Producers' Expectations and the Business Cycle*

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Abstract

This paper isolates the role of expectations on the producers' side. Positive shocks to expectations, which can be due to shocks to producers' private information, transient productivity shocks, or positive 'surprise' monetary policy shocks, are shown to cause increases in employment, output, and prices, but be inflationary only on impact and deflationary thereafter. A Friedman-rule emerges as the optimal monetary policy in this framework; however, a policy as aggressive on inflation as possible is a second-best policy.

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1 Introduction

Whereas the role of expectations along the business cycle has been demonstrated and extensively analyzed in many recent studies,[1] a feature all of them share is that they, fully or partially, focus on the consumer's side. This paper mutes the consumer (demand) channel and introduces incomplete information on the producer's side; hence, it isolates the role of expectations on the supply side of the economy and sheds light on the business cycle dynamics they generate.

From an empirical standpoint, it may be a *cliché* that each crisis is different, but few would disagree that crises (and booms alike) can be marked as either inflationary, the most prominent being the '70s, or disinflationary periods, the most prominent being the Great Depression and the recent crisis. This paper sheds some light on the latter. Its central thesis is that unjustified by fundamentals optimism on the producer's side boosts output and prices on impact, but as the (Bayesian) producer learns, output and prices gradually return to their efficient levels in a monotone fashion. Put otherwise, optimism results in output expansion and inflation on impact, but output contraction and deflation afterwards; for a high enough precision of the producer's private information, which may be reasonable or unreasonable[2], these will be accompanied by a fall in the nominal interest rates, facts in line with the debt-deflation theory developed in Fisher (1933) , although the present paper abstracts from its critical financial accelerator aspect.

The competitive (neoclassical) economy features a consumer/worker, a producer managing the consumer-owned firm, and a monetary authority. Each period is split into two stages: In the first stage only the labor market opens, while in second stage the consumption good and the nominal bond markets open.

In the model, productivity is idiosyncratic to the worker and the producer managing the firm faces uncertainty about it. Put differently, employment is an investment decision at the firm level for the producer/manager. In stage 1, the Walrasian auctioneer announces a wage-plan based on the producer's expectations about the worker's productivity and stage 2 prices; on the labor demand side, producer chooses employment at the prevailing wage while on the labor supply side the worker decides on his labor supply based on his expectations about stage 2 consumption and prices. Temporal productivity

[1]These include Beaudry and Portier (2004), Jaimovich and Rebelo (2009), Lorenzoni (2009), Christiano et al. (2010), Angeletos and La'O (2010). Chapter 9 in Veldkamp (2011) offers an overview.

[2]Section 3.1 in DellaVigna (2009) offers an overview of 'overconfidence', meant here as the overestimation of the precision of the producer's private information.

consists of a permanent and a transient i.i.d. component. What the producer looks for is total current productivity. His information set contains a noisy private signal about the current permanent productivity part along with its past realizations, as well as past realizations of aggregate productivity. The consumer knows his current (total) productivity on top of these. Given this specification, stage 2 consumption and prices will depend on the producer's expectations about current productivity and current productivity itself. Consumer knows the latter and can learn the former by observing the announced nominal wage, hence he is able to perfectly foresee stage 2 prices and consumption, rendering, thus, the analysis free of wealth effects, the presence of which motivates the works by Beaudry and Portier (2004) and Jaimovich and Rebelo (2009), and partially La'O (2010). At the same time, full price flexibility guarantees that the consumer's expectations about future will be irrelevant for the present, and, hence the 'Euler equation' channel, central in the New-Keynesian settings of Lorenzoni (2009) and Christiano et al. (2010), is shut. These features enable the analysis to focus exclusively on the producer's side and do so in a parsimonious way.

In stage 2, with output predetermined from stage 1, the consumption good and the nominal bond markets open. The nominal bond price (the inverse of the nominal interest rate) is set by a monetary authority targeting next period's inflation. Consumption good prices in turn depend on the pursued monetary policy. It is initially assumed that the monetary authority has no superior information about the state of the economy than the consumer, however this is subsequently relaxed in order to study the informational role of the nominal interest rate.

The first set of results obtained is that positive shocks to expectations ('optimism') generate positive co-movement, that is both output and employment increase, whereas shocks to the permanent productivity component cause output to underreact compared to its efficient level and employment to fall. To provide an intuition for these, in the former case, optimism results into higher wages being announced than under complete information, inducing thereby workers to increase their labor supply. In the latter case the opposite happens: producers' expectations underreact relative to the complete information case, resulting into lower wages being announced and reduced labor supply in equilibrium relative to its efficient level.

Second, for a monetary policy responding to inflation more than one-to-one, prices depend positively on output, which, as explained previously, depends positively on productivity shocks and producers' expectations. This in turn implies that productivity shocks and optimism also cause increases in prices. Intuitively, optimism implies higher nominal wages are announced resulting in higher labor supply and higher equilibrium output, which in turn induces higher prices, rendering thereby expectations partially self-fulfilling. The results about inflation, however, are different: it is shown that permanent productivity shocks are inflationary, whereas positive shocks to expectations are inflationary on impact but become deflationary after then and remain so until the economy returns to its zero-inflation steady-state. Both results, central to this paper, are entirely attributed to the Bayesian evolution of expectations. In the former case, as time evolves, producer's expectations get closer to the true productivity of the economy resulting in prices increasing at a growing rate causing thereby inflation. In sharp contrast the impact positive shocks to expectations have on future prices is lower the more distant the period in question is from the one the shock was realized as expectations converge to the true state of the economy. Hence, prices increase increase compared to their complete information level, but they do so in a decaying fashion. This implies inflation is caused on impact and deflation from the next period onwards. The same remarks can be said of output and output growth. That is, output increases on impact which is translated into positive growth, but increases in a decaying fashion thereafter, implying a negative growth rate. As mentioned earlier, in the limit the effects of shock to expectations vanish.[3]

The third set of results concerns monetary policy. Monetary policy affects prices and, in the present framework, has real affects as long as the consumer and the producer have heterogenous expectations about them. This is reflected upon the equilibrium nominal (and real) wage, which, as explained, implies output depends in part on the producer's expectations about prices. Therefore, insofar as monetary policy affects prices and there is heterogeneity in expectations about these, monetary policy also affects the real side of the economy. It turns out, that the weight placed on expectations relative to the underlying total productivity diminishes as the monetary authority becomes more aggressive on inflation. In the limit, prices are zero and the expectations' role is minimized but not eliminated as expectations continue to matter in a direct way. It is shown that equilibrium output is immune to expectations, implying the economy is at its efficient level, when the nominal interest rate does not respond at all to inflation, a 'Friedman-rule' policy. Hence, in the present framework a 'Friedman-rule' policy emerges as the optimal policy, whereas if monetary policy is at all to be pursued, it should be as aggressive on inflation as possible. In an extension to the basic framework, the monetary authority is assumed to have superior information about the following period's state, however to prevent the nominal interest rate from being fully revealing about it, it is assumed that the monetary authority

[3]The effects of transient productivity shocks are in the short-run similar to the effects of permanent productivity shocks, and, after then, resemble the ones of shocks to expectations.

transmits it with noise, which may be a measurement error or a 'surprise' monetary policy shock. Although, monetary policy recommendations mentioned above remain unchanged, it is shown that positive monetary policy shocks raise the producer's expectations and, as a result, boost output and prices, with their effect being only on impact.

The idea that incomplete information can open the door to non-neutralities of nominal factors is known at least since Phelps (1970), and was subsequently formalized in Lucas (1972).[4] The role of expectations along the business cycle has its origins at least in Pigou (1926) and was revived by Beaudry and Portier (2004). Positive co-movement in response to news is discussed in the latter and all the listed papers thus far.

The closest paper to the present one is Lorenzoni (2009) and this paper can be viewed as complementary to it. Lorenzoni (2009)[5] restricts attention to the demand side and adopts a New-Keynesian framework, whereas the present paper emphasizes the supply side[6] within a, leaving incomplete information aside, frictionless environment. A common point in the two papers is the signs of the impulse responses of output and employment to permanent productivity and expectational shocks. However, their implications for inflation are diametrically opposed, suggesting that expectational shocks within the current framework behave like transient supply shocks. Similarly, at least partially different are the implications of the two papers for monetary policy.

That news is deflationary is a point also documented in Christiano et al. (2010),[7] developed within a New-Keynesian environment. The driving force behind the result in Christiano et al. (2010) is that agents receive news about future rather than present, a distinction the present paper abstracts from. A weakness of that framework is that its results collapse if the stochastic process for technology is a random walk; the latter will be assumed here in order to illustrate the different responses to permanent and transitory productivity shocks.

The structure of the paper is as follows: Section 2 presents the model. Equilibrium results are collected in Section 3. In Section 4 the monetary authority is endowed with superior information and the effects of the nominal interest serving as a public signal are analyzed, whereas in Section 5 the

[4] Polemarchakis and Weiss (1977), Weiss (1980), Bulow and Polemarchakis (1983), and, especially, Grossman and Weiss (1982) are related papers of the early literature - Note: Connection to these to be added.

[5] However, its earlier working version Lorenzoni (2005) is closer to the present paper.

[6] Angeletos and La'O (2009), Angeletos and La'O (2010), and La'O (2010) also focus on the supply side, but their objectives are centered around the role of informational heterogeneity from which the present paper abstracts except when it comes to monetary policy.

[7] It has also been suggested in Barsky and Sims (2009).

monetary implications generated within the present framework are analyzed. Section 6 concludes.[8]

2 Environment

The economy is populated by a representative consumer supplying labor to a representative firm owned by him and managed by a producer. There is a unique non-storable consumption good. The economy is cashless and financial markets include only a market for a nominal bond whose price is set by a monetary authority. Time is discrete and infinite commencing in period 0.

Consumer's preferences are given by

$$E\sum_{t=0}^{\infty} \beta^t U(C_t, N_t)\,,$$

where period-t utility is

$$U(C_t, N_t) \;=\; \log C_t \;-\; \frac{1}{1+\zeta}\, N_t^{1+\zeta}\,.$$

C_t and N_t denote consumption and employment in period t, respectively. Agent's valuation of the future is given by $\beta \in (0,1)$.

Period-t budget constraint is

$$Q_t\, B_{t+1} \;+\; P_t\, C_t \;=\; B_t \;+\; W_t\, N_t \;+\; \Pi_t\,,$$

where Q_t and B_{t+1} denote the price and holdings of nominal bonds maturing in $t+1$, respectively, P_t and W_t denote the price of the consumption good and nominal wage in t, respectively, and Π_t denotes firm's profits that accrue to the consumer.

Firms are perfectly competitive and produce according to

$$Y_t \;=\; A_t\, N_t\,,$$

where A_t denotes the worker's productivity.[9]

The natural logarithm of productivity (henceforth lowercase letters will denote natural logarithms) consists of a permanent and a transient component related in the following way:

$$a_t \;=\; x_t \;+\; u_t\,, \tag{2.1}$$

[8] An empirical section, still in progress, is to be added.

[9] Equilibrium results are qualitatively invariant to technology being $Y_t \;=\; A_t N_t^{\alpha}$, with $\alpha \in (0,1)$, as shown in footnote 13. Monetary policy effectiveness, however, may depend on whether technology is linear or not as explained in footnote 14.

where u_t denotes the i.i.d. transient component of productivity with $u \sim N(0, \sigma_u^2)$.
The stochastic process for the permanent component x_t is given by

$$x_t = x_{t-1} + \epsilon_t , \tag{2.2}$$

where ϵ_t is an i.i.d shock and $\epsilon \sim N(0, \sigma_\epsilon^2)$.

Agents have costless access to a private signal about the permanent component of productivity

$$s_t = x_t + e_t , \tag{2.3}$$

where e_t is i.i.d. and $e \sim N(0, \sigma_e^2)$. Shocks u_t , ϵ_t , and e_t are mutually independent.

2.1 Timeline

Each period is divided into two stages. In stage 1, the producer observes the private signal s_t about the permanent productivity component, the consumer realizes his temporal productivity a_t, and the labor market opens. The producer observes a_t at the end of stage 1. In stage 2, the consumption-good and the nominal-bond markets open. The nominal interest rate is set by a monetary authority in a way analyzed in the following section. All payments materialize in stage 2 and are perfectly enforceable.

3 Equilibrium

In stage 1, following the announcement of the nominal wage by the Walrasian auctioneer, the producer chooses employment to maximize firm's expected profits, $E_t^p[\lambda_t \Pi_t]$, subject to its technology. The valuation of profits is based on the consumer's period-t Lagrange multiplier, λ_t . Henceforth, producer's expectations will refer to his expectations as of stage 1. Given the technology specified previously, the solution to the producer's problem involves the firm accommodating any labor supply at the following wage:

$$W_t = \frac{E_t^p[\lambda_t P_t A_t]}{E_t^p[\lambda_t]} . \tag{3.1}$$

In stage 1, the consumer observes the nominal wage and decides on his labor supply, whereas in stage 2, he observes the nominal bond and the consumption good prices and chooses bond holdings and consumption, respectively.[10] The consumer's optimality conditions are

$$N_t^{\zeta} = \frac{W_t}{E_{t,1}^c[P_t C_t]} \tag{3.2}$$

$$C_t = \frac{Q_t}{\beta P_t} E_t^c \left[P_{t+1} C_{t+1} \right] . \tag{3.3}$$

Equilibrium bond holdings are $B_{t+1} = 0$. As will become apparent below, the information structure implies the consumer has the same information set in stages 1 and 2, hence no stage-specific distinction is made in (3.2) and (3.3) about the consumer's expectations.

The focus will be on linear equilibria. This considerably simplifies the analysis since only first-order beliefs are relevant and agents' information processing becomes significantly more tractable.

3.1 Linear equilibrium.

In log-linear form the above equations are (where applicable, approximations are first-order around the stochastic steady-state to be characterized below)

$$w_t = E_t^p[a_t] + E_t^p[p_t] \tag{3.4}$$

$$\zeta n_t = w_t - E_t^c[p_t] - E_{t,1}^c[c_t] \tag{3.5}$$

$$c_t = -\log \beta + \log Q_t + E_{t,2}^c[c_{t+1} + \pi_{t+1}] . \tag{3.6}$$

Monetary authority. The monetary authority sets the gross nominal interest rate (equivalently, the inverse of the logarithm of the nominal bond price), $i_t = -\log Q_t$, according to the interest-rate rule

$$i_t = -\log \beta + \phi \, E_t^m[\pi_{t+1}] ,$$

where i_t denotes the nominal interest rate and π_{t+1} denotes inflation in period $t+1$, defined as $\pi_{t+1} := p_{t+1} - p_t$. The monetary policy's response to inflation ϕ will generally be assumed greater than one-to-one, that is $\phi > 1$. However, in Section 5, the no-policy case, $\phi = 0$, will also be considered.

[10] One can turn to Appendix A.1 for an analytical demonstration of the problems of the producer and the consumer.

As will be shown below, the monetary authority can acquire perfect knowledge about the current state of the economy by observing the nominal wage and the consumption good price. In later sections, the monetary authority will be endowed with superior information compared to the agents. However, to prevent the nominal interest rate from being fully revealing about the following period's state, it will be assumed that the monetary authority obtains noisy measurements of prices despite its superior information. The possibility of surprise monetary policy shocks is studied in Section 5 .

Complete information. Under complete information and ignoring the producer's private signal which would play no role, that is when $I_t^p = I_t^c = I_t^m = \big(\{a_t\}_{\tau=0}^t\big)$, where I_t^p, I_t^c, and I_t^m denote the information sets of the producer, the consumer, and the monetary authority, respectively, the real side of the economy is determined irrespectively of the monetary policy pursued. It can be confirmed that $n_t^* = 0$ and $y_t^* = a_t$.[11]

Maintaining the above simplifying assumptions about the conduction of monetary policy, the Euler equation becomes

$$E_t^c[a_{t+1}] - a_t = (\phi - 1)(E_t^c[p_{t+1}] - p_t)\,,$$

which in turn implies $p_t^* = \frac{1}{\phi-1}\, a_t$. Since the use of Kalman filter (one can also check Section 3.3) implies $E_t^c[a_{t+1}] = E_t^c[x_t] \neq a_t$, the possibility of prices being fixed in equilibrium is eliminated.[12]

Incomplete information. The *state* of the economy as of period t is given by $Z_t = \{a_t\,, s_t\}$ and *history* is $\Psi_t = \big(\{a_\tau\}_{\tau=0}^t, \{s_\tau\}_{\tau=0}^t\big)$.

It is conjectured that

$$c_t = \pi_1 E_t^p[a_t] + \pi_2 a_t \tag{3.7}$$

$$p_t = \kappa_1 E_t^p[a_t] + \kappa_2 a_t\,. \tag{3.8}$$

The information set of the producer at the beginning of stage 1 is $I_t^p = \big(\{a_\tau\}_{\tau=0}^{t-1}, \{s_\tau\}_{\tau=0}^t\big)$; all expectation operators are with respect to it unless stated otherwise. Given this, the producer's prior is $a_t \sim N(E_t^p[x_t]\,, \sigma_x^2 + \sigma_u^2)$, where σ_x is assumed to have reached its steady-state value, and together with

[11]Stationarity can be restored by using $\frac{Y_t}{A_t}$ ($y_t - a_t$ in logs).

[12]To see this point clearly, suppose that the permanent productivity component x is a constant, in which case $\{a_\tau\}_{\tau=0}^t$ serve as signals about it. Constant prices would have prevailed if either productivity a_t had been evolving as a random walk, or if the economy had been static.

$E_t^p[x_t]$, is pinned down in Section 3.3. Combining this with the above conjectures and (3.4) implies $w_t = (1 + \kappa_1 + \kappa_2)\, E_t^p[x_t]$. As the consumer knows his productivity a_t, observing the nominal wage announced by the auctioneer reveals to him producer's expectations about a_t, which implies he can extract the private information s_t the producer has. Then the consumer's information set *upon* the observation of nominal wages is $I_t^c = \Psi_t$. This trivially implies $E_{t,1}^c[c_t] = c_t$ and $E_{t,1}^c[p_t] = p_t$. The monetary authority can extract full information about the current state by observing the nominal wage in stage 1 which reveals s_t and prices in stage 2 which reveal the temporal productivity a_t. That is $I_t^m = \Psi_t$, which implies that the monetary authority and the consumer have exactly the same information in stage 2.

The above remarks coupled with (3.4) and (3.5) suggest equilibrium labor is

$$\zeta n_t = E_t^p[x_t] + E_t^p[p_t] - p_t - c_t\,.^{13} \tag{3.9}$$

As shown in Appendix A.2, combining the above conjectures with (3.9) and the market-clearing condition, $y_t = c_t$, implies

$$y_t = \frac{1}{\phi + \zeta(\phi - 1)} \left(\phi E_t^p[a_t] + \zeta(\phi - 1)\, a_t\right) \tag{3.10}$$

$$n_t = \frac{\phi}{\phi + \zeta(\phi - 1)} \left(E_t^p[a_t] - a_t\right) \tag{3.11}$$

$$p_t = \frac{1}{\phi - 1}\, y_t\,. \tag{3.12}$$

Equations (3.10) and (3.12) imply that output and prices depend positively on the producer's expectations about productivity, as well as productivity itself, with the respective weights depending on the Frisch elasticity of labor supply parametrized by ζ, and, the monetary policy parameter, ϕ.[14] The latter's effect is analyzed in detail in Section 5. Equation (3.11) implies equilibrium employment depends

[13]If technology takes the more general form $Y_t = A_t N_t^{\alpha}$, with $\alpha \in (0, 1]$, then taking the same steps implies equilibrium employment is given by $(1 + \zeta - \alpha) n_t = \log \alpha + E_t^p[x_t] + E_t^p[p_t] - p_t - c_t$. For $\alpha = 1$ this expression collapses to (3.9).

[14]The fact that in (3.11) employment depends on the distance of expectations from the underlying productivity value which implies, as can be seen in Section 5, that the monetary authority can drive the economy to its efficient level hinges on the assumed specification of preferences and technology, in particular on the former being logarithmic in consumption and the latter linear in productivity.

on the deviation of expectations about productivity from its realized value. The greater the producer's expectations about productivity, the higher output, employment, and prices will be with respect to their efficient levels.

It follows from (3.10) and (3.12) that

$$\pi_t = \frac{1}{(\phi - 1)[\phi + \zeta(\phi - 1)]} \left(\phi(E_t^p[a_t] - E_{t-1}^p[a_{t-1}]) + \zeta(\phi - 1)(a_t - a_{t-1})\right) . \quad (3.13)$$

Equation (3.13) suggests that inflation is a weighted average of the *difference* in expectations about productivity in the last two periods and the difference in productivity in the last two periods itself. The importance of this is illustrated in the following part.

3.2 Labor wedge

Following Chari et al. (2007) the labor wedge $1 - \tau_{n,t}$ is given by the ratio of the marginal rate of substitution between consumption and labor to the marginal product of labor:

$$-\frac{U_{n,t}}{U_{c,t}} = (1 - \tau_{n,t})\, MP_{n,t}$$

where $U_{n,t}$ and $U_{c,t}$ denote the marginal utility of labor and consumption, respectively, and $MP_{n,t}$ the marginal product of labor in period t. The specification of preferences and technology and market-clearing imply this is equal to

$$N_t^{1+\zeta} = 1 - \tau_{n,t} \, .$$

It can be confirmed that under complete information the labor wedge is equal to 1. Under incomplete information this will generally not be the case; taking logs and using (3.11) implies

$$\log(1 - \tau_{n,t}) = \frac{\phi(1+\zeta)}{\phi + \zeta(\phi - 1)} \left(E_t^p[a_t] - a_t\right) .$$

This shows that for $E_t^p[a_t] > a_t$, the labor wedge will be greater than 1 (log positive), and lower, otherwise, whereas it is decreasing in the monetary policy parameter, ϕ.

3.3 Equilibrium dynamics

The Kalman filter algorithm implies that producer's expectations evolve as

$$E_t^p[x_t] = (1 - k)\, E_{t-1}^p[x_{t-1}] + k\, (\theta s_t + (1 - \theta) a_{t-1}) , \quad (3.14)$$

where $k,\,\theta \in (0,1)$ and depend on variances $\sigma^2_\epsilon, \sigma^2_e, \sigma^2_u$.[15]

If a shock to the permanent productivity component $\epsilon_t = 1$ is realized, expectations about productivity in period $t+s$ increase by $1 - (1-k)^s\,(1-k\theta)$. The implied impulse response functions are

$$\frac{dy_{t+s}}{d\epsilon_t} = 1 - (1-k)^s \, \frac{\phi\,(1 - k\theta)}{\phi + \zeta(\phi - 1)} \in (0,1) \tag{3.15}$$

$$\frac{dn_{t+s}}{d\epsilon_t} = -\,(1-k)^s \, \frac{\phi\,(1 - k\theta)}{\phi + \zeta(\phi - 1)} < 0 \tag{3.16}$$

$$\frac{d\pi_{t+s}}{d\epsilon_t} = (1-k)^{s-1} \, \frac{\phi k(1 - k\theta)}{(\phi - 1)[\phi + \zeta(\phi - 1)]} > 0 \quad \text{for } s \geq 1\,. \tag{3.17}$$

The impact response of inflation is

$$\frac{d\pi_t}{d\epsilon_t} = \frac{\phi\, k\theta + \zeta(\phi - 1)}{(\phi - 1)[\phi + \zeta(\phi - 1)]}\,.$$

A permanent productivity shock is shown by (3.15) to increase output, with the increase falling short of the unit increase in the efficient level of output. Consequently, employment decreases as can be confirmed by (3.16). Both results, in line with Lorenzoni (2009), are attributed to expectations

[15] The Kalman filter implies

$$E^p_t[x_{t+1}] = (1-k)\,E^p_{t-1}[x_t] + k\,(\theta s_t + (1-\theta)a_{t-1})\,,$$

which since $E^p_t[x_{t+1}] = E^p_t[x_t]$ boils down to (3.14).

One can denote $\omega_t := \theta s_t + (1-\theta)a_{t-1} = x_t + v_t$, where $v_t \equiv \theta e_t + (1-\theta)(u_{t-1} - \epsilon_t)$ is i.i.d. and independent of ϵ_{t+1}, and $\omega \sim N(0, \sigma^2_v)$, with the relative precision of the producer's private signal be given by $\theta = \frac{\frac{1}{\sigma^2_e}}{\frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_u + \sigma^2_\epsilon}}$. It can be confirmed that $\sigma^2_v = \left(\frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_u + \sigma^2_\epsilon}\right)^{-1}$.

The steady-state value of the Kalman gain is given by

$$k = \frac{\sigma^2_x}{\sigma^2_x + \sigma^2_v}\,,$$

where $\sigma^2_x := Var_t[x_t]$ solves the Riccati equation $\sigma^2_x = \sigma^2_\epsilon \left(1 + \frac{\sigma^2_v}{\sigma^2_x}\right)$, with σ^2_v be given above. Rearranging the Riccati so as to include the limit results yields

$$\sigma^2_x = \sigma^2_\epsilon + \left(\frac{1}{\sigma^2_v} + \frac{1}{\sigma^2_x}\right)^{-1}.$$

A thorough demonstration of the Kalman filter can be found in Technical Appendix B in Ljungqvist and Sargent (2004).

As the paper focuses on the dynamics around the steady-state, it will be assumed that the prior of the agents at $t = 0$ is $x_0 \sim N(0, \sigma^2_x)$, resulting in σ^2_x and, consequently, k and θ, not being time-indexed.

underracting following a permanent productivity shock. Notably, equation (3.17) suggests productivity shocks are inflationary. At the center of this result is the positive dependence of prices on expectations for $\phi > 1$ as (3.10) and (3.12) can ascertain. Hence, as expectations converge to the new permanent productivity level, prices get closer to their efficient level, implying positive inflation throughout. This comes in sharp contrast with Lorenzoni (2009), where deflation is caused due to demand underreacting. In the limit as $s \to \infty$, all variables converge to their efficient level. Previously made remarks imply convergence is characterized by positive growth and inflation.

If a shock to the private signal $e_t = 1$ arises, expectations in period $t+s$ increase by $(1-k)^s k\theta$. The impulse response functions in this case are

$$\frac{dy_{t+s}}{de_t} = \frac{dn_{t+s}}{de_t} = (1-k)^s \frac{\phi k\theta}{\phi + \zeta(\phi - 1)} > 0 \tag{3.18}$$

$$\frac{d\pi_{t+s}}{de_t} = -(1-k)^{s-1} \frac{\phi k^2\theta}{(\phi-1)[\phi + \zeta(\phi-1)]} < 0 \quad \text{for } s \geq 1\,. \tag{3.19}$$

The impact response of inflation is

$$\frac{d\pi_t}{de_t} = \frac{\phi\, k\theta}{(\phi-1)[\phi + \zeta(\phi-1)]} > 0. \tag{3.20}$$

Equation (3.18) demonstrates the *positive co-movement* result: output and employment increase in response to a positive shock to the producer's signal. The result is due to the Walrasian auctioneer announcing higher nominal wages compared to the efficient level when producers are optimists.[16] This results in increased labor supply relative to the efficient level, obtained under complete information. In the limit as $s \to \infty$, expectations converge to the true level of productivity implying both output and employment return to their efficient levels.

As argued above, for $\phi > 1$ the price response to shocks is related positively to producers' expectations. Hence, a positive shock to the producer's private information causes increases in the price levels. Since Kalman filtering implies expectations converge monotonically to the true productivity level with the effect disappearing as $s \to \infty$, the same is true for prices, generating therefore deflation as reflected in (3.19). On impact, however, as (3.20) suggests, positive shocks to expectations are inflationary, which

[16] This implies that firms make negative profits if producers are optimists. Negative profits are subtracted in a lump-sum fashion from the consumer/owner's budget constraint, which is always satisfied. On expectation profits are zero.

is merely attributed to shocks not causing any retrospective effects. The two results combined suggest an 'overshooting'-type response to positive shocks to expectations.

Summing the above, the dynamics in the wake of a positive shock to expectations suggest convergence to the first-best is characterized by negative growth and deflation. The latter, a central result to this paper, comes in sharp contrast with Lorenzoni (2009).

If a shock to the transient component of productivity $u_t = 1$ is realized, expectations in period $t+s$, where $(s \geq 1)$ as changes in productivity affect expectations with one period lag, increase by $(1-k)^{s-1}k(1-\theta)$. The impact responses of output, employment and prices resemble those in the permanent productivity shock case, whereas from period $t+1$ onwards they resemble those in the shock to expectations case. This implies that inflation is positive in period t, depends on the parameters in $t+1$, and is negative from period $t+2$ onwards. In the limit, all effects vanish.

In period t,

$$\frac{dy_t}{du_t} = \frac{\zeta(\phi-1)}{\phi + \zeta(\phi-1)} \in (0,1) \tag{3.21}$$

$$\frac{dn_t}{du_t} = -\frac{\phi}{\phi + \zeta(\phi-1)} < 0 \tag{3.22}$$

$$\frac{d\pi_t}{du_t} = \frac{\zeta(\phi-1)}{(\phi-1)[\phi + \zeta(\phi-1)]} > 0\,. \tag{3.23}$$

The impulse responses are

$$\frac{dy_{t+s}}{du_t} = \frac{dn_{t+s}}{du_t} = (1-k)^{s-1}\frac{\phi k(1-\theta)}{\phi + \zeta(\phi-1)} > 0 \quad \text{for } s \geq 1 \tag{3.24}$$

$$\frac{d\pi_{t+s}}{du_t} = -(1-k)^{s-1}\frac{\phi k^2(1-\theta)}{(\phi-1)[\phi + \zeta(\phi-1)]} < 0 \quad \text{for } s \geq 2\,. \tag{3.25}$$

In period $t+1$,

$$\frac{d\pi_{t+1}}{du_t} = \frac{\phi\, k(1-\theta) - \zeta(\phi-1)}{(\phi-1)[\phi + \zeta(\phi-1)]}\,. \tag{3.26}$$

Depending on the parameters, the change in inflation in $t+1$ may be greater or lower than its impact or its subsequent change in $t+2$, positively or negatively signed.

4 Monetary Authority with Superior Information

In this section the assumption that the monetary authority has no superior information is lifted. In its stead it is assumed that the monetary authority knows the following period's state, that is $I_t^m = \Psi_{t+1}$. To prevent the nominal interest rate from being fully revealing about the state of the economy to prevail in the following period, this will be complemented with the additional assumption that the, otherwise fully rational, monetary authority either misreports prices or transmits 'surprise' monetary policy shocks, which will be both seen in turn.[17] This implies, as it will be shown that the nominal interest rate serves as a public signal about the following period's temporal productivity, a_{t+1}. The purpose of this section is two-fold: first, to see the informational implications per se when the monetary authority transmits its superior information, and, second, to see its equilibrium effects.

In case the monetary authority misreports the following period's price, the prevailing nominal interest rate set in $t-1$ is

$$i_{t-1} = \phi \tilde{\pi}_t \,,$$

where $\tilde{\pi}_t = \tilde{p}_t - p_t$, with

$$\tilde{p}_t = p_t + w_t \,.$$

The noise is i.i.d with $w_t \sim N(0, \sigma_w^2)$ and is independent of the shocks ϵ_t, e_t, and u_t.

In case the monetary authority transmits 'surprise' monetary policy shocks, the nominal interest rate in $t-1$ is

$$i_{t-1} = \phi \pi_t + \omega_t \,,$$

where ω is i.i.d. with $\omega_t \sim N(0, \sigma_\omega^2)$ and is independent of the shocks ϵ_t, e_t, u_t, and w_t.

Linear Equilibrium. Equilibrium is exactly like in Section 3.1 and is given by equations (3.10)-(3.13). Compared to Section 3.1 the information set of the producer is augmented by the public signal transmitted by the monetary authority, denoted z_t, and given by $\tilde{I}_t^p = \left(\{a_\tau\}_{\tau=0}^{t-1}, \{s_\tau\}_{\tau=0}^{t}\right) \cup z_t$. It is shown in Appendix A.3 that

$$E^p[a_t \,\|\, \tilde{I}_t^p] = \delta E_t^p \left[x_t \,\|\, \tilde{I}_t^p \setminus z_t\right] + (1-\delta) z_t \,,$$

where δ depends positively on the variance of the public signal transmitted by the monetary authority which in turn depends on the variance of the shock σ_w^2 and the monetary policy parameter ϕ (see also

[17] The minimum assumption required is that even if the monetary authority truthfully reveals tomorrow's state, this is not known to the producer.

Appendix A.3).[18] The public signal is given by

$$z_t = a_t + \frac{\phi + \zeta(\phi - 1)}{\zeta} w_t \tag{4.1}$$

$$z_t = a_t + \frac{\phi + \zeta(\phi - 1)}{\phi\,\zeta} \omega_t\,, \tag{4.2}$$

where (4.1) refers to the first case and (4.2) to the second one. It can be seen that (4.1) and (4.2) are the same if $w_t = \frac{1}{\phi}\omega_t$, which implies the economy's response to misreports or 'surprise' shocks is very similar.

The consumer realizes the shock as soon as he realizes his productivity in $(t, 1)$.

The reason a positive shock to the nominal interest rate rule raises the producer's expectations about productivity in the following period lies in the fact that productivity has a positive effect on prices as can be seen from (3.12). Hence, a higher nominal interest rate will overstate the following period's prices which the producer will erroneously partially attribute to an increase in productivity.

Equilibrium Dynamics. Equilibrium dynamics when shocks to ϵ_t, e_t, and u_t are realized are very similar to the ones in Section 3.3 which the reader can refer to.

Unlike in those cases, the effects of a 'misreporting' or a monetary policy shock last only one period. This is because it is a signal about a_t which consumers know and producers realize by the end of stage $(t, 1)$. Once a shock $w_t = 1$ arises, the impact responses are

$$\frac{dy_t}{dw_t} = \frac{dn_t}{dw_t} = \frac{\phi(1-\delta)}{\zeta} > 0 \tag{4.3}$$

$$\frac{dp_t}{dw_t} = \frac{d\pi_t}{dw_t} = -\frac{d\pi_{t+1}}{dw_t} = \frac{\phi(1-\delta)}{\zeta(\phi - 1)} > 0\,. \tag{4.4}$$

It can be seen from (4.3) and (4.4) that raising the nominal interest rate compared to its complete information equivalent boosts output and causes an inflationary pressure on impact and a deflationary one in the following period. These responses are in the same direction as the ones following a shock to the producer's private signal, analyzed in Section 3.3.

[18] The case analyzed in Section 3 is when $\delta = 1$.

The respective responses for a monetary policy shock ω_t are divided by ϕ as (4.2) suggests:

$$\frac{dy_t}{d\omega_t} = \frac{dn_t}{d\omega_t} = \frac{(1-\delta)}{\zeta} > 0 \tag{4.5}$$

$$\frac{dp_t}{d\omega_t} = \frac{d\pi_t}{dw_t} = -\frac{d\pi_{t+1}}{d\omega_t} = \frac{(1-\delta)}{\zeta(\phi-1)} > 0\,. \tag{4.6}$$

The above comments apply in the case of 'surprise' monetary policy shocks as well. However, in the next section it is shown that different monetary policy recommendations apply to the above cases.

5 Monetary Policy

The equilibrium wage in stage 1 depends on the producer's expectations for productivity and prices. Monetary policy can enter the real side of the economy through the latter; as (3.4) and (3.5) attest, this happens as long as the producer and the consumer have *heterogeneous* expectations about the prices to prevail in stage 2.

The ratio of weights places on producer's expectations and temporal productivity in equilibrium equations, (3.10) and (3.12), is $\frac{\phi}{\zeta(\phi-1)}$. This ratio is decreasing in ϕ. Intuitively, the greater ϕ, the lower prices will be; in the limit as $\phi \to \infty$, $p_t \to 0$ implying that monetary policy will not affect the labor market decision in stage 1, since prices are independent of the monetary policy pursued. That is the indirect channel of prices will be shut and the impact of expectations will be minimized. However, even in this limiting case, expectations continue to have an equilibrium effect through the first (direct) channel.

The assumption maintained thus far has been that $\phi > 1$, i.e. that the monetary authority's response to inflation is greater than one-to-one. For $\frac{\zeta}{1+\zeta} < \phi < 1$, positive shocks to expectations generate positive co-movement and are inflationary, while permanent productivity shocks cause negative co-movement and are deflationary. These results are the same as the ones in Lorenzoni (2009). For $0 < \phi < \frac{\zeta}{1+\zeta}$, positive shocks to expectations generate negative co-movement and are deflationary, while permanent productivity shocks generate positive co-movement and are inflationary. Whereas this case may not be necessarily appealing from an empirical standpoint, the limiting case $\phi = 0$, a 'Friedman-rule' policy, can generate interesting monetary policy implications. One can notice that for $\phi = 0$, $y_t = a_t$ and $p_t = -a_t$. That is a Friedman-rule policy completely eliminates the role of expectations and maintains the economy at its efficient (complete-information equivalent) level. As explained previously, for $0 < \phi < \frac{\zeta}{1+\zeta}$, prices depend negatively on productivity, and, hence, the indirect channel through which expectations have equilibrium effects (the price channel) mitigates the effects of the direct channel. For $\phi = 0$, the two precisely offset each other, rendering, therefore, expectations irrelevant in equilibrium. Consequently, a Friedman-rule emerges as the optimal policy in this framework. However, if monetary policy is to be pursued,[19] then it should be as aggressive on inflation as possible.

Last, the monetary policy in case the monetary authority has superior information and either misreports prices or fuels the economy with 'surprise' shocks is analyzed. In particular, again the interest is in the monetary policy parameters that bring the economy as close to its complete information level as possible. One can check from (4.3) that ϕ affects the equilibrium both directly, and indirectly through δ.

In case of the authority misreporting prices, the weight placed on the signal z_t, $1 - \delta$, depends inversely on ϕ (see also Appendix A.3). In the limit as $\phi \to \infty$, the variance of the public signal becomes infinite, resulting in $\delta \to 1$; this implies misreports have no real effects. A 'Friedman-rule' policy ensures this as well, however this is achieved through the direct channel as (4.3) suggests. Hence, both extreme policies guarantee the immunity of the real economy to price misreports.

In case of 'surprise' monetary policy shocks, ϕ matters only through δ as can be seen from (4.5). It can be seen in Appendix A.3 that the variance of the signal z_t tends to infinity only when a 'a Friedman-rule' policy is pursued, which is the unique optimal policy in this case. For $\phi > 1$, the variance of the signal increases in ϕ but is finite for $\phi \to \infty$, hence $\delta \neq 1$, which implies a policy as aggressive on inflation as possible is a second-best policy.

Notice that, despite the additional indirect presence of the monetary policy parameter ϕ through δ in the dynamics following any other type of shock when the monetary authority has access to superior information, the monetary policy implications analyzed in the no-superior information case apply here as well.

[19] As explained earlier the relative weight of expectations decreases the more aggressive the monetary authority is on inflation. As there are discontinuities for values $\phi = \frac{\zeta}{1+\zeta}$ and $\phi = 1$, attention will be restricted to values $\phi > 1$ to circumvent this issue.

6 Conclusion

This paper has studied the role of producers' expectations along the business cycle. Positive shocks to expectations ('optimism'), which can well be attributed to positive 'surprise' monetary policy shocks, were shown to cause positive co-movement and be deflationary. In such a context, the optimal monetary policy was shown to be a 'Friedman-rule' policy, whereas a policy as aggressive on inflation as possible emerged as a second-best policy.

Enriching the present environment with capital seems a rather natural extension with promising implications for the stock-market and its interaction with monetary policy.

A Omitted derivations

A.1 Agents' problems

Producer's problem. Stage 2 profits of the consumer-owned firm are given by $\Pi_t = (P_t A_t - W_t) N_t$, where $Y_t = A_t N_t$.

In stage 1, firm chooses $N_t \geq 0$ to maximize the firm's expected profits:

$$E_t^p[\lambda_t \Pi_t] \, .$$

Expectations are with respect to the information set of the producer specified in the main text. The maximization problem does not yield a solution if $W_t < \frac{E_t^p[\lambda_t P_t A_t]}{E_t^p[\lambda_t]}$, whereas any labor supply is accommodated if

$$W_t = \frac{E_t^p[\lambda_t P_t A_t]}{E_t^p[\lambda_t]} \, . \tag{A.1}$$

The case where the LHS of (A.1) is greater than the RHS implies $N_t = 0$ which in turn implies $C_t = 0$. This is a well-known issue in production economies with variable labor supply. It will be dismissed here and will be assumed that in equilibrium $C_t \, , \, N_t > 0$.

Consumer's Problem. Consumer solves the following problem:

$$\max_{C_t \, , N_t \, , B_{t+1} \geq 0} E_t^c \sum_{t=0}^{\infty} \left(\log C_t - \frac{N_t^{1+\zeta}}{1+\zeta} \right)$$

s.t.

$$P_t C_t + Q_t B_{t+1} = B_t + W_t N_t + \Pi_t \, ,$$

and the no-Ponzi constraint. The FOCs with respect to C_t, N_t, and B_{t+1} respectively are:

$$\frac{1}{C_t} = \lambda_t P_t \tag{A.2}$$

$$N_t^{\zeta} = \lambda_t W_t \tag{A.3}$$

$$Q_t = \beta \frac{E_t^c[\lambda_{t+1}]}{\lambda_t}\,, \tag{A.4}$$

where λ_t is the current-value Lagrange multiplier associated with the period-t budget constraint. Expectations are with respect to the information set of the consumer specified in the main text. No distinction has been made between stages 1 and 2 as the information structure implies the consumer has the same information set in both stages. Combining (A.2) with (A.3) and (A.2) with (A.4) yields

$$N_t^{\zeta} = \frac{W_t}{P_t C_t} \tag{A.5}$$

$$C_t = \frac{Q_t}{\beta P_t} E_t^c[C_{t+1} P_{t+1}]\,. \tag{A.6}$$

In addition, the no-Ponzi-scheme condition and the fact that nominal bonds are in zero net-supply imply $B_{t+1} = 0$ in equilibrium.

A.2 Derivation of equilibrium equations (3.10)-(3.12)

Combining (3.9) with conjectures (3.7) and (3.8) implies equilibrium labor supply is

$$n_t = \frac{1}{\zeta}(1 + \kappa_2 - \pi_1) E_t^p[a_t] - \frac{1}{\zeta}(\kappa_2 + \pi_2) a_t\,.$$

Combined with firm's technology, $y_t = a_t + n_t$, the above implies

$$y_t = \frac{1}{\zeta}(1 + \kappa_2 - \pi_1) E_t^p[a_t] + \left(1 - \frac{1}{\zeta}(\kappa_2 + \pi_2) a_t\right)\,. \tag{A.7}$$

Given the market-clearing condition $y_t = c_t$, matching coefficients in (3.7) and (A.7) implies

$$\pi_1 = \frac{1 + \kappa_2}{1 + \zeta} \tag{A.8}$$

$$\pi_2 = \frac{\zeta - \kappa_2}{1 + \zeta}\,. \tag{A.9}$$

The Euler equation (A.6) combined with market-clearing and the fact that $I_t^p = I_t^m$ can be written as

$$E_t^c[y_{t+1}] - y_t = (\phi - 1)(E_t^c[p_{t+1}] - p_t),$$

which implies (3.12). It follows then that

$$\pi_1 = (\phi - 1)\kappa_1 \tag{A.10}$$

$$\pi_2 = (\phi - 1)\kappa_2 . \tag{A.11}$$

Solving the system (A.8)-(A.11) yields the coefficients in (3.10) and (3.12):

$$\pi_1 = \frac{\phi}{\phi + \zeta(\phi - 1)} \tag{A.12}$$

$$\pi_2 = \frac{\zeta(\phi - 1)}{\phi + \zeta(\phi - 1)} \tag{A.13}$$

$$\kappa_1 = \frac{1}{\phi - 1}\pi_1 \tag{A.14}$$

$$\kappa_2 = \frac{1}{\phi - 1}\pi_2 . \tag{A.15}$$

It can also be confirmed that the Euler equation is satisfied from the stage 1 viewpoint of the consumer, since he knows that in stage 2 it will be $I_t^p = I_t^m$, which implies he can perfectly foresee not only the consumption good price, but also the nominal interest rate to prevail in stage 2.

Equation (3.11) follows from $n_t = y_t - a_t$.

A.3 Omitted derivations in Section 4

Below the case where the monetary authority misreports the following period's prices will be analyzed. The same process is followed for the case of monetary policy shocks with the results being stated at the end of this section.

Conjecture (3.8) implies that the nominal interest rate in $t-1$, i_{t-1}, serves as a public signal about productivity in t, a_t at the beginning of period t. To see this, in case the monetary authority misreports the following period's prices, agents extract $\tilde{p}_t = \frac{i_{t-1}+\phi p_{t-1}}{\phi}$, where $\tilde{p}_t = p_t + w_t$. Using this, (3.8) can be rearranged as

$$\frac{\tilde{p}_t - \kappa_1 E_t^p[a_t]}{\kappa_2} = a_t + \frac{1}{\kappa_2} w_t . \tag{A.16}$$

Since the LHS in (A.16) is known to producers and consumers in stage 1, it serves as a public signal denoted $z_t \equiv \frac{\tilde{p}_t - \tilde{\kappa}_1 E_t^p[a_t]}{\tilde{\kappa}_2}$.

It follows that as soon as the consumer realizes his productivity, he can extract the noise w_t from z_t and foresee its equilibrium effects, which the producer cannot until the end of stage 1. The heterogeneity in expectations about w_t implies it will have real effects.

It follows then that the public signal is

$$z_t \equiv \frac{(\phi+\zeta(\phi-1))\tilde{p}_t - \frac{\phi}{\phi-1}E_t^p[a_t]}{\zeta} = a_t + \frac{\phi + \zeta(\phi-1)}{\zeta} w_t . \tag{A.17}$$

The next step is to extract z_t from $E_t^p[a_t]$. Before this, note that the information sets of the producer and the consumer are $\tilde{I}_t^p = \left(\{a_\tau\}_{\tau=0}^{t-1}, \{s_\tau\}_{\tau=0}^{t}\right) \cup z_t$ and $\tilde{I}_t^c = \tilde{I}_t^p \cup \{a_t\}$, respectively. Further recall from Section 3 that the producer's prior distribution of a_t, where by 'prior' it is meant the distribution of a_t before the producer incorporates z_t in his expectations about it, is $a_t \sim N(E_t^p\left[x_t \,\|\, \tilde{I}_t^p \setminus z_t\right], \sigma_x^2+\sigma_u^2)$, where σ_x and $E_t^p\left[x_t \,\|\, \tilde{I}_t^p \setminus z_t\right]$ are pinned down in Section 3.3. One can notice that $\tilde{I}_t^p \setminus z_t = I_t^p$, with the latter be defined in Section 3. The fact that z_t is an unbiased signal of a_t and w_t is independent of the shocks in $E_t^p[x_t \,\|\, \cdot]$ implies Bayesian updating will yield (see also Chapter 2 in Veldkamp (2011)) the posterior

$$a_t \,|\, z_t \sim N\left(\delta E_t^p\left[x_t \,\|\, \tilde{I}_t^p \setminus z_t\right] + (1-\delta) z_t \,,\, \sigma_a^2\right) , \tag{A.18}$$

where $\delta = \frac{\frac{1}{\sigma_x^2+\sigma_u^2}}{\frac{1}{\sigma_x^2+\sigma_u^2}+\frac{1}{\sigma_w^2}}$ and $\sigma_a^2 = \left(\frac{1}{\sigma_x^2+\sigma_u^2} + \frac{1}{(\kappa_2)^{-2}\sigma_w^2}\right)^{-1}$, with κ_2 given by (A.15).

Substituting for $E_t^p[a_t]$, given by (A.18), in (A.17) results in

$$z_t = \left(\zeta + \frac{\phi}{\phi-1}(1-\delta)\right)^{-1} \left((\phi+\zeta(\phi-1))\tilde{p}_t - \frac{\phi}{\phi-1}\,\delta\, E_t^p\left[x_t \,\|\, \tilde{I}_t^p \setminus z_t\right]\right) . \tag{A.19}$$

Monetary policy shocks. In case the monetary authority transmits monetary policy shocks agents observe $\tilde{p}_t = \phi p_t + \omega_t$. Taking the same steps as above the public signal the monetary authority transmits is

$$z_t = a_t + \frac{\phi+\zeta(\phi-1)}{\phi\,\zeta}\,\omega_t ,$$

where

$$z_t = \left(\phi\,\zeta + \frac{\phi^2}{\phi-1}(1-\delta)\right)^{-1}\left((\phi+\zeta(\phi-1))\tilde{p}_t - \frac{\phi^2}{\phi-1}\,\delta\,E_t^p\left[x_t \,\|\, \tilde{I}_t^p \setminus z_t\right]\right).$$

The posterior distribution of $a_t \mid z_t$ is given by (A.18). However, in this case $\sigma_a^2 = \left(\frac{1}{\sigma_x^2+\sigma_u^2} + \frac{1}{(\phi\,\kappa_2)^{-2}\sigma_w^2}\right)^{-1}$, with κ_2 given by (A.15).

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