The Politics of Unemployment and Electoral Fractionalization: Empirical Evidence from OECD Countries on the "Hostage Voter" Effect[1]

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Abstract

The impact of unemployment on electoral outcomes has always been of interest for both researchers and politicians. In this paper, we study the effect of unemployment on electoral fractionalization employing a multi-party model with two dimensions of choice (policy and ideology). We show that electoral fractionalization decreases as unemployment increases. In equilibrium, unemployed voters have stronger incentives to vote for the parties making the most popular policy promises (high public spending), despite potential ideological divergence. That is, they are held "hostages". We also test our hypothesis in a sample of OECD economies, from 1960-2007, using oil price shocks as an instrument for unemployment. The results yield statistically significant support for the "Hostage-voter" hypothesis. We find that an 1% increase in the unemployment rate is associated with a 2% decrease in fractionalization. Moreover, results are robust under many alternative specifications. This documented negative relationship between unemployment and electoral fractionalization, hints the possibility that dominant parties might have a stronger preferecne for policies aiming at fiscal discipline over unemployment reduction.

Keywords: rational voting, electoral fractionalization, unemployment, public spending, coalition-proof equilibrium, instrumental variables, political cycles.

JEL classification: C22, C23, C26, C70, D72, H00, H69, J68

1 Introduction

A brief overview of electoral results in many parliamentary democracies around the world, during the second half of the twentieth century, indicates a wide variation in the degree of electoral fractionalization. This means that in some countries, there is only a small number of political parties (usually two) that receive the lion's share of votes, while in some others the votes are shared more equitably among many more parties. In other words, we observe a big variation in the degree of electoral fractionalization, even between countries with similar political system structure, such as the OECD economies. A quick look at Panel 1, in the Appendix, reveals some quite interesting stylized patterns.

First of all, one can observe that the level of electoral fractionalization in almost all OECD countries follows a pattern that resembles that of a political cycle. Secondly, there seems to be some variation in the trend of fractionalization over time. Whereas, in most western societies electoral fractionalization seems to be increasing over the last half century, there are some other countries, notably the south European states, where the trend of fractionalization seems to be reversed or stable. Therefore, an interesting set of questions arises: What are the determinants of electoral fractionalization and what is the role of economic conditions, if any, in explaining this variation in fractionalization? In particular, can unemployment account for some of this observed variation?

Furthermore, looking at the stylized facts in Panels 2 and 3 in the Appendix, one can observe a link between electoral fractionalization and unemployment. A more careful observation of the empirical evidence suggests the existence of a certain pattern between those two variables. The data clearly point out to a negative relationship between unemployment and electoral fractionalization, both across and within countries. This negative relationship might come as a surprise in the first place, since one would expect voters to punish the larger parties when unemployment is high. Therefore, in order to explain this paradox, we put forward the "Hostage-voter Effect" hypothesis. That is, we hypothesize that as unemployment rises, electoral fractionalization decreases due to the rent-seeking behavior of unemployed voters.

In parallel with our main hypothesis, we put forward an additional claim that comes out from the Greek regional electoral data, presented in Panel 2. Here we exploit the 2009-10 revision[1] of the level of greek public debt as an information shock[2] that generated expectations for a dramatic cut in public spending. That is, we observe that higher public debt is associated with higher fractionalization. The intuition is clear: higher debt implies a limited ability for the parties to credibly promise higher public spending. Thus, the rent-seeking behavior of unemployed voters is mitigated when public debt is high, since there is not much for the parties to promise. As a result, when debt is high (and expected public spending is low) unemployed voters are less likely to vote for the dominant parties, ceteris paribus. This in turn leads to a uniform increase of electoral fractionalization, as evidence from greek regional elections suggest.

Yet, contrary to the "Hostage-voter" hypothesis, this latter claim does not come as a surprise since historically high public debt is associated with an increase in fractionalization and perhaps polarization. Moreover, recent literature (Persson et al., 2007) confirms the positive relationship between debt and fractionalization. For this reason, the primary focus of our study is the "Hostage-

[1] Eurostat revised, twice within a period of six months, its estimates on the level of greek public debt upwards and by more than 10% per annum over the period extending from 2005-2010.

[2] We refer to this revision as an information shock since the upward estimate was due to misreporting occured in the past and revealed in 2010.

voter" hypothesis. In the remainder of the paper, we document this effect both theoretically and empirically.

1.1 Motivating Hypothesis: The "Hostage-Voter Effect"

We have decided to coin down the term "Hostage-voter Effect" in order to describe what appears to be a paradoxical, at a first sight, voting behavior from the part of unemployed voters. In fact, we argue that the least well-off amongst voters, the unemployed ones, are held hostages by the dominant parties. The intuition behind this hypothesis is relatively simple. Voters expect higher rents if they side with the winner of the elections, especially when public finances are healthy and allow for higher public spending (low public debt). Furthermore, voters with lower incomes, and amongst them certainly the unemployed population, exhibit a stronger rent-seeking behavior compared to employed voters with higher income. Thus, ceteris paribus, they are more likely to vote for the party that proposes more generous public spending, even if this party is ideologically distant from them, in contrast with employed voters who, given the same ideological distance from that party, they would have voted for another one. That is, the voting behavior of unemployed voters is dominated by their rent-seeking desire.

The idea of unemployment being used as a discipline device is not novel in economics. Shapiro and Stiglitz apply this idea in the labour market. Yet, our application on political economy and voting behavior is unique. If Tullock was right in asserting that "voters and buyers are essentially the same people", then it is normal to go one step forward and treat parties like firms. Instead of selling commodities they try to sell their ideologies and platforms. That is, paraphrasing K. Marx[3] who wrote that "big firms require a reserve army of unemployed", we hypothesize that big parties require "a reserve army" of (unemployed) voters in order to dominate in the political competition.

The above situation, to which we have assign the term "hostage-voter" effect, clearly portrays the dilemma which unemployed voters are caught in. Instead of punishing the dominant parties for their policies that have led them into being unemployed, as the unemployment rate increases and their chances of finding a job are becoming more slim, the more likely it is for them to vote for a dominant party due to rent-seeking behavior dominating their voting decision. This hypothesis, which might sound quite counter-intuitive for the traditional psychology based voting approach, is in line with rational voting theory, according to our model. Moreover, if it is empirically confirmed, it lays the ground to complete and perhaps revise the traditional political business cycle theory (Alesina 1987; Alesina and Roubini 1992) in two directions.

Firstly, it will complete the two-way relationship between economic and political outcomes. So far, in the existing literature (Alesina 1987; Persson et al. 2007) the impact of political institutions, such as the electoral rule, and political outcomes on real economic variables has been extensively studied and very well documented both theoretically and empirically. Nevertheless, the reverse direction of causality, that is how real economic variables affect the structure of the party system is understudied. Hence, our study aspires to bridge this gap by providing empirical and theoretical evidence on the effect of unemployment on electoral fractionalization.

Secondly, our hypothesis, conditional on being verified by empirical evidence, can provide a political "raison d'être" for unemployment. From the above discussion, it has become evident that dominant parties would have an incentive to pursue policies that favour fiscal discipline and prudence (e.g. the Stability Pact in the EU) even at the cost of unemployment being slightly above

[3] Karl Marx's initial quote was "...Big industry constantly requires a reserve army of unemployed workers..."

the natural rate, since they can exploit this "reserve army" of potential voters for own benefit. Thus, our framework allows for endogenizing political parties' preferences over unemployment and inflation in a way that traditional political business cycle theory is unable to capture.

1.2 Literature Review

Political scientists have always been interested in examining this phenomenon and have adopted various approaches to explain it. Duverger (1954), first noted that the observed variation in electoral fractionalization among countries may be attributed to the particular political institutions that characterize the political environment of each state, the most prominent of those institutions being the electoral rule. Duverger's first law asserts that "the simple majority, single ballot system favours the two-party system". That is, if a majoritarian electoral rule is applied, then we should expect a low level of electoral fractionalization, where the two larger parties should be receiving the larger share of votes. By contrast, his second hypothesis[4] suggests that "both the simple-majority system with second ballot and proportional representation favour multi-partism". Hence, when a proportional electoral rule is in place, fractionalization should be relatively higher. Individual voters are driven to vote for the larger parties in the first case, as a vote for a small party in a majoritarian system is deemed as a lost vote. In a sense, this accounts to citizens voting strategically, since voting for a minor party that has minimal chances of wining may alter the election result in their disfavor.

Nevertheless, we think that this approach does not account for the complete story. Looking again at the data, we observe the following paradox: countries with extremely stable electoral rules and solid party systems throughout their recent political history, like the United States, the United Kingdom, Germany and Sweden[5] exhibit a rather unexpected volatility in electoral fractionalization overtime. Interestingly enough and contrary to Duvergerian predictions, countries like Greece, where changes in electoral rules have been frequent over the last years and the party system is more fragile, exhibit significantly lower volatility in electoral fractionalization.

To address this issue, two strands were developed in the literature. Firstly, recent political science literature, initiated by Colomer et al. (2005), points out that the electoral rule itself might also be endogenously determined through some political processes. These in turn, might depend on electoral fractionalization. Yet, this critique is out of the scope of the present study[6].

Secondly, economic outcomes might also be linked with electoral outcomes and hence electoral fractionalization. To the support of this hypothesis there is a large and growing body of literature in economics. From a purely economic perspective, several studies have been conducted that relate electoral outcomes with macroeconomic variables. Alesina (1987), Persson and Tabelini (2003), Persson et al. (2007) and Barro (1996), have extensively analyzed the impact of institutions and other political indicators on the economic performance of a country. Specifically, Barro studies how political development and institutions affect economic performance and growth, whereas Alesina develops a rational expectations political cycles model, where partisan electoral competition affects unemployment and inflation.

More recently, Persson et. al (2007) examine the effect that the electoral rules have on government spending in parliamentary democracies. They conclude and provide empirical evidence

[4] The division of Duverger's two statements into one law and one hypothesis is due to Riker.

[5] The first two having a majoritarian electoral rule (first-past-the-post), with Germany having a mixed rule and Sweeden having a PR rule with a list.

[6] In another paper Matakos & Xefteris (2011b) explore the endogeneity of the electoral rule.

that electoral rules affect public spending indirectly through the fragmentation of the party system. Thus, they support that "proportional elections induce a more fragmented party system". As a result, they are associated with a larger incidence of coalition governments, which in turn induce higher public spending than single party governments. In fact, their results are in line with Duvergerian approach, since they suggest a causal link between electoral rules, the resulting party system fragmentation and economic outcomes.

Yet, as noted above, this approach explores the link between fractionalization and economic outcomes in one only direction. Namely, how the fractionalization of the party system affects public spending. As such, it cannot explain how economic outcomes might affect fractionalization. Therefore, we are interested in the reverse direction of the relationship between economic conditions and fractionalization. The influence of economic variables on electoral outcomes has been studied by a large variety of scholars. Both at an individual (Fiorina 1981; Kinder and Kiewiet 1979,1981; Kiewiet 1983) and at a country-level (Kramer 1971; Arcelus and Meltzer 1975; Bloom and Price 1975; Tufte 1978; Kinder Adams, Gronke 1989; Campbell 2000; Holbrook 2008), economic conditions are found to have a severe impact on determining the winner of an electoral competition. Nevertheless, the main focus of this literature is not that broad in the following sense: it might be informative on a partisan level, but it fails to capture whether economic conditions have an impact on the shape and structure of the party system itself.

Our paper differs from existing literature in two respects. Firstly, it attempts to shed some light on the reverse link between economic and electoral outcomes. Secondly, the scope of our study is more broad. We are interested in studying the impact of economic conditions on the structure of the party system, not just on the chances of re-elections of the incumbent party. For this reason, we focus on a broader definition of the electoral outcome: the fractionalization of vote shares. This is so, because electoral fractionalization can better capture the changes that occur in the party system, due to changes in the economic conditions. Hence, we set out to explore whether it does exist a reverse causal relationship between economic outcomes and fractionalization. And given that the key economic indicator is the unemployment rate, we will attempt to account for the variation in electoral fractionalization caused by changes in unemployment.

In order to formally present and empirically document our hypothesis, we proceed as follows: in Section 2 we solve a simple theoretical model of rational voting. As a preview, we can state that model's theoretical results predict that electoral fractionalization depends negatively on unemployment and positively on public debt. Then, in Section 3, we embark to test the theoretical predictions of our model applying a cross-country panel data regression analysis model on data collected from 23 "western" OECD economies during the period from 1960 until 2007. We test our empirical model under alternative econometric specifications, using both country and year fixed effects, and we perform a series of tests to account for potential endogeneity problems. To our reassurance, the model is very robust to the different specification that were tested. Moreover, our empirical findings which are statistically significant under various alternative specifications, seem to yield support for the "hostage-voter" hypothesis.

2 Theoretical Model

As stressed in the introduction, we will first solve a simple theoretical model and then test its implications by the means of an empirical study. We consider that the electoral competition takes place among four parties, in a two dimensional space. We will name the two dimensions ideology

and public spending[7], respectively. The two dimensions differ from each other in three ways. In the ideological dimension the positions of the four parties will be fixed while in the second dimension (public spending) a strategic choice will be made by each one of them[8]. Moreover, the preferences of the voters in the ideology space will be heterogeneous while all voters will share common preferences in the public spending dimension (they will all prefer high public spending compared to low public spending)[9]. Finally, the ideology dimension will be composed out of a continuum of policies while the public spending dimension will include just two alternatives: high public spending and low public spending[10].

2.1 Political Parties

After describing the nature of electoral competition, we can formally define the *party-ideology* space is as follows:

$$\Omega = \{l, L, R, r\} \subseteq [-\alpha, \alpha] \text{ such that } l < L < R < r$$

where l is a party with an extreme-left ideology, L is a party with a moderate-left ideology, R is a party with a moderate-right ideology and r is a party with an extreme-right ideology. We shall assume that a party's index $\omega \in \Omega$ coincides with its ideological position. The *ideology* space is the interval $[-\alpha, \alpha]$, that is party L's ideological position is L which is a point in $[-\alpha, \alpha]$. In order to give more structure to our model we will make the following assumption with respect to the parties' position on the ideology space.

Condition 1 ***(Symmetry)*** *Parties l and r are positioned in the extremes of the ideology space $[-\alpha, \alpha]$, that is at $l = -a$ and $r = a$, respectively. Parties L and R are symmetrically positioned at distance ϵ around zero. That is, at $L = -\epsilon$ and $R = \epsilon$ respectively.*

Each party's ideological position is public information and is known ex-ante to every agent in the society. Furthermore, each party will commit in the pre-election stage in a level of public spending (high or low). That is, each party ω can promise a level of public spending t^ω to the citizens such that $t^\omega \in \{0, T\}$ where $T > 0$.

Assume that, parties' utility function has two components. They derive utility from the ideology that is implemented, as well as from the vote share they receive. The level of public spending does not affect their welfare. Hence, under this formulation parties are both ideology driven and they also want to be represented in the parliament, since they are also interested in maximizing their vote share. Formally, their utility function takes the following form:

$$V_\omega(\pi, v_\omega) = -|\pi - \omega| + b(v_\omega),\ \omega \in \Omega$$

where v_ω is the vote share of party $\omega \in \Omega$, and π is the ideology that gets implemented by the winner, once the elections' result has been realized. That is, π also depends on the vote share

[7] Persson and Tabellini (2000)

[8] Krasa and Polborn (2010) consider an identical model of electoral competition where candidates "are exogenously committed to particular positions on some issues while they choose positions for the remaining issues". In our case, the first dimension would have been ideology and the second would been public spending. Moreover, Dziubinski and Roy (2010) consider a model with exactly two dimensions where parties are commited in one dimension but have the freedom to (credibly) choose any position in the other.

[9] This formulation of voters' preferences is exactly equivalent with Groseclose's (2007) "one and a half dimensional" preferences where "...alternatives [in our case parties]...are described by two characteristics: their position in a left-right dimension, and their position in a good-bad dimension [high-low spending], over which voters have identical preferences."

[10] In Krasa and Polborn (2010) all dimensions are binary.

allocation that results after the elections, since the party that receives the larger share of votes $v_\omega, (\omega \in \Omega)$ wins the elections. The voting rule is simple *plurality*. That is, whenever $v_L > v_\omega$ for every $\omega \in \Omega - \{L\}$ then party L wins in the electoral competition and chooses $\pi \in \Omega$. As a result $\pi = \pi^L$ where:

$$\pi^L \in \arg\max_{\pi \in \Omega} \{V_L(\pi, v_L) = -|\pi - L| + b(v_L)\}.$$

The solution to the maximization problem is straightforward: $\pi^L = L = -\epsilon$. That is, party L (the winner in our example) implements its ideology ($-\epsilon$ in our case). Therefore, since π is determined by the vote shares received by parties, we can formally define it as follows:

$$\pi : \times_{(\omega \in \Omega)} v_\omega \longrightarrow \Omega.$$

The term $b(v_\omega)$, is an increasing function with respect to v_ω and measures the benefit that each party derives by securing some votes in the parliamentary elections. This formulation of the utility function allows us to capture the negative externality that is imposed to each party if a party other than itself wins in the electoral competition and also the benefit that each party receives from being a "recognized" party that is represented in the parliament.

To conclude the discussion in this section, we note that each party's *ideology* ω, together with its *public spending proposal* t^ω comprises party's *political platform* p_ω, upon which citizens vote. That is, political competition takes place in two-dimensions: an *ideological* one (non-monetary component) and *public spending* one (monetary component). Nevertheless, given that ideology is fixed for every party, in practice, the game involves strategic decisions only in the public spending dimension.

2.2 The Voters

We consider two *continua* of voters, each one with an infinite number of voters, whose *ideological preferences* (ideal points) are single-peaked and distributed according to a *uniform* distribution U on the *ideology* interval $[-\alpha, \alpha]$. That is, $x \sim U(-\alpha, \alpha)$ in both cases. The two continua are *identical* in all respects apart from two: firstly, they can be of *unequal* size and secondly, citizens are endowed with *different* initial wealth. That is, we assume that the first continuum of voters represents a fraction $q \in [0, 1]$ of the society. These voters will be considered to be the *unemployed ones* (low initial wealth endowment - m). The second continuum will represent the remaining $(1 - q)$ society. These voters will be considered to be the employed ones (high initial wealth endowment - M). If the initial wealth endowment $y \in \{m, M\}$ with $\frac{a}{2} \leq m < M \leq a$ then the utility of an agent with ideal policy x and initial wealth endowment y (we will name this voter $\{x, y\}$) will be given by:

$$U(\pi, x, y) = -|\pi - x| + u(y + t^\pi)$$
$$\text{where } \pi \in \Omega, \text{ such that } \pi : \times_{(\omega \in \Omega)} v_\omega \longrightarrow \Omega \text{ and } x \sim U(-a, a).$$

The first component of this expression, namely $-|\pi - x|$, is the utility that individual x receives from the ideology of the winning party. The second component, $u(y + t^\pi)$, is a strictly increasing and concave utility that the same individual receives from her initial income, $y \in \{m, M\}$, and the proposed level of public spending.

2.3 The Voting Game

The voting game we consider is one of three stages. Ballots, in this model, are *secret*. Voters vote *sincerely*. All information is publicly available and known *ex ante* to all the agents of the society. That is, there is no uncertainty and no informational asymmetry in the model. The equilibrium solution concept we employ is coalition-proof Nash equilibrium. The three stages of the game are as follows:

Stage 1: Parties simultaneously decide whether or not to enter the election. If a party decides to contest the election, then, it also announces simultaneously its complete *political platform* $p_\omega = \{\omega, t^\omega\}$. That is, upon entry, the proposed *political platform* p_ω of each party becomes public information. Given that the ideology dimension of each party ω is fixed, we can abbreviate political platform $p_\omega = \{\omega, t^\omega\}$ by omitting ω.

Stage 2: Each individual votes *sincerely* and *secretly* for her most preferred platform, given parties' platform promises. Formally, sincere voting in this setup means that each voter $\{x, y\}$ faces the following maximization problem:

$$\max_{t^\omega} U(\omega, x, y) = -|\omega - x| + u(y + t^\omega).$$

The voting rule is simple **plurality**. In case of ties, each of the winning parties wins elections with equal probability.

Stage 3: Given voters' decisions on stage 2, each party receives its vote share $v_\omega \in [0, 1]$ such that $\sum_{\omega \in \Omega} v_\omega = 1$, and the voting outcome is realized. The party (or parties in case of ties) that collect the plurality of the votes win the electoral competition and implement their political platform. The winning party is denoted as π. We assume that the winner fully implements its ideology and realizes its public spending promises. Clearly, the winner depends on the vote share that each party received during elections, which in turn depends on the political platforms p_ω, $\forall$ $\omega \in \Omega$, proposed by each party. Formally we have:

$$v : \times_{(\omega \in \Omega)}\ p_\omega \rightarrow [0, 1] \text{ and } \pi : \times_{(\omega \in \Omega)} v_\omega \longrightarrow \Omega$$

Hence, we denote the winner of the electoral game as $\pi(v^*_\omega(\boldsymbol{p}^*), v^*_{-\omega}(\boldsymbol{p}^*))$, $\forall\ \omega \in \Omega$. Hereinafter, we abbreviate this expression by denoting it as $\pi(\boldsymbol{v})$, where $\boldsymbol{v}$ is the vector $(v_\omega)_{\omega \in \Omega}$.

As stated earlier, the elections' outcome depends on the distribution of ideological preferences, which is uniform in $[-a, a]$, on the positions of the four parties in the unidimensional ideology space and on the public spending proposal of each party. Hence, all that citizens do, is to vote *sincerely* for the party $\omega \in \Omega$, whose proposed policy platform p_ω maximizes their utility. As a result, by construction of the model, there is no strategical dimension in the voting decision of any individual voter.

Finally, to conclude the discussion, we will now define a key variable of interest. During the elections each party receives a vote share, denoted by $v_\omega \in [0, 1]$, and following Rae (1968) and Laakso and Taagepera (1979) we can define the electoral fractionalization of the described procedure by the following formula:

$$F = 1 - \sum_{\omega \in \Omega} (v_\omega)^2$$

2.4 Some Assumptions

Initially, in order to solve analytically our model, we need to make some extra assumptions. We will illustrate our main result in this section, with the aid of these extra assumptions, but our results can be extended for more general cases. Yet, for the time being, this particular structure of the model will be followed through the remainder of this section.

First, assume that $\alpha = \frac{1}{2}$. This assumption helps us reduce the notation of our formal analysis without loss of generality.

Also assume that the function that measures parties' utility from votes is linear. That is:

$$b(v_\omega) = v_\omega,$$

Further assume that the function measuring citizen's utility from wealth and fiscal transfers takes the following form:

$$u(y + t^\omega) = \sqrt{y + t^\omega},$$

Recall that, an individual can have an initial wealth endowment $y \in \{m, M\}$, where M is the market wage and m is the income received by state remunerations. As noted before we assume $\frac{1}{4} \leq m < M \leq \frac{1}{2}$ in order for our voters to be significantly interested in both policy dimensions (ideology and public spending).

As vote shares depend on publicly announce political platforms we can rewrite the parties' and agents' objective functions as follows:

$$V_\omega(p_\omega, p_{-\omega}) = -|\pi(p_\omega, p_{-\omega}) - \omega| + v_\omega(p_\omega, p_{-\omega}),$$
$$\text{and}$$
$$U(\omega, x, y) = -|\omega - x| + \sqrt{y + t^\omega}.$$

Also recall that, the level of public spending that parties can promise is decreasing with public debt. That is, as public debt increases, the maximum level of public spending that parties may promise decreases. Moreover, to make sure that our voters are significantly interested in both policy dimensions, we assume that $T \in (0, \frac{1}{8}]$. In practice, the above expression just says that the maximal level of transfers proposed by the parties, T, can never exceed one half of the lowest possible initial wealth endowment of the poorest members of the society.

2.5 Results

In this section, we will proceed with our theoretical analysis and provide our main results. In trying to establish our main equilibrium characterization results, we note that we are particularly interested in the class of **coalition-proof Nash** equilibria. The reason for focusing on coalition-proof equilibria is that we want to analyze a fairly competitive political environment; if coalition proofness is not guaranteed in our equilibrium then, in reality, a number of parties could wish to "coordinate" or even withdraw from the race. The main purpose of this essay is to study electoral fractionalization and its interaction with real economic variables and, thus, focusing on those equilibria provides us with the necessary framework. If, for instance, we were not restricting attention to coalition-proof equilibria, but instead allowed parties to collude, then in almost all other cases two or more parties would have an incentive to form a coalition, thus making the study of electoral fractionalization meaningless.

To prove our main result, we construct our argument in two steps. First, we propose a candidate equilibrium allocation (we call it $\boldsymbol{p}^*$) and we show that it is indeed a NE of the electoral game specified above. Finally, we show that equilibrium $\boldsymbol{p}^*$ is indeed coalition-proof.

Proposition 1 *Let* $\forall\epsilon \in (\sqrt{\frac{3}{8}} - \sqrt{\frac{1}{4}}, \frac{\frac{1}{2}-\sqrt{\frac{3}{8}}+\sqrt{\frac{1}{4}}}{3})$. *Then, for every* $q \in [0,1]$ *and every* $T \in (0, \frac{1}{8}]$, *the following political platform vector* $\boldsymbol{p}^*$*constitutes a NE of the game:* $\boldsymbol{p}^* = [\pi(\boldsymbol{v}); \boldsymbol{t}^*] = [(\frac{1}{2}L, \frac{1}{2}R); t^l = t^r = 0, t^L = t^R = T]$[11].

After proposing equilibrium $\boldsymbol{p}^*$ as a candidate for a coalition-proof equilibrium of the game, and prior to examining whether $\boldsymbol{p}^*$ is indeed such an equilibrium, we need to stress that $\boldsymbol{p}^*$ need not be the unique NE of the game. Yet, in what follows, we shall argue that it is the *unique* coalition-proof equilibrium of the game.

Proposition 2 *Let* $\epsilon \in (\sqrt{\frac{3}{8}} - \sqrt{\frac{1}{4}}, \frac{\frac{1}{2}-\sqrt{\frac{3}{8}}+\sqrt{\frac{1}{4}}}{3})$. *Then, for every* $q \in [0,1]$ *and every* $T \in (0, \frac{1}{8}]$, *the equilibrium* $\boldsymbol{p}^*$ *of Proposition 1, is the* ***unique coalition-proof*** *equilibrium of the electoral game.*

Under equilibrium $\boldsymbol{p}^*$ the indifferent voter among parties L and R is the median. This just means that the share of votes distributed among the two dominant parties is fixed and it does not vary with unemployment and public spending. Hence, any effect that those parameters have on the fractionalization index come through the changes they cause to the share of votes that is to be distributed between the extremist parties l and r and the centrist ones L and R, respectively.

Furthermore, perhaps the most interesting feature of the equilibrium structure is the fact that the two extremist parties are purely ideological and never win elections, whereas the two centrist are the dominant ones. That is extremist parties, in equilibrium, focus solely on the ideological dimension of their platform and propose low public spending, even if this costs them losing the elections. But note that, this is not imposed exogenously to the model. Nor does it happen because we assume ex ante that those parties have no chance to win in the elections. Rather, it is an optimal strategic choice made by those parties.

Someone might think that this behavior is not intuitive, since those parties could promise high public spending, especially if they are less likely to get elected and implement their promises. In a sense, their promises are empty and hence, cost-less since they are off equilibrium path. The rationale behind this behavior is that if the two extremist parties propose high public spending they will deprive votes from their "sister" parties, thus causing another less desired party to win in the elections. In that respect, their cost-less promises cause a less desired party to get elected and this imposes a negative externality to them. So, in that respect, our model says that there is no "free lunch" in politics either. And though we focus on coalition-proof equilibria in order to ensure some minimum level of political competition, we observe that even under the most competitive conditions there is still room for the parties to collude indirectly. That is why, we think that our model simple as it may appear can still capture the essential characteristics of multi-party electoral competition in representation democracies.

After this brief analysis of our coalition-proof equilibrium, we proceed by presenting our main comparative statics results on $\boldsymbol{p}^*$. That is, we will examine how electoral fractionalization varies with changes in unemployment and public spending.

[11] All proofs in the Appendix.

Proposition 3 *Assume that conditions of Proposition 2 hold. Then the electoral fractionalization index F is decreasing if: (i) the fraction of unemployed people in the economy increases (that is, as q increases), ceteris paribus; or (ii) public spending T is increasing, ceteris paribus.*

The comparative statics results of Proposition 3 confirm the "Hostage-voter" hypothesis. They can be interpreted as saying that due to the concavity of the utility function with respect to income and public spending $u(y + t^{\omega})$, unemployed voters with lower initial wealth are more likely, ceteris paribus, to vote for a party that offers high public spending due to stronger rent-seeking behavior. Given that in equilibrium, only the two centrist parties promise high public spending, it follows that an increase in either the unemployment rate or the level of public spending will result in those parties increasing their vote shares. Thus, electoral fractionalization decreases. In the empirical section that follows, we will test "Hostage-voter" effect.

2.5.1 Some Remarks

After presenting the reader with our main comparative statics result, some technical remarks with respect to our assumptions and the equilibrium structure are now in order.

Firstly, we note that the existence of some $\epsilon > 0$, such that our previous results go through is independent of the the specific functional form of $u(.)$ and $b(.)$ chosen in this example. That is, for any given concave function $u(.)$ and any convex function $b(.)$ we can always find some $\epsilon > 0$ within interval $I \subset [0, a]$ such that Propositions 1 and 2 hold true. To see this, let us abstract a bit by not setting $\alpha = \frac{1}{2}$. Then condition on ϵ becomes: $\epsilon \in (\sqrt{\frac{3a}{4}} - \sqrt{\frac{\alpha}{2}}, \frac{\alpha - \sqrt{\frac{3a}{4}} + \sqrt{\frac{\alpha}{2}}}{3})$. Consider now that $u(.)$ is linear, which is the weakest possible concave function. Then condition becomes: $\epsilon = \frac{\alpha}{4}$. That is for any strictly concave function there is always an interval, such that our results are still valid. Moreover, the interval expands as the concavity of the utility function $u(.)$ is increasing.

The reason that we have decided to focus our equilibrium analysis for those particular values of ϵ is related with the desired nature of political competition. If ϵ exceeds its upper bound, this means that the two "centrist" parties are not so close to each other anymore. Rather they are converging to the extremes in terms of ideology. But then the two centrist parties are in reality extremists, and their ideological proximity with the extremist parties drives the latter to withdraw from the political competition. In such a case, our four-party model becomes a standard two-party model where the study of fractionalization becomes trivial. If, on the other hand, ϵ falls below its lower bound then the two centrist parties converge to the centre (and to each other) that it is very hard to distinguish any ideological differences among them. In this case, the extremists do not wish to "sacrifice" a small vote share by promising low public spending in order for their neighboring moderate party to preserve some chances of winning. They prefer to promise high spending as well leading to an equilibrium in which the public spending dimension is practically cancelled out (all parties choose the same strategy). In order to study how electoral fractionalization evolves when macroeconomic variables change we need a) a multi party political competition and b) differentiation among the public spending promises of the parties'. When ϵ exceeds or falls below our presented thresholds these two conditions are never simultaneously satisfied.

Of course, it might still be very interesting to study those other cases and characterize the structure of equilibria that arise in the game, even if their analysis is not meaningful for the purposes of studying electoral fractionalization. Nevertheless, in this version of the paper, we do not provide a complete account and characterization of all the equilibria that arise under different parameter values of ϵ. Yet, we think that the study of those equilibria might also have some

important implications to other aspects of political competition. As a result, it is left as a task for future work.

3 Empirical Analysis

3.1 Introduction

In this section of the paper we will set out to test the theoretical predictions of our model using aggregate political, socio-demographic and macroeconomic data we collected for twenty-three western OECD democracies during the period 1960-2007, combining various sources. Bringing the model's predictions to data will be an interesting exercise for a variety of reasons. Firstly, whereas the theoretical predictions of our model on the impact of public spending on electoral fractionalization might seem quite intuitive, as argued before and as the data from greek regional elections suggest, our main prediction with respect to unemployment might estrange some of our readers. Hence, the need for an empirical test of the "Hostage-voter" hypothesis is indispensable. This will allow us to enrich the existing political cycles literature (Alesina, 1987) by endogenizing political parties' preferences over unemployment.

Moreover, so far and to our knowledge, in the political economics literature there has never been a comprehensive empirical study on the determinants of voting behavior and electoral fractionalization that encompasses almost all western democracies (with few exceptions). The present study, thus, is a first attempt to try to document empirically how real economic conditions might affect voting and fractionalization in western democracies. Of course, it is needless to stress once more that our current empirical study does not aspire to give a complete account on how and why citizens vote the way they do. Rather, we aspire to shed some light and provide some empirical evidence on the potential impact of economic variables, such as unemployment, as determinants of fractionalization. Surely, unemployment is not the sole determinant of electoral fractionalization, yet we think that some further empirical exploration is warranted.

Finally, since our theoretical model builds a theory of rational voting based on real economic outcomes, it is important in order for the theory to stand scrutiny to put it through a reality check. Since all our key explanatory variables are aggregate macroeconomic data we think it is imperative to provide readers with an empirical account on how well our theory performs in explaining the data. Hence, the basic estimation strategy is to conduct a cross-country panel data analysis in order to examine the degree upon which electoral fractionalization depends on economic conditions. More specifically, in what follows we will try to examine how the unemployment rate can affect electoral fractionalization in twenty-three democracies. Our initial hypothesis, that is derived from the theoretical model, is that an increase in unemployment is associated with an decrease in the level of electoral fractionalization (measured according to the Rae Index).

The remainder of the empirical section is organized as follows: Section 3.2 provides an analytic description of our main sources of data. Then, in Section 3.3 we proceed by presenting our basic estimation strategy and model. Section 3.4 discusses some preliminary OLS results and potential sources of biases and endogeneity. In Section 3.5 we deal with problems arising from endogeneity by introducing our IV's and discussing the rationale behind their use. Finally, in Section 3.6 we present and discuss our main IV results and Section 3.7 concludes the empirical component of this paper.

3.2 Data Description

In trying to test our hypothesis, we have collected and compiled observations of key political, socio-demographic and macroeconomic variables from 23 "western" OECD countries.[12] We have decided to restrict attention to this group of countries for many reasons. Firstly, we need to focus on countries with fairly stable and fair democratic institutions and electoral procedures for the most part of the second half of the twentieth century. Secondly, given that democratic elections take place approximately every four years, we had to focus on countries which have a long history of democratic and fair elections. Adding other countries that are undergoing a democratization procedure would, in fact, reduce our time span and our number of observations. Finally, our key hypothesis, the "Hostage-voter" paradox, would seem more plausible and to some extend expected in LDC's or developing economies that are under the process of democratization. Yet, we want to illustrate that even in mature democracies the "Hostage-voter" paradox still plays an important role in determining electoral outcomes. As a result, given all the above reasons, the decision to focus on OECD economies was justified.

In the remainder of this paper we will be using the observations of "The Comparative Political Data Set 1960-2007" which is a collection of political and institutional data which have been assembled in the context of the research projects "Die Handlungsspielräume des Nationalstaates" and "Critical junctures. An international comparison" directed by Klaus Armingeon and funded by the Swiss National Science Foundation. It consists of a compilation of (mostly) annual data for 23 democratic OECD countries for the period of 1960 to 2007, taken from the SourceOECD Database. In the cases of Greece, Spain and Portugal, political data were collected only for the democratic periods[13]. We also supplemented the data set with observations collected from the OECD i-Library, for reasons of consistency, plus some own calculations and data from OPEC and the US Energy Information Administration (EIA). The data on electoral fractionalization from greek regional elections are own calculations based on the official results as they appear in the official database of the Greek Ministry of Interior. We have also supplemented our calculations with data from the Eurostat Regional Yearbook and OECD online database for consistency.

The data set is suited for cross national, longitudinal and pooled time series analyses. In what follows we organize this data set into a panel, in a way that is suitable for conducting a cross-national pooled time series analysis, using for the most part of the econometric estimation, a 2SLS fixed effects estimator. Although, as the empirical analysis proceeds, we will define in more detail some of the key variables which are extensively used in our statistical analysis, for a complete description of the data set we refer the reader to the comprehensive list of all variables in Appendix C.

Finally, the data set contains some additional demographic, socio- and economic variables. A few variables have been copied from a data set collected by E. Huber, Ch. Ragin, J. Stephens, D. Brady and J. Beckfield (2004), as well as from a data set collected by D. Quinn. For a more complete description of those variables we refer our readers to the OECD online database (OECD i-Library). The data are summarized in Table 8, in Appendix B.

[12] The term "western" refers to the those member-states of OECD that joined the organization prior to the collapse of the Soviet Union and "Warsaw Pact".

[13] Data for Greece are missing during the period 1967-1973. Data for Portugal are missing until 1975, and for Spain until 1976.

3.3 The Benchmark Model

In deciding which estimating strategy to follow and how to choose our basic estimation model, firstly, we had to take into account the limitations posed by the format of the data. Even though we have a total of more that 1,000 observations from 23 countries over a time span of almost fifty years, our dependent variable, electoral fractionalization, exhibits low variation over time. The reason being, elections take place approximately every four years in almost all OECD democracies. With some notable exceptions, such as countries that have a "tradition" in calling frequent early elections, this is true for the majority of the countries in our sample. As a result, electoral fractionalization is a quite static variable. To deal with this complication we have decided to include fractionalization only on those years that elections took place. Hence, we have generated a binary variable $e_{i,t} = \{0, 1\}$ taking the value of 1 if year t was an election year in country i, and 0 otherwise. That is, our dependent variable becomes: $F_{i,t} * e_{i,t}$, where electoral fractionalization is defined by the Rae Index. Formally:

$$F_{i,t} = 1 - \sum_{n=1}^{N_{i,t}} (v_{n,i,t})^2,$$

where $v_{n,i,t}$ is the vote share of party n in country i at election year t, and $N_{i,t}$ is the total number of parties in country i that contested elections in year t. In total, our data set incorporates 322 electoral outcomes in all twenty-three countries over a period of 48 years, and as a result we obtain 322 observations for electoral fractionalization. This number of observation is completely within expectations, since it implies an average of 14 elections per country, with an election taking place on average every three and a half years.

Secondly, we had to acknowledge potential threats to the validity and consistency of our estimation results. Hence, we have decided to organize the data as a cross-country panel (pooled time series) in order to include country specific fixed effects. The main idea behind this is the fact that electoral fractionalization might as well depend on time-invariant, country-specific characteristics. As recent literature in political science and economics documents, electoral outcomes and hence, the fractionalization of the party system, can critically depend on some of those individual, time-invariant characteristics. These may include, for instance, the structure and the historical attributes of the political system, country specific demographics, such as minority and ethnic parties, irregularities and disproportionalities in the electoral rules, urbanization, the structure of the political system itself, constitutional arrangements, or even the political history of each country, to name a few. Hence, it is reasonable to treat them as time invariant parameters.

Obviously, those parameters might pose a threat to the validity of our estimations if they are not taken into consideration. Hence, in order to account for all these factors, as well as others not explicitly identified in this study, we use a fixed effects (FE) estimator which is suggested by the Hausman test as opposed to a random effects (RE) estimator. Formally, one would want to estimate the following model:

$$F_{i,t} * e_{i,t} = \beta_0 + \beta_1 (Unemployment)_{i,t} + \beta_2 (Debt/GDP)_{i,t-1} + X'_{i,t}\gamma + a_i + \lambda_t + \eta_{i,t}$$

where the dependent variable $F_{i,t} * e_{i,t}$ is the modified Rae Index of electoral fractionalization on the election year, for each country over the period 1960-2007, $X'_{i,t}$ is the set of exogenous covariates and α_i, λ_t are country and year fixed effects. Our independent variable, unemployment, is measured as a percentage of the active labor force.

We have also decided to include the lag of public debt, which is also measured as a percentage of GDP. The rationale for including it is related with our discussion on how the level of public spending depends negatively on the state's debt level d ($\frac{\partial T}{\partial d} < 0$). The more healthy public economics are, that is the lower the level of debt d is, the greater the level of public spending that a government can promise is. Since this positive relationship between debt and electoral fractionalization[14] is supported by the greek regional election data and by the theoretical predictions of our model, and since it is also documented in the existing literature (Persson et. al, 2007), we decided to include the lag.

The remaining controls include various measures of the disproportionality of the electoral rule (plurality vs. proportional rule) and electoral rule change, the degree of institutional constraints in abusing public finances, the type of government, number of parties and voter turn-out. To control for some of those, we use a series of binary variables as exogenous covariates such as the existence of single-member district plurality rule and others. Yet, since it is virtually impossible to account for all possible political parameters that might have an effect on fractionalization, we recognize that our OLS estimates might be biased towards zero, due to potential omitted variable bias (OVB).

Later on, when we will introduce our Instrumental Variables, we will try to account for those omitted exogenous covariates. But before doing so, we first want to include in the model those covariates that are available and examine how they affect estimation results, if they do so. Then, after introducing our instruments, we will provide the rationale for dropping some of them. For a complete description of the those variables, the instruments and some other key variables of the data set we have included a more detailed list in Appendix C.

Before presenting our OLS results in the section that follows, a small comment with respect to our decision to include time fixed effects is, we think, in order. As the reader is aware by now, the main aim of this study is to document the impact that real economic conditions have on voting behavior and electoral fractionalization. That is, we want to capture the effect that changes on unemployment have on electoral fractionalization. To do so, the optimal way would have been to exploit year specific, country invariant shocks on the macroeconomy. By doing so we could have been able to capture changes on unemployment that are beyond the control of domestic political competition. For instance, a global financial crisis that started in Asia and then spread around Europe could have been an ideal source of variation for unemployment that cannot be attributed to the policies of domestic governments, not to mention to the level of electoral fractionalization.

Therefore, since it would have been extremely hard to argue that the fact that the Labour Party losing 5% of its vote share to the Liberal democrats could have been the causal effect of the economic crisis and contagion that followed, exploiting those exogenous variations on output, unemployment and public debt would have been ideal for the purposes of this study. Yet, a potential source of worry comes from the fact that voting behavior and as a result electoral fractionalization may also depend on year specific political phenomena that affect voting behavior uniformly in all western democracies. An example could be the fall of the Berlin Wall, or the September 11 terrorist attacks, that had an impact on voting behavior. Of course, the incidence of such political phenomena is more rare compared to that of macroeconomic shocks, which we ideally want to exploit. Nonetheless, we have decided to include year fixed effects under almost all our chosen estimation specifications, in order to account for such phenomena, knowing that this choice will increase the difficulty of our task. On the other hand, we wanted to fully insulate our model from this kind of critique, by

[14] Observe that: $\frac{\partial T}{\partial d} < 0$ and $\frac{\partial F}{\partial T} < 0$. Hence, $\frac{\partial F}{\partial d} = \frac{\partial F}{\partial T}\frac{\partial T}{\partial d} > 0$.

making sure that our estimation strategy is correctly specified.

3.4 OLS Results

In this section, we will present the OLS results of our benchmark model as specified above. The main coefficients of interest throughout this analysis is $\beta_1 = \frac{\partial F}{\partial q}$. According to our theoretical model and the "Hostage-voter" hypothesis we expect that $\beta_1 < 0$. That is, we hypothesize that an increase in unemployment is associated with a decrease in the Rae Index of electoral fractionalization. Table 1 below presents our initial OLS estimates under seven alternative specifications.

Firstly, we note that under all specifications the coefficient on unemployment β_1, is negative as predicted by the theory. Yet, with the exception of Columns (1) and (2) where it is also statistically significant, in all other specifications it fails to be so. There are many possible explanations as to why this is happening. First of all, the inclusion of both voter turnout and the dummy for the existence of coalition government into the list of exogenous covariates reduces the magnitude of the coefficient on unemployment (β_1) almost by a half. It can be seen by comparing Columns (1) and (2) with (3) and Column (5) with (7), respectively. This is so, because the existence of a coalition government and voter turnout are, in fact, outcomes that are either determined by the voting procedure (coalition government) for instance, or they are determined simultaneously with electoral fractionalization (voter turnout). As such, it is electoral fractionalization, which summarizes the fragmentation of partisan power in a political system, that determines whether a country will have a coalition government and not vice versa.

Yet, political scientists argue that the political tradition of a country on having frequently coalition governments (e.g. Germany or Netherlands) can have a positive effect on electoral fractionalization, in direct contrast with single-party government tradition (e.g. USA or UK[15]). But, to the extend that this tradition is a time invariant country-specific characteristic of the respective political system, which we believe it is the case, then it is captured by including country fixed effects. Moreover, if a country's history of having, or not, coalition governments is not time invariant, then according to Duverger (1954) and the vast strand of literature developed after his seminal work, the electoral rule crucially affects the fragmentation of the party system and hence, the probability of ending up with a coalition government in lower when Plurality electoral rules are applied. In fact, a simple check in our data set reveals a strong, negative and statistically significant correlation between plurality rule and incidence of coalition government (-0.558). As a result, we decide to exclude those two variables from the list with the exogenous covariates, especially in light of the fact that we continue to include the variable on the electoral rule in all subsequent specifications of the model.

[15] Remember that the time span of our study expands until 2007.

Table 1: *Basic OLS Model and Alternative Specifications*
Dependent Variable: *Rae Index of Electoral Fractionalization;*

	OLS	OLS	OLS	OLS	OLS	OLS	OLS
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Unemployment (%)	-.3519	-.3519	-.1816	-.0199	-.0435	0.0485	-.0205
	(0.156)**	(0.2039)*	(0.2657)	(0.1736)	(0.1676)	(0.1544)	(0.163)
Debt/GDP (%)	0.039	0.039	0.0154	-.0309	- - -	-.0232	-.0289
	(0.0222)*	(0.0305)	(0.044)	(0.0232)		(0.0254)	(0.0225)
L.Debt/GDP (%)	- - -	- - -	- - -	- - -	-.0255	- - -	- - -
					(0.0234)		
Social Expenditures/GDP (%)	0.2458	0.2458	0.1903	- - -	- - -	- - -	- - -
	(0.189)	(0.2322)	(0.2231)				
Dummy Plurality ER	-4.3209	-4.3209	-4.6463	-2.3093	3.1011	-2.5386	-2.4889
	(3.4908)	(0.5808)***	(1.2164)***	(2.1076)	(2.3164)	(1.7892)	(2.1583)
Dummy High Institut. constraints	-1.1514	-1.1514	-1.3898	2.2912	1.5203	1.5847	1.9858
	(1.4789)	(1.9940)	(1.5221)	(2.1278)	(2.0329)	(1.9906)	(2.0463)
Voter Turnout (%)	- - -	- - -	-.2450	- - -	- - -	-.1181	-.1525
			(0.1459)*			(0.1236)	(0.1262)
Dummy Coalition Gov't	- - -	- - -	-.3322	- - -	- - -	0.8069	- - -
			(1.4625)			(0.9954)	
Country Fixed Effects	Y	Y	Y	Y	Y	Y	Y
Year Fixed Effects	N	N	N	Y	Y	Y	Y
Const.	71.5042	71.5042	87.6248	70.3172	66.9462	85.4845	89.1142
	(4.5822)***	(3.2215)***	(12.0107)***	(2.5924)***	(2.4275)***	(12.0471)***	(12.2614)***
Obs.	135	135	132	224	217	219	224
R^2	0.9197	0.9197	0.9266	0.932	0.9318	0.9399	0.9342
F statistic	45.3932						

Table 1: Note: *Clustered Standard Errors at the Country level reported in parantheses, apart from Col.(1) where Robust Standard Errors are reported. (***) Statistically significant at the 1% level; (**) Statistically significant at the 5% level; Statistically significant at the 10% level.*

Secondly, another potential reason that our estimated coefficients reduce in magnitude and become not statistically significant is the inclusion of year fixed effects in all specifications from Column (4) and onwards. It is, thus, normal to expect that the inclusion of year fixed effects will increase the measurement error and hence the bias towards zero. Nevertheless, as we have discussed in the previous section, we still think that the correct specification of the model should include both country- and year-specific fixed effects. So, we have decided to include them in all subsequent specifications of both the OLS and the 2SLS models.

Finally, the main reason that our estimates on coefficient β_1 appear to be biased towards zero and not statistically significant is related with reverse causality. As we have stressed extensively in the introduction, there is a large strand in the literature that documents how policies and political outcomes can affect unemployment and other economic variables (e.g. inflation). That is, we have reasons to hypothesize that not only unemployment might have an effect electoral fractionalization, but also at the same time electoral outcomes can affect unemployment. Different parties might

pursue different policies with respect to unemployment, or enact different legislation with respect to minimum wage and unemployment benefits. Therefore, apart from the OVB problem we have discussed earlier and relates to the existence of some unobserved political processes that potentially affect fractionalization, we also run into an endogeneity problem. That is, our main independent variable of interest, unemployment, might be endogenous. With this in mind, in the next section we introduce the solution to these potential threats to the validity of our estimates: instrumental variable regressions.

3.5 Accounting for Endogeneity and Bias: Introducing Oil Price Shocks

As discussed previously, there are potential sources of worry about our OLS estimates presented in the preceding section. Therefore, we have decided to estimate the relationship between unemployment and electoral fractionalization using an IV estimator. The reason for choosing this specific estimator is related with problems of endogeneity and OVB that a standard OLS estimator would otherwise have. Since recent literature in political economy has documented that political cycles on the one hand (Barro and Alesina), and the fragmentation of the party system on the other (Persson, Tabelini et al.), can indeed have an effect on economic performance and macroeconomic variables, such as unemployment rate and public spending, it would have been difficult to estimate the direction of causality and get consistent and unbiased estimates with standard OLS models. To put it more simply, it would very well be the case that those macroeconomic variables affect the voting behavior of the agents and as a result, the fractionalization in the share of votes that parliamentary parties receive and the final outcome of the electoral process. But, it may also be the other way around. That is, the outcome of the elections and the resulting fragmentation of the party system can, in turn, have an effect on unemployment.

In order to deal with this potential endogeneity problem, we are pursuing a strategy of instrumenting for unemployment, using the shocks on imported oil prices from 1960 and onwards. The idea of using shocks on the price of oil as an instrument for unemployment is first discussed by Levitt (2001), in a paper where he summarizes the strategies for identifying the causal link between crime and unemployment. The difficulty in identifying the causal effect of unemployment on crime is similar with ours, since, there are unobserved policies that, just like in our case, affect simultaneously both the crime rates and unemployment. Furthermore, reverse causality also makes its appearance. Levitt cites a study by Raphael and Winter-Ebmer (2000) at the US State level, which are equivalent in size with most European states, where they use two instruments for unemployment: the closing of military bases and the shocks on oil price. They argue that oil price shocks can have an effect on unemployment and thus, they decide to include it as an instrument.

But apart from Raphael and Winter-Ebmer, we also have good reasons for including oil price shocks as an instrument for unemployment. A quick look at Panel 4, in the Appendix, reveals a certain pattern between oil prices an unemployment. Moreover, we argue that shocks on oil prices are not related neither with our exogenous covariates, such the electoral rule and the institutional constraints, nor with other unobserved, and therefore omitted, political variables that might affect fractionalization. Since it is very hard to argue that the shocks on oil prices can cause changes in the electoral rule, or the structure of the party system in any given country, we think that it is an instrument that satisfies the exclusion restriction.

Of course, someone might argue that extremely high oil prices might cause public unrest and wide scale social disruption that might lead to a regime change or violent events. But, so far and to our knowledge, there is no evidence that oil prices can create such social dynamics which in turn

might affect the party system. Parallel to that, we are focusing our analysis on the most advanced western democracies of the world, where especially after the seventies there is no recorded evidence of wide civil unrest or uprising cause by oil prices that has led to a restructuring of the party system. All countries in our data set had fair, democratic and peaceful electoral competitions uninterruptedly from 1970 and onwards. This observation supplements our argument that the effect of oil prices in electoral fractionalization is solely coming through economic conditions and in particular unemployment.

Furthermore, we argue that our instrument is independent of the outcome, electoral fractionalization. That is, shocks on oil prices not only affect fractionalization only via unemployment but also electoral fractionalization cannot have an effect on the oil price shocks. On this latter point, there is a growing amount of empirical evidence that suggest that oil prices follow a pattern that is hardly affected by the voting behavior of voters in any OECD economy, not to mention electoral fractionalization. Pindyck (1999) and Barnett and Vivanco (2003) show evidence on mean-reversion whereas Cashin, Liang and McDermott (2000) and Engel and Valdes (2000) find evidence of persistence. Bartsch (2006) makes the point that the international oil prices show very weak mean reversion.

Thus, we think that It would be extremely hard to argue that changes in the fragmentation of the party system in any OECD economy can predict or affect the shocks on the price of oil. This argument would amount to claiming that elections in a country can affect the price of oil. But, even in the extreme case that one is willing to claim that political outcomes in oil exporting countries can drive the shocks on the price of oil, this argument would not apply in our data set since we do not include major oil exporting countries, such as Russia or Saudi Arabia. The only oil exporting country in our data set, which could potentially affect the oil prices, is Norway. Yet, as a robustness check, we conduct our analysis in a restricted sample that excludes the oil exporting countries. Since we do not get significantly different results, we conclude that there is no reason for further worry.

Having decided to proceed in our empirical analysis using real imported oil prices as our exogenous instrument for unemployment, we formally want to estimate the following 2SLS model:

$$(Unemployment)_{i,t} * e_{i,t} = b_0 + b_1 P_{i,t-1} + X'_{i,t}\gamma + a_i + \lambda_t + \varepsilon_{i,t}$$
and
$$F_{i,t} * e_{i,t} = \beta_0 + \beta_1(\hat{U}nempl.)_{i,t} + X'_{i,t}\gamma + a_i + \lambda_t + \nu_{i,t},$$

where $P_{i,t-1}$ is the Producer Price Index-weighted (PPI) real price of imported crude oil at refinery and $X'_{i,t}$ is the set of controls discussed in the previous section. Again, our coefficient of interest β_1, is expected to have a negative sign as predicted by the theory. Yet, we also expect that $b_1 < 0$, which means that in the effect of an increase in the oil price is negative(positive) to unemployment(employment).

Initially, our expectation on the sign of b_1 might sound counter-intuitive. Yet, it is consistent with a "Phillips curve" type of argument. Furthermore, looking at the data in Panel 4, we can get a first indication on the expected sign of b_1. In fact, we will argue that, if our model is estimated correctly, b_1 must be small in magnitude, yet a negative number. In order to justify this expectation, we need to go back one step and present the findings of recent literature with respect to the effect that oil shocks have on unemployment.

Firstly, there is a growing consensus among the literature, that the effects of an oil price shock on unemployment were not constant overtime. Especially after the 1974 oil crisis, the impact of

oil price shocks on the macroeconomy has changed. In a early paper, Sweder van Wijnbergen (1985) presents a theoretical model of intertemporal analysis, where there is disequilibrium in the goods and labour market in the first period, in order to study the effect of oil price shocks on the labour market and the current account. Although his analysis focuses mainly on the 1974-75 oil crisis, it can provide some critical insight on what might have happened after the seventies. He argues that in the very short run the supply shock in the goods market, caused by an increase in oil prices, dominates and causes unemployment to increase. Yet, in the medium and long run the labour market adjusts. The increase in the price level causes real wage to fall below the worker's productivity and hence firms are willing to employ more and produce more, given demand. Then, if the hike in inflation is met by a cut in the rate of interest, investment will increase and there will be excess demand in both labour and goods markets. Especially for the developing economies, that run CA deficits the effect of increased investment on employment will be larger[16]. If this increase in investment is also coupled by expansionary fiscal policy then the long-run effect of an increase in oil prices on employment is positive. That is, the initial negative shock that drives unemployment upwards, is quickly reversed by monetary, and perhaps fiscal, adjustment. The paper also makes the claim that the stylized facts on the US economy during the 1975 oil crisis confirm this analysis. Unemployment in the US reached its peak in mid 1975 to rebound shortly afterwards and start declining due to the effect on the real wage.

One potential explanation for this is the ability of enterprises and industry to better absorb the costs of higher oil prices by means of passing the effect on the consumers or by having stocks of output so that it can suspend production or even shift production to less oil intensive goods. To the extend that consumers are more adversely affected by rising oil prices compared to the industry, and to the extend that degree of substitutability is large enough, we can expect the effect of falling real wage to dominate over rising costs of production. Hence, after an initial negative shock on employment, we can expect that soon enough employment picks up again.

In support of this latter hypothesis comes the paper by Keane and Prasad (1996), who study the effects of oil prices on real wage and employment, using panel data from the National Longitudinal Survey of Young Men in the US. In their paper, they find that while the short-run effect of an oil price increase on employment is negative, the long-run effect is in fact positive. The main reason for this finding is that increases in the price of oil unambiguously cause real wage to decline at the aggregate level in all sectors of industry. Moreover, they estimate that the fall on real wage is on the level of 4-5% for a standard deviation increase in the price of oil (19% approximately). Since the effect of an oil price increase on labour demand depends on the degree of substitutability between oil and labour, they explain their finding by noting that labour and oil are not gross but net substitutes. As a result, the fall in real wage is to be expected. Hence, they conclude that it is those changes in relative wages and the sectoral changes in the labour market induced by the higher oil prices that can explain the positive effect on employment.

This idea, that structural changes and increased flexibility in the labour market induced by oil price changes can explain the differential impact on the economy across episodes between the 1970s and nowadays, is made by Blanchard and Gali (2007). Applying structural VAR techniques in a sample of advanced OECD economies they try to estimate the impact of an oil price shock to unemployment, inflation and output. In this sense their findings, given their time span and the set of countries are directly relevant to our present study. In general, they derive three main

[16] We note that at the time of this analysis refers to (mid seventies) the developed European Union incorporated only 6 member states and many of our OECD economies (Greece, Spain, Portugal and others) were considered to be developing ones.

conclusions: Firstly, the impact of oil price shocks on the macroeconomy has changed over time and in particular after 1980. In that respect the episodes of 1974 are not identical with those that followed afterwards. Secondly, they attest that oil price shocks overtime have driven labour markets to become more flexible, and thirdly, that due the shocks the share of oil in production has decreased overtime.

In particular, with respect to unemployment and real wage, they show that the magnitude and the direction vary between the episodes of 1973 and 1979 and the episodes in early 2000. While unemployment rises sharply during the episodes of the 1970s, this trend reverses in the 1999-2000 episode where unemployment falls in a response to an increase in the oil prices. Moreover, in the 2002-2005 episode unemployment is declining while oil prices are still on the rise. Given that our sample contains data after 1970 for the majority of the countries, it is normal to expect b_1 to be slightly negative. Finally, their VAR estimations yield similar results. It is estimated that CPI inflation and wage inflation reach their peak at Q.4 after the incidence of an oil episode. This implies that due to more flexible labour markets and the fall in the real wage, employment starts to pick up during Q.4 and unemployment starts to decline in the same time. In addition, they show that this trend of increase in employment and decline in unemployment is more charecteristic of the late 1990s oil shocks, especially for countries like Japan, France and Germany.

In light of the above, our estimates on coefficient b_1 should not come as a surprise, given that we use the one year lagged PPI price of imported oil as our exogenous instrument. Table 2 below, summarizes the first stage results of our 2SLS model, while Table 3 compares our benchmark 2SLS model with the OLS. Columns (4) and (5) of Table 2 are the first-stage regressions of the 2SLS models presented in Columns (2) and (4) in Table 3, respectively. Columns (2) and (3) of Table 2 represent the first-stages of regressions in Columns (1) and (2) in Table 4, respectively.

More specifically, in the first three columns of Table 2, we use the restricted sample of oil importing countries. In Columns (4) and (5) we use the full data set. Yet, our estimates on the impact of an oil price increase on unemployment do not change. In all five specifications we find the effect to be negative and statistically significant at the 1% level. Furthermore, the F statistic on the excluded instrument is above the value of 10, under all alternative specifications. Since the estimates on our first stage regressions have an economic interpretation, and moreover they are highly statistically significant under all specifications, we decide to use the lag or real oil prices as our instrumental variable.

Table 2: *First Stage Regressions of Benchmark 2SLS Model*
Dependent Variable: *Unemployment rate (%)*

	OLS	OLS	OLS	OLS	OLS
	(1)	(2)	(3)	(4)	(5)
L.Imported Oil Price (real)	-.0088	-.0088	-.0087	-.0082	-.0082
	$(0.0023)^{***}$	$(0.0023)^{***}$	$(0.0026)^{***}$	$(0.0023)^{***}$	$(0.0023)^{***}$
L.Debt/GDP (%)	0.0475	0.0475	0.0548	0.0457	0.0457
	$(0.015)^{***}$	$(0.015)^{***}$	$(0.0127)^{***}$	$(0.0148)^{***}$	$(0.0148)^{***}$
Dummy Plurality ER	- - -	5.5656	2.5882	- - -	5.4399
		$(0.7167)^{***}$	$(1.0446)^{**}$		$(0.722)^{***}$
Dummy High Institut. constraints	- - -	- - -	-1.6668	- - -	- - -
			(1.0559)		
Year Fixed Effects	Y	Y	Y	Y	Y
Country Fixed Effects	Y	Y	Y	Y	Y
Obs.	211	211	208	220	220
Sample Size	Oil Import.	Oil Import.	Oil Import.	Full	Full
R^2	0.7958	0.7958	0.8075	0.7969	0.7969
F statistic (excluded instrument)	14.02	14.02	10.76	12.30	12.30

Table 2: *Clustered Standard Errors at the Country level reported in parantheses. (***) Statistically significant at the 1% level; (**) Statistically significant at the 5% level; Statistically significant at the 10% level.*

In the next table (Table 3), we compare our 2SLS estimates on the effect that unemployment has on electoral fractionalization, with those under OLS. We compare Columns (1) with (2), (3) with (4) and (5) with (6). In all three specifications, the coefficient on unemployment is negative and statistically significant at the 5% level. As expected, the OLS estimates were biased towards zero whereas under 2SLS the coefficient increases in magnitude and becomes a more negative number, as our theoretical model predicts.

Furthermore, with the exception of Columns (1) and (2) our OLS results failed to be statistically significant under various alternative specifications. In fact, when we estimate the impact of unemployment on fractionalization using a 2SLS estimator, under all specifications, our coefficient estimates become statistically significant. We attribute the improvement in our estimates in both the reduction of OVB, and the fact that by instrumenting unemployment with real oil prices we account for endogeneity.

Table 3: *Comparison of 2SLS and OLS Models under Alternative Specifications*
Dependent Variable: *Rae Index of Electoral Fractionalization;*
Instrumental Variable: *PPI Imported Oil Prices at Refinery*

	OLS1	IV1	OLS2	IV2	OLS3	IV3
	(1)	(2)	(3)	(4)	(5)	(6)
Unemployment (%)	-.0702	-1.6221	-.0702	-1.6221	-.0435	-1.6320
	(0.1709)	(0.6763)**	(0.1709)	(0.6763)**	(0.1676)	(0.6782)**
L.Debt/GDP (%)	-.0205	0.0571	-.0205	0.0571	-.0255	0.0643
	(0.0211)	(0.0522)	(0.0211)	(0.0522)	(0.0234)	(0.0529)
Dummy Plurality ER	- - -	- - -	-3.8809	-1.3706	3.1011	-24.9690
			(1.2775)***	(2.6330)	(2.3164)	(2.2638)***
Dummy High Institut. constraints	- - -	- - -	- - -	- - -	1.5203	-1.4899
					(2.0329)	(2.7795)
Other Controls	N	N	N	N	N	N
Year Fixed Effects	Y	Y	Y	Y	Y	Y
Country Fixed Effects	Y	Y	Y	Y	Y	Y
Const.	71.8913	58.2954	68.1551	59.6660	66.9462	84.6091
	(2.5181)***	(1.9061)***	(2.1913)***	(3.3564)***	(2.4275)***	(3.0022)***
Obs.	220	220	220	220	217	217
Instrument	No	L.1 Price	No	L.1 Price	No	L.1 Price
Sample	Full	Full	Full	Full	Full	Full
R^2	0.9321	0.8659	0.9321	0.8659	0.9318	0.8651
F statistic (1^{st} Stage)		12.30		12.30		12.30

Table 3: *Clustered Standard Errors at the Country level reported in parantheses. (***) Statistically significant at the 1% level; (**) Statistically significant at the 5% level; Statistically significant at the 10% level.*

One last comment before proceeding to the next section, where we present and discuss in more detail our 2SLS estimates, relates with the use of the full sample of countries. Since in the discussion we argued in favour of using only the set of oil importing countries, we want to take the opportunity here to reassure the reader that the results we obtain in Table 3 do not change dramatically once we restrict our sample to include oil importing countries only.

3.6 Results of Instrumental Variable Regressions

The estimated results of the 2SLS model, that were presented above in Table 3, are repeated below in Table 4 for the restricted sample, which now includes only the oil importing OECD countries. Columns (1) and (2) in Table 4 present the estimates of the *exact same* model as in Columns (4) and (6) in Table 3. The only difference is in the size of our sample. In this table, we restrict our attention only to those countries that are net importers of oil.

A first look reveals that in both cases the coefficient of interest on unemployment has the same sign and it is almost equal in magnitude (-1.62 in Table 3 compared to -1.46 in Table 4). Moreover, it is statistically significant even at the 1% level, when it comes to our specification that includes all the possible exogenous control variables. In addition, the F statistic on the excluded instruments is once more above the desired threshold. In this sense, it is reassuring to see that our model is very robust in this alteration of the sample size. Furthermore, apart from our key explanatory variable unemployment, public debt has a positive sign (approximately .06), as predicted by our theoretical model and in accordance with our empirical findings in Panel 2. That is, $\beta_1 < 0$ and $\beta_2 > 0$. Nevertheless, only the coefficient on unemployment is statistically significant, the reason being the absence of a good instrument for public debt[17]. Yet, since we believe it is a determinant of electoral fractionalization according to our theoretical model, we find it necessary to include it. Column (3) compare our IV results with the basic OLS model.

The results in Columns (1) and (2) will be the main focus of this section. The coefficient on unemployment can be read as saying that an one percentage point increase in the unemployment rate between elections can be associated with an 1.45 percentage points decrease in electoral fractionalization. Given that the average fractionalization level in the sample countries is 0.72, we can say that an 1% increase in the unemployment rate is associated with a 2% decrease in electoral fractionalization. Or, to put it differently, an one percentage point variation in unemployment between election years can account for the 16% of the variation in fractionalization, on average. For a sole real economic variable, we think that the effect is quite impressive and yields support to our "Hostage-voter" hypothesis.

Also, with respect to our second variable of interest, we find a positive association between public debt and fractionalization. Although it is not statistically significant, the sing of the coefficient β_2 is in line with previous results by Persson, Tabelini et al. (2007). Yet, the difference between our estimates and theirs is that we hypothesize the direction of causality running the other way. But, since we don't instrument for public debt, we only confine ourselves in noting this positive, and to some extend expected, relationship between public debt and fractionalization without making any further inference on the direction of causality.

Furthermore, the signs on our other control variables are in line with the predictions of the literature. The coefficient on the electoral rule dummy is negative and highly statistically significant. This estimate reads as follows: the change from a pure PR-list system ($D_i = 0$) to a first-past-the-post, single-member district Plurality rule ($D_i = 1$) is associated with a decrease in electoral fractionalization by 25 percentage points, or approximately 1/3 of the fractionalization level, on average. This finding is in accordance with Duverger's Law that relates the fragmentation of the party system with the electoral rule.

[17] In the section that follows, we discuss more extensively the problems faced in identifying a valid instrument for public debt as well.

Table 4: *Benchmark 2SLS Model on Restricted Sample*
Dependent Variable: *Rae Index of Electoral Fractionalization;*
Instrumental Variable: *PPI Imported Oil Prices at Refinery*

	2SLS(2)	2SLS(3)	OLS-3
	(1)	(2)	(3)
Unemployment (%)	-1.4670	-1.4509	-.0702
	(0.5735)**	(0.5451)***	(0.1767)
L.Debt/GDP (%)	0.0528	0.0582	-.0232
	(0.049)	(0.0489)	(0.0248)
Dummy Plurality Rule	-1.4067	-25.5136	2.8671
	(2.4855)	(1.7741)***	(2.5046)
Dummy High Institut. constraints	- - -	-1.0904	1.5625
		(2.4783)	(2.0266)
Country Fixed Effects	Y	Y	Y
Year Fixed Effects	Y	Y	Y
Const.	59.2052	84.1763	57.4329
	(3.1600)***	(2.8308)***	(3.1188)***
Obs.	211	208	208
Instrument	L.1 Price	L.1 Price	L.1 Price
Sample Size	Oil Import.	Oil Import.	Oil Import.
R^2	0.8818	0.883	0.9331
F statistic (1^{st} Stage)	14.025	10.755	

Table 4: Note: *Clustered Standard Errors at the Country level reported in parantheses. (***) Statistically significant at the 1% level; (**) Statistically significant at the 5% level; Statistically significant at the 10% level.*

Although the results that we will present in Tables 6 and 7 under alternative specifications are qualitatively equivalent, since they have the same sign across columns and they are numerically close, we have decided to focus on the previous two. The reason is that the instrument we use to estimate the effect of unemployment on fractionalization is the simplest one possible. That is, we use the one year lagged value of the price of imported oil at refinery, weighted by the Produce Price Index (PPI!) with 2005 as a base year. We call this price, real oil price. The reason of not employing more indexation on our measure of oil price shocks is that we fear that introducing more complicated techniques will compromise our data and allow room from extra critique directed at our instruments. In a sense, if real oil prices do not have a statistically significant effect on electoral fractionalization, then it would be hard to argue that it is the instrument and not the chosen indexation or technique that caused this relationship to be statistically significant. But, as the reader can see in Table 5 below, even with the simplest possible measure of oil price shocks, our reduced form OLS results confirm our intuition on the effect that the oil prices have on electoral fractionalization, via unemployment. Formally, we estimate the following model:

$$F_{i,t} * e_{i,t} = \gamma_0 + \gamma_1 P_{i,t-1} + X'_{i,t}\gamma + a_i + \lambda_t + \xi_{i,t}$$

Table 5: Reduced Form OLS of the Benchmark TSLS Model
Dependent Variable: *Rae Index of Electoral Fractionalization;*

	OLS	OLS	OLS	OLS	OLS	OLS
	(1)	(2)	(3)	(4)	(5)	(6)
L.Imported Oil Price (real)	0.0134	0.0134	0.0131	0.0129	0.0129	0.0126
	(0.0048)***	(0.0048)***	(0.0044)***	(0.0046)***	(0.0046)***	(0.0041)***
L.Debt (% of GDP)	-.0170	-.0170	-.0212	-.0168	-.0168	-.0214
	(0.0204)	(0.0204)	(0.0217)	(0.0211)	(0.0211)	(0.0226)
Dummy Plurality ER	- - -	-8.5425	-4.8574	- - -	-8.3727	-4.9794
		(0.8554)***	(2.0861)**		(0.8561)***	(2.1058)**
Dummy High Institut. constraints	- - -	- - -	1.2099	- - -	- - -	1.3279
			(2.1415)			(2.1098)
Country Fixed Effects	Y	Y	Y	Y	Y	Y
Year Fixed Effects	Y	Y	Y	Y	Y	Y
Const.	64.7447	78.8264	64.3372	70.4317	79.4756	64.3224
	(1.3638)***	(1.1865)***	(1.6404)***	(1.5054)***	(1.6927)***	(1.6820)***
Obs.	220	220	217	211	211	208
Sample Size	Full	Full	Full	Oil Import.	Oil Import.	Oil Import.
R^2	0.9346	0.9346	0.9343	0.9355	0.9355	0.9353
F statistic						

Table 5: Note: *Clustered Standard Errors at the Country level reported in parantheses. (***) Statistically significant at the 1% level; (**) Statistically significant at the 5% level; Statistically significant at the 10% level.*

Since both β_1 and $b_1$are negative, we expect $\gamma_1 > 0$. In fact, a quick look in Table 5, confirms our expectation. The first three Columns, correspond to the reduced form relationships of the TSLS models presented in Table 3 (Full Sample), whereas Columns (4)-(6) depict the same relationships for the TSLS models of Table 4 (Net Oil Importers). Although excluding net oil exporters reduces slightly the magnitude of γ_1, in all six cases it is positive and statistically significant at the 1% level. Hence, we confirm our initial choice of real oil prices as an instrument.

Furthermore, what is even more reassuring, is the fact that our TSLS model is extremely robust to the use of alternative specifications to measure the oil price shocks. In Table 6, we present the estimates of our Model in Table 4 using two alternative instruments: the de-trended component of the real oil price and the predicted residuals of an AR(2) process on the real oil price. In the first case to detrend the oil prices we use the Hoddrick-Prescott filter with smoothing parameter 6.25, given that we have yearly observations. The main reason for using the detrended component of oil prices is the ability to capture the shocks on oil prices better. We present the estimates of this model on Column (5) on the restricted sample. Since the coefficient estimates are statistically significant at the 1% level, negative and almost equal in magnitude (-1.7 compared to -1.46) we think that there is no need to elaborate further on the Hoddrick-Prescott technique. After all, this is not the main point of the paper. As long as our results are qualitatively equally important, we are happy to include it as a robustness check but we don't think it warrants further analysis.

In the remaining columns, we use the predicted residuals on oil price, after we allow it follow an AR(2) process. There is extensive literature documenting that oil prices might follow a mean-reverting process. Formally we estimated the following model:

$$P_{i,t} = \phi_{i,0} + \phi_{i,1} P_{i,t-1} + \phi_{i,2} P_{i,t-2} + u_{i,t},\ i = 1, ..., 23$$

so that our First-Stage Regression becomes:

$$(Unemployment)_{i,t} * e_{i,t} = b_0 + b_1 \hat{u}_{i,t} + X'_{i,t}\gamma + a_i + \lambda_t + \varepsilon_{i,t},$$

where $\hat{u}_{i,t}$ is the predicted residual of the above AR(2) process on $P_{i,t}$ and all the other controls remain the same as before.

Once more, we observe that our estimation results do not change dramatically compared to our estimates in Tables 3 and 4. All the coefficients on unemployment are highly statistically significant and the estimated values are all negative and quite close in magnitude, ranging from -1.41 to -1.78. Moreover, in all cases the F statistic on the excluded instrument is larger than the desired critical value. As a result, we have decided to include our estimates as a further robustness check, but we think that more discussion about the AR(2) is not warranted in this section. We only commend that choice of AR(2) was far from being arbitrary. Given that we observed systematic and significant auto-correlation on all the residuals until the second lag, we concluded that an AR(2) process was the optimal one to get the most accurate estimations on the real oil shocks.

Finally, to conclude this section, we include a final table in the Appendix, where a robustness check is performed with respect to the choice of our 2SLS estimator. As we have argued in the beginning of this section, we think that the just identified 2SLS model is the optimal choice to predict the effect of unemployment on fractionalization operating through oil price shocks. Yet, in Table 7 we estimate our benchmark model using the GMM estimator. Once more, the estimates on the coefficient β_1 are negative and quite similar in magnitude, ranging from -1.5 to -1.87. But this time, they marginally fail to be statistically significant at the 5% level. Also another complication related with the GMM method is that we don't have enough clusters (23), given that we use year and country dummy variables in order to construct clustered standard errors. For these reasons, we conclude that the 2SLS model is the optimal one. Regardless, our results are robust under both specifications, which allows us to proceed in their causal interpretation.

Table 6: Robustness of Estimation Results under Alternative Instruments
Dependent Variable: *Rae Index of Electoral Fractionalization;*
Instrumental Variable: *PPI Imported Oil Prices at Refinery*

	IV-1	IV-2	IV-2b	IV-3	IV-4	IV-5
	(1)	(2)	(3)	(4)	(5)	(6)
Unemployment (%)	-1.4509	-1.4171	-1.4841	-1.7844	-1.7117	-1.9377
	(0.5451)***	(0.5895)**	(0.667)**	(0.6774)***	(0.6387)***	(0.8074)**
L.Debt/GDP (%)	0.0582	0.0562	0.0537	0.0692	0.0735	0.0729
	(0.0489)	(0.0514)	(0.0543)	(0.0574)	(0.0571)	(0.061)
Dummy Plurality ER	-25.5136	-25.6090	-1.3626	-.5911	-24.7774	-.6060
	(1.7741)***	(1.9060)***	(2.7435)	(2.9410)	(2.0784)***	(3.1202)
Dummy High Institut. constraints	-1.0904	-1.0255	- - -	- - -	-1.5915	- - -
	(2.4783)	(2.5304)			(2.7602)	
Country Fixed Effects	Y	Y	Y	Y	Y	Y
Year Fixed Effects	Y	Y	Y	Y	Y	Y
Const.	84.1763	84.1361	59.1981	59.0730	84.4862	59.6062
	(2.8308)***	(2.8228)***	(3.2120)***	(3.6188)***	(2.9949)***	(3.8118)***
Obs.	208	208	211	211	208	220
Instrument	L.1 Price	Res.AR(2)	Res.AR(2)	L.Res.AR(2)	L. Trend	L.Res.AR(2)
Sample	Oil Import.	Oil Import.	Oil Import.	Oil Import.	Oil Import.	Full
R^2	0.883	0.8854	0.8805	0.8552	0.8623	0.8363
F statistic (1^{st} Stage)	10.755	11.345	11.778	11.455	11.243	11.381

Table 6: Note: *Clustered Standard Errors at the Country level reported in parantheses. (***) Statistically significant at the 1% level; (**) Statistically significant at the 5% level; Statistically significant at the 10% level.*

3.7 Discussion of the Results

The importance of our empirical findings stems out from two factors. Firstly, our results yield support for the "Hostage-Voter" Hypothesis and its main underlying idea. Because, in putting forward this claim we have shown in our theoretical model the significant impact that the macro-economy has on voting behavior, electoral outcomes and the party system, as measured by the level of electoral fractionalization. Therefore, it is quite important to note that our empirical estimations yield extremely strong support to the findings of the theory.

In specific, in our sample the average variation in the unemployment rate between elections is 3.9 percentage points, whereas the average variation on the level of electoral fractionalization is 9.5 percentage points. Given that one percentage point increase in unemployment is causing a minimum of 1.45 percentage points decrease in electoral fractionalization, it is easy to see that a standard deviation in unemployment can account for almost the 2/3 of one standard deviation in electoral fractionalization. Or equivalently, one standard deviation change in unemployment can be associated with more than 8 percentage points change in the distribution of electoral power between parties.

Even though we acknowledge that there might be other factors, for instance the electoral rule, that affect fractionalization and might counteract with unemployment, thus, reducing the variation in fractionalization, this is still an extremely astonishing number. Especially given the fact that we are only accounting for a sole real economic variable. If we also account for the effect of public debt, then, we can get an idea of how important the role of the economy is in determining the electoral behavior and fractionalization. In fact, the economy seems to be the single most important determinant of electoral behavior in western OECD democracies.

Secondly, despite the fact that the importance of the economy in determining electoral outcomes is documented, the degree upon which this happens, in association with the "Hostage-voter" hypothesis, allows us to revisit the traditional political business cycle theory. That is, our study empirically documents that unemployed voters are held "hostages" by the dominant parties, which exploit the fact that their voting behavior is mainly driven by rent-seeking. As a result, an increase in unemployment causes a reduction of electoral fractionalization. Hence, the dominant parties might benefit from higher unemployment, under certain circumstances. And, although we do not go as far as to suggest that big parties have an incentive to pursue higher unemployment, we note that our empirical findings provide some reasoning as to why some parties might favour fiscal discipline and debt reduction polices over policies that aim to reduce unemployment.

This stands in direct contrast with common wisdom as expressed in the traditional "opportunistic" model of political business cycle theory, developed by Nordhaus (1975). Nordhaus' model predicts that parties out of pure opportunistic motives would have an incentive to reduce unemployment before the elections. Yet, Alesina and Roubini find that empirical evidence are not in accordance with Nordhaus' opportunistic model predictions. Instead, they find support for Alesina's "partisan rational expectations" model of political business cycles.

Our model, in essence, bridges the gap between those two models. While it is a model of opportunistic parties, its predictions on the relationship between unemployment and the political cycle are more in line with the partisan rational expectations model. The main reason for this is the approach we follow, by reversing the link between political outcomes and the economy. In the Alesina model parties have exogenously given, differential preferences on unemployment and inflation. In our model parties' preferences on unemployment and debt are derived from the effect that the macroeconomy has on electoral fractionalization. That is, we endogenize partisan preferences on the macroeconomy and we explain how the data can fit the opportunistic model. In that respect, our models results are similar with the Alesina partisan model of political cycles yet, they offer a further insight on why parties might have differential preferences on macroeconomic variables. Therefore, our model is not antagonistic with theirs. Rather, it helps complete the political business cycle theory by exploring both directions of the link between partisan politics and the macroeconomy.

4 Concluding remarks

To sum up, in this paper we have attempted to give a more comprehensive explanation to the phenomenon of varying electoral fractionalization in western OECD democracies, that will take into account the impact of economic factors on electoral behavior. Rather than arguing in the traditional way, that the degree of electoral fractionalization depends solely upon the existing electoral rule, we have developed and solved a theoretical model that relates electoral fractionalization with economic conditions. In specific, we have attempted to show how opportunistic voting can

affect electoral fractionalization in a democracy. To this aim, we have built a theoretical model of political competition among four parties.

The major contribution of the model is the formal formulation of the "Hostage-voter" hypothesis and of the relationship between unemployment, voting behavior and electoral fractionalization, which we have found to be negative. Of course, this is not to say that opportunistic, rent-seeking voting behavior is the sole determinant of electoral fractionalization in a democracy. There might be other factors, such as ideology or institutional design, that affect citizens' voting decisions, which in turn determine the level of fractionalization. Yet, as our theoretical model suggests, there is a significant relationship between economic conditions, such as unemployment and public debt, and fractionalization. Additionally, the model also offers and interesting insight, based on rational choice theory, as to why extremist parties target the more ideologically driven voters, while centrist ones focus more on tangible political promises, such as increased public spending.

Apart from the theoretical predictions, our model also makes a significant empirical contribution. That is, our empirical findings yield strong support to the predictions of the model and document both the occurrence of the "Hostage-voter effect", and the impact of the macroeconomy on electoral fractionalization. In that respect, our study is unique since it is the first to document both theoretically and empirically the relationship between economic conditions and the fragmentation of the party system. In fact, we find a statistically significant and quantitatively important relationship between unemployment, debt and electoral fractionalization. Moreover, it appears to be extremely robust to variations of the model.

Furthermore, the empirical findings in support of the "Hostage-voter" effect and the impact that unemployment has on electoral fractionalization allows us to study the link between economic conditions and electoral outcomes in both directions, thus contributing in enriching and completing the theory of political business cycles. And even though our paper does not explicitly provides a theoretical model of political cycles, it clear hints the intuition behind such a model. Since some political parties, not due to ideology but driven solely by opportunistic motives, might prefer to exercise fiscal discipline more than reducing unemployment, then, our empirical findings provide the ground to reconcile the stylized facts and "partisan rational expectations" political cycles theory with the traditional opportunistic one. Hence, our paper contributes to that aim as well, although we leave its formal exploration for future research.

Finally, on a more technical scope, our paper provides further support to the existing literature for using real oil price shocks as an instrument for unemployment. Given that there is a limited amount of studies which document the use of this instrument, our results add further justification to this growing body of empirical literature. Moreover, although not the main purpose of this study, our empirical results confirm the findings of other studies about the differential impact of oil price shocks on employment both between the short-run and the long-run and also between the late 1970s and the early 1990s. Nevertheless, our study leaves open some interesting questions that still require further research. Thus, we will attempt to address them in a future endeavour.

5 References

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6 Appendix A: Proofs

Proposition 1. We need to check that the proposed equilibrium $\boldsymbol{p}^*$ is indeed a symmetric NE. For symmetricity, we simply apply the definition. Then, to prove that it is a NE, we will have show that no party has an incentive to deviate unilaterally from its equilibrium strategy, that is: $\forall \omega \in \Omega$, $V_\omega(p^*_\omega, p^*_{-\omega}) > V_\omega(p'_\omega, p^*_{-\omega})$, $\forall\ p'_\omega,\ p^*_{-\omega}$.

In order to proceed, we need first to calculate the vote share that each party receives under the proposed equilibrium. To do so, we have to identify the voter who is indifferent between voting for party r or R, R or L and L or l respectively. Then, we can compute the vote share for each party under $\boldsymbol{p}^*$. We begin by identifying the indifferent voter between parties r and R. Formally, the following equality must hold[18]:

$U_{y,x}(p_r) = U_{y,x}(p_R) \iff U_{y,x}(\frac{1}{2}, t^r = 0) = U_{y,x}(\epsilon, t^R = T)$

$-|\frac{1}{2} - x| + u(y + t^r) = -|\epsilon - x| + u(y + t^R) \iff$

$-(\frac{1}{2} - x) + u(y + 0) = -(x - \epsilon) + u(y + T) \iff$

$x = \frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}[u(y + T) - u(y + 0)]$ or $x = \frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}(\sqrt{y + T} - \sqrt{y})$

Given that a fraction q of the electorate has $y = m$ and the remaining $1 - q$ has $y = M$, and given that the two continua of voters are identical in all other respects, we can then compute the "aggregate" indifferent voter:

$x = \frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}[q(\sqrt{m + T} - \sqrt{m}) + (1 - q)(\sqrt{M + T} - \sqrt{M})]$.

Then, all voters to the right of x will vote for party r whereas all voters to the left of x (and till voter ϵ) will voter for party R. By a symmetric argument a similar analysis applies when we compare the voter who is indifferent between parties L and l.

We now repeat the same exercise for parties R and L[19]. Again, the following equality must hold:

$U_{y,x}(p_L) = U_{y,x}(p_R) \implies U_{y,x}(-\epsilon, t^L = T) = U_{y,x}(\epsilon, t^R = T)$.

Assume that $x \in (0, \epsilon]$, then $U_{y,x}(-\epsilon, T) = U_{y,x}(\epsilon, T) \implies -| - \epsilon - x| + u(y + T) = -|\epsilon - x| + u(y + T) \implies$

$x = 0 \implies x \notin (0, \epsilon]$.

Also, if $x \in [-\epsilon, 0)$, then $U_{y,x}(-\epsilon, T) = U_{y,x}(\epsilon, T) \implies -| - \epsilon - x| + u(y + T) = -|\epsilon - x| + u(y + T) \implies$

$x = 0 \implies x \notin [-\epsilon, 0)$.

Hence, since both equalities result in contradictions, we conclude that the indifferent voter is $x = 0$. Then, we can compute the (symmetric) vote share allocation that is realized under $\boldsymbol{p}^*$:

$v_L(\boldsymbol{p}^*) = v_R(\boldsymbol{p}^*) = \frac{1}{4} - \frac{\epsilon}{2} + \frac{1}{2}[q(\sqrt{m + T} - \sqrt{m}) + (1 - q)(\sqrt{M + T} - \sqrt{M})] + \epsilon =$

$= \frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}[q(\sqrt{m + T} - \sqrt{m}) + (1 - q)(\sqrt{M + T} - \sqrt{M})]$ and

$v_l(\boldsymbol{p}^*) = v_r(\boldsymbol{p}^*) = \frac{1}{4} - \frac{\epsilon}{2} - \frac{1}{2}[q(\sqrt{m + T} - \sqrt{m}) + (1 - q)(\sqrt{M + T} - \sqrt{M})]$.

The realized outcome, given the above vote share allocation, is clearly $\pi(\boldsymbol{v}) = (\frac{1}{2}L, \frac{1}{2}R)$. Then, we can compute the resulting utility outcomes for all parties. There are as follows:

$V_R(\boldsymbol{p}^*) = -\frac{1}{2}|\epsilon - \epsilon| - \frac{1}{2}| - \epsilon - \epsilon| + v_R(\boldsymbol{p}^*) =$

$= -\epsilon + \frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}[q(\sqrt{m + T} - \sqrt{m}) + (1 - q)(\sqrt{M + T} - \sqrt{M})]$

$V_L(\boldsymbol{p}^*) = -\frac{1}{2}|\epsilon - (-\epsilon)| - \frac{1}{2}| - \epsilon - (-\epsilon)| + v_L(\boldsymbol{p}^*) =$

$= -\epsilon + \frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}[q(\sqrt{m + T} - \sqrt{m}) + (1 - q)(\sqrt{M + T} - \sqrt{M})] = V_R(\boldsymbol{p}^*)$

[18] We need only examine the case where $x \in [\epsilon, \frac{1}{2}]$, since by single-peakedness and the fact that in equilibrium party r offers $p^*_r = (t^r_{\mathrm{x}}, t^r_{\bar{x}}) = (0, 0)$, any voters to the left of ϵ will never face the dillemma between L and r for that matter.

[19] We focus on the voters on the interval $[-\epsilon, \epsilon]$ since the other voters are not relevant for this comparison.

and equivalently by symmetry we have:
$V_r(\boldsymbol{p}^*) = -\frac{1}{2}|\epsilon - \frac{1}{2}| - \frac{1}{2}| - \epsilon - \frac{1}{2}| + v_r(\boldsymbol{p}^*) =$
$= -\frac{1}{2} + \frac{1}{4} - \frac{\epsilon}{2} - \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})]$ and
$V_l(\boldsymbol{p}^*) = -\frac{1}{2}|\epsilon - (-\frac{1}{2})| - \frac{1}{2}| - \epsilon - (-\frac{1}{2})| + v_l(\boldsymbol{p}^*) =$
$= -\frac{1}{2} + \frac{1}{4} - \frac{\epsilon}{2} - \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})]$

We will now examine unilateral deviations by parties r and R, since then, by a symmetry argument the same holds also true for l and L respectively. Consider $\boldsymbol{p}' = (p'_R, p^*_{-R})$, such that $p'_R \neq p^*_R$. That is, $p'_R = (\epsilon, t^R = 0)$, will never bring extra votes to R. In fact it will cause it to lose votes. To see this check that for the following is true $\forall\ q \in [0,1]$ and $x \in [0, \frac{1}{2}]$:

$U_{y,x}(p'_R) = U_{y,x}(p^*_L) \iff -|\epsilon - x| + \sqrt{y} = -|-\epsilon - x| + \sqrt{y+T} \implies \hat{x} = \frac{1}{2}(\sqrt{y+T} - \sqrt{y})$.Hence, all voters to the left of $\hat{x}$ will voter either for party L or l. That is, party R loses $\frac{1}{2}(\sqrt{y+T} - \sqrt{y})$ votes to party L and an equal amount to party r from this deviation.

But now observe that the realized vote share for party R is the following[20]:
$v_R(\boldsymbol{p}') = \frac{1}{4} + \frac{\epsilon}{2} - \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})]$
implying that
$v_R(\boldsymbol{p}') < \frac{1}{4} + \frac{\epsilon}{2} + [q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})] = v_L(\boldsymbol{p}')$.
Also observe the following:
$v_r(\boldsymbol{p}') = \frac{1}{4} - \frac{\epsilon}{2}$ and
$v_l(\boldsymbol{p}') = \frac{1}{4} - \frac{\epsilon}{2} - \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})]$.

Since the vote-share allocation between parties L and l under $\boldsymbol{p}'$ is the same, as under $\boldsymbol{p}^*$ we have that:
$v_L(\boldsymbol{p}') > v_R(\boldsymbol{p}') > v_r(\boldsymbol{p}') > v_l(\boldsymbol{p}')$

Hence, party L wins with certainty and we can compute the utility resulting from this deviation for party R.
$V_R(\boldsymbol{p}') = -|-\epsilon - \epsilon| + v_R(\boldsymbol{p}') = -2\epsilon + \frac{1}{4} + \frac{\epsilon}{2} - \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})] \implies$
$\implies V_R(\boldsymbol{p}') < V_R(\boldsymbol{p}^*)$.

So, we conclude that there is no profitable deviation for party R (and L respectively), since strategy $(t^R = 0)$ (and $t^L = 0$ respectively) does not do any better against p^*_{-R} (p^*_{-L}) in terms of altering the outcome of the elections (again party L wins).

Now, as a next step, consider unilateral deviations by extremist parties. Due to symmetry we need only focus on party r. Let $\tilde{\boldsymbol{p}} = (\tilde{p}_r, p^*_{-r})$ such that $\tilde{p}_r = (\frac{1}{2}, t^r = T)$. Then, by a computation directly analogous as above, one can find the indifferent voter $x \in [0, \frac{1}{2}]$ who in this case is $x = \frac{1}{4} + \frac{\epsilon}{2}$. To see this just set $U_{y,x}(\tilde{p}_r) = U_{y,x}(p^*_R) \iff -|\frac{1}{2} - x| + u_{y,x}(y+T) = -|\epsilon - x| + u(y+T)$ and observe that the $u(.)$ terms in both sides cancel out since under $\tilde{\boldsymbol{p}}$, we have $t^r = t^R = T$. Then, it is checked that the following vote share allocation is realized:
$v_r(\tilde{\boldsymbol{p}}) = \frac{1}{4} - \frac{\epsilon}{2} > v_r(\boldsymbol{p}^*) = \frac{1}{4} - \frac{\epsilon}{2} - \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})]$, but
$v_R(\tilde{\boldsymbol{p}}) = \frac{1}{4} + \frac{\epsilon}{2} < v_R(\boldsymbol{p}^*) = v_L(\boldsymbol{p}^*) = v_L(\tilde{\boldsymbol{p}})$.
Since also $v_l(\tilde{\boldsymbol{p}}) = v_l(\boldsymbol{p}^*) < v_L(\boldsymbol{p}^*) = v_L(\tilde{\boldsymbol{p}})$, we conclude that party L wins again with certainty[21].

But now, after computing utility for party r, observe that the following is always true:

[20] In practice, what happens is that the indifferent voter, between parties R and r, x is "moving" more to the left in the $[-\frac{1}{2}, \frac{1}{2}]$ axis from $\frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}[q(\sqrt{m(s)+2T_d} - \sqrt{m(s)}) + (1-q)(\sqrt{M+2T_d} - \sqrt{M})]$ to $\frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}[q(\sqrt{m(s)+T_d} - \sqrt{m(s)}) + (1-q)(\sqrt{M+T_d} - \sqrt{M})]$ or even to $\frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}[q(\sqrt{m(s)} - \sqrt{m(s)}) + (1-q)(\sqrt{M} - \sqrt{M})] = \frac{1}{4} + \frac{\epsilon}{2}$. As a result the vote share of party R is shrinking.

[21] Again, as shown in Lemma 1 party r can never win any votes on the $[-\frac{1}{2}, 0]$ no matter what it does, hence, the vote share allocation between parties l and L remains unchanged.

$V_r(\tilde{\boldsymbol{p}}) = -|-\epsilon-\frac{1}{2}| + v_r(\tilde{\boldsymbol{p}}) = -(\frac{1}{2}+\epsilon)+\frac{1}{4}-\frac{\epsilon}{2}$. Then $V_r(\tilde{\boldsymbol{p}}) < V_r(\boldsymbol{p}^*) \iff$
$-(\frac{1}{2}+\epsilon)+\frac{1}{4}-\frac{\epsilon}{2} < -\frac{1}{2}+\frac{1}{4}-\frac{\epsilon}{2}-\frac{1}{2}[q(\sqrt{m+2T}-\sqrt{m})+(1-q)(\sqrt{M+2T}-\sqrt{M})] \iff$
$\iff \epsilon > \frac{1}{2}[q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})]$

which is always true $\forall\epsilon \in (\sqrt{\frac{3}{8}}-\frac{1}{2}, \frac{1-\sqrt{\frac{3}{8}}}{3})$.

To see this note that, as shown in Lemma 1, $\forall T \in (0, \frac{1}{8}]$ and $\forall q \in [0,1]$ we can always compute:
$\max_{T,q} ||q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})|| =$
$= \sqrt{\frac{1}{4}+\frac{1}{8}}-\sqrt{\frac{1}{4}} = \sqrt{\frac{3}{8}}-\sqrt{\frac{1}{4}}$[22]. Hence, by strengthening the inequality above we have:
$V_r(\tilde{\boldsymbol{p}}) < V_r(\boldsymbol{p}^*) \iff \epsilon > \frac{1}{2}\max_{T,q} ||q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})|| = \frac{1}{2}(\sqrt{\frac{3}{8}}-\frac{1}{2})$,

which is always true $\forall\epsilon \in (\sqrt{\frac{3}{8}}-\frac{1}{2}, \frac{1-\sqrt{\frac{3}{8}}}{3})$.

Hence, we conclude that $\tilde{\boldsymbol{p}} = (\tilde{p}_r, p^*_{-r})$ cannot be a profitable deviation for r, since $\forall\epsilon$ we have $V_r(\tilde{\boldsymbol{p}}) < V_r(\boldsymbol{p}^*)$.

So now, consider the following deviation: $\hat{\boldsymbol{p}} = (\hat{p}_r, p^*_{-r})$ such that $\hat{p}_r = \{\varnothing\}$ and r does **not** enter elections, so that only parties l, L and R compete. Then it is easy to compute the vote shares.

$v_r(\hat{\boldsymbol{p}}) = 0, v_R(\hat{\boldsymbol{p}}) = \frac{1}{2}$,
$v_L(\hat{\boldsymbol{p}}) = v_L(\boldsymbol{p}^*) = \frac{1}{4}+\frac{\epsilon}{2}+\frac{1}{2}[q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})]$ and
$v_l(\hat{\boldsymbol{p}}) = v_l(\boldsymbol{p}^*) = \frac{1}{4}-\frac{\epsilon}{2}-\frac{1}{2}[q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})]$.
Clearly $v_R(\hat{\boldsymbol{p}}) > v_L(\hat{\boldsymbol{p}}) > v_l(\hat{\boldsymbol{p}})$ and now party R wins elections with certainty. But now observe that: $V_r(\hat{\boldsymbol{p}}) = -|\epsilon-\frac{1}{2}| + v_r(\hat{\boldsymbol{p}}) = -\frac{1}{2}+\epsilon$.

But then $V_r(\hat{\boldsymbol{p}}) > V_r(\boldsymbol{p}^*) \iff$
$-\frac{1}{2}+\epsilon > -\frac{1}{2}+\frac{1}{4}-\frac{\epsilon}{2}-\frac{1}{2}[q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})] \iff$
$\iff \epsilon > \frac{1}{4}-\frac{\epsilon}{2}-\frac{1}{2}[q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})] \iff$
$3\epsilon > \frac{1}{2}-[q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})]$.

Recalling that $\max_{T,q} ||q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})|| = \sqrt{\frac{3}{8}}-\sqrt{\frac{1}{4}}$

we can strengthen[23] the above inequality by replacing $[q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})]$ with $\sqrt{\frac{3}{8}}-\frac{1}{2}$:

Hence, $V_r(\hat{\boldsymbol{p}}) > V_r(\boldsymbol{p}^*) \iff \epsilon > \frac{1}{3}(\frac{1}{2}-\sqrt{\frac{3}{8}}+\frac{1}{2})$ which is never satisfied $\forall\epsilon \in (\sqrt{\frac{3}{8}}-\frac{1}{2}, \frac{1-\sqrt{\frac{3}{8}}}{3})$.

That is, $\hat{\boldsymbol{p}} = (\hat{p}_r, p^*_{-r})$ cannot be a profitable deviation for r either, and by a symmetric argument, for l as well. Hence, neither of the two extremist parties has an incentive to deviate unilaterally from $\boldsymbol{p}^*$. So, we conclude that $\boldsymbol{p}^*$ is a NE of the electoral game. This completes the proof. ■

In proving Proposition 2, and for the purposes of expositional simplicity, we will first characterize which equilibria fail to be coalition-proof. Hence, we prove the following statement.

Lemma 1 *Let $\epsilon \in (\sqrt{\frac{3}{8}}-\sqrt{\frac{1}{4}}, \frac{1-\sqrt{\frac{3}{8}}}{3})$. Then, any non-symmetric strategy profile can never be a coalition-proof Nash equilibrium of the electoral game.*

Proof. To prove this claim, consider the case that in equilibrium parties do not play symmetric strategies and as a result the equilibrium is decisive. That is, one of the two centrist parties

[22] It can be checked that this expression obtains its maximal value when $q = 1$, $m = \frac{1}{4}$and $T_d = \frac{1}{8}$.
[23] By strengthening we mean that the RHS value is diminishing and hence the inequality is more easily statisfied for lower values of ϵ.

wins with certainty. Clearly, this is the only format that any decisive equilibrium can take, given symmetric party-positions and symmetric distribution[24]. Even if party l (or r) proposes $\hat{p}_l = (-\frac{1}{2}, t^l = T)$ (or $\hat{p}_r = (\frac{1}{2}, t^r = T)$) and the others propose zero public spending, party l (or r) never wins in an asymmetric equilibrium.

Therefore, assume that either party L or R win elections with certainty. We will only show the result for party L. Then, the same reasoning can be applied to party R. Consider equilibrium $\dot{\boldsymbol{p}} = (\dot{p}_L, \dot{p}_{-L})$, where party L wins with certainty. We will show that is not a coalition-proof equilibrium. Obviously since party L wins the plurality of the vote the following is true: $v_L(\dot{\boldsymbol{p}}) > v_{-L}(\dot{\boldsymbol{p}}), \forall\ -L \in \Omega - \{L\}$. We compute:

$V_l(\dot{\boldsymbol{p}}) = -|-\epsilon-(-\frac{1}{2})| + v_l(\dot{\boldsymbol{p}}) = -\frac{1}{2} + \epsilon + v_l(\dot{\boldsymbol{p}})$, $V_L(\dot{\boldsymbol{p}}) = -|-\epsilon-(-\epsilon)| + v_L(\dot{\boldsymbol{p}}) = v_L(\dot{\boldsymbol{p}})$,

$V_R(\dot{\boldsymbol{p}}) = -|-\epsilon-\epsilon| + v_R(\dot{\boldsymbol{p}}) = -2\epsilon + v_R(\dot{\boldsymbol{p}})$, and $V_r(\dot{\boldsymbol{p}}) = -|-\epsilon-\frac{1}{2}| + v_r(\dot{\boldsymbol{p}}) = -\frac{1}{2} - \epsilon + v_r(\dot{\boldsymbol{p}})$.

Now, we only need to show that there exist at least one group of two or more parties that have a profitable joint deviation strategy. We choose one such group, namely, group $\{r, R\}$. Consider for a moment that parties r and R jointly deviate from $\dot{p}_R$, $\dot{p}_r$ to p_R^* and p_r^* respectively. Then the resulting vote shares are:

$v_r(\dot{p}_{l,L},\ p_R^*, p_r^*) = \frac{1}{4} - \frac{\epsilon}{2} - \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})]$,

$v_R(\dot{p}_{l,L}, p_R^*, p_r^*) = \frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})]$, whereas

$v_L(\dot{p}_{l,L}, p_R^*, p_r^*) \leq \frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})]$,

$v_l(\dot{p}_{l,L}, p_R^*, p_r^*) \geq \frac{1}{4} - \frac{\epsilon}{2} - \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})]$

$\forall\ \dot{p}_l, \dot{p}_L \neq p_l^*, p_L^*$.

To strengthen our point further, consider that this deviation does not cause party R to win. Rather, it ties with L. But even in this case, observe that the following is true:

$V_R(\dot{p}_{l,L}, p_R^*, p_r^*) = -\epsilon + v_R(\dot{p}_{l,L}, p_R^*, p_r^*)$.

Then, $V_R(\dot{p}_{l,L}, p_R^*, p_r^*) > V_R(\dot{\boldsymbol{p}}) \iff -\epsilon + v_R(\dot{p}_{l,L}, p_R^*, p_r^*) > -2\epsilon + v_R(\dot{\boldsymbol{p}})$,

which is always true given that $v_R(\dot{p}_{l,L}, p_R^*, p_r^*) > v_R(\dot{\boldsymbol{p}})$ (recall R is losing in $\dot{\boldsymbol{p}}$). Also compute, $V_r(\dot{p}_{l,L}, p_R^*, p_r^*) = -\frac{1}{2} + v_r(\dot{p}_{l,L}, p_R^*, p_r^*)$.

Then $V_r(\dot{p}_{l,L}, p_R^*, p_r^*) > V_r(\dot{\boldsymbol{p}}) \iff -\frac{1}{2} + v_r(\dot{p}_{l,L},\ p_R^*, p_r^*) > -\frac{1}{2} - \epsilon + v_r(\dot{\boldsymbol{p}}). \iff \epsilon > v_r(\dot{\boldsymbol{p}}) - v_r(\dot{p}_{l,L},\ p_R^*, p_r^*)$.

But observe that, given that R was losing under $\dot{\boldsymbol{p}}$, the following is true $\forall\ T \in (o, \frac{1}{8}]$, $q \in [0,1]$ and ϵ :

$v_r(\dot{\boldsymbol{p}}) - v_r(\dot{p}_{l,L},\ p_R^*, p_r^*) \leq q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M}) \leq$

$\leq \max_{T,q} ||q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})|| = \sqrt{\frac{3}{8}} - \frac{1}{2} < \epsilon$, which is always true.

That is, $V_r(\dot{p}_{l,L}, p_R^*, p_r^*) > V_r(\dot{\boldsymbol{p}})$ and as a result this deviation is profitable for both r and R. Moreover, as stressed above this deviation is a NE of the continuation game. Hence, an equilibrium where party L wins with certainty can never be a coalition-proof equilibrium of the game. By a symmetric argument, the same is true for any equilibrium where the other moderate party R wins with certainty. It can never be a coalition-proof equilibrium of the game. This concludes the proof of the Lemma. ■

By Lemma 1, we can now restrict attention only to non-decisive equilibria.

Proposition 2. To complete the proof of Proposition 2 we need to show two more things. First, that the (symmetric) non-decisive NE $\boldsymbol{p}^*$ proposed in Proposition 1, is indeed coalition-proof. Secondly, we need to show that it does not exist any other symmetric coalition-proof

[24] We note that by definition, in any non-decisive equilibrium parties must play symmetric strategies. Given that parties are also symmetrically positioned around the mean it is clear that in such a case two or four parties win elections with equal probability.

equilibrium of the game, apart from $\boldsymbol{p}^*$. For part (i) of the claim consider that there are nine symmetric candidate coalition-proof equilibria, with three possible outcomes: the two moderates win with probability $\frac{1}{2}$, all parties win with equal probability $\frac{1}{4}$[25], and the two extremists win with probability $\frac{1}{2}$. Let us start from the first class of equilibria: clearly the Nash equilibrium $\boldsymbol{p}^*$, which we have proposed in Proposition 1, belongs in this class. We need to show that this NE is also a coalition-proof equilibrium. To do so, we have to check for multi-party deviations.

Consider any multi-party joint deviation that results in a change in the outcome of the election from $\pi(\boldsymbol{v}(\boldsymbol{p}^*)) = (\frac{1}{2}L, \frac{1}{2}R) = 0$ to any other outcome where one party wins with certainty. Given that two parties (l and L) are positioned to the left and the other two (r and R) to the right of outcome $\pi(\boldsymbol{v}(\boldsymbol{p}^*))$, any change that moves away from this will have at least two "losers" losing at least ϵ in utility terms from this change in ideology being implemented. This utility loss cannot be compensated by any increase in their vote shares since by assumption $\epsilon > \max||q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})||$, the latter being the maximum possible utility gain for any party due to an increase in vote shares caused by a switch in strategies. Hence, the only deviations worth examining are those made by parties with bliss-points that are on the same side with respect to the equilibrium outcome $\pi(\boldsymbol{v}(\boldsymbol{p}^*))$. That is, a deviating coalition containing at the same time both a moderate and an extremist party (such as l with R or r with L), with bliss points on opposite sides, can never be profitable for all its members. Hence, given that under $\boldsymbol{p}^*$parties L and R are elected with equal probability and outcome $\pi(\boldsymbol{v}) = (\frac{1}{2}L, \frac{1}{2}R)$ is realized, the only joint deviations that move away from this outcome and at the same time are worthy of consideration are those by groups $\{l, L\}$ and $\{r, R\}$ respectively. By symmetry we need only check for one them, say $\{r, R\}$. Consider all possible joint deviation strategies $(\bar{p}_r, \bar{p}_R) \neq (p_r^*, p_R^*)$. That is: $(\bar{p}_r, \bar{p}_R) \in \{(T, 0); (T, T); (0, 0)\}$

Then, observe that under all possible combinations the following is true:

$v_R(\bar{\boldsymbol{p}}) < v_R(\boldsymbol{p}^*) = v_L(\boldsymbol{p}^*) = v_L(\bar{\boldsymbol{p}})$ and $v_r(\bar{\boldsymbol{p}}) > v_r(\boldsymbol{p}^*)$.

As a result any joint deviation by parties r and R to one of the above strategies will result in party L wining with certainty. But then:

$V_R(\bar{\boldsymbol{p}}) = -|-\epsilon-\epsilon| + v_R(\bar{\boldsymbol{p}}) = -2\epsilon + v_R(\bar{\boldsymbol{p}}) < -\epsilon + v_R(\boldsymbol{p}^*)$, or

$V_R(\bar{\boldsymbol{p}}) = -|\frac{1}{2} - \epsilon| + v_R(\bar{\boldsymbol{p}}) = -\frac{1}{2} + \epsilon + v_R(\bar{\boldsymbol{p}}) < -\epsilon + v_R(\boldsymbol{p}^*) \implies$

$\implies V_R(\bar{\boldsymbol{p}}) < V_R(\boldsymbol{p}^*)$ for every $\epsilon \in (\sqrt{\frac{3}{8}} - \sqrt{\frac{1}{4}}, \frac{1-\sqrt{\frac{3}{8}}}{3})$.

Also consider the joint deviation where party r chooses not to enter the electoral competition and R offers transfers ($t^R = T$). This deviation causes party R to win with certainty since it collects half of the votes, but it is not individually rational for party r, as we have shown in Proposition 1[26]. Hence, any joint deviation by parties r and R cannot be profitable for both them, since clearly R is always worse-off. By analogy the same is true for any joint deviation made by parties l and L.

Now, consider deviations that leave the outcome unchanged, that is $\pi(\boldsymbol{v}) = (\frac{1}{2}L, \frac{1}{2}R)$. These deviations, due to the symmetric structure of the problem, have to involve symmetric deviation strategies otherwise the outcome will change and as shown above this can never be a coalition-proof equilibrium. We need only examine two more cases:

Firstly, the one where the two extremist parties l and r play mirror strategies $(t^l, t^r) = (T, T)$. That is they offer the maximum possible transfers. This deviation increases their respective vote share from $v_l(\boldsymbol{p}^*) = v_r(\boldsymbol{p}^*) = \frac{1}{4} - \frac{\epsilon}{2} - \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})]$, as computed in Prop. 1, to $\frac{1}{4} - \frac{\epsilon}{2}$ since now all four parties offer the maximum possible transfers, whereas

[25] This occurs when all four parties choose not to contest the electoral competition and hence, they win with equal probability.

[26] Now all parties with the exception of r play equilibrium strategies p^*_{-r} and the analysis laid out in Proposition 1 is relevant.

their utility from the ideology outcome that is implemented remains unchanged due to the fact that the outcome of the election remains the same (parties L and R win with probability $\frac{1}{2}$). Clearly, this is a profitable joint deviation. But then, it is easily checked that this deviation fails to be an equilibrium of the continuation game since each one of the two extremist parties has an incentive to renege and revert back to its original strategy p_l^* and p_r^* respectively which guarantees a favorable change in the outcome that yields utility gain of ϵ which by assumption is strictly greater than the potential utility loss due to a smaller voter share, in this case computed to be $\frac{1}{2}[q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})]$.

Secondly, consider equilibrium $\check{\boldsymbol{p}}$ where parties L and R play the same strategies as in equilibrium $\boldsymbol{p}^*$, that is $\check{p}_{L,R}=p_{L,R}^*$ but parties l and r do not enter the electoral race, that is $p_{l,r}=\{\varnothing\}$. Then, in such a case, clearly $v_l(\check{\boldsymbol{p}})=v_r(\check{\boldsymbol{p}})=0$ and $v_L(\check{\boldsymbol{p}})=v_R(\check{\boldsymbol{p}})=\frac{1}{2}$, which implies that: $\pi(\boldsymbol{v}(\check{\boldsymbol{p}}))=\pi(\boldsymbol{v}(\boldsymbol{p}^*)=\frac{1}{2}(L+R)$. But now, observe that under $\check{\boldsymbol{p}}$ we have: $V_l(\check{\boldsymbol{p}})=V_r(\check{\boldsymbol{p}})=-\frac{1}{2}$.

So, consider the following joint deviation by parties l and r: they both play strategies p_l^* and p_r^* respectively. The vote shares that each party receives are identical with those computed in Proposition 1:

$v_l(\boldsymbol{p}^*)=v_r(\boldsymbol{p}^*)=\frac{1}{4}-\frac{\epsilon}{2}-\frac{1}{2}[q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})]>0 \implies$
$V_l(\boldsymbol{p}^*)=V_r(\boldsymbol{p}^*)=-\frac{1}{2}+\frac{1}{4}-\frac{\epsilon}{2}-\frac{1}{2}[q(\sqrt{m+T}-\sqrt{m})+(1-q)(\sqrt{M+T}-\sqrt{M})]>-\frac{1}{2} \implies$
$V_l(\boldsymbol{p}^*)>V_l(\check{\boldsymbol{p}})$ and $V_r(\boldsymbol{p}^*)>V_r(\check{\boldsymbol{p}})$.

Note that now, all parties play strategies p_ω^*. Clearly, this deviation is a NE of the continuation game, since as shown in Proposition 1 no party can have a profitable unilateral deviation from equilibrium $\boldsymbol{p}^*$. Hence, we conclude that equilibrium $\check{\boldsymbol{p}}$ cannot be a coalition-proof equilibrium of the game, since there is a profitable joint deviation by parties l and r which is a NE of the continuation game. This concludes the first part part of the argument, that is equilibrium $\boldsymbol{p}^*$ is indeed coalition-proof.

Now for part (ii), we focus on the remaining two classes of symmetric equilibria. Let us consider equilibrium $\boldsymbol{p}'$ where all parties win with equal probability. That is, $p_\omega=\{\varnothing\}$, and $v_\omega(\boldsymbol{p}')=\frac{1}{4}$, $\forall\ \omega\in\Omega$. Then, $\forall p_l,p_r$ there always exist a profitable deviation for the coalition containing the two moderate parties $\{L,R\}$ that makes both of them strictly better off. Consider deviations symmetric $p_L^*\neq p'_L$ and $p_R^*\neq p'_R$. Then,

$v_L(p_L^*,p_R^*,p'_{l,r})=\frac{1}{2}>v_L(\boldsymbol{p}')=v_l(\boldsymbol{p}')=\frac{1}{4}$, and by symmetry
$v_R(p_L^*,p_R^*,p'_{l,r})=\frac{1}{2}>v_R(\boldsymbol{p}')=v_r(\boldsymbol{p}')=\frac{1}{4}$.

This implies that parties L and R win elections with probability $\frac{1}{2}$. But then observe that:

$V_R(\boldsymbol{p}')=\frac{1}{4}(-|\epsilon-\epsilon|-|-\epsilon-\epsilon|-|\frac{1}{2}-\epsilon|-|-\frac{1}{2}-\epsilon|)+v_R(\mathbf{p}')=-\frac{\epsilon}{2}-\frac{1}{8}+\frac{\epsilon}{4}-\frac{1}{8}-\frac{\epsilon}{4}+\frac{1}{4}=-\frac{\epsilon}{2}$,
whereas

$V_R(p_L^*,p_R^*,p'_{l,r})=\frac{1}{2}(-|\epsilon-\epsilon|-|-\epsilon-\epsilon|)+v_R(p_L^*,p_R^*,p'_{l,r})=-\epsilon+\frac{1}{2} \implies$
$V_R(p_L^*,p_R^*,p'_{l,r})>V_R(\boldsymbol{p}')$ and by symmetry $V_L(p_L^*,p_R^*,p'_{l,r})>V_L(\boldsymbol{p}')$ as well.

Thus, p_L^*, p_R^* is a profitable deviation for both moderate parties which is also a NE of the continuation game since either of the two parties cannot achieve a better outcome by deviating unilaterally[27], which means that $\boldsymbol{p}'$ cannot be a coalition-proof Nash equilibrium.

Finally, consider the case where in equilibrium $\hat{\boldsymbol{p}}$ the two extremist parties, l and r win with probability $\frac{1}{2}$. Then, $\forall p_l,p_r$ consider again the same as before deviation by the coalition $\{L,R\}$. That is, $p_L^*\neq\hat{p}_L$ and $p_R^*\neq\hat{p}_R$. Then, the following is true:

$v_L(p_L^*,p_R^*,\hat{p}_{l,r})\geq\frac{1}{4}+\frac{\epsilon}{2}>v_L(\hat{\boldsymbol{p}})$ and by symmetry $v_R(p_L^*,p_R^*,\hat{p}_{l,r})\geq\frac{1}{4}+\frac{\epsilon}{2}>v_R(\hat{\boldsymbol{p}})$.

Also note that, since by symmetry

[27] Recall that extremist parties l and r also play mirror strategies.

$v_L(p_L^*, p_R^*, \hat{p}_{l,r}) = v_R(p_L^*, p_R^*, \hat{p}_{l,r}) > \frac{1}{4}$, and $v_l(p_L^*, p_R^*, \hat{p}_{l,r}) = v_r(p_L^*, p_R^*, \hat{p}_{l,r})$,

parties L and R now win elections with equal probability one half. But then, compare the following:

$V_R(\hat{\boldsymbol{p}}) = -\frac{1}{2} + v_R(\hat{\boldsymbol{p}})$ and $V_R(p_L^*, p_R^*, \hat{p}_{l,r}) = -\epsilon + v_R(p_L^*, p_R^*, \hat{p}_{l,r}) \implies$

$\implies V_R(p_L^*, p_R^*, \hat{p}_{l,r}) > V_R(\hat{\boldsymbol{p}})$, given $\epsilon < \frac{1}{2}$ and $v_R(p_L^*, p_R^*, \hat{p}_{l,r}) > v_R(\hat{\boldsymbol{p}})$.

By an analogous point, the same is also true for party L, that is, $V_L(p_L^*, p_R^*, \hat{p}_{l,r}) > V_L(\hat{\boldsymbol{p}})$. Hence, we conclude that this is a profitable deviation for both moderate parties and $\hat{\boldsymbol{p}}$ cannot be a coalition-proof equilibrium of the game. Finally, note that, as we have shown above, equilibrium $\boldsymbol{p}^*$ is the only symmetric and coalition-proof equilibrium where the two moderate parties win with probability $\frac{1}{2}$.

Bringing both parts of the argument together, we have shown that *non-symmetric* equilibria of the game **fail** to be *coalition-proof*, whereas, from the *symmetric* ones, that involve symmetric equilibrium strategies, the *unique coalition-proof* NE of the game is the equilibrium $\boldsymbol{p}^*$ of Proposition 1. This completes the proof. ■

Proposition 3. Define $F = 1 - \sum_{\omega \in \Omega} (v_\omega)^2$. Then, we have computed the (symmetric) vote share allocation that is realized under $\boldsymbol{p}^*$, in proving Proposition 1. That is:

$v_L(\boldsymbol{p}^*) = v_R(\boldsymbol{p}^*) = \frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})]$

and

$v_l(\boldsymbol{p}^*) = v_r(\boldsymbol{p}^*) = \frac{1}{4} - \frac{\epsilon}{2} - \frac{1}{2}[q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})]$.

Hence, we can compute $F = 1 - [2v_L(\boldsymbol{p}^*)^2 + 2v_l(\boldsymbol{p}^*)^2] = 1 - 2[v_L(\boldsymbol{p}^*)^2 + v_l(\boldsymbol{p}^*)^2]$.

For notational simplicity define the function:

$z(q, m, T) = q(\sqrt{m+T} - \sqrt{m}) + (1-q)(\sqrt{M+T} - \sqrt{M})$,

for $q \in (0,1)$, $m \in (\frac{1}{4}, M)$ and $T \in (0, \frac{1}{8}]$.

Also recall that we have so far assumed that: $\frac{\partial T(d)}{\partial d} < 0$. We can, then, rewrite F as follows:

$F = 1 - 2[(\frac{1}{4} + \frac{\epsilon}{2} + \frac{1}{2}z(q,m,T))^2 + (\frac{1}{4} - \frac{\epsilon}{2} - \frac{1}{2}z(q,m,T))^2] =$

$= 1 - 2[(\frac{1}{4} + \frac{\epsilon}{2})^2 + (\frac{1}{4} + \frac{\epsilon}{2})z(.) + \frac{1}{4}z^2(.) + (\frac{1}{4} - \frac{\epsilon}{2})^2 - (\frac{1}{4} - \frac{\epsilon}{2})z(.) + \frac{1}{4}z^2(.)] =$

$= 1 - 2[\frac{1}{2}z^2(q,m,T) + \epsilon z(q,m,T) + (\frac{1}{4} + \frac{\epsilon}{2})^2 + (\frac{1}{4} - \frac{\epsilon}{2})^2]$.

For simplicity let $(\frac{1}{4} + \frac{\epsilon}{2})^2 + (\frac{1}{4} - \frac{\epsilon}{2})^2 = C$, a constant, so that F becomes:

$F = 1 - [z^2(q,m,T) + 2\epsilon z(q,m,T) + 2C]$.

Now, one can proceed in proving our main result by straightforward computation of $\frac{\partial F}{\partial q}$ and $\frac{\partial F}{\partial d}$.

For (i) we have to show that $\frac{\partial F}{\partial q} < 0$. First compute: $\frac{\partial F}{\partial q} = \frac{\partial F}{\partial z(.)}\frac{\partial z(.)}{\partial q} = -[2z(q,m,T) + 2\epsilon]\frac{\partial z(.)}{\partial q}$. But observe that $\frac{\partial z(.)}{\partial q} = (\sqrt{m+T} - \sqrt{m}) - (\sqrt{M+T} - \sqrt{M}) > 0$ by concavity since $m < M$ and $T > 0$. Hence, we have: $-2z(q,m,T) + 2\epsilon < 0$, $\frac{\partial z(.)}{\partial q} > 0$ and consequently $\frac{\partial F}{\partial q} < 0$.

For (ii) we have to show that $\frac{\partial F}{\partial T} < 0$. First compute $\frac{\partial F}{\partial T} = \frac{\partial F}{\partial z(.)}\frac{\partial z(.)}{\partial T} = -[2z(q,m,T) + 2\epsilon]\frac{\partial z(.)}{\partial T}$. Since we know that $-2z(q,m,T) + 2\epsilon < 0$, we need only show that $\frac{\partial z(.)}{\partial T} > 0$. But observe that:

$\frac{\partial z(.)}{\partial T} = \frac{q}{\sqrt{m+T}} + \frac{1-q}{\sqrt{M+T}} > 0$. Hence, $\frac{\partial F}{\partial T} < 0$. This completes both parts of the proof. ■

7 Appendix B

7.1 Graphs and Figures

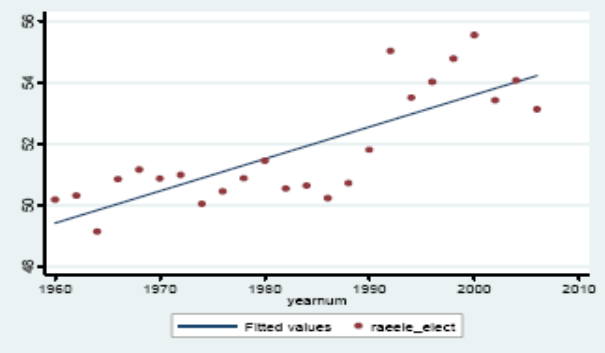


Fig.1: USA

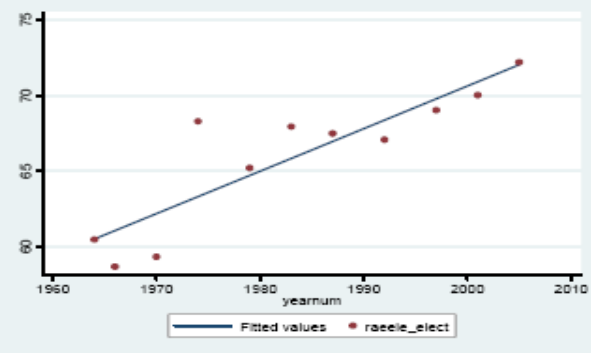


Fig.2: United Kingdom

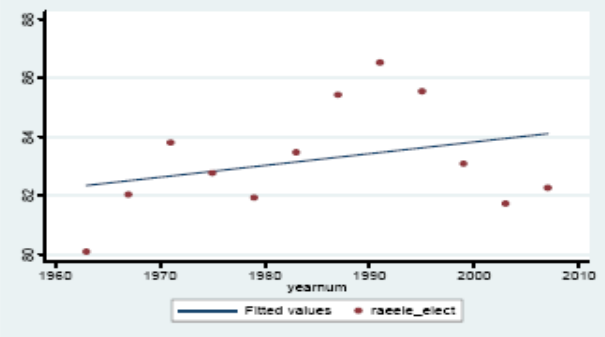


Fig. 3: Switzerland

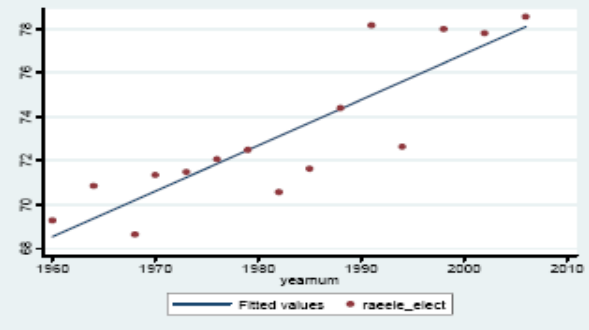


Fig. 4: Sweden

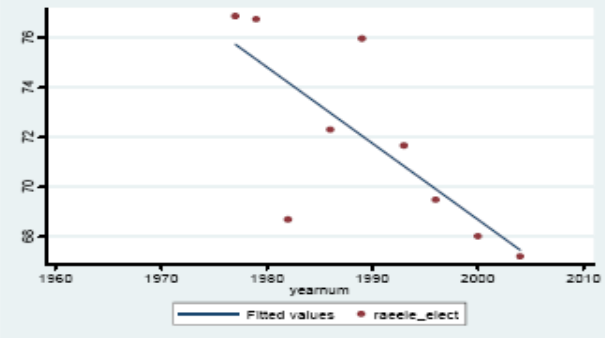


Fig. 5: Spain

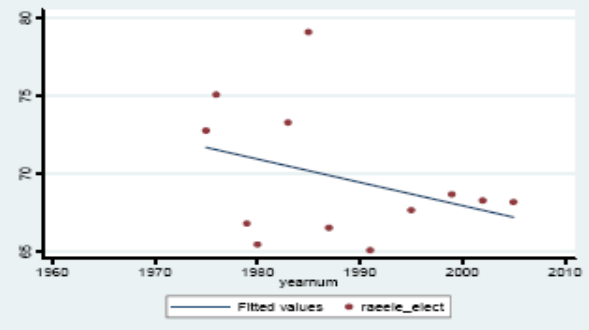


Fig. 6: Portugal

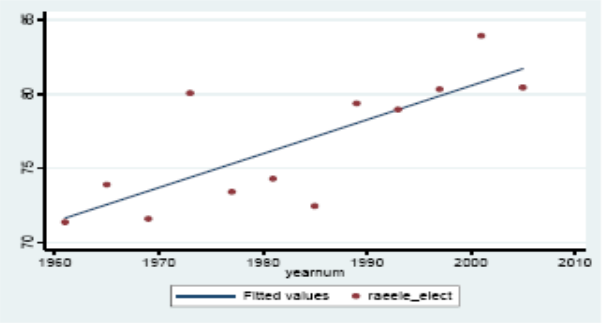


Fig. 7: Norway

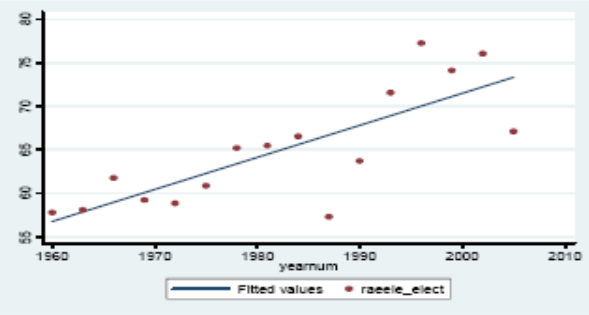


Fig. 8: New Zealand

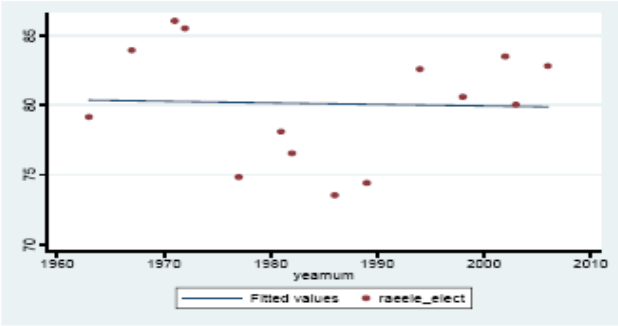


Fig. 9: Netherlands

Fig. 10: Luxemburg

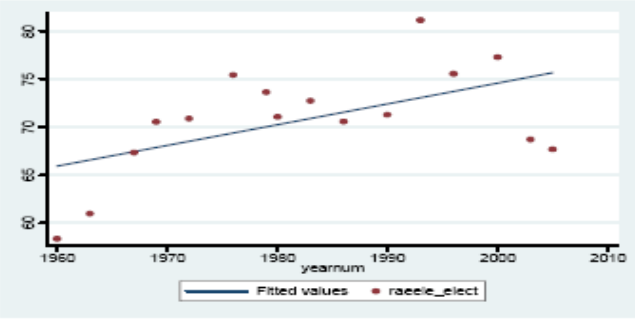


Fig. 11: Japan

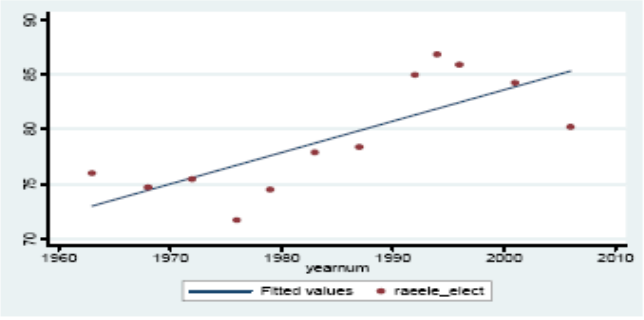


Fig. 12: Italy

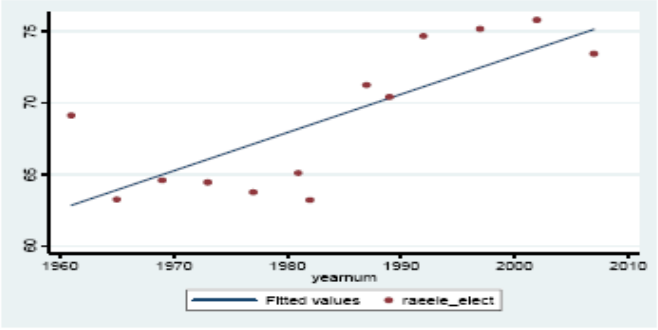


Fig. 13: Ireland

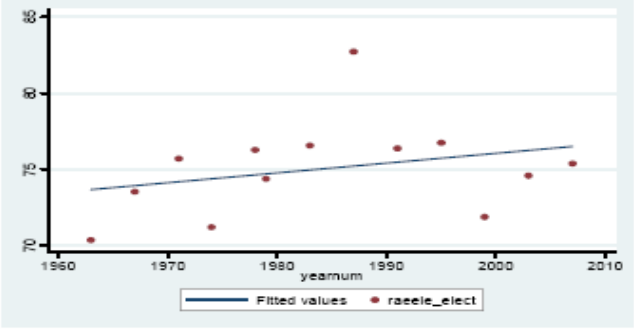


Fig. 14: Iceland

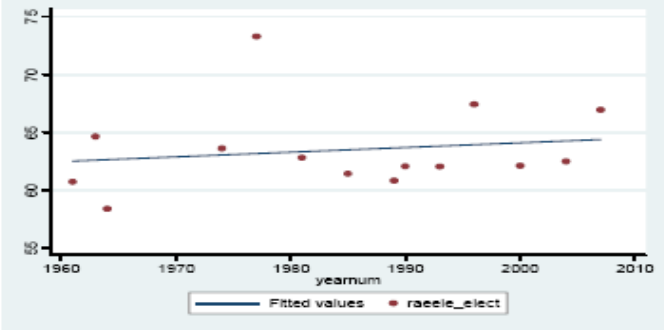


Fig. 15: Greece

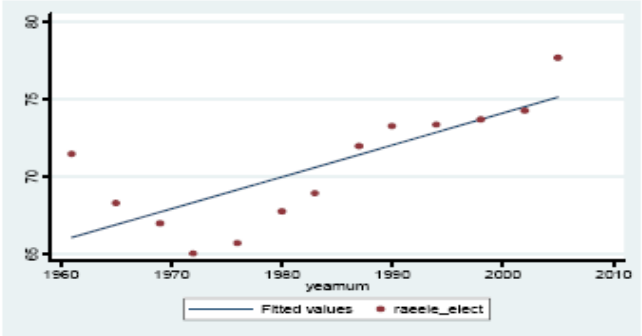


Fig. 16: Germany

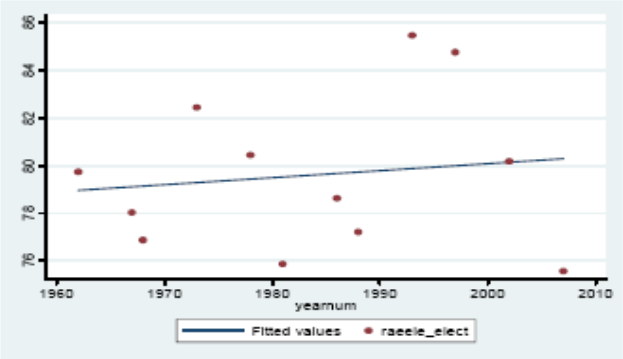


Fig.17: France

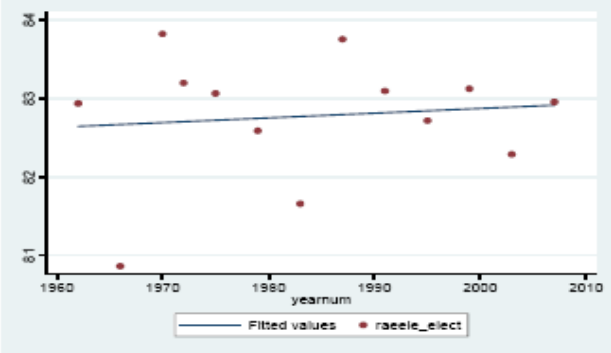


Fig.18: Finland

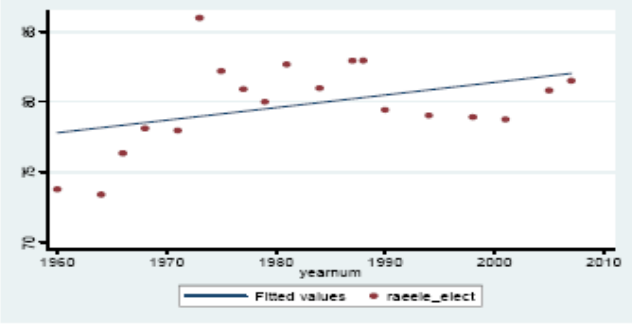


Fig. 19: Denmark

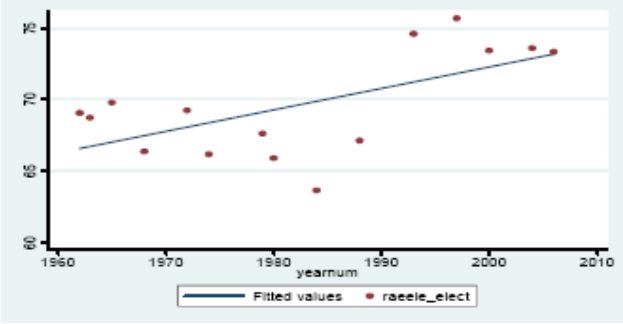


Fig. 20: Canada

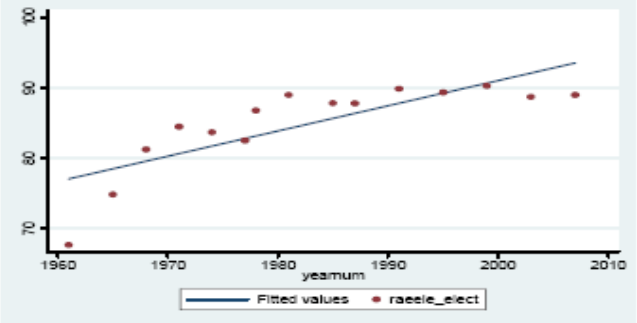


Fig. 21: Belgium

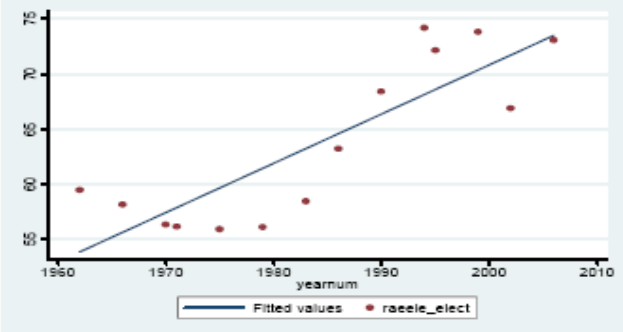


Fig. 22: Austria

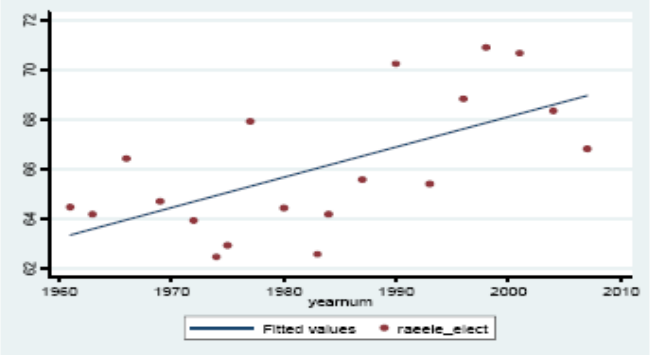
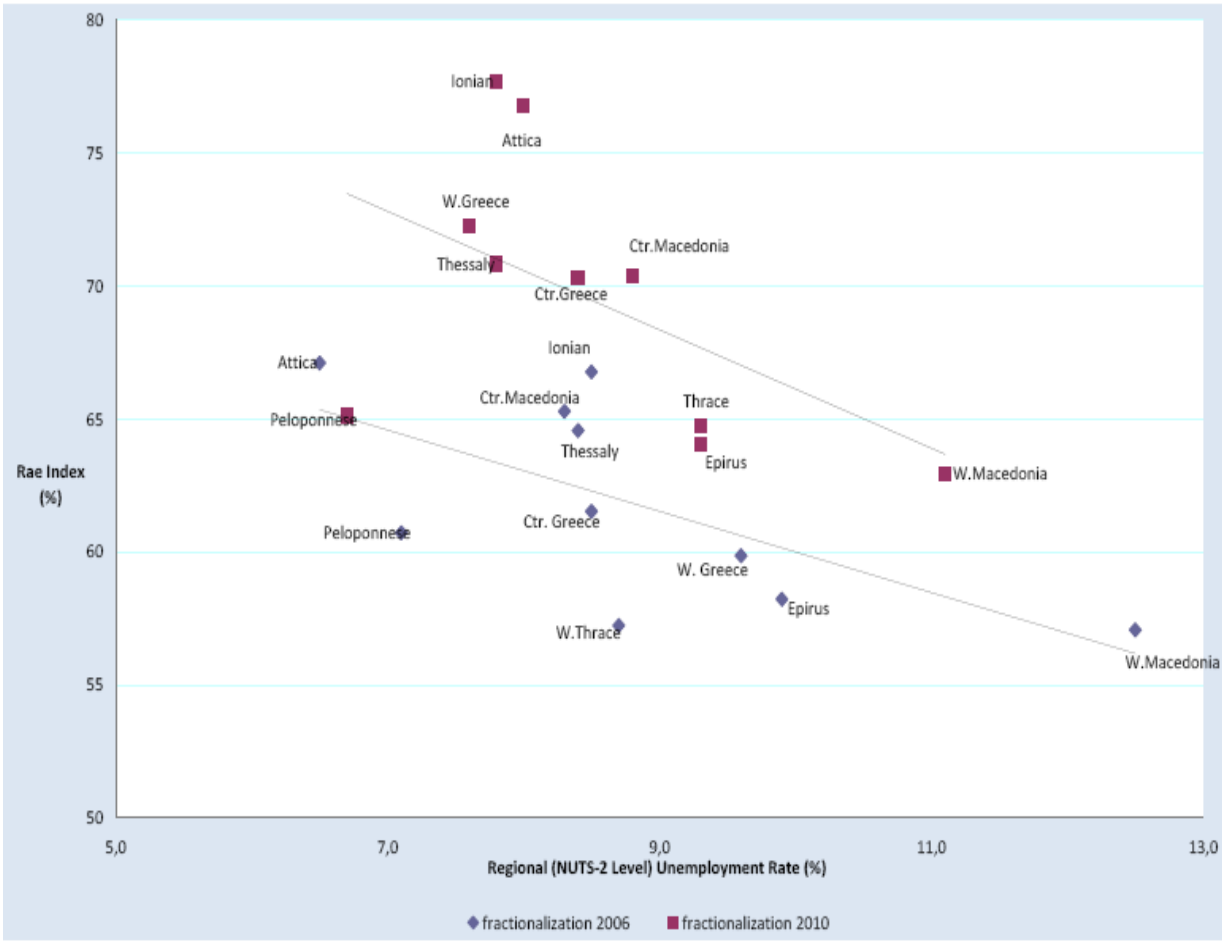
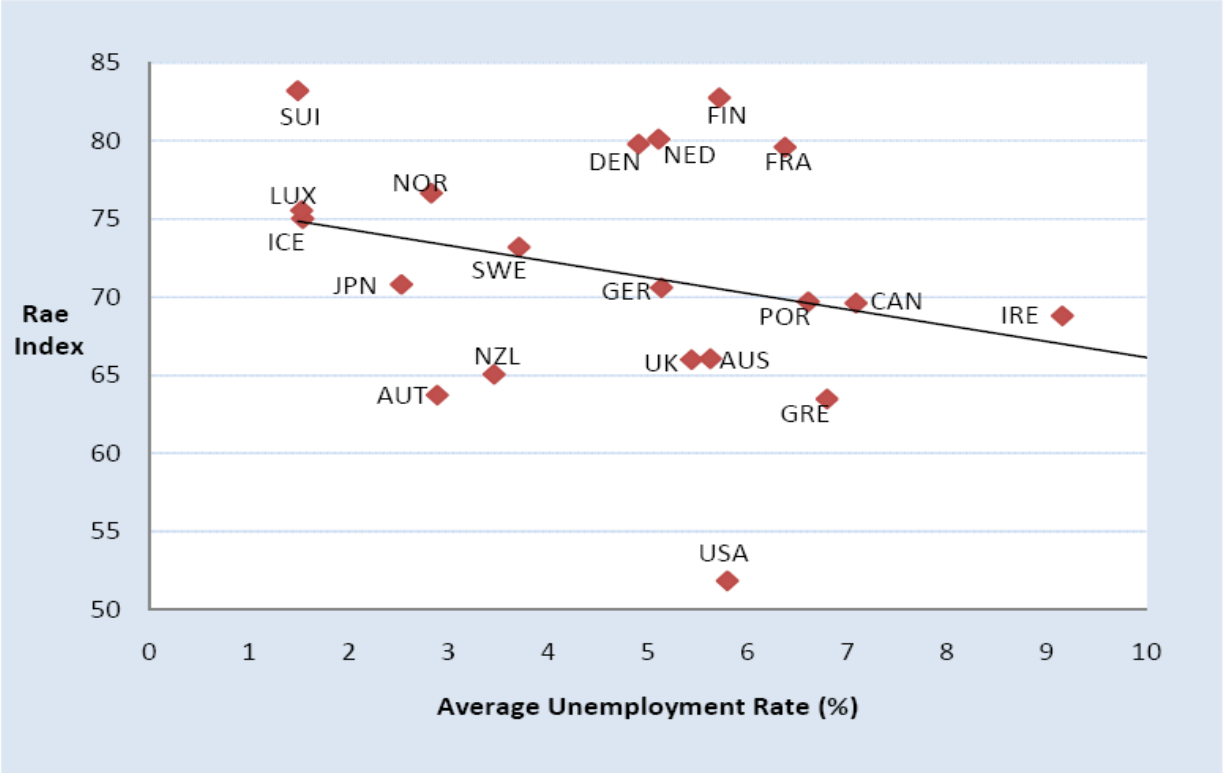


Fig. 23: Australia

Figure 1: Fractionalization Levels in 23 OECD Countries from 1960-2007



Unemployment, Public Debt and Electoral Fractionalization (Rae Index) in Greek Regional Elections of 20



Average Electoral Fractionalization (Rae Index in %) and Unemployment Rates in 23 OECD Countries fro

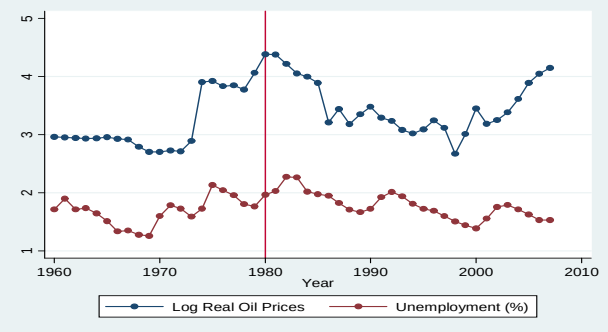


Fig.1: USA

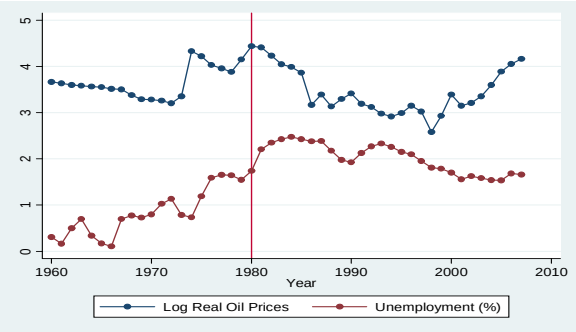


Fig.2: United Kingdom

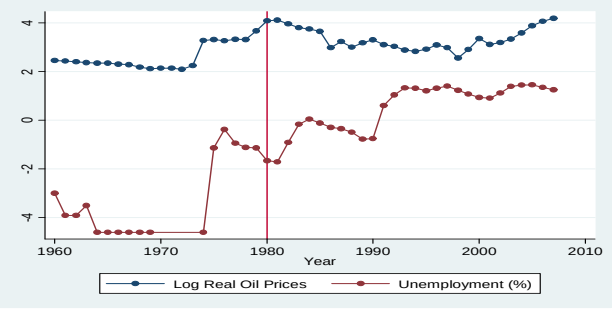


Fig. 3: Switzerland

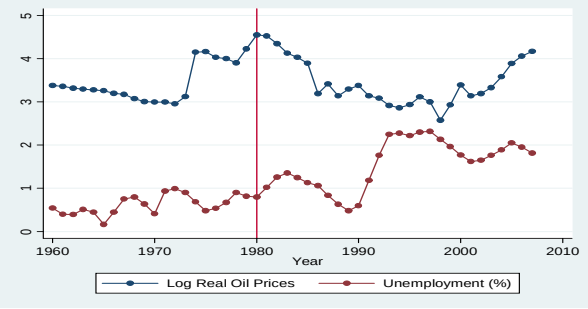


Fig. 4: Sweden

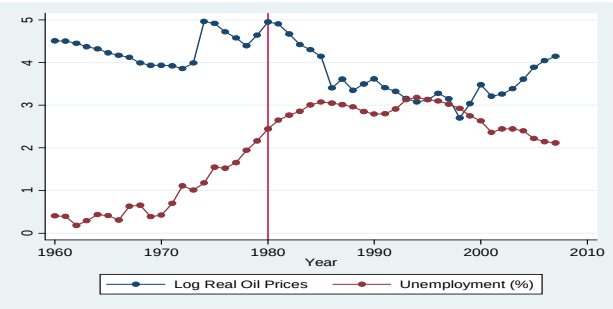


Fig. 5: Spain

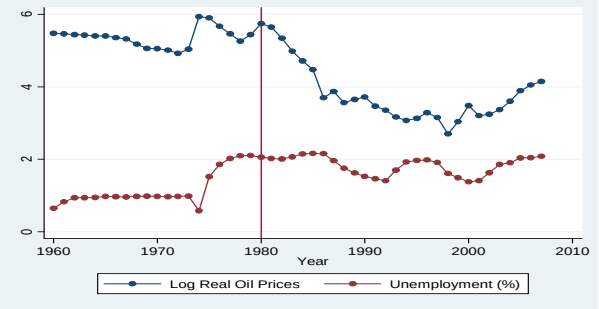


Fig. 6: Portugal

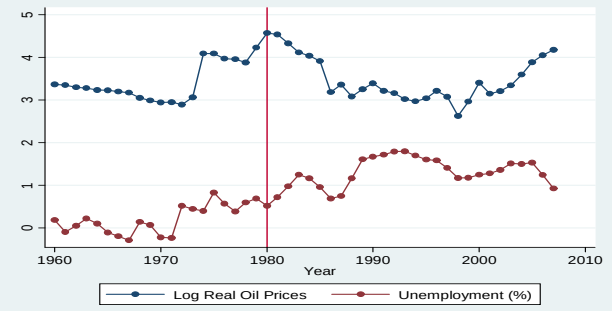


Fig. 7: Norway

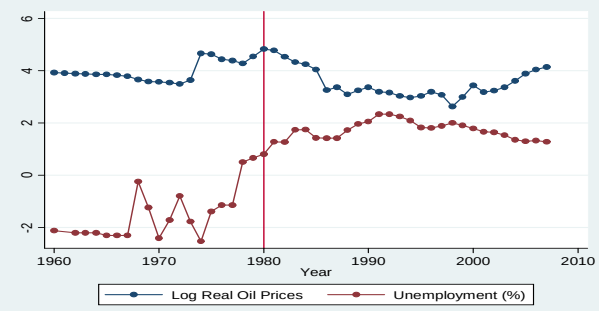


Fig. 8: New Zealand

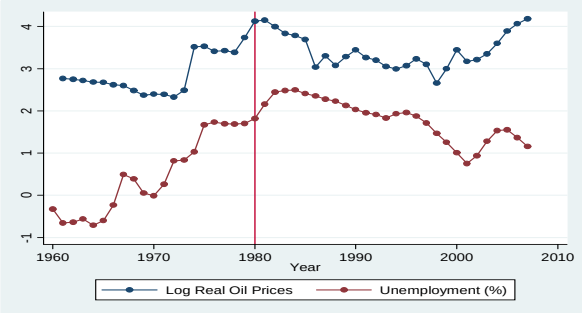


Fig. 9: Netherlands

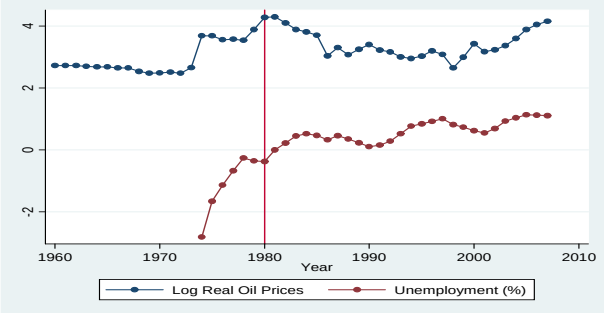


Fig. 10: Luxemburg

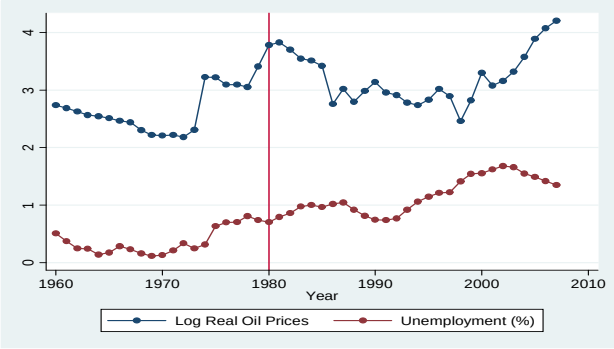


Fig. 11: Japan

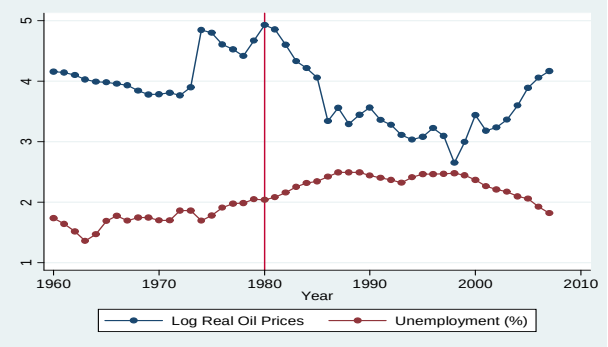


Fig. 12: Italy

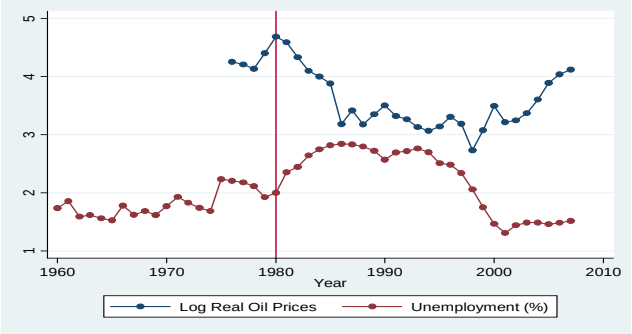


Fig. 13: Ireland

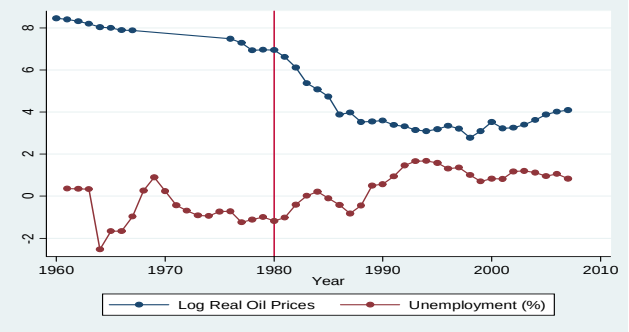


Fig. 14: Iceland

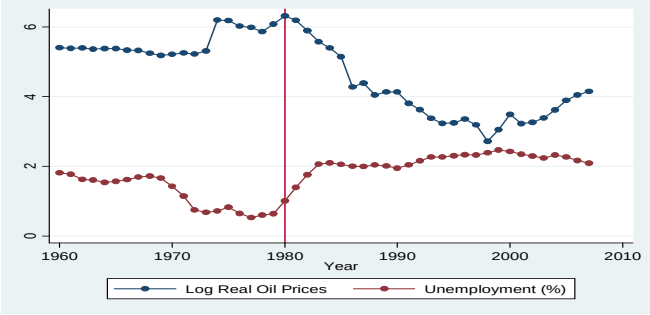


Fig. 15: Greece

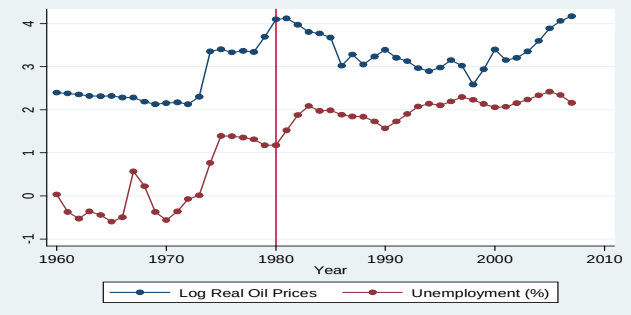


Fig. 16: Germany

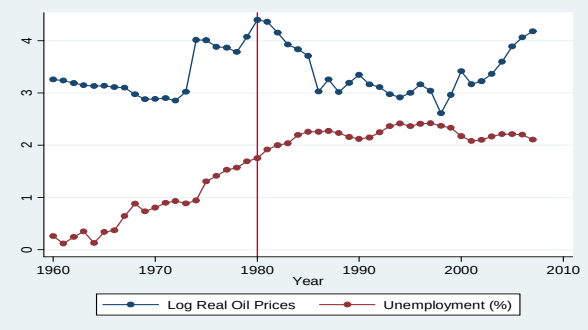


Fig.17: France

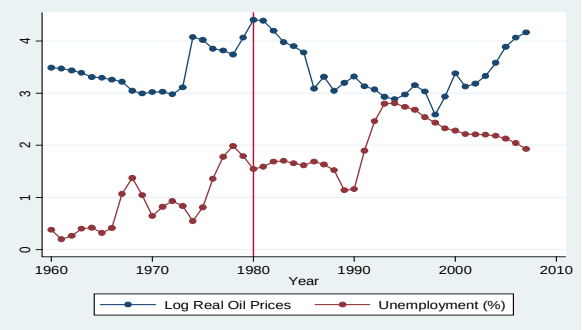


Fig.18: Finland

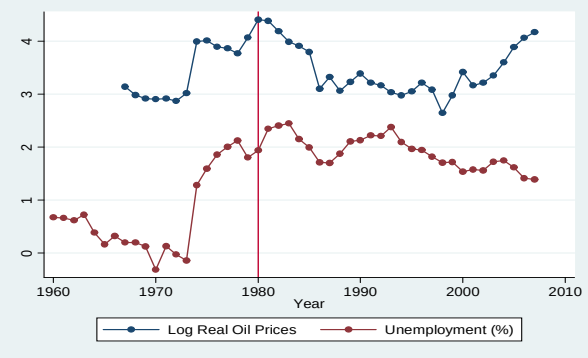


Fig. 19: Denmark

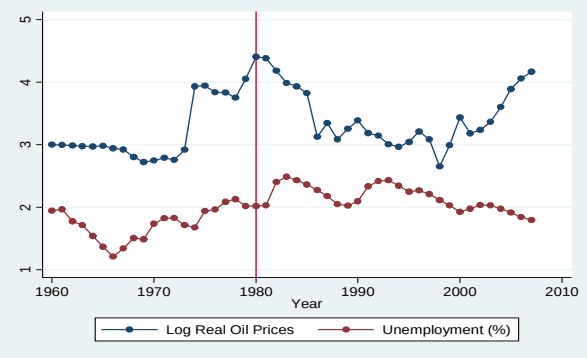


Fig. 20: Canada

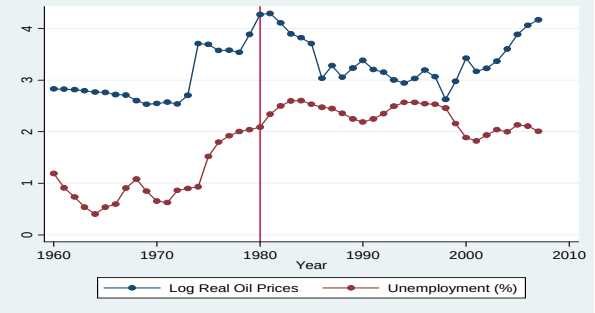


Fig. 21: Belgium

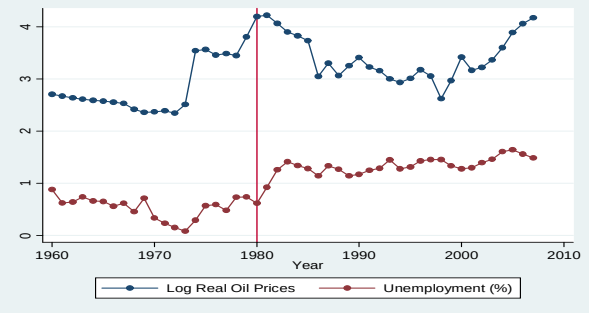


Fig. 22: Austria

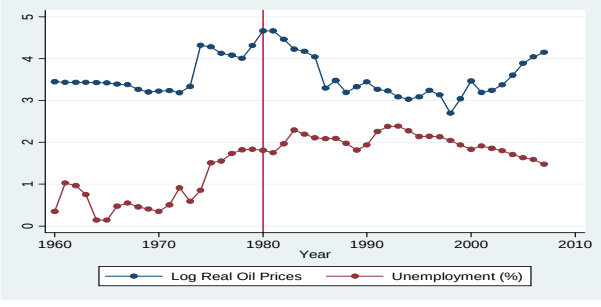


Fig. 23: Australia

7.2 Tables of Summary Statistics and Results

Table 7: Robustness of the Model under Alternative Econometric Specification
Dependent Variable: *Rae Index of Electoral Fractionalization;*
Instrumental Variable: *PPI Imported Oil Prices at Refinery*

	2SLS-3	GMM1	GMM2	GMM3	GMM4	GMM5
	(1)	(2)	(3)	(4)	(5)	(6)
Unemployment (%)	-1.4509	-1.5033	-1.6945	-1.7177	-1.8710	-1.6945
	(0.5451)***	(0.8347)*	(0.9329)*	(0.9083)*	(0.9811)*	(0.9329)*
L.Debt/GDP (%)	0.0582	0.0664	0.0729	0.0553	0.0635	0.0729
	(0.0489)	(0.0525)	(0.0559)	(0.0529)	(0.0553)	(0.0559)
Dummy Plurality Rule	-25.5136	-26.4545	-25.9240	-13.3134	-12.5900	- - -
	(1.7741)***	(2.6075)***	(2.9082)***	(4.2112)***	(4.5079)***	
Dummy High Institut. constraints	-1.0904	-.4553	-.8611	- - -	- - -	-.8611
	(2.4783)	(2.3665)	(2.5445)			(2.5445)
Year Fixed Effects	Y	Y	Y	Y	Y	Y
Country Fixed Effects	Y	Y	Y	Y	Y	Y
Const.	84.1763	82.0337	82.3982	72.1462	71.2970	56.4742
	(2.8308)***	(2.4547)***	(2.6099)***	(3.7560)***	(3.6949)***	(4.1994)***
Obs.	208	208	217	211	220	217
Instrument	L.1 Price	L.1 Price	L.1 Price	L.1 Price	L.1 Price	L.1 Price
Sample Size	Oil Import.	Oil Import.	Full	Oil Import.	Full	Full
R^2	0.883	0.8744	0.8549	0.8473	0.8292	0.8549
F statistic (1^{st} Stage)	10.76	12.85	11.63	11.51	10.92	11.63

Table 7: Note: *Clustered Standard Errors at the Country level reported in parantheses. (***) Statistically significant at the 1% level; (**) Statistically significant at the 5% level; Statistically significant at the 10% level.*

Variable	Obs	Mean	Std. Dev.	Min	Max
Rae Index of Fractionalization	322	71,68795	9,455858	49,15	90,28
Effective Party Index	322	3,987919	1,507375	1,97	10,29
Unemployment Rate (%)	1089	5,2591	3,977986	0	24,17
Debt/GDP (%)	768	57,83388	29,29046	4,64	175,27
Right Government	1063	39,25569	39,05281	0	100
Centrist Government	1063	25,52948	31,63897	0	100
Left Government	1063	32,52319	37,23355	0	100
New Government Dummy	1062	0,217514	0,41275	0	1
Voter Turn-out	1065	78,8493	13,12697	35	95,8
Employment Ratio	1029	66,50389	8,500277	43,8	103,2
Industrial Employment (%)	1104	0,319995	0,068356	0,173656	0,493255
Coalition Government Dummy	1012	0,592885	0,49154	0	1
Institutional Constraints Dummy	1056	0,373106	0,483859	0	1
Plurality ER Dummy	1067	0,169635	0,375487	0	1

Table 8: Summary Statistics of Key Variables