Is Multimedia Convergence To Be Welcomed?

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Abstract

This paper considers the consumer implications of the process of convergence across multimedia and telecoms markets. Convergence starts when one firm begins to sell products in hitherto separate horizontal markets competing against rivals active in just one or other of the markets. Convergence creates a strategic link between the markets which alters the price levels; creates the possibility of bundle prices; and creates winners and losers in the population. Partial convergence (e.g. a merged provider of telephony and internet services vs. independent sellers of telephony or internet broadband) lowers prices in the less competitive sector, raises them in the more competitive sector and raises the total prices paid by consumers active in both sectors as compared to the counter-factual of no convergence. Full convergence (e.g. multiple firms offering TV and internet bundles) leads to deep discounts for bundle purchases but no reductions in stand alone prices paid by consumers in only one of the converging sectors.

Keywords: Multimedia; Consumer Surplus; Bundling; Pay TV; Internet; Telecommunications; Convergence

JEL Classification D43, L13

1 Introduction

Across the globe the process of multimedia convergence is very much in the public eye. Sky (part of Rupert Murdoch’s News Corporation) and Virgin are two UK companies of substantial size which have recently begun competing aggressively head-to-head in multiple technology markets. Figure 1 illustrates the resultant triple-play offers including broadband, TV and fixed telephony

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at bundle discounts. Comcast of the US is following the same strategy. In other countries this process of convergence is yet to begin or only partially complete. This paper asks which consumers are the winners and which the losers from this convergence process. Further, is convergence led by short run economic profits or long run technology imperatives? In short for whom does convergence create value and for whom destroy it?

Figure 1: Adverts for bundles from the main two providers of pay TV and broadband in the UK: Sky and Virgin Media. Comcast in the US offers a similar package.

Convergence occurs when one firm, through merger or organic growth, begins to sell products in hitherto separate horizontal markets. An example would be a provider of Pay TV services acquiring a local telephony company or an internet provider. By converging the merged firm is in a position to strategically link the pricing between markets: adjusting the price levels and introducing bundle reductions.

Initially the merged firm may be competing against rivals who have yet to converge. I refer to this as a situation of partial convergence. If full convergence is reached then firms with full product lines compete head-to-head with their mixed bundle offers (Figure 1).

In building a model to understand the implications of convergence I explicitly note that the main source of differentiation is often at the firm level – and not at the product level. For example, in the case of convergence of TV and internet any given channel, such as CNN, appears the same on any firm which carries it. The same is broadly true for broadband internet access. However one firm may have better customer service or take less time to organize engineers. This is firm level differentiation. As differentiation is between firms, a consumer buying more
than one product from the same firm enjoys economies of scope. For example, buying pay TV and internet broadband from one provider allows one to deal with just one customer service department, receive one bill, establish one standing order, and have one process for the resolution of complaints.

To allow distributional effects of convergence to be most readily understood the model focuses on two types of consumer. Firstly there are ‘small’ consumers who are only interested in a subset of the products being converged: they are either in the market for good X or good Y. These separate markets may not be equally competitive. Secondly there are ‘large’ consumers who desire the whole set of products being converged: both X and Y. Large consumers harness economies of scope if they go to only one firm. Thus a merged firm can seek to target large and small consumers differentially using mixed bundle tariffs or by adjusting relative price levels; while independent rivals have fewer tools with which to respond to diverse potential consumers.

The analysis reveals that partial convergence lowers the consumer surplus of the ‘small’ customer who seeks to buy only the more competitive of the components. Partial convergence (a merged firm selling X and Y competing with separate X and Y sellers) links a more competitive good X with a relatively less competitive good Y. The strategic interaction created by the partial convergence ensures that the price of good X when sold alone rises. The price of the bundle of both products also rises - so the large buyers pay more under partial convergence than they did pre-convergence. However these consumers benefit from the economies of scope as they save on hassle and time costs. Consumers of good Y only (the less competitive good) are winners from partial convergence: their prices fall from all providers. Independent sellers of good Y wish to keep large and small consumers. But the large consumers, the ones buying both goods, see that X is more expensive and so the set of both would be too unless the independent Y seller responds by lowering her price. Independent sellers of Y are therefore forced to lower their prices to mitigate the X price rise which pulls the price of component Y down across the industry. The profits of the unmerged good Y seller fall as a result of partial convergence. Should the business involve substantial fixed costs this could force the unmerged firm out of business and so constitutes a theory of potential harm.

Only once full convergence is attained do the large buyers (purchasers of both goods X and Y) enjoy serious deep bundle discounts - their consumer surplus rises. However the small buyers see their prices revert to pre-convergence levels. By using sophisticated bundle tariffs the competing firms can separate the large from the small consumers. Competition is intense for the large consumers, but reverts back to its previous intensity for the small guy. Hence good Y only consumers who benefit from lower prices under partial convergence will see these gains removed
upon full convergence. But overall compared to the pre-convergence case, full convergence is to be welcomed: at least large buyers gain even if small buyers stand still.

The model is presented in Section 2; Section 3 analyses the process of convergence and Section 4 extends the model to endogenise the process of convergence and probe the model’s robustness. The final section concludes with proofs contained in the Appendix.

1.1 Convergence and the Bundling Literature

There is a substantial literature which explores the motivation a monopolist has to bundle goods. The literature offers two very clear insights. The first, captured by Adams and Yellen (1976) and since extended substantially, is that bundling can be used to lower the variance of valuations for those who buy both goods. This allows a bundle price to capture more of the surplus and so increase profits. The second insight is that volume discounts are a general part of non-linear pricing (Mussa and Rosen 1978).

The analysis in this paper abstracts from these price discrimination related motivations for bundling. Instead we consider the strategic rationale for bundling and how it pertains to the process of convergence. It is known that tying can be used by a dominant firm with a monopoly in one market to leverage the market power across into another market (Whinston (1990), Carbajo et al. (1990), Carlton and Waldman (2002) and Nalebuff (2004)). However our interest is in the case in which no firm is a monopolist in one of its markets and bundling can be matched by rivals. Bundling in such competitive markets has been taken up in fewer papers. And an analysis applicable to the process of convergence is, to my knowledge, offered in only two other papers: Nalebuff (2000) and Granier and Podesta (2008).

Nalebuff (2000) considers demand for a system of products: all buyers require all components. As a result the distributional question of surplus amongst differing groups of consumer which we analyse here cannot be addressed. The Nalebuff analysis considers differentiation at the product and not firm level. This implies that consumers derive no economies by purchasing from one firm, beyond any bundled price reduction which might be offered.¹ Nalebuff analyses how strategic competition would play out between a (converged) firm offering only the bundle of components against independent providers of differentiated versions of each component. The bundle seller has the profit advantage as a result of the coordination it has as to the price of the overall bundle. The rival component sellers do not internalise the losses of the other parts of the bundle they are sold with and so push prices above the merged firm, so losing market share.

In our paper this price result applies for the less competitive market, but not for the more

¹The model is motivated by Microsoft Office rather than the multimedia examples we discuss.
competitive market where prices fall. Indeed the price falls most for the independent seller. Secondly we have captured the effect on the equilibrium prices of the consumer economies of scope which the subset of buyers who buy both components receive. Thus we are able to document who the winners and losers will be from the process of convergence.

Granier and Podesta (2008) continue the investigation in Nalebuff (2000) by also focusing on product (not firm) differentiation and assuming all consumers require the bundle. They extend the analysis to allow consumers to differ in their correlation between valuations for the two goods.

The economic literature has more to say if full convergence is reached in an industry. Bundling between duopolists in the case of product specific, rather than firm specific, preferences are analysed in among others, Matutes and Regibeau (1992) and Reisinger (2006). However these papers assume that all consumers buy the bundle preventing a distributional analysis between different consumer groups. Further differentiation is at the product level so that the economies of scope inherent in the convergence processes we have discussed cannot be captured.

Thanassoulis (2007) considers bundling between duopolists allowing for different consumer groups and firm-specific differentiation. There it is shown that, in the full convergence setting, à la carte pricing (no bundling) would increase consumer surplus as compared to the firms being able to offer mixed bundles.\(^2\) However the process of convergence is not analysed and so its distributional effects cannot be addressed.

2 The Model

Consider a process of convergence in the market for two goods: \(X\) and \(Y\). Firms \(X_1\) and \(X_2\) both sell the same good \(X\) only which they can produce at the same constant marginal cost. Firms \(Y_1\) and \(Y_2\) both sell good \(Y\) only, with again neither having an a priori cost advantage over the other. We will analyse three different ownership structures for these 4 firms. The first will be the pre-convergence case with all four firms competing separately. Next we will analyse partial convergence in which firms \(X_2\) and \(Y_2\) have merged to form firm 2 which competes with separate firms \(X_1\) and \(Y_1\). Finally we will analyse full convergence in which \(X_1\) and \(Y_1\) also merge to form firm 1 which competes with the (merged) firm 2.

The firms are differentiated at the firm and not product level. Thus the four firms all compete

\(^2\)Armstrong and Vickers (forthcoming) extend the analysis of Thanassoulis (2007) to the case of general consumer demand as opposed to large versus small buyers. The results are all confirmed in this richer setting; though the identity of the winners and losers is now arguably made less transparent.
in the market for customer service where they offer a differentiated service. The introduction
argued that this is a close description for many multimedia sectors which are converging. For
example in the case of TV: ESPN is identical on rival platforms, but firms are differentiated by
items such as their customer service and support, speed of engineer callout, clarity of bill etc.
The market for customer service is modelled as a Hotelling line of unit length. Firms X1 and
Y1 are located at point 0. While firms X2 and Y2 are located at opposite ends of the line at
coordinate 1.

We suppose that consumers are uniformly distributed along the Hotelling line. To study
the effect of convergence on consumer surplus generally we introduce three different types of
consumer. Thus at each location \( \theta \) along the Hotelling line there exist consumer types \( AX, AY \)
and \( B \).

The proportion \( AX \) of consumers are only in the market for one unit of good \( X \). The strength
of competition in the good \( X \) market is captured by the taste parameter \( \lambda_X \). Thus an \( AX \) type
consumer located at \( \theta \) who buys from firm X2 incurs a taste cost of \( \lambda_X (1 - \theta) \).

Similarly the proportion \( AY \) of consumers desire one unit of good \( Y \). The parameter \( \lambda_Y \)
captures the strength of competition in the good \( Y \) market. We assume that \( \lambda_X \leq \lambda_Y \) so that
good \( X \) is the more competitive of the two markets (the firms are seen as closer substitutes).

All other consumers, a proportion \( B \), desire the bundle of both good \( X \) and good \( Y \). These
are the ‘large’ consumers who purchase both of the converging technologies. Suppose, in the
pre convergence case, that a \( B \) type consumer located at \( \theta \) decides to visit the separately owned
firms X1 and Y1 to purchase the two required goods. In this case the consumer incurs taste
cost \( \lambda_X \theta + \lambda_Y \theta \).

In the case of convergence the taste costs of \( AX \) and \( AY \) consumers are unaffected. However
we capture that consumers enjoy economies of effort or scope in buying multiple goods from
only one firm (one required connection and engineer callout, one bill etc.) Thus the taste cost
parameter for a \( B \) consumer buying two goods from one firm is denoted \( \lambda_B \) and we assume that
\( \lambda_B < \lambda_X + \lambda_Y \). Figure 2 depicts a \( B \) type consumer choosing between going to the merged firm
2 versus buying \( X \) from X1 and \( Y \) from the independent Y1.

The assumption that \( X2 \) should merge with \( Y2 \) is motivated by the observation that firms
contemplating merger are likely to prefer merger partners whose products are attractive to their
core customers. Finally, in our benchmark analysis we assume that the merger does not make
the firm specific disutility any worse than either of the two merging parties. That is we have

\[
\lambda_B = \max \{ \lambda_X, \lambda_Y \} \tag{1}
\]

This would be so in the case of TV/internet if, for example, convergence via merger does not
3 Analysis Of The Process Of Convergence

3.1 The Pre Convergence Case

As a benchmark we analyse the market outcomes before any convergence has occurred. \( X \) and \( Y \) are sold by separate firms, and the large \( B \) type buyers must visit at least two firms. There is therefore no strategic link between the good \( X \) and \( Y \) markets. We therefore have standard Hotelling competition for both \( X \) and \( Y \). The following result is derived by standard analysis (so its proof is omitted):

\[ \text{Proposition 1} \quad \text{In the pre convergence case there is a unique equilibrium. In this equilibrium:} \]

1. Firms \( X1 \) and \( X2 \) charge a margin of \( \lambda_X \) over cost. Firms \( Y1 \) and \( Y2 \) charge a margin of \( \lambda_Y \) over costs. Consumers split themselves between the firms symmetrically as prices are equal.

2. Type \( B \) consumers pay a margin of \( \lambda_X + \lambda_Y \) above cost.

3. Firm \( X1 \) makes a profit of \( (AX + B) \frac{1}{2} \lambda_X \). Similarly for the other firms.
3.2 Partial Convergence

Suppose that $X_2$ and $Y_2$ have merged to form firm 2. This firm has converged - but is in competition with separate firms $X_1$ and $Y_1$. We denote the margin above cost charged for $X$ by firm $X_1$ as $\rho_X$, while $Y_1$ charges margin $\rho_Y$. Partial convergence provides the potential for firm 2 to offer a bundled tariff whereas the competition cannot. The merged firm 2 therefore sells $X$, $Y$, or the bundle of both goods at margins $\{x, y, b\}$.

Given margins set by the firms, the indifferent $AX$ type consumer will be located at $\hat{\theta}_X$, and similarly the indifferent $AY$ type consumer at $\hat{\theta}_Y$ where

$$2\lambda_X \hat{\theta}_X = \lambda_X + x - \rho_X \quad \text{and} \quad 2\lambda_Y \hat{\theta}_Y = \lambda_Y + y - \rho_Y \quad (2)$$

A $B$ consumer who goes to firm 2 to buy both goods gains an economy of scale by incurring taste cost proportional to $\lambda_B = \max \{\lambda_X, \lambda_Y\}$. To facilitate the determination of the equilibrium we first solve for the equilibrium under the following no-hybrid purchases assumption. We will then show that the strategies which support the equilibrium are robust to relaxing the assumption. Thus the equilibrium found will hold generally.

**No-hybrids assumption** In equilibrium the ‘large’ $B$ type consumers either go to 2 or buy the set $\langle X_1, Y_1 \rangle$.

Thus in equilibrium (and in small deviations around equilibrium) we start by assuming that the consumers do not form hybrid bundles combining a product from 2 and a product from either $X_1$ or $Y_1$. We will subsequently drop this assumption and show that it is without loss of generality - there can be no equilibrium in this model in which type $B$ consumers strictly prefer to form hybrid bundles. Under the no hybrids assumption the marginal type $B$ consumer between 2 and $\langle X_1, Y_1 \rangle$ is at $\hat{\theta}_B$ where

$$\hat{\theta}_B (\lambda_X + \lambda_Y + \lambda_B) = \lambda_B + b - \rho_X - \rho_Y \quad (3)$$

The market shares can be used to derive the firms’ profit functions:

$$\Pi_2 (x, y, b) = AX \cdot x \left(1 - \hat{\theta}_X\right) + AY \cdot y \left(1 - \hat{\theta}_Y\right) + B \cdot b \left(1 - \hat{\theta}_B\right) \quad (4)$$

$$\Pi_{X1} (\rho_X) = \rho_X \left[AX \cdot \hat{\theta}_X + B \cdot \hat{\theta}_B\right] \quad (5)$$

The profit function for $Y_1$ is analogous to $\Pi_{X1}$. The firms seek to maximise their profits. The strategic competition can be solved to yield:

**Lemma 1** Under the no-hybrids assumption the price equilibrium of the market is given by:
1. Firm 2 does not offer a special bundled tariff and sets margins equal to

\[ x = \frac{1}{2} (\lambda_X + \rho_X) \quad ; \quad y = \frac{1}{2} (\lambda_Y + \rho_Y) \quad \text{(6)} \]

\[ b = \frac{1}{2} (\lambda_X + \lambda_Y + \rho_X + \rho_Y) = x + y \]

2. Firms X1 and Y1 set margins equal to

\[
\begin{pmatrix} 
\rho_X \\
\rho_Y 
\end{pmatrix} = \begin{pmatrix} 
\lambda_X \\
\lambda_Y 
\end{pmatrix} + \frac{q}{2} \left( \frac{\beta (\lambda_Y - \lambda_X)}{4(\alpha_X \alpha_Y + \beta (\alpha_X + \alpha_Y))} + 2 \beta^2 \right) \begin{pmatrix} 
\frac{3}{2} (\alpha_Y + \beta) \\
-\frac{1}{2} \beta 
\end{pmatrix} \quad \text{(7)}
\]

where

\[ \alpha_X = \frac{AX}{2\lambda_X} \quad ; \quad \alpha_Y = \frac{AY}{2\lambda_Y} \quad ; \quad \beta = \frac{B}{\lambda_X + 2\lambda_Y} \quad \text{(8)} \]

**Proof.** See Appendix for all proofs.

It may appear surprising that the merged entity (2) does not seek to offer a price discount for bundle good consumers. To see the intuition behind the result note that as type \( B \) consumers do not form hybrids (initially by assumption) firm 2 is able to target \( B \) consumers with the price \( b \) and the \( AX \) consumers with the price \( x \) (and similarly for price \( y \)). Thus the prices \( x \) and \( b \) serve two different populations and these prices can be set with reference to the elasticities of demand of each of these populations.

Take as a benchmark that 2 offers no bundle reduction and sets \( b = x + y \). By virtue of the economies of scope \( B \) consumers enjoy in buying from a merged firm, 2 will serve a greater proportion of large consumers than it does of the small \( A \) type consumers. This will act to lower the elasticity of demand (defined as \( \frac{\delta q}{\delta p} \)) as it raises volumes \( (q) \). Hence this elasticity effect for the \( B \) consumers is a force for a higher bundle price as any price reductions are felt on a larger volume.

However a small price reduction wins sales of both components \( X \) and \( Y \) from large consumers, and not just one component. That is \( \frac{\delta q}{\delta p} \) is larger in magnitude as two goods are sold to \( B \) types for a given price reduction, as compared to one good to an \( AX \) type. This acts to raise the elasticity of demand and so is a force for a lower bundle price.

These two effects work in opposite directions and so keep any bundle price reduction small. In the case of the standard Hotelling model with uniform consumer demand these two effects exactly offset each other and so the bundle price exactly equals the sum of the pure component prices.

Note that though prices are additive, the large \( B \) type consumers perceive a sub-additive bundle overall as they incur lower taste/transport costs from the bundle purchase. Lemma 1 makes the point that if a large converged firm is competing with independent rivals when
differentiation is at the firm level, then the economies of scope generated for the consumer by dint of convergence are sufficient to maximise profits. There is no need to reduce value by introducing large bundle price reductions.

A role for some bundle price reductions would reappear if we relaxed the simplification that small AX, AY consumers cannot be upgraded to become purchasers of both products at a sufficiently tempting price. It is a simple corollary of the price discrimination literature that in this case a seller would offer volume discounts to induce those with marginal valuations to buy both products X and Y. The volume discount motivation is well understood (Mussa and Rosen 1978) and operates independently of the strategic environment. We have instead shown that there is no strategic reason, in this model for sub-additive bundling.

To complete our analysis of the partial-convergence market outcomes we now drop the no-hybrid assumption. The economies of scope B consumers enjoy in buying both their component products from one firm ensure that the analysis is unchanged:

**Lemma 2** Under partial convergence:

1. There can be no equilibrium in which B consumers strictly prefer to form hybrid bundles combining goods from 2 with X1 or Y1.

2. Any interior equilibrium must satisfy Lemma 1.

3. An interior equilibrium exists at least locally if the margins captured in (7) satisfy

\[
(2\lambda_Y - \lambda_X)\rho_X - 2\lambda_X\rho_Y < \lambda_X^2
\]  

This condition is satisfied in the special case of the component good markets being equally competitive \((\lambda_X = \lambda_Y)\).

The fact that large B consumers choose not to form hybrid bundles is not surprising. Multimedia markets are often casually referred to as one-stop shopping markets. This model generates one-stop shopping behaviour endogenously. It is possible for the market outcome to be a corner solution however in which prices adjust so that B consumers are indifferent between forming a hybrid bundle and buying both goods at 2. Condition (9) gives a sufficient condition for this not to be the case. In the rest of the paper I restrict attention to parameters which allow for a non-corner (interior) equilibrium - this includes the special case of the X and Y markets being equally competitive \((\lambda_X = \lambda_Y)\).
3.2.1 Market Analysis

The distributional and profit effects of moving from no convergence to partial convergence can now be established:

**Proposition 2** Comparing partial convergence achieved by the merger forming 2 to the no convergence benchmark we have:

<table>
<thead>
<tr>
<th></th>
<th>AX consumers</th>
<th>AY consumers</th>
<th>B consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prices</strong></td>
<td>↑</td>
<td>↓</td>
<td>↑</td>
</tr>
<tr>
<td><strong>Firm 2 market share</strong></td>
<td>↑</td>
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<td>↑</td>
</tr>
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</table>

And partial convergence lowers the profits of (unmerged) firm Y1.

Proposition 2 shows that the strategic link created by partial convergence between markets X and Y creates winners and losers in the population. Partial convergence leads to lower consumers surplus (higher prices) for consumers in the more competitive sector, coupled with higher consumer surplus (lower prices) for consumers in the less competitive sector. The large B type consumers see higher retail prices whomever they buy from. These higher prices will be in part offset by the reductions in the taste cost which the merger has afforded for those who buy from 2. Overall the consumer surplus outcome for B consumers is ambiguous.\(^3\) The rest of this section tries to explain the economic intuition behind these outcomes.

We begin our analysis of the strategic effects of partial convergence by first turning to firm X1 which sells the more competitive of the two goods. As a result of the merger creating firm 2, B consumers who purchase from X1 have a lower elasticity of demand than the AX consumers \((\lambda_B = \lambda_Y \geq \lambda_X)\). If they were to transfer to buying from 2 then they would, given the economies of scope, also move their good Y purchases to 2. Thus the B type consumers see a greater differentiation between \(X1, Y1\) and 2 than the small AX consumers see between X1 and 2. Therefore the partial convergence has lowered the aggregate elasticity of demand faced by firm X1 and so she would raise her price. This price rise leads firm 2 to also raise its component X price, though not by as much. This follows as prices are strategic complements. Thus AX consumers pay more and 2 wins a greater market share here even though it raises its price to these consumers.

Next consider firm Y1. Both B and AY consumers incur the same taste cost in moving to 2 due to the fact that Y was the less competitive product and type B consumers harness economies of scope in going to 2 \((\lambda_B = \lambda_Y)\). However the competitive position of Y1’s product...
as regards the large $B$ consumers has been damaged by $X1$’s price rise. As $B$ consumers who buy from $Y1$ also buy from $X1$, and $X1$ has raised her price, the demand from $B$ consumers is diminished. To counter this effect $Y1$ must counteract some of the price rise from $X1$ by lowering her price. She doesn’t counteract it all as such a price drop is loss making for the small $AY$ type consumers. On balance therefore $Y1$ lowers her price, but not by as large a margin as $X1$ raised hers.

Returning to firm 2, the $AY$ consumers see a price reduction from firm $Y1$ and so 2 responds by lowering her component $Y$ price. Thus $AY$ consumers benefit. The sum of the prices from $X1$ and $Y1$ has risen above the pre-convergence level. Therefore 2’s optimal response is to also allow her bundle price to rise. Therefore type $B$ consumers pay higher prices, however those that buy from 2 save taste costs (harness convenience savings) from the economies of scope arising from the fact that we have firm specific (rather than product specific) differentiation.

Finally the strategic link between markets $X$ and $Y$ has forced firm $Y1$ to lose market share amongst the large $B$ buyers, and to lower the prices charged to $AY$ buyers so that profits unambiguously decline. If $Y1$ incurs large fixed costs of operation then clearly $Y1$ could be driven out of business by the merger of 2.

Figure 3 explores one numerical example and analyses the consumer surplus implications for different relative competitiveness values of markets $X$ and $Y$. The graph illustrates that large $B$ consumers can lose or gain from partial convergence in consumer surplus terms. Similarly the figure illustrates that the overall population consumer surplus effects can also either go up or down. So though partial convergence leads to winners and losers, an overall ranking between the two states is not available.

### 3.3 Full Convergence

We now consider the case of full convergence: $X1$ and $Y1$ merge to form firm 1 selling both goods $X$ and $Y$ which competes with the merged firm 2. Both firms are now in a position to strategically link the two markets. As a result we will see that both firms will offer deeply discounted bundle prices to those buying both goods:

**Lemma 3** The unique symmetric equilibrium in the case of full convergence is given by both firms 1 and 2 setting margins

$$x = \lambda_X, \quad y = \lambda_Y, \quad b = \lambda_B := \lambda_Y$$

(10)

Under full convergence the competing firms are in a position to sub-additively bundle and so target different consumer groups with different prices. The $B$ consumers do not form hybrids
Figure 3: A graph of the consumer surplus changes from partial convergence as compared to the benchmark of no convergence. The proportions of consumers assumed for this numerical simulation were $AX = 0.2 = AY$, $B = 0.6$. For a range of values of $\frac{\lambda_X}{\lambda_Y}$ lying within $(0, \frac{1}{2})$ condition (9) is broken implying that the equilibrium is given by a corner solution.

due to the economies of scope that they enjoy by purchasing both products from a single firm; so the three consumer types are strategically separated. The small $AX$ and $AY$ consumers therefore receive the pre-convergence price.

The greater the economies of scope $B$ consumers harness in purchasing from one firm, the less differentiated the competing firms are perceived to be. In the partial convergence case a $B$ consumer forming the bundle $\langle X1, Y1 \rangle$ had to incur taste cost of $(\lambda_X + \lambda_Y)$ times distance. With full convergence this is lowered to $\lambda_B$. This reduction in differentiation between the two bundles acts to push the bundle price down below the pre-convergence sum of the component prices. Under condition (1) generated by assuming that merger does not damage a firm beyond the worse of the merger partners, the more competitive of the goods ($X$) is sold as an upgrade at cost (with no added margin) if the less competitive good ($Y$) is purchased.

**Proposition 3** Comparing all three possible convergence scenarios:
Partial convergence benefits large B consumers as compared to the pre-convergence benchmark. However small consumers gain nothing once convergence has run its course as the mixed bundled tariffs separates these consumers from the large B type. The interim of partial convergence leads to bundle prices rising and the price of hitherto competitive components rising also as compared to the pre-convergence benchmark. However recall that bundle consumers gain through the convenience of having one vendor (lower incurred taste costs) so that their overall surplus change is ambiguous. The purchasers of the hitherto less competitive good (Y) see lower prices under partial convergence.

4 Analysis Of Model Extensions

4.1 Endogenising Convergence

Is the partial convergence stage only transitory? Is full convergence in the short run profit interests of the firms? This question is important as the main negative effects on consumers arose under partial convergence (higher prices for one component and the bundle, reduced profits for one of the unmerged rivals resulting in the possibility of predation). This section asks whether under partial convergence the unmerged rivals X1 and Y1 would decide to merge also, moving the industry to full convergence. I explore this question by endogenising the convergence process. We will see that partial convergence – with all its negative consumer surplus outcomes – is unfortunately an equilibrium outcome.

This section will therefore show that full convergence is not in the short run profit interests of the competing firms. This result begs the question of why convergence is seen in some markets (such as the UK). We discuss this below. First consider the following two stage game.

Stage 1. Firms X1 and Y1 decide if they wish to merge. Simultaneously firms X2 and Y2 also decide if they would like to merge.
Stage 2. The independent firms compete non-collusively in prices.

I restrict attention to the leading special case of goods $X$ and $Y$ being equally competitive ($\lambda_X = \lambda_Y := \lambda$). Lemmas 1 and 3 and Proposition 1 allow us to derive the following pay-off matrix:

\[
\begin{array}{c|cc|c|c}
 & \text{Don’t merge} & \text{(joint profit)} & \text{Merge} \\
\hline
\text{Don’t merge} & \frac{1}{2} & AX \cdot \lambda + AY \cdot \lambda + B \cdot 2\lambda & AX \cdot \frac{1}{2} + AY \cdot \frac{1}{2} + \frac{2}{5}B \cdot 2\lambda \\
\text{Merge} & AX \cdot \frac{1}{2} + AY \cdot \frac{1}{2} + \frac{2}{5}B \cdot 2\lambda & AX \cdot \frac{1}{2} + AY \cdot \frac{1}{2} + \frac{1}{5}B \cdot 2\lambda & \frac{1}{2} \\
\end{array}
\]

Inspection of the game matrix allows us to rank the profits of each game outcome:

\[
\begin{align*}
[\Pi_2]_{\text{full convergence}} & < [\Pi_{X2} + \Pi_{Y2}]_{\text{partial convergence (1 merged)}} \\
& < [\Pi_{X2} + \Pi_{Y2}]_{\text{no convergence}} < [\Pi_2]_{\text{partial convergence (2 merged)}}
\end{align*}
\]

Hence we have:

**Proposition 4** The two stage convergence game has one type of pure strategy equilibrium: partial convergence.

If one pair of firms merges they capture two strategic advantages over their unmerged rivals. Firstly they are able to offer large $B$ consumers economies of scope and so grow their market share amongst this group with no price reductions. The second effect is that the merged firm can coordinate targeted pricing at the different subgroups in the population. Both of these advantages lead to the merged firm growing their profit as compared to their unmerged rivals. Hence no convergence is not a Nash equilibrium as partial convergence dominates it.

The unmerged firm would not rather merge herself in response as full convergence leads to intense bundle on bundle competition for the large $B$ buyers. The price to these large buyers therefore plummets which lowers everyone’s profits. On the other hand remaining separate creates a strategic effect: as the independent firms do not internalise the other’s gain from large $B$ type buyers, they do not overall price as aggressively as they would do if they merged. As
competition is in prices which are strategic complements this also eases some of the competitive pressure created by the merged firm. Thus it prevents profits falling to fully merged levels.

Hence partial convergence is the unique pure strategy equilibrium of the dynamic convergence game. As a result there are advantages to being the first firm to merge as the greater share of profits will then be secured.

This result is seemingly at odds with the situation in the UK which has arguably achieved full convergence. In the UK Rupert Murdoch’s Sky and Virgin Media compete head to head with multimedia bundles across the whole country. The competition is characterised by the deep bundle discounts predicted by this model (see Figure 1 and refer to Lemma 3). This model would suggest that this mode of competition is going to be value destroying with larger (short run) profits available to both if one of the firms dropped out of the bundle market.

There may be a number of reasons, outside the current model, which might make convergence an imperative not withstanding the short run profit loss. Perhaps leading amongst these are future research on product development. If product development requires interaction between the different digital technologies for example then it may be impossible for either firm not to converge. Further, in general firms might be facing competition from many both merged and unmerged rivals. Thus the optimal strategic response would depend upon who was the closest strategic competitor. This argument is potentially applicable to the US where the main multimedia providers (Comcast, Time Warner etc.) compete in some States and not in others. Next coordination problems might make playing the asymmetric pure strategy equilibrium difficult. That is firms might wish not to coordinate and instead fight a war of attrition in the hope that the rival will withdraw.

In the UK the current state of competition between Virgin Media and Sky appears to have the feel of a war of attrition which Virgin Media appears to be losing. As evidence for this consider the (indexed) share prices graphed in Figure 4. Since merging with a cable operator in early 2006 Virgin Media has seen its share price tumble as compared to Sky’s. It is possible that future R&D might save the situation for Virgin Media. However the war of attrition might be running its course with most recently Sky making an offer to buy up much of Virgin’s programming output.\footnote{There is of course also a mixed strategy equilibrium. In this case there is a positive probability of miscoordination in which case full convergence can result.}

Figure 4: The indexed share prices of Sky and Virgin Media. Both share prices set at 100 on 1st June 2006. Source: Thomson Datastream.

4.1.1 Aggressive Bundling

The dynamic game in this section offers a new theory of potential harm arising from bundling. It is known that tying can be used to force rivals out of business. However the literature has explained this by considering a dominant firm which sells two products competing with firms selling only one of the products. By bundling the dominant firm is able to credibly become more aggressive and so can force a rival out of business (Whinston (1990) and Nalebuff (2004)). Here however no firm has an exogenous advantage. If one firm merged and sought to practice aggressive bundling then rivals could merge also. However, we have shown that such a response would not be profitable. If one converged firm formed, rival firms would rather remain independent than converge also. This is purely the result contained in Proposition 4 that partial convergence is the pure strategy equilibrium state. In addition we have shown through Proposition 2 that partial convergence lowers the profits of the unmerged firm in the less competitive sector (firm Y1). If the profit reduction was sufficient to make Y1 unable to pay her fixed costs then she would be forced to exit - to the detriment of consumers. Thus bundling can have anticompetitive effects, even in this ex ante symmetric model.
4.2 Multiple Products

Suppose, more generally, that within each of markets $X$ and $Y$ there are multiple components which might be bought. This section will address how our results extend into this more complicated setting. Further, a natural question of interest is whether convergence will distort the set of products that consumers buy. This section indicates that the pricing equilibrium in this richer setting is for firms to use two part-tariffs. The variable price is set at cost while the fixed fee allows the firms to recoup margins equal to those determined in the core model. Thus we will see that convergence does not distort consumption patterns of those who purchase in equilibrium; and the consumer surplus implications of convergence remain as in the core analysis.

To develop the reported insights I augment the core model as follows. Suppose that there are $N$ different goods within market $X$. Thus a consumer’s purchases can be denoted by the vector $X = (X_1, ..., X_N)$ with $X_i$ denoting the quantity of good $i$ from market $X$ purchased. In the case of pay TV $X_i$ would be one of the channels which could be purchased. We suppose that consumers one-stop shop for goods within a given market, but can source from one firm for market $X$ goods and a different firm for market $Y$ goods. The cost of serving a consumer with demand $X$ is

$$k_X + c_X \cdot X$$

which captures a per consumer fixed cost of $k_X$ plus the constant marginal cost per good selected. The costs are the same across firms. The costs differ between markets $X$ and $Y$. We allow firms to set two-part tariffs.

As before, type $AX$ consumers are uniformly distributed between $X1$ and $X2$, and incur taste costs proportional to $\lambda_X$ in purchasing from either firm. Independently of location, $\theta$, the $AX$ consumers have a second type variable: $\varphi_X$. This captures a preference between the types of good in market $X$. The consumer has gross utility of $U(\varphi_X, X)$ if she buys $X$. Let $v_{AX}(\varphi_X)$ denote the maximal surplus available to this consumer arising from marginal cost pricing:

$$v_{AX}(\varphi_X) = \max_X \{U((\varphi_X, X)) - c_X \cdot X\}$$

The $AY$ consumers are modelled similarly in their purchases from market $Y$. Type $B$ consumers buy goods from both markets $X$ and $Y$ and so have gross utility of $U(\varphi_B, X, Y)$. The utilities of all the consumers are assumed high enough that all consumers take part by purchasing some volume of at least one good in equilibrium. This model is a natural extension of Armstrong and Vickers (2001).

**Lemma 4** The margins found in the core model (Proposition 1 and Lemmas 1 and 3) are equilibria of the model with multiple goods in each market. Thus the price charged by firm $X1$
is given by

\[ p_X(X) = [\text{Fixed cost per consumer}] + [\text{Variable costs}] + [\text{margin from our core model}] \]

\[ = k_X + c_X \cdot X + [\text{margin from our core model}] \]

And analogously for the other firms in all convergence states.

**Proof.** That Proposition 1 (pre-convergence) extends to this setting is a corollary of Armstrong and Vickers (2001 Proposition 5). If one assumes that B consumers one-stop shop then Lemma 3 (full convergence) is also confirmed by Armstrong and Vickers (2001 Proposition 5). The equilibrium is robust to relaxing the one-stop shop assumption as the market is split equally between the two merged firms and so no B consumer would prefer to form a hybrid bundle by splitting purchases between the two sellers as she would unambiguously raise her taste costs.

For the case of partial convergence we proceed as follows. Suppose firms could observe each consumers’ type, though not her location, and suppose that the firms are charging tariffs as in the lemma. We aim to show no firm has an incentive to deviate from the proposed tariffs. The firms are competing in utilities to attract consumers. Therefore the optimal response to the rivals’ prices will be to set marginal prices at cost, thus maximising gross utility, and then extract the desired amount of surplus as a fixed fee given a desired utility level. With such a tariff the AX consumers have maximal utility given by \( v_{AX}(\varphi_X) \) no matter who they buy from, and analogously for AY and B. Thus the problem is identical to the core model and so the margins offered from the core model maximise profits. Now note that these prices are independent of the observed types \( \{\varphi_X, \varphi_Y, \varphi_B\} \) and so the result holds for all types as required.

Therefore two part tariffs prevent participating consumers having to accept sub-optimal volumes: the degree of convergence has no effect on the marginal prices. The fixed fees consumers pay however behave exactly as in the core model. Partial convergence raises the fixed fee for those buying only products X (more competitive market), and lowers it for Y consumers (less competitive market) and makes no difference for B consumers.

**4.3 General Consumer Economies of Scope**

The analysis so far has assumed that when a firm merges it offers a no worse level of service than the worse of the two merging firms: this generated condition (1). We now make the economies of scope general replacing (1) by the assumption

\[ \max(\lambda_X, \lambda_Y) \leq \lambda_B \leq \lambda_X + \lambda_Y \]  

The pre-convergence case is unaffected as with no converged firm no consumer is in a position to harness economies of scope. Let us therefore turn to the case of partial convergence. Here the
only possible interior equilibrium remains one in which \( B \) consumers do not form hybrid bundles. The more competitive good \( (X) \) rises in price as does the bundle as above. However whereas before the component good \( Y \) fell in price, now it doesn’t fall as far. And if the economies of scope are sufficiently modest \( (\lambda_B \text{ close to } \lambda_X + \lambda_Y) \), the component price of \( Y \) may actually rise as compared to the pre-convergence benchmark. Thus quite possibly all consumers pay more for the goods if the industry takes the form of partial convergence. We collect these results in the following proposition which is proved in the appendix.

**Proposition 5**  Suppose that in the case of partial convergence, the large \( B \) type consumers have economies of scope given by (11). In this case:

1. With pre-convergence Proposition 1 is unaffected.

2. Under partial convergence:

   (a) There can be no equilibrium in which \( B \) consumers form hybrid bundles.

   (b) Proposition 2 holds for consumers \( AX \) and \( B \).

   (c) Prices to consumers \( AY \) are higher than given in Lemma 1 and may lie above the pre-convergence case.

3. Under full convergence, Proposition 3 holds except for the ranking of the partial convergence \( Y \) prices.

Recall that for the \( Y1 \) firm, partial convergence has linked the two markets and so her type \( B \) consumers now consider both the price of good \( Y1 \) and of good \( X1 \) when deciding whether to buy or to instead swap to the merged firm and harness the economies of scope. Firm \( X1 \) raises its price due to the same intuition as given after Proposition 2. This creates pressure on \( Y1 \) to lower her price so as not to lose too many \( B \) consumers. This was the force described above which resulted in good \( Y \) prices falling under assumption (1) on the economies of scope. However, as the economies of scope become more modest the attraction of going to the merged firm \( 2 \) falls. Thus firm \( Y1 \) needs to lower her price less to counteract the rise \( X1 \) has posted. If \( \lambda_B \) becomes high enough (economies of scope small enough) then \( Y1 \) will find that the \( B \) consumers actually have a lower elasticity of demand in moving to \( 2 \) than they did before partial convergence, not withstanding the rise in \( X1 \) prices. This is because \( \lambda_B \) becomes sufficiently larger than \( \lambda_Y \). At this point \( Y1 \) can be in a position to even raise price. For this reason buyers of only good \( Y \) need not see their consumer surplus rise from partial convergence if the economies of scope enjoyed by the large type \( B \) buyers are small.
5 Conclusions

Is multimedia convergence to be welcomed? From a starting position of no convergence at all then for consumers full convergence is to be welcomed. Those consumers who desire both of the goods being converged (TV and internet, or TV and mobile telephony for example) enjoy deep price reductions on the bundle. These consumers also benefit from the economies of scope inherent in sourcing from just one firm. The ‘small’ consumers who desire only a subset of the converged products do no better, and no worse, than the no convergence case. Thus these consumers are left behind, but as the ‘large’ consumers gain, a gain is recorded overall.

The firms themselves do not however gain from full convergence. At least in the short run as compared to partial convergence. Full convergence leads to intense competition for the ‘large’ consumers who require the set of converging technologies. This prompts steep bundle discounts. Short run profits would sum to more if only one firm fully converged and the other remained separate. However to achieve this requires one firm to capitulate as the merged firm will capture the lion’s share of the profits. Without such coordination a war of attrition may result. Alternatively the long run imperatives of R&D in digital technologies may force the industry to convergence - irrespective of the short run costs in terms of lost profits.

If firms solely maximise their short run profits then the process of convergence is liable to stop at the point of partial convergence: one merged firm competing with non-merged rivals. This is exactly because full convergence raises consumer surplus by lowering profits. If a market stops at partial convergence then the whole convergence process is, as far as consumers are concerned, at best a mixed bag. And it can certainly be overall bad for consumers as compared to no convergence at all. Under partial convergence the prices paid by ‘large’ consumers who want all the converging products rise in comparison to the pre-convergence benchmark. This is so whether these buyers go to the merged firm or the independent competitors. The market share of the merged firm rises amongst this group of buyers regardless as they harness the economies of scope available from purchasing from just one firm. Overall therefore the consumer surplus effect on these consumers is ambiguous, even though they pay more than if there was no convergence at all.

Partial convergence creates a means to link a more competitive market with a less competitive one. As a result prices for the more competitive good rise. Consumers who only wish to purchase this good see their consumer surplus decline. Prices for the less competitive good are pulled down. So ‘small’ consumers of one of the converging goods lose out while small consumers of the other good gain. Therefore whether convergence is to be welcomed will depend upon whether there is a reason to favour one type of consumer to another.
A Proofs From Core Model

Proof of Lemma 1. Firm 2’s profit function is given by (4). Note that the terms in \(x, y\) and \(b\) are independent of each other as no \(B\) types ever buy hybrid bundles including 2’s products by assumption. Hence the profit function is a negative quadratic in its margin terms \(\{x, y, b\}\) which can therefore be maximised to give

\[
x = \frac{1}{2}(\lambda_X + \rho_X); y = \frac{1}{2}(\lambda_Y + \rho_Y); b = \frac{1}{2}(\lambda_X + \lambda_Y + \rho_X + \rho_Y) = x + y
\]

Therefore there is no sub-additive bundling here as no consumers make hybrid purchases.

Now consider \(X1\) whose profit function is given by (5). This is a negative quadratic in \(\rho_X\) and so the first order condition for the maximal point is

\[
0 = \frac{AX}{2\lambda_X} \cdot (\lambda_X + x - 2\rho_X) + \frac{B}{\lambda_X + \lambda_Y + \lambda_B} \cdot (\lambda_B + b - 2\rho_X - \rho_Y)
\]

\(\text{ (12) }\)

where we have substituted in for \(x\) and \(b\). Using the change of notation

\[
\alpha_X := \frac{AX}{2\lambda_X} \quad \text{ and } \quad \beta := \frac{B}{\lambda_X + \lambda_Y + \lambda_B}
\]

we have the system of equations (for firms \(X1\) and \(Y1\))

\[
\begin{bmatrix}
\frac{3}{2} (\alpha_X + \beta) & \frac{1}{2} \beta \\
\frac{1}{2} \beta & \frac{3}{2} (\alpha_Y + \beta)
\end{bmatrix}
\begin{bmatrix}
\rho_X \\
\rho_Y
\end{bmatrix}
= \begin{bmatrix}
\frac{3}{2} \lambda_X \alpha_X + \beta \left( \lambda_B + \frac{1}{2} \lambda_X + \frac{1}{2} \lambda_Y \right) \\
\frac{3}{2} \lambda_Y \alpha_Y + \beta \left( \lambda_B + \frac{1}{2} \lambda_X + \frac{1}{2} \lambda_Y \right)
\end{bmatrix}
\]

These can be solved to give

\[
\begin{bmatrix}
\rho_X \\
\rho_Y
\end{bmatrix}
= \begin{bmatrix}
\frac{3}{2} (\alpha_Y + \beta) & -\frac{1}{2} \beta \\
-\frac{1}{2} \beta & \frac{3}{2} (\alpha_X + \beta)
\end{bmatrix}^{-1}
\begin{bmatrix}
\frac{3}{2} \lambda_X \alpha_X + \beta \left( \lambda_B + \frac{1}{2} \lambda_X + \frac{1}{2} \lambda_Y \right) \\
\frac{3}{2} \lambda_Y \alpha_Y + \beta \left( \lambda_B + \frac{1}{2} \lambda_X + \frac{1}{2} \lambda_Y \right)
\end{bmatrix}
\]

Focusing on the margin charged for good \(X\) (the other is found by swapping the \(X\) and \(Y\) terms) we have

\[
\rho_X = \lambda_X + \frac{1}{\frac{3}{2} (\alpha_X + \alpha_Y) + 2\beta^2}
\begin{bmatrix}
-\frac{9}{4} \lambda_X \alpha_Y \beta - 2\lambda_X \beta^2 + \frac{3}{2} (\alpha_Y + \beta) \beta \left( \lambda_B + \frac{1}{2} \lambda_X + \frac{1}{2} \lambda_Y \right) \\
-\frac{9}{4} \lambda_Y \alpha_Y \beta - \frac{1}{2} \beta^2 \left( \lambda_B + \frac{1}{2} \lambda_X + \frac{1}{2} \lambda_Y \right)
\end{bmatrix}
\]

Focusing on the brace term we can simplify this to

\[
\beta \alpha_Y \left\{ \frac{3}{2} \lambda_B - \frac{3}{2} \lambda_X \right\} + \beta^2 \left\{ \lambda_B - \frac{3}{2} \lambda_X + \frac{1}{2} \lambda_Y \right\}
\]

We now use the fact that \(X\) is the weakly more competitive market so that \(\lambda_X \leq \lambda_Y = \lambda_B\) which gives (7).
Proof of Lemma 2. Part 1. Suppose, for a contradiction, that some type $B$ consumers strictly prefer to buy the hybrid $(X2,Y1)$. There are therefore two cutoffs for type $B$ consumers. Those with small $\theta$’s buy from $X1$ and $Y1$. At $\hat{\theta}_X$ the $B$ consumer swaps to the hybrid $(X2,Y1)$. Then at $\hat{\theta}_h$ the consumer swaps from the hybrid to the bundle from 2. We have

$$\langle X2,Y1 \rangle \sim 2 \iff x + \lambda_X \left(1 - \hat{\theta}_h\right) + \rho_Y + \lambda_Y \hat{\theta}_h = b + \left(1 - \hat{\theta}_h\right) \lambda_B$$

$$\hat{\theta}_h \left(\lambda_B + \lambda_Y - \lambda_X\right) = b - x - \lambda_X - \rho_Y + \lambda_B$$

The equilibrium pricing behaviour under this assumption can be determined. Firm $X1$ is straightforward as this firm never sells (by assumption) part of a bundle product:

$$\Pi_{X1}(\rho_X) = (AX + B) \rho_X \hat{\theta}_X$$

$$\frac{\partial \Pi_{X1}}{\partial \rho_X} = \left(\frac{AX + B}{2\lambda_X}\right) \{\lambda_X + x - 2\rho_X\} \Rightarrow \rho_X = \frac{1}{2} (x + \lambda_X) \quad (13)$$

For firm 2 note that

$$\Pi_2(x,y,b) = AX \cdot x \left(1 - \hat{\theta}_X\right) + AY \cdot y \left(1 - \hat{\theta}_Y\right) + B \left\{b \left(1 - \hat{\theta}_h\right) + x \left(\hat{\theta}_h - \hat{\theta}_X\right)\right\}$$

$$= (AX + B) \cdot x \left(1 - \hat{\theta}_X\right) + AY \cdot y \left(1 - \hat{\theta}_Y\right) + B \cdot (b - x) \left(1 - \hat{\theta}_h\right)$$

Differentiating we have

$$\frac{\partial \Pi_2}{\partial y} = \frac{A(Y)}{2\lambda_Y} \{\lambda_Y - 2y + \rho_Y\} \Rightarrow y = \frac{1}{2} (\rho_Y + \lambda_Y) \quad (14)$$

For the bundle price we have

$$\frac{\partial \Pi_2}{\partial b} = \frac{B}{\lambda_B + \lambda_Y - \lambda_X} \{\lambda_Y - 2b + 2x + \rho_Y\} \Rightarrow b = x + \frac{1}{2} (\rho_Y + \lambda_Y) \quad (15)$$

Note that again $b = x + y$ and so there is no bundling here. For the $x$ component we have

$$\frac{\partial \Pi_2}{\partial x} = \left(\frac{AX + B}{2\lambda_X}\right) \{\lambda_X - 2x + \rho_X\} + \left(\frac{B}{\lambda_B + \lambda_Y - \lambda_X}\right) \{- (\lambda_Y - b + x + \rho_Y) + (b - x)\}$$

$$= \left(\frac{AX + B}{2\lambda_X}\right) \{\lambda_X - 2x + \rho_X\} \text{ using (15) } \Rightarrow x = \frac{1}{2} (\lambda_X + \rho_X)$$

Combining with (13) we therefore have

$$x = \rho_X = \lambda_X \Rightarrow \hat{\theta}_X = \frac{1}{2}$$

Finally for firm $Y1$ we have

$$\Pi_{Y1}(\rho_Y) = \left[AY \cdot \hat{\theta}_Y + B \cdot \hat{\theta}_h\right] \rho_Y$$

$$\frac{\partial \Pi_{Y1}}{\partial \rho_Y} = \frac{AY}{2\lambda_Y} \{\lambda_Y - y + 2\rho_Y\} + \frac{B}{\lambda_B + \lambda_Y - \lambda_X} \{b - x - \lambda_X - 2\rho_Y + \lambda_B\}$$

$$= \frac{3AY}{4\lambda_Y} (\lambda_Y - \rho_Y) + \frac{B}{\lambda_B + \lambda_Y - \lambda_X} \left(\frac{1}{2} \lambda_Y - \frac{3}{2} \rho_Y + \lambda_B - \lambda_X\right)$$

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This can be solved to give the optimal margin for $Y$ to charge:

$$\rho_Y = \lambda_Y + \frac{(\lambda_B - \lambda_X - \lambda_Y)}{\nu} \left( \frac{2}{1 + \frac{AY \lambda_B + \lambda_Y - \lambda_X}{2\lambda_Y}} \right)$$

(16)

We only need the fact that $\lambda_B < \lambda_X + \lambda_Y$ to deduce that $\rho_Y < \lambda_Y$ therefore

$$\rho_Y < y < \lambda_Y \Rightarrow \tilde{\theta}_Y > \frac{1}{2}$$

Our maintained assumption that some $B$ consumers purchase hybrids requires $\frac{1}{2} = \tilde{\theta}_X < \tilde{\theta}_B \Rightarrow \tilde{\theta}_X < \tilde{\theta}_B \Rightarrow \rho_Y < \lambda_B - \lambda_X$$

(17)

Now use $\lambda_B = \max \{\lambda_X, \lambda_Y\}$. If $\lambda_X \geq \lambda_Y$ then $\lambda_B = \lambda_X$ and our requirement would be that $\rho_Y < 0$ which is impossible. Therefore restrict attention to the case $\lambda_X < \lambda_Y$; hence $\lambda_B = \lambda_Y$ and our restriction becomes

$$\lambda_X < \lambda_Y - \rho_Y = \frac{2\lambda_X}{3 \left( 1 + \frac{AY \lambda_B - \lambda_X}{2\lambda_Y} \right)} < \frac{2}{3} \lambda_X$$

Another contradiction. Hence hybrid bundles cannot form part of the equilibrium as stated.

Part 2 now follows as we have shown that an interior equilibrium cannot have $B$ consumers forming hybrid bundles, hence they must buy both products from 2 or from $X_1$ and $Y_1$.

For part 3 we establish when the prices given in Lemma 1 do not create an incentive for $B$ consumers to form the hybrid bundle $(X_2, Y_1)$. This is the relevant deviation as $\tilde{\theta}_X < \frac{1}{2} < \tilde{\theta}_Y$. Note that $(X_2, Y_1) \succ (X_1, Y_1) \Leftrightarrow \theta > \tilde{\theta}_X$. Therefore our use of $(X_1, Y_1)$ as the alternative to 2 is valid if $\theta_B < \tilde{\theta}_X$. The required condition is therefore

$$\frac{\rho_X + \rho_Y - \lambda_Y}{\lambda_X + 2\lambda_Y} > \frac{\rho_X - \lambda_X}{2\lambda_X}$$

$$\Rightarrow (2\lambda_Y - \lambda_X) \rho_X - 2\lambda_X \rho_Y < \lambda_X^2$$

(18)

Note that if $\lambda_X = \lambda_Y = \lambda$ for any population splits then $\rho_X = \rho_Y = \lambda$ and so the left hand side would equal $-\lambda^2$ which is strictly less than $\lambda^2$, the right hand side.

**Proof of Proposition 2.** Using the fact that $\lambda_X < \lambda_Y$, (7) implies that $\rho_X > \lambda_X$ and so (6) implies $\rho_X > x > \lambda_X$. $\lambda_X$ is the pre convergence case yielding the price and market share results for $AX$. The margin charged to $AY$ consumers is given by $\rho_Y < \lambda_Y$ from (7). (6) then implies that $\rho_Y < y < \lambda_Y$. For $B$ consumers note that (7) implies that $\rho_X + \rho_Y > \lambda_X + \lambda_Y$. Therefore (6) implies that $\rho_X + \rho_Y > b > \lambda_X + \lambda_Y$.
For the profit calculation insert the first order condition for $Y_1$ (analogous to 12) into the profit function for $Y_1$ (analogous to 5). This yields

$$\Pi_{Y_1}^{\text{partial}} = \rho_Y^2 \left[ \frac{AY}{2\lambda_Y} + \frac{B}{2\lambda_Y + \lambda_X} \right] < \frac{\lambda_Y}{2} \left[ \frac{AY + B}{1 + \frac{\lambda_X}{2\lambda_Y}} \right] < \frac{\lambda_Y}{2} \left[ AY + B \right] = \Pi_{Y_1}^{\text{pre}}$$

**Proof of Lemma 3.** First let us assume one stop shopping so that all $B$ consumers always go to only one store. The game is now symmetric and so we consider symmetric equilibria. The margins $x$ are only paid by type $AX$ consumers, $b$ by type $B$ and so on. Thus the model collapses to a standard Hotelling one with linear costs for each consumer type. This has a unique equilibrium as the profit functions are negative quadratics in price (a result of the uniform distribution) and so (10) is the Hotelling equilibrium.

Now relax the one stop shop assumption. At any symmetric price equilibrium firms 1 and 2 share the market. We wish to show that should firm 1 lower its component price by some small $\varepsilon > 0$ then no type $B$ consumers would decide to create hybrid bundles. Suppose 1 were to lower the margin on $Y$ by $\varepsilon$. Buying both goods from firm 1 gives a higher utility to $(X_2,Y_1)$ for all type $B$ consumers with

$$\theta \leq \frac{\lambda_X}{\lambda_B - \lambda_Y + \lambda_X} + \frac{x + y - b}{\lambda_B - \lambda_Y + \lambda_X} - \frac{\varepsilon}{\lambda_B - \lambda_Y + \lambda_X} = 1 + \frac{x + y - b}{\lambda_X} - \frac{\varepsilon}{\lambda_X}$$

As $\lambda_B = \lambda_Y$ (by assumption of the model), $b \leq x + y$ and for small $\varepsilon$, firm 1 for the bundle is preferred to $(X_2,Y_1)$ if $\theta \leq 1$. Similarly the bundle from firm 2 is preferred to the hybrid bundle $(X_2,Y_1)$ for small $\varepsilon$ if $\theta \geq \frac{1}{2}$. Thus in any small deviation from symmetric prices the type $B$ consumers would one stop shop. Therefore the price equilibrium found assuming one stop shopping is the only possible price equilibrium of the more general setting.

**B Analysis of General Economies of Scope**

**Proof of Proposition 5.** The pre-convergence case is clear. Consider partial convergence under the assumption that $B$ consumers do not form hybrid bundles. Repeating the proof of Lemma 1 yields unchanged margins for the merged firm at

$$x = \frac{1}{2} (\lambda_X + \rho_X); y = \frac{1}{2} (\lambda_Y + \rho_Y); b = x + y$$

The independent firms $X_1$ and $Y_1$ set margins given by

$$\begin{pmatrix} \rho_X \\ \rho_Y \end{pmatrix} = \begin{pmatrix} \lambda_X \\ \lambda_Y \end{pmatrix} + \frac{\beta}{\Delta} \left( \frac{1}{2} \beta (\lambda_Y - \lambda_X) + \left( \frac{3}{2} \lambda_Y + \beta \right) (\lambda_B - \lambda_X) \right)$$

$$\left( \frac{1}{2} \beta (\lambda_X - \lambda_Y) + \left( \frac{3}{2} \lambda_X + \beta \right) (\lambda_B - \lambda_Y) \right)$$

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where $\Delta$ is the appropriate determinant:

$$
\Delta = \frac{9}{4} (\alpha_X \alpha_Y + \beta (\alpha_X + \alpha_Y)) + 2\beta^2
$$

and $\alpha_X = \frac{AX}{2\lambda_X}$; $\alpha_Y = \frac{AY}{2\lambda_Y}$; $\beta = \frac{B}{\lambda_X + \lambda_Y + \lambda_B}$ as before.

Now note that $\rho_X > \lambda_X$ as $\lambda_Y \geq \lambda_X$ and $\lambda_B \geq \lambda_Y$ by assumption. Clearly if $\lambda_X \approx \lambda_Y$ then good $Y$ prices can rise above the pre-convergence benchmark of $\lambda_Y$. Whereas if economies of scope are strong so that $\lambda_B \approx \lambda_Y$ then good $Y$ prices fall below the pre-convergence benchmark. As $y = \frac{1}{2} (\lambda_Y + \lambda_B)$ firm 2 moves its component $Y$ price in the same direction as $Y1$, though not as far. Next $\rho_X + \rho_Y > \lambda_X + \lambda_Y$ so that the bundle price rises. [A local equilibrium as described here can exist for some parameter values using the same derivation used for (9)].

To show that $B$ consumers cannot form hybrid bundles in equilibrium return to the proof of Lemma 2. The substitution of (16) into (17) yields that an equilibrium with hybrid purchases requires

$$
\lambda_Y + \lambda_X - \lambda_B < \frac{3}{2} \frac{AY}{B} (\lambda_B + \lambda_Y - \lambda_X) \left( \frac{\lambda_B - \lambda_X - \lambda_Y}{\lambda_Y} \right)
$$

which is a contradiction. Thus hybrid equilibria with even modest economies of scope can be ruled out.

Part 3 is immediate from the price results for part 2.

References


