Tournaments and Liquidity Constraints for the Agents

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Abstract. A celebrated result in the theory of tournaments is that relative performance evaluation (tournaments) is a superior compensation method to absolute performance evaluation (piece rate contracts) when the agents are risk-averse, the principal is risk-neutral or less risk-averse than the agents and production is subject to common shocks that are large relative to the idiosyncratic shocks. This is because tournaments get closer to the first best by filtering common uncertainty. This paper shows that, surprisingly, tournaments are superior even when agents are liquidity constrained so that transfers to them cannot fall short of a predetermined level. The rationale is that, by providing insurance against common shocks through a tournament, payments to the agents in unfavorable states increase and payments in favorable states decrease which enables the principal to satisfy tight liquidity constraints for the agents without paying any ex ante rents to them, while simultaneously providing higher-power incentives than under piece rates.

Keywords: Piece rates, tournaments, liquidity constraints.

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1. Introduction

Even though linear contracts are only a proxy of the theoretically optimal non-linear contracts, they are popular in several occupations or industries (e.g., sales, physician contracts with HMOs, contracts between processors and farmers, and faculty raises), partly because they are simple to design and easy to implement and enforce.\(^1\) The most common linear contracts are the piece rate contract and the cardinal tournament. Under the piece rate contract each agent is evaluated according to his absolute performance or according to his performance against a predetermined standard, while under the tournament each agent is evaluated relative to the performance of his peers. In particular, under both schemes each agent receives a base payment and a bonus payment, but the bonus payment is determined by absolute performance in piece rates and by relative performance in tournaments.\(^2\) Following the footsteps of Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983) and others, the comparison of these two alternative evaluation methods has been the subject of current literature (Tsoulouhas (1999), Wu and Roe (2005 and 2006), Marinakis and Tsoulouhas (2007) and Tsoulouhas and Marinakis (2007)). This comparison is important because it allows us to contrast the efficiency of absolute performance evaluation against relative performance evaluation.

Absent liquidity considerations, when agents are risk averse and production is subject to sufficiently large common shocks the tournament is a superior incentive scheme to the piece rate. This is because the tournament uses the information generated by the performance of the group of participating agents as a whole, while the piece rate does not. Specifically, if the disturbance in the output of each agent is correlated with the disturbances in the output of the other agents, the information contained in the average production can be very useful to the principal in creating a contract which is a step closer to the First Best. Moreover, under the tournament, if the principal is risk-neutral or is less risk-averse than the agent, an insurer–insured relationship can be developed between principal and agent allowing for a Pareto improvement of the contract. That is, the principal will offer insurance to the risk averse agent by filtering away the common shock from his responsibility. Insurance will make the agent more tolerant to a higher-power incentive scheme and, therefore, the agent is expected to increase his effort level.

\(^1\)The non-linearity of the theoretically optimal contract is partly due to the fact that contracts accommodate all possible events. Holmström and Milgrom (1987), however, have argued that schemes that adjust compensation to account for rare events may not provide correct incentives in ordinary high probability circumstances.

\(^2\)The base payment ensures agent participation and the bonus provides incentives to perform. Under tournament an agent receives a bonus if his performance is above that of his peers, and a penalty otherwise.
One might conjecture that the superiority of tournaments over piece rates may not survive under liquidity constraints. Marinakis and Tsoulouhas (2007) have shown that the optimality of tournaments over piece rates breaks down when the risk-neutral principal is subject to a limited liability (bankruptcy) constraint, which limits the payments a principal can make, provided that the liquidation value of the principal’s enterprise is sufficiently small. This is so because tournaments increase payments in unfavorable states, but these are the states in which the limited liability constraint comes into play. The intuition is that contracts with risk neutrality and limited liability for the principal look very much like those that would have been obtained with risk aversion. In other words, if the principal is concerned about the allocation of profit across states, he will no longer offer insurance against common shocks via tournaments and will resort to piece rate contracts or fixed performance standards. This paper investigates the optimality of tournaments over piece rates when the agent, instead, is subject to a liquidity constraint which introduces ex post limitations on the minimum payment the agent can accept or the maximum penalty that can be imposed on him (Innes 1990, 1993a and 1993b). The liquidity constraint prevents the principal from compensating the agent by an amount smaller than a predetermined level in all states of nature.

The models used by Lazear and Rosen (1981), Green and Stokey (1983), Meyer and Vickers (1997) and others, allow the payments to the agents to be negative. In particular, under both the piece rate and the tournament payment schemes, if the agents produced a sufficiently low output they would usually have to pay the principal. Thus, according to the standard literature, if the production of an agent is sufficiently low the principal will penalize the agent by imposing a negative compensation and acquire whatever output the agent produced. This is certainly inconsistent with what we observe in reality.

The liquidity constraint is partly an institutional constraint on contracts. It is imposed by law for several industries in numerous countries. Such legislation aims at removing the burden of excessive penalties imposed on agents for negative outcomes beyond their control, rather than at maximizing social welfare. However, a liquidity constraint for the agent may alter the choice the principal makes between tournaments and absolute performance contracts. This can be due to a number of reasons. Some of these reasons are in favor of tournaments and some are in favor of piece rates. First, by increasing payments to the agents in unfavorable states, tournaments are more likely to satisfy tight liquidity constraints for the agents. Second, by providing insurance, tournaments may satisfy the liquidity constraints for the agents without paying rents to them. This is so because tournaments increase the compensation to the agents in unfavorable states but they reduce the payments in favorable states. By contrast piece rates may pay the agents ex ante rents when the liquidity constraints
are tight (i.e., when the minimum required payment to the agents is high), which reduces the principal’s profit. If piece rates pay ex ante rents to the agents, they could be dominant over tournaments from the principal’s perspective only if implemented effort under piece rates were higher. But, in general, tournaments allow the principal to implement higher-power incentives than piece rates, which enhances the dominance of tournaments. Third, agents may be unable to pay for insurance especially in low states of nature if the liquidity constraints are tight, which works against tournaments. Fourth, the attitude of the principal and the agents toward risk may change. Liquidity constraints may make the agents more tolerant to risk, in the sense that if the agents know that their liability is limited, they may become indifferent among the range of states over which the liquidity constraint is binding. This is certainly in accord with Laffont and Martimort (2002) who state (see p.121):

“A limited liability constraint on transfers implies higher-powered incentives for the agent. It is almost the same as what we would obtain by assuming that the agent is a risk lover. The limited liability constraint on transfers somewhat convexifies the agent’s utility function.”

On the other hand, the liquidity constraints for the agents are expected to make the principal care about the allocation of payments and, hence, profit across states to satisfy the liquidity constraints and ensure agent participation. When the principal becomes less tolerant to risk, while agents simultaneously become more tolerant to risk and, therefore, they are not willing to pay enough for insurance, the principal may find it suboptimal to offer insurance to the agent through a tournament and may resort to piece rates again. Thus, in all, it is not a priori clear if tournaments, which are normally superior over piece rates when production is subject to common shocks, maintain their superiority under liquidity constraints for the agents.

Our analysis shows that, surprisingly, in the presence of sufficient common uncertainty a principal contracting with risk averse agents will prefer to offer a tournament even when agents are liquidity constrained. This finding is diametrically opposite to the result for the case when the principal, instead, is subject to limited liability. The rationale for this result follows directly from the discussion above. It turns out that by providing insurance against common shocks through a tournament, so that payments to the agents in unfavorable states increase and payments in favorable states decrease, the principal can satisfy tight liquidity constraints for the agents without paying any ex ante rents to them while simultaneously providing them with higher-power incentives than under piece rates. The individual rationality constraints for the agents are always binding under tournaments, whereas under piece rates they are non-binding (that is, the agents receive ex ante rents) when the liquidity
constraints for the agents are really tight (that is, when the minimum payment required to satisfy the liquidity constraints is high). This finding establishes our claim that the principal can satisfy tight liquidity constraints for the agents without paying any ex ante rents to them under tournament. Our second claim, that the principal can implement higher-power incentives under tournament, follows from the fact that the piece rate contract cannot be defined for a piece rate larger than one (in the sense that the principal would not make an offer such that marginal cost exceeded marginal revenue) whereas the tournament is defined for a larger bonus factor. The larger the minimum payment satisfying an agent’s liquidity constraint, the higher the power of incentives the principal provides. In other words, the principal counterbalances the increase in the base payment, which is required to satisfy the liquidity constraint, with higher-power incentives in order to curb agent rents and in order to reduce the likelihood that output is low. Tournaments provide the principal with added flexibility in the determination of this power when the liquidity constraints are really tight.

On the other hand, regardless of whether the principal offers a piece rate or a tournament, the liquidity constraints for the agents are non-binding (that is, in some sense, agents receive ex post rents) when the minimum payment required to satisfy the liquidity constraints is low. In that case, the analysis is similar to the benchmark case in Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983), and tournaments are optimal under sufficient common uncertainty.

The empirical application that stems from our analysis is that firms should adopt relative performance evaluation via tournaments over absolute performance evaluation via piece rates regardless of whether the agents are liquidity (wealth) constrained or not. This finding enhances the generality of the results obtained in Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983). For instance, in the case of processor companies contracting with farmers who most often are liquidity constrained, processors need not fear that the farmers’ liquidity issues detract from the superiority of tournaments.

Even though the issue we analyze has been largely overlooked by the current tournament literature, the introduction of liquidity constraints on the agent side is not novel. Bhattacharya and Guasch (1988) examine the efficiency of tournaments with heterogeneous agents. They argue that tournaments that are based on comparisons across ability levels are more efficient than tournaments that are based on comparisons within cohorts of similar ability agents. However, this result is reversed when agents are subject to limited liability (liquidity) constraints, because tournaments with comparisons across cohorts are more likely to lead to negative payments. Kim (1997) analyzes a setting with a risk neutral principal and a risk neutral agent when the agent’s liability is limited. He shows that the optimal contract

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3Wealth constraints can certainly be a concern in contracts for salesmen as well.
is a bonus contract in which the principal and the agent share the output, and the agent receives an additional fixed bonus only when output is greater than some predetermined level. Demougin and Garvie (1991) examine two forms of constraints for risk neutral agents: non-negativity constraints for the transfers to the agents and ex post individual rationality constraints for the agents. They show that the principal cannot implement the First Best and agents earn informational rents. Courty and Marschke (2002) analyze a framework with liquidity constraints and budget balancing. They show that when the difference in agent budgets is large enough, the liquidity and budget balancing constraints bind, thereby reducing the effectiveness of incentives. Demougin and Fluet (2003) focus on examining the cost of providing incentives through rank-order tournaments when agents care about the fairness of their payoffs relative to that of others, and agents are subject to limited liability which makes rents possible. They show that the presence of more envious contestants reduces the principal’s cost of providing incentives, when rents must be paid, because the agents will motivate themselves to perform even with lower rents from the principal. Kräkel (2007) analyzes a model with risk-neutral agents who are subject to limited liability, but face no common uncertainty inflicted on their productive activities, to make the point that the Lazear and Rosen (1981) finding of equal incentive efficiency for piece rates and for rank-order tournaments does not necessarily carry over when limited liability is introduced. In particular, piece rates dominate tournaments if idiosyncratic risk is high. This is an intuitive result because, even absent limited liability, a tournament would be suboptimal by introducing idiosyncratic noise from the activity of other agents onto the payment to any given agent. Therefore, the introduction of limited liability should not change that, but it should change the specification of the piece rate. Namely, given risk-neutrality, limited liability should entail a move from the "selling the enterprise to the agent" solution (i.e., a piece rate of 1) to a piece rate of less than one, because the liquidity constraint prevents the sale of the enterprise to the agent. We differ from Kräkel in a number of important respects. We assume the existence of sufficient common uncertainty which provides scope for tournaments. We also assume that agents are risk-averse to incorporate the insurance aspect of tournaments, and we show that tournaments in our setting are dominant over piece rates with or without limited liability. Last but not least, note that similar to Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983) and Malcomson (1984) we are not looking for the optimal contract, instead, we contrast the efficiency properties of absolute to relative performance evaluation.

Section 2 presents our model, section 3 presents the benchmark case without liquidity constraint and section 4 presents our results when the agents are liquidity constraint. Section 5 determines the dominant compensation scheme and section 6 concludes.
2. Model
A principal signs a contract with \( n \) homogeneous agents.\(^4\) Each agent \( i \) produces output according to the production function \( x_i = a + e_i + \eta + \varepsilon_i \), where \( a \) is the agent’s known ability, \( e_i \) is his effort, \( \eta \) is a common shock and \( \varepsilon_i \) is an idiosyncratic shock. The idiosyncratic shocks, \( \varepsilon_i \), and the common shock follow independent distributions. Each agent’s effort and the subsequent realizations of the shocks are private information to him, but the output obtained is publicly observed. The principal compensates agents for their effort based on their outputs by using a piece rate contract or a tournament. Agent preferences are represented by a CARA utility function \( u(w_i, e_i) = \exp(-u_i + \frac{1}{2a}e_i^2) \), where the agent’s coefficient of absolute risk aversion is set equal to 1 for simplicity. The cost of effort is measured in monetary units. Each agent has a reservation utility \( -\exp(-\bar{u}) \).

3. Piece Rates and Tournaments without Liquidity Constraints
We start by deriving the optimal contractual variables for the piece rate and the tournament without liquidity constraints for the agents. We assume that the total production disturbance, \( \varepsilon_i + \eta \), follows a normal distribution with zero mean and variance equal to \( c = \frac{\sigma^2}{2} \), and the idiosyncratic shock, \( \varepsilon_i \), follows a normal distribution with zero mean and variance equal to \( d = \frac{\beta^2}{2} \).

The piece rate contract \((R)\) is the payment scheme in which the compensation to the \( i^{\text{th}} \) agent is \( w_i = b_R + \beta_Rx_i \), where \((b_R, \beta_R)\) are the contractual variables to be determined by the principal. The principal determines these parameters by backward induction. Thus, the principal calculates each agent’s expected utility

\[
EU_R = -\exp\left(-b_R - \beta_R (a + e_i) + \frac{e_i^2}{2a} + \frac{\beta_R^2\sigma^2}{2\sqrt{2\pi}}\right).
\]  

To ensure the compatibility of the contract with agent incentives to perform, the principal calculates the effort level that maximizes (1). First order conditions yield

\[
e_i = a\beta_R.
\]  

To ensure the compatibility of the contract with agent incentives to participate, the principal selects the value of the base payment, \( b_R \), that satisfies the agent’s individual rationality constraint with equality so that the agent receives no rents but still accepts the

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\(^4\)Tsoulouhas and Marinakis (2007) examine a model with ex post heterogeneous agents to make the point that agent heterogeneity compromises the insurance function of tournaments.

\(^5\)As will become obvious in the remaining analysis, this assumption on the variance simplifies the exposition.

\(^6\)Note that the concavity of the utility function implies that first order conditions are sufficient.
contract. The agents individual rationality constraint satisfies $EU_R = -\exp(-\bar{u})$, where $EU_R$ is determined by (1) and (2). Solving for $b_R$ implies

$$b_R = \bar{u} + \frac{c}{\sqrt{2\pi}} - a \frac{\beta}{2} - a \beta R.$$  

(3)

Thus, by choosing the piece rate $\beta_R$, the principal can precisely determine the agent’s effort because the agent will optimally set his effort according to (2). In addition, by setting $b_R$ in accordance with (3) the principal can induce agent participation at least cost. That is, agent incentives to perform are only determined by the piece rate $\beta_R$, whereas agent incentives to participate are determined by the base payment $b_R$.

Given conditions (2) and (3) the principal maximizes his expected total profit

$$ET\Pi_R = \sum_{i=1}^{n} [Ex_i - Ew_i] = n \left[ a + a \beta_R - a \frac{\exp\left(\frac{c}{\sqrt{2\pi}} + \frac{a^2}{2}\right)}{\beta R - \bar{u}} \right].$$  

(4)

The solution to this problem satisfies

$$\beta_R = \frac{a}{a + \frac{c}{\sqrt{2\pi}}}. $$  

(5)

Condition (3) then implies

$$b_R = \bar{u} - a \frac{\exp\left(\frac{c}{\sqrt{2\pi}} + \frac{a^2}{2}\right)}{\beta R - \bar{u}}. $$  

(6)

Given conditions (5) and (4) expected profit per agent is

$$E\Pi_R = a + \frac{a^2}{2 a + \frac{c}{\sqrt{2\pi}}} - \bar{u}. $$  

(7)

The tournament (T) is the payment scheme in which the compensation to each agent is determined by a relative performance evaluation. Specifically, $w_i = b_T + \beta_T (x_i - \bar{x})$, where $\bar{x}$ is the average output obtained by all agents and $(b_T, \beta_T)$ are the contractual variables to be determined by the principal. Under a tournament the agent’s expected utility is

$$EU_T = -\exp\left(-b_T - \beta_T \frac{n-1}{n} (a + e_i) + \beta_T \frac{1}{n} \sum_{j \neq i} (a + e_j) + \frac{e_i^2}{2a} + \frac{1}{2} \frac{n-1}{n} \frac{\beta_T^2}{\sqrt{2\pi}} \right). $$  

(8)

The effort level that maximizes (8) satisfies

$$e_i = \frac{n-1}{n} a \beta_T. $$  

(9)
Further, the individual rationality constraint $EUT = -\exp(-\overline{u})$ implies

$$b_T = \overline{u} + \frac{1}{2} \frac{n - 1}{n} \left( \frac{n - 1}{n} a + \frac{d}{\sqrt{2\pi}} \right) \beta_T^2. \tag{10}$$

Then, given conditions (9) and (10), the principal maximizes expected total profit

$$ET\Pi_T = n \left[ a + \frac{n - 1}{n} a\beta_T - \frac{1}{2} \frac{n - 1}{n} \left( \frac{n - 1}{n} a + \frac{d}{\sqrt{2\pi}} \right) \beta_T^2 - \overline{u} \right]. \tag{11}$$

The solution to the principal’s maximization problem satisfies

$$\beta_T = \frac{a}{\frac{n - 1}{n} a + \frac{d}{\sqrt{2\pi}}}, \tag{12}$$

therefore,

$$b_T = \overline{u} + \frac{1}{2} \frac{a^2}{a + \frac{n - 1}{n} \frac{d}{\sqrt{2\pi}}}. \tag{13}$$

Given (12) and (11) expected profit per agent is

$$E\Pi_T = a + \frac{1}{2} \frac{a^2}{a + \frac{n - 1}{n} \frac{d}{\sqrt{2\pi}}} - \overline{u}. \tag{14}$$

By comparing (4) to (14) it can easily be shown that

$$E\Pi_T > E\Pi_R \iff \frac{n}{n - 1} d < c, \tag{15}$$

that is, tournaments are superior when total uncertainty is large relative to the idiosyncratic uncertainty (equivalently, when common uncertainty is relatively large) and when the number of agents is large. This so because tournaments eliminate common uncertainty but they add the average individual noise of others. It is also straightforward to show that

$$\beta_T > \beta_R \tag{16}$$

and

$$b_T > b_R. \tag{17}$$

The rationale behind (17) is that the expected bonus payment under tournament is zero, whereas that under piece rate is positive. Therefore, agents expect to be compensated for effort through the base payment in a tournament. The intuition behind (16) is that the principal implements higher-power incentives when common uncertainty is removed from
the responsibility of the agent under tournament.

4. Piece Rates and Tournaments with Liquidity Constraints

Next we turn to the case with liquidity constraints for the agents. The liquidity constraint is

$$w_i \geq \bar{w},$$  

(18)

where $\bar{w}$ is the minimum permissible payment. The liquidity constraints for the agents necessitate a support for the production shocks which is bounded below and above. The support must be bounded below so that in the worst possible output state the liquidity constraints are still satisfied (obviously they cannot be satisfied with an output space which is unbounded below). For a similar reason, the support must be bounded above to eliminate the case when the payment under tournament is below the minimum required to satisfy the liquidity constraint when average output is unbounded above.⁷ With bounded support for the production shocks one might expect that the First Best is always implementable by punishing the agent severely for outcomes outside the support (see p. 140 in Bolton and Dewatripont (2004)). Note, however, that the liquidity constraints of the agents prevent severe punishment of them.

The requirement of bounded support eliminates unbounded distributions such as the normal we used in section 3 (which is typically used in the literature for the setting without liquidity constraints). The normal distribution is necessary to obtain a closed form solution for the case without liquidity constraints. Further, a truncated normal distribution provides neither a closed form solution nor a numerical one. However, we were able to obtain significant insight through a numerical analysis by assuming that the idiosyncratic and the common shocks follow independent uniform distributions, in which case the sum of these shocks follows a triangular distribution. Specifically, the idiosyncratic shocks, $\varepsilon_i$, follow independent uniform distributions with support $[-d,d]$ and, therefore, the total production shock, $v_i \equiv \varepsilon_i + \eta$, follows a triangular distribution with density $f(\cdot)$, the support of which is assumed to be $[-c,c]$ with zero mean. The following lemmata apply to piece rates and tournaments with liquidity constraints.

**Lemma 1** Under piece rates, when the agents are subject to liquidity constraints in addition to individual rationality constraints, at least one of the individual rationality and the liquidity constraints for each agent binds depending on the values of parameters $w, \bar{w}, a$ and $c$.

⁷An alternative approach would be to consider a modification of the payment schemes such that the agent still receives the minimum payment required to satisfy his liquidity constraint, $\bar{w}$; specifically, consider $\max\{w_i, \bar{w_i}\}$ where $\bar{w}_i$ is determined by the scheme. However, the analysis in this case is intractable.
Proof. The proof is straightforward by noting that if both constraints were non-binding, then, the principal would reduce the payments to the agent until one of the two constraints became binding (that is, until the agent received no rents in an ex ante or in an ex post sense). As shown in section 3, solving without the liquidity constraint for each agent (in which case the individual rationality constraint is obviously binding) implies that the contractual variables \((b_R, \beta_R)\) satisfy conditions (6) and (5) and therefore the payment \(w_i\) may or may not satisfy the liquidity constraint in all states depending on the values of parameters \(w, \bar{w}, a\) and \(c\). Therefore, when the individual rationality constraint is binding, the liquidity constraint is binding or non-binding (the latter when \(w\) is relatively low). Solving without the individual rationality constraint (in which case the liquidity constraint is obviously binding in the lowest possible state) implies that the payments to the agent may or may not satisfy the individual rationality constraint depending on the values of parameters \(w, \bar{w}, a\) and \(c\) again. Therefore, when the liquidity constraint is binding the individual rationality constraint is binding or non binding (the latter when \(w\) is relatively large).

Lemma 2 Under tournaments, when the agents are subject to liquidity constraints in addition to individual rationality constraints, and assuming that the regularity condition \((n - 1)a > nd\) holds, the individual rationality constraint for each agent is always binding and the liquidity constraint for each agent is binding or non-binding depending on the values of parameters \(w, \bar{w}, a\) and \(d\).

Proof. First, similar to Lemma 1, the two constraints cannot simultaneously be non-binding. Solving without the individual rationality constraint (in which case the liquidity constraint is obviously binding in the lowest possible state) implies that \(b_T = w + \beta_T d\). This is so because \(w_i = b_T + \beta_T (x_i - \bar{x}) = w\) and, given that \(\epsilon_i \in [-d, d]\), if the number of agents is sufficiently large \(x_i - \bar{x} \overset{D}{\sim} \text{uniform}[-d, d]\). Then, since the principal’s profit per agent is \(\Pi_i = x_i - b_T = a - w + \left(\frac{a - 1}{n}a - d\right) \beta_T + \eta + \epsilon_i\), it follows that expected profit per agent is \(E\Pi_T = a - w + \left(\frac{a - 1}{n}a - d\right) \beta_T\). To maximize this expected profit the principal chooses the maximum \(\beta_T\) that satisfies the individual rationality constraint with equality so that the agent accepts the contract. Therefore, the individual rationality constraint is always binding. As shown in section 3, solving without the liquidity constraint (in which case the individual rationality constraint is obviously binding) implies that the contractual variables \((b_T, \beta_T)\) satisfy (13) and (12) and therefore the payment \(w_i\) may or may not satisfy the liquidity constraint in all states depending on the values of parameters \(w, \bar{w}, a\) and \(d\). Therefore, when the individual rationality constraint is binding, the liquidity constraint is binding or non binding (the latter when \(w\) is relatively low).

Note that the regularity condition \((n - 1)a > nd\) requires that agents are of sufficiently high
ability. The proof of Lemma 2, then, shows that the principal's profit is increasing in the bonus factor $\beta_T$. The rationale why the individual rationality constraint is always binding for the tournament case but not for the piece rate case is that profit is decreasing in the piece rate $\beta_R$. Therefore, unlike the tournament case in which the principal benefits by increasing the bonus factor $\beta_T$ until it yields no rents to the agent, in the piece rate case the principal may prefer to provide the agent with rents in order to increase his profit. Thus, there is a fundamental difference between tournaments and piece rates in this respect, which drives the results in our paper.

We start by analyzing the piece rate case. The piece rate scheme can be written as $w_i = b_R + \beta_R (a + e_i + v_i)$. As Lemma 1 indicates, the individual rationality constraint can be binding or not. Because of this, the procedure for determining the contractual variables is somewhat different than the one we followed above for the case without liquidity constraints (without liquidity constraints the individual rationality constraints are always binding). With liquidity constraints, we determine the base payment $b_R$ through these constraints, and the piece rate $\beta_R$ from the profit maximizing condition. Then we check whether this solution satisfies the individual rationality constraints.

Clearly, if the payment satisfies the liquidity constraint (18) in the lowest possible state, then, it satisfies the constraint in all states because the payment scheme is increasing in the state. Therefore, if the constraint is binding in the lowest state, then it is non-binding in all states. From the agent's perspective, given that the principal controls incentives through the payment scheme, the worst state is the one in which the principal provides him no incentives to perform and the production state turns out to be the worst, that is, $e_i = 0$ and $v_i = -c$. In the remaining analysis we focus on the case when the liquidity constraint is binding in the lowest possible state.\(^\text{8}\) Therefore, the principal will set

$$b_R = w - \beta_R (a - c).$$

The expected utility for the agent is

$$EU_i = -\int_{-c}^c \exp (-\beta_R v_i) f(v_i) dv_i \exp \left( -w - \beta_R c - \beta_R e_i + \frac{e_i^2}{2a} \right).$$

To provide correct incentives to the agent, the principal calculates the effort level $e_i$ that

\(^{8}\text{Recall that the liquidity constraint is an institutional constraint which prohibits penalizing the agent for obtaining a low output, and it should hold regardless of whether the contract is optimal or not. The agent's optimal response under the contract should not be included in the calculation of the required base wage, because we cannot assume the optimal contract in setting up the constraint. Instead, the constraint determines the agent's optimal response under contract and the optimal contract.}\)
maximizes (20). First order conditions yield

$$-\int_{-c}^{c} \exp (-\beta_R v_i) f(v_i) dv_i \exp \left(-w - \beta_R c - \beta_R e_i + \frac{e_i^2}{2a}\right) \left(-\beta_R + \frac{e_i}{a}\right) = 0$$

(21)

and, because \(\int_{-c}^{c} \exp (-\beta_R v_i) f(v_i) dv_i\) and \(\exp \left(-w - \beta_R c - \beta_R e_i + \frac{e_i^2}{2a}\right)\) cannot be equal to zero, it follows that

$$e_i = a\beta_R.$$  

(22)

The principal’s profit per agent is \(\Pi_i = (1 - \beta_R) x_i - b_R = a + a\beta_R - a\beta_R^2 - w - \beta_R c + (1 - \beta_R) v_i\). Then the expected profit per agent is

$$E\Pi_R = a + a\beta_R - a\beta_R^2 - w - \beta_R c.$$  

(23)

Maximizing the expected profit with respect to \(\beta_R\) yields

$$\beta_R = \frac{a - c}{2a}.$$  

(24)

Hence, given the contractual variables and the optimal effort level for the agent, the expected profit per agent is

$$E\Pi_R = \frac{5}{4} a + \frac{1}{4} c^2 - \frac{1}{2} c - w.$$  

(25)

Note that condition (25) indicates that the principal will make an offer only if \(w\) is relatively low, otherwise production is unprofitable.

Given conditions (19) and (24), the individual rationality constraint requires

$$-\int_{-c}^{c} \exp \left(-\frac{a - c}{2a} v_i\right) f(v_i) dv_i \exp \left(-w - \frac{3}{4} c - \frac{5}{8} c^2 + \frac{1}{4} a\right) \geq -\exp(-\pi).$$

(26)

Clearly, (26) may or may not hold, depending on the values of parameters \(w, \bar{w}, a\) and \(c\). If it holds, then the contractual variables to be offered by the principal satisfy (19) and (24). If (26) does not hold, that is, if \(\beta_R\) in (24) violates the individual rationality constraint, then the individual rationality constraint is binding. In this case, \(\beta_R\) must be determined through the individual rationality constraint with equality. Given (20), (22) and the density function for \(v_i\), the individual rationality constraint is written as

$$\int_{-c}^{c} \exp (-\beta_R v_i) \frac{e - |v_i|}{e^2} dv_i \exp \left(\frac{1}{2} a\beta_R^2 - \beta_R c\right) \exp(-w) = \exp(-\pi).$$

(27)
Given that $c > 0$, (27) is equivalent to

$$
\frac{1 + \exp (2\beta Rc) - 2 \exp (\beta Rc) \exp (-\beta Rc)}{\beta^2 Rc^2} \exp \left( \frac{1}{2} a^2 \beta^2 Rc - \beta Rc \right) \exp (\overline{w} - w) - 1 = 0. \quad (28)
$$

A closed form solution for $\beta_R$ is impossible to obtain from (28). As a result we have to rely on computational methods in order to determine the piece rate values $\beta_R$ which are individually rational. Our computations proceed as follows: We derive the contractual variables from equations (19) and (24) assuming that the liquidity constraint is binding in the lowest state and ignoring the individual rationality constraint. Then we check if the individual rationality constraint (26) is satisfied by the solution (in which case it is non-binding) or if it is violated (in which case it is binding). If (26) is found to be binding, then the piece rate $\beta_R$ is derived by the solution of (28) using a Newton algorithm and $b_R$ is still determined by (19). In this case, when we have multiple solutions for $\beta_R$, we keep the one maximizing the principal’s profit. If (26) is found to be non-binding we keep the solutions from equations (19) and (24).

Next, we turn to the tournament case. Recall that under the tournament the compensation to each agent is $w_i = b_T + \beta_T (x_i - \overline{x})$, which can be written as $w_i = b_T + \beta_T (e_i - \overline{e}) + \beta_T \vartheta_i$, where $\vartheta_i \equiv e_i - \overline{e}$, with $\overline{e}$ denoting the average effort and $\overline{e}$ denoting the average idiosyncratic shock. Given that the agents are homogeneous, the contract is uniform for all agents and the optimal effort level is equal in equilibrium for all agents. Thus, the compensation to each agent can be expressed as $w_i = b_T + \beta_T \vartheta_i$. As shown in the proof of Lemma 2,

$$
b_T = \overline{w} + \beta_T d. \quad (29)
$$

Similar to piece rates, if the liquidity constraint is binding in the lowest state, then it is non-binding in all states, because the payment under tournament is also increasing in the state. The agent’s expected utility is

$$
EU_i = -\int_{-d}^{d} \exp (-\beta_T \vartheta_i) f(\vartheta_i) d\vartheta_i \exp \left( -\overline{w} - \beta_T d - \beta_T \frac{n-1}{n} e_i + \beta_T \frac{1}{n} \sum_{j \neq i}^{n} e_j + \frac{e_i^2}{2a} \right). \quad (30)
$$

The effort level that maximizes the agent’s expected utility satisfies

$$
-\int_{-d}^{d} \exp (-\beta_T \vartheta_i) f(\vartheta_i) d\vartheta_i \exp \left( -\overline{w} - \beta_T d - \beta_T \frac{n-1}{n} e_i + \beta_T \frac{1}{n} \sum_{j \neq i}^{n} e_j + \frac{e_i^2}{2a} \right) \cdot \left( -\beta_T \frac{n-1}{n} + \frac{e_i}{a} \right) = 0. \quad (31)
$$

Because the product of the first two terms in the equation cannot be equal to zero, it follows
that
\[ e = \frac{n - 1}{n} a\beta_T. \]  

(32)

Given Lemma 2, which states that the individual rationality constraint is always binding under tournaments with liquidity constraints, the principal chooses the value of the piece rate \( \beta_R \) that satisfies the agent’s individual rationality constraint with equality. Thus, (29) and (32) imply

\[ EU_i = -\int_{-d}^{d} \exp(-\beta_T \vartheta_i) f(\vartheta_i) d\vartheta_i \exp \left( -w - \beta_T d + \frac{1}{2} \left( \frac{n - 1}{n} \right)^2 a\beta_T^2 \right) = -\exp(-\overline{u}). \]  

(33)

Note that in equilibrium \( x_i - \overline{x} = \vartheta_i \overset{D}{\rightarrow} uniform[-d, d] \), when the number of agents is sufficiently large. Hence, \( \int_{-d}^{d} \exp(-\beta_T \vartheta_i) f(\vartheta_i) d\vartheta_i \) converges to

\[ \int_{-d}^{d} \exp(-\beta_T \vartheta_i) \frac{1}{2d} d\vartheta_i = \frac{\exp(\beta_T d) - \exp(-\beta_T d)}{2\beta_T d} \]  

(34)

Then, (33) becomes

\[ -\frac{\exp(\beta_T d) - \exp(-\beta_T d)}{2\beta_T d} \exp \left( -w - \beta_T d + \frac{1}{2} \left( \frac{n - 1}{n} \right)^2 a\beta_T^2 \right) = -\exp(-\overline{u}) \iff \]

\[ \exp \left( \frac{1}{2} \left( \frac{n - 1}{n} \right)^2 a\beta_T^2 - \beta_T d \right) \exp(\overline{u} - w) = \frac{2\beta_T d}{\exp(\beta_T d) - \exp(-\beta_T d)}. \]  

(35)

Clearly, similar to the piece rate case, equation (35) has no closed form solution. A solution can only be obtained by computational methods (recall that we use a Newton algorithm). The principal’s profit per agent is \( \Pi_i = x_i - b_T = a - w + \left( \frac{n - 1}{n} a - d \right) \beta_T + \varepsilon_i + \eta \). Hence, given the optimal base payment and the optimal effort level for the agent, the expected profit per agent is

\[ E\Pi_T = a - w + \left( \frac{n - 1}{n} a - d \right) \beta_T. \]  

(36)

where \( \beta_T \) can only be determined numerically by solving (35).

5. The Dominant Contract Under Liquidity Constraints

The principal’s decision about which compensation scheme to offer depends entirely on expected profits. Clearly, under both schemes, expected profits decline when a liquidity constraint is introduced in addition to the other constraints. Our analysis indicates that these profits decline faster under piece rates as the liquidity constraint becomes tighter. The in-
tuition behind our result is that the liquidity constraint distorts the agent’s incentives to perform because it reduces the penalty the principal can impose for unfavorable outcomes. Therefore, the principal needs to provide higher-power incentives. By filtering common shocks from the responsibility of the agent, tournaments make the agent more tolerant to higher-power incentives, hence, it is easier for the principal to implement higher-power incentives under tournament than under piece rates. Moreover, the piece rate $\beta_R$ cannot exceed 1 (i.e., because marginal cost cannot exceed marginal revenue). By contrast, the bonus factor $\beta_T$ can exceed 1 which enables the implementation of higher-power incentives.

Figure 1 illustrates that tournaments are dominant over piece rates when liquidity constraints are introduced. In particular, panel (a) shows that expected profit is always strictly larger under tournament regardless of the value of $w$, that is, regardless of how tight the liquidity constraint is. Note that in our numerical analysis we assume that condition (15) holds, that is, we assume that common uncertainty is sufficiently large relative to the idiosyncratic uncertainty. For the case without liquidity constraints expected profits per agent are calculated by using conditions (7) and (14). For the case with binding liquidity constraints expected profits per agent are calculated by using condition (23) where $\beta_R$ is determined either by (24) or by the numerical solution of (28), and condition (36) where, again, $\beta_T$ is determined numerically by (35). Obviously, for the range over which the liquidity constraint is non-binding, expected profit is flat and independent of $w$. We confirmed this result for all possible values of common uncertainty that satisfy condition (15). A sufficient increase in the minimum permissible wage $w$ decreases the expected profit under both schemes, but it does so much faster under piece rates. In fact, piece rates cannot be defined at all after a critical value of $w$ is passed (see point B in panel (a)), because the principal needs to offer a piece rate larger than 1 to provide correct incentives to the agent. However, given that $\beta_R$ cannot exceed 1, piece rates cannot be defined. In interpreting the results depicted in Figure 1, note that for $w$ in the range up to A the individual rationality constraint under piece rates is binding and the liquidity constraint is non-binding. For $w$ in the AB range the individual rationality constraint under piece rates is binding or non-binding and the liquidity constraint is binding. Under tournaments, the individual rationality constraint is always binding (see Lemma 2). Lastly, for $w$ in the range up to C the limited liability constraint under tournament is non-binding.

Panel (b) indicates that the base payment is always larger under tournament, but it increases with $w$, that is, when the minimum acceptable payment increases the base payment must also increase to provide correct incentives to the agent to participate. Further, panel (c) indicates that both the piece rate $\beta_R$ and the bonus factor $\beta_T$ increase when $w$ increases.

9In other words, the principal does not find it profitable to make an offer that the agent will accept.
Figure 1: The expected profit per agent and the contractual variables for the piece rate contract and the tournament.
There are two reasons for this: First, because the base payment increases when $w$ increases, the principal must provide the agents with higher-power incentives in order to exert more effort and make up in lost profit due to the increase in the base payment. Second, when $w$ increases, the principal provides the agents with higher-power incentives in order to minimize the likelihood that output is low and the principal is forced by the liquidity constraint to pay the minimum acceptable wage to the agent when, absent the constraint, it would have been optimal to pay less or impose a penalty. Again, note that piece rates are not defined for a piece rate above 1, whereas under tournaments the principal can continue to provide incentives to the agents through a bonus factor larger than 1, which explains the increased dominance of tournaments over piece rates for large values of the minimum acceptable payment $w$.

6. Conclusion

A familiar result in the principal-agent literature is that when agents are risk averse and production is subject to relatively large common shocks the tournament is a superior compensation scheme to the piece rate. The superiority of tournaments over piece rates may not survive under liquidity constraints. Prior research (for instance, Marinakis and Tsoulouhas (2007) for limited liability on the principal) would lead someone to expect the same result even when limited liability is imposed on the agent instead of the principal. In addition, one might also expect that limited liability would make the agents more tolerant to risk (in the sense that liquidity constraints convexify the agent’s utility function) and the principal less tolerant to risk (in the sense that the principal cares about the allocation of payments across states in order to satisfy the liquidity constraints). The reduced interest of agents in getting insurance, as well as the reduced ability of the principal to provide it, might also diminish the scope for tournaments. However, there is a fundamental difference between limited liability on the principal side and limited liability on the agent side. Under limited liability for the principal, the agents cannot be suckered by the prospect of payments the principal cannot make, therefore, the principal introduces a constraint to provide correct incentives to the agents. The constraint puts a maximum on the payment to the agents in low states and, hence, the solution looks like that if the principal were risk averse. Under liquidity constraints for the agents, instead, the agents will not sign a contract with the principal unless it satisfies these constraints and the principal incorporates the constraints to make sure that the agents participate. The constraints put a minimum on the payments to the agents in low states. Our analysis builds on this fact to show that in the presence of common uncertainty a principal contracting with risk averse agents will prefer to offer a tournament even when agents are liquidity constrained.

The rationale for our result is that by providing insurance against common shocks through a tournament, so that payments to the agents in unfavorable states increase and
payments in favorable states decrease, the principal can satisfy tight liquidity constraints for the agents without paying any ex ante rents to them while simultaneously providing them with higher-power incentives than under piece rates. The larger the minimum payment satisfying an agent’s liquidity constraint, the higher the power of incentives the principal provides. In other words, the principal counterbalances the increase in the base payment, which is required to satisfy the liquidity constraint, with higher-power incentives in order to curb agent rents and in order to reduce the likelihood that output is low. Tournaments provide the principal with added flexibility in the determination of this power.
References


