Abstract

We examine a semi-parametric model of downward nominal and real wage rigidities, specialising it to the case of proportional rigidities, and analyse its econometric foundations, identification, and estimation, noting that, without any parametric assumptions on its error structures and the anticipated inflation distribution, the model can be estimated using maximum likelihood. We use simulations to examine the model’s small sample properties and compare them to those of a truly parametric, likelihood based, model when the assumptions in the latter are right and when they are wrong. We conclude that the semi-parametric approach offers a viable and robust alternative to parametric specifications.

JEL Classification: J31 Keywords: downward nominal wage rigidity, downward real wage rigidity.
1 Introduction

Non-classical features of the labour market and their implications for macroeconomic models and outcomes have attracted research attention for many decades. One major strand of the academic literature focuses on the measurement of wage rigidities.\(^1\) A second group of papers take wage rigidities for granted and examines their implications for macroeconomic performance, inflation dynamics, and the conduct of monetary policy.\(^2\) Wage (and price) rigidities are now considered important enough to incorporate in state-of-the-art macroeconomic models.\(^3\) This paper belongs to the first literature. It makes a contribution by proposing and assessing (relative to extant alternatives) a semi-parametric method for measuring both nominal and real wage rigidities.

The cyclical behaviour of the real wage rate will continue to attract attention for a number of reasons. However, the extent to which real wage cyclicality can speak about possible nominal wage rigidities is no longer a concern in the literature. Following McLaughlin (1994) and the development of micro data sets, more direct approaches have been adopted. These focus on observed wage-change distributions, attempting to discern patterns that might be consistent with rigidities of various kinds. The earliest studies focus specifically on nominal rigidity but more recent approaches also deal with the measurement of real wage rigidities, menu costs and other patterns of behaviour.

An important point of departure for the modern approaches to the measurement of rigidities is the symmetry of the wage-change distribution. It seems reasonable that, as inflation abates and nominal wage-change distributions shift to the left, the nominal wage-change distribution could be distorted by possible downward nominal wage rigidity (DNWR). Thus, early studies look for increased asymmetries during lower inflation periods. A number of standard symmetry measures have been used. These include the skewness coefficient, the difference between the median and the mean, symmetrically differenced histograms and the Lebow et al. (1995) thinness measure. McLaughlin (2000) also considers other measures such as signed ranks, runs, signed gaps and tripples. A non-parametric, consistent, test of the symmetry of a probability density function, free of most of the difficulties that plague simpler symmetry measures, is that of Ahmad and Li

\(^1\)In early Keynesian models, downward nominal wage rigidity was associated with movements along the labour demand curve and a countercyclical real wage. This is one reason for the focus of so many papers on the cyclicality of the real wage. These papers include the earliest studies in the area by Dunlop (1938) and Tarsis (1939) and, more recently, those by Solon et al. (1984) and Abraham and Haltiwanger (1995).

\(^2\)Early papers that consider the interaction between wage rigidities, inflation and macroeconomic performance include Schultz (1939), Samuelson and Solow (1960), and Tobin (1972). The themes in these papers and more contemporary concerns with the appropriate form of monetary policy, are pursued in inter alia Lebow et al. (1995), Fortin (1996), Akerlof et al. (1996), and Alquist et al. (2006).

\(^3\)A recent example is Blanchard and Gali (2007).
(1997). Using this test, Christofides and Stengos (2001) study nominal wage-change distributions in the PSID for ‘stayers’, i.e. individuals who remain in the same position \textit{and} with the same employer\textsuperscript{4}, and, consistent with DNWR, find more asymmetries as inflation moderates.

These early uses of symmetry are relative: They do not insist that the wage-change distribution be symmetric, only that it become less symmetric at lower rates of inflation. By contrast, other uses of symmetry are absolute. Card and Hyslop (1997) argue that most wage determination models imply symmetry. Assuming that the portion of the wage-change distribution above the median is not contaminated by rigidities, they propose its use as the no-rigidity counterfactual; departures from this counterfactual distribution below the median would suggest rigidities. This insight has conditioned many studies in the area. For instance, it is part of the important initiative in the International Wage Flexibility Project (IWFP). The measure of downward real wage rigidity (DRWR) in the IWFP relies on the symmetry of the wage-change distribution.\textsuperscript{5}

Another method for estimating nominal and real wage rigidity is the Maximum Likelihood approach and its ab initio reliance on specific distributions. Frequently, the assumed distributions are normal, which are, of course, symmetric.\textsuperscript{6} Altonji and Devereux (2000) is one influential paper in this genre but the papers in the November 2007 issue of the Economic Journal are also prominent examples.\textsuperscript{7} In addition to its well-known statistical properties, the ML approach has the advantage that it can incorporate a measurement error process in the observed wage-change data. Also, by its very nature, it provides an easy interpretation of the parameters involved.\textsuperscript{8}

The use of symmetry and parametric assumptions about underlying distributions leads to a powerful approach that has produced a wealth of empirical findings across many countries, sectors, types of workers and firms. These assumptions may have come at a cost\textsuperscript{9} and so it is important to explore ways in which the goal of quantifying DNWR and DRWR can be reached using alternative methodologies that do not rely on particular distributions and/or symmetry.

\textsuperscript{4}Most studies define a ‘stayer’ more loosely, as someone who has the same employer across survey points. Since individuals with the same employer may not be in the same position through time, these studies overestimate the degree of wage flexibility.

\textsuperscript{5}For a description of the nominal and real wage rigidity measures used in the IWFP, see Dickens et al (2007, pp. 206-207) and Du Cajun et al (2007, p. 44).

\textsuperscript{6}An exception which relies on the generalised hyperbolic distribution is Behr and Pötter (2005).

\textsuperscript{7}See Goette et al. (2007), Bauer et al. (2007), Devicien ti et al. (2007) and Barwell and Schweitzer (2007).

\textsuperscript{8}The mixed method of moments estimator used in the IWFP and described in Dickens et al (2006, section B1) deals with measurement error using a specific symmetric distribution, the two-sided Weibul.

\textsuperscript{9}Any choice of a particular distribution, symmetric or otherwise, may be inappropriate, with possible complications for estimation and inference. McLaughlin (1999, p.130) actually questions the assumption of symmetry, noting reasons why wage-change distributions may be asymmetric. The IWFP measure of real rigidity requires not only overall symmetry in the wage-change distribution but also other conditions which may make it impossible to calculate it (Dickens et al, note 9), or lead to inaccuracies (Du Cajun et al, p.44). For a discussion on how DRWR may affect wage adjustment more generally, see Christofides and Neachou (2007, p. 700).
In this paper we delve more deeply into the semi-parametric approach proposed in Christofides and Nearghou (2007). This model makes no parametric assumptions about the nature of the underlying error distributions or the anticipated inflation distribution (AID) and, in particular, it does not assume symmetry. We specialise this approach to a model of proportional DNWR and DRWR rigidity, so as to be more comparable with the parametric approaches contained in some of the papers mentioned above. Beginning with no parametric assumptions on error distributions, the discretisation inherent in the approach leads naturally to a likelihood-based estimation approach.

For this more special case, we explore its econometric foundations, identification conditions, and carry out simulations designed to explore the cost of inappropriately imposing parametric assumptions and symmetry. Thus, we are able to compare the small sample properties of our model to those of a truly parametric approach when the parametric assumptions are correct and when they are wrong. The application in Christofides and Nearghou (2007) involved data which should be free of error and our simulation design here does not explore this dimension, concentrating instead on the fundamental features of these approaches. The semi-parametric proportional DNWR and DRWR approach involves unobserved heterogeneity with respect to the prevailing wage-setting regime; in the context of the assumption of proportional DNWR and DRWR, the constant proportionality factor may differ across the two types of rigidities and within time, though for ease of presentation our simulations build in the same degree of real and nominal rigidity. The semiparametric approach involves observed mean heterogeneity with respect to notional wage growth (across and within time) and with respect to the expected inflation rate (across time only). The observed heterogeneity dimensions are explored in our

---

10 A noteworthy non-parametric method for the study of wage rigidity is Holden and Wulfshberg (2007).
11 Estimation using maximum likelihood has important advantages over the regression approach used originally by Kahn (1997), as well as everyone else who has implemented the location-histogram approach since then. On the one hand it allows one to introduce constraints on the parameters to be estimated in a much easier fashion, and in particular to constrain the heights of the bins of the estimated probability histograms to sum-up to one. Furthermore, to allow non-linearities in the distortions due to rigidity, such as those implied by proportionality.
12 Christofides and Nearghou (2007) use regression analysis for the estimation of their model of DNWR and DRWR and menu costs, relying on the standard parametric assumptions for hypothesis tests. On the other hand, they do not assume proportional downward rigidities, thereby achieving greater flexibility but at the cost of not being able to summarise in a single number the degree of either type of rigidity.
13 We also use simulations to show that when DRWR is present, then using the sample median of the actual wage growth distribution as a location standardisation parameter - as is the standard practice for location standardisation in the presence of only DNWR - could lead to biased estimates. This occurs when the distortion of the actual WGD due to DRWR extends beyond the median of the notional WGD.
14 The IWFP studies purge data of possible errors by making assumptions about patterns in the error terms. The papers in the Economic Journal make parametric assumptions about error distributions and estimate the relevant parameters. Our approach here parallels that in the IWFP in that it works with clean, or presumed purged, data.
simulations.

Section 2 outlines the semi-parametric model of DNWR and DRWR. Section 3 examines how, through a process of standardisation of the location of the wage growth distributions across time and of their discretisation, we end up analytically and with no imposed parametric assumptions at a likelihood-based estimation specification. Section 4 spells out the simulation experiments we conduct. These are designed to check the internal features and properties of the semi-parametric approach and to compare its small-sample properties to those of the comparable parametric approach. Section 5 concludes.

2 The model

Unit of observation The unit of observation considered here is the wage agreement, denoted by \( \iota \), between an employer and an employee,\(^{15}\) determining the growth rate of the nominal wage over a certain period. We denote this growth rate by \( \dot{\omega}_\iota \).

Downward wage rigidity: generic mechanism We postulate that there exists a rigidity-free, or ‘notional’, nominal wage growth rate, denoted by \( \dot{\omega}^N_\iota \), associated with wage agreement \( \iota \).

The relationship between this and the actual wage growth rate for agreement \( \iota \), when the actual growth rate may be constrained by the presence of downward wage rigidity, can be described generically as follows:

\[
\dot{\omega}_\iota = \begin{cases} 
\dot{\omega}^N_\iota, & \dot{\omega}^N_\iota \geq b_\iota \\
\delta_\iota b_\iota + (1 - \delta_\iota) \dot{\omega}^N_\iota, & \dot{\omega}^N_\iota < b_\iota 
\end{cases}
\]  

(1)

where \( b_\iota \) is the relevant rigidity bound for the wage-setting regime that applies for the particular wage agreement, and \( \delta_\iota \) an indicator variable that takes the value of 1 if rigidity is binding, and the value of 0 otherwise.

Here we assume that there are three types of mutually exclusive wage-setting regimes that may apply for each agreement \( \iota \); a flexible regime \( (R_\iota = f) \), a regime characterised by DNWR \( (R_\iota = n) \), and another by DRWR \( (R_\iota = r) \), where \( R_\iota \) is the variable that records the regime

\(^{15}\)The definition of the employer and employee sites will depend on the nature of the wage agreement, which could either be collective or individual. In the first case, the ‘employer’ could either be a single employer (firm of public sector agency) or a group of such individual employer entities, and the ‘employee’ one or more labour unions. In the second case, the ‘employee’ would be a single individual, and the ‘employer’ the employer entity that hires the individual.
that applies. The rigidity bounds corresponding to each regime are the following

\[
b_i = \begin{cases} 
    -\infty, & R_i = f \\
    0, & R_i = n \\
    \dot{P}_e, & R_i = r
\end{cases}
\]

(2)

where \( \dot{P}_e \) is the rate of inflation that is anticipated by the employee side\(^{16} \) to prevail during the period wage agreement \( \iota \) will be in force. Thus, when the flexible regime is the one that applies, then there is no effective downward rigidity bound, and, so from (1), the actual and notional wage growth rates coincide

\[
\dot{w}_i = \dot{w}_i^f = \dot{w}_i^N
\]

(3)

When the DNWR regime is applicable, then the value of zero is the common rigidity bound to all agreements negotiated under this regime, and, from (1), the relationship between the actual and notional wage growth rates reduces to

\[
\dot{w}_i = \dot{w}_i^N = \begin{cases} 
    \dot{w}_i^N, & \dot{w}_i^N \geq 0 \\
    (1 - \delta_i) \dot{w}_i^N, & \dot{w}_i^N < 0
\end{cases}
\]

(4)

Finally, when the DRWR regime is applicable, then the relevant rigidity bound is \( \dot{P}_e \), which may vary across \( \iota \), it is unobservable and thus treated here as a random variable. Then from (1), the relationship between the actual and notional wage growth rates for this case is given by

\[
\dot{w}_i = \dot{w}_i^F = \begin{cases} 
    \dot{w}_i^N, & \dot{w}_i^N \geq \dot{P}_e \\
    \delta_i \dot{P}_e + (1 - \delta_i) \dot{w}_i^N, & \dot{w}_i^N < \dot{P}_e
\end{cases}
\]

(5)

The wage-setting regime is typically unobservable by the modeler, although, known to the negotiating parties. Therefore, here it is treated as a form of unobservable heterogeneity within the population of wage agreements, and thus \( R_i \) as a random variable.

From the above it is clear that the nature of the distribution of \( \dot{w}_i \) is determined by the joint distribution of \( \left( \dot{w}_i^N, \dot{P}_e, \delta_i, R_i \right) \), which may be heterogeneous across wage agreements. Therefore the assumptions made next refer the conditional distribution of \( \left( \dot{w}_i^N, \dot{P}_e, \delta_i, R_i \right) \) on a set of heterogeneity characteristics, recorded by the vector \( x_i \).

\(^{16}\)This belief may be also shared by the employer side.
Wage setting regimes  Regarding the distribution of \( R_i \), and the probability that agreement \( \iota \) may be negotiated under a particular regime, this may depend on the preferences of the employee negotiating party.\(^{17}\) So, for example, in the case of collective bargaining, unions may seek to avoid real wage cuts, thus negotiate under DRWR. On the other hand, an individual may be sensitive to nominal wage cuts and thus negotiate under DNWR. Therefore, in general, we could allow for heterogeneity, captured by \( x_i^R (\subset x_i) \), and so define

\[
p^{R^*} (x_i^R) \equiv \Pr (R_i = R^* | x_i^R) , \ R^* \in \{ f, n, r \} \tag{6}
\]

Here, for simplicity, we will assume homogeneity, thus define

\[
p^{R^*} (x_i^R) = p^{R^*} \equiv \Pr (R_i = R^* ) , \ R^* \in \{ f, n, r \} \tag{7}
\]

The rigidity-free nominal wage growth rate  The notional wage growth rate for agreement \( \iota \) is assumed to be given by

\[
\dot{w}_i^N = \mu^N (x_i^N) + \varepsilon_i^N \tag{8}
\]

where \( \mu^N (x_i^N) \) is the conditional expectation function (CEF) of \( \dot{w}_i^N \), evaluated at the vector of characteristics \( x_i^N (\subset x_i) \), that gives the mean notional wage growth rate within the sub-population of wage agreements that share the same characteristics as agreement \( \iota \), and \( \varepsilon_i^N \) the deviation of \( \dot{w}_i^N \) from that mean.

In the absence of any type of rigidity, the nominal wage growth rate is likely to reflect the value of inflation that is anticipated to prevail during the time period agreement \( \iota \) will be effective. Thus \( x_i^N \) could include information that can be used to form inflation expectations,\(^{18}\) recorded by \( x_i^P (\subset x_i^N) \), and given that the inflation level typically varies through time, \( \mu^N (x_i^N) \) could also be expected to do so.

Furthermore, there may be variation in \( \mu^N (x_i^N) \) within the wage agreements that are effective in the same time period as the notional wage growth rate may reflect additional characteristics that can vary within time. For example, at the aggregate level, productivity and demand shocks, and, at the individual level, measures of human capital. Here we will assume that we can partition the sub-population of wage agreements that are all effective in a particular period into \( S \) mean-homogeneous subgroups, indexed by \( s \).\(^{19}\) We can then define \( \mu^N_{is} \) to be the

---

\(^{17}\)We recall that here we assume that the wage setting regimes are mutually exclusive.

\(^{18}\)Such as, past inflation levels and values of inflation leading indicators.

\(^{19}\)For example, at the aggregate level, \( s \) could indicate industries, and at the individual level, the completed
mean notional wage growth rate for the agreements that are effective in period \( t \) and belong to heterogeneity group \( s \)

\[
\mu^N \left( x^N_t \right) = \mu^N_{ts} \quad (9)
\]

\[
t = t \left( x^N_t \right), \quad t \in \{1, \ldots, T\} \quad (10)
\]

\[
s = s \left( x^N_t \right), \quad s \in \{1, \ldots, S\} \quad (11)
\]

where \( t \left( \cdot \right) \) and \( s \left( \cdot \right) \) are used generically to map any vector of characteristics into the time period the agreement is effective, and into the label of the within \( t \) mean heterogeneous group.

For the error term we are going to assume that it is IID across all wage agreements,

\[
\epsilon^N_{i,t} \mid x^{N}_{i,t} \overset{IID}{\sim} f^N \left( \cdot \mid x^{N}_{i,t}; 0, \sigma^2_N \right) = f^N \left( \cdot ; 0, \sigma^2_N \right) \quad (12)
\]

\[
E \left( \epsilon^N_{i,t} \mid x^{N}_{i,t} \right) = E \left( \epsilon^N_{i,t} \right) = 0 \quad (13)
\]

\[
Var \left( \epsilon^N_{i,t} \mid x^{N}_{i,t} \right) = Var \left( \epsilon^N_{i,t} \right) = \sigma^2_N \quad (14)
\]

therefore

\[
\dot{w}^N_{i,t} \mid x^{N}_{i,t} \overset{IID}{\sim} f^N \left( \cdot \mid x^{N}_{i,t}; \mu^N \left( x^N_{i,t} \right), \sigma^2_N \right) \quad (15)
\]

**The rigidity bounds** The *distribution* of the rigidity bound \( b_i \) is only well defined when \( R_i = r_i^{20} \) and in that case it coincides with the distribution of the anticipated inflation rate \( \dot{P}^e_i \).

Given the earlier discussion about the role of inflation expectation in the determination of the level of the notional wage growth rate, we can write

\[
\dot{P}^e_i = \mu^P \left( x^P_i \right) + \epsilon^e_i
\]

where \( \mu^P \left( x^P_i \right) \) is the conditional expectation function of \( \dot{P}^e_i \), evaluated at the vector \( x^P_i \), and \( \epsilon^e_i \) the deviation of \( \dot{P}^e_i \) from that mean, interpreted as an idiosyncratic expectation error. Thus we assume that there is no mean heterogeneity across agreements that are effective in the same time period, and so \( \mu^P \left( x^P_i \right) \) can only vary across time. We define \( \mu^P_i \) to be the parameter that...
records this mean for period $t$

$$\mu^P(x^P_t) = \mu^P_t, \quad t = t(x^N_i) \quad (16)$$

For the idiosyncratic expectation error we are going to assume that it is IID across all wage agreements

$$\varepsilon^P_i \mid x^P_i \sim_{\text{IID}} g(\cdot \mid x^P_i; 0, \sigma^2_P) \quad (17)$$

$$E(\varepsilon^P_i \mid x^P_i) = E(\varepsilon^P_i) = 0 \quad (18)$$

$$\text{Var}(\varepsilon^P_i \mid x^P_i) = \text{Var}(\varepsilon^P_i) = \sigma^2_P \quad (19)$$

therefore

$$\hat{\dot{P}}_i \mid x^P_i \sim_{\text{IID}} g(\cdot \mid x^P_i; \mu^P(x^P_i), \sigma^2_P) \quad (20)$$

We will also assume that, conditional on $x_i$, the error terms $\varepsilon^N_i$ and $\varepsilon^P_i$ are independent within $i$, and so $\hat{w}^N_i$ and $\hat{\dot{P}}_i$ will be independent within and across $i$.

**Variable indicating whether rigidity is binding** Generally, the distribution of the binary indicator variable $\delta_i$ could be heterogeneous with respect to the wage setting regime that applies for agreement $i$, and also with respect to a number of characteristics, captured by the vector $x^\delta_i \subset x_i$. For example, in the case of a collective agreement, the vector $x^\delta_i$ could include factors affecting the bargaining power of the union(s) involved in the negotiations (such as the union size), while in the case of individual agreements, characteristics such as the extent of unionisation and coverage of collective agreements in the sector that the employer operates. From the definition of the downward rigidity mechanism, there is also heterogeneity with respect to the notional wage growth rate and the rigidity bound when this is random. Here we assume a very simple type of heterogeneity where the distribution is different only according to whether the notional wage growth rate is above or below the rigidity bound that applies.

So, under DNWR, we can write

$$\Pr(\delta_i = 1 \mid \hat{w}^N_i, R_i = n, x^\delta_i) = \begin{cases} 0, & \hat{w}^N_i \geq 0 \\ \rho^n(x^\delta_i), & \hat{w}^N_i < 0 \end{cases} \quad (21)$$
and, under DRWR:

\[
\Pr \left( \delta_i = 1 | \dot{w}^N_i, P^e_t, R_s = r, x^\delta_i \right) = \begin{cases} 
0 & \dot{w}^N_i \geq P^e_t \\
\rho^e \left( x^\delta_i \right) & \dot{w}^N_i < P^e_t
\end{cases}
\]  

(22)

The first line of both expressions simply reflects the fact\textsuperscript{21} that when the notional wage growth rate is greater than the respective rigidity bound, then rigidity is not binding and the actual and notional wage growth rates coincide. The second line defines that, conditional on the notional wage growth rate being below the rigidity bound that applies, then this probability \( \rho \) does not depend on the specific values of wage growth and rigidity bound (although it may depend on \( x^\delta_i \)). This is the ‘proportional’ type of downward rigidity.

We could complete the specification of the distribution of \( \delta_i \) by adding that in the case of the flexible regime we have\textsuperscript{22}

\[
\Pr \left( \delta_i = 1 | \dot{w}^N_i, R_s = f, x^\delta_i \right) = \rho^f = 0
\]  

(23)

Here we will assume

\[
\rho^{R^*} \left( x^\delta_i \right) = \rho^{R^*}, \ R^* \in \{ n, r \}, \ s = s \left( x^N_i \right), \ s \in \{ 1, \ldots, S \}
\]

that is, heterogeneity with respect to \( s \)....

**Summing up** Given the simple structure of heterogeneity, for the remaining of the paper we opt to use a simpler notation where the subscript \( i \) is now replaced by the set of subscripts \( tsi \), where \( t \) and \( s \) have already been defined, and \( i \) is defined as the wage agreement index within sub-population \( ts \). Then, the basic equations of the model become:

\[
\dot{w}^N_{tsi} = \mu^N_{ts} + \varepsilon^N_{tisi} \ \sim \ f^N_{ts} (\cdot ; \mu^N_{ts}, \sigma^2_N)
\]  

(24)

\[
P^e_{tsi} = \mu^P_t + \varepsilon^P_{tisi} \ \sim \ g_t (\cdot ; \mu^P_t, \sigma^2_P)
\]  

(25)

\textsuperscript{21}See first lines of (4) and (5), respectively.

\textsuperscript{22}This has no practical meaning as, in this case, the actual wage growth rate is always equal to the notional.
It is important to note, in view of the material that follows, that the respective distribution of each of these two random variables is only characterised by mean heterogeneity. Thus the only distributional characteristic that varies across the heterogeneity groups that apply in each case is the location of the distribution, while its the shape remains the same.

**PDF of actual wage growth rate** The PDF of the distribution of actual wage growth rates within sub-population $ts$, denoted by $f_{ts}$, could be expressed as the mixture of the (conditional) PDF’s characterising the distribution of wage growth rates within each sub-population

$$f_{ts} (\dot{w}) = \begin{cases} p^f f_{ts}^N (\dot{w}) + p^n f_{ts}^n (\dot{w}) + p^r f_{ts}^r (\dot{w}) & , \ w \neq 0 \\ p^n \Pr (\dot{w}_{tsi} = 0 | R_{tsi} = n) & , \ w = 0 \end{cases}$$

where, based on (4) and (5) we can write, respectively:

$$f_{ts}^n (\dot{w}) = \begin{cases} f_{ts}^N (\dot{w}) & , \ \dot{w} > 0 \\ f_{ts}^N (\dot{w}) - \rho_n^s f_{tsi}^N (\dot{w}) & , \ \dot{w} < 0 \\ \rho_n^s F_{tsi}^N (0) & , \ \dot{w} = 0 \end{cases}$$

and

$$f_{ts}^r (\dot{w}) = f_{ts}^N (\dot{w}) - f_{ts}^N (\dot{w}) [1 - G_t (\dot{w})] \rho_r^s + f_{ts}^N (\dot{w}) g_t (\dot{w}) \rho_g^r$$

In the above expressions, $F_{tsi}^N (\cdot)$ and $G_t (\cdot)$ are the cumulative distribution functions of $\dot{w}_{tsi}^N$ and $\dot{P}_{tsi}^e$ respectively. The second line of (38) simply reflects that the actual WGD has a mass point at value zero, that is due to the presence of DNWR.

Expression (28) can become more specific if additional assumptions are made. In particular, if the support of $\dot{P}_{tsi}^e$ is a subset of the support of $\dot{w}_{tsi}^N$ \(^{23}\) i.e.

$$g_t (\dot{p}) > 0 \quad , \ \min (\dot{P}_{tsi}^e) \leq \dot{p} \leq \max (\dot{P}_{tsi}^e)$$

\(^{23}\)This could be justified if we took the view that the change in nominal wage in the absence of rigidities, i.e. the notional wage growth rate, reflects the anticipated change in prices, as well as changes in other factors, such as individual specific productivity and demand shocks

$$\dot{w}_{tsi}^N = \dot{P}_{tsi}^e + \tau_{tsi}$$

where the term $\tau_{tsi}$ accumulates the effect of all other factors and has zero mean, or its mean is small relative to the variance of $\dot{w}_{tsi}^N$. 

11
where
\[
\min (\dot{w}_{tsi}) \leq \min (\dot{P}_{tssi}) \quad \& \quad \max (\dot{P}_{tssi}) \leq \max (\dot{w}_{tsi})
\]  
then equation (28) can be rewritten as follows
\[
f_{ts}^r (\dot{w}) = \begin{cases} 
    f^N_{ts} (\dot{w}) & , \; \dot{w} > \max (\dot{P}_{tssi}) \\
    f^N_{ts} (\dot{w}) - \rho_s f^N_{ts} (\dot{w}) & , \; \dot{w} < \min (\dot{P}_{tssi}) \\
    f^N_{ts} (\dot{w}) - f^N_{ts} (\dot{w}) [1 - G_t (\dot{w})] \rho_s \mu + F^N_{ts} (\dot{w}) g_t (\dot{w}) \rho_s & , \; \min (\dot{P}_{tssi}) \leq \dot{w} \leq \max (\dot{P}_{tssi}) 
\end{cases}
\]  
\]  

3 Estimation
In this Section we describe a method of estimating the rigidity parameters in the model presented in Section 2. We recall that the distributional assumptions made there about the joint distribution of the components of the mechanism that produces the actual wage growth rate, i.e. \((\dot{w}_i^N, \dot{P}_i^e, \delta_i, R_i)\), did not go as far as to define a parametric family for the actual wage growth distribution (WGD), therefore the approach described here could be described as semi-parametric. The basic idea underlying this is the approximation of the continuous distribution of the actual wage growth rate with a discrete distribution, and the search for the presence of distortions in the shape of the latter that are consistent with the presence of rigidity. Its implementation involves three stages: firstly, the standardisation of the location of the actual WGDs across sub-populations, then, the discretisation of the location-standardised actual WGDs and, finally, the estimation\(^{24}\) of the model for the probability mass functions (PMF’s) of the resulting discrete distributions which, by construction, involve the rigidity parameters. Next we look at the details of the implementation of each stage in turn.

3.1 Details of method

Standardisation The purpose of this exercise is to standardise the location of the actual WGD for each sub-population such that the underlying notional WGDs are identical across sub-populations.

\(^{24}\) We will consider estimation using maximum likelihood. A regression approach is discussed in Christofides and Nearchou (2007).
We define the location-standardised variables

\[ \tilde{w}_{tsi} \equiv \hat{w}_{tsi} - \lambda_{ts} \sim \tilde{f}_t (\tilde{w}) \]  
\[ \tilde{w}_{tsi}^N \equiv \hat{w}_{tsi}^N - \lambda_{ts} \sim \tilde{f}_t^N (\tilde{w}) \]  

where \( \lambda_{ts} \) is the location parameter of the notional WGD for sub-population \( ts \) that is chosen such that the location-standardised notional WGDs are identical across \( ts \).

Using \( \lambda_{ts} \), we also define the location-standardised versions of the variables that record the anticipated inflation rate, and the wage growth rates under the three wage-setting regimes;

\[ \tilde{\bar{p}}_{tsi}^e \equiv \hat{\bar{p}}_{tsi}^e - \lambda_{ts} \sim \tilde{g}_t (\tilde{p}) \]  
\[ \tilde{w}_{tsi}^f \equiv \hat{w}_{tsi}^f - \lambda_{ts} \sim \tilde{f}_t^N (\tilde{w}) \]  
\[ \tilde{w}_{tsi}^n \equiv \hat{w}_{tsi}^n - \lambda_{ts} \sim \tilde{f}_t^n (\tilde{w}) \]  
\[ \tilde{w}_{tsi}^r \equiv \hat{w}_{tsi}^r - \lambda_{ts} \sim \tilde{f}_t^r (\tilde{w}) \]  

Given that the effect of the standardisation is to shift all distributions relating to a particular sub-population by the same distance (\( = \lambda_{ts} \)) from their original location it follows that similar relationships to those in (38)-(28) and (31), that hold for the original distributions, will hold for the location-standardised distributions. That is, it must be true that

\[ \tilde{f}_{ts} (\tilde{w}) = \begin{cases} 
   p^f \tilde{f}_t^N (\tilde{w}) + p^n \tilde{f}_t^n (\tilde{w}) + p^r \tilde{f}_t^r (\tilde{w}) & , \tilde{w} \neq -\lambda_{ts} \\
   p^n \Pr (\tilde{w}_{tsi} = -\lambda_{ts} | R_{tsi} = n) & , \tilde{w} = -\lambda_{ts}
\end{cases} \]  

where

\[ \tilde{f}_t^n (\tilde{w}) = \begin{cases} 
   \tilde{f}_t^N (\tilde{w}) & , \tilde{w} > -\lambda_{ts} \\
   \tilde{f}_t^N (\tilde{w}) - \rho_s \tilde{f}_t^n (\tilde{w}) & , \tilde{w} < -\lambda_{ts} \\
   \rho_s \tilde{F}_t^N (-\lambda_{ts}) & , \tilde{w} = -\lambda_{ts}
\end{cases} \]
Figure 1: Location standardisation.

\[
\tilde{f}_{ts}(\tilde{w}) = \tilde{f}^N(\tilde{w}) - \tilde{f}^N(\tilde{w}) \left[ 1 - \tilde{G}_t(\tilde{w}) \right] \rho_s + \tilde{F}^N(\tilde{w}) \tilde{g}_t(\tilde{w}) \rho_s^c \tag{40}
\]

\[
= \begin{cases} 
\tilde{f}^N(\tilde{w}), & \tilde{w} > \max \left( \tilde{P}_{te}^{\pi_s} \right) \\
\tilde{f}^N(\tilde{w}) - \rho_s^c \tilde{f}^N(\tilde{w}), & \tilde{w} < \min \left( \tilde{P}_{te}^{\pi_s} \right) \\
\tilde{f}^N(\tilde{w}) - \tilde{f}^N(\tilde{w}) \left[ 1 - \tilde{G}_t(\tilde{w}) \right] \rho_s^c + \tilde{F}^N(\tilde{w}) \tilde{g}_t(\tilde{w}) \rho_s^c, & \min \left( \tilde{P}_{te}^{\pi_s} \right) \leq \tilde{w} \leq \max \left( \tilde{P}_{te}^{\pi_s} \right)
\end{cases} \tag{41}
\]

The purpose of this type of standardisation is to facilitate the comparison of the shapes of the WGDs across sub-populations, and, ultimately, the identification of the rigidity parameters. To see how this could be achieved, we note that in the absence of rigidity the standardised actual WGDs will coincide with the corresponding standardised notional WGDs, and thus by construction (see (35)), be identical across \( t \). On the other hand, in the presence of rigidity, the standardised actual WGDs from each sub-population will be distorted, possibly in a different way, depending on the location of the notional WGD with respect to the relevant rigidity bounds; the point zero and the support of the anticipated inflation distribution (AID). Then, the comparison of the parts of the standardised WGDs across sub-populations that are expected to be distorted differently in the presence of rigidity would provide information about the size of the distortions that ultimately could lead to the identification of the rigidity parameters.

An example of implementation of the standardisation stage is depicted in Figure 1. For simplicity we present a case where only DNWR affects the wage adjustment process. The top
diagram shows the PDFs of the notional (dashed line) and actual (solid line) WGDs from two sub-populations before standardisation, and the bottom diagram the corresponding PDFs after standardisation takes place. In this example we have assumed that in sub-population 1 the inflation level is high enough so that no part of the notional WGD lies below zero, and therefore the PDF of the notional \( f_1^N \) and actual \( f_1 \) WGDs are identical for this case. On the other hand, the inflation level in sub-population 2 is assumed to be low enough so that a part of the notional distribution lies below zero. As a result, for this case, the notional and actual PDFs do not coincide for all levels of wage growth rates, but instead the characteristic distortion in the shape of the PDF of the actual WGD \( f_2 \) is visible for the part that corresponds to the non-positive part of the support of the notional WGD.

Standardisation is assumed to take place using as standardisation parameters the values \( \lambda_1 \) and \( \lambda_2 \) for sub-populations 1 and 2, respectively, which are the values of the same-order-quantiles of the unstandardised notional WGDs in the two sub-populations. As we can see in the bottom diagram, the PDF of the standardised actual WGD in sub-population 1 \( \tilde{f}_1 \) coincides with the PDF of the standardised notional WGD \( \tilde{f}_1^N \), which, by assumption, is the same for both sub-populations. On the other hand, the shape of the PDF of the standardised actual WGD for sub-population 2 \( \tilde{f}_2 \) preserves the distortion in its shape, except that instead of this extending to the left - and including - point zero, it now extends to the left and includes point \(-\lambda_2\), the standardised value of zero for sub-population 2.\(^{25}\)

**Discretisation** At this stage we define the approximation to the standardised actual WGD. This involves the partition of the support of \( \tilde{w}_{tsi} \) into \( J \) successive sub-intervals of equal length,

\(^{25}\)The standardisation procedure described here is a convenient tool to develop the semi-parametric estimator. Its sole purpose is to alter the location of the actual WGDs so that the notional WGDs corresponding to their location-standardized versions are identical across sub-populations, while, at the same time, preserving the relative position of the notional distribution and the distributions of the rigidity bounds, i.e. the point zero, in the case of DNWR (degenerate distribution), and the distribution of anticipated inflation rates, in the case of DRWR. In order to satisfy this condition, the (distributions of) the rigidity bounds must be shifted by the same distance as the actual WGD. Then, as a result, the position of the distortions introduced to the shape of the unstandardised actual WGD due to the presence of the two types of rigidity relative to the position of the unstandardised notional WGD, will be the same as the position of the distortions to the standardised actual WGD relative to the position of the standardised notional WGD. We note that this approach has also been used by others in the literature who adopted the location-histogram approach, e.g. Kahn (1997) and Beissinger and Knoppik (2001). Alternatively, as in Christofides and Nearchou (2007) and Christofides and Nearchou (n.d.), one could proceed directly to the ‘discretisation’ stage, defining year specific bins relative to the position of some location parameter of the notional distribution for each year.
denoted by $B_j$, which we refer to as ‘bins’

$$B_j \equiv [\eta_{j-1}, \eta_j) \quad , \quad j = 1, \ldots, J$$

(42)

$$\eta \equiv \eta_j - \eta_{j-1} \quad \forall j$$

(43)

$$B_j \cap B_\zeta = 0 \quad , \quad j \neq \zeta$$

(44)

$$\Pr \left( \tilde{w}_{tsi} \in \bigcup_{j=1}^{J} B_j \right) = 1$$

(45)

and define the (ordered) discrete random variable $y_{tsi}$ whose value is equal to the value of the index of the bin that contains $\tilde{w}_{tsi}$:

$$y_{tsi} \equiv \{ j, \ j \in \{1, \ldots, J\} : \tilde{w}_{tsi} \in B_j \}$$

(46)

$$y_{tsi} \sim f_{yts} (\cdot)$$

(47)

where $f_{yts} (\cdot)$ is the PMF, or probability histogram, of $y_{tsi}$:

$$f_{yts} : \mathbb{N}+ \to \{P_{jts}\}_{j=1,\ldots,J}$$

(48)

$$P_{jts} \equiv \Pr (y_{tsi} = j) = \Pr (\tilde{w}_{tsi} \in B_j) = \int_{\eta_{j-1}}^{\eta_j} f_{ts} (\tilde{w}) \, d\tilde{w}$$

(49)

We also define

$$P_j^N \equiv \Pr (\tilde{w}_{tsi}^N \in B_j)$$

(50)

$$\pi_{jts} \equiv \Pr \left( \tilde{P}_{tsi}^c \in B_j \right)$$

(51)

to be the probabilities that the standardised notional wage growth rate, and standardised anticipated inflation rate of individual $i$ for sub-population $ts$, respectively, fall in bin $j$. Finally, we introduce the following notation to indicate the value of index $j$ for ‘special’ bins that will
be needed for the exposition of the material that follows:

\[ J_{ts}^0 \equiv \{ j, j \in \{1, \ldots, J\} : -\lambda_{ts} \in B_j \} \]  
\[ J_{ts}^P \equiv \{ j, j \in \{1, \ldots, J\} : E\tilde{P}_{tsi} \in B_j \} \]  
\[ \bar{J}_{ts}^P \equiv \{ j, j \in \{1, \ldots, J\} : \max (\tilde{P}_{tsi}) \in B_j \} \]  
\[ J_{ts}^P \equiv \{ j, j \in \{1, \ldots, J\} : \min (\tilde{P}_{tsi}) \in B_j \} \]  

i.e. \( J_{ts}^0 \) is the index value of the bin that contains \(-\lambda_{ts}^N\) (the standardised value of zero), \( J_{ts}^P \) the index value of the bin that contains the standardised value of mean anticipated inflation for sub-population \( ts \), and \( \bar{J}_{ts}^P \) and \( J_{ts}^P \) the index values of the leftmost and rightmost bins, respectively, that contain standardised values of anticipated inflation for sub-population \( ts \).\(^{26}\)

An example of implementation of the discretisation stage is depicted in Figure 2. The solid line in the top diagram represents the PDF of a standardised WGD from a high inflation period, where DNWR had no effect on its shape. On the other hand, the solid line in the bottom

\(^{26}\)Note that the bins that contain values of the standardised anticipated inflation distribution (i.e. values of \( j \) s.t. \( \pi_{j,ts} > 0 \)) must have limits that satisfy the condition \( \{ \eta_j > \min (\tilde{P}_{tsi}) \ \& \ \eta_{j-1} < \max (\tilde{P}_{tsi}) \}. \)
diagram represents the PDF of a standardised WGD from a low inflation period, where DNWR had produced distortions that are visible for \( \tilde{w}_{tsi} \leq -\lambda_{ts} \). Discretisation is implemented with \( 12 (= J) \) bins. In both diagrams the bin that includes the standardised value of anticipated inflation has index value \( j = 6 \), therefore \( J_{ts}^P = 6 \) in both cases. Furthermore, for the case depicted in the bottom diagram, the standardised value of zero, i.e. \( -\lambda_{ts} \), is located in bin \( j = 3 \) therefore, for this case, \( J_{ts}^0 = 3 \).

Given the relationships that exist between the PDFs of the standardised notional and actual WGDs, and the standardised AID, similar relationships can be derived for the PMFs of their respective discrete approximations. Using (38)-(41), we can write \( P_{jts} \), defined in (49), as follows

\[
P_{jts} = \int_{\eta_{j-1}}^{\eta_j} \tilde{f}_{ts} (\tilde{w}) \, d\tilde{w}
\]

\[
= \int_{\eta_{j-1}}^{\eta_j} \left[ p^f \tilde{f}^N (\tilde{w}) + p^n \tilde{f}^n_{ts} (\tilde{w}) + p^r \tilde{f}^r_{ts} (\tilde{w}) \right] \, d\tilde{w}
\]

\[
= p^f P^N_j + p^n p^n_{jts} + p^r P^r_{jts}
\]  

(56)

where

\[
P^N_j = \int_{\eta_{j-1}}^{\eta_j} \tilde{f}^N (\tilde{w}) \, d\tilde{w}
\]  

(57)

\[
P^n_{jts} = \int_{\eta_{j-1}}^{\eta_j} \tilde{f}^n_{ts} (\tilde{w}) \, d\tilde{w} = \begin{cases} 
  p^N_j, & j > J^P_{ts} \\
  p^N_j - \rho^n_s P^N_j, & j < J^P_{ts} \\
  p^N_j + \rho^n_s \sum_{\xi < j} P^N_\xi, & j = J^P_{ts}
\end{cases} 
\]

(58)

and

\[
P^r_{jts} = \int_{\eta_{j-1}}^{\eta_j} \tilde{f}^r_{ts} (\tilde{w}) \, d\tilde{w}
\]

(59)

\[
= p^N_j - P^N_j \left[ 1 - \tilde{G}_t (\eta_j) \right] \rho^r_s + \left( \sum_{\xi > j} P^N_\xi \right) \left[ \tilde{G}_t (\eta_j) - \tilde{G}_t (\eta_{j-1}) \right] \rho^r_s
\]

(60)

\[
= p^N_j - P^N_j \left( \sum_{\xi > j} \pi_{it} \right) \rho^r_s + \left( \sum_{\xi < j} P^N_\xi \right) \pi_{jts} \rho^r_s
\]

(61)

\[
= \begin{cases} 
  p^N_j, & j > J^P_{ts} \\
  p^N_j - \rho^n_s P^N_j, & j < J^P_{ts} \\
  p^N_j - P^N_j \left( \sum_{\xi > j} \pi_{it} \right) \rho^r_s + \left( \sum_{\xi < j} P^N_\xi \right) \pi_{jts} \rho^r_s, & j_{ts} \leq j \leq J^P_{ts}
\end{cases}
\]

(62)
Then, combining, we can write:

\[ P_{jts} = P_j^N - \]

\[ - p^n \rho^n P_j^N I_{(j < J_0)} + p^n \rho^n \left( \sum_{\zeta < j} P_{\zeta}^N \right) I_{(j = J_0)} \]

\[ - p^r \rho^r P_j^N \left( \sum_{\zeta > j} \pi_{\zeta jts} \right) + p^r \rho^r \left( \sum_{\zeta < j} P_{\zeta}^N \right) \pi_{jts} \]

\[ (63) \]

**Construction of the likelihood function**

Given that \( \tilde{w}_{tsi} \) are independent across \( i \) and \( t \), the same must be true for \( \tilde{w}_{tsi} \) and \( y_{tsi} \). It then follows that the likelihood function for the sample of \( y_{tsi} \)'s is given by

\[ L_2 = \prod_{t=1}^{T} \prod_{s=1}^{S} \prod_{i=1}^{n_{ts}} \prod_{j=1}^{J} P_{jts}^{I_{(y_{tsi} = j)}} \]

\[ (64) \]

and the corresponding log-likelihood function is by

\[ \ln L_2 = \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{i=1}^{n_{ts}} \sum_{j=1}^{J} I_{(y_{tsi} = j)} \ln P_{jts} \]

\[ (65) \]

At this point we note that the problem of maximising this log-likelihood is not tractable because the allocation of the probability mass of the AID across the bins that contain values of anticipated inflation may be different across sub-populations (the \( \pi \)'s in (63) are also indexed by \( ts \)). This allocation is determined by the position of the AID relative to the notional WGD, since by assumption, the shape of the AID distribution is the same across years and only its location is allowed to vary. Therefore, in general, \( \pi_{jts} \equiv \Pr \left( \tilde{P}_{tsi}^e \in B_j \right) \) can be expected to vary across \( ts \), and given that within each \( ts \) the collection of probabilities \( \{ \pi_{jts} \}_{j=1,\ldots,J} \) and \( \rho^r_{ts} \) always appear multiplied to each other in the log-likelihood function above, this means that they cannot be identified separately.

The variation in the relative position of the notional WGD and AID would not pose a problem if the location of the standardised value of mean anticipated inflation \( (E\tilde{P}_{tsi}^e) \) relative to the endpoints of the bin that contained it (i.e. the bin indexed by \( J_{ts}^P \)) were fixed across \( ts \),
i.e.\textsuperscript{27}

\[ E \bar{P}_{tsi} - \eta_{J^P_{ts} - 1} = c \iff \eta_{J^P_{ts}} - E \bar{P}_{tsi} = \eta - c \]  

(66)

We subsequently refer to this as the ‘AID identification condition’. Under this, the probability mass of AID that would fall in the bin that contained the standardised mean anticipated inflation would be the same across sub-populations

\[ \pi_{jts} = \Pr \left( \bar{P}_{tsi} \in B_j \right) = \pi_0 \quad , \quad j = J^P_{ts} \]  

(67)

and the same would also be true for all bins that contained values of anticipated inflation, i.e.

\[ \pi_{jts} = \Pr \left( \bar{P}_{tsi} \in B_j \right) = \pi_q \quad , \quad q \equiv j - J^P_{ts} \quad \& \quad j \in \{J^P_{ts}, \ldots, J^P_{ts}\} \]  

(68)

where the index \(q\) gives the relative position of bin \(j\) in the support of the standardised AID.

Equation (61) would then become

\[ P_{jts}^r = P^N_j - P^N_j \left( \sum_{\xi > j} \pi_{\xi - J^P_{ts}} \right) \rho^s_n + \left( \sum_{\zeta < j} P^N_\zeta \right) \pi_{j - J^P_{ts}} \rho^s_n \]  

(69)

and equation (56) would simplify to the following

\[ P_{jts} = P^N_j + D^N_{jts} + D^r_{jts} \]  

(70)

where

\[ D^N_{jts} = -\frac{\rho^m_n}{\rho^m_n} \left( \sum_{\xi > j} P^N_\xi \right) I_{(j > J^N_{ts})} + \frac{\rho^m_n}{\rho^m_n} \left( \sum_{\zeta < j} P^N_\zeta \right) I_{(j = J^N_{ts})} \]  

(71)

and

\[ D^r_{jts} = -\frac{\rho^r_n}{\rho^r_n} P^N_j \left( \sum_{\xi > j} \pi_{\xi - J^P_{ts}} \right) + \frac{\rho^r_n}{\rho^r_n} \left( \sum_{\zeta < j} P^N_\zeta \right) \pi_{j - J^P_{ts}} \]  

(72)

\textsuperscript{27}Given that the bin width is the same for all bins, this is equivalent to having the difference between any two values taken by the standardised value of mean anticipated inflation across sub-populations to be a multiple of the bin width: \(|E \bar{P}_{tsi}^{(1)} - E \bar{P}_{tsi}^{(2)}| = \nu \cdot \eta, \quad \nu \in \mathbb{N}_+, \quad (t_1, s_1) \neq (t_2, s_2)\).
The log-likelihood function would then be given by

\[
\ln L_2 = \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{i=1}^{n_{ts}} \sum_{j=1}^{J} I(y_{tsi}=j) \ln \left( P_{jts}^N + D_{jts}^n + D_{jts}^r \right)
\]  

(73)

For this case, the unknown parameters to be estimated through the maximisation of the log-likelihood function would then be

\[
\{ \rho_{ss}^{nn} \}_{s=1}^{S} \text{ : Overall (or unconditional) DNWR effect}
\]
\[
\{ \rho_{ss}^{rr} \}_{s=1}^{S} \text{ : Overall (or unconditional) DRWR effect}
\]
\[
\{ P_{j}^{N} \}_{j=1}^{J} \text{ : probability-histogram of standardised notional distribution}
\]
\[
\{ \pi_{q} \}_{q \in Q} \text{ : probability-histogram of standardised AID}
\]

where

\[
Q \equiv \{ q = j - J_{ts}^{P} : j \in \{ J_{ts}^{P}, \ldots, \bar{J}_{ts}^{P} \} \}
\]  

(75)

is defined to be the set of values taken by the index \( q \).

3.2 Identification strategy

Identification requires variation across sub-populations in terms of the part of the respective actual WGDs that is distorted by rigidity. The identification information then comes from the comparison of the shape of the part of the standardised actual WGD of some sub-population that is expected to be distorted by rigidity if present, with the same part of the standardised actual WGD of another sub-population that is not expected to be distorted by rigidity, even if rigidity is present. If the shapes are identical, in some statistical sense, then this is (statistical) evidence for the absence of rigidity. If, on the other hand, the shapes differ, in some statistical sense, in the way predicted when a certain type of rigidity is present, then this is evidence in favour of the presence of that particular type of rigidity.

We provide detailed proof for the sufficiency of identifying conditions for each type of downward rigidity that may be present in Appendix A.

\[ \text{For example, if the number of bins covered by the support of the AID is equal to three (in all periods), then } q \in Q = \{-1,0,1\}, \text{ where } q = 0 \text{ corresponds to the bin that contains mean anticipated inflation in the given period (i.e. } j = J_{ts}^{P}) \text{, } q = -1 \text{ to the bin located one position to its left, and } q = 1 \text{ to the bin located one position to its right.} \]
3.3 Empirical implementation issues

**Standardisation of actual wage growth rate data** The methodology described above requires the standardisation of the location of the distribution underlying the data by subtracting from each data point a quantity \( \lambda_{ts} \) which is equal to the value of some location parameter of the *notional* distribution, such as the mean or some quantile. However, such population parameter values are typically neither known, nor can they be directly estimated as we do not observe the notional wage growth rates.

A way forward is to consider implementing the standardisation using an estimate of such a location parameter (of the *notional* WGD) that is based on *actual* wage growth rate data. This is feasible as certain location parameters of the actual WGD coincide with the corresponding location parameters of the notional; in particular, this is true for the quantiles of the actual WGD that lie in the region above the maximum of the rigidity bounds that apply, e.g. \( \max \left( 0, \max_i \left( \hat{P}_{t}^{u} \right) \right) \) when both types of rigidity are present. In such a case, the methodology described in Section 3 is only valid asymptotically, and this is true when a consistent estimator of the location parameter is used. In Section 4 we investigate the implications of this in small samples.

**Location and spread of the anticipated inflation distribution** The methodology requires knowledge of the approximate location of the AID. The required estimates of the mean anticipated inflation rate could be based on inflation level forecasts, in turn based on some time series econometric model of inflation. Furthermore, the estimates of the \( \pi \)'s should reveal which bins include (standardised) values of the anticipated inflation rate. So as an empirical strategy, one should start the model specification allowing for a more extensively spread AID than might initially think it is the case.

**Treatment of measurement error** The methodology described above does not account for the presence of measurement error in the observed wage growth rates, which can be a serious problem for survey data. If measurement error is an issue, in view of the data to be analysed, one could proceed as described in Dickens et al. (2007), where a preliminary stage of ‘cleaning’ the data preceedes the stage of implementation of the method.
4 Simulations

With the Monte Carlo simulation exercises that follow we investigate the properties of the semi-parametric estimator that was presented in the previous section and compare its performance to that of the parametric estimator considered in Goette et al. (2007), which here is implemented in a measurement-error free setup.

4.1 Basic setup

The DGP  The structure of the model underlying the Data Generating Process (DGP) of the simulated data is the same as the one discussed in Section 2. We recall that in the general setup where both DNWR and DRWR are allowed to be present, the actual wage growth rate $\dot{w}_{tsi}$ for agreement $i$ in sector $s$ \footnote{Here we think of $s$ as recording some partition of the economy into ‘sectors’.} for period $t$ is given by

$$\dot{w}_{tsi} = I(R_{tsi}=f) \dot{w}^N_{tsi} + I(R_{tsi}=n) \dot{w}^n_{tsi} + I(R_{tsi}=r) \dot{w}^r_{tsi}, \hspace{1em} i = 1, \ldots, n_{ts}$$

(76)

where $\dot{w}^N_{tsi}$, $\dot{w}^n_{tsi}$, and $\dot{w}^r_{tsi}$ record the value of the actual wage growth rate $\dot{w}_{tsi}$ when this is determined under the flexible, DNWR, and DRWR wage setting regimes, respectively, and parameters $p^f$, $p^n$ and $p^r$, are the probabilities that determine the mixture of wage setting regimes within the population for the given experiment.

Starting off with the assumptions about the notional wage growth rate $\dot{w}^N_{tsi}$, this is given by

$$\dot{w}^N_{tsi} = \mu^N_{ts} + \epsilon^N_{tsi}$$

(77)

where the mean notional wage growth rate $\mu^N_{ts}$ is assumed here to reflect the level of inflation that prevails in $t$, denoted by $\dot{P}$, and the cumulative effect of a number factors affecting real wage growth in sector $s$ within that time period, such as productivity and demand shocks, captured by $\dot{\tau}_{ts}$, i.e.

$$\mu^N_{ts} = \dot{P} + \dot{\tau}_{ts}$$

(78)

For our purposes we fix $\dot{P}$ to be equal to $t$, which means that we generate data from low, medium, and high inflation periods given that $t = 1, \ldots, 10$. On the other hand $\dot{\tau}_{ts}$ takes values randomly according to one of two specifications
\[ \dot{\tau}_{ts} \in \{-2, 2\}, \quad \Pr (\dot{\tau}_{ts} = -2) = 0.5 \quad (\text{Specification A}) \]
\[ \dot{\tau}_{ts} \overset{IID}{\sim} Unif (-2, 2) \quad (\text{Specification B}) \]

Thus, summing up, \( \mu_{\dot{\tau}_{ts}} \) takes values randomly according to

\[ \mu_{\dot{\tau}_{ts}} \in \{t - 2, t + 2\}, \quad \Pr (\mu_{\dot{\tau}_{ts}} = t - 2) = 0.5 \quad (\text{Specification A}) \]
\[ \mu_{\dot{\tau}_{ts}} \overset{IID}{\sim} Unif (t - 2, t + 2) \quad (\text{Specification B}) \]

Given the definition of \( \mu_{\dot{\tau}_{ts}} \), the error \( \epsilon_{\dot{\tau}_{tsi}} \) can be interpreted as the term that accumulates the inflation expectation error \( = \dot{P}^e_{tsi} - \dot{P}_t \) as well as idiosyncrasies of employer \( i \) in factors affecting real wage growth \( = \dot{\tau}_{tsi} - \dot{\tau}_{t} \). Here it is assumed to follow one of two specifications

\[ \epsilon_{\dot{\tau}_{tsi}} \overset{IID}{\sim} N (0, \sigma_N^2) \quad (\text{Specification A}) \]
\[ \epsilon_{\dot{\tau}_{tsi}} = a + b z_{tsi} \quad z_{tsi} \overset{IID}{\sim} \chi^2_{20} \quad (\text{Specification B}) \]

where the parameters \( a \) and \( b \) are chosen so that the median and variance of the resulting distribution coincide with that of the distribution under Specification A, \( ^{30} \)

\[ \text{median} (a + b z_{tsi}) = 0 \]
\[ \text{Var} (a + b z_{tsi}) = 4.1 \]

The PDF of \( \epsilon_{\dot{\tau}_{tsi}} \) under the two specifications is displayed in Figure 3; under Specification A the notional WGD is Normal (and symmetric), while under Specification B, non-Normal and positively skewed.

The value of the actual wage growth rate determined under the DNWR regime is generated according to the rule

\[ \hat{w}_{tsi}^{n} = \begin{cases} \hat{w}_{tsi}^{N}, & \hat{w}_{tsi}^{N} \geq 0 \\ (1 - \delta_{tsi}^{n}) \hat{w}_{tsi}^{N}, & \hat{w}_{tsi}^{N} < 0 \end{cases} \]
\[ \delta_{tsi}^{n} \overset{IID}{\sim} \text{Bernoulli} (\rho_{n}^{A}) \]

Similarly the value of the actual wage growth rate determined under the DRWR regime is

\(^{30}\)With the choice of degrees of freedom to equal 20, we control for the degree of non-symmetry (positive skewness) of the distribution.
generated according to the rule

\[
\hat{w}^r_{tisi} = \begin{cases} \\
\hat{w}^N_{tisi}, & \hat{w}^N_{tisi} \geq \hat{P}^e_{tisi} \\
\delta^r_{tisi} \hat{P}^e_{tisi} + (1 - \delta^r_{tisi}) \hat{w}^N_{tisi}, & \hat{w}^N_{tisi} < \hat{P}^e_{tisi}
\end{cases}
\] (87)

The heterogeneity in the distribution of the \( \delta \)'s exists only with respect to the wage setting regime (\( n \) or \( r \)) and the sector \( s \). Despite that this is exactly the type of heterogeneity that is accounted for in the estimation procedure, we choose values of \( (p^n_s, \rho^n_s) \) and \( (p^r_s, \rho^r_s) \), so that \( \rho^n_{tsi} \) and \( \rho^r_{tsi} \), respectively, are both equal to 0.2 for all \( s \), as shown in Table 1. This strategy has two purposes; on the one hand to improve the readability of the tables of results produced, and on the other, to avoid any distortions in the results from a possible relationship between the value of the population parameter and the performance of its estimator.\(^{31}\)

Finally, the values of the anticipated inflation rates across \( t, s, \) and \( i \) are generated as Normal

\section*{Notes}

\(^{31}\)Some initial simulation exercises that are not reported here have suggested that larger values of \( \rho \) are more efficiently estimated than smaller values, in terms of the RMSE obtained.
IID random variables, assuming further that these are unbiased

\[ \hat{P}_{tsi} \sim_{IID} N \left( \hat{P}_t, \sigma^2_p \right) , \quad \sigma_p = 0.5 \]  

(89)

In each experiment, the sample size for each period is fixed \( n_t = 1000 \), with equal number of observations \( n_{ts} \) from each sector within each period, i.e. \( n_{ts} = n_t / S \). In all experiments the number of periods is also fixed \((T = 10)\), while the number of sectors \((S)\) is left as a simulation parameter that could take values in \( \{1, 5, 10\} \).

**Details of implementation of semiparametric estimator**  For the implementation of the semiparametric approach we need to specify the value of the location parameter that is used for the standardisation, and the details regarding the partition of the support of the notional distribution for the implementation of the ‘discretisation’ stage.

**Lambda**  With regard to the choice of standardisation parameter, this is left as a simulation parameter. We consider three different specifications:

\[ \lambda_{ts} = \hat{m}_{ts}^N \]  (Specification A)

\[ \lambda_{ts} = \hat{m}_{ts} \]  (Specification B)

\[ \lambda_{ts} = \hat{q}_{\alpha,ts} \]  (Specification C)

(90)

where \( \hat{m}_{ts} \) is the sample median for sample from sub-population \( ts \), and \( \hat{q}_{\alpha,ts} \) a sample percentile whose order \( \alpha \) is chosen according to the following rule:

\[ \alpha = \max_{ts} \left( q^{-1}(\hat{q}_{\alpha,ts}) \right) \]  (91)

\[ \hat{q}_{\alpha,ts} = \min_i \left( \hat{w}_{tsi} : \hat{w}_{tsi} > \max \left( 0, \max_i \left( \hat{P}_{tsi} \right) \right) \right) \]  (92)

where \( q^{-1} (\cdot) \) is the inverse quantile function. In this way \( \alpha \) is the minimum quantile order that ensures that across all sub-populations the respective quantile of order \( \alpha \) lies in the part of the corresponding actual WGD that is not distorted by rigidity. Thus under Specification A standardisation is based on a population location parameter, whereas in the remaining two specifications on estimates of location parameters that are based on the actual wage growth data.
Bins  Regarding the second choice, for all simulation exercises executed we have set the bin width ($= \eta \equiv \eta_j - \eta_{j-1}$) equal to 1% and the number of bins ($= J$) equal to 17. Furthermore the bin limits take values from the set \{..., -2.5, -1.5, -0.5, 0.5, 1.5, 2.5, ...\}. Given our choice of value for $\sigma_N = 4.1$, the probability histograms resulting from this specification of discretisation could cover above 96% of the probability mass of the underlying continuous distribution. For each subsample, the location of the bins (or, equivalently, the value of $\eta_0$) is chosen in order to maximise the amount of data that the resulting histogram covers.\footnote{For example, if standardisation is based on the population median ($\lambda_{ts} = m^N_{ts}$) and the notional WGD is symmetric, so that the standardised value of the median ($\tilde{m}_{ts} \equiv m^N_{ts} - \lambda_{ts}$) is zero, then a choice of equal number of bins to the left and to the right of point zero would likely ensure that the resulting (standardised) histogram covers the maximum amount of data points of a given sample. In such a case, the bin limits would take values $\eta_j \in \{-8.5, -7.5, ..., 0.5, 0.5, 7.5, 8.5\}$, and so $\eta_0 = -8.5$.}

Likelihood function  We estimate the likelihood function that is given by (71), (72), and (73). We assume knowledge of the mean anticipated inflation rate ($\bar{P}_{ts} \equiv E P^e_{tsi} = \hat{P}_t$), as well as of the minimum and maximum values in its support.

Details of implementation of parametric estimator  In all experiments, the likelihood defined by expressions (38), (27) and (28) is adapted to reflect the choice of Normality for the notional WGD and the AID. We assume knowledge of the mean anticipated inflation rate ($\bar{P}_{ts} \equiv E P^e_{tsi} = \hat{P}_t$), therefore only $\sigma_p$ is the only unknown parameter regarding the AID.

4.2 Results

Tables 2-3  In Tables 2 and 3 we report results from experiments where the notional wage growth rates are generated as IID Normal random variables, and both types of rigidity are present. The results include the mean and root mean squared error (RMSE) of the estimates of $\rho^{ss}_{ts}$ and $\rho^{rr}_{ts}$ obtained from the 300 replications of each experiment. Each experiment executed has three variants with respect to the number of within-period heterogeneity groups ($= S$) that exist; Table 2 includes the results for $S$ equal to 1 and 5, and Table 3 for the case $S$ is equal to 10.

Columns (1)-(4) include results about the semiparametric estimator that is proposed here, and the underlying experiments differ with respect to the standardisation parameter ($\lambda_{ts}$) used in the implementation of the method (columns 1-3), and whether the identification condition regarding the location of AID is satisfied (columns 3-4).\footnote{The AID identification condition is satisfied under Specification A regarding $\mu^N_{ts}$ (and $\hat{\tau}_{ts}$), but not under Specification B. Under Specification A the mean anticipated inflation rate is always located within the middle of the support. In particular, the column (1) exper-}
iment could be considered as the benchmark (ideal) case since standardisation is based on the population value of a location parameter of the notional distribution (the notional median), and the AID identification condition is satisfied. The experiments underlying columns (2) and (3) differ from (1) in that the standardisation parameter used is the sample median and the sample percentile \( \hat{q}_{a,t,s} \) defined in (92), respectively, and the experiment underlying column (4) differs from (1) in that the standardisation parameter used is \( \hat{q}_{a,t,s} \) and the AID identification condition is not satisfied.

The results in column (1) for the estimates of \( \rho \) for both rigidity mechanisms appear very close to the population value. There is some evidence of positive bias, as all means reported are above the population value of the parameter estimated.

The comparison of the results across experiments with different value of \( S \) suggests a decrease in the efficiency of the estimator as the number of heterogeneity groups increases, seen in the increase of the RMSE. This affects mostly the DNWR estimator. There is no visible effect in the mean of the estimates (especially for the DRWR estimates), prompting us to conclude that the increase in RMSE is primarily due to increase in the variance.

This result could be expected. Increasing the number of sectors means fewer observations in each sub-sample (\( n_{ts} \)), which, in this case, results to a decrease in the precision of the estimation of the shape of the actual probability histograms. This should reflect in the estimation of the rigidity parameters and the notional probability histograms, which are the parameters that enter the likelihood function and whose interaction gives rise to the actual probability histograms.

In the experiment underlying column (2), standardisation is carried out with the sample median. We note that, given the design of the experiments, it is possible that for some sub-populations the underlying wage growth distributions are affected beyond the notional median to a varying degree. When this happens, the actual median will differ, not always by the same amount, from the notional median, and so the sample median is inappropriate in this case to be used for location standardisation. This is evident from the deterioration of the results in this column relative to column (1), especially on the results for the measure of DRWR.

In the experiment underlying the results in column (3), the order of the sample quantile is chosen so that it is located in a part of the actual WGD that is not distorted by rigidity, and thus appropriate to be used for standardisation. Thus the comparison of columns (1) and (3) provides us with information on the implications of using an estimate rather the population value of the location parameter. The results suggest loss of efficiency, as one might expect, with the bin that contains it, while under Specification B it is random.
some increase in the positive bias. This seems to be stronger for the DNWR measure, especially for $S = 1, 5$.

The experiment underlying the results in column (4) emulates the most realistic scenario, where the location parameter has to be estimated and the AID identifying condition is not satisfied. The comparison of columns (3) and (4) thus gives information on the implications of the AID identifying condition not being satisfied, as they both use the same estimated standardisation parameter. The results suggest a further loss of efficiency from that lost from the use of an estimate rather than the population value of the location parameter, which mostly affects the DRWR measure. We conclude by noting that from both (3) and (4), an increase in the number of heterogeneity groups within each period for a given sample size results to a loss of efficiency, as it was for column (1).\textsuperscript{34}

Given that the same DGP underlies the data used to produce the results in columns (4) and (5) - for the parametric estimator, - these results provide information on how the semiparametric estimator compares to the (correctly specified) parametric one. As one would expect, the parametric estimator performs better, with smaller bias and, for most of the parameters estimated, a smaller RMSE. However, overall, the results in the two columns are not very different.

Tables 4-5 In Tables 4 and 5 we report results from experiments that correspond to the columns (1), (4)-(6) in Tables 2 and 3. Here the real wage growth rates are generated as non-Normal non-symmetrically IID random variables, thus the comparison with the corresponding results in Tables 2 and 3 can provide information on the effect of non-Normality and non-symmetry on the performance of the estimators. In the cases of the parametric estimator, Normality is still assumed.

The properties of the semiparametric estimator appear similar to the Normality case; the results are relatively close to the population value, suggesting positive bias. Also, there is loss of efficiency as we move from using the population value to using an estimate of the location parameter used for standardisation, and as the number of $S$-heterogeneity groups increases.

The parametric estimator now appears to underestimate the size of DNWR and overestimate the size of DRWR, showing particularly high loss of efficiency in the estimation of the latter (sharp increase of the RMSE).

Clearly, in this setup the semiparametric estimator outperforms the other two estimators.\textsuperscript{34}

\textsuperscript{34}In these cases, the increase in $S$ has the additional effect of reducing the precision in the estimation of the standardisation parameter, which contributes to the loss of efficiency in the estimation of the $\rho$'s.
Tables 6-7 In Tables 6 and 7 we report results that correspond to columns (1), (4)-(6) in Tables 2 and 3. Further, in the case of Table 6 only DNWR is present ($p^r = 0$), and in the case of Table 7 only DRWR is present ($p^n = 0$).

Regarding the properties of the semiparametric estimator for each type of rigidity, these appear similar to those as in the case where both types of rigidity are present. It is interesting to note here that the DNWR parameter appears to be estimated more accurately when both types of rigidity are present than in the case where only DNWR is present.

The parametric estimator outperforms the other two estimators under both scenarios of rigidity present.

5 Conclusion

The simulation results presented above suggest that the semiparametric estimator considered in this paper can be a viable alternative to existing estimators of the size of downward wage rigidity, nominal or real.

Its performance is comparable to that of a well specified parametric estimator when the notional wage growth distribution is Normal. Furthermore, it appears to be robust to deviations from Normality.

On the other hand the parametric estimator only outperforms the semiparametric estimator when it is correctly specified.

References


30


31


<table>
<thead>
<tr>
<th></th>
<th>$\rho^n_s$</th>
<th>$\rho^n_r$</th>
<th>$p^n_s$</th>
<th>$p^n_r$</th>
<th>$\rho^{mn}_s$</th>
<th>$\rho^{rr}_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DNWR$</td>
<td>0.2</td>
<td>-</td>
<td>1</td>
<td>0</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>$DRWR$</td>
<td>-</td>
<td>0.2</td>
<td>0</td>
<td>1</td>
<td>-</td>
<td>0.2</td>
</tr>
<tr>
<td>$DNWR, DRWR$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 1: Simulation parameters

<table>
<thead>
<tr>
<th></th>
<th>S/P</th>
<th>P/M</th>
</tr>
</thead>
<tbody>
<tr>
<td># heterogeneity groups = 1</td>
<td>(1)  (2)  (3)  (4)  (5)</td>
<td>(1)  (2)  (3)  (4)  (5)</td>
</tr>
<tr>
<td>$\rho_1^{mn}$</td>
<td>0.2084 0.2181 0.2118 0.2105 0.1992</td>
<td>0.0176 0.0259 0.0304 0.0222 0.0130</td>
</tr>
<tr>
<td>$\rho_1^{rr}$</td>
<td>0.2059 0.1575 0.2067 0.2126 0.2045</td>
<td>0.0159 0.0447 0.0178 0.0252 0.0461</td>
</tr>
<tr>
<td># heterogeneity groups = 5</td>
<td>S/P</td>
<td>P/M</td>
</tr>
<tr>
<td>(1)  (2)  (3)  (4)  (5)</td>
<td>(1)  (2)  (3)  (4)  (5)</td>
<td></td>
</tr>
<tr>
<td>$\rho_1^{mn}$</td>
<td>0.2073 0.2193 0.2105 0.2106 0.2004</td>
<td>0.0355 0.0435 0.0363 0.0387 0.0228</td>
</tr>
<tr>
<td>$\rho_2^{mn}$</td>
<td>0.2122 0.2280 0.2177 0.2128 0.1983</td>
<td>0.0384 0.0516 0.0433 0.0444 0.0227</td>
</tr>
<tr>
<td>$\rho_3^{mn}$</td>
<td>0.2083 0.2205 0.2126 0.2173 0.2005</td>
<td>0.0345 0.0446 0.0376 0.0416 0.0243</td>
</tr>
<tr>
<td>$\rho_4^{mn}$</td>
<td>0.2060 0.2185 0.2101 0.2110 0.2006</td>
<td>0.0345 0.0440 0.0381 0.0359 0.0237</td>
</tr>
<tr>
<td>$\rho_5^{mn}$</td>
<td>0.2114 0.2231 0.2131 0.2123 0.1985</td>
<td>0.0353 0.0480 0.0406 0.0422 0.0234</td>
</tr>
<tr>
<td>$\rho_1^{rr}$</td>
<td>0.2053 0.1570 0.2065 0.2138 0.2002</td>
<td>0.0242 0.0500 0.0275 0.0301 0.0248</td>
</tr>
<tr>
<td>$\rho_2^{rr}$</td>
<td>0.2060 0.1563 0.2074 0.2132 0.1993</td>
<td>0.0248 0.0505 0.0289 0.0302 0.0241</td>
</tr>
<tr>
<td>$\rho_3^{rr}$</td>
<td>0.2040 0.1522 0.2060 0.2128 0.2018</td>
<td>0.0235 0.0333 0.0277 0.0299 0.0333</td>
</tr>
<tr>
<td>$\rho_4^{rr}$</td>
<td>0.2052 0.1556 0.2069 0.2121 0.2015</td>
<td>0.0255 0.0510 0.0274 0.0299 0.0258</td>
</tr>
<tr>
<td>$\rho_5^{rr}$</td>
<td>0.2036 0.1544 0.2070 0.2118 0.1994</td>
<td>0.0238 0.0522 0.0274 0.0306 0.0307</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>AID IC</th>
<th>yes</th>
<th>yes</th>
<th>yes</th>
<th>no</th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td>satisfied</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$\hat{q}_{50}$</td>
<td>$\hat{q}_{50}$</td>
<td>$\hat{q}_{\alpha}$</td>
<td>$\hat{q}_{\alpha}$</td>
<td>-</td>
</tr>
</tbody>
</table>

NB: For each parameter, the first line records the mean and the second (smaller font) the RMSE. The number of replications is 300.

Table 2: DNWR & DRWR - symmetric notional WGD
<table>
<thead>
<tr>
<th># heterogeneity groups = 10</th>
<th>S/P</th>
<th>P/M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>( \rho_{1n}^{\text{nn}} )</td>
<td>0.2111</td>
<td>0.2225</td>
</tr>
<tr>
<td></td>
<td>0.0528</td>
<td>0.0610</td>
</tr>
<tr>
<td>( \rho_{2n}^{\text{nn}} )</td>
<td>0.2124</td>
<td>0.2249</td>
</tr>
<tr>
<td></td>
<td>0.0538</td>
<td>0.0637</td>
</tr>
<tr>
<td>( \rho_{3n}^{\text{nn}} )</td>
<td>0.2169</td>
<td>0.2244</td>
</tr>
<tr>
<td></td>
<td>0.0492</td>
<td>0.0559</td>
</tr>
<tr>
<td>( \rho_{4n}^{\text{nn}} )</td>
<td>0.2091</td>
<td>0.2218</td>
</tr>
<tr>
<td></td>
<td>0.0541</td>
<td>0.0626</td>
</tr>
<tr>
<td>( \rho_{5n}^{\text{nn}} )</td>
<td>0.2100</td>
<td>0.2247</td>
</tr>
<tr>
<td></td>
<td>0.0517</td>
<td>0.0612</td>
</tr>
<tr>
<td>( \rho_{6n}^{\text{nn}} )</td>
<td>0.2083</td>
<td>0.2210</td>
</tr>
<tr>
<td></td>
<td>0.0485</td>
<td>0.0537</td>
</tr>
<tr>
<td>( \rho_{7n}^{\text{nn}} )</td>
<td>0.2112</td>
<td>0.2248</td>
</tr>
<tr>
<td></td>
<td>0.0534</td>
<td>0.0650</td>
</tr>
<tr>
<td>( \rho_{8n}^{\text{nn}} )</td>
<td>0.2122</td>
<td>0.2230</td>
</tr>
<tr>
<td></td>
<td>0.0517</td>
<td>0.0626</td>
</tr>
<tr>
<td>( \rho_{9n}^{\text{nn}} )</td>
<td>0.2132</td>
<td>0.2274</td>
</tr>
<tr>
<td></td>
<td>0.0524</td>
<td>0.0665</td>
</tr>
<tr>
<td>( \rho_{10n}^{\text{nn}} )</td>
<td>0.2062</td>
<td>0.2219</td>
</tr>
<tr>
<td></td>
<td>0.0509</td>
<td>0.0588</td>
</tr>
<tr>
<td>( \rho_{1r}^{\text{rr}} )</td>
<td>0.2031</td>
<td>0.1515</td>
</tr>
<tr>
<td></td>
<td>0.0320</td>
<td>0.0578</td>
</tr>
<tr>
<td>( \rho_{2r}^{\text{rr}} )</td>
<td>0.2026</td>
<td>0.1513</td>
</tr>
<tr>
<td></td>
<td>0.0309</td>
<td>0.0576</td>
</tr>
<tr>
<td>( \rho_{3r}^{\text{rr}} )</td>
<td>0.2055</td>
<td>0.1492</td>
</tr>
<tr>
<td></td>
<td>0.0310</td>
<td>0.0200</td>
</tr>
<tr>
<td>( \rho_{4r}^{\text{rr}} )</td>
<td>0.2017</td>
<td>0.1513</td>
</tr>
<tr>
<td></td>
<td>0.0321</td>
<td>0.0387</td>
</tr>
<tr>
<td>( \rho_{5r}^{\text{rr}} )</td>
<td>0.2048</td>
<td>0.1514</td>
</tr>
<tr>
<td></td>
<td>0.0341</td>
<td>0.0580</td>
</tr>
<tr>
<td>( \rho_{6r}^{\text{rr}} )</td>
<td>0.2031</td>
<td>0.1508</td>
</tr>
<tr>
<td></td>
<td>0.0300</td>
<td>0.0575</td>
</tr>
<tr>
<td>( \rho_{7r}^{\text{rr}} )</td>
<td>0.2024</td>
<td>0.1489</td>
</tr>
<tr>
<td></td>
<td>0.0307</td>
<td>0.0587</td>
</tr>
<tr>
<td>( \rho_{8r}^{\text{rr}} )</td>
<td>0.2026</td>
<td>0.1520</td>
</tr>
<tr>
<td></td>
<td>0.0323</td>
<td>0.0562</td>
</tr>
<tr>
<td>( \rho_{9r}^{\text{rr}} )</td>
<td>0.2064</td>
<td>0.1535</td>
</tr>
<tr>
<td></td>
<td>0.0333</td>
<td>0.0566</td>
</tr>
<tr>
<td>( \rho_{10r}^{\text{rr}} )</td>
<td>0.2055</td>
<td>0.1510</td>
</tr>
<tr>
<td></td>
<td>0.0296</td>
<td>0.0570</td>
</tr>
</tbody>
</table>

AID IC satisfied | yes | yes | yes | no | no |
\( \lambda \) | \( \hat{q}_{50} \) | \( \hat{q}_{50} \) | \( \hat{q}_{\alpha} \) | \( \hat{q}_{\alpha} \) | -

NB: For each parameter, the first line records the mean and the second (smaller font) the RMSE. The number of replications is 300.

Table 3: DNWR & DRWR - symmetric notional WGD
<table>
<thead>
<tr>
<th># heterogeneity groups = 1</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S/P</td>
<td>P/M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>$\rho_1^{nm}$</td>
<td>0.2069</td>
<td>0.2044</td>
<td>0.1844</td>
</tr>
<tr>
<td></td>
<td>0.0305</td>
<td>0.0237</td>
<td>0.0267</td>
</tr>
<tr>
<td>$\rho_1^{rr}$</td>
<td>0.2034</td>
<td>0.2081</td>
<td>0.3637</td>
</tr>
<tr>
<td></td>
<td>0.0151</td>
<td>0.0223</td>
<td>0.2140</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th># heterogeneity groups = 5</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S/P</td>
<td>P/M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>$\rho_1^{nm}$</td>
<td>0.2039</td>
<td>0.2060</td>
<td>0.1885</td>
</tr>
<tr>
<td></td>
<td>0.0397</td>
<td>0.0395</td>
<td>0.0304</td>
</tr>
<tr>
<td>$\rho_2^{nm}$</td>
<td>0.2046</td>
<td>0.2057</td>
<td>0.1848</td>
</tr>
<tr>
<td></td>
<td>0.0391</td>
<td>0.0419</td>
<td>0.0317</td>
</tr>
<tr>
<td>$\rho_3^{nm}$</td>
<td>0.2066</td>
<td>0.2093</td>
<td>0.1884</td>
</tr>
<tr>
<td></td>
<td>0.0413</td>
<td>0.0441</td>
<td>0.0311</td>
</tr>
<tr>
<td>$\rho_4^{nm}$</td>
<td>0.2077</td>
<td>0.2065</td>
<td>0.1848</td>
</tr>
<tr>
<td></td>
<td>0.0409</td>
<td>0.0437</td>
<td>0.0338</td>
</tr>
<tr>
<td>$\rho_5^{nm}$</td>
<td>0.2049</td>
<td>0.2023</td>
<td>0.1881</td>
</tr>
<tr>
<td></td>
<td>0.0447</td>
<td>0.0416</td>
<td>0.0325</td>
</tr>
<tr>
<td>$\rho_1^{rr}$</td>
<td>0.2009</td>
<td>0.2137</td>
<td>0.3457</td>
</tr>
<tr>
<td></td>
<td>0.0252</td>
<td>0.0334</td>
<td>0.1962</td>
</tr>
<tr>
<td>$\rho_2^{rr}$</td>
<td>0.2018</td>
<td>0.2121</td>
<td>0.3547</td>
</tr>
<tr>
<td></td>
<td>0.0236</td>
<td>0.0317</td>
<td>0.2074</td>
</tr>
<tr>
<td>$\rho_3^{rr}$</td>
<td>0.2022</td>
<td>0.2125</td>
<td>0.3508</td>
</tr>
<tr>
<td></td>
<td>0.0239</td>
<td>0.0321</td>
<td>0.2037</td>
</tr>
<tr>
<td>$\rho_4^{rr}$</td>
<td>0.2014</td>
<td>0.2124</td>
<td>0.3510</td>
</tr>
<tr>
<td></td>
<td>0.0235</td>
<td>0.0301</td>
<td>0.2039</td>
</tr>
<tr>
<td>$\rho_5^{rr}$</td>
<td>0.2006</td>
<td>0.2113</td>
<td>0.3501</td>
</tr>
<tr>
<td></td>
<td>0.0246</td>
<td>0.0326</td>
<td>0.2035</td>
</tr>
</tbody>
</table>

AID IC satisfied: yes no no

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$q_{50}$</th>
<th>$\hat{q}_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>

NB: For each parameter, the first line records the mean and the second (smaller font) the RMSE.

The number of replications of each experiment is 300.

Table 4: DNWR & DRWR – non-symmetric notional WGD
# heterogeneity groups = 10

<table>
<thead>
<tr>
<th></th>
<th>S/P</th>
<th>P/M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(4)</td>
</tr>
<tr>
<td>$\rho_{1n}$</td>
<td>0.2104</td>
<td>0.2090</td>
</tr>
<tr>
<td></td>
<td>0.0572</td>
<td>0.0563</td>
</tr>
<tr>
<td>$\rho_{2n}$</td>
<td>0.2067</td>
<td>0.2109</td>
</tr>
<tr>
<td></td>
<td>0.0535</td>
<td>0.0616</td>
</tr>
<tr>
<td>$\rho_{3n}$</td>
<td>0.2020</td>
<td>0.2069</td>
</tr>
<tr>
<td></td>
<td>0.0491</td>
<td>0.0583</td>
</tr>
<tr>
<td>$\rho_{4n}$</td>
<td>0.2077</td>
<td>0.2043</td>
</tr>
<tr>
<td></td>
<td>0.0577</td>
<td>0.0392</td>
</tr>
<tr>
<td>$\rho_{5n}$</td>
<td>0.2004</td>
<td>0.2100</td>
</tr>
<tr>
<td></td>
<td>0.0521</td>
<td>0.0008</td>
</tr>
<tr>
<td>$\rho_{6n}$</td>
<td>0.2042</td>
<td>0.2072</td>
</tr>
<tr>
<td></td>
<td>0.0603</td>
<td>0.0383</td>
</tr>
<tr>
<td>$\rho_{7n}$</td>
<td>0.2060</td>
<td>0.2049</td>
</tr>
<tr>
<td></td>
<td>0.0593</td>
<td>0.0550</td>
</tr>
<tr>
<td>$\rho_{8n}$</td>
<td>0.2059</td>
<td>0.2046</td>
</tr>
<tr>
<td></td>
<td>0.0567</td>
<td>0.0575</td>
</tr>
<tr>
<td>$\rho_{9n}$</td>
<td>0.2027</td>
<td>0.2075</td>
</tr>
<tr>
<td></td>
<td>0.0594</td>
<td>0.0386</td>
</tr>
<tr>
<td>$\rho_{10n}$</td>
<td>0.2065</td>
<td>0.2027</td>
</tr>
<tr>
<td></td>
<td>0.0590</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\rho_{1r}$</th>
<th>$\rho_{2r}$</th>
<th>$\rho_{3r}$</th>
<th>$\rho_{4r}$</th>
<th>$\rho_{5r}$</th>
<th>$\rho_{6r}$</th>
<th>$\rho_{7r}$</th>
<th>$\rho_{8r}$</th>
<th>$\rho_{9r}$</th>
<th>$\rho_{10r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.2054 0.2248</td>
<td>0.2090 0.2248</td>
<td>0.2107 0.2248</td>
<td>0.2095 0.2248</td>
<td>0.2095 0.2248</td>
<td>0.2095 0.2248</td>
<td>0.2095 0.2248</td>
<td>0.2095 0.2248</td>
<td>0.2095 0.2248</td>
<td>0.2095 0.2248</td>
</tr>
<tr>
<td></td>
<td>0.0353 0.0476</td>
<td>0.0353 0.0476</td>
<td>0.0353 0.0476</td>
<td>0.0353 0.0476</td>
<td>0.0353 0.0476</td>
<td>0.0353 0.0476</td>
<td>0.0353 0.0476</td>
<td>0.0353 0.0476</td>
<td>0.0353 0.0476</td>
<td>0.0353 0.0476</td>
</tr>
</tbody>
</table>

AID IC satisfied

$\lambda = q_{50}$, $\hat{q}_{\alpha}$

NB: For each parameter, the first line records the mean and the second (smaller font) the RMSE.

The number of replications of each experiment is 300.

Table 5: DNWR & DRWR – non-symmetric notional WGD
<table>
<thead>
<tr>
<th># heterogeneity groups = 1</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S/P</td>
<td>P/M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(4)</td>
<td>(5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_1^{nm} )</td>
<td>0.2102</td>
<td>0.2121</td>
<td>0.2001</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0197</td>
<td>0.0222</td>
<td>0.0101</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th># heterogeneity groups = 5</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S/P</td>
<td>P/M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(4)</td>
<td>(5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_1^{nm} )</td>
<td>0.2142</td>
<td>0.2133</td>
<td>0.2007</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0434</td>
<td>0.0426</td>
<td>0.0244</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_2^{nm} )</td>
<td>0.2132</td>
<td>0.2132</td>
<td>0.1994</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0455</td>
<td>0.0438</td>
<td>0.0243</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_3^{nm} )</td>
<td>0.2165</td>
<td>0.2165</td>
<td>0.1981</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0428</td>
<td>0.0415</td>
<td>0.0231</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_4^{nm} )</td>
<td>0.2113</td>
<td>0.2100</td>
<td>0.2005</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0435</td>
<td>0.0414</td>
<td>0.0223</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_5^{nm} )</td>
<td>0.2133</td>
<td>0.2163</td>
<td>0.1980</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0435</td>
<td>0.0466</td>
<td>0.0233</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th># heterogeneity groups = 10</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S/P</td>
<td>P/M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(4)</td>
<td>(5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_1^{nm} )</td>
<td>0.2109</td>
<td>0.2161</td>
<td>0.2013</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0586</td>
<td>0.0392</td>
<td>0.0351</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_2^{nm} )</td>
<td>0.2123</td>
<td>0.2118</td>
<td>0.1995</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0594</td>
<td>0.0604</td>
<td>0.0341</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_3^{nm} )</td>
<td>0.2160</td>
<td>0.2173</td>
<td>0.2011</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0564</td>
<td>0.0582</td>
<td>0.0355</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_4^{nm} )</td>
<td>0.2018</td>
<td>0.2072</td>
<td>0.2001</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0591</td>
<td>0.0628</td>
<td>0.0331</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_5^{nm} )</td>
<td>0.2191</td>
<td>0.2205</td>
<td>0.1990</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0585</td>
<td>0.0629</td>
<td>0.0346</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_6^{nm} )</td>
<td>0.2104</td>
<td>0.2137</td>
<td>0.2020</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0564</td>
<td>0.0553</td>
<td>0.0345</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_7^{nm} )</td>
<td>0.2062</td>
<td>0.2131</td>
<td>0.2032</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0545</td>
<td>0.0597</td>
<td>0.0337</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_8^{nm} )</td>
<td>0.2136</td>
<td>0.2162</td>
<td>0.2004</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0627</td>
<td>0.0637</td>
<td>0.0322</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_9^{nm} )</td>
<td>0.2161</td>
<td>0.2161</td>
<td>0.1956</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0615</td>
<td>0.0395</td>
<td>0.0324</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_{10}^{nm} )</td>
<td>0.2131</td>
<td>0.2105</td>
<td>0.1988</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0574</td>
<td>0.0587</td>
<td>0.0336</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| AID IC satisfied           | yes                            | no                             | no                             |                               |                               |
| \( \lambda \)              | \( q_{50} \)                   | \( \hat{q}_\alpha \)          | -                             |                               |                               |

NB: For each parameter, the first line records the mean and the second (smaller font) the RMSE. The number of replications of each experiment is 300.

Table 6: DNWR – symmetric notional WGD
<table>
<thead>
<tr>
<th># heterogeneity groups = 1</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S/P</td>
<td>P/M</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(4)</td>
</tr>
<tr>
<td>$\rho_{1}^{rr}$</td>
<td>0.2086</td>
<td>0.2146</td>
</tr>
<tr>
<td></td>
<td>0.0167</td>
<td>0.0259</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th># heterogeneity groups = 5</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S/P</td>
<td>P/M</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(4)</td>
</tr>
<tr>
<td>$\rho_{1}^{rr}$</td>
<td>0.2073</td>
<td>0.2136</td>
</tr>
<tr>
<td></td>
<td>0.0237</td>
<td>0.0297</td>
</tr>
<tr>
<td>$\rho_{2}^{rr}$</td>
<td>0.2080</td>
<td>0.2135</td>
</tr>
<tr>
<td></td>
<td>0.0245</td>
<td>0.0297</td>
</tr>
<tr>
<td>$\rho_{3}^{rr}$</td>
<td>0.2050</td>
<td>0.2126</td>
</tr>
<tr>
<td></td>
<td>0.0242</td>
<td>0.0294</td>
</tr>
<tr>
<td>$\rho_{4}^{rr}$</td>
<td>0.2081</td>
<td>0.2132</td>
</tr>
<tr>
<td></td>
<td>0.0244</td>
<td>0.0304</td>
</tr>
<tr>
<td>$\rho_{5}^{rr}$</td>
<td>0.2061</td>
<td>0.2127</td>
</tr>
<tr>
<td></td>
<td>0.0249</td>
<td>0.0311</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th># heterogeneity groups = 10</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S/P</td>
<td>P/M</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(4)</td>
</tr>
<tr>
<td>$\rho_{1}^{rr}$</td>
<td>0.2052</td>
<td>0.2225</td>
</tr>
<tr>
<td></td>
<td>0.0304</td>
<td>0.0435</td>
</tr>
<tr>
<td>$\rho_{2}^{rr}$</td>
<td>0.2030</td>
<td>0.2209</td>
</tr>
<tr>
<td></td>
<td>0.0304</td>
<td>0.0433</td>
</tr>
<tr>
<td>$\rho_{3}^{rr}$</td>
<td>0.2070</td>
<td>0.2244</td>
</tr>
<tr>
<td></td>
<td>0.0320</td>
<td>0.0463</td>
</tr>
<tr>
<td>$\rho_{4}^{rr}$</td>
<td>0.2039</td>
<td>0.2238</td>
</tr>
<tr>
<td></td>
<td>0.0320</td>
<td>0.0432</td>
</tr>
<tr>
<td>$\rho_{5}^{rr}$</td>
<td>0.2064</td>
<td>0.2255</td>
</tr>
<tr>
<td></td>
<td>0.0356</td>
<td>0.0453</td>
</tr>
<tr>
<td>$\rho_{6}^{rr}$</td>
<td>0.2051</td>
<td>0.2206</td>
</tr>
<tr>
<td></td>
<td>0.0322</td>
<td>0.0417</td>
</tr>
<tr>
<td>$\rho_{7}^{rr}$</td>
<td>0.2072</td>
<td>0.2215</td>
</tr>
<tr>
<td></td>
<td>0.0323</td>
<td>0.0422</td>
</tr>
<tr>
<td>$\rho_{8}^{rr}$</td>
<td>0.2049</td>
<td>0.2217</td>
</tr>
<tr>
<td></td>
<td>0.0292</td>
<td>0.0425</td>
</tr>
<tr>
<td>$\rho_{9}^{rr}$</td>
<td>0.2080</td>
<td>0.2248</td>
</tr>
<tr>
<td></td>
<td>0.0315</td>
<td>0.0447</td>
</tr>
<tr>
<td>$\rho_{10}^{rr}$</td>
<td>0.2062</td>
<td>0.2249</td>
</tr>
<tr>
<td></td>
<td>0.0308</td>
<td>0.0441</td>
</tr>
</tbody>
</table>

AID IC satisfied yes no no
$\lambda$  $q_{50}$  $\hat{q}_{\alpha}$ -

NB: For each parameter, the first line records the mean and the second (smaller font) the RMSE.
The number of replications of each experiment is 300.

Table 7: DRWR – symmetric notional WGD
A Identification: Proofs

A.1 DNWR

The data from a particular sub-population, indexed by $t_s$,

$$\{\dot{w}_{tst}\}_{t=1,\ldots,n_t}$$  (93)

allows us to estimate the set of probabilities

$$\{P_{jts}\}_{j=1,\ldots,J}$$  (94)

i.e. the height of the bins of the probability histogram corresponding to the standardised actual WGD\(^35\) (the ‘actual’ probability histogram) for that sub-population. We recall from (58) that, in the presence of DNWR, the quantities above relate to the parameter of interest $\rho_s^{nn}$ and the notional bin heights $\{P^N_j\}_{j=1,\ldots,J}$, which are ancillary parameters, in the following way

$$P_{jts} = \begin{cases} 
    P_j^N, & j > J^0_{ts} \\
    (1 - \rho_s^{nn}) P_j^N, & j < J^0_{ts} \\
    P_j^N + \rho_s^{nn} \sum_{\zeta<j}^N P_\zeta^N, & j = J^0_{ts}
\end{cases}$$  (95)

The following two propositions provide sufficient conditions for the identification of $\rho_s^{nn}$ for the cases where $S = 1$ (homogeneity case w.r.t. $s$) and $S > 1$ (heterogeneity case w.r.t. $s$). To simplify the notation we drop subscript $s$ in the discussion of the homogeneity case.

**Proposition 1 (Homogeneity case)** In the case where $S = 1$, conditions on the data on actual wage growth rates

$$\{\dot{w}_{ti}\}_{t=1,\ldots,T, i=1,\ldots,n_t}$$  (96)

that are sufficient for the identification of the rigidity parameter $\rho^{nn}$ in the semiparametric model described above, are the following:

1. $T \geq 2$, and

2. for at least one pair of periods $t_1, t_2 \in \{1, \ldots, T\}$, where $t_1 \neq t_2$, the following are satisfied:

\(^{35}\)That is, the PMF of $y_t$.\]
(a) \( 1 < J_{t_1}^0 \leq J \), i.e. the ‘actual’ probability histogram in \( t = t_1 \) is distorted by DNWR,

(b) \( J_{t_2}^0 < J_{t_1}^0 \), i.e. at least one of the bins distorted in \( t = t_1 \) is not distorted in \( t = t_2 \).

**Proof.** Without loss of generality we consider the case where

\[
T = 2
\]

\[
1 < J_2^0 < J_1^0 \leq J
\]

i.e. the probability histograms underlying the data from the two periods are both distorted by DNWR, since both \( J_1^0 \) > 1 and \( J_2^0 \) > 1, but not all bins whose height is distorted in \( t = 1 \) are also affected in \( t = 2 \). An example that adheres to this case is depicted in Figure 4, where the solid line represents the PDF of the standardised notional WGD (\( \tilde{f}^N \)), the wider bins correspond to the ‘actual’ probability histogram of for period 1 \((P_{j_1}, j = 1, \ldots, J)\), and the narrower bins to the ‘actual’ probability histogram for period 2 \((P_{j_2}, j = 1, \ldots, J)\). In this example we have \( J = 9 \). In period 1 the bin that contains the standardised value of zero has index value \( j = 5 \) (= \( J_1^0 \)), and for period 2 the corresponding bin has index value \( j = 4 \) (= \( J_2^0 \)). For ease of reference we mark the bins - in both histograms - that are not distorted by rigidity\(^{36}\) with stripes. We also use a dashed horizontal line to indicate the height of the corresponding bin in the notional probability histogram for those values of \( j \) where at least one of the bins is distorted.

For the general case defined by (97) and (98), the available data

\[
\left\{ \tilde{w}_{ti} \right\}_{t=1,2, i=1,\ldots,n_t}
\]

allows us to obtain the collection of estimates

\[
\left\{ \hat{P}_jt \right\}_{j=1,\ldots,J}, \quad t = 1, 2
\]

where, in period \( t = 1 \), the estimated quantities satisfy

\[
P_{j_1} = \begin{cases} 
P_j^N, & j = J_1^0 + 1, \ldots, J \quad \text{(if} \ J_1^0 < J) \\ 
(1 - \rho^m) P_j^N, & j = 1, \ldots, J_1^0 - 1 \\ 
P_j^N + \rho^m \sum_{\zeta<j} P_{\zeta}^N, & j = J_1^0 
\end{cases}
\]

\(^{36}\)Whose height is, therefore, the same as the height of the corresponding bins of the ‘notional’ probability histogram; this, by assumption, is the same for both periods.
and, in period $t = 2$

$$P_{j2} = \begin{cases} 
  P_j^N, & j = J_2^0 + 1, \ldots, J_1^0, \ldots, J \\
  (1 - \rho^{nn}) P_j^N, & j = 1, \ldots, J_2^0 - 1 \\
  P_j^N + \rho^{nn} \sum_{\zeta < j} P_{\zeta}^N, & j = J_2^0 
\end{cases} \quad (102)$$

Thus, for the example depicted in Figure 4, we see that the wider bins - that correspond to the period 1 histogram - have equal height to the corresponding notional bins (and are, therefore, marked by stripes) for $j \geq 5 \ (= J_1^0)$, the bin indexed by $j = 5$ is taller than the corresponding notional bin (and not marked by stripes), and all bins indexed by $j < 5$ are shorter than the corresponding notional bins (and also not marked by stripes). A similar pattern applies to the period 2 (narrower) bins, except that in that case the three different type of bins discussed above - equal, taller, shorter than the corresponding notional bins - are defined with respect to the bin indexed by $j = 4 \ (= J_2^0)$.

The probabilities from the two periods which are associated with the bin that contains the standardised value of point zero in period 1, i.e. the bin indexed by $j = J_1^0$, satisfy

$$P_{j1} = P_j^N + \rho^{nn} \sum_{\zeta < j} P_{\zeta}^N \quad (103)$$

$$P_{j2} = P_j^N \quad (104)$$
Combining these two, we can write

\[ \rho_{nn} = \frac{P_{j_1} - P_{j_2}}{\sum_{\zeta<j} P_{N\zeta}} , \quad j = J_0^1 \]  

(105)

Then, using the result in the top row on the RHS of (102), we can write the denominator as a function of the quantities identified by the data from period \( t = 2 \);

\[ \sum_{\zeta<J_0^0} P_{N\zeta} = 1 - \sum_{\zeta\geq J_0^0} P_{N\zeta} = 1 - \sum_{\zeta\geq J_0^0} P_{\zeta 2} \]  

(106)

and, therefore, substituting back into (105), we could express the rigidity parameter as a function of quantities that are estimable

\[ \rho_{nn} = \frac{P_{j_1} - P_{j_2}}{1 - \sum_{\zeta\geq j} P_{\zeta 2}} , \quad j = J_0^0 \]  

(107)

The above result is self explanatory: the numerator gives the magnitude of the probability mass that, due to DNWR, is missing, in total, from the bins that lie to the left of the bin that contains the standardised value of zero in period 1, and correspond to the area of the un-standardised notional WGD that lies below zero.\(^{37}\) On the other hand, the denominator gives the probability mass that would be allocated to those bins in the absence of DNWR. Thus their ratio gives the probability of the event a negative notional wage growth rate not being realised conditional on the event the notional growth rate being negative, which is the formal definition of \( \rho_{nn} \). This ratio is often referred to in this literature as the ‘sweep-up ratio’, which, in the case of proportional DNWR, is equal to \( \rho_{nn} \).

We conclude this part by making three additional points:

First, we note from (102) that the data from period \( t = 2 \) allow the identification of \( \{ P_{Nj} \}_{j = J_0^2 + 1, \ldots, J_0^1, \ldots} \). From (101) we also note that

\[ P_{j1} = (1 - \rho_{nn}) P_{Nj} , \quad j = 1, \ldots, J_0^1 - 1 \]  

(108)

Therefore we can substitute expression (107) for \( \rho_{nn} \) in (108) and solve for \( P_{Nj} \) to write the remaining probabilities of the ‘notional’ probability histogram as a function of identifiable quan-

\(^{37}\)More precisely, below \( \eta_{J_0^0 - 1} + \lambda_1 \).
Secondly, we note that an alternative identification strategy is available in the special case where, in addition to the conditions in (98), it is also true that \( J_0^1 - J_0^2 > 1 \), i.e. the difference in the value of the index of the bins that contain the standardised values of zero in the histograms from the two periods is greater than one. In that case, for those bins indexed by \( j = J_2^0 + 1, \ldots, J_1^0 - 1 \), i.e. the bins ‘in between’ the bins indexed by \( j = J_2^0 \) and \( j = J_1^0 \), it is true that

\[
P_{j_1} = (1 - \rho^{nn}) P_N^j \tag{109}
\]
\[
P_{j_2} = P_N^j \tag{110}
\]

therefore we could write

\[
\rho^{nn} = \frac{P_{j_2} - P_{j_1}}{P_{j_2}}, \quad J_2^0 < j < J_1^0 \tag{111}
\]

where, as in (107), the quantities on the RHS of this expression are all identified by the available data.

Thirdly, we note that both formulas in (107) and (111) are also valid in the case where \( J_2^0 \leq 1 \), i.e. the location of the second distribution is in the positive orthant and far from point zero such that its probability histogram cannot be distorted by the presence of DNWR. In that case we have

\[
P_{j_2} = P_N^j, \quad j = 1, \ldots, J \tag{112}
\]

**Proposition 2** (Heterogeneity case) In the case where \( S > 1 \), conditions on the data on actual wage growth rates

\[
\{\hat{w}_{tsi}\}_{t=1,\ldots,T, i=1,\ldots,n_t} \tag{113}
\]

that are sufficient for the identification of the rigidity parameters \( \rho^{nn}_s \) in the semiparametric model described above, are the following:

1. The conditions of Proposition 1 are satisfied for some \( s^* \in \{1, \ldots, S\} \).
2. For \( s \neq s^* \) it is true that \( 1 < J_{ts}^0 \leq J \) for at least one value of \( t \in \{1, \ldots, T\} \), i.e. there exists at least one period for each heterogeneity group other than \( s^* \) whose ‘actual’ probability histogram is distorted by DNWR.

**Proof.** From above, the conditions of Proposition 1 are also sufficient for the identification of the standardised notional probability histogram \( \{P_N^j\}_{j=1,\ldots,J} \). Then we can recover the
remaining $\rho_{nn}^s$'s by applying

$$\rho_{nn}^s = \frac{P_j^N - P_{jts}^N}{P_j^N}, \ s \neq s^*, \ 1 \leq j < J_{ts}^0$$

where the condition $1 \leq j < J_{ts}^0$ states that the probability mass $P_{jts}$ from the ‘actual’ histogram of sub-population $ts$ is distorted. 

A.2 DRWR

In this case, the data from a particular sub-population, indexed by $ts$,

$$\{\dot{w}_{tsi}\}_{i=1,\ldots,n_t}$$

allows us to estimate the ‘actual’ probability histogram bin heights

$$\{P_{jts}\}_{j=1,\ldots,J}$$

that satisfy

$$P_{jts} = \begin{cases} 
    P_j^N, & j > J_{ts}^P \\
    (1 - \rho_{rr}^s) P_j^N, & j < J_{ts}^P \\
    P_j^N - P_j^N \rho_{rr}^s \left( \sum_{\xi > j} \pi_{\xi - J_{ts}^P} \right) + \left( \sum_{\zeta < j} P_{\zeta}^N \right) \rho_{rr}^s \pi_{j - J_{ts}^P}, & J_{ts}^P \leq j \leq J_{ts}^P 
\end{cases}$$

(116)

The following two propositions provide sufficient conditions for the identification of $\rho_{rr}^s$ for the cases where $S = 1$ (homogeneity case w.r.t. $s$) and $S > 1$ (heterogeneity case w.r.t. $s$). As before, to simplify the notation we drop subscript $s$ in the discussion of the homogeneity case.

**Proposition 3 (Homogeneity case)** Conditions on the data on actual wage growth rates

$$\{\dot{w}_{ti}\}_{t=1,\ldots,T, \ i=1,\ldots,n_t}$$

(117)

that are sufficient for the identification of the rigidity parameter $\rho_{rr}^s$ in the semiparametric model described above, are the following:

---

38 Here we assume that the location of AID is ‘standardised’ in the way discussed at the end of Section 3.1.

39 A simplified version of the relationship given in (70)-(72), where $p^n = 0$ (and $p', p^t > 0$).
1. $T \geq 2$, and

2. for at least one pair of periods $t_1, t_2 \in \{1, \ldots, T\}$, where $t_1 \neq t_2$, the following are satisfied:

   (a) $1 \leq J_{t_1}^p < \bar{J}_{t_1}^p \leq J$, i.e. the support of the AID in $t = t_1$ is a subset of the support of the notional WGD in that period,

   (b) $\bar{J}_{t_2}^p < J_{t_1}^p$, i.e. the support of the AID in $t = t_2$ does not overlap with the support of the AID in $t = t_1$ and lies to its left, therefore the bins that contain the support of AID in $t = t_1$ are not distorted in $t = t_2$ by DRWR.

**Proof.** Without loss of generality we consider the case where

\begin{align}
T &= 2 \\
1 &\leq J_{t_1}^p < \bar{J}_{t_1}^p \leq J \\
1 &\leq \bar{J}_{t_2}^p < J_{t_1}^p
\end{align}

i.e. the underlying probability histograms for the data from the two periods are both distorted by DRWR, but the bins that contain the support of the AID in period $t = 1$ are not distorted in period $t = 2$. An example that adheres to the above assumptions is depicted in Figure 5,\(^{40}\) which shows the PDF of the standardised notional WGD, as well as the ‘actual’ probability histograms for periods 1 and 2, where $J_{t_1}^p = 6$ and $J_{t_2}^p = 2$. Furthermore, the support of the (standardised) AID covers three bins such that, for period 1, $J_{t_1}^p = 5$ and $\bar{J}_{t_1}^p = 7$, and, for period 2, $J_{t_2}^p = 2$ and $\bar{J}_{t_2}^p = 4$.

For the general case described by (118)-(120), the available data

\begin{align}
\{\hat{w}_{ti}\}_{t=1,2, i=1,\ldots,n_t}
\end{align}

allows us to obtain the collection of estimates

\begin{align}
\left\{\hat{P}_{jt}\right\}_{j=1,\ldots,J}, \quad t = 1, 2
\end{align}

\(^{40}\)The format of the design is the same to that of Figure 4, which depicted the DNWR example.
where in period 1, the estimated quantities satisfy

\[
P_{j1} = \begin{cases} 
    P_N^j, & j = J_1^P + 1, \ldots, J \\
    (1 - \rho^{rr}) P_N^j & \text{for } j = 1, \ldots, J_1^P - 1 \\
    P_N^j - P_N^j \rho^{rr} \left( \sum_{\xi=j+1}^{J_1^P} \pi_\xi - J_1^P \right) + 
    \left( \sum_{\zeta<j} P_N^\zeta \right) \rho^{rr} \pi_{j-J_1^P} & \text{for } j = J_1^P, \ldots, \bar{J}_1^P
\end{cases}
\]

and, in period 1

\[
P_{j2} = \begin{cases} 
    P_N^j, & j = J_2^P + 1, \ldots, J \\
    (1 - \rho^{rr}) P_N^j & \text{for } j = 1, \ldots, J_2^P - 1 \\
    P_N^j - P_N^j \rho^{rr} \left( \sum_{\xi=j+1}^{J_2^P} \pi_\xi - J_2^P \right) + 
    \left( \sum_{\zeta<j} P_N^\zeta \right) \rho^{rr} \pi_{j-J_2^P} & \text{for } j = J_2^P, \ldots, \bar{J}_2^P
\end{cases}
\]

In Figure 5 we see that for period 1 (wider bins), the bins indexed by \( j > 7 (= \bar{J}_1^P) \) have equal height to the notional bins (see first row of (123)), the bins indexed by \( j < 5 (= J_1^P) \) exhibit a deficit of probability mass relative to the notional bins (see second row of (123)), while the bins indexed by \( 5 \leq j \leq 7 \) - that contain the support of the AID in period 1 - exhibit either deficit or surplus (see third row of (123)): the leftmost bin \( (j = 5) \) exhibits a deficit, while the other two \( (j = 6, 7) \) a surplus. The same pattern can be seen in the histogram for the period 2 except that, for that case, the three types of bins discussed above have different location: the bins that have equal height to the notional bins are indexed by \( j > 4 (= \bar{J}_2^P) \), the one that exhibits deficit
is indexed by \( j = 1(<j^P_2 = 2) \), and, finally, the bins indexed by \( 2 \leq j \leq 4 \) - that contain the support of AID for that period - exhibit either deficit or surplus.

From (123) and (124) it then follows that, for the bin that contains the maximum value of the support of the AID in period \( t = 1 \), i.e. for \( j = \bar{J}^P_1 \), the associated probabilities from the two periods satisfy

\[
P_{j1} = P^N_j - P^N_j \left( \sum_{\xi=j+1}^{J^P_1} \pi_{\xi-j^P_1} \right) \rho^{rr} + \left( \sum_{\zeta<j} P^N_{\zeta} \right) \rho^{rr} \pi_{j-j^P_1} \tag{125}
\]

\[
= P_j^N + \left( \sum_{\zeta<j} P^N_{\zeta} \right) \rho^{rr} \pi_{j-j^P_1} \tag{126}
\]

\[
P_{j2} = P_j^N \tag{127}
\]

Furthermore, for period \( t = 2 \) and \( j = \bar{J}^P_1 \), it is also true that (see Figure 5, for \( j = 7 \)):

\[
1 - \sum_{\zeta \geq j} P_{\zeta} = 1 - \sum_{\zeta \geq j} P^N_{\zeta} = \sum_{\zeta < j} P^N_{\zeta} \tag{128}
\]

Combining the above, we can write

\[
\rho^{rr} \pi_{j-j^P_1} = \frac{P_{j1} - P_{j2}}{1 - \sum_{\zeta \geq j} P_{\zeta}} \quad , \quad j = \bar{J}^P_1 \tag{129}
\]

In the same way, using (123) and (124), we can also write, for \( j = \bar{J}^P_1 - 1 \) (i.e. for the bin located one position to the left of the bin indexed by \( j = \bar{J}^P_1 \) - see Figure 5, for \( j = 6 \)) the following:

\[
P_{j1} = P^N_j - P^N_j \left( \rho^{rr} \pi_{j-j^P_1} \right) + \left( \sum_{\zeta<j} P^N_{\zeta} \right) \rho^{rr} \pi_{j-j^P_1} \tag{130}
\]

\[
P_{j2} = P_j^N \tag{131}
\]

\[
\sum_{\zeta<j} P^N_{\zeta} = 1 - \sum_{\zeta \geq j} P_{\zeta} \tag{132}
\]
and therefore, combining (130)-(132), we can write

\[ \rho^{rr} \pi_{j - J_1^p} = \frac{P_{j_1} - P_{j_2} + P_{j_2} \rho^{rr} \pi_{j_1^p - J_1^p}}{1 - \sum_{\zeta > j} P_{\zeta 2}}, \quad j = J_1^p - 1 \]  

(133)

At this point it is clear that expressions (129) and (133) share the same structure, being special cases of the following expression

\[ \rho^{rr} \pi_{j - J_1^p} = \frac{P_{j_1} - P_{j_2} + P_{j_2} \sum_{\xi = j + 1}^{J_1^p} \rho^{rr} \pi_{\xi - J_1^p}}{1 - \sum_{\zeta > j} P_{\zeta 2}} \]  

(134)

In fact, following the same line of thought as to the one used to derive (129) and (133), we can show that the following are true for all \( j = J_1^p, \ldots, \bar{J}_1^p \)

\[ P_{j_1} = P_{j_1}^N - P_{j_2}^N \sum_{\xi = j + 1}^{J_1^p} \rho^{rr} \pi_{\xi - J_1^p} + \left( \sum_{\zeta < j} P_{\zeta}^N \right) \rho^{rr} \pi_{j - J_1^p} \]  

(135)

\[ P_{j_2} = P_{j_2}^N \]  

(136)

\[ \sum_{\zeta < j} P_{\zeta}^N = 1 - \sum_{\zeta > j} P_{\zeta 2} \]  

(137)

This would then lead us to derive expression (134), which is therefore true for all these values of \( j \), that is

\[ \rho^{rr} \pi_{j - J_1^p} = \frac{P_{j_1} - P_{j_2} + P_{j_2} \sum_{\xi = j + 1}^{J_1^p} \rho^{rr} \pi_{\xi - J_1^p}}{1 - \sum_{\zeta > j} P_{\zeta 2}}, \quad j = J_1^p, \ldots, \bar{J}_1^p \]  

(138)

In this expression we note that its RHS includes several probabilities from the two ‘actual’ probability histograms, i.e. a subset of the probabilities in (122), that can be identified from the available data, as well as the sum of terms \( \rho^{rr} \pi_{\xi - J_1^p} \) for \( \xi = j + 1, \ldots, \bar{J}_1^p \).

This sum is equal to zero for \( j = \bar{J}_1^p \), and as we can see from (129), the term \( \rho^{rr} \pi_{j_1^p - J_1^p} \) can be written as a function of only identifiable quantities.

For the bin immediately to its left, indexed by \( j = \bar{J}_1^p - 1 \), this sum will be equal to \( \rho^{rr} \pi_{j_1^p - J_1^p} \), therefore we can also write \( \rho^{rr} \pi_{j - J_1^p} \) for \( j = \bar{J}_1^p - 1 \) as a function of only identifiable quantities.

Repeating this sequence of calculations, moving one bin at a time to the left, one can see that it is possible to express the RHS of (138) only as a function of identifiable quantities.
Then, using that
\[
\sum_{\xi = J^P_1}^{J^P} \pi_{\xi - J^P_1} = 1 \tag{139}
\]
we can obtain \(\rho^{rr}\) by summing-up the quantities on the RHS of (138) for \(j = J^P_1, \ldots, J^P_1\):
\[
\rho^{rr} = \sum_{\xi = J^P_1}^{J^P} \rho^{rr} \pi_{\xi - J^P_1} \tag{140}
\]

Having been able to write \(\rho^{rr}\) as a function of identifiable quantities, it is then possible to recover the AID probabilities, \(\{\pi_q\}_{q \in \Omega}\), and the remaining notional probabilities (for \(j < J^P_1\)). First we note that we can substitute the RHS of (140) into (138) and solve for \(\pi_{j - J^P_1}\), therefore in this way the probabilities in the 'AID' probability histogram can also be identified.

Furthermore, we note from (124) that the data from period \(t = 2\) allow the identification of \(\{P^N_j\}_{j = J^P_{t+1}, \ldots, J^P_1, \ldots, J^P}\). From (123) we also note that
\[
P_{j1} = (1 - \rho^{rr}) P^N_j, \quad j = 1, \ldots, J^P_{1} - 1 \tag{141}
\]
Therefore we can substitute expression (140) for \(\rho^{rr}\) in (141) and solve for \(P^N_j\) to write the remaining probabilities of the 'notional' probability histogram as a function of identifiable quantities.

We conclude this part by making two additional observations. Firstly, we observe that in the special case where \(J^P_1 = J^P_1 = J^P\), i.e. the support of the AID in \(t = 1\) is confined within the width of a single bin, then for \(j = J^P_1\) it is true that
\[
\pi_{j - J^P_1} = \pi_0 = 1 \tag{142}
\]
\[
\sum_{\xi = j + 1}^{J^P} \pi_{\xi - J^P_1} = 0 \tag{143}
\]
therefore expression (138) simplifies to the following
\[
\rho^{rr} \pi_{j - J^P_1} = \rho^{rr} = \frac{P_{j1} - P_{j2}}{1 - \sum_{\xi > j} P_{\xi2}}, \quad j = J^P_1 \tag{144}
\]
This is similar to the result presented in expression (107) for the case of DNWR, where the
support of the distribution of the rigidity bounds is degenerate at point zero and therefore also confined within the width of a single bin, the one indexed by \( j = J^0_t \).

Secondly, in the special case where \( J^P_t - \tilde{J}_1^P > 1 \), an alternative identification strategy is also available. In that case we have, for \( \tilde{J}_2^P < j < J^P_1 \)

\[
P_{j1} = (1 - \rho^{rr}) P_j^N
\]

\[
P_{j2} = P_j^N
\]

and therefore we can write

\[
\rho^{rr} = \frac{P_{j2} - P_{j1}}{P_{j2}}, \quad j = \tilde{J}_2^P + 1, \ldots, J^P_1 - 1
\]

where the RHS includes only identifiable quantities.

**Proposition 4** (Heterogeneity case) In the case where \( S > 1 \), conditions on the data on actual wage growth rates

\[
\{\dot{w}_{tsi}\}_{t=1,\ldots,T, i=1,\ldots,n_t}
\]

that are sufficient for the identification of the rigidity parameters \( \rho^{rr}_s \) in the semiparametric model described above, are the following:

1. The conditions of Proposition 3 are satisfied for some \( s^* \in \{1, \ldots, S\} \).
2. For \( s \neq s^* \) it is true that \( 1 < \tilde{J}_{ts}^P \leq J^P \) for at least one value of \( t_1 \in \{1, \ldots, T\} \), i.e. there exists at least one period for each heterogeneity group other than \( s^* \) whose ‘actual’ probability histogram is distorted by DRWR.

**Proof.** The details of the proof are available on request.

**A.3 DNWR and DRWR**

In this case, the data from a particular populating, indexed by \( ts \),

\[
\{\dot{w}_{tsi}\}_{i=1,\ldots,n_t}
\]

allows us to estimate the bin heights of the ‘actual’ probability histogram

\[
\{P_{jts}\}_{j=1,\ldots,J}
\]
that satisfy
\[
P_{jts} = P^N_j + D^m_{jts} + D^r_{jts}
\] (151)

where

\[
D^m_{jts} = -\rho_{sn}^m P^N_j I (j < J^0_{ts}) + \rho_{sn}^m \left( \sum_{\zeta < j} P^N_{\zeta} \right) I (j = J^0_{ts})
\] (152)

\[
D^r_{jts} = -\rho_{sr}^r P^N_j \left( \sum_{\zeta > j} \pi_{\zeta-J^0_{ts}} \right) + \rho_{sr}^r \left( \sum_{\zeta < j} P^N_{\zeta} \right) \pi_{j-J^0_{ts}}
\] (153)

The following two propositions provide sufficient conditions for the identification of \( \rho_{sn}^m \) and \( \rho_{sr}^r \) for the cases where \( S = 1 \) (homogeneity case w.r.t. \( s \)) and \( S > 1 \) (heterogeneity case w.r.t. \( s \)). To simplify the notation we drop subscript \( s \) in the discussion of the homogeneity case.

**Proposition 5** *Conditions on the data on actual wage growth rates*

\[
\{\dot{w}_{ti}\}_{t=1,\ldots,T, i=1,\ldots,n_t}
\] (154)

that are sufficient for the identification of the total-rigidity parameters \( \rho^m \) and \( \rho^r \) in the semi-parametric model described in Section 2, are the following:

1. \( T \geq 2 \),

2. for at least one pair of periods \( t_1, t_2 \in \{1,\ldots,T\} \), where \( t_1 \neq t_2 \), the following are satisfied:

   (a) \( 1 \leq J^P_{t_1} < J^P_{t_2} \leq J \) and \( J^0_{t_1} < J^P_{t_1} \), i.e. the bins of the ‘actual’ probability histogram in \( t = t_1 \), that contain the support of AID are a subset of the support of the notional WGD in that period, and are only distorted by DRWR, while the remaining bins that lie to their left are possibly distorted by DNWR,

   (b) \( J^P_{t_2} < J^P_{t_1} \) and \( J^0_{t_2} < J^P_{t_2} \), i.e. the bins that contain the support of AID in \( t = t_1 \) are not distorted in \( t = t_2 \) by either DRWR or DNWR, while any remaining bins that lie to their left are possibly distorted by DNWR, and

---

\[^{41}\text{See page 20. We also assume that the location of AID is ‘standardised’ in the way discussed at the end of Section 3.1.}\]
3. for at least one pair of periods $t_3, t_4 \in \{1, \ldots, T\}$, where $t_3 \neq t_4$ and, possibly, $t_3, t_4 \in \{t_1, t_2\}$, the following are satisfied:

(a) $1 < J^0_{t_3} \leq J$, i.e. the ‘actual’ probability histogram in $t = t_3$ is distorted by DNWR,

(b) $J^0_{t_3} < J^0_{t_4}$, i.e. at least one of the bins distorted by DNWR in $t = t_3$ is not distorted in $t = t_4$ by this type of rigidity.

**Proof.** Without loss of generality we assume that

$$T = 2$$

(155)

$$1 \leq J^P_{t_1} < J^P_{t_1} \leq J$$

and

$$1 < J^0_{t_1} < J^0_{t_1}$$

(156)

$$1 \leq J^P_{t_2} < J^P_{t_2} \leq J$$

and

$$1 < J^0_{t_2} < J^0_{t_1}$$

(157)

$$J^0_{t_2} \neq J^0_{t_1}$$

(158)

i.e. the underlying probability histograms for the data from the two periods are distorted by both DNWR and DRWR, such that the bins that contain the support of the AID in period $t = 1$ are not distorted in period $t = 2$ by either of the two types of rigidity, and also, different sets of bins are distorted by DNWR in the two periods. Assumption (158) includes two sub-cases, namely

Case 1: $J^0_{t_2} < J^0_{t_1}$

(159)

Case 2: $J^0_{t_1} < J^0_{t_2}$

(160)

Here we will prove sufficiency of the above conditions for Case 1.

Without loss of generality, in addition to (155)-(158), we assume further that

$$J^0_{t_2} + 1 = J^P_{t_2} = J^0_{t_1} = J^P_{t_1} - 1$$

(161)

Therefore, for Case 1 above, assumptions (155)-(159) and (161) imply the following order for the index values:

$$1 \leq J^P_{t_2} < J^0_{t_2} \leq J^P_{t_2} \leq J^0_{t_1} < J^P_{t_1} \leq J$$

(162)

42This is the case where the largest possible number of bins are distorted by both types of rigidity in the two periods, i.e. we have the least possible amount of information about the ‘notional’ probability histogram that satisfy assumptions (155)-(159).
This case is depicted in Figure 6, where

\[
\begin{array}{cccc}
J_1^P & J_2^P & J_3^P & J_1^0 \\
\text{Period } t_1 = 1: & 5 & 6 & 7 & 4 \\
\text{Period } t_2 = 2: & 2 & 3 & 4 & 3
\end{array}
\]

The available data

\[
\{\dot{\omega}_{ti}\}_{t=1,2, i=1,\ldots,n_t},
\]

allows us to obtain the collection of estimates

\[
\left\{\hat{P}_j^t\right\}_{j=1,\ldots,J}, \quad t = 1, 2
\]

where in period 1, the estimated quantities satisfy

\[
P_{j1} = \left\{
\begin{array}{l}
P_j^N, \quad j = \bar{J}_1^P + 1, \ldots, J \\
P_j^N - P_j^N\rho_{rr} \left(\sum_{\xi=j+1}^{J_1^P} \pi_{\xi} - J_1^P\right) + \\
\quad \left(\sum_{\zeta<j} P_j^N\right)\rho_{rr}\pi_{j} - J_1^P
\end{array}
\right\}, \quad j = \bar{J}_1^P, \ldots, \bar{J}_1^P
\]

\[
P_j^N - \rho_{rr} P_j^N + \rho_{nn} \left(\sum_{\zeta<j} P_j^N\right), \quad j = \bar{J}_1^P - 1 = J_1^0
\]

\[
P_j^N - \rho_{rr} P_j^N - \rho_{nn} P_j^N, \quad j = 1, \ldots, \bar{J}_1^0 - 1
\]

and, in period 2
Figure 6: Identification, DNWR and DRWR case.

\[ \begin{aligned}
P_{j_2} &= \left\{ 
\begin{array}{ll}
    P_j^N & , \; j = J_2^P + 1 (= J_1^P) , \ldots , J \\
    P_j^N + \rho^\tau \pi_{j_2^P - j_2^P} \left( \sum_{\zeta < j} P_\zeta^N \right) & , \; j = J_2^P (= J_1^P - 1 = J_1^0) \\
    P_j^N - P_j^N \rho^\tau \pi_{j_2^P - j_2^P} + \left( \sum_{\zeta < j} P_\zeta^N \right) \rho^\tau \pi_{j_2^P - j_2^P} + \rho^m \left( \sum_{\zeta < j} P_\zeta^N \right) & , \; j = J_2^P - 1 = J_2^0 \\
    P_j^N - \rho^m P_j^N - \rho^\tau P_j^N & , \; j = 1 , \ldots , J_2^P - 1
\end{array} \right. \\
\end{aligned} \] 

(Bins \( j = J_1^P , \ldots , J \)): First we note from the top row of (166) that the data from period 2 allows us to estimate directly the bins of the ‘notional’ probability histogram indexed by \( j = J_1^P , \ldots , J \) (in Figure 6: \( j = 5 , \ldots , 9 \)):

\[ \{ P_{j_2} \}_{j = J_2^P + 1 , \ldots , J} = \{ P_j^N \}_{j = J_1^P , \ldots , J} \]  (167)
Furthermore, from (165), we see that the bins of the ‘actual’ probability histogram in period 1 that contain the support of AID (in Figure 6: \( j = 5, 6, 7 \)) are only distorted by DRWR. Therefore combining the information from two periods we can employ the same arguments used in the proof of proposition 3, to write \( \rho^{rr} \) and \( \{ \pi_q \}_{q \in Q} \) as functions of identifiable quantities.

**Bin** \( j = J_1^P - 1 = J_0^1 \): With regard to the identification of \( \rho^{nn} \), we will make use of the information about \( P_{j1} \) and \( P_{j2} \) for \( j = J_0^1 \) (\( = J_1^P - 1 = J_2^P \)) - (in Figure 6: \( j = 4 \)) - in addition to the results about \( \rho^{rr} \) and \( \{ \pi_q \}_{q \in Q} \). Copying from (165) and (166) we have, for this bin

\[
P_{j1} = P_{j1}^N - \rho^{rr} P_{j1}^N + \rho^{nn} \left( \sum_{\zeta < j} P_{\zeta}^N \right) \tag{168}
\]

\[
P_{j2} = P_{j2}^N + \rho^{rr} \pi_{\bar{J}_P^2 - J_2^P} \left( \sum_{\zeta < j} P_{\zeta}^N \right) \tag{169}
\]

Using

\[
\left( \sum_{\zeta < j} P_{\zeta}^N \right) = 1 - P_{j1}^N - \left( \sum_{\zeta = J_1^P}^J P_{\zeta}^N \right), \quad j = J_1^P - 1
\]

and (167), we can solve (169) for \( P_{j2}^N \), where the RHS includes only identifiable quantities:

\[
P_{j2}^N = \frac{P_{j2} - \rho^{rr} \pi_{J_2^P - J_2^P} \left[ 1 - \left( \sum_{\zeta = J_1^P}^J P_{\zeta}^N \right) \right]}{1 - \rho^{rr} \pi_{J_2^P - J_2^P}}, \quad j = J_1^P - 1
\]

Therefore, the expression we obtain by solving (168) for \( \rho^{nn} \)

\[
\rho^{nn} = \frac{P_{j1} - P_{j1}^N + \rho^{rr} P_{j1}^N}{1 - P_{j1}^N - \left( \sum_{\zeta = J_1^P}^J P_{\zeta}^N \right)}, \quad j = J_1^P - 1 = J_0^1
\]

will also include only identifiable quantities on its RHS. ■

**Bins** \( j = 1, \ldots, J_1^P - 2 \): Given that \( \rho^{nn} \) and \( \rho^{rr} \) are identifiable, it is straightforward to show that the remaining bins of the ‘notional’ probability histogram, i.e. with index values \( j = 1, \ldots, J_1^P - 2 \), are also identifiable (in Figure 6: \( j = 1, 2, 3 \)). The easiest way is by solving the expression in the last row of (165) for \( P_{j}^N \)

\[
P_{j}^N = \frac{P_{j1}}{1 - \rho^{rr} - \rho^{nn}}, \quad j = 1, \ldots, J_1^P - 2 \tag{172}
\]
We conclude this section by noting that, in order to check in practice whether the sufficient conditions for identification stated above are satisfied for the available data, one must know the values of \( \{ J_0^t, J_P^t, \bar{J}_P^t \}_{t=1,...,T} \). With regard to \( J_0^t \), this is easy to calculate. On the other hand, for \( \{ d_t^P, \bar{J}_P^t \}_{t=1,...,T} \) one would have to estimate \( E \hat{P}_{et} \) (for all \( t \)), which would determine the value of \( J_P^t \), and then make assumptions about the number of bins that, in addition to \( J_P^t \), also contain values of the support of AID.

**Proposition 6** (Heterogeneity case) In the case where \( S > 1 \), conditions on the data on actual wage growth rates

\[
\{ \hat{w}_{tsi} \}_{t=1,...,T, i=1,...,n_t}
\]

that are sufficient for the identification of the rigidity parameters \( \rho_{nn}^s \) and \( \rho_{rr}^s \) in the semiparametric model described above, are the following:

1. The conditions of Proposition 5 are satisfied for some \( s^* \in \{1,\ldots,S\} \).
2. For each \( s \neq s^* \) there exist values \( t_1, t_2 \in \{1,\ldots,T\} \), where not necessarily \( t_1 \neq t_2 \), for which it is true that \( 1 < J_0^{t_1 s} < J_P^{t_1 s} \leq J \) and \( 1 < J_0^{t_2 s} \leq J \), i.e. (a) there exists at least one period for each heterogeneity group other than \( s^* \) where the corresponding ‘actual’ probability histogram is distorted, at least in some part of it, only by DRWR, and, (b) there is at least one period for each heterogeneity group other than \( s^* \) where the corresponding ‘actual’ probability histogram is distorted by DNWR (and possibly, by DRWR in the same part as well).

**Proof.** The details of the proof are available on request. ■