Strategic Managerial Compensation Arising From Product Market Competition*

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Abstract: We develop a model in which, first, firms design compensation schemes for their managers while subject to moral hazard and, second, firms compete in Bertrand or Cournot fashion in the product market. We derive the strategic properties of managerial compensation levels and incentives. We show that the implications include: greater systematic risk may weaken the incentives of one firm while strengthening those of a competitor; an increase in the idiosyncratic risk of a firm causes all its competitors to adjust their incentives; and strategic considerations may account for the rise in U.S. CEO pay and the use of incentives.
1 Introduction

Should a manager’s compensation scheme be influenced by that of a competitor’s manager? If so, what are the implications for corporate finance? We demonstrate that, due to product market competition, managerial compensation levels and incentives across firms in an industry are strategically related. Consequently, we show that a change in the corporate environment has two effects: a direct effect that arises due to the standard agency problem; and a strategic effect which takes into account the strategic manner in which firms respond to one another. This results in novel implications about the properties of managerial compensation schemes: a reversal may occur in the conventional wisdom about risk and incentives; changes in the corporate environment of a firm cause all firms in the industry to adjust their compensation schemes; and a ratcheting effect of compensation levels and incentives may occur in response to changes in the corporate environment common to all firms in the industry, such as systematic risk.

We study an industry composed of two heterogeneous firms that are engaged in strategic product market competition (Bertrand or Cournot). Each risk-neutral firm (principal) hires a risk-averse manager (agent) to operate the firm. The marginal cost of production of a firm is a function of the effort exerted by its manager, a firm-specific shock which captures idiosyncratic risk, and an industry-wide shock which captures systematic risk. Following Raith (2003), firms reward their managers on the basis of the extent to which they reduce costs of production.1 We show there are countervailing effects determining the strategic properties of managerial incentives. To characterize those properties and implications thereof, we derive reasonable conditions under which managerial incentives are strategic complements or substitutes.2 These conditions relate to the type of competition in the product market, the

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1In Section 3.2, we discuss in detail the implications of having firms reward their managers using profits instead.

2Strategic variables have the following taxonomy (Fudenberg and Tirole 1984; Bulow, Geanakoplos, and Klemperer 1985): they are strategic complements if an aggressive move by one firm engenders aggressive moves by its rivals (in our context, if a firm strengthens its incentives, its rival responds by strengthening its incentives); and they are strategic substitutes if an aggressive move by one firm engenders defensive moves by its rivals (in our context, if a firm strengthens its incentives, its rival responds by weakening its incentives).
demand for the product, and the properties of the strategic variables being used by firms when competing in the product market. If demand is linear and additively separable, then managerial incentives inherit the properties of the strategic variables (prices or quantities, depending on the type of competition). If firms operate in a perfectly competitive market, then managerial incentives are solely determined by the usual trade-off between risk sharing and the provision of incentives.

The manner in which managerial incentives respond to a change in the corporate environment may be decomposed into direct and strategic effects. The direct effect represents a firm’s response holding fixed its rival’s incentives, while the strategic effect represents a firm’s reaction to the change in its rival’s incentives. Thus, a large direct effect for one firm translates into a large strategic effect for its rival.

There are numerous implications stemming from the strategic nature of managerial incentives arising from product market competition. Consider the common wisdom originating from Holmstrom (1979) and Holmstrom and Milgrom (1987) that an increase in risk should be associated with a weakening of managerial incentives. We show this may no longer hold due to the strategic effect. Suppose there is an increase in systematic risk and managerial incentives are strategic substitutes. Both firms have a tendency to weaken their incentives due to risk-sharing considerations (the direct effect). However, because managerial incen-

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3Sundaram, John, and John (1996) devise an empirical means by which to determine whether the strategic variables being used by firms in the product market are strategic substitutes or complements. Their competitive strategy measure (CSM) is the correlation between the change in a firm’s profit margin (which is the change in net income over the change in net sales) and the change in its competitors’ net sales (whereby all firms in the same 4-digit SIC industry are included as competitors). If CSM is negative (positive), then the strategic variables are strategic substitutes (complements, respectively). The average CSM in their sample is \(-0.02\) (with a median of \(-0.02\)), implying that their average sample firm competes on the basis of strategic substitutes. Kedia (2006) uses the same procedure as in Sundaram, John, and John to study a sample of 656 firms distributed over 196 4-digit SIC industries during the period 1984 to 1991. Kedia finds that 29 percent of the 4-digit SIC industries in the sample do not engage in strategic interaction, 21 percent compete with only strategic complements, and 21 percent compete with only strategic substitutes, while the remainder compete with combinations of strategic complements and substitutes. Overall, Kedia finds that industries with competition in prices among differentiated goods are more likely to compete in strategic complements; industries where firms compete in market share and where substantial investment is required in plant and equipment are more likely to compete in strategic substitutes; and industries with no strategic interaction are likely to be those with a large number of small firms, and those with low entry and exit barriers, suggesting they are perfectly competitive.
tives are strategic substitutes, if a firm weakens its incentives, its rival has a tendency to respond by strengthening its incentives (the strategic effect). When the strategic effect is strong enough, one firm weakens its incentives while another strengthens them. Our model therefore provides a potential novel explanation as to why the relationship between risk and incentives is ambiguous in the empirical literature. As emphasized by Prendergast (2002), some empirical studies find that the link is positive (Core and Guay 1999; Oyer and Schaefer 2005; Rajgopal, Shevlin, and Zamora 2006); some find that the link is insignificant (Garen 1994; Yermack 1995; Bushman, Indjejikian, and Smith 1996; Ittner, Larcker, and Rajan 1997; Conyon and Murphy 2000); and others find that the link is negative (Lambert and Larcker 1987; Aggarwal and Samwick 1999a; Jin 2002).

Another implication of the strategic nature of managerial incentives is that all firms in the industry react to changes in the corporate environment that should otherwise just affect one firm. For example, consider an increase in the idiosyncratic risk of a firm, which causes the firm to weaken the incentives of its manager (as in a standard agency model); if managerial incentives are strategic complements, then the rival responds by also weakening the incentives of its manager. Therefore, firms may be adapting their compensation schemes not because of a change in their own environment, but because their competitors experienced a change in their corporate governance, for example.

We show in the Appendix that managerial compensation levels are also strategic. This has ramifications in the context of benchmarking CEO pay, which is prevalent (Bizjak, Lemmon, and Naveen 2008; Faulkender and Yang 2010). We argue it is optimal for a firm to take into account the compensation schemes of peer groups not just to ensure the CEO does not have an incentive to leave the firm (i.e., to satisfy the CEO’s reservation utility), but also to implement the strategic mechanisms we identify.

Finally, if managerial compensation schemes are strategic complements, then a ratcheting effect occurs as compensation schemes react to changes in the corporate environment com-

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4 In a random sample of 100 S&P 500 firms, Bizjak, Lemmon, and Naveen (2008) find that 96% use benchmarking in setting CEO pay.
mon to all firms in the industry. For example, consider a decline in systematic risk, which leads both firms to strengthen their incentives due to risk-sharing (the direct effect); since incentives are strategic complements, each firm responds to its rival by further strengthening its incentives (the strategic effect). Extrapolating these forces to a setting populated by many firms, as each firm in the industry reacts strategically to each competitor, the overall response is magnified as it permeates the entire industry. Thus, seemingly small changes in systematic risk can lead to dramatic changes in compensation levels and incentives. The same holds true for any change in the corporate environment that affects all firms in the industry, such as the onset of rapid technological change or the passing of Sarbanes-Oxley, for example. Hence, strategic considerations may have contributed towards the dramatic rise in U.S. CEO pay and the use of incentives (Gabaix and Landier 2008; Frydman and Saks 2010).

The paper is organized as follows. Section 2 reviews the literature. Section 3 describes the setup of the model. Section 4 solves for the strategic properties of managerial incentives. Section 5 derives theoretical implications of these strategic properties, and Section 6 discusses their empirical ramifications. Section 7 concludes. An Appendix contains the proofs of all propositions and derives the strategic properties of managerial compensation levels, which parallel those of incentives.

2 Literature Review

We complement four literatures in corporate finance and the theory of the firm. The first literature pertains to the design of compensation schemes among firms facing principal-agent problems that are engaged in product market competition. In Raith (2003), an endogenous number of firms compete in prices along a Salop circle. In Baggs and de Bettignies (2007), two firms compete in prices at opposite ends of a Hotelling line. These models have homogeneous

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5Gabaix and Landier (2008) argue the rise in U.S. CEO pay is attributable to the increase in market capitalization. An alternative explanation is provided by the Lake Wobegon effect that no firm wants to admit having a CEO who is below average (Hayes and Schaefer 2009).
(or symmetric) firms and are devoid of the strategic considerations we explore. The closest model to ours is due to Aggarwal and Samwick (1999b), wherein two firms compete in Bertrand or Cournot fashion, which compares as follows. First, in our framework, uncertainty arises from shocks that affect the firm’s cost of production, whereas in their model it is added onto the profit function of the firm without providing a microfoundation. This uncertainty is the source of risk that the principal must balance in providing managerial incentives. Second, their model is not solvable when managers exert effort, such that the results they discuss pertain to a setup in which there are no agency problems. Third, in their model, managers are risk-neutral, so the role of risk cannot be examined; whereas in our model managers are risk-averse. Finally, and most importantly, firms are symmetric in their model and the strategic considerations we identify are not addressed.

The second literature to which we are related links the severity of the principal-agent problem to the extent of competition in the product market. Hart (1983), Scharfstein (1988), Hermalin (1992), Schmidt (1997), and Raith (2003) determine the impact of competition (often measured by the number of firms) on the provision of managerial effort. However, these papers do not enable compensation contracts to influence competition in the strategic manner that we identify. We discuss the empirical ramifications pertaining to this literature in Section 6 in the context of our model.

The third literature concerns examining the broad ways in which managers should be compensated in light of product market competition. Fershtman and Judd (1987) and Sklivas (1987) derive the extent to which the agent’s compensation scheme should be made contingent on performance measures other than profit, finding that positive weight should be placed on sales due to product market competition. These models are devoid of moral hazard (or adverse selection) and risk aversion.\(^6\)

The fourth literature shows that debt contracts influence product market competition. Bolton and Scharfstein (1990), Rotemberg and Scharfstein (1990), and Maksimovic (1988)\(^6\) See also Vickers (1985) and Katz (1991). Fumas (1992) extends this literature to include relative performance evaluation in a model similar to Aggarwal and Samwick (1999b).
demonstrate how capital structure changes the intensity of competition. In these models, firms initially choose their capital structure and then compete in the product market. Debt serves to commit managers to be more or less aggressive depending on parameter values. Our model provides analogous findings on the relationship between compensation contracts and product market competition.

Finally, our paper relates to a broad question posed in game theory as to whether strategic substitutability or complementarity in a static framework translates into strategic substitutability or complementarity in a dynamic framework, e.g., Echenique (2004) and Vives (2009).

3 The Model

3.1 Timing and Structure of the Game

There are two firms (the principals) and two managers (the agents) in an industry. Each principal hires an agent to operate the firm, and the manager-firm pairings are labeled 1 and 2. The firms compete in Cournot or Bertrand fashion with products that are substitutes. Following Raith (2003), managerial effort $e_i$ reduces the (constant) marginal cost of production. For $i = 1, 2$, the marginal cost of firm $i$ is

$$c_i = c - (\theta_i + \theta)e_i - \varepsilon_i - \varepsilon,$$

where $\varepsilon_i$ is a firm-specific normal shock with mean zero and variance $\sigma_i^2$ that is independent across firms, $\varepsilon$ is a normal shock common to both firms with mean zero and variance $\sigma^2$, and $c$ is an industry-wide parameter (that reflects the technology of the industry). The sensitivity of marginal cost to managerial effort has a firm-specific component, $\theta_i$, and an industry-wide

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$^7$The literature that combines product market competition with agency problems is typically ambiguous as to exactly who constitutes the agent. The agent may be the CEO, a product line manager, or a plant manager, for example. When discussing some empirical implications of the model, we interpret the agent as the CEO.
component, \( \theta \). The volatility \( \sigma_i^2 \) measures the firm-specific (i.e., idiosyncratic) risk of firm \( i \); and \( \sigma^2 \) measures industry-wide risk, which may thus be interpreted as systematic risk. For ease of exposition, we assume that \( \varepsilon_i \) and \( \varepsilon \) are independently distributed.\(^9\) Both agents have the effort cost function \( e_i^2 / 2 \) and reservation utility \( r \). As is common in the agency literature, both agents have constant absolute risk aversion (CARA) preferences with the coefficient of absolute risk aversion \( R \).\(^{10}\)

The timing of the game is as follows:

1. Agents are exogenously matched to principals.

2. Principals design contracts that satisfy individual rationality and incentive compatibility. The contracts are observed by the two firms.

3. Agents accept or reject the contracts and exert unobservable effort if they accept.

4. Firms engage in Cournot or Bertrand competition.\(^{11}\)

5. Shocks (and thereby costs of production) are realized and agents are compensated.

We solve the game using backwards induction as follows. First, we derive the manager’s optimal effort policy as a function of the incentives he is offered by the firm. Second, we derive the Nash equilibrium prices or quantities as a function of managerial incentives. Third, we solve the principal’s problem, which entails choosing the compensation scheme of the manager that maximizes firm profit subject to the conditions that the manager signs the contract (individual rationality) and exerts the desired level of effort (incentive compatibility). In designing its compensation scheme, each firm anticipates that the strength of the incentives

\(^8\)The common factor \( \theta \) may reflect the stage of the industry’s life cycle, for example: when the industry is young, there are many opportunities to experience efficiency gains via learning (in which case \( \theta \) is large); but when the industry is mature, most learning has already taken place, so there are few opportunities remaining to cut costs (in which case \( \theta \) is small).

\(^9\)The analysis does not change if firm-specific and industry-wide risks are correlated.

\(^{10}\)The analysis does not change if the agents have heterogeneous costs of effort, reservation utilities, and risk aversion parameters.

\(^{11}\)We presume in our exposition that the firms make the pricing or quantity decisions. However, the managers would make the same choices.
offered its manager affects not just its optimal price or quantity policy, but also that of its rival.

3.2 The Manager’s Effort Problem

This stage of the game is the same regardless of the type of product market competition (Bertrand or Cournot). Following Raith (2003), the principal compensates the agent according to the extent to which he reduces the firm’s marginal cost of production. Specifically, the contract takes the linear form

\[ t_i = \alpha_i + \beta_i (c - c_i), \]

(2)

where \( \alpha_i \) represents the agent’s salary and \( \beta_i \) the agent’s incentives. Let \( \beta = (\beta_1, \beta_2) \) denote the vector of managerial incentives. The term \( c - c_i = (\theta_i + \theta) e_i + \varepsilon_i + \varepsilon \) is the performance measure by which the manager is evaluated. Thus, firms potentially differ along two dimensions: the precision of the performance measure \( 1/(\sigma_i^2 + \sigma^2) \); and the sensitivity of marginal cost to managerial effort \( \theta_i + \theta \).

Making the manager’s compensation contingent on the extent of the cost reduction is informationally efficient given that effort reduces cost (Holmstrom 1979). However, equivalently, we could have assumed that the agent’s compensation depends on realized profit and the compensation scheme entails relative performance evaluation (RPE). This can be done as follows, which parallels the procedure proposed by Raith (2003). The realized profit of firm \( i \) depends on its own and rival’s costs, \( c_i \) and \( c_j \). By solving the system of realized profits with respect to marginal costs, we can express each firm’s marginal cost as a function of its own and rival’s profits. Thus, we can substitute \( c_i \) into the linear compensation scheme (2) to obtain a compensation scheme that depends on the realized profits of firms \( i \) and \( j \).

The certainty equivalent of agent \( i \) is
\[ CE_i = \alpha_i + \beta_i(\theta_i + \theta)e_i - R\beta_i^2(\sigma_i^2 + \sigma^2)/2 - e_i^2/2. \]  

(3)

This yields the effort policy

\[ e_i = \beta_i(\theta_i + \theta), \]  

(4)

such that the expected marginal cost of firm \( i \) is given by

\[ E(c_i) = c - \beta_i(\theta_i + \theta)^2. \]  

(5)

The stronger are managerial incentives, the smaller is the firm’s (expected) marginal cost of production. By virtue of our timing structure, expected profits and optimal prices or quantities only depend on expected costs.

### 3.3 Product Market Demand

Following Singh and Vives (1984), suppose that \( U(q_1, q_2) \) is a strictly concave and strictly monotone utility function representing the preferences of a representative consumer. Let \( q = (q_1, q_2) \) and \( p = (p_1, p_2) \) denote the vector of quantities and prices, respectively. The representative consumer maximizes \( U(q) - pq \), which gives rise to an inverse demand system \( p_i = d_i(q) \). Inverse demand functions are downward sloping, \( \partial d_i/\partial q_i < 0 \), and the cross effects, \( \partial d_i/\partial q_j \), are negative because the goods are substitutes. The inverse demand system can be inverted to yield a direct demand system \( q_i = D_i(p) \). Direct demand functions are downward sloping, \( \partial D_i/\partial p_i < 0 \), and \( \partial D_i/\partial p_j \) is positive since the products are substitutes. Furthermore, we assume that the “own effect” \( |\partial D_i/\partial p_i| \) or \( |\partial d_i/\partial q_i| \) is larger than the “cross effect” \( |\partial D_i/\partial p_j| \) or \( |\partial d_i/\partial q_j| \). The expected gross profit of firm \( i \) in terms of prices is

\[ \pi_i(p) = (p_i - (c - \beta_i(\theta_i + \theta)^2)) D_i(p); \]  

(6)
and in terms of quantities is

$$\hat{\pi}_i(q) = (d_i(q) - (c - \beta_i(\theta_i + \theta))^2)q_i. \tag{7}$$

To ensure second-order conditions are satisfied, we assume that expected gross profit functions are strictly concave in their own strategic variable:

**Assumption 1 (Concavity)**

$$\frac{\partial^2 \pi_i}{\partial p_i^2} < 0 \text{ for all } p \text{ and } \frac{\partial^2 \hat{\pi}_i}{\partial q_i^2} < 0 \text{ for all } q.$$

We make the following assumptions to ensure that reaction functions in the product market are well-behaved and have slopes less than one in absolute value so as to obtain a unique price or quantity Nash equilibrium:

**Assumption 2 (Stability and Uniqueness)**

$$\frac{\partial^2 \pi_i}{\partial p_i^2} + \left| \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \right| < 0 \text{ for all } p \text{ and } \frac{\partial^2 \hat{\pi}_i}{\partial q_i^2} + \left| \frac{\partial^2 \hat{\pi}_i}{\partial q_i \partial q_j} \right| < 0 \text{ for all } q.$$

We assume prices are strategic complements and quantities are strategic substitutes:

**Assumption 3 (Strategic Substitutability and Complementarity)**

$$\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} > 0 \text{ for all } p \text{ and } \frac{\partial^2 \hat{\pi}_i}{\partial q_i \partial q_j} < 0 \text{ for all } q.$$

We introduce the following definitions. As stated earlier, the products sold by the two firms are substitutes, implying $\partial \pi_i / \partial p_j > 0$ and $\partial \hat{\pi}_i / \partial q_j < 0$. In the context of Bertrand competition, we say that if an increase in the rival’s price raises the firm’s profit at an increasing rate, then the products exhibit *increasing substitutability*, i.e. $\partial^2 \pi_i / \partial p_j^2 \geq 0$; and

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12With linear demand, it is always the case that prices are strategic complements and quantities are strategic substitutes when the products are substitutes.
if it does so at a decreasing rate, then they exhibit *decreasing substitutability*, i.e. \( \partial^2 \pi_i / \partial p_j^2 \leq 0 \). For increasing (decreasing) substitutability, we need a demand function that is convex (concave, respectively) in the rival’s price, i.e., \( \partial^2 D_i / \partial p_j^2 \geq (\leq)0 \). If demand is linear in the rival’s price, then the effect is absent, such that \( \partial^2 \pi_i / \partial p_j^2 = 0 \). Decreasing substitutability arises if demand is derived from a CES utility with an elasticity of substitution between 1 and 2, while increasing substitutability arises with an elasticity of substitution in excess of 2, for example.

Finally, we say that the demand function \( D_i(p) \) exhibits *supermodularity* (submodularity) with respect to prices if \( \partial^2 D_i / (\partial p_i \partial p_j) \geq (\leq)0 \). This affects the strength of strategic complementarity \( \partial^2 \pi_i / (\partial p_i \partial p_j) \) with respect to marginal cost since \( \partial^2 \pi_i / (\partial p_i \partial p_j) = (p_i - c_i) \partial^2 D_i / (\partial p_i \partial p_j) + \partial D_i / \partial p_j \). A lower marginal cost \( c_i \) (due to stronger incentives \( \beta_i \)) makes the degree of strategic complementarity stronger if and only if demand is supermodular with respect to prices. Intuitively, an increase in the rival’s price benefits the firm; with supermodularity (submodularity), the higher is the firm’s price, the more (less, respectively) the firm benefits from the increase in the rival’s price. If demand is additively separable, then the effect disappears, such that \( \partial^2 D_i / (\partial p_i \partial p_j) = 0 \).

### 4 Strategic Managerial Incentives

#### 4.1 Bertrand Competition in the Product Market

Firm \( i \) maximizes expected gross profit \( \pi_i(p) = (p_i - (c - \beta_i(\theta_i + \theta)^2)) D_i(p) \) with respect to its price \( p_i \) to yield the first-order condition (FOC)

\[
\frac{\partial \pi_i}{\partial p_i} = (p_i - (c - \beta_i(\theta_i + \theta)^2)) \frac{\partial D_i}{\partial p_i} + D_i(p) = 0.
\]  

\(^{13}\)Similarly, in the context of Cournot competition, we say that if an increase in the rival’s quantity decreases the firm’s profit at an increasing rate, then the products exhibit *increasing substitutability*, i.e. \( \partial^2 \pi_i / \partial q_j^2 \leq 0 \); and if it does so at a decreasing rate, then they exhibit *decreasing substitutability*, i.e. \( \partial^2 \pi_i / \partial q_j^2 \geq 0 \).
The second-order condition (SOC) is satisfied by virtue of Assumption 1. We denote the unique price equilibrium by \( \{ p^*_i(\beta), p^*_j(\beta) \} \). Let \( \pi_i(p^*_i(\beta), p^*_j(\beta), \beta_i) \) denote the gross profit of firm \( i \) at the optimum. We discuss below the three mechanisms by which the managerial incentives offered to manager \( i \) influence the gross profit of firm \( i \).

The following lemma derives the manner in which managerial incentives affect equilibrium prices:

**Lemma 1 (Incentives and Prices with Bertrand Competition)** The equilibrium price of firm \( i \) is decreasing in the incentives of its manager and the incentives of its rival’s manager, i.e. \( \partial p^*_i/\partial \beta_i < 0 \) and \( \partial p^*_j/\partial \beta_j < 0 \).

Suppose firm \( i \) strengthens the incentives of its manager. This spurs the manager of firm \( i \) to exert greater effort, which lowers the firm’s expected marginal cost and thereby leads the firm to charge a lower price for its product, i.e. \( \partial p^*_i/\partial \beta_i < 0 \). Given the strategic complementarity of prices, the rival responds by lowering its price, \( \partial p^*_j/\partial \beta_j < 0 \).

We now derive the equilibrium set of managerial incentives. The principal associated with firm \( i \) maximizes the firm’s net profit (i.e., net of the agent’s expected total compensation)

\[
\max_{\{\alpha_i^B, \beta_i^B, e_i^B\}} \pi_i^B - E(t_i^B), \tag{9}
\]

subject to the individual rationality (IR) constraint of the agent:

\[
\alpha_i^B + \beta_i^B(\theta_i + \theta)e_i^B - R(\beta_i^B)^2(\sigma_i^2 + \sigma^2)/2 - (e_i^B)^2/2 \geq r, \tag{10}
\]

and the incentive compatibility (IC) constraint of the agent given by the effort policy \( e_i^B = \beta_i^B(\theta_i + \theta) \). The IR constraint binds at the optimum, yielding the expected total compensation \( E(t_i^B) = r + R(\beta_i^B)^2(\sigma_i^2 + \sigma^2)/2 + (e_i^B)^2/2 \). Applying the effort policy, the expected total compensation of the agent becomes

\[
E(t_i^B) = r + (\beta_i^B)^2[(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)]/2. \tag{11}
\]
The principal’s net profit thereby becomes

\[
\pi_i^B - E(t_i^B) = (p_i^*(\beta^B) - (c - \beta_i^B(\theta_i + \theta)^2)) D_i(p^*(\beta^B)) - E(t_i^B) = \pi_i^B(p_i^*(\beta^B), p_j^*(\beta^B), \beta_i^B) - (\beta_i^B)^2[(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)]/2 - r. \tag{12}
\]

The FOC with respect to \(\beta_i^B\) yields

\[
\frac{\partial}{\partial \beta_i^B} \left( \pi_i^B - E(t_i^B) \right) = \frac{\partial \pi_i^B}{\partial p_i^*} \frac{\partial p_i^*}{\partial \beta_i^B} + \frac{\partial \pi_i^B}{\partial p_j^*} \frac{\partial p_j^*}{\partial \beta_i^B} + \frac{\partial \pi_i^B}{\partial \beta_i^B} - \beta_i^B[(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)] = 0. \tag{13}
\]

The incentives of the manager affect the principal’s net profit through four different channels. The first two operate via the product market by influencing the firm’s and rival’s prices, while the latter two arise in an isolated principal-agent problem (i.e., they represent the usual trade-off between risk-sharing and incentives). First, there is the term \(\frac{\partial \pi_i^B}{\partial p_i^*} \frac{\partial p_i^*}{\partial \beta_i^B}\), which captures the extent to which the gross profit of the firm is influenced by the impact of managerial incentives on its own price. Because the firm chooses in a later stage of the game the price that maximizes its gross profit, this effect, by the envelope theorem, disappears. Second, there is the term \(\frac{\partial \pi_i^B}{\partial p_j^*} \frac{\partial p_j^*}{\partial \beta_i^B}\), which captures the extent to which the gross profit of the firm is influenced by the impact of managerial incentives on its rival’s price. We label this the strategic effect, which arises because \(\frac{\partial p_i^*}{\partial \beta_i^B} \neq 0\), following from the fact that \(\frac{\partial^2 \pi_i^B}{\partial p_j^* \partial p_i^*} \neq 0\). The strategic effect is negative: when firm \(i\) offers stronger incentives to its manager, the rival’s price decreases \(\frac{\partial p_j^*}{\partial \beta_i^B} < 0\) by Lemma 1 because prices are strategic complements, which leads to a decline in firm \(i\)’s gross profit \(\frac{\partial \pi_i^B}{\partial p_j^*} > 0\) since the products are substitutes. Third, there is the term \(\frac{\partial \pi_i^B}{\partial \beta_i^B} > 0\), which measures the extent to which the manager’s effort reduces the firm’s expected marginal cost. Fourth, there is the term \(-\beta_i^B[(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)]\): an increase in incentives induces greater effort and exposes the risk-averse agent to more risk, both of which cause the principal to enhance the agent’s expected total compensation. Overall, the benefit from strengthening incentives originates from the reduction in marginal cost, while the cost is manifested via the (negative)
strategic effect and increased compensation.

The managerial incentives of firm $i$ respond to a change in the managerial incentives of firm $j$ as follows:

$$
\frac{d\beta_i^B}{d\beta_j^B} = \frac{\frac{\partial^2\pi_i^B}{\partial p_i^* \partial p_j^*} \times \frac{\partial p_j^*}{\partial p_j^*} \frac{\partial p_i^*}{\partial p_i^*} \frac{\partial p_j^*}{\partial p_j^*} \frac{\partial^2\pi_i^B}{\partial \beta_i^B \partial \beta_j^B}}{\frac{\partial^2\pi_i^B}{\partial (p_j^*)^2} \left( \frac{\partial p_j^*}{\partial \beta_i^B} \right)^2 + \frac{\partial^2\pi_i^B}{\partial p_j^* \partial p_i^*} \frac{\partial p_j^*}{\partial p_j^*} \frac{\partial p_i^*}{\partial p_i^*} \frac{\partial p_j^*}{\partial p_j^*} \frac{\partial^2\pi_i^B}{\partial \beta_i^B \partial \beta_j^B} - [(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)]} < 0
$$

Second-order condition (SOC) of the principal’s problem (14)

To characterize the strategic properties of managerial incentives, we determine the conditions under which they are strategic substitutes or complements. Suppose firm $j$ offers stronger incentives $\beta_j$ to its manager. How should firm $i$ respond in terms of its incentives $\beta_i$? There are three (possibly) opposing effects that are present in the numerator of (14). To better understand these three effects, we examine the second term in (13), which constitutes the strategic effect and from which the three effects are derived. The strategic effect is negative because when firm $i$ strengthens its incentives $\beta_i$, it lowers its marginal cost and its own price, which in turn triggers an aggressive price response on the part of its rival $j$, lowering firm $i$’s profit. How is the strategic effect influenced by a change in $\beta_j$, such as an increase in $\beta_j$?

First, because firm $j$ commits to being more aggressive when setting its price in the next stage, firm $i$’s best-response, given that prices are strategic complements, is to also lower its price. This is achieved by strengthening the incentives of its manager, i.e., $\beta_i$ increases, which is captured by the first term in the numerator of (14).
Second, the second term in the numerator of (14) is negative if \( \partial^2 p_j^* / (\partial \beta_i \partial \beta_j) \leq 0 \), which arises if demand is supermodular, \( \partial^2 D_i / (\partial p_i \partial p_j) \geq 0 \). This follows from the proof of Lemma 1, and in particular equation (22), from which we infer

\[
\frac{\partial^2 p_j^*}{\partial \beta_i \partial \beta_j} = \frac{1}{\Delta} \left( (\theta_j + \theta)^2 \frac{\partial^2 D_j}{\partial p_i \partial p_j} \right) \frac{\partial D_i}{\partial p_i} (\theta_i + \theta)^2,
\]

(15)

where \( \Delta \equiv \frac{\partial^2 \pi_1}{\partial p_i^2} \frac{\partial^2 \pi_2}{\partial p_j^2} - \frac{\partial^2 \pi_1}{\partial p_i \partial p_2} \frac{\partial^2 \pi_2}{\partial p_j \partial p_1} \). If demand is supermodular, then \( \partial^2 p_j^* / (\partial \beta_i \partial \beta_j) \leq 0 \) given that \( \Delta > 0 \) from Assumption 2 and \( \partial D_i / \partial p_i < 0 \), implying that an increase in \( \beta_j \) leads to a higher \( |\partial p_j / \partial \beta_i| \) (recall that this term is negative). This hurts firm \( i \)'s profit more, inducing firm \( i \) to respond by weakening its incentives \( \beta_i \) in order to mitigate this negative effect. If, on the other hand, demand is submodular, then firm \( i \) responds by strengthening its incentives. Therefore, the second effect operates in the same direction as the first effect if demand is submodular. The second effect is absent if demand is additively separable.

Third, when firm \( i \) becomes more aggressive, it triggers an even more aggressive response on the part of firm \( j \) in the pricing stage of the game. This hurts firm \( i \)'s profit when demand exhibits decreasing substitutability. To see this, note that the profit of firm \( i \) is increasing in the price of its rival \( p_j \), and, under decreasing substitutability, it is concave in \( p_j \). The strategic effect in this case becomes more negative when firm \( j \) strengthens its incentives \( \beta_j \).\(^{14}\) Firm \( i \) loses more by strengthening its incentives, so it lowers \( \beta_i \). This third effect, which is captured by the third term in the numerator of (14), operates in the opposite direction from the first effect. Under increasing substitutability, however, both effects operate in the same direction. This third effect is absent if demand is linear in the rival’s price.

To summarize, if demand functions exhibit strong supermodularity and decreasing substitutability, relative to the effect of strategic complementarity, then the second and third terms in the numerator of (14) dominate the first term, such that managerial incentives are strategic substitutes, i.e., \( d\beta_i^B / d\beta_j^B < 0 \). On the other hand, if demand functions exhibit submodularity and increasing or weakly decreasing substitutability, all three effects

\(^{14}\)That is, \( p_j \) moves down the concave profit function, so the slope increases.
point in the same direction, such that managerial incentives are strategic complements, i.e., $d\beta_i^B / d\beta_j^B > 0$. The next proposition states our findings.\(^{15}\)

**Proposition 1 (Strategic Incentives with Bertrand Competition)** Managerial incentives are strategic substitutes if demand functions exhibit strong supermodularity and decreasing substitutability; and they are strategic complements if demand functions exhibit submodularity and increasing or weakly decreasing substitutability.

If demand is linear and additively separable in prices (e.g., $D_i = A - p_i + \gamma p_j$), then only the first term in the numerator of (14) remains (the other two become zero), such that incentives are strategic complements (as are prices). In other words, in this case, managerial incentives inherit the properties of the strategic variables (prices) being utilized by firms when competing in the product market.

Moreover, if firms operate in a perfectly competitive market, then their actions (prices) are no longer strategic; that is, firms are price-takers. In this case, managerial incentives are no longer strategic, such that $d\beta_i^B / d\beta_j^B = 0$ (since we have that $\frac{\partial^2 \pi^B}{\partial p_j \partial p_i} = 0$ and $\frac{\partial^2 \pi^B}{\partial (p_j)^2} = 0$).

The distinction between an agent’s reservation utility $r$ and compensation scheme $(\alpha_i, \beta_i)$ are important. Irrespective of the strategic properties of managerial incentives, it is always the case that each agent earns his reservation utility. Therefore, if a firm weakens the incentives of its manager in response to an action by its competitor, this may be accompanied by an adjustment in the manager’s salary so as to ensure retention.

\(^{15}\)Given the continuity of the principal’s profit function with respect to $\beta$ and the fact that $\beta_i$ lies in a compact set, existence of equilibrium in stage 1 of the game is guaranteed if the principal’s profit function is quasi-concave in $\beta_i$. This arises if the degree of constant relative risk aversion $R$ and/or the variances $\sigma^2$ and $\sigma_i^2$ (which do not affect equilibrium prices $p^*$) are large enough; see the denominator of (14). This also implies that we have single-valued best-responses, which according to Proposition 1, slope either up or down. For a unique equilibrium $\{\beta_1^B, \beta_2^B\}$ of managerial incentives to exist, we require that $|\frac{d\beta_i^B}{d\beta_j^B}| < 1$. With linear demand, this condition holds for a wide range of parameter values, but we do not have general conditions on fundamentals that would guarantee this condition and hence uniqueness. In any case, the comparative statics we perform below in $(\beta_1, \beta_2)$ space can be performed locally around any stable equilibrium, without changing our insights qualitatively.
4.2 Cournot Competition in the Product Market

The analysis in this sub-section parallels the scenario with Bertrand competition.\textsuperscript{16} Firm $i$ maximizes expected gross profit $\hat{\pi}_i(q) = (d_i(q) - (c - \beta_i(\theta_i + \theta)^2))q_i$ with respect to its quantity $q_i$ to yield the FOC

$$\frac{\partial \hat{\pi}_i}{\partial q_i} = \frac{\partial d_i}{\partial q_i}q_i + d_i(q) - (c - \beta_i(\theta_i + \theta)^2) = 0. \quad (16)$$

The SOC is satisfied by virtue of Assumption 1. We denote the unique quantity equilibrium by $\{q^*_i(\beta), q^*_j(\beta)\}$. The following lemma derives the manner in which managerial incentives affect equilibrium quantities:

**Lemma 2 (Managerial Incentives and Quantities with Cournot Competition)** The equilibrium quantity of firm $i$ is increasing in the incentives of its manager and decreasing in the incentives of its rival’s manager, i.e. $\partial q^*_i/\partial \beta_i > 0$ and $\partial q^*_i/\partial \beta_j < 0$.

The principal’s objective is

$$\hat{\pi}_i^C - E(t_i^C) = (d_i(q^*(\beta^C)) - (c - \beta_i^C(\theta_i + \theta)^2))q^*_i(\beta^C) - E(t_i^C) = \hat{\pi}_i^C(q^*_i(\beta^C), q^*_j(\beta^C), \beta^C_i) - (\beta^C_i)^2[(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)]/2 - r. \quad (17)$$

The FOC with respect to $\beta_i^C$ yields

$$\frac{\partial (\hat{\pi}_i^C - E(t_i^C))}{\partial \beta_i^C} = \frac{\partial \hat{\pi}_i^C}{\partial q_i^*} \frac{\partial q_i^*}{\partial \beta_i^C} + \frac{\partial \hat{\pi}_i^C}{\partial q_j^*} \frac{\partial q_j^*}{\partial \beta_i^C} + \frac{\partial \hat{\pi}_i^C}{\partial \beta_i^C} - \beta_i^C[(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)] = 0. \quad (18)$$

\textsuperscript{16}Kreps and Scheinkman (1983) study a two-stage game in which, first, firms choose capacities (that constrain how much they may produce) and, second, firms compete in prices. The authors show that, under mild assumptions about demand, the unique equilibrium outcome is the Cournot outcome.
The managerial incentives of firm $i$ respond to a change in the managerial incentives of firm $j$ as follows:

\[
\frac{d\beta^C_i}{d\beta^C_j} = -\frac{\frac{\partial^2 \hat{\pi}_i^C}{\partial q_j^* \partial q_i^*} \times \frac{\partial q_j^*}{\partial \beta^C_j} + \frac{\partial^2 \hat{\pi}_i^C}{\partial \beta^C_j \partial \beta^C_i} \times \frac{\partial q_j^*}{\partial \beta^C_j} \times \frac{\partial q_i^*}{\partial \beta^C_i} - [(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)]}{\frac{\partial q_j^*}{\partial \beta^C_j}^2 \frac{\partial q_i^*}{\partial \beta^C_i}} < 0.
\]  \hspace{1cm} (19)

Second-order condition (SOC) of the principal’s problem

The intuition underlying the strategic properties of managerial incentives is akin to that we offered in the Bertrand competition case, with one exception. From (16), the quantity equilibrium $\{q_1^*(\beta), q_2^*(\beta)\}$ is linear in the marginal cost of each firm and thereby managerial incentives $\beta$. Thus, there is no second effect (i.e., demand supermodularity or submodularity does not play a role), implying only the first and third effects remain.\(^\text{17}\) The next proposition summarizes our findings.\(^\text{18}\)

**Proposition 2 (Strategic Incentives with Cournot Competition)** Managerial incentives are strategic substitutes if demand functions exhibit decreasing or weakly increasing substitutability; and they are strategic complements if demand functions exhibit strongly increasing substitutability.

If demand is linear in the rival’s quantity (e.g., $d_i = B - q_i - \nu q_j$), then managerial incentives are strategic substitutes (as are quantities). Overall, then, if demand is linear and

---

\(^\text{17}\)By contrast, prices are not linear in marginal costs, so the second effect is present in the Bertrand case.

\(^\text{18}\)Vives (2009) obtains similar conditions to ours in a model in which, in the first stage, firms make capacity investments that lower marginal cost and, in the second stage, firms compete in Cournot fashion. For a unique equilibrium $\{\beta^C_1, \beta^C_2\}$ of managerial incentives to exist, we require that $|\frac{\partial \beta^C}{\partial \beta^C_j}| < 1$. With linear demand, this condition holds for a wide range of parameter values.
additively separable, irrespective of the type of competition, managerial incentives inherit the properties of the strategic variables (prices or quantities) being utilized by firms when competing in the product market.

Similarly, as with Bertrand competition, if firms operate in a perfectly competitive market, then managerial incentives are not strategic.

5 Theoretical Implications of Strategic Managerial Incentives

The manner in which managerial incentives respond to a change in the corporate environment may be decomposed into direct and strategic effects. The direct effect represents the amount by which a firm’s incentives respond to the change in the corporate environment holding fixed its rival’s incentives. The strategic effect represents the amount by which a firm’s incentives react to the change in its rival’s incentives. Therefore, the direct and strategic effects are interrelated, the latter to some degree reflecting the former. A large direct effect for one firm typically translates into a large strategic effect for the other firm.

We first consider a change in the corporate environment that is common to both firms, and then a change in the corporate environment that is specific to one firm. For ease of exposition, we focus on the role of risk, such that the former pertains to systematic risk, while the latter pertains to idiosyncratic risk. However, all the arguments we put forth are applicable across a broad range of changes in the corporate environment, such as those brought about by shifts in managerial practices and corporate governance. For example, when discussing a change in systematic risk, we could alternatively be referring to the passing of Sarbanes-Oxley (in the sense that it affects all firms in the industry); and when discussing a change in the idiosyncratic risk of a firm, we could alternatively be referring to a change in the composition of its board of directors.
5.1 Strategic Managerial Incentives and Systematic Risk

Consider an increase in systematic risk $\sigma^2$. We will show that the direct effect yields the "traditional" response that one would expect from a standard agency model, while the strategic effect operates in the same or opposite direction from the direct effect depending on whether managerial incentives are strategic complements or substitutes, respectively. If managerial incentives are strategic substitutes and the strategic effect is sufficiently strong, then the strategic effect may overpower the direct effect to yield an "unusual" response for one firm and a "traditional" response for another. If managerial incentives are strategic complements, then the strategic effect operates in the same direction as the direct effect, such that both firms experience "traditional" responses; however, the responses are stronger than would be predicted by a standard agency model, leading to a \textit{ratcheting} effect.

5.1.1 Managerial Incentives are Strategic Substitutes

Suppose managerial incentives are strategic substitutes and firms compete in Bertrand fashion.$^{19}$ Holding constant the managerial incentives of firm $j$, from (13), we infer that the best-response incentives curve of firm $i$ shifts as follows in response to the increase in systematic risk:$^{20}$

$$\frac{d\beta_i^B}{d\sigma^2} = \frac{\beta_i^B R}{\frac{\partial^2 \pi_i^B}{\partial (p_j^*)^2} \left( \frac{\partial p_j^*}{\partial \beta_i^B} \right)^2 + \frac{\partial^2 \pi_i^B}{\partial p_j^* \partial p_i^*} \frac{\partial p_j^*}{\partial \beta_i^B} \frac{\partial \beta_i^B}{\partial \beta_i^B} - [(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)]} < 0.$$  \hfill (20)

Second-order condition (SOC) of the principal’s problem

As expected, greater systematic risk implies lower incentives, all else being equal. But

$^{19}$See Proposition 1 for the conditions under which this arises. For ease of exposition, we focus on the case in which firms compete in Bertrand fashion. The same arguments apply if firms compete in Cournot fashion.

$^{20}$Note that this expression does not describe the change in equilibrium incentives; rather, it describes the way in which each best-response incentives curve shifts.
in this strategic environment, all else cannot be equal, because the rival responds. Owing to the strategic substitutability of managerial incentives, in equilibrium, it may very well be the case that one firm has strengthened the incentives offered its manager. For this to occur, heterogeneous responses to a change in $\sigma^2$ are required (i.e., $d\beta_i^B/d\sigma^2$ is different from $d\beta_j^B/d\sigma^2$).

![Figure 1: Increase in Systematic Risk when Managerial Incentives are Strategic Substitutes and the Strategic Effect is Strong](image)

Figure 1 illustrates our argument. The curve $BR_i$ represents the best-response function of firm $i$: it describes the manner in which firm $i$ responds to the managerial incentives being offered by firm $j$. The initial equilibrium is $(\beta_1^*, \beta_2^*)$. Suppose systematic risk $\sigma^2$ increases, leading to the new equilibrium $(\beta_1^{**}, \beta_2^{**})$. Interestingly, firm 1 offers stronger incentives, $\beta_1^{**} > \beta_1^*$, although risk has increased. What is the intuition? An increase in systematic risk leads to a downward shift in both best-response incentives curves. This is the direct effect. However, the best-response incentives curve of firm 2, $BR_2$, decreases more than $BR_1$.
because the firms are heterogeneous; by inspecting (20), one notices that the potential source of heterogeneous responses may be differences in idiosyncratic risk $\sigma_i^2$ or the sensitivity of marginal cost to managerial effort $\theta_i$, for example. Hence, when systematic risk increases, both firms weaken their incentives, holding the rival’s managerial incentives constant. But when the rival offers weaker incentives, the best-response of a firm is to strengthen its incentives since managerial incentives are strategic substitutes. If the strategic effect is strong enough, as it is in Figure 1 for firm 1, then it outweighs the direct effect, the net result of which is a positive relationship between systematic risk and incentives for firm 1 (and a negative relationship between systematic risk and incentives for firm 2).

The direct and strategic effects may be decomposed as follows using Figure 1. The initial equilibrium is point A. The move from point A to point $B_1$ represents the amount by which the managerial incentives of firm 1 decline due to the downward shift in $BR_1$ (brought about by the increase in systematic risk) in the absence of a response by firm 2, thus it captures the direct effect associated with firm 1. Similarly, the move from point A to point $B_2$ is the amount by which the incentives of firm 2 decline due to the downward shift in $BR_2$ in the absence of a response by firm 1, thus it captures the direct effect of firm 2. The direct effects are negative, in agreement with agency theory which posits that incentives should become weaker in light of greater risk. The new equilibrium is point C. The move from point $B_1$ to point $C$ represents the amount by which the incentives of firm 1 strengthen due to the response by firm 2 to the increase in systematic risk, capturing the strategic effect of firm 1. Because firm 2 weakens its incentives significantly as a consequence of the increase in systematic risk (i.e., the downward shift in $BR_2$ is large), firm 1 responds by strengthening its incentives considerably since incentives are strategic substitutes. In this example, the strategic effect is strong enough to overpower the direct effect, such that firm 1 experiences a net increase in its incentives (from $\beta_1^*$ to $\beta_1^{**}$). The move from point $B_2$ to point $C$ represents the amount by which the incentives of firm 2 weaken due to the response by firm 1 to the increase in systematic risk, capturing the strategic effect of firm 2.
To identify which firm may strengthen its managerial incentives in response to an increase in systematic risk, consider the following. In Figure 1, firm 2 has a large response (direct effect) to the increase in risk, which leads firm 1 to have a strong strategic effect; and firm 1 has a small response (direct effect) to the increase in systematic risk, which leads firm 2 to have a weak strategic effect. The net effects are that firm 1 strengthens its incentives while firm 2 weakens them. Therefore, the firm that is less sensitive to systematic risk (i.e., the one with the small direct effect) is the one for which we obtain an unusual response.

In Raith (2003), piece rates are positively correlated with the variance of firm profit across markets that differ in product substitutability, market size, or entry costs. Therefore, in Raith, a positive link between risk and incentives arises across heterogeneous industries. By contrast, we may obtain positive and negative links across heterogeneous firms in the same industry.

If managerial incentives are strategic substitutes, but the strategic effect does not overpower the direct effect, then the strategic effect serves to dampen the response of managerial incentives to a change in the corporate environment common to all firms in the industry. Figure 2 illustrates an increase in systematic risk, which moves the equilibrium from point $A$ to point $C$. The moves from point $A$ to points $B_1$ and $B_2$ represent the (negative) direct effects of firms 1 and 2. The moves from points $B_1$ and $B_2$ to point $C$ represent the (positive) strategic effects. For each firm, because its rival weakens its incentives, the strategic response is to strengthen incentives (since incentives are strategic substitutes). However, in neither case is the strategic effect strong enough to overpower the direct effect. Thus, the strategic effect dampens the extent to which the incentives of each firm decline as a consequence of the increase in systematic risk. This scenario arises if firms are homogeneous, or they are
not extensively heterogeneous.

Figure 2: Increase in Systematic Risk when Managerial Incentives are Strategic Substitutes and the Strategic Effect is Weak

5.1.2 Managerial Incentives are Strategic Complements

Suppose managerial incentives are strategic complements. Then the strategic effect operates in the same direction as the direct effect. Thus, strategic considerations serve to amplify the response of managerial incentives to a change in the corporate environment common to both firms. Consider an increase in systematic risk $\sigma^2$, as illustrated in Figure 3. The moves from point $A$ to points $B_1$ and $B_2$ representing the direct effects of firms 1 and 2 are relatively weak. In the absence of strategic considerations, neither firm would respond significantly to the increase in systematic risk. However, when we take into account the strategic effects represented by the moves from points $B_1$ and $B_2$ to point $C$, we see that the net changes in incentives are considerable. Extrapolating these findings into industries characterized by multiple firms competing against each other, we infer that a drastic ratcheting effect may
occur. That is, seemingly small changes in the corporate environment common to all firms in the industry can lead to dramatic changes in managerial incentives. As each firm in the industry reacts strategically to each other firm, the overall response is magnified as it permeates the entire industry.

Figure 3: Increase in Systematic Risk when Managerial Incentives are Strategic Complements

5.2 Managerial Incentives and Idiosyncratic Risk

When there is a change in the idiosyncratic risk of a firm, the best-response incentives curve of that firm shifts, while the other firm experiences a movement along its best-response incentives curve. Thus, in light of strategic considerations, all firms in the industry respond to changes in a firm’s idiosyncratic risk. To illustrate this, suppose managerial incentives are strategic complements and consider an increase in the idiosyncratic risk $\sigma_1^2$ faced by firm 1. This shifts downward the best-response incentives curve of firm 1, and represents a movement along the best-response incentives curve of firm 2. Firm 1 weakens its incentives
due to risk-sharing considerations, and firm 2 responds by weakening its incentives since managerial incentives are strategic complements, as shown in Figure 4. In a traditional agency model, there would be no change in the incentives offered by firm 2 (that is, there is no strategic effect). In broad terms, this means that when a firm experiences a change in its specific corporate environment that leads it to adapt its compensation scheme, all the firm’s competitors react by adapting their compensation schemes; if managerial incentives are strategic complements (substitutes), then their reactions move in the same (opposite, respectively) direction.

Figure 4: Increase in the Idiosyncratic Risk of Firm 1 when Managerial Incentives are Strategic Complements

6 Empirical Implications of Strategic Managerial Incentives

Our model suggests numerous avenues for future empirical research. First, the strategic nature of managerial compensation schemes is itself empirically testable. That is, we should
find that a change in the compensation scheme offered by a firm should cause its rivals to respond by changing their compensation schemes. These predictions are testable in terms of both compensation levels and incentives. Because firms typically engage in various forms of competition utilizing a wide variety of strategic tools spanning a number of products and industries, observed changes in compensation schemes reflect the agglomeration of all these (direct and strategic) effects. Indeed, Kedia (2006) finds that 29 percent of 4-digit SIC industries use both strategic complements and substitutes when competing. Hence, we postulate that the most practical approach is to test directly the strategic relationship of compensation levels and incentives, rather than attempt to infer them from the characteristics of the product markets in which the firms are competing. If the compensation schemes of a collection of firms are not found to be strategic, then the model predicts that those firms are engaged in perfect competition, or the strategic effects we identified are weak. Kedia finds that 29 percent of 4-digit SIC industries are not engaged in strategic interaction, suggesting they are perfectly competitive.

Second, the strategic property of compensation levels has ramifications in terms of the setting and formulation of benchmark pay (Bizjak, Lemmon, and Naveen 2008; Faulkender and Yang 2010). In evaluating the importance of benchmarking, the empirical literature has appealed to the agency theoretic prediction that, at the optimum, an agent should be offered his reservation utility (in a standard model with moral hazard). Our analysis suggests there is also a strategic motive stemming from the product market. In a random sample of 100 S&P 500 firms, Bizjak, Lemmon, and Naveen study proxy statements to find that peer groups are typically based on industry and size, suggesting they may be competitors; and that the majority of firms using benchmarking target their pay levels at or above the median of their peer group. The key point we make is that benchmarking (via formal and informal means) may be implemented not only to ensure the CEO remains with the firm by setting reservation wages, but also as a consequence of the fact that the firms are competing strategically in the marketplace.
Third, the model predicts that if compensation schemes are strategic complements, then a ratcheting effect of CEO compensation levels and incentives occurs. Specifically, consider a change in the corporate environment common to all firms in the industry that leads them to increase their compensation levels and incentives (the "direct" effect). Due to strategic complementarity, each firm responds to the other by further increasing its compensation (the "strategic" effect). It is thereby possible that strategic considerations contributed towards the dramatic rise in U.S. CEO pay, which increased sixfold between 1980 and 2003, as well as the sharp rise in pay-performance sensitivity during the mid-1980s to 2005 (Gabaix and Landier 2008; Frydman and Saks 2010).

Fourth, in examining the empirical link between managerial compensation schemes and changes in the corporate environment, we propose that such changes be decomposed into components that are firm-specific versus common to all firms whenever possible. For example, firm risk should be decomposed into idiosyncratic and systematic risk. The model predicts that managerial incentives respond to a change in a firm’s specific corporate environment (e.g., idiosyncratic risk) in the usual fashion consistent with agency theory, whereas managerial incentives may respond to a change in the corporate environment common to all firms in the industry (e.g., systematic risk) in a fashion that dispels traditional agency theory.

Fifth, future empirical work should test the prediction that a change in the corporate environment specific to a firm may cause not just the firm itself to adjust its compensation scheme, but also cause the firm’s rivals to adjust their compensation schemes due to the strategic effects we identified. We showed that while an increase in the idiosyncratic risk of a firm leads it to weaken its managerial incentives (in accord with traditional agency theory), the rival responds by strengthening (weakening) its incentives if compensation schemes are strategic substitutes (complements, respectively). In this sense, idiosyncratic risk may become endowed with characteristics previously solely attributed to systematic risk. Another example would be a change in a firm’s board of directors that leads the firm to change
the compensation scheme of its CEO. The firm’s competitors may react by adapting their compensation schemes even though they were not directly affected by the change in the firm’s board; such reactions would not relate to the retention concern, but instead the fact that managers may have to be incentivized more or less aggressively to be successful in the product market.

Sixth, the relationship between managerial compensation schemes and the corporate environment common to all firms in the industry may be asymmetric across firms, suggesting that a broad set of moderating factors should be considered in empirical studies of executive compensation. For example, the model predicts that, if firms are heterogeneous and managerial incentives are strategic substitutes, then an increase in systematic risk may strengthen the incentives offered by one set of firms and weaken the incentives offered by another set. It is important to emphasize that this holds true across firms in the same industry, and not just across industries. This may explain why some empirical studies find that the relationship between risk and incentives is positive (Core and Guay 1999; Oyer and Schaefer 2005; Rajgopal, Shevlin, and Zamora 2006); some find that the link is insignificant (Garen 1994; Yermack 1995; Bushman, Indjejikian, and Smith 1996; Ittner, Larcker, and Rajan 1997; Conyon and Murphy 2000); and others find that the link is negative (Lambert and Larcker 1987; Aggarwal and Samwick 1999a; Jin 2002). Because there are numerous sources of heterogeneity across firms (both within and across industries), empirical researchers will have to determine which firm characteristics significantly moderate the relationship between managerial incentives and firm risk. One such moderating factor may be firm size. In general, any event that affects all firms in the industry (such as a regulatory change, e.g. Sarbanes-Oxley) may lead firms to adapt their compensation schemes in opposite directions, not because firms are fundamentally heterogeneous (which in and of itself is not sufficient to generate asymmetry), but because of the strategic effects we uncovered.

Finally, there is considerable evidence that an increase in the extent of competition is associated with a strengthening of managerial incentives (DeFond and Park 1999; Hubbard
and Palia 1995; Cunat and Guadalupe 2005, 2009; Karuna 2007). A number of theoretical papers use different models to examine the link between product market competition and pay-performance sensitivity, and predict either an ambiguous or positive relationship (Raith 2003; Schmidt 1997; Hart 1983; Scharfstein 1988; Hermelin 1992). The strategic considerations we identified may shed light on this debate. Karuna (2007) examines three industry characteristics emphasized by Raith (2003) that determine the extent to which an industry is competitive: the cost of entry, market size, and the degree of product differentiation. A recurring theme of our model is that, due to firm heterogeneity and strategic behavior, changes in the corporate environment may lead to asymmetric responses in managerial compensation across firms in the same industry. Therefore, future empirical work should consider allowing for this possibility by examining firm-specific moderating factors in testing the relationship between the extent of competition and compensation.

7 Conclusion

This paper demonstrated that managerial compensation schemes are strategic due to product market competition. We derived reasonable conditions under which managerial compensation levels and incentives are strategic substitutes or complements. These conditions relate to the form of competition (Bertrand or Cournot), the characteristics of the demand for the product, and the properties of the strategic variables being wielded by firms when competing in the product market. If firms operate in a perfectly competitive market, then compensation schemes are not strategic. We demonstrated that the strategic nature of managerial compensation schemes has implications for the ways in which firms adapt their compensation in response to changes in their own corporate environment, the environments of their rivals, and the environment common to all firms operating in the industry. The ramifications of strategic compensation schemes are numerous. We emphasized the follow-

\footnote{A related study is due to Joh (1999), who finds that, in Japan, managerial compensation is positively linked to industry profit, the positive effect being stronger in competitive industries than in concentrated industries.}
ing: the conventional wisdom may no longer hold about the precision of the performance measure and managerial incentives; the corporate environment specific to one firm influences the design of compensation schemes across all firms in the industry; and a ratcheting effect of compensation may occur that amplifies otherwise small changes in the corporate environment common to all firms in the industry.

Our findings suggest the following avenues for future theoretical research. First, our predictions about the strategic nature of managerial compensation schemes should naturally extend to an environment with more than two firms. The strategic relationship between the compensation schemes of any pair of firms is a function of how the demands for their products are inter-related and whether the strategic variables utilized by the firms in the product market are strategic substitutes or complements.

Second, one may expand the set of strategic tools being wielded by firms when competing in the product market. We focused on the traditional forms of competition emphasized in microeconomics, namely price and quantity competition. However, our basic arguments should also be applicable under other, perhaps more exotic, forms of competition. For example, the realm of competition may include capital expenditures; R&D expenses to improve the quality of the product and lower costs of production; advertising expenses to increase demand; and the setting of product characteristics, such as design, features, and quality. One would then have to determine whether such strategic variables are strategic complements or substitutes, and the extent to which the strategy of one firm affects the demand of another.

Third, along the lines of Bolton and Scharfstein (1990), Rotemberg and Scharfstein (1990), and Maksimovic (1988), future theoretical work should aim to uncover the strategic properties of capital structures that arise from product market competition. Just as firms react to one another’s compensation schemes, one may find that firms react to one another’s capital structures. The literature shows that debt serves as a commitment device that disciplines management towards behaving with a more aggressive or defensive posture depending
on the characteristics of the product market. Thus, we postulate that, under certain condi-
tions, firms make capital structure decisions to incentivize their managers to respond to and
be more successful against their competitors in the product market.
A Appendix: Proofs of Lemmas and Propositions

A.1 Proof of Lemma 1

We invoke the Implicit Function Theorem using the system of first order conditions given by (8):

\[
\begin{pmatrix}
\frac{\partial p_i^*}{\partial \beta_i} & \frac{\partial p_i^*}{\partial \beta_j} \\
\frac{\partial p_j^*}{\partial \beta_i} & \frac{\partial p_j^*}{\partial \beta_j}
\end{pmatrix}
= - \left( \begin{pmatrix}
\frac{\partial^2 \pi_i}{\partial p_i^2} & \frac{\partial^2 \pi_i}{\partial p_j^2} \\
\frac{\partial^2 \pi_j}{\partial p_i^2} & \frac{\partial^2 \pi_j}{\partial p_j^2}
\end{pmatrix}\right)^{-1}
\begin{pmatrix}
\frac{\partial D_i}{\partial p_i} (\theta_i + \theta)^2 & 0 \\
0 & \frac{\partial D_j}{\partial p_j} (\theta_j + \theta)^2
\end{pmatrix}
\]

\[
= - \frac{1}{\Delta} \begin{pmatrix}
\frac{\partial^2 \pi_i}{\partial p_i^2} \frac{\partial p_i^*}{\partial p_i} (\theta_i + \theta)^2 - \frac{\partial^2 \pi_i}{\partial p_j^2} \frac{\partial p_i^*}{\partial p_j} (\theta_i + \theta)^2 & \frac{\partial \pi_i}{\partial p_i} \frac{\partial D_i}{\partial p_i} (\theta_i + \theta)^2 \\
\frac{\partial \pi_j}{\partial p_j} \frac{\partial p_j^*}{\partial p_i} (\theta_i + \theta)^2 - \frac{\partial \pi_j}{\partial p_j} \frac{\partial p_j^*}{\partial p_j} (\theta_i + \theta)^2 & \frac{\partial^2 \pi_j}{\partial p_j^2} \frac{\partial p_j^*}{\partial p_j} (\theta_j + \theta)^2
\end{pmatrix},
\]

where \( \Delta = \frac{\partial^2 \pi_i}{\partial p_i^2} \frac{\partial p_i^*}{\partial p_i} - \frac{\partial^2 \pi_i}{\partial p_j^2} \frac{\partial p_i^*}{\partial p_j} \) is the determinant of the Jacobian of the system of first-order conditions. From Assumption 2, \( \Delta \) is strictly positive. It then follows that

\[
\frac{\partial p_i^*}{\partial \beta_i} = - \frac{1}{\Delta} \left( \frac{\partial^2 \pi_i}{\partial p_i^2} \frac{\partial D_i}{\partial p_i} (\theta_i + \theta)^2 \right) > 0;
\]

\[
\frac{\partial p_i^*}{\partial \beta_j} = \frac{1}{\Delta} \left( \frac{\partial^2 \pi_i}{\partial p_i^2} \frac{\partial D_i}{\partial p_j} (\theta_j + \theta)^2 \right) > 0;
\]

\[
\frac{\partial p_j^*}{\partial \beta_i} = \frac{1}{\Delta} \left( \frac{\partial^2 \pi_j}{\partial p_j^2} \frac{\partial D_i}{\partial p_i} (\theta_i + \theta)^2 \right) > 0;
\]

\[
\frac{\partial p_j^*}{\partial \beta_j} = - \frac{1}{\Delta} \left( \frac{\partial^2 \pi_j}{\partial p_j^2} \frac{\partial D_i}{\partial p_j} (\theta_j + \theta)^2 \right) < 0.
\]

The above signs follow from Assumption 1 and the facts that prices are strategic complements and demand is downward sloping in its own price. Also note that the strength of \( |\frac{\partial p_i^*}{\partial \beta_j}| \) depends positively on how strong the strategic complementarity \( \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \) is. The strength of the strategic complementarity in turn depends on \( \beta_i \). If demand is supermodular, \( \frac{\partial^2 D_i}{\partial p_j \partial p_i} > 0 \), then the higher the \( \beta_i \) the higher the \( |\frac{\partial p_i^*}{\partial \beta_j}| \). The opposite
holds when demand is submodular, \( \partial^2 D_i / (\partial p_j \partial p_i) < 0 \).

For use in the proof of Proposition 1, note, using (22), that \( \partial p_i^* / \partial \beta_j \) does not depend on \( \beta_j \), i.e., \( \partial^2 p_i / \partial \beta_j^2 = 0 \) and \( \partial^2 p_i / \partial \beta_i^2 = 0 \). However, \( \partial p_i^* / \partial \beta_j \) does depend on \( \beta_i \), as we have argued above.

### A.2 Proof of Proposition 1

We would like to determine how \( \beta_j \) affects \( \beta_i \) (the slope of principal \( i \)'s reaction function with respect to incentives). We will totally differentiate the first order condition (13). The first term in (13) is zero from the envelope theorem, the second term is the strategic effect, the third term captures the direct effect of incentives on production cost and the last term is the cost associated with incentivizing the agent. The strategic term \( \frac{\partial \pi_i^B}{\partial p_j} \frac{\partial p_i^*}{\partial \beta_i} \) depends on \( \beta = (\beta_1, \beta_2) \) as follows: first, \( \beta \) affects the equilibrium prices \( (p_1^*, p_2^*) \) and thus the profit function \( \pi_i^B \), which suggests that it affects the term \( \partial \pi_i^B / \partial p_j^* \) and second, \( \beta \) affects the way equilibrium prices react to changes in incentives, i.e., the term \( \partial p_j^* / \partial \beta_i^B \). Therefore, differentiating the strategic effect with respect to either \( \beta_i \) or \( \beta_j \) will yield three terms. Furthermore, the third term in (13), since cost reduction depends only on \( \beta_i \) linearly, is independent of \( \beta \).

Thus, by totally differentiating (13) with respect to \( \beta_i \) and \( \beta_j \) we obtain:

\[
\left\{ \frac{\partial^2 \pi_i}{\partial p_j^2} \left( \frac{\partial p_j}{\partial \beta_i} \right)^2 + \frac{\partial^2 \pi_i}{\partial p_j \partial p_i} \frac{\partial p_j}{\partial \beta_i} \frac{\partial p_i}{\partial \beta_i} + \frac{\partial^2 \pi_i}{\partial p_i} \frac{\partial^2 p_j}{\partial \beta_i^2} - [(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)] \right\} d\beta_i +
\left\{ \frac{\partial^2 \pi_i}{\partial p_j^2} \frac{\partial p_j}{\partial \beta_j} \frac{\partial p_i}{\partial \beta_i} + \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \frac{\partial p_i}{\partial \beta_j} \frac{\partial \pi_i}{\partial \beta_i} + \frac{\partial^2 \pi_i}{\partial p_j^2} \frac{\partial^2 p_j}{\partial \beta_i^2} \right\} d\beta_j = 0.
\]

From Lemma 1 we know that \( \partial^2 p_j / \partial \beta_i^2 = 0 \). Then, the above expression reduces to
\[
\left\{ \frac{\partial^2 \pi_i}{\partial eta_i^2} \left( \frac{\partial p_j}{\partial eta_i} \right)^2 + \frac{\partial^2 \pi_i}{\partial p_j \partial p_i} \frac{\partial p_j}{\partial eta_i} + \left[ (\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma_j^2) \right] \right\} d\beta_i + \left\{ \frac{\partial^2 \pi_i \partial p_j}{\partial p_j \partial p_i} + \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \frac{\partial p_i}{\partial eta_i} \frac{\partial p_j}{\partial eta_i} + \frac{\partial^2 p_j}{\partial p_j \partial eta_i} \right\} d\beta_j = 0 \Leftrightarrow \]

\[
\frac{d\beta_i}{d\beta_j} = -\frac{\frac{\partial^2 \pi_i}{\partial p_j^2} \frac{\partial p_j}{\partial \beta_i} - \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \frac{\partial p_i}{\partial \beta_i} + \frac{\partial^2 p_j}{\partial p_j \partial \beta_i} \left[ (\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma_j^2) \right]}{\frac{\partial^2 \pi_i}{\partial p_j^2} \left( \frac{\partial p_i}{\partial \beta_i} \right)^2 + \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \frac{\partial p_i}{\partial \beta_i} \frac{\partial p_j}{\partial \beta_i} - \left[ (\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma_j^2) \right]}.
\]

If \(|d\beta_i/d\beta_j| < 1\) we obtain a unique equilibrium \((\beta^*_1, \beta^*_2)\). The denominator of \(d\beta_i/d\beta_j\) is the second derivative of the profit function with respect to \(\beta_i\), which is negative due to the SOC of the principal’s problem. The sign of the numerator is ambiguous and we discuss the three effects (stemming from the three different terms in the numerator) in the main text.

### A.3 Proof of Lemma 2

We invoke the Implicit Function Theorem using the system of first-order conditions given by (16):

\[
\begin{pmatrix} \frac{\partial q_i}{\partial \beta_1} & \frac{\partial q_i}{\partial \beta_2} \\ \frac{\partial q_2}{\partial \beta_1} & \frac{\partial q_2}{\partial \beta_2} \end{pmatrix} = -\begin{pmatrix} \frac{\partial^2 \pi_1}{\partial q_1^2} & \frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} \\ \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_2}{\partial q_2^2} \end{pmatrix}^{-1} \begin{pmatrix} (\theta_1 + \theta)^2 & 0 \\ 0 & (\theta_2 + \theta)^2 \end{pmatrix} = -\frac{1}{\Delta} \begin{pmatrix} \frac{\partial^2 \pi_2}{\partial q_2^2} & -\frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} \\ -\frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_1}{\partial q_1^2} \end{pmatrix} \begin{pmatrix} (\theta_1 + \theta)^2 & 0 \\ 0 & (\theta_2 + \theta)^2 \end{pmatrix} = -\frac{1}{\Delta} \begin{pmatrix} \frac{\partial^2 \pi_2}{\partial q_2^2} (\theta_1 + \theta)^2 & -\frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} (\theta_2 + \theta)^2 \\ -\frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} (\theta_1 + \theta)^2 & \frac{\partial^2 \pi_2}{\partial q_1^2} (\theta_2 + \theta)^2 \end{pmatrix},
\]

37
where \( \Delta \equiv \frac{\partial^2 \hat{\pi}_i}{\partial q_i^2} \frac{\partial^2 \hat{\pi}_j}{\partial q_j^2} - \frac{\partial^2 \hat{\pi}_i}{\partial q_i \partial q_j} \frac{\partial^2 \hat{\pi}_j}{\partial q_j \partial q_i} \) is the determinant of the Jacobian of the system of first-order conditions. From Assumption 2, \( \Delta \) is strictly positive. It then follows that

\[
\frac{\partial q_i^*}{\partial \beta_i} = -\frac{1}{\Delta} \left( \frac{\partial^2 \hat{\pi}_j}{\partial q_j^2} (\theta_i + \theta)^2 \right) = \frac{1}{\Delta} \left( \frac{\partial^2 d_j}{\partial q_j^2} q_j + 2 \frac{\partial d_j}{\partial q_j} (\theta_i + \theta)^2 \right) > 0; \tag{23}
\]

\[
\frac{\partial q_j^*}{\partial \beta_j} = \frac{1}{\Delta} \left( \frac{\partial^2 \hat{\pi}_i}{\partial q_i^2} (\theta_j + \theta)^2 \right) = \frac{1}{\Delta} \left( \frac{\partial^2 d_i}{\partial q_i^2} q_i + \frac{\partial d_i}{\partial q_j} (\theta_j + \theta)^2 \right) < 0. \tag{24}
\]

The above signs follow from Assumption 1 and the fact that quantities are strategic substitutes. Note that the above derivatives do not depend on \( \beta \).

### A.4 Proof of Proposition 2

Following a similar logic as in the proof of Proposition 1, we totally differentiate (18) with respect to \( \beta_i \) and \( \beta_j \) in order to determine the slope of the reaction functions:

\[
\left\{ \frac{\partial^2 \hat{\pi}_i}{\partial q_i^2} \frac{\partial q_i}{\partial \beta_i} \right\}^2 + \frac{\partial^2 \hat{\pi}_i}{\partial q_i \partial q_i} \frac{\partial q_i}{\partial \beta_i} + \frac{\partial^2 q_i}{\partial \beta_i \partial \beta_i} - [(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)] + \frac{\partial^2 \hat{\pi}_i}{\partial \beta_i^2} \right\} d\beta_i +
\]

\[
\left\{ \frac{\partial^2 \hat{\pi}_i}{\partial q_j^2} \frac{\partial q_j}{\partial \beta_j} + \frac{\partial^2 \hat{\pi}_i}{\partial q_i \partial q_j} \frac{\partial q_i}{\partial \beta_j} + \frac{\partial^2 q_j}{\partial \beta_j \partial \beta_j} + \frac{\partial^2 \hat{\pi}_i}{\partial \beta_i \partial \beta_j} + \frac{\partial^2 \hat{\pi}_i}{\partial \beta_i^2} \right\} d\beta_j = 0.
\]

From (24), \( \partial^2 q_i / \partial \beta_i^2 \) and \( \partial^2 q_j / (\partial \beta_i \partial \beta_j) \) are also zero (incentives do not affect the slope of \( q_i^* \) with respect to \( \beta_i \) or \( \beta_j \)). Then, the above expression becomes

\[
\left\{ \frac{\partial^2 \hat{\pi}_i}{\partial q_i^2} \frac{\partial q_i}{\partial \beta_i} \right\}^2 + \left\{ \frac{\partial^2 \hat{\pi}_i}{\partial q_i \partial q_i} \frac{\partial q_i}{\partial \beta_i} + \frac{\partial^2 q_i}{\partial \beta_i \partial \beta_i} - [(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)] \right\} d\beta_i +
\]

\[
\left\{ \frac{\partial^2 \hat{\pi}_i}{\partial q_j^2} \frac{\partial q_j}{\partial \beta_j} + \frac{\partial^2 \hat{\pi}_i}{\partial q_i \partial q_j} \frac{\partial q_i}{\partial \beta_j} + \frac{\partial^2 q_j}{\partial \beta_j \partial \beta_j} + \frac{\partial^2 \hat{\pi}_i}{\partial \beta_i \partial \beta_j} + \frac{\partial^2 \hat{\pi}_i}{\partial \beta_i^2} \right\} d\beta_j = 0 \Leftrightarrow
\]

\[
\frac{d \beta_i}{d \beta_j} = -\frac{\frac{\partial^2 \hat{\pi}_i}{\partial q_i \partial q_i} \frac{\partial q_i}{\partial \beta_i} + \frac{\partial^2 \hat{\pi}_i}{\partial q_i \partial q_j} \frac{\partial q_i}{\partial \beta_j} + \frac{\partial^2 \hat{\pi}_i}{\partial q_j \partial q_j} \frac{\partial q_j}{\partial \beta_i} + \frac{\partial^2 \hat{\pi}_i}{\partial q_j \partial q_j} \frac{\partial q_j}{\partial \beta_j} - [(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)]}{\frac{\partial^2 \hat{\pi}_i}{\partial q_i^2} \frac{\partial q_i}{\partial \beta_i} \frac{\partial q_i}{\partial \beta_i} + \frac{\partial^2 \hat{\pi}_i}{\partial q_i \partial q_i} \frac{\partial q_i}{\partial \beta_i} \frac{\partial q_i}{\partial \beta_i} - [(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)]}.
\]

The denominator of \( d \beta_i / d \beta_j \) is the second derivative of the profit function with respect to
\(\beta_i\), which is negative due to the SOC of the principal’s problem. The sign of the numerator is ambiguous and we discuss the two effects (stemming from the two different terms in the numerator) in the main text.

**B Appendix: Strategic Managerial Compensation Levels**

We show that the expected total compensation levels of managers exhibit the same strategic properties as managerial incentives. To achieve this, we transform the firm’s problem into one in which compensation levels instead of incentives are being chosen. To begin with, apply the binding individual rationality (IR) constraint, 

\[E(t_i) = r + (\beta_i)^2[(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)]/2,\]

resulting in an expression for the manager’s incentives as a function of his expected total compensation:

\[\beta_i = \left(\frac{2(E(t_i) - r)}{(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)}\right)^{1/2}.\]  

(25)

The firm’s expected marginal cost becomes

\[E(c_i) = c - \left(\frac{2(E(t_i) - r)}{(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)}\right)^{1/2} (\theta_i + \theta)^2.\]  

(26)

The greater is the manager’s expected total compensation, the smaller is the firm’s marginal cost of production.

We consider Bertrand competition. The case with Cournot competition is similar, so it is omitted. For the sake of brevity, we do not state the equivalent assumptions that are required, which can be inferred from the corresponding cases with managerial incentives.

As before, firm \(i\)’s expected gross profit is given by 

\[\pi_i(p) = (p_i - (c - \beta_i(\theta_i + \theta)^2)) D_i(p).\]

Applying equation (26), it becomes

\[\pi_i(p) = \left(p_i - \left(c - \left(\frac{2(E(t_i) - r)}{(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)}\right)^{1/2} (\theta_i + \theta)^2\right)\right) D_i(p).\]  

(27)
The FOC with respect to the firm’s price is
\[
\frac{\partial \pi_i}{\partial p_i} = \left( p_i - \left( c - \frac{2(E(t_i) - r)}{(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)} \right)^{1/2} \right) \frac{\partial D_i}{\partial p_i} + D_i(p) = 0. \tag{28}
\]

The following lemma derives the manner in which compensation levels affect equilibrium prices:

**Lemma 3 (Compensation and Prices with Bertrand Competition)** The equilibrium price of firm \(i\) is decreasing in the expected total compensation of its manager, i.e. \(\partial p_i^*/\partial E(t_i) < 0\), and decreasing in the expected total compensation of its rival’s manager, since prices are strategic complements, i.e. \(\partial p_i^*/\partial E(t_j) < 0\).

**Proof.** Define the matrix
\[
M \equiv \begin{pmatrix}
\left( \frac{2(E(t_1) - r)}{(\theta_1 + \theta)^2 + R(\sigma_1^2 + \sigma^2)} \right)^{-1/2} \frac{\partial D_1}{\partial p_1} (\theta_1 + \theta)^2 & 0 \\
0 & \left( \frac{2(E(t_2) - r)}{(\theta_2 + \theta)^2 + R(\sigma_2^2 + \sigma^2)} \right)^{-1/2} \frac{\partial D_2}{\partial p_2} (\theta_2 + \theta)^2
\end{pmatrix}
\]

We invoke the Implicit Function Theorem:
\[
\left( \begin{array}{c}
\frac{\partial p_i^*}{\partial E(t_1)} \\
\frac{\partial p_i^*}{\partial E(t_2)}
\end{array} \right) = -\left( \begin{array}{cc}
\frac{\partial^2 \pi_1}{\partial p_1^2} & \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} \\
\frac{\partial^2 \pi_2}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi_2}{\partial p_2^2}
\end{array} \right)^{-1} M = -\frac{1}{\Delta} \left( \begin{array}{cc}
\frac{\partial^2 \pi_1}{\partial p_1^2} & -\frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} \\
-\frac{\partial^2 \pi_2}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi_2}{\partial p_2^2}
\end{array} \right) M
\]
\[
= -\frac{1}{\Delta} \begin{pmatrix}
\frac{\partial^2 \pi_1}{\partial p_1^2} \left( \frac{2(E(t_1) - r)}{(\theta_1 + \theta)^2 + R(\sigma_1^2 + \sigma^2)} \right)^{-1/2} \frac{\partial D_1}{\partial p_1} (\theta_1 + \theta)^2 & -\frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} \left( \frac{2(E(t_2) - r)}{(\theta_2 + \theta)^2 + R(\sigma_2^2 + \sigma^2)} \right)^{-1/2} \frac{\partial D_1}{\partial p_2} (\theta_1 + \theta)^2 \\
-\frac{\partial^2 \pi_2}{\partial p_2 \partial p_1} \left( \frac{2(E(t_1) - r)}{(\theta_1 + \theta)^2 + R(\sigma_1^2 + \sigma^2)} \right)^{-1/2} \frac{\partial D_2}{\partial p_1} (\theta_1 + \theta)^2 & \frac{\partial^2 \pi_2}{\partial p_2^2} \left( \frac{2(E(t_2) - r)}{(\theta_2 + \theta)^2 + R(\sigma_2^2 + \sigma^2)} \right)^{-1/2} \frac{\partial D_2}{\partial p_2} (\theta_2 + \theta)^2
\end{pmatrix},
\]
where \(\Delta \equiv \frac{\partial^2 \pi_1}{\partial p_1^2} \frac{\partial^2 \pi_2}{\partial p_2^2} - \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} \frac{\partial^2 \pi_2}{\partial p_2 \partial p_1}\) is the determinant of the Jacobian of the system of first-order conditions. The remainder follows as in the case with managerial incentives. \(\blacksquare\)

The intuition is the same as in the case with managerial incentives. If firm \(i\) enhances the compensation of its manager, this lowers the firm’s expected marginal cost and thereby price due to greater managerial effort, i.e. \(\partial p_i^*/\partial E(t_i) < 0\). Given the strategic complementarity of prices, the rival responds by lowering its price, \(\partial p_j^*/\partial E(t_i) < 0\).
We now derive the equilibrium set of expected total compensation levels. The principal’s objective is

\[ \pi_i^B - E(t_i^B) = \left( p_i^* (E(t_i^B)) - \left( c - \left( \frac{2 (E(t_i^B) - r)}{(\theta_i + \theta)^2 + R(\sigma_i^2 + \sigma^2)} \right)^{1/2} \right) \right) D_i (p^*(E(t^B))) - E(t_i^B) \]

\[ \equiv \pi_i^B (p_i^* (E(t_i^B)), p_j^* (E(t_j^B)), E(t_i^B), E(t_j^B)) - E(t_i^B). \]  

(29)

The FOC with respect to \( E(t_i^B) \) yields

\[ \frac{\partial (\pi_i^B - E(t_i^B))}{\partial E(t_i^B)} = \frac{\partial \pi_i^B}{\partial p_i^*} \frac{\partial p_i^*}{\partial E(t_i^B)} + \frac{\partial \pi_i^B}{\partial p_j^*} \frac{\partial p_j^*}{\partial E(t_i^B)} + \frac{\partial \pi_i^B}{\partial E(t_i^B)} - 1 \]

using (28) =

\[ \frac{\partial \pi_i^B}{\partial p_j^*} \frac{\partial p_j^*}{\partial E(t_i^B)} + \frac{\partial \pi_i^B}{\partial E(t_i^B)} - 1 = 0. \]  

(30)

As in the case with managerial incentives, there are four distinct channels through which the manager’s expected total compensation affects the principal’s objective. First, there is the term \( \frac{\partial \pi_i^B}{\partial p_i^*} \), which disappears since the firm chooses in a later stage of the game the price that maximizes gross profit. Second, there is the term \( \frac{\partial \pi_i^B}{\partial p_j^*} \), which captures the extent to which the firm’s gross profit is influenced by the impact of its manager’s compensation on its rival’s price (the "strategic" effect). Third, there is the term \( \frac{\partial \pi_i^B}{\partial E(t_i^B)} > 0 \), which measures the extent to which the manager’s effort reduces the firm’s expected marginal cost. Fourth, there is the term \(-1\), reflecting the direct cost associated with enhancing the manager’s compensation.

The following proposition derives the strategic properties of managerial compensation levels:  

**Proposition 3 (Strategic Compensation with Bertrand Competition)** The managerial compensation of firm \( i \) responds to a change in the managerial compensation of firm \( j \)

\[^{22}\text{For a unique equilibrium of expected total compensation levels } \{E(t_1^B), E(t_2^B)\} \text{ to exist, we require that } \left| \frac{dE(t_1^B)}{dE(t_2^B)} \right| < 1. \text{ With linear demand, this condition holds for a wide range of parameter values.}\]
as follows:

$$\frac{dE(t_B^i)}{dE(t_B^j)} = -\frac{\partial^2 \pi_i}{\partial p_j^2 \partial E(t_i) \partial E(t_j)} + \frac{\partial^2 \pi_i}{\partial p_j \partial p_i \partial E(t_j) \partial E(t_i)} \frac{\partial p_j}{\partial E(t_i)} + \frac{\partial^2 \pi_i}{\partial p_j \partial p_i \partial E(t_i) \partial E(t_j)} \frac{\partial p_j}{\partial E(t_i)} + \frac{\partial^2 \pi_i}{\partial p_j \partial p_i \partial E(t_j) \partial E(t_i)} \left( \frac{\partial p_j}{\partial E(t_i)} \right)^2 + \frac{\partial^2 \pi_i}{\partial p_j \partial p_i \partial p_i \partial E(t_i) \partial E(t_j)} \frac{\partial p_j}{\partial E(t_i)} \frac{\partial p_j}{\partial E(t_i)} \frac{\partial p_j}{\partial E(t_i)}.$$

(31)

The rest is the same as the statement in Proposition 1, where the three effects (similar to the three terms in the numerator of (31)) are presented and discussed.

**Proof.** We totally differentiate (30) with respect to both compensation levels in order to determine the slope of the reaction functions:

$$\left\{ \frac{\partial^2 \pi_i}{\partial p_j^2 \partial E(t_i)^2} + \frac{\partial^2 \pi_i}{\partial p_j \partial p_i \partial E(t_j) \partial E(t_i)} \frac{\partial p_j}{\partial E(t_i)} + \frac{\partial^2 \pi_i}{\partial p_j \partial p_i \partial E(t_i) \partial E(t_j)} \left( \frac{\partial p_j}{\partial E(t_i)} \right)^2 + \frac{\partial^2 \pi_i}{\partial p_j \partial p_i \partial p_i \partial E(t_i) \partial E(t_j)} \frac{\partial p_j}{\partial E(t_i)} \frac{\partial p_j}{\partial E(t_i)} \frac{\partial p_j}{\partial E(t_i)} \right\} dE(t_i) + \left\{ \frac{\partial^2 \pi_i}{\partial p_j^2 \partial E(t_i) \partial E(t_j)} + \frac{\partial^2 \pi_i}{\partial p_j \partial p_i \partial E(t_j) \partial E(t_i)} \frac{\partial p_j}{\partial E(t_i)} + \frac{\partial^2 \pi_i}{\partial p_j \partial p_i \partial E(t_j) \partial E(t_i)} \left( \frac{\partial p_j}{\partial E(t_i)} \right)^2 + \frac{\partial^2 \pi_i}{\partial p_j \partial p_i \partial p_i \partial E(t_i) \partial E(t_j)} \frac{\partial p_j}{\partial E(t_i)} \frac{\partial p_j}{\partial E(t_i)} \frac{\partial p_j}{\partial E(t_i)} \right\} dE(t_j) = 0.$$

The term $\partial^2 \pi_i / (\partial E(t_i) \partial E(t_j))$ is zero since the cost reduction in firm $i$ does not depend on the compensation of the rival firm. The remainder follows as in the proof of Proposition 1.

$\blacksquare$
References


