Consumer theory with bounded rational preferences

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June 28, 2010

Abstract
Building on the work of Shafer (1974), this paper provides a continuous bivariate representation theorem for preferences that need not be complete or transitive. Applying this result to the problem of choice from competitive budget sets allows for a proof of the existence of a demand correspondence for a consumer who has preferences within this class that are also convex. Similarly to the textbook theory of utility maximization, this proof also uses the Maximum Theorem. With an additional mild convexity axiom that conceptually parallels uncertainty aversion, the correspondence reduces to a function that satisfies WARP.

Keywords: incomplete & intransitive preferences, representation, demand.
JEL: D0

1 Introduction
This paper is concerned with the consumer’s problem of choosing the best among the feasible alternatives when her preferences over the latter are not assumed complete or transitive. It is motivated by experimental evidence that favors bounded rationality in this sense of limitations to the consumer’s decisiveness and consistency[1] and builds on the literature generated by the seminal work of Sonnenschein (1971), and particularly on the innovative paper of Shafer (1974). The demand theory presented here is one that highlights weighing as the fundamental behavioral principle in competitive choice environments where deviations from the utility maximization paradigm of completeness and transitivity are in place.

The paper’s building block is a generalization in the direction of incomplete preferences of a continuous representation theorem due to Shafer (1974), which delivers a bivariate preference function that fully characterizes strict preferences in a general metric space. The properties of this representation are then compared to those of other results in the relevant literature and it is argued that the preference function generally performs better than weak-utility (Peleg (1970) and multi-utility (Kochov (2007), Evren and Ok (2007)) when the criteria of generality, tractability and characterization of strict preferences are considered jointly.

We then restrict the consumption set to the neoclassical $\mathbb{R}^n_+$, assume that preferences are hemicontinuous and convex, employ this representation theorem to maximize the prefer-

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* I am indebted to Robert Evans for very kind and helpful advice. I also thank Jose Apesteguia, Eddie Dekel, Jayant Ganguli, Nikos Gerasimou, David K. Levine, R. Duncan Luce, Fabio Maccheroni, Sonje Reiche, Ariel Rubinstein, Aldo Rustichini, Nicholas Yannelis, a referee and participants at SAET 2009 (Ischia) and RES 2010 (Surrey) for comments or questions. Any errors are my own.

1 See, for example, Anderson (2003) and Kivetz and Simonson (2000).
ence function over competitive budget sets and prove the existence of a demand correspondence for which the optimal feasible alternatives are the maximal (i.e. undominated) ones. As Mas-Colell (1974) first observed (see also Kim and Richter (1986)), this result is implicit in the work of Sonnenschein (1971). Yet, the proof offered here is very different and comes about by bringing together two suitably generalized results of Shafer (1974) on the one hand and the celebrated Maximum Theorem of Berge on the other. Hence, this proof of demand existence is an analogue to the textbook proof of the model with complete and transitive preferences that is based on utility maximization and the Maximum Theorem.

Not surprisingly, the demand correspondence obtained by maximizing incomplete and intransitive preferences does not satisfy the Weak Axiom of Revealed Preference (WARP) in general. However, an identification of the pattern of WARP-“irrationality” is offered as a simple corollary with the intuitive prediction that if alternatives $x$ and $y$ violate WARP over two different budget sets then it must follow that $x$ and $y$ are either indifferent or incomparable. The final task we carry out is to reduce this demand correspondence to a single-valued demand function by introducing the mild convexity axiom of ambivalence aversion, which predicts that if two alternatives $x$ and $y$ are indifferent or incomparable, then there exists some convex combination of $x$ and $y$ that is preferred to $x$ or $y$. From the conceptual point of view this axiom parallels that of ambiguity aversion introduced by Schmeidler (1989) in choice under uncertainty.

2 Preference Representation

2.1 Definitions and Result

Throughout this section the consumer is assumed to have preferences over alternatives belonging to a metric space $X$. Her strict preferences, indifference and weak preferences are captured by the binary relations $\succ$, $\sim$ and $\succeq := \succ \cup \sim$ on $X$ respectively, while the inverses of $\succ$ and $\succeq$ are denoted $\prec$ and $\preceq$. For notational convenience, $(x, y) \in \succeq$ is usually written $x \succeq y$ and this applies to every relation considered here. It is assumed that $\succ$ is asymmetric and $\sim$ is reflexive and symmetric, so that $\succeq$ is merely reflexive. In particular, the two axioms that are typically imposed on $\succeq$ and which are not employed here are completeness and transitivity, which require that for all $x, y \in X$ and all $x, y, z \in X$, $x \succeq y$ or $y \succeq x$ and $(x \succeq y, y \succeq z) \Rightarrow x \succeq z$, respectively. These preferences are commonly referred to in the literature as rational. Since neither completeness nor transitivity will be assumed in what follows, we will refer to these preferences as bounded rational.

In the absence of completeness, the relation $\succsim := \{ (x, y) : x \not\succeq y$ and $y \not\succeq x \}$ which stands for the consumer’s incomparability relation is nonempty and, by construction, also irreflexive and symmetric. Let us now introduce the relation $\simcirc := \sim \cup \succsim$, which is reflexive and symmetric by definition. We will refer to $\simcirc$ as the consumer’s ambivalence relation because it consists of pairs over which she is either indifferent or indecisive. Ambivalence is identically equal to $(X \times X) \setminus (\succ \cup \prec)$ and is the maximal (with respect to set inclusion)
reflexive and symmetric relation on $X$ given the asymmetric relation $\succ$. It is emphasized that ambivalence is not a relabeling of indifference. Indifference is reflexive and symmetric by assumption and in view of the above remark it is contained in the ambivalence relation. The latter is reflexive and symmetric by definition, and this is true even when $\sim$ is an equivalence relation, which happens when $\succeq$ is transitive and therefore a preorder. The two notions coincide if and only if $\succeq$ is complete, in which case $\preceq \equiv \emptyset$.

The strict preference relation $\succ$ is said to have an open graph if $\succ$ is an open subset of $X \times X$ with the product topology. Importantly, it was shown first by Shafer (1974) and then by Bergstrom, Parks, and Rader (1976) that completeness or transitivity of $\succeq$ are not necessary for $\succ$ to have this continuity property.

We can now state and prove the representation theorem, which generalizes that of Shafer (1974) in the direction of incomplete preferences and substantially improves the partial extensions of that result in relation to incompleteness that were provided by Shafer and Sonnenschein (1975) and Bergstrom, Parks, and Rader (1976). The argument is a variation of Shafer’s (1974) constructive idea and crucially depends on the observation that $\sim$ is reflexive and symmetric.

**Theorem 1**

An asymmetric relation $\succ$ has an open graph on a metric space $X$ if and only if there exists a continuous function $P : X \times X \to \mathbb{R}$, unique up to odd and strictly increasing transformations, such that for all $x, y \in X$

$$P(x, y) > 0 \iff x \succ y$$

$$P(x, y) = 0 \iff x \sim y$$

$$P(x, y) = -P(y, x).$$

**Proof.**

Suppose $\succ$ has an open graph and let $P$ be defined by

$$P(x, y) := \begin{cases} d((x, y), \sim) & \text{if } x \succ y \text{ or } x \sim y \\ -d((y, x), \sim) & \text{if } y \succ x \text{ or } y \sim x \end{cases}$$

(1)

where $d(\cdot)$ is a metric that generates the product topology on $X \times X$ and $d((x, y), \sim) = \inf_{(w, z) \in \sim} d((x, y), (w, z))$. The open-graph property of $\succ$ and the fact that $\sim \equiv (X \times X) \setminus (\succ \cup \preceq)$ establish that $\sim$ is closed in $X \times X$ and therefore that $P$ is continuous. The three properties of $P$ follow directly from (1) and the reflexivity and symmetry of $\sim$, while its uniqueness up to odd-monotonic transformations is straightforward. The converse implication is true because, as noted in Bergstrom, Parks, and Rader (1976), for a function $P$ with the above properties, the set $\{P^{-1}(z) : z > 0\} = \{(x, y) : x \succ y\}$ is open in $X \times X$ since $P$ is continuous.

**Remark.** In the skew-symmetric bilinear theory of Fishburn (1982) the bivariate function $\phi$ established there is unique up to similarity transformations, i.e. unique up to positive
scalar multiplication. The bivariate function $\mathcal{P}$ of Theorem 1 is not bilinear in general and hence its uniqueness properties are richer, in direct analogy with the uniqueness properties of linear vs. non-linear utility representations.

The function $\mathcal{P}$ of Theorem 1 also generalizes the utility function concept for preferences that are incomplete or intransitive. Given its binary structure, it would be natural for $\mathcal{P}$ to be called a preference function, because with each ordered pair $(x, y) \in X \times X$ it associates a positive, negative or zero value whenever $(x, y) \in \succ$, $(x, y) \in \prec$ or $(x, y) \notin \succ$ and $(x, y) \notin \prec$ respectively. This motivates the following definition, which supersedes the one proposed by Vind (1991).

**Definition 1**

If $\succ$ is an asymmetric relation on $X$, then $\mathcal{P} : X \times X \to \mathbb{R}$ is a preference function if for all $x, y \in X$, $\mathcal{P}(x, y) > 0 \iff x \succ y$, $\mathcal{P}(x, y) = 0 \iff (x \not\succ y, y \not\prec x)$ and $\mathcal{P}(x, y) = -\mathcal{P}(y, x)$.

In light of this definition, $\mathcal{P}$ of Theorem 1 is a preference function satisfying $\mathcal{P}(x, y) = 0 \iff x \sim y$. In Shafer’s (1974) original representation theorem, $\succeq$ satisfies completeness but not transitivity and the preference function $k : \mathbb{R}_+^n \times \mathbb{R}_+^n \to \mathbb{R}$ introduced there is such that $k(x, y) \geq 0 \iff x \succeq y$ and $k(x, y) = -k(y, x)$ so that $k(x, y) = 0 \iff x \sim y$. If $\succeq$ satisfies both completeness and transitivity (and $X$ is separable and connected), then the open graph property of $\succ$ implies the existence of a continuous utility function $u : X \to \mathbb{R}$ with $x \succeq y \iff u(x) \geq u(y)$ for all $x, y \in X$, so that one can define $\mathcal{P}^*$ by $\mathcal{P}^*(x, y) := u(x) - u(y)$. It is easily verified that $\mathcal{P}^*$ satisfies the properties of Definition 1. This fact supports the claim made above that preference functions generalize utility functions when completeness and/or transitivity are not assumed. Of course, the original idea behind this generalization (certainly so for intransitive preferences) is due to Shafer (1974).
2.2 Discussion

We now turn to a comparison between Theorem 1 and other representations of incomplete preferences that are already available in the literature. To start with, Peleg (1970), assuming transitivity and imposing further some hemicontinuity and order-separability restrictions on the strict preference relation \(\succ\), showed that there exists a continuous “weak” utility function \(v : X \to \mathbb{R}\) such that, for any \(x, y \in X, x \succ y \Rightarrow v(x) > v(y)\). This weak utility representation, however, does not characterize strict preferences, because knowledge of the values of the function does not lead to a reconstruction of the strict preference relation \(\succ\). To compare Peleg’s \(v\) with \(\mathcal{P}\) of Theorem 1, consider the (continuous) function \(\psi : X \times X \to \mathbb{R}\) defined by \(\psi(x, y) := v(x) - v(y)\). Clearly then,

\[ x \succ y \Rightarrow \psi(x, y) > 0 \quad \text{and} \quad y \succ x \Rightarrow \psi(x, y) < 0, \quad (2) \]

while the equivalence in the above relations is ruled out by incompleteness. However, it is an implication of the properties of the preference function \(\mathcal{P}\) that

\[ x \succ y \Leftrightarrow \mathcal{P}(x, y) > 0 \quad \text{and} \quad y \succ x \Leftrightarrow \mathcal{P}(x, y) < 0. \quad (3) \]

A comparison of (2) and (3) shows that \(\mathcal{P}\) completely characterizes \(\succ\) whereas the weak utility function \(v\) only partially so, despite the additional property of \(\succ\)-transitivity that it relies on.

Applying Shafer’s (1974) representation theory above to the problem of equilibrium existence in an abstract economy with incomplete and intransitive preferences, Shafer and Sonnenschein (1975) observed that the open-graph property of Theorem 1 implies the existence of a continuous function \(V : X \times X \to \mathbb{R}_+\) defined by \(V(x, y) := d((x, y), \succ^c)\), where \(\succ^c := (X \times X) \setminus \succ\) and \(d(\cdot)\) is again a product metric. This function has the properties that, for all \(x, y \in X, V(x, y) > 0 \Leftrightarrow x \succ y\) and \(V(x, y) = 0 \Leftrightarrow x \not\succ y\). While these were sufficient for the authors’ purposes, one observes that \(V\) takes non-negative values and therefore it is not a preference function. Thus, \(V\) does not characterize \(\succ\) and cannot be used to elicit this relation’s maximal elements. Theorem 1 demonstrates that introducing the ambivalence relation \(\sim\) allows for this device of Shafer and Sonnenschein (1975), which was also later characterized by Bergstrom, Parks, and Rader (1976), to be substantially improved upon at no cost.

Kochov (2007) and Evren and Ok (2007) showed that \(\succeq\) is an incomplete preorder with a closed graph in \(X \times X\) if and only if there exists an infinite set \(\mathcal{U}\) of continuous real-valued functions \(u\) on \(X\) such that, for all \(x, y \in X, x \succeq y \Leftrightarrow u(x) \geq u(y)\) for all \(u \in \mathcal{U}\), provided \(X\) is compact. This multi-utility result assumes transitivity, which Theorem 1 does not. Furthermore, the open-graph property of the asymmetric relation \(\succ\) and the closed graph property of the preorder \(\succeq\) are not related in the present context. Finally, while this “multi-utility” representation allows for a full characterization of the preorder \(\succeq\) at the cost of

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2A discussion of two difficulties associated with this axiom when \(\succeq\) is an incomplete preorder is provided in Gerasimou (2010).

3This is Kochov’s (2007) formulation. Evren and Ok (2007) provide a one-way axiomatization assuming that \(X\) is locally compact.
introducing an infinite number of functions, the preference function \( P \) of Theorem 1 fully characterizes the asymmetric relation \( \succ \) only, but it does so in a more general (i.e. without assuming transitivity), tractable and perhaps also intuitive way. For demand theory, it is shown next that working with \( P \), or equivalently, focusing on \( \succ \) rather than on \( \succeq \), is actually sufficient.

3 Consumer Demand

3.1 Demand Correspondence

From now on we assume that the consumption set is \( X = \mathbb{R}^n_+ \). Furthermore, we let \( Y := \mathbb{R}^n_{++} \) denote the set of strictly positive price-income ratios that the consumer faces, with generic element \( p \in Y \). We define also the budget correspondence \( B : Y \to X \) by \( B(p) = \{ x \in X : px \leq 1 \} \), which, as well-known, is continuous, convex- and compact-valued. In addition to the *sine qua non* axioms concerning the asymmetry of \( \succ \) and the reflexivity and symmetry of \( \sim \), the following two axioms will also be imposed on preferences, the restrictions applying to every element \( x \in X \).

**Axiom 1 (Hemicontinuity)**

\( \mathcal{U}_\succ(x) := \{ y \in X : y \succ x \} \) and \( \mathcal{L}_\succ(x) := \{ y \in X : x \succ y \} \) are open in \( X \).

**Axiom 2 (Convexity)**

\( \mathcal{U}_\succ(x) \) is convex.

Axioms 1 and 2 are the familiar (upper- and lower-) hemicontinuity and convexity restrictions on the preference relation \( \succ \) respectively. Using Theorem 1 and these two axioms we can prove the existence of an upper-hemicontinuous and compact-valued demand correspondence consisting of the maximal (undominated) elements of the asymmetric relation \( \succ \). As noted in the introduction, this result is implicit in Sonnenschein (1971). Instead of using the KKM Theorem and non-empty intersection arguments that were employed in that paper, however, preferences here are maximized by means of the preference function \( P \), much in the spirit of neoclassical consumer theory.

**Theorem 2**

*Under Axioms 1 and 2 there exists an upper-hemicontinuous, nonempty- and compact-valued correspondence \( x : Y \to X \) such that, for all \( p \in Y \), \( x(p) = \{ x \in B(p) : P(x,y) \geq 0 \ \forall \ y \in B(p) \} \).*

**Proof.**

First, from the proof of the Lemma in Shafer (1974) we know that under Axioms 1 and 2 the relation \( \succ \) has an open graph, so that, from Theorem 1, there exists a continuous preference function \( P \) on \( X \). Next, note the following generalizations in the direction of
incomplete preferences of Theorems 2 and 3 in [Shafer (1974)] that come about by utilizing
Theorem 1 (the proofs follow Shafer’s original arguments and appear in the Appendix):

**Lemma 1**
For \( p \in Y \), there exists \( x \in B(p) \) such that \( P(x, y) \geq 0 \) for all \( y \in B(p) \).

**Lemma 2**
For \( p \in Y \), it holds that \( x \in B(p) \) satisfies \( P(x, y) \geq 0 \) for all \( y \in B(p) \) if and only if there
exists a continuous function \( h : Y \times X \rightarrow R \) such that \( h(p, x) = \max\{h(p, z) : z \in B(p)\} \).

Finally, observe that, since the function \( h \) of Lemma 2 is continuous and so is the budget
correspondence \( B \), it follows from Berge’s Maximum Theorem that there exists a con-
tinuous function \( m : Y \rightarrow R \) such that \( m(p) = \max\{h(p, z) : z \in B(p)\} \) and an upper-
hemicontinuous compact-valued correspondence \( x : Y \rightarrow X \) satisfying \( x(p) = \argmax\{h(p, z) : z \in B(p)\} \). From Lemma 2 we know that \( x(p) = \{x \in B(p) : P(x, y) \geq 0 \forall y \in B(p)\} \) while Lemma 1 shows that \( x(p) \neq \emptyset \).

In the context of intransitive but complete preferences, [Shafer (1974)] interpreted the
function \( h \) of Lemma 2 as the consumer’s price-dependent “utility” function. One can check
that irrespectively of whether preferences are complete or not, for \( p \in Y \) and \( x, y \in B(p) \),
\( x \succ y \Rightarrow h(p, x) \geq h(p, y) \) so that \( h(p, \cdot) \) is weakly increasing with respect to \( \succ \) but not
strictly increasing in general. Yet, in any budget set the maximizers of \( h \) coincide with the
maximal elements of \( \succ \). With this interpretation, and observing that \( m : Y \rightarrow R \) defined by \( m(p) = \max\{h(p, z) : z \in B(p)\} \) in the proof of Theorem 2 above is an optimal value
function, one would indeed be tempted to interpret \( m \) as the consumer’s “indirect utility”
function in this setting. However, by construction of the function \( h \) in the proof of Lemma
2 follows that \( m(p) = 0 \) for all \( p \in Y \). Hence, \( m \) is trivially both non-increasing and non-
decreasing in prices and therefore cannot really be interpreted in this way.

The next observation shows that when the demand correspondence \( x(\cdot) \) of Theorem 2
violates WARP, the violation is caused by consumer ambivalence, which is an intuitive but
nevertheless interesting fact.

**Corollary 1**
Suppose there exist \( p, p' \in Y \) and \( x, y \in X \) such that (i) \( x \in x(p) \) and \( y \in B(p) \setminus x(p) \); (ii)
\( y \in x(p') \) and \( x \in B(p') \). Then \( x \sim y \).

**Proof.**
By assumption, \( P(x, y) \geq 0 \) and \( P(y, x) \geq 0 \). Thus, \( P(x, y) = 0 \) so that \( x \sim y \).

### 3.2 Demand Function

Desirable as it may be to have a general theory of consumer choice as the one put forward
in Theorem 2, since it is actually the case that the consumer can only afford one feasible
consumption bundle from each budget set it is also desirable to know under what additional condition(s) the model with incomplete and intransitive preferences predicts that the above demand correspondence reduces to a single-valued demand function. To this end, we introduce the following axiom.

**Axiom 3 (Ambivalence Aversion)**

If \( x \sim y \), then there exists \( \alpha \in (0, 1) \) such that \( z := \alpha x + (1 - \alpha)y \) satisfies \( z \succ x \) or \( z \succ y \).

If a consumer does not have a strict preference between two alternatives \( x \) and \( y \) and she is ambivalence-averse, then some weighted average of \( x \) and \( y \) (which will generally depend on both of them) makes her better off than \( x \) or \( y \). If, for example, a consumer is about to buy ice-cream for the first time and is offered the choice between a cup of vanilla, a cup of chocolate and a cup containing both in equal shares, then given that she’s never tried vanilla and chocolate before, she may prefer the mixture because it gives her the opportunity to diversify across the two primitive alternatives so that whichever of the two turns out to be undesirable for her, she can consume the part containing the other (presumably more desirable) one. Her preferences therefore adapt to ambivalence in a way that parallels the axiom of uncertainty aversion (hedging) introduced by Schmeidler (1989) in choice under ambiguity.

A testable implication of ambivalence aversion is that when the decision maker chooses between two alternatives that are either not well-understood or generate decision conflict at the same time when mixtures of them are also feasible, certain mixtures will be preferable to the original options and the weights that determine these mixtures will generally be biased toward the option that is more familiar to her. In the ice-cream example, if the consumer likes chocolate but hasn’t tried vanilla before, she may prefer a cup where the analogy is 3/4 chocolate and 1/4 vanilla. This prediction is also in line with Ellsberg-paradox findings, where preferences have been shown to favor risky alternatives that are well understood compared to uncertain ones that are not. The framework there, of course, is one of genuine probabilistic uncertainty whereas here uncertainty is implicit, but there is an analogy between the two that is perhaps noteworthy.

**Theorem 3**

Under Axioms 1–3, \( p \in Y \) implies there exists \( x \in B(p) \) such that \( \mathcal{P}(x, y) > 0 \) for all \( y \in B(p) \) with \( y \neq x \).

**Proof.**

Let \( p \in Y \). From Theorem 2 there exists \( x \in B(p) \) such that \( \mathcal{P}(x, y) \geq 0 \) for all \( y \in B(p) \). Suppose \( w \in B(p) \) also satisfies \( \mathcal{P}(w, y) \geq 0 \) for all \( y \in B(p) \). It follows that \( \mathcal{P}(x, w) = 0 \) which in turn is equivalent to \( x \sim w \). From Axiom 3 and the fact that \( B(p) \) is convex there exists \( z \in B(p) \) such that \( z \succ x \) or \( z \succ w \), or equivalently \( \mathcal{P}(z, x) > 0 \) or \( \mathcal{P}(z, w) > 0 \), which is a contradiction. \( \blacksquare \)
Thus, under the conditions of Theorem 3 the upper-hemicontinuous demand correspondence \( x(\cdot) \) of Theorem 2 becomes a continuous demand function. Given prices \( p, p' \in Y \), if \( x(p) = x, x \neq y \) and \( y \in B(p) \), then \( \mathcal{P}(x, y) > 0 \) and hence \( x(p') = y \) implies \( x \not\in B(p') \). Therefore, the demand function associated with Theorem 3 satisfies WARP.

The only paper that we are aware of which has provided demand-existence results similar to Theorem 3 is that by Shafer (1975), where the preference relations that generate WARP- and SARP-rational demand functions that exhaust the budget were characterized. However, completeness in that case was replaced by local completeness, which requires that if \( x \not\in \text{cl}[\mathcal{U}_{\succ}(y)] \), where \( \text{cl}(\cdot) \) denotes closure, then there exists \( \beta \in [0, 1) \) such that \( \beta x + (1 - \beta)y > x \). Yet, it generally holds that \( \text{cl}[\mathcal{U}_{\succ}(y)] \neq \mathcal{U}_{\succ}(y) \) and therefore the behavioral restrictions that this axiom places are not clear. In contrast, the ambivalence aversion axiom that is used in Theorem 3 is arguably behavioral in nature and although similar to local completeness in its convexity-like structure, the two are independent. Furthermore, Shafer’s (1975) results rely on the budget-exhaustive property of the demand functions. By contrast, existence in Theorem 3 is proved without this assumption and therefore local non-satiation restrictions on preferences were also not imposed. Finally, unlike Shafer (1975), the demand function here is ultimately established through the use of a preference function.

To interpret Theorem 3 we provide the following narrative, where although alternatives are indivisible and therefore, strictly speaking, no convex combinations exist (and hence the narrative can’t be thought of as a formal example), the central idea is nevertheless preserved. Suppose a consumer is about to buy a computer as a gift for her child. She goes to a computer store and tells the salesman about her plan. He informs her that there are three types of computers, “desktops” (d), “laptops” (l) and “palmtops” (p). She understands that desktops dominate laptops and laptops dominate palmtops in computing power, while palmtops dominate laptops and laptops dominate desktops in portability.

Keeping this in mind, to make a choice she first assigns a numerical value from 0 (lowest) to 2 on each of the two attributes of the three available alternatives as follows (the first entry in brackets corresponds to the assigned value for computing power and the second to that for portability): \( d = (2, 0), l = (1, 1), p = (0, 2) \). With perfect information about her child’s attitude toward computing power and portability a transitive ordering of the above triple would occur, either such that

\[
d \succ l, \ l \succ p, \ d \succ p
\]  \hspace{1cm} (4)

or

\[
p \succ l, \ l \succ d, \ p \succ d.
\]  \hspace{1cm} (5)

Thus, she would buy the desktop in the first case and the palmtop in the latter case. However, she is unaware of the criterion that matters most for her child and is therefore reluctant to make a choice according to either (4) or (5). In particular, due to the conflicting values over the two attributes, in all three pairs she is ambivalent, so that \( d \sim l, l \sim p \) and \( d \sim p \). Ambivalence-averse as she is, however, she looks for weighted averages in every pair. There
is none for \((d,l)\) and \((l,p)\) but there is one for \((d,p)\), namely the laptop, since \(l = \frac{1}{2}d + \frac{1}{2}p\).

This option, in particular, offers a balanced choice in terms of both computing power and portability. Reasoning in this way, she leaves the store with the laptop, much like experimental subjects that behave in accordance with the “compromise effect” ([Simonson](1989)).

4 Concluding Remarks

Motivated by the scepticism surrounding the principal rationality axioms of complete and transitive preferences, in this paper we revisited the consumer’s problem when both these axioms were removed. It was shown that the two basic pillars of this theory, preference representation and demand, can be appropriately modified to accommodate this weaker axiomatic framework. In particular, the existence of a demand correspondence was proved by maximizing a preference function, much in the spirit of the textbook theory of the consumer. Furthermore, a single-valued and well-behaved demand function was established under an assumption that allows for a new interpretation of the consumer’s problem and its solution. In sharp contrast to the neoclassical consumer who maximizes utility and makes choices by completely preordering the feasible alternatives and weighing when indifferent, the proposed consumer makes choices by weighing when either indecisive or indifferent. She is therefore “rational” in that she maximizes her preferences to single out the best alternative, but also “bounded rational” because her preferences incorporate the imperfections of indecisiveness and inconsistency, both of which are inherent properties of human decision making.

Appendix

Proof of Lemma 1.

Let \(B : Y \rightarrow X\) be a continuous compact-valued correspondence and \(P : X \times X \rightarrow \mathbb{R}\) be a continuous function. Fix \(p \in Y\) and let \(S : B(p) \rightarrow \mathbb{R}\) be defined by \(S(y) = \max\{P(w,y) : w \in B(p)\}\). Since \(P\) is continuous and \(B(p)\) is compact, \(S\) is continuous. In view of these facts, from Berge’s Theorem follows that \(E : B(p) \rightarrow B(p)\) defined by \(E(y) = \arg\max\{S(y) : y \in B(p)\}\) is an upper-hemicontinuous and compact-valued correspondence, and from Weierstrass’ Theorem it is also nonempty-valued. Let \(G : B(p) \rightarrow B(p)\) be defined by \(G(y) = \text{co}[E(y)]\), where \(\text{co}(\cdot)\) denotes convex hull. \(G\) is convex-valued and from the above it is also nonempty-valued, upper-hemicontinuous and compact-valued (see Corollary 5.33 and Theorem 17.35 in [Aliprantis and Border](2006)). From Kakutani’s Theorem there exists \(x \in B(p)\) such that \(x \in G(x)\).

Since \(G(x) = \text{co}[E(x)]\) is convex and \(X = \mathbb{R}^n_+\), from Carathéodory’s Theorem there exist \(m \leq n + 1\) points \(x_1, \ldots, x_m\) in \(B(p)\) and real numbers \(\alpha_1, \ldots, \alpha_m\) with \(\alpha_i \geq 0\) and \(\sum_{i=1}^m = 1\) such that \(x_i \in E(x)\) for all \(i \leq m\) and \(x = \sum_{i=1}^m \alpha_i x_i\). Suppose \(P(x_i, x) > 0\) for all \(i \leq m\). From Theorem 1 this is equivalent to \(x_i \in U_\succ (x)\) and since \(U_\succ (x)\) is convex from Axiom
2 and $x = \sum_{i=1}^{m} a_i x_i$, it also implies $x \in U_\infty(x)$, a contradiction. There exists then some $x_j$, $j \in \{1, \ldots, m\}$, such that $P(x_j, x) \leq 0$. But since $x_i \in E(x)$ for all $i \leq m$, it is true that $P(x_j, x) = P(x_i, x)$ for all $i \neq j$. Thus, $P(x_i, x) \leq 0$, or equivalently, $P(x, x_i) \geq 0$ for all $i \leq m$. From the definition of $x_i$ then follows that $P(x, y) \geq 0$ for all $y \in B(p)$.

**Proof of Lemma 2.**

Let the function $h : Y \times X \rightarrow \mathbb{R}$ be defined by $h(p, x) = \min\{P(x, y) : y \in B(p)\}$. Since $P$ and $B$ are continuous, $h$ is continuous. Suppose $x \in B(p)$ is such that $P(x, y) \geq 0$ for all $y \in B(p)$. Since $P(x, x) = 0$, it follows that $h(p, x) = \min\{P(x, y) : y \in B(p)\} = 0$. Furthermore, for $z \in B(p)$, $z \neq x$, it holds that $h(p, z) = \min\{P(z, y) : y \in B(p)\} \leq \min\{P(x, y) : y \in B(p)\} = h(p, x) = 0$. Thus, if $x \in B(p)$ is such that $P(x, y) \geq 0$ for all $y \in B(p)$, then $h(p, x) \geq h(p, z)$ for all $z \in B(p)$.

Now suppose $x \in B(p)$ and $h(p, x) \geq h(p, z)$ for all $z \in B(p)$. Let $w \in B(p)$ be such that $P(w, y) \geq 0$ for all $y \in B(p)$. It follows from above that $h(p, w) = 0$, while $h(p, x) \geq h(p, w)$ is also true by assumption. But since it is necessarily true that $h(p, x) \leq 0$ because $x \in B(p)$, this implies $h(p, x) = h(p, w) = 0$, so that $P(x, y) \geq 0$ for all $y \in B(p)$ too.

**References**


