-1}, \quad (A.1.7)$$

This series gives the effect of a tax change on the number of firms that join jurisdiction $A$. Using the ratio test, this series is convergent only if $\Delta > \frac{2(\alpha - 1)}{t}. \quad (30)$ If this condition is satisfied, then it can be shown that the series converges to $\frac{t}{2(\alpha - 1)}$. Notice that this is the absolute of the derivative of $n_{fA}$, from (10), with respect to $\tau_{fA}$.

\footnotetext{30}{According to the ratio test, if $\left|\frac{a_{k+1}}{a_k}\right|$ approaches a number less than one as $k$ approaches infinity, then $\sum k a_k$ converges. Otherwise, the series diverges. In the case we analyse we have that

$$L = \lim_{k \to \infty} \left|\frac{a_{k+1}}{a_k}\right| = \frac{2(\alpha - 1)}{t \Delta}, \quad (A.1.8)$$

and so $L < 1$ if and only if $\Delta > \frac{2(\alpha - 1)}{t}.$}
If now $\Delta \leq \frac{2(\alpha - 1)}{t}$ then the series diverges. In this case, a small tax increase results in jurisdiction A loosing all the firms. Hence, such a deviation is not profitable. This proves that (A.1.5) is indeed an equilibrium. The revenue functions, excluding the cost of the public good, in this equilibrium are given by

$$R_A = \frac{\alpha - 1}{t} \text{ and } R_B = 0.$$  \hspace{1cm} (A.1.9)

Collecting the intervals of $\Delta$ and the corresponding optimal responses one arrives at the tax rates, shares and revenue functions given in Proposition 1.

\[\square\]

**A.2 Proof that the revenue function may be discontinuous and not quasi-concave.**

Suppose, for simplicity, that $x_B = 0$. Given this, jurisdiction A’s first order condition is

$$\frac{\partial R_A}{\partial x_A} = \begin{cases} \frac{[2tx_A - 3(\alpha - 1)][2tx_A - 5(\alpha - 1)]}{9[tx_A - 2(\alpha - 1)]^2} - x_A, & \text{if } x_A \geq \frac{3(\alpha - 1)}{t} \\ -x_A, & \text{if } x_A \leq \frac{3(\alpha - 1)}{t}. \end{cases} \hspace{1cm} (A.2.10)$$

Let $R_1(x_A) = \frac{[2tx_A - 3(\alpha - 1)][2tx_A - 5(\alpha - 1)]}{9[tx_A - 2(\alpha - 1)]^2}$ and $R_2(x_A) = x_A$ be the two components of (A.2.10) when $x_A \geq \frac{3(\alpha - 1)}{t}$. It can be shown that $R_1 \left( \frac{3(\alpha - 1)}{t} \right) \geq R_2 \left( \frac{3(\alpha - 1)}{t} \right)$ if and only if $t \geq 9(\alpha - 1)$.

Now we investigate the slopes of $R_1(x_A)$ and $R_2(x_A)$ for $x_A \geq \frac{3(\alpha - 1)}{t}$. To do so we first compute the derivatives of $R_1$ and $R_2$ with respect to $x_A$ that are given by

$$R'_1 = \frac{2t(\alpha - 1)^2}{9[tx_A - 2(\alpha - 1)]^3} \text{ and } R'_2 = 1. \hspace{1cm} (A.2.11)$$

Since $R'_1 > 0$ and $R'_2 > 0$ for all $x_A \geq \frac{3(\alpha - 1)}{t}$, it is the case that both $R_1$ and $R_2$ are increasing in this interval of $x_A$. Also, $R_2$ increases at a constant rate, while the rate of increase of $R_1$ is decreasing in $x_A$. We know now that $R_1 \left( \frac{3(\alpha - 1)}{t} \right) \geq R_2 \left( \frac{3(\alpha - 1)}{t} \right)$ if and only if $t \geq 9(\alpha - 1)$ and that these functions behave according to (A.2.11). In the construction of the net revenue function $R_A$ there are, thus, two cases to consider.

**Case 1:** $t > 9(\alpha - 1)$. As already noted, when $t > 9(\alpha - 1)$, $R_1$ starts out above $R_2$. Following from (A.2.11), it is also the case that $R'_1 \left( \frac{3(\alpha - 1)}{t} \right) > R'_2 \left( \frac{3(\alpha - 1)}{t} \right)$ if and only if $t > \frac{9(\alpha - 1)}{2}$. Since now $R'_1$ decreases in $x_A$ it must be that $R_1$ and $R_2$ intersect once in the interval $\left[ \frac{3(\alpha - 1)}{t}, \infty \right)$. At the point of intersection it must be that $\frac{\partial R_A}{\partial x_A} = 0$. To the left of the intersection point it is the case that $\frac{\partial R_A}{\partial x_A} > 0$ and to the right that $\frac{\partial R_A}{\partial x_A} < 0$.

**Case 2:** $t < 9(\alpha - 1)$. When $t < 9(\alpha - 1)$, $R_1$ starts out below $R_2$. If $t < \frac{9(\alpha - 1)}{2}$, then $R'_1 < R'_2$ for all $x_A$, which implies that $R_1$ and $R_2$ never intersect. In this case $\frac{\partial R_A}{\partial x_A} < 0$ for all $x_A \geq \frac{3(\alpha - 1)}{t}$. If $\frac{9(\alpha - 1)}{2} < t < 9(\alpha - 1)$, then initially (that is, for $x_A$’s close to $\frac{3(\alpha - 1)}{t}$) $R'_1 > R'_2$ and eventually $R'_1 < R'_2$. When $x_A = \tilde{x} \equiv (3t)^{-1} \left[ \left( 6t (\alpha - 1)^2 \right)^{1/3} + 6(\alpha - 1) \right]$, it is the case that $R'_1(\tilde{x}) = R'_2(\tilde{x})$. It can then be shown that $R_1(\tilde{x}) \geq R_2(\tilde{x})$ if and only if $\alpha \leq 1 + .11487t$, or $t \geq \tilde{t} \approx 8.705(\alpha - 1)$. The implication, if $t < \tilde{t}$, is that $R_1$ and $R_2$ never intersect. In this case $\frac{\partial R_A}{\partial x_A} < 0$ for all $x_A > \frac{3(\alpha - 1)}{t}$. If now $9(\alpha - 1) > t \geq \tilde{t}$, then $R_1$ and $R_2$ intersect twice and, therefore, it is the case the first order condition holds with equality, $\frac{\partial R_A}{\partial x_A} = 0$, twice in the
interval $\left[\frac{3(\alpha-1)}{t}, \infty\right)$. The first intersection gives a local minimum whereas the second gives a local maximum.

Turning now to the case in which $x_A \in \left[0, \frac{3(\alpha-1)}{t}\right]$, one sees from (??) that $R_A$ is decreasing.

We can conclude that if $t < \tilde{t} \approx 8.705(\alpha-1)$ the net revenue function is everywhere decreasing. If $8.705(\alpha-1) \approx \tilde{t} < t < 9(\alpha-1)$ the net revenue function is decreasing in $\left[0, \frac{3(\alpha-1)}{t}\right]$ and exhibits a sideways S shape for $x_A \geq \frac{3(\alpha-1)}{t}$, where it attains a unique local maximum. If $t > 9(\alpha-1)$ the net revenue function is decreasing in $\left[0, \frac{3(\alpha-1)}{t}\right]$ and inverse U-shaped for $x_A \geq \frac{3(\alpha-1)}{t}$, where it attains a unique local maximum.

\[\square\]

**A.3 Proof of Proposition 2.**

There are three cases to consider. Recall that one assumption that we maintain throughout the paper is $\alpha < t + 1$ (otherwise, the shopper side also tips).

**Case 1:** High externality, $\alpha \geq 2t/9 + 1$. First, suppose that $x_A \geq \frac{3(\alpha-1)}{t}$. Jurisdiction $B$ would not want to go from $x_B = 0$, where $R_B(0) > 0$, to $x_B = x_A + \varepsilon$. To see this, suppose that $x_A = \frac{3(\alpha-1)}{t}$. The net revenue of jurisdiction $B$ at $x_B = x_A + \varepsilon$ is arbitrarily close to $\frac{\alpha-1}{t} - \frac{9(\alpha-1)^2}{2\varepsilon^2}$, which is negative if and only if $t \leq \frac{9(\alpha-1)}{2\varepsilon}$. Higher $x_A$ than $\frac{3(\alpha-1)}{t}$ will reduce the net revenue at $x_B = x_A + \varepsilon$ even further, up to $x_B = x_A + \frac{3(\alpha-1)}{t}$.

Also, the net revenue function of jurisdiction $B$ is decreasing in $x_B \geq x_A + \frac{3(\alpha-1)}{t}$ (where sharing takes place). As we showed in the proof of lemma 2 the net revenue function is decreasing in this region when the rival jurisdiction sets its public good equal to zero (and $t$ is low, as it is in the case we are examining). Now the public good of the rival is higher than zero which implies that the net revenue of jurisdiction $B$ must be decreasing. These arguments suggest that if $x_A \geq \frac{3(\alpha-1)}{t}$ jurisdiction $B$’s best response is to set $x_B = 0$.

Second, suppose that $x_A \leq \frac{3(\alpha-1)}{t}$. If jurisdiction $B$ sets $x_B = 0$ its net revenue is $R_B = 0$, because the firm side has tipped in favor of $A$. If it sets $x_B = x_A + \varepsilon$ its revenue is arbitrarily close to $R_B = \frac{\alpha-1}{t} - \frac{2\varepsilon^2}{2}$. Such a deviation is profitable if $x_A < \frac{\sqrt{2(\alpha-1)}}{\sqrt{t}}$. If $x_A \geq \frac{\sqrt{2(\alpha-1)}}{\sqrt{t}}$, jurisdiction $B$ does not want to deviate from $x_B = 0$. Moreover, $\frac{\sqrt{2(\alpha-1)}}{\sqrt{t}} \leq \frac{3(\alpha-1)}{t}$ if and only if $\alpha \geq 2t/9 + 1$. As we said above, $R_B$ is decreasing for $x_B \geq x_A + \frac{3(\alpha-1)}{t}$. These arguments suggest that if $x_A \geq \frac{\sqrt{2(\alpha-1)}}{\sqrt{t}}$, then jurisdiction $B$’s best response is to set $x_B = 0$. If $x_A < \frac{\sqrt{2(\alpha-1)}}{\sqrt{t}}$, jurisdiction $B$’s best response is to set $x_B = x_A + \varepsilon$.

Given jurisdiction $B$’s best response function, jurisdiction $A$’s best response is to set either $x_A = \frac{\sqrt{2(\alpha-1)}}{\sqrt{t}}$ or $x_A = 0$. Both choices yield zero net revenue. We assume that it chooses the first one. Moreover, public good investment is higher than in a one-sided market, i.e., $\frac{\sqrt{2(\alpha-1)}}{\sqrt{t}} > \frac{4}{\pi}$.

**Case 2:** Medium externality, $0.11t + 1 \approx \bar{\alpha} \leq \alpha < 2t/9 + 1$. Jurisdiction $B$ would always want to deviate from $x_B = 0$ to $x_B = x_A + \varepsilon$, if $x_A = \frac{3(\alpha-1)}{t}$. Hence, by continuity, this
deviation will always be profitable for \( x_A \)’s greater than \( \frac{3(\alpha - 1)}{t} \), but close enough to \( \frac{3(\alpha - 1)}{t} \) and for \( x_A \leq \frac{3(\alpha - 1)}{t} \). For \( x_A \)’s significantly higher than \( \frac{3(\alpha - 1)}{t} \), such a deviation is unprofitable. For example, if \( x_A = \frac{\sqrt{2(\alpha - 1)}}{\sqrt{t}} \), (where now \( \sqrt{\frac{2(\alpha - 1)}{t}} \geq \frac{3(\alpha - 1)}{t} \)) a deviation to \( x_B = x_A + \varepsilon \), will yield net revenue arbitrarily close to zero. Using an argument similar to the one in case 1 above, we can show that \( R_B \) is decreasing for \( x_B \geq x_A + \frac{3(\alpha - 1)}{t} \). These arguments suggest that jurisdiction B’s best response if \( x_A \geq \tilde{x} \in \left[ \frac{3(\alpha - 1)}{t}, \frac{\sqrt{2(\alpha - 1)}}{\sqrt{t}} \right] \) is \( x_B = 0 \) and if \( x_A < \tilde{x} \), it is \( x_B = x_A + \varepsilon \).

Jurisdiction A’s optimal response is either to set \( x_A = \tilde{x} \) (with \( R_A > 0 \)) or to set \( x_A = 0 \) (with \( R_A = 0 \)). We can show that \( R_A (\tilde{x}) > 0 \) as follows. The difference in the net revenues between the high and the low public good jurisdictions when \( x_B = 0 \) and \( x_A \geq \frac{3(\alpha - 1)}{t} \) is equal to \( \frac{x_A^2}{t} - \frac{x_A^2}{2} \). This difference is greater than zero if and only if \( x_A \leq \frac{3}{2} \). This inequality holds since \( \frac{\sqrt{2(\alpha - 1)}}{\sqrt{t}} \leq \frac{3}{2} \) if and only if \( t \geq \frac{9(\alpha - 1)}{4} \). Since the net revenue of the low public good jurisdiction is strictly positive, when \( x_B = 0 \) and \( x_A \geq \frac{3(\alpha - 1)}{t} \), it must be that the net revenue of the high public good jurisdiction is also strictly positive.

In this equilibrium, the firm side does not tip. \( \tilde{x} > \frac{4}{9} \), which implies that the degree of vertical differentiation is higher than in the one-sided model. This can be proved as follows. When \( t \leq 6.75(\alpha - 1) \), then \( \frac{3(\alpha - 1)}{t} \geq \frac{4}{9} \) and since \( \tilde{x} > \frac{3(\alpha - 1)}{t} \) it must be that \( \tilde{x} > \frac{4}{9} \). Now suppose that \( t > 6.75(\alpha - 1) \). \( \tilde{x} \) satisfies (uniquely) \( R_B (x_B = 0) - \left( \frac{\alpha - 1}{t} - \frac{x_B^2}{2} \right) = 0 \). \( R_B (x_B = 0) \) monotonically increases in \( x_A \) and \( \frac{\alpha - 1}{t} - \frac{x_B^2}{2} \) monotonically decreases. We have shown that at \( x_A = \frac{4}{9} \) the difference is negative if and only if \( t < 9(\alpha - 1) \). This implies that \( \tilde{x} > \frac{4}{9} \). Finally, \( \frac{dx}{dt} < 0 \), because \( \frac{dR_B}{dt} > 0 \) (when \( x_A > \frac{3(\alpha - 1)}{t} \)) and \( \frac{d}{dt} \left( \frac{\alpha - 1}{t} - \frac{x_B^2}{2} \right) < 0 \). The same comparative static applies in the next case.

**Case 3:** Low externality, \( 1 \leq \alpha < \bar{\alpha}, \approx .11t + 1 \). In this case, the net revenue function of jurisdiction A has a local maximum in \( \left[ \frac{3(\alpha - 1)}{t}, \infty \right) \), assuming that \( x_B = 0 \) (see lemma 2). This maximum is attained at \( x_A^* \). There exists an \( \tilde{x} \in \left( \frac{3(\alpha - 1)}{t}, \frac{\sqrt{2(\alpha - 1)}}{\sqrt{t}} \right) \), such that jurisdiction B does not want to set \( x_B = x_A + \varepsilon \), for all \( x_A \geq \tilde{x} \). Also, \( R_B \) is decreasing for \( x_B > x_A^* \), as we have argued in the proof of Proposition 2. These arguments suggest that the best response of jurisdiction B is to set \( x_B = 0 \) if \( x_A \geq \max \{ x_A^*, \tilde{x} \} \) and if \( x_A < \max \{ x_A^*, \tilde{x} \} \) to set either \( x_B = x_A + \varepsilon \) or \( x_B = x_B^* \), where \( x_B^* \) satisfies the first order condition in \( \left[ \frac{3(\alpha - 1)}{t} + x_A, \infty \right) \). Jurisdiction A, in turn, will either set \( x_A = 0 \) or \( x_A = \max \{ x_A^*, \tilde{x} \} \). This can be explained as follows. If \( x_A < \max \{ x_A^*, \tilde{x} \} \), and given B’s best response, A will become the low public good jurisdiction and the best it can do is to set \( x_A = 0 \). On the other hand, any \( x_A \) strictly greater than \( \max \{ x_A^*, \tilde{x} \} \) is suboptimal. In this case, B’s best response is \( x_B = 0 \). If \( \max \{ x_A^*, \tilde{x} \} = x_A^* \), the sub-optimality follows from the fact that \( x_A^* \) is a local maximum and the fact, from lemma 2, that the net revenue function of A is either a sideways S or inverse U-shaped in this region. Hence, any \( x_A > x_A^* \) will lower the net revenue. If \( \max \{ x_A^*, \tilde{x} \} = \tilde{x} \), then, given the above argument, we are already on the decreasing part of A’s net revenue function, but such an over-investment is needed to prevent the rival from becoming the high public good jurisdiction. Nevertheless, \( x_A > \tilde{x} \) is excessive and consequently \( x_A = \tilde{x} \).
Jurisdiction $A$ is better off following the latter strategy, i.e., $x_A = \max \{x^*_A, \bar{x} \}$, and becoming the high public good jurisdiction, while $x_B = 0$. When the firm side does not tip (which is the case) the difference in net revenue between the high and low public good jurisdictions is, $\frac{x_A}{3} - \frac{x_B^2}{2}$, when $x_B = 0$. Jurisdiction $A$ is better off becoming the high public good jurisdiction if and only if $\frac{x_A}{3} \geq \frac{x_B^2}{2} \iff x_A \leq \frac{2}{3}$.

Moreover, as we have proved in Proposition 2, the maximum $x^*_A$ is equal to $\frac{4}{9} \approx .444$ (when $t \to \infty$) and the maximum $\bar{x}$ is when $t$ is arbitrarily close to $8.705 (\alpha - 1)$, which is arbitrarily close to .48. Both are less than $\frac{2}{3}$, so jurisdiction $A$ is better off becoming the high public good jurisdiction. Jurisdiction $B$ chooses $x_B = 0$. Finally, for low $t$, $x_A = \bar{x} > \frac{4}{9}$ and for high $t$, $x_A = x^*_A < \frac{4}{9}$.

References


