Labor-market Volatility in a Matching Model with Worker Heterogeneity and Endogenous Separations

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Abstract

This paper studies the business-cycle behavior of a matching model that allows for heterogeneity in worker productivity and endogenous separations. A general shortcoming of the models that incorporate variation in the separation rate is that they predict a countercyclical vacancy rate, which is at odds with the data. I show that allowing for worker heterogeneity into the model helps reconcile the model with the data. For reasonable amounts of variation in the separation rate, the model can generate sufficiently large variation in the unemployment rate and much higher volatility in the vacancy rate than implied by models that incorporate variation in the separation rate, but assume a homogeneous labor force. With reasonable parameter values, the model not only does not predict a countercyclical vacancy rate, but also generates fluctuations in both the unemployment and vacancy rate that are consistent with the data.

JEL classification: E32; E24

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1 Introduction

An issue that has received considerable attention is whether search and matching models can generate business-cycle fluctuations in key labor-market variables that are in line with the data. This interest has been triggered by the work of Shimer (2005) who argued persuasively that the textbook matching model fails to account for the observed volatility of unemployment and vacancy rates. Recently, the extent to which the incorporation of variation in the separation rate, which in the data accounts for about one third of the business-cycle variation in the unemployment rate, can improve the model’s ability to match the data, received a lot of attention. However, a serious shortcoming of the models that allow for variation in the separation rate is that the increase volatility of the unemployment rate, due to countercyclical movements in the separation rate, comes at the cost of a much lower volatility in the vacancy rate. This is because a rise in unemployment due to a spike in job destruction, lowers the duration of vacancies and thus the average vacancy cost, thereby stimulating the growth of vacancies during bad times. As a consequence, these models predict a countercyclical vacancy rate, which is at odds with the data.

This failure rests on the assumption that the workers who are laid off in a recession can form new more productive matches even if aggregate productivity is still low. The most widely used model that allows for variation in the separation rate is due to Moretensen and Pissarides (1994). In the MP model, in a recession firms become more selective about the idiosyncratic productivity of the matches they wish to retain, resulting in a higher separation rate. The labor force is homogeneous and match productivity is drawn from an exogenous distribution, which is unrelated to aggregate productivity. Consequently, spikes in job destruction have no impact on the composition of unemployed in terms of productivity. Instead, they just raise the number of unemployed, thereby raising the arrival rate or searching workers to vacant firms, and thus stimulating job creation.

In this paper I argue that since the workers that are laid off in in bad times are those that are no longer sufficiently productive to be worthwhile for firms to stay matched with them, it is conceptually more appealing to write down a model in which spikes in job destruction dampen instead of facilitating job creation. The model I consider
offers an improvement along this dimension by incorporating heterogeneity in worker productivity. In the model I study the rise in the mass of matches that are no longer worthwhile to operate during bad times, and as a consequence, the rise in unemployment due to the destruction of such matches, does not facilitate the creation of new matches, unless aggregate productivity improves. This is because the workers who are laid off in downturns are those that are not productive enough to be worthwhile for firms to hire them. Instead, the larger flow of low-productivity workers into unemployment due to a spike in job destruction congests the market making more difficult for firms to recruit.

I show that with with a reasonable variation in the separation rate the model can generate sufficiently large variation in the unemployment rate and much higher volatility in the vacancy rate than what is implied by other models that allow for fluctuations in the separation rate. With reasonable parameter values the model not only does not predict a countercyclical vacancy rate, but also generates fluctuations in both the unemployment and the vacancy rate that are consistent with the data.

A growing number of papers respond to Shimer’s (2005) critique by attempting to generate higher volatility for the unemployment rate by incorporating separation rate fluctuations. In some cases, such as in Yashiv (2006) and Mortensen and Nagypal (2007a) the incorporation is carried out mechanically by allowing for the separation rate to follow an exogenous stochastic process. Others, such as Mortensen and Nagypal (2007b) and Silva and Toledo (2009) follow Mortensen and Pissarides (1994) and model variations in the separation as a result of the optimizing response of firms to changes in aggregate productivity. Surprisingly enough, allowing for endogenous instead of exogenous fluctuations in the separation rate, as in the MP model does not contribute to more volatility in the vacancy rate. As Pissarides (2009) points out, this is due to the envelope property characterizing optimal job destruction. The jobs that are destroyed when there is a negative productivity shock are those whose productivity is the reservation one, i.e., whose expected profit is zero. The firm is indifferent between making these jobs active or inactive, thus countercyclical movements in job destruction have no impact on the volatility of the surplus firms expect to generate from opening vacancies. Papers that consider exogenous changes in the separation rate find more volatility in job creation, because an
exogenous increase in the separation rates translates into an increase in the rate at which future profits are discounted. However, this discount rate effect is very small, thus both the model with exogenous and the model with endogenous fluctuations in the separation rate grossly underpredict the volatility of the vacancy rate.

The literature also proposes several variations of the textbook model that can help amplify the response of vacancies. In general, whenever separation shocks are excluded, the model fails to account for both the variability of vacancies and unemployment observed in the data, unless some sort of wage stickiness is introduced into the model. The main response of the literature to the unemployment volatility puzzle was therefore to study more closely the type of wage contract used in the model. However, the view that wage stickiness is the answer to the puzzle was recently challenged by Pissarides (2009) who demonstrates that there is as much cyclicity in the empirical wage equations for new jobs as in the Nash wage equation of the standard matching model. Pissarides analysis therefore suggests that wage stickiness is not the answer to increasing the model’s unemployment volatility, and that future work should consider other extensions. One such extension proposed by Pissarides is the incorporation of a fixed hiring cost. However, as I demonstrate in this paper, with fixed hiring costs the model overpredicts the volatility of the vacancy rate and underpredicts the volatility of the unemployment rate, suggesting that it misses on the share of variation in market tightness due to variation in the separation rate. My paper contributes to the literature by showing that incorporating heterogeneity in worker productivity in a model with endogenous variation in the separation rate goes a long way in improving the models ability to match jointly the behavior of vacancies and unemployment.

In Section 2 I describe the model under study. Section 3 characterizes the steady state equilibrium. In Section 4 I present comparative static results that characterize the response of key labor-market variables to aggregate productivity shocks. In section 5 I illustrate some comparative static results in some variations of the matching model found in the literature and compared them to the results of my model. Section 6 further discusses the model’s results by presenting some quantitative results. Finally, section 7

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1 For an extensive review of this literature see Mortensen and Nagypal (2007a) and Pissarides (2009).
2 See for example, Hall (2005), Hall and Milgrom (2008).
2 The Model

Consider an economy with N different types of workers indexed by $i \in \{1, ..., N\}$, and a large number of firms. Each type of worker accounts for $1/N$-th of the total population. In any period a worker may be either matched with a firm or unemployed, while a firm may be either matched with a worker and producing or unmatched and posting a vacancy.

When a worker of type $i$ is matched with a firm, he can produce $yx^i$ units of output, where $y$ is aggregate productivity and $x^i$ is worker-specific productivity. The structure of the worker-specific productivity component is such that $x^1 < x^2 < .... < x^N$. Unemployed workers receive a flow benefit of $b$ per period. Firms that post a vacancy pay a recruiting cost of $c$ per period.

Let $u^i_t$ and $v_t$ denote the number of unemployed workers of type $i$ and posted vacancies, respectively, in period $t$. The total number of contacts between searching workers and firms in period $t$ is determined by a matching function $m(v_t, u_t)$, where $u_t = \sum_{i=1}^{N} u^i_t$ gives the total number of unemployed workers in period $t$. The probability that a worker contacts a firm can be written as $m(\theta_t)$, where $\theta_t = \frac{v_t}{u_t}$ measures the tightness of the labor market. Likewise, a vacancy is contacted by a worker (of any type) with probability $q(\theta_t)$ and by a worker of type $i$ with probability $q(\theta_t) \frac{u^i_t}{u_t}$.

Once a searching worker and firm meet, they negotiate on a contract that divides the surplus of the match between the worker and the firm in fixed proportions in line with Nash Bargaining. The worker’s bargaining weight is $\beta$ and the disagreement point is the severance of the match. Let $S^i_t$ denote the surplus of a match with a worker of type $i$ in period $t$. The worker and the firm will agree to continue the match only if $S^i_t > 0$. While they will separate if separation is jointly optimal, in which case $S^i_t \leq 0$. Aside from the jointly optimal separations, matches also face a risk of exogenous separation with a probability $s$. 
2.1 Value Functions

The values of unemployment and unemployment for a worker of type \( i \), solve

\[
U^i_t = b + \gamma E_t \left[ m(\theta) \beta S^i_{t+1} + U^i_{t+1} \right] \tag{1}
\]

\[
W^i_t = w^i_t + \gamma E_t \left[ s \beta S^i_{t+1} + W^i_{t+1} \right] \tag{2}
\]

where \( \gamma = \frac{1}{1+r} \) is the discount factor. The value of a match with a worker of type \( i \) to a firm is given by

\[
J^i_t = y_t x^i - w^i_t + \gamma E_t \left[ s (1-\beta) S^i_{t+1} + J^i_{t+1} \right] \tag{3}
\]

Finally, the value of a vacancy satisfies:

\[
V_t = -c + \gamma E_t \left[ (1-\beta) \sum_{i=1}^{N} \frac{u^i_t}{u_t} S^i_{t+1} + V_{t+1} \right] \tag{4}
\]

2.2 Lows of Motion

The unemployment rate of a worker of type \( i \) evolves according to

\[
u^i_{t+1} = \left( \frac{1}{N} - u^i_t \right) s - u^i_t m(\theta) I + \delta^i \tag{5}
\]

where \( \delta^i \) captures discrete jumps from employment to unemployment due to endogenous job destruction, and \( I \) is an indicator function which takes the value of 1 if the next period’s state is such that the worker can form a match that generates a positive surplus and 0 otherwise. More precisely: \( \delta^i = 0 \) and \( I = 1 \), if \( S^i_{t+1} > 0 \), but \( \delta^i = (1-s) \left( \frac{1}{N} - u^i_t \right) \) and \( I = 0 \), otherwise.

2.3 Equilibrium

In free entry equilibrium \( V_t = 0 \) holds for all \( t \); thus \( \theta_t \) is determined by:

\[
\frac{c}{q(\theta_t)} = \gamma E_t (1-\beta) \sum_{i=1}^{N} \frac{u^i_t}{u_t} S^i_{t+1} \tag{6}
\]

The match surplus satisfies:

\[
S^i_t = \max \{ y_t x^i - b + \gamma E_t S^i_{t+1} [1 - s - \beta m(\theta)], 0 \} \tag{7}
\]

Equations (5) to (7) determine the equilibrium path of \( \theta_t \) for given realizations of the aggregate state and distribution of unemployment across worker types \( \{u^1_t, u^2_t, ..., u^N_t\} \).
3 Steady State

Next, I characterize the steady state equilibrium where the distribution of workers across unemployment and the aggregate state $y$ is constant.

In a steady state equilibrium there exists a cutoff productivity $x^n$ such that $S^i \geq 0$ if and only if $x^i \geq x^n$. To understand why notice that the steady-state surplus of a match with a worker of type $i$ satisfies

$$S^i = \max\{\frac{yx^i - b}{r + s + \beta m(\theta)}, 0\}$$

Hence, $S^i \geq 0$ if and only if $x^i \geq x^n$, where $x^n = \frac{b}{y}$, gives the reservation productivity, which is decreasing in $y$. It follows that it is optimal for a firm to form or continue a match with a worker only if his productivity is higher than $x^n$.

The steady-state distribution of workers across unemployment is given by:

$$u^i = \frac{1}{N} \frac{s}{(s + m(\theta))}$$

if $i \geq n$, and

$$u^i = \frac{1}{N}$$

otherwise. The total number of unemployed is given by

$$u = \frac{(N - n + 1)}{N} \frac{s}{(s + m(\theta))} + \frac{n - 1}{N}$$

which gives:

$$u = \frac{s + Rm(\theta)}{s + m(\theta)}$$

(9)

$R = \frac{(n-1)}{N}$ is the share of the labor force with productivity below the reservation productivity. Since, $x^n$ is decreasing in $y$ then $R$ is also decreasing in $y$.

The total number of separations can be calculated as $\sum_{i=n}^{N} s(\frac{1}{N} - u^i) + \sum_{i=1}^{n-1} (\frac{1}{N} - u^i)$. Dividing this by the total number of employed workers, $1-u$, gives the average separation rate denoted by $\tilde{s}$:

$$\tilde{s} = \frac{s + R(1 - s)}{1 - R}$$

(10)

Evidently, $\tilde{s}$ is increasing in $R$. Hence, the negative relation between the reservation productivity and aggregate productivity implies also a negative relation between the average separation rate and aggregate productivity.
Since only workers whose productivity is above the reservation productivity will get hired once they contact a vacancy, the average job finding rate differs from the contact rate $m(\theta)$. In particular, the average job finding rate can be calculated as $\tilde{m} = \sum_{i=n}^{N} \frac{u^i}{u} m(\theta)$, which gives:

$$\tilde{m} = \frac{sm(\theta)(1-R)}{s + Rm(\theta)}$$

The free-entry condition that determines the steady-state value of $\theta$ is,

$$\frac{c}{q(\theta)} = \gamma(1 - \beta) U \sum_{i=n}^{N} S^i$$

where $U \equiv \frac{u^i}{u} \equiv \frac{1}{N} \sum_{i} \frac{u^i}{u}$ for all $i \geq n$.

The main difference between this model and other models that allow for separation-rate fluctuations is the presence of $U$ in the free entry condition. $U$ measures the probability that the productivity of the worker the searching firm will encounter will be higher than the reservation productivity. Notice that $U$ is decreasing in $R$, which means that the probability of encountering a worker that is worthwhile for the firm to hire is lower the higher the reservation productivity. This, coupled with the fact that the reservation productivity, $x_n(y) = \frac{b}{y}$, is decreasing in $y$, implies that countercyclical movements in the reservation productivity, and thus in the separation rate, lead to countercyclical movements in vacancy duration, thereby amplifying the response of vacancy creation to aggregate productivity shocks. Unlike other models that allow for fluctuations in the separation rate, in this model, a rise in unemployment due a spike in the separation rate does not facilitate job creation. This is because the workers who are laid off in downturns are those whose productivity is not high enough to be worthwhile for the firms to hire them, unless aggregate productivity improves. Instead, these workers congests the market making more difficult for firms to locate workers that are worthwhile for the firms to hire, as captured by the negative relation between $R$ and $U$. The presence of $U$ in the free entry condition therefore captures an additional source of fluctuations in vacancy creation which is absent from existing models that incorporate countercyclical movements in the separation rate.

For the results below, it is also useful to characterize average labor productivity and the replacement ratio, i.e., the ratio of the opportunity cost of employment to the worker
to this average labor productivity. Let the $\tilde{y}$ denote the average productivity among the employed. This is measured as $\tilde{y} = \frac{\sum_{i=1}^{N} (\frac{1}{1-u})yx^i}{N - (n-1)}$, and can be expressed as:

$$\tilde{y} = \frac{y \sum_{i=n}^{N} x^i}{N - (n - 1)}$$

(13)

Finally, the replacement ratio in the model is $\tilde{b} = \frac{b}{\tilde{y}}$.

4 Elasticities

Next, I derive comparative static results that describe how the key labor-market variables in the model respond to changes in aggregate productivity.

By taking logs of the free-entry condition in (12) and differentiating the result with respect to $\ln y$ one obtains the following expression for the elasticity of market tightness with respect to aggregate productivity:

$$\frac{\partial \ln \theta}{\partial \ln y} = \frac{1}{M} \left[ \frac{\partial \ln U}{\partial \ln y} + \frac{\partial \ln \Delta}{\partial \ln y} \right]$$

(14)

where $\Delta = \sum_{n}^{N} (yx^i - b)$, $M = \frac{\alpha(x+s) + \beta m(\theta)}{(r+s+\beta m(\theta))}$, and $\alpha$ is the elasticity of the matching function with respect to unemployment. Suppose that the labor force is homogeneous, so that $x^i = x$ for all $i$, and that $x > \frac{b}{y}$ so that there are no endogenous separations. Under these assumptions the model becomes equivalent to the canonical model analyzed in Shimer (2005) (which I henceforth label as canonical), in which there is only a constant separation rate $s$ and all workers are identical. The elasticity of market tightness with respect to aggregate productivity in this case is given by

$$\frac{\partial \ln \theta}{\partial \ln y} = \frac{1}{M} \frac{\partial \ln \Delta^c}{\partial \ln y}$$

(15)

where $\Delta^c = yx - b$. Assuming that both models are calibrated in the same way then it should be the case that $x = \frac{\sum_{n}^{N} x^i}{N - (n-1)}$ so that $\frac{\partial \ln \Delta^c}{\partial \ln y} = \frac{\partial \ln \Delta}{\partial \ln y}$. However, even in this case the elasticity in (14) differs from the one in (15) by the term $\frac{\partial \ln U}{\partial \ln y}$. If this term is positive then the elasticity of the v-u ratio with respect to aggregate productivity in the model with endogenous separation and heterogeneous labor force is higher than that in the canonical model.
By taking logs and differentiating with respect to $\ln y$ we obtain
\[
\frac{\partial \ln U}{\partial \ln y} = -\left[\left(\frac{(1-\alpha)Rm}{s + Rm}\right) \frac{\partial \ln \theta}{\partial \ln y} + \left(\frac{m(\theta) \frac{n}{N}}{s + Rm}\right) \frac{\partial \ln n}{\partial \ln y}\right]
\] (16)
The first term reflects the fact that a fall in the v-u ratio due to a fall in aggregate productivity raises unemployment thus making it easier for firms to locate workers they are willing to recruit. The second term captures the impact of countercyclical movements in the reservation productivity. In a recession firms become more selective about the productivity of the workers they wish to continue being matched with, resulting in spike in job destruction. The workers that enter unemployment due to this spike in job destruction are those that are no longer sufficiently productive for firms to be willing to recruit them, unless aggregate productivity improves. These workers congest the market making it more difficult for the firms to locate workers who are more productive and thus better suited for their vacancies. Evidently, if the second effect dominates, so that $\frac{\partial \ln U}{\partial \ln y} > 0$, then the rise in unemployment in a recession has a smaller positive impact on job creation than in the canonical model, thus raising the elasticity of the v-u ratio with respect to aggregate productivity.

Substituting (16) into (14) gives:
\[
\frac{\partial \ln \theta}{\partial \ln y} = \left[-\frac{m(\theta) \frac{n}{N}}{s + Rm} \frac{\partial \ln n}{\partial \ln y} + \frac{\partial \ln \Delta}{\partial \ln y}\right] \left[M + \frac{(1-\alpha)m(\theta)R}{s + Rm}\right]
\] (17)

The elasticity of unemployment with respect to aggregate productivity can be written as:
\[
\frac{\partial \ln u}{\partial \ln y} = \frac{\partial \ln U}{\partial \ln y} - \frac{(1-\alpha)m(\theta)}{s + m(\theta)} \frac{\partial \ln \theta}{\partial \ln y}
\] (18)
In the canonical model the elasticity of unemployment with respect to aggregate productivity is given by the second term in (18), only. This means that if $\frac{\partial \ln U}{\partial \ln y} > 0$, then for the same response of market tightness, the elasticity of unemployment in this model is higher than in the canonical model. With (16) substituted in, the elasticity becomes
\[
\frac{\partial \ln u}{\partial \ln y} = -\frac{(1-\alpha)m(\theta)}{s + m(\theta)} \left[\frac{s(1-R)}{s + Rm(\theta)} \frac{\partial \ln \theta}{\partial \ln y} + \frac{m(\theta) \frac{n}{N}}{s + Rm(\theta)} \frac{\partial \ln n}{\partial \ln y}\right]
\] (19)
The first term captures the effect of changes in aggregate productivity on the unemployment rate through the impact of such changes on the v-u ratio. The second term reflects...
the changes in unemployment due to countercyclical movements in the reservation productivity. Clearly, the negative response of the reservation productivity implies a larger elasticity of the unemployment with respect to aggregate productivity shocks than in the canonical model.

The elasticity of the job finding rate, \( \hat{m} \), with respect to aggregate productivity can be expressed as:

\[
\frac{\partial \ln \hat{m}}{\partial \ln y} = \left[ \frac{s(1 - \alpha)}{s + Rm(\theta)} \right] \frac{\partial \ln \theta}{\partial \ln y} - \frac{n}{N(1 - R)} \left[ \frac{s + m(\theta)}{s + Rm(\theta)} \right] \frac{\partial \ln n}{\partial \ln y} \quad (20)
\]

The job finding rate responds to changes in aggregate productivity due to the impact of such changes on both the v-u ratio (first term) and the reservation productivity (second term).

The separation rate responds to aggregate productivity shocks due to the impact of such changes on the reservation productivity. Specifically,

\[
\frac{\partial \ln \hat{s}}{\partial \ln y} = \frac{n}{N} \frac{1}{(1 - R)(s + R(1 - s))} \frac{\partial \ln n}{\partial \ln y} \quad (21)
\]

Finally, the elasticity of vacancies with respect to aggregate productivity satisfies

\[
\frac{\partial \ln v}{\partial \ln y} = \frac{\partial \ln \theta}{\partial \ln y} + \frac{\partial \ln u}{\partial \ln y} \quad (22)
\]

which gives:

\[
\frac{\partial \ln v}{\partial \ln y} = \frac{1}{M} \frac{\partial \ln \Delta}{\partial \ln y} \left[ \frac{s + \alpha m(\theta)}{s + m(\theta)} \right] + \frac{\partial \ln U}{\partial \ln y} \left[ \frac{1}{M} \frac{s + \alpha m(\theta)}{s + m(\theta)} - 1 \right] \quad (23)
\]

The second term in (23) is absent from the corresponding elasticity in the canonical model. Apparently, if \( \frac{\partial \ln U}{\partial \ln y} > 0 \), the elasticity in (23) is higher than that in the canonical model, only if the term in the second bracket is positive. However, using Shimer’s (2005) parameter values, I find that this term is small, but negative. Despite the fact that the countercyclical movements in job creation costs (i.e., \( \frac{\partial \ln U}{\partial \ln y} > 0 \)) amplify the response of the v-u ratio, the volatility of vacancies is still smaller than that in the canonical model.

To see why notice from (14) and (18) that an increase in \( \frac{\partial \ln U}{\partial \ln y} \) raises \( \frac{\partial \ln n}{\partial \ln y} \) by less than it raises \( \frac{\partial \ln \theta}{\partial \ln y} \), meaning that it has a positive impact on \( \frac{\partial \ln u}{\partial \ln y} \). Specifically, a 1 percent increase in \( \frac{\partial \ln U}{\partial \ln y} \) raises the former by only 1 percent and the latter by \( \frac{1}{M} > 1 \)
percent. However, since the unemployment rate is negatively correlated with $m(\theta)$, the \( \frac{1}{M} \) percent increase in the elasticity of the v-u ratio implies an additional increase in the elasticity of the unemployment rate by \( \frac{1}{M} (1-\alpha) m(\theta) (1 - \frac{s}{s + m(\theta)}) \) percent, as captured by the second term in (18). Using Shimer’s parameter value this percentage increase is sufficiently large so that the increase in the elasticity of the unemployment rate dominates the increase in the elasticity of the v-u ratio. Thus, the model extended to include heterogeneity in worker productivity and endogenous job destruction shocks has the potential to explain the volatility of unemployment, and to generate a large enough change in market tightness relative to that in labor productivity, but does not do better than the canonical model in explaining the behavior of vacancies.

This does not mean, however, that these features have no effect on the model’s business cycle performance. As shown in Section 5, the model with a constant separation rate can match the volatility of vacancies by introducing a large enough hiring cost, but still fails to account for the volatility of unemployment. Countercyclical movements in the separation rate are therefore necessary in order to explain the volatility of unemployment, as it is also suggested by evidence. Hence, matching models that feature countercyclical movements in the separation rate have more potential to jointly match the variation in unemployment and vacancies. However, as mentioned earlier, a general shortcoming of these models is that countercyclical movements in the separation rate induce procyclical movements in vacancy duration, which dampen the response by vacancies. As I show in Section 6, since the model developed in this paper offers an improvement along this dimension, it outperforms existing models when it comes to explaining jointly the behavior of unemployment and vacancies. Specifically, with a reasonable variation in the separation rate, the model can generate sufficiently large variation in the unemployment rate and much higher volatility in the vacancy rate than that in existing models that incorporate fluctuations in the separation rate.

To better understand why this model performs better in explaining jointly the behavior of vacancies and unemployment note that by substituting (16) into (23) we can write the elasticity of the vacancy rate with respect to aggregate productivity as:

\[
\frac{\partial \ln v}{\partial \ln y} = \left[ 1 - \frac{(1 - \alpha)m(\theta)}{s + m(\theta)} \right] \frac{s(1 - R)}{s + R(m(\theta))} \frac{\partial \ln \theta}{\partial \ln y} + \frac{m(\theta)\frac{m(\theta)}{N}}{s + R(m(\theta))} \frac{\partial \ln n}{\partial \ln y} \quad (24)
\]
The first term in (24) shows that a higher elasticity of the v-u ratio implies a higher elasticity of the vacancy rate. The second term shows for a given response of market tightness, the countercyclical movements in the reservation productivity dampen the volatility of the vacancy rate, which is the general shortcoming of models that incorporate variation in the separation rate. However, these countercyclical movements in the separation rate are necessary in order for the model to match the volatility of unemployment with a reasonable amount of variation in the v-u ratio. To the extend that countercyclical movements in the reservation productivity induce countercyclical movements in vacancy duration, then, as explained above, \( \frac{\partial \ln \theta}{\partial \ln y} \) is higher in this model than both in the canonical model and other models that allow for variation in the separation rate. Thus, for a given response of the separation and unemployment rate, this model has more potential to generate a large enough change in the vacancy rate compared to existing models that incorporate separation rate fluctuations, as I show in Section 6.

It is also worth mentioning here that the exact correspondence between changes in aggregate productivity and changes in labor productivity that is present in the canonical model does not hold in a model with endogenous separations. Specifically, a percentage increase in the aggregate component of productivity translates into a smaller percentage increase in the average productivity of labor in a model with endogenous than in the canonical model. This is because in a model with endogenous separations an increase in aggregate productivity lowers the reservation productivity, thereby lowering average labor productivity. Since in the model developed here separations are endogenous, this feature is present here as well. The elasticity of labor productivity with respect to aggregate productivity is given by

\[
\frac{\partial \ln \tilde{y}}{\partial \ln y} = 1 + \left[ \frac{n}{N(1-R)} + \frac{bn}{y \sum N x^i} \right] \frac{\partial \ln n}{\partial \ln y} \tag{25}
\]

which is less than one.

This divergence between aggregate and labor productivity creates an additional channel through which the model can generate a larger change in the vacancy rate relative to that in labor productivity than that in the canonical model. When confronting the model with the data, the appropriate measure of the changes in a variable \( z \) relative to
the changes in labor productivity is given by

$$\frac{\Delta_y \ln z}{\Delta_y \ln \tilde{y}} = \frac{\partial \ln z}{\partial \ln \tilde{y}} / \frac{\partial \ln y}{\partial \ln y}$$

(26)

Since for a given volatility of aggregate productivity the volatility of labor productivity is smaller, the change in variable $z$ relative to that in labor productivity is larger in this model than that in the canonical model. Moreover, since both types of models are calibrated to match the empirical volatility of the average productivity of labor, $y$-shocks are larger in this model than in the canonical model. In turn, larger $y$-shocks generate larger fluctuations in the key labor market variables.

5 Variations of the textbook model

In this section I explore some variations of the textbook matching model that have been proposed in the literature. For the purpose at hand, I review the results of models that allow for separation rate fluctuations. I also explore the business cycle properties of a matching model with a constant separation rate, as in the canonical model, but with a fixed hiring costs. Pissarides (2009) demonstrates that the incorporation of a fixed hiring cost into the canonical model improves the model’s ability to explain the volatility of the vacancy rate considerably. This is because in the presence of a fixed hiring cost the average vacancy cost does not rise in proportion to market tightness. However, Pissarides does not tackle the question of how well the model explains the volatility of the unemployment rate. In Section 5.2 I tackle this question.

5.1 Job destruction shocks

The literature considers variations of the textbook matching model in which separations are either exogenous or endogenous. Shimer (2005) and more recently Mortensen and Nagypal (2007a), allow the separation rate to follow an exogenous stochastic process, and explore the implications of variations in the separation rate for labor market volatility. It is conceptually more appealing to model separations as an endogenous outcome of the optimizing response of firms to changes in aggregate productivity. For this reason,
Mortensen and Nagypal (2007b) study the business cycle performance of a generalized version of the MP model with endogenous separations. In what follows I present comparative static results that characterize the response of key labor-market variables to aggregate productivity shocks, both in the model with endogenous and the model with exogenous variation in the separation rate.

5.1.1 Endogenous job destruction shocks

In the MP model with endogenous separations the economy is populated by *ex ante* identical risk-neutral workers of measure one and firms of a large number. All agents discount the future income flows at the rate $r$. Each worker-firm employment match produces output $yx$, where $x$ is match-specific idiosyncratic productivity and $y$ is aggregate productivity. The idiosyncratic productivity is drawn from a common distribution $F(\cdot)$. In each period, a switch in the value of idiosyncratic productivity occurs with a probability $\delta$. The flow opportunity cost of employment to the worker and the flow cost of posting a vacancy are $b$ and $c$, respectively. The flow of new matches is determined by an aggregate matching function $m(v, u)$, where $v$ and $u$ give the number of vacancies and unemployed workers respectively. The elasticity of the matching function with respect to unemployment is $\alpha$. Wages are determined according to Nash bargaining with $\beta$ being the worker’s bargaining share. The payoff to the firm and the worker from being matched is increasing in the value of idiosyncratic productivity. Thus, the worker and the firm choose to adopt a reservation policy: they choose to form and continue any match that has an idiosyncratic productivity $x \geq p$. The reservation productivity $p$ is lower when aggregate productivity $y$ is higher.

As shown in Mortensen and Nagypal (2007b) the MP model with endogenous changes in job destruction has no trouble generating a large enough negative response in the separation rate relative to changes in labor productivity, and generates sufficient volatility in the unemployment rate. In fact, the model generates too much of a relative change in the separation rate. However, the model grossly underpredicts the change in the v-u ratio relative to changes in labor productivity. As the authors also argue, by over-predicting the change in the separation rate and under-predicting the change in the v-u ratio the
model clearly misses on the share of variation in the unemployment rate due to its two margins.

A serious shortcoming of the model that follows from its failure to generate sufficiently large fluctuations in the v-u ratio is that it predicts a countercyclical vacancy rate. To see why note that the elasticity of vacancies in the MP model is given by

$$\frac{\partial \ln v}{\partial \ln y} = \left[ 1 - \frac{(1 - a)\hat{m}}{\hat{s} + \hat{m}} \right] \frac{\partial \ln \theta}{\partial \ln y} + \frac{\hat{m}}{\hat{s} + \hat{m}} \left[ \frac{F'(p)p}{F(p)(1 - F(p))} \right] \frac{\partial \ln p}{\partial \ln y}$$

(27)

where $\hat{m} = m(\theta)(1 - F(p))$, is the average job finding rate, and $\hat{s} = sF(p)$, is the average separation rate.\(^3\) This expression is similar to the expression in (24) that gives the elasticity of vacancies implied by the model with endogenous separations and heterogeneity in worker productivity, developed in this paper. The first term in (27) captures the effect of changes in aggregate productivity on the unemployment rate due to the impact of these changes on the v-u ratio and corresponds to the first term in (24). The second term in (27) captures the impact of countercyclical movements in reservation productivity and corresponds to the second term in (24). Given the negative impact of fluctuations in the reservation productivity on the volatility of the vacancy rate, the model can generate larger fluctuations in the vacancy rate than the canonical model only if it generates a sufficiently larger change in the v-u ratio relative to changes in aggregate productivity than in the canonical model. However, this is not the case. In fact, the response of the v-u ratio to aggregate productivity shocks is observationally equivalent to the one in the canonical model with constant separations. In particular,

$$\frac{\partial \ln \theta}{\partial \ln y} = \frac{1}{M^c} \frac{\partial \ln \Delta^e}{\partial \ln y}$$

(28)

where $\Delta^e = \int_p (yx - b)dF(x)$, $M^c = \frac{c(r + \delta) + b\hat{m}}{(r + \delta + b\hat{m})}$.\(^3\)

The reason the incorporation of endogenous changes in the separation rate does not contribute to more volatility in the v-u ratio than that in the canonical model is the envelope property that characterizes the optimal job destruction. The jobs that are destroyed

\(^3\)This is the generalized version of the MP model studied in Mortensen and Nagypal (2007b) in which initial match productivity is also drawn from a distribution, instead of just being the highest possible, as assumed in the MP model. For simplicity, I assume that the distribution of initial productivity is the same as the subsequent distribution, $F(\cdot)$.\(^4\)
are those whose productivity is the reservation one, i.e., whose expected profit is zero. Thus, the firm is indifferent between making these jobs active or inactive, meaning that changes in the reservation productivity have no impact on the profits firms expect to generate from opening vacancies.

Table 1: Model results in Mortensen and Nagypal (2007b)

<table>
<thead>
<tr>
<th></th>
<th>$\epsilon_{\bar{y}y}$</th>
<th>$\epsilon_{v\bar{y}}$</th>
<th>$\epsilon_{u\bar{y}}$</th>
<th>$\epsilon_{\tilde{m}\bar{y}}$</th>
<th>$\epsilon_{\tilde{s}\bar{y}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>baseline</td>
<td>1.72</td>
<td>-1.96</td>
<td>-3.68</td>
<td>0.70</td>
<td>-3.25</td>
</tr>
<tr>
<td>$\tilde{b} = 0.55$</td>
<td>2.29</td>
<td>-9.15</td>
<td>-11.40</td>
<td>1.18</td>
<td>-11.10</td>
</tr>
<tr>
<td>$\beta = 0.05$</td>
<td>2.02</td>
<td>0.50</td>
<td>-1.52</td>
<td>1.21</td>
<td>-0.43</td>
</tr>
<tr>
<td>$\epsilon_{\tilde{b}\tilde{y}}$</td>
<td>2.13</td>
<td>-2.43</td>
<td>-4.56</td>
<td>0.87</td>
<td>-4.04</td>
</tr>
<tr>
<td>$\tilde{b} = 0.55$</td>
<td>4.44</td>
<td>-17.80</td>
<td>-22.20</td>
<td>2.30</td>
<td>-21.60</td>
</tr>
<tr>
<td>$\beta = 0.05$</td>
<td>3.27</td>
<td>0.81</td>
<td>-2.46</td>
<td>1.96</td>
<td>-0.69</td>
</tr>
<tr>
<td>data</td>
<td>7.56</td>
<td>3.68</td>
<td>-3.88</td>
<td>2.34</td>
<td>-1.97</td>
</tr>
<tr>
<td>canonical</td>
<td>1.72</td>
<td>1.35</td>
<td>-0.45</td>
<td>0.50</td>
<td></td>
</tr>
</tbody>
</table>

Table 1, borrowed from Mortensen and Nagypal (2007b), reports the model-implied elasticities of the key labor-market variables with respect to aggregate productivity ($y$) and labor productivity ($\tilde{y}$) implied by the MP model with endogenous separations (labeled as baseline in the table). The parameter values used to derive these results are those used by Shimer (2005). The table also reports the relevant responses for two variations of the baseline parameterization, the relevant responses implied by the canonical model based on Table 3 in Shimer (2005), and the relevant empirical responses based on Table 1 in Shimer (2005) (labeled as data). As can be seen, for the baseline parameterization the volatility of the v-u rate is much smaller than what is empirically observed, and the vacancy rate is countercyclical. The first variation sets the replacement ratio to 0.55 as opposed to 0.4

As Mortensen and Nagypal (2007b) point out, the empirical equivalent to the elasticity of $x$ with respect to change in $y$ in the model with exogenous separations is the OLS coefficient $\rho_{xy} \frac{\sigma_y}{\sigma_x}$, where $\rho_{xy}$ is the correlation between $\ln x$ and $\ln y$ and $\sigma_x$ is the standard deviation of $\ln x$. Table 1 reports the same OLS coefficients.
in Shimer (2005). Although this improves the change in the v-u ratio, mainly because it lowers the elasticity of labor productivity with respect to aggregate productivity, it comes at the cost of an unrealistically high response of the separation and unemployment rate, which leads to a strongly countercyclical vacancy rate. The second variation shows that allowing for a very small value for the worker’s bargaining share of 0.05 also leads to a decrease in the elasticity of labor productivity with respect to aggregate productivity, and thus also increases the volatility of the v-u ratio. This variation is more promising in the sense that the implied variation in the separation rate remains reasonable and therefore the vacancy rate is procyclical. Still, the magnitude of variation in the vacancy rate is by far smaller than in the data.\footnote{Mortensen and Nagypal (2007b) also report the results of two additional variations of the baseline case in which the distribution of the initial match productivity differs from the distribution of the continuation productivity. The purpose of these two additional variations is to show that if the expected flow surplus under the initial distribution is smaller than under the subsequent distribution then the model-implied elasticity of market tightness is larger than in the canonical (Shimer (2005)) model. For these two variations the vacancy rate is procyclical but it’s volatility is still very small. For the rest of the variables, the predicted volatilities are in relative magnitude and size correct, but the absolute magnitude of the responses is smaller than in both the canonical model and the data.}

### 5.1.2 Exogenous job destruction shocks

The papers that consider fluctuations in the exogenous separation rate \( s \) as a potential source of labor market volatility allow \( s \) to follow an exogenous stochastic process that is negatively correlated with aggregate productivity, while keeping the rest of the assumptions as in the canonical model. In particular, in these models all workers and firms are identical and each worker-firm employment match produces output \( yx \), where \( x \) a constant productivity level and \( y \) is aggregate productivity. The flow opportunity cost of employment to the worker and the flow cost of posting a vacancy are again \( b \) and \( c \), respectively, and the flow of new matches is determined by \( m(v, u) \), which has an elasticity with respect to unemployment \( \alpha \). Wages are determined by Nash bargaining and \( \beta \) is the worker’s bargaining share. Future income flows are discounted at rate \( r \).

Surprisingly enough, models that allow for the separation rate to follow an exogenous
stochastic process perform better than the MP model with endogenous separations in explaining the volatility of vacancies. This is because the fluctuations in the exogenous separation rate $s$ are equivalent to fluctuations in the rate at which firms discount future profits, $r$. Notice from equation (8) that $s$ and $r$ enter the surplus function in the same way, meaning that $r+s$ is the total discount rate agents apply to profits. Hence, countercyclical movements in $s$ imply that the rate at which firms discount future profits is higher in recessions and lower in booms, which in turn, helps amplify the volatility of vacancies.

The elasticity of the v-u ratio with respect to $y$ in the model with exogenous changes in the separation rate is given by:

$$\frac{\partial \ln \theta}{\partial \ln y} = \frac{1}{M} \left[ \frac{\partial \ln \Delta^c}{\partial \ln y} - \frac{s}{r+s+\beta m(\theta)} \frac{\partial \ln s}{\partial \ln y} \right]$$

(29)

The second term in the bracket capture this discount rate effect. The elasticity of unemployment is given by

$$\frac{\partial \ln u}{\partial \ln y} = -\frac{1}{s+m(\theta)} \frac{\partial \ln \theta}{\partial \ln y} + \frac{m(\theta)}{s+m(\theta)} \frac{\partial \ln s}{\partial \ln y}$$

(30)

Evidently, changes in exogenous separation rate amplify the response of unemployment. The second term captures the impact of separation rate shocks on the elasticity of unemployment and corresponds to the second term in (19), whereas the first term gives the impact of procyclical movements in the v-u ratio and is equivalent to the first term in (19). Finally, the elasticity of vacancies is given by

$$\frac{\partial \ln v}{\partial \ln y} = \left[ \frac{s+\alpha m(\theta)}{s+m(\theta)} \right] \frac{1}{M} \frac{\partial \ln \Delta^c}{\partial \ln y} - \frac{\partial \ln s}{\partial \ln y} \left[ \frac{1}{M} \left( \frac{s(s+\alpha m(\theta))}{(r+s+\beta m(\theta))(s+m(\theta))} - \frac{m(\theta)}{s+m(\theta)} \right) \right]$$

(31)

To assess the model’s performance in explaining the business-cycle fluctuations of the key labor-market variables, I perform the following exercise. I parameterize the model as in Shimer (2005) and choose a value for $\frac{\partial \ln s}{\partial \ln y}$ for which the model-implied elasticity of the unemployment rate with respect to labor productivity is equivalent to that in the data. Based on Shimer’s estimates, reported in Table 1, the latter is equal to $-3.88$. I then compare the predicted volatilities with those in the data.

The results of this exercise are summarized in the first column of Table 2. To match the elasticity of the unemployment rate requires setting $\frac{\partial \ln s}{\partial \ln y} = -3.59$ (denoted by $\epsilon_{sy}$ in
Table 2: Model results with exogenous shocks to separations

<table>
<thead>
<tr>
<th></th>
<th>$\epsilon_{sy}$</th>
<th>$\epsilon_{\theta y}$</th>
<th>$\epsilon_{my}$</th>
<th>$\epsilon_{vy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>model</td>
<td>$b = 0.4$</td>
<td>-3.59</td>
<td>2.05</td>
<td>0.58</td>
</tr>
<tr>
<td>model</td>
<td>$b = 0.7$</td>
<td>-3.12</td>
<td>3.73</td>
<td>1.04</td>
</tr>
<tr>
<td>data</td>
<td></td>
<td>-1.97</td>
<td>7.56</td>
<td>2.34</td>
</tr>
</tbody>
</table>

The table), which is much higher than relevant empirical estimate of $-1.97$. It follows that
the model with exogenous changes in the separation rate requires an unrealistically high
response of the separation rate to changes in aggregate productivity in order to match
the volatility of the unemployment rate. But, more importantly, despite the fact that
the assumed elasticity of the separation rate is much higher than the data, the resulting
elasticity of the v-u ratio is by far smaller than that in the data. In particular, the
model predicts an elasticity of 2.05, while empirically it is equal to 7.56. The elasticity of
vacancies is $-1.82$, which is slightly smaller than the -1.96 of the MP model (reported in
Table 1), but still negative.

One way to increase the changes in the vacancy rate relative to changes in labor
productivity is to increase the replacement ratio. Shimer (2005) sets the replacement ratio
to 0.4, but Hagedorn and Manovskii argue that Shimer’s choice is too low, because it does
not allow for the value of leisure or home production. They suggest a replacement ratio
of 0.955, which seems implausibly large and $\beta = 0.052$, which seems implausibly small.
Mortensen and Nagypal (2007a) show that using these values and letting the rest of the
parameters take the values used by Shimer yields an elasticity of market tightness with
respect to labor productivity of 26.83, which is implausibly large. While Hagedorn and
Manovskii’s replacement ratio seems too large, Hall (2006) argues that Shimer’s suggested
replacement ratio is too low. Based on empirical literature on household consumption and
demand, Hall suggests a value of about 0.7.

The second column of Table 2 shows the results when Hall’s suggestion is used. With
the higher replacement ratio the required elasticity of the separation rate is slightly
smaller, but still much larger than that in the data. Moreover, the elasticity of the
v-u ratio is larger, but still much smaller than that in the data.

The results of this exercise suggest that for reasonable parameter values the discount rate effect on the volatility of the expected surplus from opening a vacancy and therefore on the volatility of the v-u ratio is very small. Consequently, the large negative response of the unemployment rate due to an exogenous reduction in the separation rate is accompanied by a countercyclical response in the vacancy rate. The small response of the v-u ratio also explains why matching the volatility of unemployment requires unrealistically large changes in the separation rate. By underpredicting the change in the in the v-u ratio clearly the model misses on the share of variation in the unemployment rate due to changes in the job-finding rate. Thus, to match the volatility of the unemployment rate requires a larger volatility in the separation rate.

5.2 Hiring costs

Pissarides (2009) argues that a feature of the canonical model that diminishes the response of job creation to cyclical productivity shocks is the proportional relation between tightness and the cost of posting vacancies. Suppose there is a positive productivity shock. Firms post more vacancies at cost \( c \) each, but because the duration of vacancies increases that average vacancy cost rises in proportion to the rise in tightness, which dampens the growth of vacancies. He proposes an alternative specification of vacancy costs, which assumes that in addition to the posting cost \( c \) of the canonical model there is an additional fixed cost of vacancy posting, denoted by \( H \). This fixed cost could be the cost of setting up the position, or the cost of interviewing and negotiating with the worker.

It is straightforward to show that when we introduced a fixed hiring cost in the canonical model with constant separation rate the elasticity of the v-u ration with respect to aggregate productivity becomes

\[
\frac{\partial \ln \theta}{\partial \ln y} = \frac{\partial \ln \Delta^c}{\partial \ln y} \left[ \frac{(\theta^a + H)(r + s + \beta m(\theta))}{(\theta^a(\alpha(r + s) + \beta m(\theta)) + H\beta(1 - \alpha)m(\theta))} \right]
\]

If \( H = 0 \) then the term in the bracket becomes equal to \( \frac{1}{M} \) and the elasticity is equivalent to the one in the canonical model, given in equation (15). However, for \( H > 0 \), the term in the bracket is larger than \( \frac{1}{M} \), which implies that the elasticity of the v-u ratio is higher.
than in the canonical model.

Since procyclical movements in the job finding rate induce countercyclical movements in the unemployment rate, the elasticity of the unemployment rate is also larger than in the canonical model. Specifically,

\[
\frac{\partial \ln u}{\partial \ln y} = \frac{(1 - \alpha)m(\theta)}{s + m(\theta)} \frac{\partial \ln \theta}{\partial \ln y}
\]

(33)

so that a larger \(\frac{\partial \ln \theta}{\partial \ln y}\) implies a larger \(\frac{\partial \ln u}{\partial \ln y}\).

The question that follows is whether the variation in the v-u ratio is large enough to predict the observed variation in the unemployment, in the absence of separation rate shocks. The short answer to this question is no, since empirically changes in the separation rate account for about one third in the variation in the unemployment rate. Nevertheless, to investigate this possibility, I choose the value of \(H\) that matches the empirical elasticity of the v-u ratio with respect to labor productivity, and then examine whether the model-implied volatilities of the unemployment and vacancy rates are consistent with those in the data. Once again, for the sake of comparability, the rest of the parameters take the values used in Shimer (2005).

Table 3: Model results with fixed hiring costs

<table>
<thead>
<tr>
<th>(H)</th>
<th>(\epsilon_{my})</th>
<th>(\epsilon_{vy})</th>
<th>(\epsilon_{uy})</th>
</tr>
</thead>
<tbody>
<tr>
<td>model</td>
<td>0.67</td>
<td>2.12</td>
<td>5.59</td>
</tr>
<tr>
<td>data</td>
<td>2.34</td>
<td>3.68</td>
<td>-3.88</td>
</tr>
</tbody>
</table>

The results of this exercise are summarized in Table 3. To match the elasticity of the v-u ratio requires setting \(H = 0.67\).\(^6\) As expected, the volatility of the job finding rate is close to that in the data. However, the model overpredicts the volatility of the vacancy rate and underpredicts the volatility of the unemployment rate. This suggests that fluctuations in the separation rate are necessary for the implied volatility in the unemployment and

\(^6\)Note that if we set a higher replacement ratio than the 0.4 used in Shimer (2005) the value of \(H\) that matches the empirical volatility of the v-u ratio is smaller, but the implied volatilities of the rest of the variables remain unchanged. For instance, if we set the replacement ratio to 0.7, then \(H = 0.24\), but values of the elasticities remain as reported in Table 3.
vacancy rates to be consistent with the data. Recall, that job destruction shocks amplify the response of unemployment to changes in labor productivity, but have a negative impact on the volatility of vacancies. Introducing such shocks can therefore improve the model’s performance in explaining jointly the behavior of vacancies and unemployment.

6 Some Quantitative Results

Next, I present quantitative results of the model with endogenous fluctuations in the separation rate and heterogeneity in worker productivity outlined in Sections 2 to 4. In my baseline calculations I use parameter values used by Shimer (2005) to facilitate direct comparability of my results with the results of Shimer’s canonical model and the variations discussed above. Hence, aggregate productivity is normalized to $y = 1$ and the quarterly discount rate is $r = 0.012$. I set the elasticity parameter to $\alpha = 0.72$, and let worker’s bargaining power take the same value, $\beta = 0.72$. The value of leisure, $b$, the exogenous separation rate, $s$, and the vacancy cost $c$, are set so that: 1) the implied average quarterly separation rate (endogenous and exogenous together) is 0.10, 2) the implied job finding rate is 1.355, 3) the replacement ratio, $\bar{b}$ equals 0.40.

Table 4 reports the model-implied elasticities of the key labor-market variables both with respect to aggregate and with respect to labor productivity, when the elasticity of the reservation productivity, $\epsilon_{ny}$, is such that $\epsilon_{uy} \tilde{y} = -3.88$, as derived based on Table 1 of Shimer (2005) which reports the empirical elasticities. The table also reports the empirical equivalents of the rest of the elasticities based on Table 1 of Shimer (2005), the results of the canonical model, and the results of the model with exogenous changes in the separation rate, when $\epsilon_{sy}$ is such that $\epsilon_{uy} \tilde{y} = -3.88$, as above.\(^7\) There is no obvious empirical counterpart to which the value of $R$ should be matched. For this reason, I derive results for three different values of $R$. I set the number of worker types to $N = 100$ and then set $n = 2$, $n = 3$ and $n = 4$ so that $R = 1, 2$ and 3 percent respectively.

As can be seen, the volatility of the separation rate that matches the empirical volatil-\(^7\) Since for a given volatility of the separation rate the volatility of vacancies implied by the model with exogenous changes in separation rate is larger than in the MP model with endogenous changes in the separation rate, I report only the results of the model with exogenous changes.
Table 4: Model results at different values for $n$- match $\epsilon_{u\tilde{y}}$

<table>
<thead>
<tr>
<th>$R$</th>
<th>$\epsilon_{\theta\tilde{y}}$</th>
<th>$\epsilon_{v\tilde{y}}$</th>
<th>$\epsilon_{\tilde{s}\tilde{y}}$</th>
<th>$\epsilon_{\tilde{m}\tilde{y}}$</th>
<th>$m(\theta)$</th>
<th>$s$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>4.03</td>
<td>1.03</td>
<td>$-1.64$</td>
<td>3.56</td>
<td>1.61</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>2%</td>
<td>4.03</td>
<td>1.01</td>
<td>$-1.61$</td>
<td>3.59</td>
<td>2.12</td>
<td>0.08</td>
<td>0.055</td>
</tr>
<tr>
<td>3%</td>
<td>3.97</td>
<td>0.91</td>
<td>$-1.55$</td>
<td>3.53</td>
<td>3.55</td>
<td>0.07</td>
<td>0.05</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$R$</th>
<th>$\epsilon_{\theta\tilde{y}}$</th>
<th>$\epsilon_{v\tilde{y}}$</th>
<th>$\epsilon_{\tilde{s}\tilde{y}}$</th>
<th>$\epsilon_{\tilde{m}\tilde{y}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>5.21</td>
<td>1.33</td>
<td>$-2.11$</td>
<td>4.61</td>
</tr>
<tr>
<td>2%</td>
<td>5.18</td>
<td>1.30</td>
<td>$-2.07$</td>
<td>4.61</td>
</tr>
<tr>
<td>3%</td>
<td>5.03</td>
<td>1.15</td>
<td>$-1.96$</td>
<td>4.48</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$m(\theta)$</th>
<th>$s$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td>7.56</td>
<td>3.68</td>
<td>$-1.97$</td>
</tr>
<tr>
<td>canonical</td>
<td>1.72</td>
<td>1.35</td>
<td>0.50</td>
</tr>
<tr>
<td>ex. sep. shocks</td>
<td>2.05</td>
<td>$-1.82$</td>
<td>$-3.59$</td>
</tr>
</tbody>
</table>

ity of the unemployment rate is smaller than that of the model with exogenous shocks to separations, and very close to that in the data. The resulting elasticity of the v-u ratio is also much larger than those of both the canonical model and the model with exogenous separation rate shocks, and much closer to that in the data. For this reason, unlike the model with exogenous, and the MP model with endogenous changes in the separation rate, this model predicts a procyclical vacancy rate. Although the model clearly out-performs existing models when it comes to explaining the behavior of vacancies, it still underpredicts the volatility of vacancies.

One way to increase the changes in the vacancy rate relative to changes in labor productivity is to increase the replacement ratio. In Table 5, I set the replacement ratio to 0.7, as suggested by Hall(2006), and choose the value of $\epsilon_{ny}$, for which $\epsilon_{u\tilde{y}} = -3.88$, as in the data. By increasing the replacement rate the elasticity of labor productivity with respect to aggregate productivity declines so that the implied change in market tightness relative to changes in labor productivity is larger. This reduces the value of $\epsilon_{ny}$ that matches the elasticity of the unemployment rate with respect to labor productivity. In turn, a smaller value of value of $\epsilon_{ny}$ implies a larger elasticity of vacancies with respect to
Table 5: Model results at a higher replacement ratio

<table>
<thead>
<tr>
<th>$R$</th>
<th>$\epsilon_y$</th>
<th>$\epsilon_v$</th>
<th>$\epsilon_{\tilde{y}}$</th>
<th>$\epsilon_{\tilde{m}_y}$</th>
<th>$m(\theta)$</th>
<th>$s$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>5.41</td>
<td>2.48</td>
<td>-1.46</td>
<td>3.61</td>
<td>1.61</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>2%</td>
<td>5.38</td>
<td>2.46</td>
<td>-1.49</td>
<td>3.61</td>
<td>2.12</td>
<td>0.08</td>
<td>0.055</td>
</tr>
<tr>
<td>3%</td>
<td>3.25</td>
<td>1.27</td>
<td>-0.99</td>
<td>-1.98</td>
<td>3.55</td>
<td>0.07</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$R$</th>
<th>$\epsilon_{\tilde{y}}$</th>
<th>$\epsilon_{\hat{y}}$</th>
<th>$\epsilon_{\tilde{y}}$</th>
<th>$\epsilon_{\tilde{m}_y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>7.17</td>
<td>3.29</td>
<td>-1.94</td>
<td>4.78</td>
</tr>
<tr>
<td>2%</td>
<td>7.15</td>
<td>3.27</td>
<td>-1.98</td>
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<tr>
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<td>6.95</td>
<td>3.07</td>
<td>-1.93</td>
<td>4.63</td>
</tr>
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</table>

| data  | 7.56            | 3.68            | -1.97           | 2.34            |
| ex. sep. shocks | 3.73            | -0.15           | -3.12           | 1.04            |

aggregate productivity and thus with respect to labor productivity. As can be seen, this variation improves the results considerably. Not only the volatilities of the separation rate, the v-u ratio and the unemployment rate are consistent with the data, but also that of the vacancy rate, whereas the higher replacement ratio does not improve much the results of the model with exogenous changes in the separation rate. As shown in the last column of Table 5, at a higher replacement ratio the volatility of the separation rate that matches the observed volatility of the unemployment rate in the model with exogenous separation rate shocks is still higher than in the data, while the model still grossly underpredicts the volatility of vacancy creation.

Recall also that in the MP model, an increase in the replacement ratio improves the elasticity of market tightness with respect to labor productivity significantly, but this improvement comes at the cost a much higher elasticity of the separation rate and thus unemployment rate to changes in aggregate productivity. As can be seen in Table 1 an increase in the replacement rate from 0.4 to 0.55 doubles the elasticity of the v-u ratio but the implied elasticity of the separation rate is more than 10 times the empirical one, and the implied elasticity of the unemployment rate is more than 5 times the empirical one.
Notice also that the model has no trouble generating a large enough response in the job finding rate relative to labor productivity. In fact, the model generates too much of a relative change in the job finding rate. The excess volatility of the job finding rate is due to fluctuations in the reservation productivity. Allowing for smaller fluctuations in the reservation productivity would lower the volatility in the job finding rate but the implied volatilities of the v-u ratio, unemployment rate, and vacancy rate would be smaller. Although the model over-predicts the relative change in the job finding rate it still outperforms other models that allow for separation rate variations when it comes to explaining jointly the behavior of the job finding rate and other key labor market variables. In Table 6 I report the results of a different exercise. I choose the value of $\epsilon_{ny}$ that matches the volatility of the job finding instead of the unemployment rate. To facilitate direct comparison I also report the results of the model of exogenous changes in the separation rate when the value of $\epsilon_{sy}$ is such that the volatility of the job finding rate is consisted with the data. As expected, in this case the model under-predicts the volatility of unemployment and vacancies, but does much better than the model with exogenous separation rate shocks. For the latter to match the volatility of the job finding

<table>
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<tr>
<th>$R$</th>
<th>$\epsilon_{\theta y}$</th>
<th>$\epsilon_{vy}$</th>
<th>$\epsilon_{\delta y}$</th>
<th>$\epsilon_{uy}$</th>
<th>$m(\theta)$</th>
<th>$s$</th>
<th>$u$</th>
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<td>-1.75</td>
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<th>$\epsilon_{vy}$</th>
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<tbody>
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<td>-1.92</td>
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<tr>
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<td>1.36</td>
<td>-0.99</td>
<td>-1.91</td>
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<td></td>
</tr>
<tr>
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<td>3.25</td>
<td>1.27</td>
<td>-0.99</td>
<td>-1.98</td>
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<table>
<thead>
<tr>
<th></th>
<th>$\epsilon_{ny}$</th>
<th>$\epsilon_{nv}$</th>
<th>$\epsilon_{uy}$</th>
<th>$\epsilon_{vy}$</th>
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</thead>
<tbody>
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<td>-59.2</td>
<td>-70.2</td>
<td>-67.5</td>
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</table>

Table 6: Model results at different values for $n$- match $\epsilon_{ny}$
rate requires unrealistically high fluctuations in separations and unemployment, leading
to a strongly countercyclical behavior in vacancies. Since the volatility of the v-u ratio
is smaller in the MP model than the model with exogenous separation rate shocks, the
volatility of the separation rate would be even larger in this case, leading to an even more
strongly countercyclical behavior in vacancies.

7 Conclusion

In this paper, I have examined the business cycle properties of a version of the widely
used matching model of endogenous variation in the separation rate due to Mortensen
and Pissarides (1994). The main difference between the model I have studied and the
MP model is that in the latter the labor force is homogenous, whereas I allow for hetero-
geneity in worker productivity. Allowing for variation in the separation rate is empirically
relevant, since one third of the business-cycle fluctuations in the unemployment rate are
due to fluctuations in the separation rate. However, the MP model can generate suffi-
ciently large variation in the unemployment rate, but at the cost of a much less volatile
vacancy rate than in the data. I demonstrate that allowing for heterogeneity in worker
productivity improves the model’s ability to explain jointly the behavior of vacancies and
unemployment considerably.
References


