Minimum Wage Spillover Effects and Social Welfare in a Model of Stochastic Job Matching *

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Abstract

In this paper, I carry out a welfare analysis of the minimum wage in the framework of a Pissarides-type stochastic job matching model. I explore the role of the minimum wage in a labor market with trading externalities and present the necessary and sufficient condition for a minimum wage hike to be welfare-improving. The characterization of minimum wage spillover effects in this context leads to an interesting result: there is a direct link between the welfare effects and spillover effects of a minimum wage. This theoretical finding suggests that the welfare impact of minimum wage changes can be inferred from the empirical observation of spillover effects on the wage distribution.

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1 Introduction

Recent theoretical and empirical studies have challenged the orthodox view about the effects of the minimum wage on labor market outcomes, thereby curbing the “stigma” of inefficiency attached to this labor market institution. Following the influential work of Card and Krueger (1995), labor economists have come to acknowledge that moderate increases in the level of the wage floor can mitigate wage inequality without adversely affecting employment. Despite the broad consensus regarding the beneficial effects of the minimum wage on the welfare of labor market participants, it is evident that evaluating the minimum wage as a policy instrument necessitates a complete characterization of its impact on the distribution of wages, unemployment, and social welfare. In this paper, I examine these issues in the context of a search and matching model with match-specific productivity and wage bargaining. My principal objective is to analyze the equilibrium effects of minimum wage changes on labor market outcomes, describe the channels through which such changes impact on social welfare, and derive specific conditions for a welfare improving minimum wage.

The existing theoretical literature on minimum wage effects focuses on addressing the empirical regularities presented in the survey of minimum wage research by Card and Krueger (1995), who report a lack of significant employment losses, the existence of a spike at the legislated minimum, and possible effects on the wage distribution above the level of the minimum wage (spillover effects). To provide a theoretical explanation for these stylized facts, Card and Krueger hint at monopsony models of the labor market (see Manning [2003] for a survey on monopsony). Based on the assumption that monopsony power is generated by informational or search frictions, these models propose convincing theoretical arguments for some of the stylized facts associated with the minimum wage. However, their analytical framework is problematic in several critical respects.

First, most of the contributions to the literature are ad hoc modifications of existing models, which were not developed to account for the existence of a wage floor. As a result, their theoretical predictions account for some empirical regularities, but fail to explain other relevant stylized facts. For example, the studies by van den Berg and Ridder (1998) and van den Berg (2003) build on the equilibrium search model of Burdett and Mortensen (1998). Following the standard Burdett and Mortensen (1998) analysis, these studies assume homogeneous workers and firms and on-the-job search. This framework gives rise to wage dispersion, however, the resulting equilibrium wage distribution is at odds with empirical observation: it is increasing and does not exhibit a spike at the minimum wage. Bontemps, Robin, and van den Berg (2000) address these problems by introducing heterogeneity in the Burdett and Mortensen (1998) framework. Their assumption of productivity dispersion across firms gives predictions that are consistent with empirical evidence on minimum wage effects at the expense of analytical complexity.

Moreover, the assumption of exogenous contact rates between searching individuals and firms −employed in these studies− raises an additional theoretical consideration: is it plausible to examine the impact of the minimum wage in a setting where labor demand is exogenous? Clearly, in order to conduct policy analysis and discuss efficiency issues in the determination of a welfare optimizing minimum wage, the basic equilibrium search setting has to be augmented by a matching function. Mortensen (2000) shows that it is straightforward to incorporate features of the matching model into models of wage posting and create a general equilibrium framework within which the analysis of the equilibrium effects of policy intervention is possible. Such extensions, however, render the models less tractable, and thus, less amenable to a formal derivation and characterization of the conditions for welfare enhancing policy measures.

Flinn (2006) addresses these limitations within an alternative general equilibrium setting capable of capturing most of the empirical regularities associated with the minimum wage.[1] He builds a model that relies on the existence of match-specific productivity, job search, constant returns to scale matching technology, and worker-firm bargaining over match-specific rents. The equilibrium which results from the imposition or the uprating of the minimum wage exhibits the following features: positive or negative employment effects; a wage distribution that has a spike at the minimum wage and is continuous to the right of it. Flinn (2006) uses this theoretical framework to empirically assess the welfare impact of minimum wages. His estimations suggest that the workers' share of the surplus is different from their corresponding elasticity of the matching function, which is an indication of underlying inefficiency (see Hosios [1990]). Contrary to Hosios (1990), Flinn (2006) considers the bargaining power of the workers to be a primitive parameter and treats the minimum wage as the policy variable. He argues that the social planner can alleviate the existing inefficiency in the labor market and improve welfare by increasing the workers' "effective" bargaining power through increases in the minimum wage. Although this claim sounds plausible and quite intuitive, Flinn (2006) does not derive analytical conditions under which minimum wages are welfare enhancing.

In this paper, I extend the theoretical framework of Flinn (2006) to account for the welfare effects of minimum wage increases. My analysis assumes continuous time, a stationary environment, match-specific productivity, job search, Nash bargaining, a constant returns to scale matching technology, a fixed mass of workers, and an endogenously determined population of firms. Within this framework, I present the behaviour of agents in the presence and in the absence of a binding minimum wage and characterize the corresponding decentralized equilibrium allocations. Supposing that the minimum wage is exogenously given, I compare steady state social welfare in the unconstrained and constrained cases and argue that the existence of an exogenous wage floor implies a suboptimal equilibrium allocation when the Hosios (1990) condition is satisfied, that is, when the workers' share of the surplus is equal to their matching function elasticity. In this sense, there is only scope for a minimum wage to improve welfare if the Hosios condition does not hold.

Under the assumption that the minimum wage is the only policy instrument available to the social planner, I show how social welfare varies in response to minimum wage changes and derive the necessary and sufficient condition for a minimum wage hike to be welfare

[1] A partial equilibrium version of the same model is presented in Flinn (2002).

enhancing. Moreover, I demonstrate that for incremental changes in the minimum wage this condition depends only on the relationship between the workers' bargaining power and the elasticity of the matching function, which are assumed to be primitive parameters.

Apart from the theoretical explanation of the spike in the wage distribution at the minimum wage, this theoretical framework is flexible enough to account for minimum wage spillover effects. The introduction or the uprating of a binding minimum wage implies an increase or a decrease in the value of search, which in turn affects accepted wages. I derive the condition for the existence of spillovers and show that there is a direct relationship between changes in social welfare and spillovers. My main conclusion is that one can infer the welfare impact of the minimum wage by examining the post-change wage distribution.

This paper is organized as follows. In Section 2, I describe the behaviour of agents and characterize the equilibrium allocations in the unconstrained and constrained cases. The framework in the unconstrained case is a standard Pisarides-type model of stochastic job matching; its presentation is included as a benchmark for the constrained case. In my discussion of equilibrium welfare, I assume that the minimum wage is exogenously given −beyond the control of the social planner. In Section 3, I treat the minimum wage as a policy instrument and discuss the implications of changes in its level for social welfare and accepted wages. I also include a brief discussion on the employment effects of the minimum wage. Section 4 concludes with a discussion of the implications of my findings for empirical research on the minimum wage. Technical details and derivations are gathered in the Appendix.

2 The Model

The model is set up in continuous time and assumes a stationary labor market environment. The labor market is populated by a mass of workers normalized to 1. The mass of firms in the market is endogenously determined: the decision of each firm to enter the market and post a vacancy is determined by the expected value of a filled job position and by the cost of posting a vacancy. If a firm decides to enter, it can employ at most one worker. In the steady state, a measure u of workers are unemployed and a mass v of job positions are unfilled.

The matching technology is given by the matching function $M(u,v)$, which has the following properties: it is continuous, nonnegative, increasing in both arguments, and concave, with $M(u,0) = M(0,v) = 0$ for all (u,v). I also assume that M displays constant returns to scale.The constant returns to scale property of the matching technology implies that the arrival rate of offers for unemployed workers, denoted λ_w, and firms with vacancies, denoted λ_e, can be expressed in terms of labor market tightness, $\theta = \frac{v}{u}$:

$$\lambda_w = \theta q(\theta) \text{ and} \tag{1}$$

$$\lambda_e = q(\theta), \tag{2}$$

where $\theta q(\theta)$ increases with θ and $q(\theta)$ decreases with θ. For the sake of exposition, I assume that the stocks of searchers on each side of the market are mapped to matches by a Cobb-Douglas function. This implies that the elasticity of matching with respect to unemployment, given by

$$\eta = \frac{\partial \ln (M(u,v))}{\partial \ln (u)},$$

is constant and independent of market tightness.[2] Finally, an implication of the constant returns to scale matching technology is that $\eta \in (0,1)$.

When a firm meets a potential employee, they both observe the productive value of the match, z, which is drawn randomly from a predetermined distribution, $G(z)$; G is assumed to be continuous with infimum $\underline{z}$ and supremum $\overline{z}$. The realized match value is divided between the firm and the worker using the generalized Nash bargaining framework.

Given that my ultimate goal is the characterization of labor market outcomes in the steady state, I assume that agents have no preference for the present, that is, agents' common discount rate, r, goes to zero. This simplifies the analysis and allows me to directly compare steady-state solutions without considering the dynamic version of the same model. Consummated matches are destroyed at an exogenously given rate $\delta \in (0,1)$, in which case workers join the unemployment pool receiving a constant instantaneous payoff $b > 0$, and firms advertise their vacant job positions incurring an instantaneous cost $c > 0$. In the labor market setting considered in this paper, employed individuals do not receive alternative offers of employment, that is, there is no on the job search.

2.1 Individual Behaviour without a Minimum Wage

In this part, I outline the basic features of the theoretical setting without a minimum wage $-$the unconstrained case. This is a standard Pissarides (2000) model of stochastic job matching, so the exposition is brief.

2.1.1 Decisions of Individual Workers and Firms

Consider a worker and a firm whose match has productivity value z; let $w(z)$ denote the wage paid to the worker and $z - w(z)$ the profit flow of the firm. Not all matches between unemployed workers and vacant firms are mutually beneficial. There is some common reservation productivity value, denoted z^*, below which neither the firm nor the worker will want to trade. A random worker-firm contact results in an acceptable match if their joint productivity value is $z \geq z^*$, which happens with probability $1 - G(z^*)$.

It can be shown (see the Appendix) that the steady-state flow value of unmatched

[2]The assumption that the matching function is Cobb-Douglas facilitates the presentation of the model's predictions as it allows me to consider the elasticity of matching with respect to unemployment as a primitive parameter. However, the analysis can be generalised to account for any constant returns to scale matching technology by expressing this elasticity as a function of market tightness, $\eta(\theta)$.

workers and firms, denoted U and V, respectively, are given by

$$U = b + \lambda_w \left(1 - G\left(z^*\right)\right) E\left(\frac{w\left(z\right) - U}{\delta} | z \geq z^*\right), \tag{3}$$

$$V = -c + \lambda_e \left(1 - G\left(z^*\right)\right) E\left(\frac{z - w\left(z\right) - V}{\delta} | z \geq z^*\right). \tag{4}$$

U represents the average income of an unemployed worker and is equal to the sum of a flow term, b, plus an expected "capital gain" due to a change in employment status. Similarly, V represents the average rent of a vacant job and is equal to the expected profit, if the job is filled, minus the cost of posting the vacancy. In equilibrium, the free entry and exit of firms in the labor market drives rents from vacant jobs to zero. This equilibrium condition for the supply of vacant jobs ($V = 0$) implies that

$$E\left(\frac{z - w\left(z\right)}{\delta} | z \geq z^*\right) = \frac{c}{\lambda_e \left(1 - G\left(z^*\right)\right)}. \tag{5}$$

2.1.2 Wage determination

For the division of the match value between the firm and the worker, I use the Nash bargaining framework. The worker's reservation wage is equal to her flow value of unemployment, U, while the firm's reservation profit flow is equal to the flow value of holding a vacancy, which in equilibrium is equal to zero due to the free entry condition. Therefore, for any match with value $z \geq z^*$, the flow surplus S is defined as

$$S\left(z\right) = \left[w - U\right] + \left[z - w\right] = z - U,$$

and the wage is given by

$$w = \arg\max_{w} \left[w - U\right]^{\beta} \left[z - w\right]^{1-\beta}, \text{ that is,}$$

$$w\left(z\right) = \beta z + \left(1 - \beta\right) U, \tag{6}$$

where β is the worker's bargaining power. The worker receives his reservation wage, U, plus a fraction β of the match specific flow surplus, S, while the firm receives its reservation profit flow, $V = 0$, plus a share $(1 - \beta)$ of the flow surplus S.

From the above, it is straightforward that a match with productivity z is mutually acceptable if and only if the wage and the profit flow are greater than or equal to the respective outside options of the paired worker-firm, which implies that the reservation productivity is given by

$$z^* = U. \tag{7}$$

In this setting, both the reservation productivity and the reservation wage are equal to the workers' outside option, U.

2.1.3 Equilibrium

At stationary equilibrium, a mass of workers are unemployed. The unemployment rate, u, is found by equalizing the flow of entries into and exits from the unemployment pool. At every point in time, an unemployed worker finds a vacant job with probability λ_w and is hired if the match productivity value exceeds the threshold $z^* = U$. The exit rate from unemployment is equal to $\lambda_w [1 - G(U)]$ and the number of unemployed workers finding a job amounts to $\lambda_w [1 - G(U)] u$. The number of job destructions at every point in time is given by $\delta (1 - u)$. Equating these flows gives equilibrium unemployment/ unemployment rate:[3]

$$u = \frac{\delta}{\delta + \lambda_w [1 - G(U)]}. \tag{8}$$

So far, the flow value of unemployment was considered as exogenously given. Having determined the wage offer and the reservation productivity, I can now calculate the flow value of search:

$$U = b + \beta \lambda_w \int_U^{\bar{z}} \left(\frac{z - U}{\delta} \right) dG(z). \tag{9}$$

Similarly, the job creation condition (equation [5]) can be expressed as follows

$$c = (1 - \beta) \lambda_e \int_U^{\bar{z}} \left(\frac{z - U}{\delta} \right) dG(z). \tag{10}$$

Combining equations (9) and (10), I get a reduced-form equilibrium relation between the value of unemployed workers and market tightness

$$U = b + \frac{\beta}{(1 - \beta)} c\theta, \tag{11}$$

which, when substituted into (10), gives

$$\frac{\delta c}{\lambda_e [1 - G(U)]} = (1 - \beta)(z^e - b) - \beta c \theta,$$

where $z^e = E(z | z \geq z^*)$. Substituting λ_e from equation (2), yields the job creation condition

$$(1 - \beta)(z^e - b) = \beta c \theta^* + \frac{\delta c}{q(\theta^*) [1 - G(z^*)]}, \tag{12}$$

which can be solved implicitly for tightness at decentralized labor market equilibrium.

Equilibrium market tightness can be substituted into (11) to compute the flow value of unemployment, which is equal to the reservation productivity and the reservation wage. These values can then be used in combination with equation (8), to calculate equilibrium unemployment and vacancies. Therefore, a labor market equilibrium is characterized by the triplet (u^*, U^*, θ^*), which satisfies equations (8), (11), and (12).

[3]The population of workers is normalized to 1, so there is no need to distinguish between unemployment (the measure of unemployed workers) and the unemployment rate (the proportion of unattached workers relative to the aggregate worker population). Henceforth, I use the term unemployment.

In equilibrium, for any match value on the acceptable range, the wage offer is given by (6). Using the wage offer equation and the distribution of match values, I calculate the wage distribution, denoted $H(w)$. First, I solve the wage equation (6) for match specific productivity, z:

$$z = x(w) = \frac{w - (1-\beta) U}{\beta},$$

where $x(w)$ is the inverse function of $w(z)$. Meetings of firms with workers lead to acceptable matches if the realized match value z is greater than or equal to the reservation match value, z^*, and the wage is greater than or equal to the reservation wage, U; since $z^* = U$, a match is accepted with probability $[1 - G(U)]$. The equilibrium distribution of wages is a simple mapping from the match productivity distribution, $G(z)$, into $H(w)$, conditional on the match value (and the wage) being on the acceptable range

$$H(w|w \geq U) = \frac{G(x(w)) - G(U)}{1 - G(U)}. \tag{13}$$

After simple differentiation, I get the following expression for the wage density

$$h(w) = \left\{ \begin{array}{cc} \frac{\beta^{-1} g(x(w))}{1-G(U)} & \text{for } w \geq U \\ 0 & \text{for } w < U \end{array} \right\}. \tag{14}$$

Therefore, the accepted wage density is determined by: the workers' bargaining power, β; the lowest accepted productivity/wage, which is equal to the flow value of unemployment, $w^* = z^* = U$; and the primitive parameters of the match specific productivity distribution.

2.1.4 Efficiency

The equilibrium allocation presented above suffers from trading externalities, which are not appropriately accounted for by searchers. If an additional worker joins the unemployment pool, each firm has a higher probability of filling its vacant position, but each unemployed worker has a lower probability of finding a job. Similarly, an extra vacancy increases the job-finding rate for unemployed workers, but decreases the rate at which vacant firms encounter potential employees. Hence, there are congestion effects within each group of searchers and positive externalities between the two groups. In the setting considered in this paper, externalities can arise through the entry or exit of firms in the labor market and through the decision of searchers to accept or reject a match. In a similar framework, Hosios (1990) demonstrates that welfare maximization requires that search externalities are appropriately recognized in the allocation of surplus shares to job seekers and vacancy creators. The condition for efficiency derived in Hosios (1990) is a standard point of reference in the literature, so the present exposition can be brief.[4]

Steady state social welfare, denoted SW, is defined as aggregate output, net of search

[4]For the derivation of the efficiency conditions in a model of stochastic job matching, see Pissarides (2000).

costs −the cost of posting vacancies:

$$SW = ub + (1-u)\, z^e - cv.$$

Note that in this definition of welfare I include the return on leisure received by job seekers, b, the surplus of accepted matches −which is divided between the firm and its employee− expressed in terms of expected productivity, $z^e = E\,(z|z \geq z^*)$, and finally, I subtract the costs incurred by firms holding a vacancy. Recalling that $v = \theta u$, I obtain a more convenient expression for social welfare:

$$SW = ub + (1-u)\, z^e - \theta uc, \tag{15}$$

A social planner who wished to maximize welfare would solve the following problem

$$\max_{\theta, z^*, u} SW \qquad \text{s.t. } u = \frac{\delta}{\delta + \theta q\,(\theta)\,[1 - G\,(z^*)]}$$

Comparing the first order conditions of the social planner's problem to the equilibrium equations presented above suggests that the decentralized equilibrium allocation is efficient if and only if workers' bargaining power, β, is equal to the elasticity of the matching function with respect to unemployment, η,

$$\beta = \eta. \tag{16}$$

This is the Hosios (1990) condition for efficiency in the decentralized equilibrium. The intuition is that efficient job creation and job acceptance require that the share of the surplus captured by workers is equal to their marginal contribution to the creation of match opportunities.

2.2 Individual Behaviour under a Binding Minimum Wage

I now consider the predictions of the model when the behaviour of agents is constrained by the presence of a minimum wage. For the analysis in this section, the minimum wage is assumed to be an exogenous feature of the labor market environment, rather than a policy variable set by the social planner. In this sense, I do not examine how a varying minimum wage affects steady state outcomes, but rather adjust the framework presented above to account for the existence of an exogenously given wage floor. In the following section, I relax this assumption, consider the minimum wage as a policy variable, and perform a comparative statics exercise.

Assume that the minimum wage, m, is set at a level that affects the behaviour of agents in the labor market, $m > w^* = z^* = U$, that is, the minimum wage is greater than the reservation productivity/wage, otherwise it is not binding. The flow value of unemployment under a minimum wage, denoted U_m, is considered as given for the time being. Wages are still determined by worker-firm negotiations modeled by the Nash bargaining framework.

For any match with value z, the corresponding wage is given by the solution to

$$w_c = \arg\max_{w} \left[w - U_m\right]^{\beta} \left[z - w\right]^{1-\beta},$$

which implies that

$$w_c(z) = \beta z + (1-\beta) U_m. \tag{17}$$

However, the behaviour of agents is now constrained by the presence of a minimum wage. Let us assume full compliance. From the point of view of the firm, acceptable matches are those with productivity value weakly greater than the minimum wage, $z \geq m$, otherwise the firm incurs a negative profit flow. The legislated wage floor imposes an additional restriction: acceptable matches are those that pay wages at least as large as the minimum wage, $w_c(z) \geq m$. Therefore,

$$z \geq m \text{ and } w_c(z) = \beta z + (1-\beta) U_m \geq m,$$
$$\text{or } z \geq m \text{ and } z \geq \frac{m - (1-\beta) U_m}{\beta}.$$

Denote $\hat{z}$ the value of z that gives a Nash-bargained wage equal to the minimum

$$w_c(\hat{z}) = m. \tag{18}$$

Combining equations (17) and (18), it is possible to determine the value of $\hat{z}$

$$\hat{z} = \frac{m - (1-\beta) U_m}{\beta}.[5] \tag{19}$$

Given an exogenous value of search, U_m, it could happen that the Nash-bargained wage is below the minimum, $w_c(z) < m$, although the corresponding match productivity is above it, $z \geq m$: this can happen for $z \in [m, \hat{z})$. Clearly, the match is acceptable from the firm's point of view; however, as long as the wage is lower than the minimum, the match cannot be consummated. In this case, the firm could raise the wage to m, thus rendering the match acceptable, and still make non-negative profits. Hence, the wage for all contacts with productivity weakly greater than the minimum is given by

$$w(z) = \max\left\{\beta z + (1-\beta) U_m, m\right\} \forall z \geq m,$$

which implies

$$w(z) = \left\{ \begin{array}{c} \beta z + (1-\beta) U_m, \text{ for } z \in [\hat{z}, \overline{z}] \\ m, \text{ for } z \in [m, \hat{z}) \end{array} \right\}. \tag{20}$$

For values of the match specific productivity on $[m, \hat{z})$, the worker receives a share of the surplus strictly greater than his bargaining power. Flinn (2006) describes this as an increase in the "effective" bargaining power of the worker. To formalize Flinn's idea, I define the

[5] It is straightforward to show that this analysis is relevant only if $\hat{z} \geq m$; otherwise, the minimum wage is not binding and the interactions of agents are given by the unconstrained case.

"effective" bargaining power and then use it to examine the implications of the minimum wage for equilibrium outcomes:

Definition 1 *Define the "effective" bargaining power of workers, ε, to be the average share of the match surplus received by employed workers*

$$\varepsilon = \frac{w^e - U_m}{z^e - U_m}, \tag{21}$$

where z^e and w^e denote the average match specific productivity and the average accepted wage under a binding wage floor: $z^e = E\left(z|z \geq m\right)$ and $w^e = E\left(w\left(z\right)|z \geq m\right)$.

Contrary to the bargaining power, β, which is a primitive parameter of the model, the "effective" bargaining power, ε, is endogenously determined in the constrained decentralized equilibrium; it depends on β, m, and U_m. In the Appendix, I demonstrate that changes in the minimum wage have a direct and an indirect impact on $\varepsilon\left(\beta, m, U_m\right)$. A minimum wage hike increases the average wage, which implies a direct positive effect on the value of $\varepsilon\left(\beta, m, U_m\right)$. A minimum wage hike also affects the value of search, which in turn determines $\varepsilon\left(\beta, m, U_m\right)$. Therefore, the total effect of the minimum wage on the "effective" bargaining power of workers depends on U_m, which is endogenously determined.

The above Definition suggests that the upper bound of ε is 1, while its lower bound is β: a non binding minimum wage, $m_0 \leq U_m = U$, implies that $\varepsilon\left(\beta, m_0, U\right) = \beta$. A binding minimum wage, $m > U_m$, raises the constrained average wage above the unconstrained average wage, which implies that $\varepsilon\left(\beta, m, U_m\right) > \beta$. Therefore, the existence of a binding minimum wage raises the "effective" bargaining power of workers above β.

2.2.1 Equilibrium under a Binding Minimum Wage

If the minimum wage is binding at stationary equilibrium, $m > U_m$, then the reservation productivity/wage is $z^* = w^* = m$. Equilibrium unemployment is computed by equating the flows into and out of the unemployment pool. The number of job destructions is given by $\delta\left(1-u\right)$ just as in the unconstrained case. The exit rate from unemployment, however, is affected by the minimum wage imposition: a binding minimum wage increases the reservation productivity value, and thus, truncates the distribution of match specific productivity values. The new exit rate from unemployment is equal to $\lambda_w\left[1 - G\left(m\right)\right]$ and the number of unemployed workers finding a job amounts to $\lambda_w\left[1 - G\left(m\right)\right]u$. Equilibrium of flows implies

$$\delta\left(1-u\right) = \lambda_w\left[1 - G\left(m\right)\right]u.$$

Therefore, equilibrium unemployment under the minimum wage is given by

$$u = \frac{\delta}{\delta + \lambda_w\left[1 - G\left(m\right)\right]}. \tag{22}$$

Clearly, the higher reservation productivity implies that fewer firm-worker contacts will result in jobs. However, equilibrium unemployment is also dependent on the contact rate, which is endogenously determined in the model I consider and may be greater or lower than the unconstrained equilibrium contact rate. Therefore, the overall effect of the minimum wage on equilibrium unemployment is ambiguous.[6]

The flow value of an unemployed worker is

$$U_m = b + \lambda_w \left(1 - G\left(m\right)\right) E\left(\frac{w\left(z\right) - U_m}{\delta} | z \geq m\right), \tag{23}$$

and substituting $w\left(z\right)$ from (20):

$$U_m = b + \lambda_w \left(\frac{\int_m^{\hat{z}} \left(m - U_m\right) dG\left(z\right) + \beta \int_{\hat{z}}^{\overline{z}} \left(z - U_m\right) dG\left(z\right)}{\delta}\right). \tag{24}$$

The flow value of a vacant firm is

$$V = -c + \lambda_e \left[1 - G\left(m\right)\right] E\left(\frac{z - w\left(z\right) - V}{\delta} | z \geq m\right).$$

The free entry of firms guarantees that in equilibrium all profit opportunities from new jobs are exploited, thus driving V to zero. This implies

$$c = \lambda_e \left(1 - G\left(m\right)\right) \left[E\left(\frac{z - w\left(z\right)}{\delta} | z \geq m\right)\right] \tag{25}$$

and substituting $w\left(z\right)$ from equation (20) and λ_e from equation (2) yields the job creation condition:

$$c = q\left(\theta^*\right) \left(\frac{\int_m^{\hat{z}} \left(z - m\right) dG\left(z\right) + \left(1 - \beta\right) \int_{\hat{z}}^{\overline{z}} \left(z - U_m\right) dG\left(z\right)}{\delta}\right), \tag{26}$$

which can be solved implicitly for tightness at the constrained decentralized equilibrium.

Equilibrium market tightness can be substituted into (24) to compute the flow value of unemployment −this is no longer the minimal acceptable productivity/ wage, but rather an "implicit" reservation wage that determines equilibrium wages. These values can then be used in combination with equation (22), to calculate equilibrium unemployment and vacancies. Therefore, a labor market equilibrium is characterized by the triplet (u^*, θ^*, U_m^*), which satisfies equations (22), (24), and (26).

There is an alternative and more convenient way of presenting the constrained equilibrium allocation. If I introduce (21) as an additional equilibrium equation, I can characterize the constrained equilibrium allocation based on equilibrium equations that are algebraically similar to the unconstrained equilibrium equations ([11] and [12]) with the notable difference that the bargaining power, β, is now replaced by the "effective" bargaining power, ε.

Use expression (21) to substitute the average wage into (24) and (26), so that the value

[6]The conditions under which a minimum wage has adverse employment effects are presented in the following section.

of search and the job creation condition are expressed in terms of ε:

$$U_m = b + \varepsilon \lambda_w \int_m^{\overline{z}} \left(\frac{z - U_m}{\delta} \right) dG(z) \tag{27}$$

and

$$c = (1 - \varepsilon) \lambda_e \int_m^{\overline{z}} \left(\frac{z - U_m}{\delta} \right) dG(z) \,. \tag{28}$$

Combining equations (27) and (28), I get a reduced-form relation between the value of unemployed workers and market tightness that holds in the constrained equilibrium

$$U_m = b + \frac{\varepsilon}{(1 - \varepsilon)} c\theta, \tag{29}$$

which, when substituted into (28), gives

$$\frac{\delta c}{q(\theta)\left[1 - G(m)\right]} = (1 - \varepsilon)(z^e - b) - \varepsilon c\theta, \tag{30}$$

where $z^e = E(z|z \geq m)$.

To provide a rationale for this alternative representation of equilibrium relations between the endogenous variables of the model, note that the constrained equilibrium equations can be reduced to the corresponding unconstrained equilibrium equations by setting the "effective" bargaining power equal to β. Hence, using Definition 1, the equilibrium allocation can be expressed in a way that encompasses both the constrained and the unconstrained cases: a labor market equilibrium is characterized by the quadruplet $(\varepsilon^*, u^*, \theta^*, U_m^*)$, which satisfies equations (21), (22), (29), and (30).

Let us now examine how wages are distributed in the constrained equilibrium. In the presence of a binding minimum wage, wages are given by (20). Contacts with match values on $[\hat{z}, \overline{z}]$ result in jobs paying wages weakly greater than the minimum. Solving the wage function in terms of the match-specific productivity

$$z = x(w; m) = \frac{w - (1 - \beta) U_m}{\beta},$$

where $x(w; m)$ is the inverse function of $w(z)$ for any given minimum wage, m. Meetings of firms with workers lead to acceptable matches if the realized match value z is greater than or equal to the minimum wage, which happens with probability $\left[1 - G(m)\right]$. Therefore, the equilibrium distribution of wages that are strictly greater than the minimum can be expressed as a simple mapping from the match specific productivity distribution, $G(z)$, into $F(w)$, conditional on the match value (and wage) being on the acceptable range

$$F(w|w > m) = \frac{G(x(w; m)) - G(m)}{1 - G(m)}.$$

All contacts with $z \in [m, \hat{z})$ result in jobs offering the minimum wage. This implies that the observed wage distribution will exhibit a mass point/ spike at the legislated minimum wage. The magnitude of the spike is given by the mass of matches offering the minimum wage, $G(\hat{z}) - G(m)$, conditional on them being on the acceptable range, that is,

$$F(w|w=m) = \frac{G(\hat{z}) - G(m)}{1 - G(m)}.$$

From the above analysis, it is straightforward that the density function of observed wages in the constrained steady state is

$$f(w;m) = \left\{ \begin{array}{cc} \frac{\beta^{-1} g(x(w;m))}{1-G(m)} & \text{for } w > m \\ \\ \frac{G(\hat{z})-G(m)}{1-G(m)} & \text{for } w = m \\ \\ 0 & \text{for } w < m \end{array} \right\}. \quad (31)$$

2.2.2 Welfare and Efficiency

I now examine the efficiency properties of the constrained equilibrium allocation. Note that in this section the minimum wage is not considered as a choice (policy) variable, but rather as an exogenously given constraint. The implications of a varying minimum wage for the efficiency of the labor market equilibrium allocation and the calculation of the "welfare maximizing" minimum wage level are deferred until the next section.

Steady state social welfare in the presence of a binding minimum wage is denoted SW_m and is defined as in the unconstrained case (equation [15]), that is, aggregate output net of search costs with the only difference that match acceptance is now constrained by the minimum wage, $z \geq z^* \geq m$. Therefore,

$$SW_m = ub + (1-u) z^e - \theta uc, \quad (32)$$

where the expected productivity of accepted matches is given by $z^e = E(z|z \geq z^*)$.

An omniscient social planner who wished to maximize social welfare, but had no control over the minimum wage level, would solve the following problem

$$\max_{\theta,.z^*,u} \{SW_m = ub + (1-u) z^e - \theta uc\}$$

$$\text{subject to } u = \frac{\delta}{\delta + \theta q(\theta)[1 - G(z^*)]} \text{ and } z^* \geq m.$$

It is evident that the wage floor constrains the set of reservation productivity values that can be chosen by the social planner.

Substituting the first constraint into SW_m, the planner's problem takes the following form

$$\max_{\theta,z^*} \left\{ SW_m = \frac{\delta b + \theta q(\theta)[1 - G(z^*)] z^e - \delta c\theta}{\delta + \theta q(\theta)[1 - G(z^*)]} \right\} \text{ subject to } z^* \geq m.$$

Setting to zero the partial derivatives of SW with respect to θ and z^*, yields the following conditions that have to be satisfied for welfare to be maximized in the constrained equilibrium:

$$(1-\eta)(z^e - b) = \eta c\theta^* + \frac{\delta c}{q(\theta^*)[1-G(z^*)]}, \tag{33}$$

$$z^* = b + \frac{\eta c\theta}{(1-\eta)}, \tag{34}$$

$$\text{and } z^* \geq m, \tag{35}$$

where η is the elasticity of the matching function with respect to unemployment.

Let us now compare equation (30) to (33), and equation (29) to (34). Clearly, if the "effective" bargaining power is equal to the elasticity of the matching function with respect to unemployment,

$$\varepsilon = \eta, \tag{36}$$

the first two equations coincide. However, this condition is not sufficient for equations (29) and (34) to coincide. It also has to be the case that $z^* = U_m \geq m$, but this would render the minimum wage non-binding. Under a binding minimum wage, $z^* \geq m > U_m$ and the matching function elasticity exceeds the "effective" bargaining power: simple comparison of equations (29) and (34) leads to $\eta > \varepsilon$. This suggests that constraint (35) is binding, so the social planner's constrained optimum corresponds to a corner solution.

When the social planner maximizes social welfare, his objective is to achieve optimal job creation and optimal job acceptance. The existence of a binding minimum wage imposes a constraint on the set of reservation productivity values −that determine job acceptance. In the unconstrained case, the reservation productivity is endogenously determined and equals the value of search. By contrast, in the constrained case, the cutoff level of the match specific productivity is always equal to the level of the wage floor, which has to be greater than the value of search, otherwise the minimum does not bind. As a result, the constrained optimum corresponds to inefficiently low job acceptance.

Turning now to job creation, the "effective" bargaining power of workers, ε, by definition (equation [21]), has to be greater than their actual bargaining power, β, when the minimum wage binds. Suppose the decentralized equilibrium achieves the planner's constrained optimum. If the minimum wage is binding, then $\beta < \varepsilon < \eta$. The implication of this inequality is that in the constrained optimum firms receive, on average, a higher share of the aggregate surplus, $(1-\varepsilon)$, than in the unconstrained optimum, $(1-\eta)$. Therefore, the constrained optimum corresponds to inefficiently high job creation in comparison to the unconstrained optimum.

The above discussion is summarized in Proposition 1.

Proposition 1 *If the minimum wage truncates the match-productivity distribution above the cutoff productivity level that corresponds to the unconstrained social optimum, then any constrained equilibrium allocation is suboptimal.*

3 The Effects of Minimum Wage Changes: Welfare, Wages, Unemployment

After analyzing how an exogenous wage floor impacts on the behaviour of agents, I now examine the implications of a varying minimum wage for unemployment, accepted wages, and the welfare of labor market participants.

3.1 Welfare and Efficiency

In the previous section, I described the predictions of the model treating m as an exogenous constraint. I now adopt a different perspective and consider m to be the policy variable chosen by the social planner.

According to the analysis above, if wages are negotiated in a decentralized fashion, the condition for the efficiency of the equilibrium allocation is given by the equality of the workers' bargaining power to the elasticity of the matching function with respect to unemployment. Let us consider an efficient labor market. It is obvious that in this setting there is no scope for a minimum wage to increase social welfare. More importantly, in such a labor market, the minimum wage has a distortionary effect.

Suppose now that the bargaining power parameter is not equal to the elasticity of the matching function with respect to unemployment, $\beta \neq \eta$, which implies that search externalities are not taken into account by individual agents resulting in market failure and a suboptimal decentralized equilibrium allocation. This inefficiency necessitates an intervention by the social planner to restore efficiency in the market. The discussion above indicates that search externalities can be internalized by equating the bargaining power of workers to their matching function elasticity, so that the Hosios condition is satisfied. However, the social planner has no control over the division of the surplus between the matched firm-worker. The only way the planner can intervene in this setting is by varying the level of the minimum wage. Could such an intervention mitigate existing inefficiencies and increase welfare? If so, under what conditions?

The objective of the social planner is the maximization of social welfare (equation [32])[7] with respect to the policy variable under his control, that is, the minimum wage. The social planner's decisions impact on the individual behaviour of agents in the labor market, given by the equilibrium equations (21), (22), (29), and (30). These reactions of agents have to be factored into the planner's maximization problem. Therefore, the planner maximizes social welfare (equation [32]) subject to equations (21), (22), (29), and (30). In the Appendix, I demonstrate that the planner's problem yields a first order condition that can be summarized as follows:

$$\frac{dSW_m}{dm} = A\frac{dU_m}{dm} = 0, \tag{37}$$

where $A > 0$ is given by equation (74) in the Appendix. Proposition 2 provides a useful interpretation of this result:

Proposition 2 *Social welfare and the flow value of unemployment change in the same*

[7] To consider the welfare effect of a minimum wage imposition, one could simply set $m = U$.

direction after a minimum wage increase.

Social welfare is maximized if the flow value of unattached workers is maximized, so I only need to examine how changes in the minimum wage affect the flow value of unemployment. Taking the differentials of equations (21), (29), (30) and solving for $\frac{dU_m}{dm}$, I conclude that:

$$\frac{dU_m}{dm} \overset{sgn}{=} \left[G\left(\hat{z}\right) - G\left(m\right)\right]\left(\eta - \varepsilon\right) - \eta\left[1 - \varepsilon\right]\left(m - U_m\right) g\left(m\right). \quad (38)$$

[8]

To interpret this result, I identify the effects of an increase in the wage floor on the value of search: there is a positive effect stemming from the higher "effective" bargaining power of workers, a negative effect due to lower job creation (caused by the fall in the surplus of firms), and finally one additional negative effect due to lower job acceptance (implied by the higher cutoff productivity level). The first term on the right-hand side of equation (38) shows that the positive effect of the higher match-surplus, captured by the mass of workers receiving the minimum $\left[G\left(\hat{z}\right) - G\left(m\right)\right]$, dominates the corresponding negative effect on job-creation as long as the marginal contribution of workers to the creation of match opportunities exceeds their average or "effective" share of the surplus: i.e. $\eta > \varepsilon$. The second term on the right-hand side of (38) corresponds to the negative effect of the minimum wage rise on job acceptance: the incrementally higher wage floor eliminates $g\left(m\right)$ matches with surplus $\left(m - U_m\right)$.

Using the above relationship, I determine the conditions under which the minimum wage has a positive or a negative effect on the flow value of unemployment. Simple inspection of equation (38) gives a sufficient condition for the minimum wage to be welfare decreasing. This result is summarized in Proposition 3:

Proposition 3 *If the minimum wage binds and the workers' "effective" bargaining power* (ε) *is weakly greater than the elasticity of the matching function with respect to unemployment* (η)*, any increase in the minimum wage results in a decrease in the flow value of unemployment* (U_m) *and in social welfare* (SW_m)*.*

Proof. Under a binding minimum wage, $m > U$ and $\hat{z} > m$. If $\varepsilon > \eta$, then

$$\left(G\left(\hat{z}\right) - G\left(m\right)\right)\left(\eta - \varepsilon\right) - \eta\left(1 - \varepsilon\right)\left(m - U_m\right) g\left(m\right) < 0,$$

which implies (from equation [38])

$$\frac{dU_m}{dm} < 0.$$

From Proposition 2, the direction of social welfare changes in response to minimum wage variations is the same as the direction of changes in the flow value of unemployment. ■

Proposition 3 is only a sufficient (but not necessary) condition for changes in the minimum wage to decrease welfare. In this sense, the minimum wage can have a negative welfare effect even in cases where the workers' "effective" bargaining power is lower than

[8] For the derivation, see Lemma 1 in the Appendix.

the elasticity of the matching function with respect to unemployment. Moreover, this result suggests that there is scope for the minimum wage to increase welfare only if $\eta > \varepsilon$. This necessary condition for positive welfare effects is summarized in Corollary 1:

Corollary 1 *A minimum wage increase can only improve social welfare if the workers' "effective" bargaining power (ε) is lower than the elasticity of the matching function with respect to unemployment (η).*

Suppose that the labor market we examine exhibits this feature, that is, $\eta > \varepsilon$. What is the necessary and sufficient condition under which an increase in the minimum wage increases welfare? Clearly, I need to identify the parameter values for which changes in the minimum wage have a positive impact on U_m, and consequently, on welfare.

According to equation (38), an important determinant of the minimum wage effect is the mass of matches paying the minimum wage $(G(\hat{z}) - G(m))$, that is, the spike at the level of the legislated wage floor.[9] It is evident that the magnitude of the spike is determined by the shape of the match specific productivity distribution as well as the gap between $\hat{z}$ and m. For expositional purposes, I present the following simplification:

Definition 2 *For any binding minimum wage, m, define x_m to be the level of productivity, which corresponds to the average match specific productivity density, $g(x_m)$, on the interval $[m, \hat{z}]$. Clearly, $g(x_m)$ satisfies*

$$(G(\hat{z}) - G(m)) = \int_m^{\hat{z}} g(z)\, dz = (\hat{z} - m)\, g(x_m) = \frac{(1-\beta)}{\beta}(m - U_m)\, g(x_m). \tag{39}$$

Let us assume that the minimum wage is binding and the necessary condition for positive welfare effects is satisfied $\eta > \varepsilon$. Given these assumptions, what is the additional restriction on the parameter values that guarantees a positive welfare effect? To answer this question, rewrite expression (38) using the above definition

$$\frac{dU_m}{dm} \overset{sgn}{=} \frac{(1-\beta)}{\beta}(\eta - \varepsilon)\, g(x_m) - \eta(1-\varepsilon)\, g(m).$$

Postulate that

$$\frac{dU_m}{dm} \geq 0,$$

which implies

$$\frac{(1-\beta)}{\beta}(\eta - \varepsilon)\, g(x_m) - \eta(1-\varepsilon)\, g(m) \geq 0.$$

This inequality can be solved in terms of the workers' "effective" bargaining power (ε):

$$\varepsilon \leq \frac{\eta\left[g(x_m) - \beta g(x_m) - \beta g(m)\right]}{g(x_m) - \beta g(x_m) - \beta\eta g(m)}. \tag{40}$$

[9] Formally, the spike is given by the mass of match-values paying the minimum wage conditional on them being on the acceptable range, that is, $\frac{G(\hat{z})-G(m)}{1-G(m)}$.

Suppose the minimum wage is $m_0 \geq U_m$. If it binds ($m_0 > U_m$), then $\varepsilon(m_0) > \beta$; if not ($m_0 = U_m$), then $\varepsilon(m_0) = \beta$. A minimum wage increase from m_0 to m implies $\varepsilon(m) > \varepsilon(m_0) > \beta$.

Combining these assumptions with inequality (40), equation (38), and Proposition 2, I obtain the necessary and sufficient condition for positive welfare effects, summarized in Proposition 4:

Proposition 4 *For any initial wage floor, $m_0 \geq U_m$, a higher binding minimum wage, $m > m_0$, increases U_m and social welfare if and only if the following condition holds*

$$\varepsilon(m) \leq \frac{\eta\left[g(x_m) - \beta g(x_m) - \beta g(m)\right]}{g(x_m) - \beta g(x_m) - \beta\eta g(m)}, \tag{41}$$

where $\beta \leq \varepsilon(m_0) < \varepsilon(m)$ and $\frac{\eta[g(x_m)-\beta g(x_m)-\beta g(m)]}{g(x_m)-\beta g(x_m)-\beta\eta g(m)} < \eta$.

In other words, if the level of m is such that ε satisfies inequality (41), the minimum wage has a welfare improving effect. Proposition 4 indicates that the necessary and sufficient condition for the minimum wage to increase social welfare depends on the primitive parameters of the model and the distribution of match-specific productivity.

Exploiting the continuity of $G(z)$, the above necessary and sufficient condition can be simplified. Corollary 2 presents a more practically useful version of this condition that holds in the limit:

Corollary 2 *There is a binding minimum wage, m, that increases the value of search and social welfare if and only if*

$$\beta < \frac{\eta}{1+\eta}. \tag{42}$$

Proof. Assume that the existing minimum wage, m_0, just binds: $m_0 \approx U_m$, which means that $\varepsilon(m_0) \approx \beta$. From condition (41), a marginally higher minimum wage $m > m_0$ (with $\varepsilon(m) > \beta$) increases the value of search, $\frac{dU_m}{dm} > 0$, if

$$\beta \approx \varepsilon(m_0) < \varepsilon(m) \leq \frac{\eta\left[g(x_m) - \beta g(x_m) - \beta g(m)\right]}{g(x_m) - \beta g(x_m) - \beta\eta g(m)},$$

which gives

$$\beta < \frac{\eta\left[g(x_m) - \beta g(x_m) - \beta g(m)\right]}{g(x_m) - \beta g(x_m) - \beta\eta g(m)}$$

and

$$\frac{\beta\eta}{\eta-\beta} < \frac{g(x_m)}{g(m)}. \tag{43}$$

The continuity of $G(z)$ and Definition 2 imply that

$$\lim_{m\to m_0} \frac{g(x_m)}{g(m)} = 1. \tag{44}$$

Substituting equation (44) into (43):

$$\frac{\beta\eta}{\eta-\beta}<1,$$

which if rearranged gives condition (42).

If condition (42) holds, there is an $\varepsilon(m)>\beta$ corresponding to a binding minimum wage, $m>m_0$, such that condition (41) is satisfied. Condition (41) guarantees that this minimum wage is welfare improving. ■

Under the above condition, a marginally binding minimum wage increases the value of search as long as the match specific productivity distribution is continuous. This result implies that the social planner can assess the welfare impact of a minimum wage imposition being agnostic about the distribution of match specific productivities and relying solely on the relationship between the bargaining power parameter (β) and the elasticity of the matching function with respect to unemployment (η). It should be emphasized, however, that condition (42) cannot be used to evaluate discrete changes in the minimum wage: it is a limiting condition that determines the sign of $\frac{dU_m}{dm}$ at $m\to U_m$.

If condition (42) holds, then the value of search is increasing in the minimum wage at m_0, where m_0 satisfies $\varepsilon(m_0)=\beta$. Proposition 3 suggests that the value of search is decreasing at m_1, where m_1 satisfies $\varepsilon(m_1)=\eta$. The value of search is continuous, so there exists at least one value of the minimum wage, $m^*\in(m_0,m_1)$, such that $\frac{dU_m}{dm}|_{m=m^*}=0$. Therefore, there is at least one $m^*\in(m_0,m_1)$ at which the value of search and social welfare are (locally) maximized. From the above analysis, m^* satisfies

$$\varepsilon(m^*)=\frac{\eta\left[g(x_m)-\beta g(x_m)-\beta g(m^*)\right]}{g(x_m)-\beta g(x_m)-\beta\eta g(m^*)}. \tag{45}$$

This is a necessary condition for a welfare maximizing minimum wage, as m^* may correspond to a local, rather than a global social welfare maximum.

The discussion on the efficiency of the constrained labor market equilibrium in the previous section facilitates the interpretation of this result. When workers receive a low share of the surplus in comparison to their matching function elasticity (low β relative to η), the expected wage is too low resulting in inefficiently high job acceptance. This low value of workers' bargaining power also implies that firms have an incentive to flow into the market and post vacancies, as their share of the match surplus is greater than their corresponding value of matching function elasticity; therefore, we also observe inefficiently high job creation. To alleviate this inefficiency one has to raise the bargaining power of workers so that the Hosios condition is satisfied. However, in the setting I consider, the policymaker has no control of the division of the rents in any given match. The only policy instrument available to the social planner is the minimum wage. The introduction or uprating of a binding minimum wage increases the "effective" bargaining power of workers and imposes a restriction on the acceptable match productivity, thus decreasing job acceptance. Moreover, the lower average share of the surplus captured by firms decreases the inflow of firms in the market leading to lower job creation and decreasing the contact rate. This policy is welfare improving to the extent that the positive effects outweigh the negative ones.

The relative magnitude of these positive and negative effects depends on the shape of the match productivity distribution as well as the values of the bargaining power parameter and the elasticity of matching with respect to unemployment. Corollary 2 presents a limiting condition, which can be used to determine the welfare impact of a minimum wage imposition without knowledge of the match productivity distribution. In general, the condition for a welfare improving minimum wage imposition depends on the shape of the match-specific productivity density.

3.2 Minimum Wage Effects on Accepted Wages

In the context of this model, the imposition of a binding minimum wage or the uprating of an already binding wage floor may have three effects on employed workers. First, due to the higher reservation productivity, some matches become unprofitable and are terminated −the standard disemployment effect. Second, some workers receiving a wage below the current minimum have their pay increased to the level of the mandated wage floor −the compliance effect. Finally, the minimum wage change may result in the renegotiation of wages strictly above the current minimum −the spillover effect. In this part, I analyze the compliance and spillover effects relegating the discussion of minimum wage employment effects to the next subsection.

For purposes of discussion, I consider the effect of a minimum wage change from m_1 to m_2, assuming that $m_2 > m_1$. Wages under the two different levels of the wage floor m_1 and m_2 are denoted w_1 and w_2, respectively, and are given by equation (20), reproduced below

$$w_i(z) = \left\{ \begin{array}{c} \beta z + (1-\beta) U_{m_i}, \text{ for } z \in [\hat{z}_i, \overline{z}] \\ m_i > \beta z + (1-\beta) U_{m_i}, \text{ for } z \in [m_i, \hat{z}_i) \end{array} \right\}, \tag{46}$$

where $i \in \{1, 2\}$.

Suppose that the minimum wage increase has no spillover effects. Conditional on a worker being matched with the same employer under both minimum wage levels, his earnings under the new minimum, $w_2(z)$, should equal his initial earnings, $w_1(z)$, plus the compliance effect, denoted $CP(z; m_1, m_2)$. Algebraically,

$$w_2(z) = w_1(z) + CP(z; m_1, m_2) =$$
$$= w_1(z) + \max(m_2 - w_1, 0) = \left\{ \begin{array}{c} w_1(z), \text{ for } z \in [\hat{z}_2, \overline{z}] \\ m_2, \text{ for } z \in [m_2, \hat{z}_2) \end{array} \right\}.$$

All matches with productivity $z \in [\hat{z}_2, \overline{z}]$ pay the same wage under both minimum wages. Simple comparison of the wage equations on the this range of z values suggests that

$$U_{m_1} = U_{m_2}.$$

In other words, if there are no spillover effects, the minimum wage uprating does not affect the value of search.

Let us now assume that the minimum wage uprating considered above has spillover effects. Conditional on a worker being matched with the same employer under both minimum wage levels, the minimum wage uprating has a spillover effect on this worker's wage if and only if

$$w_2(z) \neq w_1(z) \text{ for } w_1(z), w_2(z) > m_2 > m_1.$$

Equation (46) implies that this can only happen if

$$U_{m_1} \neq U_{m_2}.$$

The magnitude of the spillover effect, denoted $SP(z; m_1, m_2)$, is given by

$$SP(z; m_1, m_2) = w_2(z) - w_1(z) = (1 - \beta)(U_{m_2} - U_{m_1}), \text{ for } z \in [\hat{z}_2, \overline{z}].$$

Therefore, the minimum wage uprating has positive spillover effects when the value of search increases ($U_{m_1} < U_{m_2}$) and negative spillover effects when the value of search decreases ($U_{m_1} > U_{m_2}$).

This result is summarized in Proposition 5:

Proposition 5 *A minimum wage increase has positive (negative) spillover effects on accepted wages if and only if the value of search under the new minimum is higher (lower) than the value of search under the initial minimum.*

Combining this result with the analysis in the previous subsection, leads to the following result

Corollary 3 *A minimum wage increase has positive (negative) spillovers if and only if it increases (decreases) social welfare.*

Proof. Proposition 2 suggests that the value of search and social welfare change in the same direction after a minimum wage increase. Therefore, if the minimum wage increase has a positive (negative) effect on social welfare and the value of search, it will have positive (negative) spillovers (see Proposition 5). Now, if the minimum wage increase has positive (negative) spillovers, from Proposition 5 the value of search has increased (decreased), and, from Proposition 2, social welfare has increased (decreased). ■

This result suggests that it is possible to infer the welfare impact of minimum wage changes from the empirical observation of spillover effects on the wage distribution.

Notwithstanding the practical significance of the above result, the assumption that all agents are ex ante homogeneous implies that the value of search is the same for all workers. This means that the magnitude of the spillover effect is independent of the productivity of the worker-firm pair: the workers just affected by the minimum wage (low productivity matches) experience the same spillover effect (in absolute terms) as the workers further up the wage distribution (high productivity matches). One could claim that this is an important caveat of my approach, since low-wage workers and high-wage workers should not be similarly affected by the minimum wage change.

However, this is a conceptual issue and not a limitation of the framework presented in this paper. Let us revisit the concept of the spillover effect and define it as the percentage change (instead of the absolute change) in the accepted wage. This alternative definition gives a pattern of monotonically diminishing spillover effects.

The model can also be modified to account for diminishing (in absolute terms) spillover effects if I assume that the aggregate labor market consists of several distinct submarkets with different match-productivity distributions and different populations of agents. In other words, the assumption of ex-ante homogeneity of workers is partially relaxed: workers are ex-ante homogeneous within submarkets and ex-ante heterogeneous between submarkets. The framework presented in this paper describes the interactions and characterizes the equilibrium allocation in each individual submarket.

To examine the effects of the minimum wage in this context, let us first consider a very simple scenario. There are two submarkets, one with high-productivity and a second one with low-productivity; the corresponding match-specific productivity distributions are denoted G_H and G_L, respectively. Let us also assume that G_H first order stochastically dominates G_L. The minimum wage is binding in the low-productivity submarket and non-binding in the high-productivity submarket. The extensive analysis of minimum wage effects presented above suggests that in the low-productivity submarket all wages above the minimum wage are uniformely affected by the minimum wage change (constant spillover). On the other hand, in the high-productivity submarket wages are unaltered. Therefore, this simple extension of the basic model gives rise to an aggregate wage structure that exhibits spillovers up to a certain wage and no spillovers above that wage level.

3.3 Minimum Wage Effects on Unemployment

This model predicts that the imposition of a binding minimum wage or the uprating of an already binding wage floor impacts on equilibrium unemployment in two ways: first, it increases the cutoff productivity level implying that fewer firm-worker contacts are consummated; in addition to this effect, the minimum wage change affects the contact rate, which is endogenously determined in this model and can be greater or lower than the initial equilibrium contact rate. Hence, the overall effect of a minimum wage change on equilibrium unemployment is ambiguous. Formally,

$$\frac{du}{dm} = -\Delta_1 \frac{d\theta}{dm} + \Delta_2,$$

where $\Delta_1 > 0$ and $\Delta_2 > 0$.[10] It is evident that when $\frac{d\theta}{dm} < 0$, equilibrium unemployment increases in response to a minimum wage increase $\left(\frac{du}{dm} > 0\right)$. It can be shown that, when

[10]The derivation is included in the Appendix.

the value of search increases $\left(\frac{dU_m}{dm} > 0\right)$, market tightness is decreasing $\left(\frac{d\theta}{dm} < 0\right)$.[11] This is only a sufficient condition for increasing equilibrium unemployment, as market tightness can be decreasing even in cases where $\frac{dU_m}{dm} < 0$.

Proposition 6 *If an increase in the minimum wage raises the value of search, it will also raise equilibrium unemployment.*

An immediate implication of this result is the following necessary condition for a decrease in equilibrium unemployment:

Corollary 4 *Equilibrium unemployment can only decrease in response to a minimum wage hike if the value of search decreases.*

These results should not come as a surprise. The size of workers in the labor market is assumed to be fixed. Therefore, changes in unemployment are determined by the entry and exit of firms. A higher value of search reflects a higher expected wage, that is, lower firm profitability, and thus, fewer firms in the labor market, which in turn implies higher unemployment and lower employment.

Empirical research on this topic, however, indicates that minimum wage increases do not have a significant disemployment effect.[12] How could I reconcile my findings with this stylized fact?

I argue that extending this framework to account for endogenous worker participation can deliver predictions that are consistent with empirical observation regarding the effect of the minimum wage on the mass of employed workers. Suppose the minimum wage increases the value of search; in this case, one would expect an inflow of individuals in the labor market, which would imply an increase in the mass of job seekers. The higher probability of meeting unmatched employees induces more firms to enter the market and post vacancies. In equilibrium, the population of firms and workers in the labor market increases, but the mass of vacancies relative to unemployed workers decreases −due to lower firm profitability under the higher minimum wage. This scenario suggests that a minimum wage hike can increase labor market participation and result in a higher level of employment.

It is notable that this increase in employment is not the result of a decrease in unemployment, but rather the consequence of a rise in the number of meetings caused by the larger population of workers in the labor market. To determine equilibrium unemployment, we would have to consider the effect of the minimum wage change on market tightness and participation. If the minimum wage hike raises the value of search, then we would expect an increase in labor market participation and at the same time a decrease in labor market tightness. This would imply an increase in unemployment. Therefore, the minimum wage increase results in both higher employment and higher unemployment in equilibrium.

While the principal objective of our study is the analysis of minimum wage impacts on social welfare, understanding how minimum wage changes affect employment and unemployment is also of great importance. In light of empirical evidence on the employment

[11] See Lemma 2 in the Appendix.

[12] See Card and Krueger (1995) and Stewart (2004).

effects of the minimum wage, endogenizing the decision of workers to participate in the labor market appears to be an attractive extension. However, this extension is by no means trivial, so I choose to leave it to further research.

4 Conclusion

Despite the large number of empirical studies on the effects of the minimum wage on labor market outcomes, theoretical research on this topic has been hampered by the inherent difficulty of modeling labor market events in a general equilibrium framework. The contribution of this paper to the existing literature lies in the formulation of a tractable general equilibrium model within which it is possible to conduct a formal analysis of the minimum wage impact on social welfare and produce testable results.

In a search and matching model with Nash bargaining and match-specific productivity, I investigate the implications of the minimum wage for the efficiency of the equilibrium allocation. The work of Hosios (1990) on trading externalities indicates that for labor market efficiency the share of match surplus allotted to labor market participants −workers or firms− should be equal to their marginal contribution to the creation of match opportunities. I demonstrate that if the Hosios condition holds, then the decentralized equilibrium allocation without a minimum wage is Pareto superior to any equilibrium allocation under a binding minimum wage. Therefore, unless the labor market is inefficient, there is no scope for a minimum wage to increase welfare. I assume that the labor market allocation suffers from trading externalities −the bargaining power of workers is too low to satisfy the Hosios condition. In this context, I explain that a minimum wage hike can raise the "effective" bargaining power of the workers, and thus, mitigate the inefficiency of the decentralized equilibrium allocation.

The analysis draws heavily on the work of Flinn (2006), who alludes to this minimum wage effect without providing an explicit derivation. In a setting very similar to the one employed in Flinn (2006), I describe the mechanism through which the minimum wage can improve efficiency and derive the necessary and sufficient condition under which an increase in the minimum wage is welfare enhancing. I show that the welfare impact of the minimum wage is dependent on the relationship between the Nash bargaining power parameter and the elasticity of the matching function. Based on this finding, I specify a Hosios-type condition for constrained efficiency.

Moreover, I extend Flinn's (2006) framework to describe the phenomenon of minimum wage spillover effects. Variation in the level of the minimum wage has an impact on the value of unemployed workers. Given that wages are determined through Nash bargaining, if a minimum wage hike alters the value of search, it also affects accepted wages further up the wage distribution, thus causing spillover effects. This analysis relates to my discussion on efficiency as social welfare and the value of search change in the same direction in response to a minimum wage increase. Combining these findings, I demonstrate that the minimum wage improves social welfare if and only if it has positive spillover effects.

My main conclusion is that spillover effects characterize the impact of the minimum wage on the welfare of labor market participants, so the welfare effects of minimum wage changes can be inferred by the observation of spillovers on the wage distribution. In this way, I establish theoretically the view that minimum wage welfare effects can be empirically assessed by the comparison of pre- and post-change wage distributions. This is an interesting result as it provides a testable prediction that calls for further empirical research.

5 Appendix

5.1 Decisions of Individual Workers and Firms without a Minimum Wage

This discussion complements the analysis in Section 2.1.1. Let Y_f and V_f denote the present discounted value (PDV) of a firm with a filled and an unfilled job, respectively. Using the discount rate, r, the PDV of a filled job satisfies

$$Y_f - V_f = \frac{(z - w(z)) - rV_f}{r + \delta}, \tag{47}$$

while the PDV of an unfilled job satisfies

$$rV_f = -c + \lambda_e (1 - G(z^*)) E (Y_f - V_f | z \geq z^*). \tag{48}$$

Substituting (47) into (48) gives

$$rV_f = -c + \lambda_e (1 - G(z^*)) E \left(\frac{z - w(z) - rV_f}{r + \delta} | z \geq z^* \right). \tag{49}$$

Defining the steady state flow value of a vacant job to be $V = rV_f$ and taking the limit of (49) as r goes to zero leads to

$$V = -c + \lambda_e (1 - G(z^*)) E \left(\frac{z - w(z) - V}{\delta} | z \geq z^* \right).$$

Similarly, denote Y_w and V_w the present discounted value of an employed and an unemployed worker, respectively. Using the discount rate, r, the expected PDV of an employed worker satisfies

$$Y_w - V_w = \frac{w(z) - rV_w}{r + \delta}, \tag{50}$$

while the expected PDV of an unemployed worker satisfies

$$rV_w = b + \lambda_w (1 - G(z^*)) E ((Y_w - V_w) | z \geq z^*). \tag{51}$$

Substituting (50) into (51) gives

$$rV_w = b + \lambda_w (1 - G(z^*)) E \left(\frac{w(z) - rV_w}{r + \delta} | z \geq z^* \right).$$

Defining the steady state flow value of an unemployed worker to be $U = rV_w$ and taking the limit of the above expression as r tends to zero:

$$U = b + \lambda_w (1 - G(z^*)) E \left(\frac{w(z) - U}{\delta} | z \geq z^* \right). \tag{52}$$

5.2 Equilibrium Effects of Changes in the Minimum Wage

A change in the minimum wage affects simultaneously the "effective" bargaining power of workers, the value of search, market tightness, and unemployment. The impact of the minimum wage on the equilibrium values of these variables is determined endogenously: one needs to examine the simultaneous effect of the minimum wage on the set of equilibrium equations (21), (22), (29), and (30). Taking the total differential of each equilibrium equation yields a system of four equations, which can be solved in terms of $\frac{d\varepsilon}{dm}$, $\frac{dU_m}{dm}$, $\frac{d\theta}{dm}$, and $\frac{du}{dm}$.

To consider the effects of minimum wage changes on equilibrium unemployment, take the total differential of equation (22):

$$\frac{du}{dm} = -\Delta_1 \frac{d\theta}{dm} + \Delta_2, \tag{53}$$

where

$$\Delta_1 = \frac{u\,(1 - G\,(m))\, q\,(\theta)\,(1-\eta)}{[\delta + \theta q\,(\theta)\,(1 - G\,(m))]} \text{ and} \tag{54}$$

$$\Delta_2 = \frac{u\theta q\,(\theta)\, g\,(m)}{[\delta + \theta q\,(\theta)\,(1 - G\,(m))]}. \tag{55}$$

Clearly, the variation of equilibrium unemployment in response to minimum wage changes is dependent on the effect of m on θ. If $\frac{d\theta}{dm} < 0$, equilibrium unemployment increases after a minimum wage hike; if $\frac{d\theta}{dm} > 0$, the effect of m on u is ambiguous.

Let us now examine the implications of a varying minimum wage for the other endogenously determined variables. Take the total differential of the reduced-form relation between the value of search and market tightness, given by equation (29),

$$\frac{dU_m}{dm} = \frac{c\varepsilon}{(1-\varepsilon)}\frac{d\theta}{dm} + \frac{c\theta}{(1-\varepsilon)^2}\frac{d\varepsilon}{dm}. \tag{56}$$

Similarly, the total differential of the job creation condition, given by equation (30), is:

$$\begin{aligned} &-[1 - G\,(m)]\,\{\eta\,[(1-\varepsilon)\,(z^e - U_m)] + \theta\varepsilon c\}\,\frac{d\theta}{dm} - \\ &-[1 - G\,(m)]\,\theta\,[(z^e - b) + \theta c]\,\frac{d\varepsilon}{dm} - \\ &-g\,(m)\,\theta\,[(1-\varepsilon)\,(m - U_m)] = 0. \end{aligned} \tag{57}$$

It is evident that both $\frac{dU_m}{dm}$ and $\frac{d\theta}{dm}$ depend on the impact of m on the "effective" bargaining power of workers; the "effective" bargaining power of workers is defined by equation (21). Substituting the average wage and average match specific productivity into (21) gives

$$\varepsilon = \frac{(m - U_m)\,(G\,(\hat{z}) - G\,(m)) + \beta \int_{\hat{z}}^{\overline{z}} (z - U_m)\, dG\,(z)}{\int_m^{\overline{z}} (z - U_m)\, dG\,(z)}. \tag{58}$$

The total differential of the above equation is:

$$\int_{m}^{\overline{z}} (z - U_m)\, dG(z) \frac{d\varepsilon}{dm} = -\left[(1-\varepsilon)(1-G(m)) - (1-\beta)(1-G(\hat{z}))\right] \frac{dU_m}{dm} + \\ + \left[G(\hat{z}) - G(m) - (1-\varepsilon)(m - U_m)\, g(m)\right]. \tag{59}$$

Exploiting equations (56), (57), and (59) it is possible to determine how the equilibrium values of U_m and θ vary with m. First, use equation (57) to substitute $\frac{d\varepsilon}{dm}$ into (56) and solve for $\frac{d\theta}{dm}$

$$\frac{d\theta}{dm} = -\left(\frac{(1-\varepsilon)^2 (z^e - b + \theta c)}{c(\eta - \varepsilon)\left[(1-\varepsilon)(z^e - b) - \theta\varepsilon c\right]}\right) \frac{dU_m}{dm} - \\ - \left(\frac{\theta(1-\varepsilon)(m - U_m)\, g(m)}{(1-G(m))(\eta-\varepsilon)\left[(1-\varepsilon)(z^e - b) - \theta\varepsilon c\right]}\right). \tag{60}$$

From equation (60), substitute $\frac{d\theta}{dm}$ into equation (56)

$$\frac{dU_m}{dm} = \frac{c\theta}{(1-\varepsilon)} \frac{\left[1-G(m)\right](z^e - U_m)\left[\eta - \varepsilon\right]}{\left[1-G(m)\right]\left\{\eta\left[(1-\varepsilon)(z^e - b) - \theta\varepsilon c\right] + \theta\varepsilon c\right\}} \frac{d\varepsilon}{dm} - \\ - \frac{c\theta}{(1-\varepsilon)} \frac{\varepsilon(1-\varepsilon)(m - U_m)\, g(m)}{\left[1-G(m)\right]\left\{\eta\left[(1-\varepsilon)(z^e - b) - \theta\varepsilon c\right] + \theta\varepsilon c\right\}}. \tag{61}$$

Now use equation (59) to substitute $\frac{d\varepsilon}{dm}$ into equation (61)

$$B_1 \frac{dU_m}{dm} = B_2 \left[(G(\hat{z}) - G(m))(\eta - \varepsilon) - \eta(1-\varepsilon)(m - U_m)\, g(m)\right], \tag{62}$$

where

$$B_1 = 1 + \frac{c\theta}{(1-\varepsilon)} \frac{\left[\eta - \varepsilon\right]\left[(1-\varepsilon)(1-G(m)) - (1-\beta)(1-G(\hat{z}))\right]}{\left[1-G(m)\right]\left\{\eta\left[(1-\varepsilon)(z^e - b) - \theta\varepsilon c\right] + \theta\varepsilon c\right\}} \text{ and} \tag{63}$$

$$B_2 = \frac{c\theta}{(1-\varepsilon)\left[1-G(m)\right]\left\{\eta\left[(1-\varepsilon)(z^e - b) - \theta\varepsilon c\right] + \theta\varepsilon c\right\}}. \tag{64}$$

Equation (62) gives a reduced-form relation between $\frac{dU_m}{dm}$ and its determinants. The following Lemma simplifies the relation that determines the dependence of U_m on m:

Lemma 1 *The direction of change in the value of search after an increase in the minimum wage is given by*

$$\frac{dU_m}{dm} \overset{sgn}{=} (G(\hat{z}) - G(m))(\eta - \varepsilon) - \eta(1-\varepsilon)(m - U_m)\, g(m). \tag{65}$$

Proof. Clearly, it has to be the case that $B_1 \overset{sgn}{=} B_2$, where B_1 and B_2 are given by equation (63) and equation (64), respectively. Inspection of equation (64) suggests that B_2 is positive, so for (65) to hold, B_1 must be positive.

B_1 can be expressed as follows

$$B_1 = \frac{(1-\varepsilon)[1-G(m)]\eta[(1-\varepsilon)(z^e-b)-\theta\varepsilon c]+(1-\beta)(1-G(\hat{z}))\theta\varepsilon c}{(1-\varepsilon)[1-G(m)]\{\eta[(1-\varepsilon)(z^e-b)-\theta\varepsilon c]+\theta\varepsilon c\}}+$$
$$+\eta\theta c\frac{(1-\varepsilon)[1-G(m)]-(1-\beta)(1-G(\hat{z}))}{(1-\varepsilon)[1-G(m)]\{\eta[(1-\varepsilon)(z^e-b)-\theta\varepsilon c]+\theta\varepsilon c\}},$$

which is positive if

$$(1-\varepsilon)[1-G(m)]-(1-\beta)[1-G(\hat{z})]\geq 0.$$

Substitute $(1-\varepsilon)[1-G(m)]$ and $(1-\beta)[1-G(\hat{z})]$ from equations (26) and (28), respectively:

$$(1-\varepsilon)[1-G(m)]-(1-\beta)(1-G(\hat{z}))=$$
$$=\frac{\delta c}{q(\theta)\frac{\int_m^{\overline{z}}(z-U_m)dG(z)}{[1-G(m)]}}-\frac{\delta c-q(\theta)\int_m^{\hat{z}}(z-m)\,dG(z)}{q(\theta)\frac{\int_{\hat{z}}^{\overline{z}}(z-U_m)dG(z)}{[1-G(\hat{z})]}}=$$
$$=\frac{[1-G(\hat{z})]\int_m^{\hat{z}}(z-m)\,dG(z)}{\int_{\hat{z}}^{\overline{z}}(z-U_m)\,dG(z)}+$$
$$+\frac{\delta c}{q(\theta)}\left[\left(\frac{\int_m^{\overline{z}}(z-U_m)\,dG(z)}{[1-G(m)]}\right)^{-1}-\left(\frac{\int_{\hat{z}}^{\overline{z}}(z-U_m)\,dG(z)}{[1-G(\hat{z})]}\right)^{-1}\right].$$

Given that $\frac{\int_l^L zdG(z)}{[1-G(l)]}$ is decreasing in l, the above expression is positive for $\hat{z}\geq m$. Therefore, B_1 is positive. ■

Exploiting Lemma 1, I determine the sufficient condition for $\frac{d\theta}{dm}<0$, summarized in Lemma 2:

Lemma 2 *If the value of search is increasing in the minimum wage* $\left(\frac{dU_m}{dm}>0\right)$*, market tightness is decreasing in the minimum wage* $\left(\frac{d\theta}{dm}<0\right)$*.*

Proof. Inspection of equation (60) suggests that $\frac{dU_m}{dm}>0$ and $(\eta-\varepsilon)>0$ imply $\frac{d\theta}{dm}<0$. From (65), $(\eta-\varepsilon)>0$ is a necessary condition for $\frac{dU_m}{dm}>0$. Therefore, if $\frac{dU_m}{dm}>0$, then $(\eta-\varepsilon)>0$. Hence, $\frac{dU_m}{dm}>0$ is a sufficient condition for $\frac{d\theta}{dm}<0$. ■

5.3 Properties of the "Effective" Bargaining Power

The "effective" bargaining power of workers is defined as the average share of the match surplus received by employed workers, equation (21). Substituting the average wage and average match specific productivity from Definition 1 into (21) gives the relation between the "effective" bargaining power and its determinants, equation (58), which is reproduced here for convenience:

$$\varepsilon=\frac{(m-U_m)(G(\hat{z})-G(m))+\beta\int_{\hat{z}}^{\overline{z}}(z-U_m)\,dG(z)}{\int_m^{\overline{z}}(z-U_m)\,dG(z)}.$$

It is evident that ε is a function of β, m, and U_m: $\varepsilon = \phi(\beta, m, U_m)$. If the minimum wage is not binding ($m \leq U_m = U$), then $\varepsilon = \phi(\beta, m, U) = \beta$. The intuition is that in the absence of a binding wage floor all workers capture a share of the surplus equal to their bargaining power, β. If the minimum wage is binding ($m > U_m$), the workers capture a share of the surplus greater than their bargaining power, so $\varepsilon = \phi(\beta, m, U_m) > \beta$.

To examine how $\varepsilon = \phi(\beta, m, U_m)$ varies with β, m, and U_m, I take its total differential

$$d\varepsilon = \frac{\partial \phi(\beta, m, U_m)}{\partial \beta} d\beta + \frac{\partial \phi(\beta, m, U_m)}{\partial m} dm + \frac{\partial \phi(\beta, m, U_m)}{\partial U_m} dU_m$$

or alternatively

$$\begin{aligned} \int_m^{\overline{z}} (z - U_m)\, dG(z)\, d\varepsilon = & \left[\int_{\hat{z}}^{\overline{z}} (z - U_m)\, dG(z)\right] d\beta + \\ & + \left[G(\hat{z}) - G(m) - (1-\varepsilon)(m - U_m) g(m)\right] dm - \\ & - \left[(1-\varepsilon)(1 - G(m)) - (1-\beta)(1 - G(\hat{z}))\right] dU_m. \end{aligned} \tag{66}$$

Clearly, the "effective" bargaining power of workers is decreasing in the value of search (U_m). Any factor that affects U_m in the constrained decentralized equilibrium should also have an indirect effect on ε. In the context of this paper, I only consider the equilibrium effects of changes in the minimum wage, so $d\beta = 0$ and all other factors (e.g. unemployment benefits) that could influence ε indirectly through U_m are assumed to be given.

To account for this indirect effect of the minimum wage on ε, I substitute dU_m from equation (61) into equation (66). This gives

$$Q_1 d\varepsilon = Q_2 dm, \tag{67}$$

where

$$\begin{aligned} Q_1 = & \frac{c\theta}{(1-\varepsilon)} \frac{\left[(1-\varepsilon)(1-G(m)) - (1-\beta)(1-G(\hat{z}))\right]\left[1-G(m)\right](z^e - U_m)\left[\eta - \varepsilon\right]}{\left[1-G(m)\right]\left\{\eta\left[(1-\varepsilon)(z^e - b) - \theta\varepsilon c\right] + \theta\varepsilon c\right\}} + \\ & + \int_m^{\overline{z}} (z - U_m)\, dG(z), \end{aligned}$$

and

$$\begin{aligned} Q_2 = & \frac{c\theta\left[(1-\varepsilon)(1-G(m)) - (1-\beta)(1-G(\hat{z}))\right]\varepsilon(m - U_m) g(m)}{\left[1-G(m)\right]\left\{\eta\left[(1-\varepsilon)(z^e - b) - \theta\varepsilon c\right] + \theta\varepsilon c\right\}} + \\ & + \left[G(\hat{z}) - G(m) - (1-\varepsilon)(m - U_m) g(m)\right]. \end{aligned} \tag{68}$$

Using the proof of Lemma 1, it is straightforward to show that Q_1 is positive. Therefore, the dependence of ε on the minimum wage is only influenced by the sign of Q_2. Rewrite equation (68) as follows

$$Q_2 = Q_{2A} + Q_{2B},$$

where

$$Q_{2A} = \frac{c\theta\left[(1-\varepsilon)(1-G(m))-(1-\beta)(1-G(\hat{z}))\right]\varepsilon(m-U_m)g(m)}{[1-G(m)]\left\{\eta\left[(1-\varepsilon)(z^e-b)-\theta\varepsilon c\right]+\theta\varepsilon c\right\}} \text{ and} \tag{69}$$

$$Q_{2B} = \left[G(\hat{z})-G(m)-(1-\varepsilon)(m-U_m)g(m)\right]. \tag{70}$$

Clearly, $Q_{2A}>0$ (see the proof of Lemma 1), so the sign of Q_2 depends on the sign of Q_{2B}. If $Q_{2B}\geq 0$, then ε is increasing in m. If $Q_{2B}<0$, then the dependence of ε on the minimum wage is ambiguous in sign. Therefore,

$$\left[G(\hat{z})-G(m)-(1-\varepsilon)(m-U_m)g(m)\right]\geq 0 \tag{71}$$

is a sufficient condition for $\frac{d\varepsilon}{dm}>0$.

Combining this sufficient condition for the "effective" bargaining power to be increasing in the minimum wage with Lemma 1 leads to the following result:

Lemma 3 *If the value of search is increasing in the minimum wage, the "effective" bargaining power of workers is also increasing in the minimum wage.*

Proof. Lemma 1 suggests that $\frac{dU_m}{dm}>0$ if and only if

$$\left[G(\hat{z})-G(m)\right](\eta-\varepsilon)-\eta(1-\varepsilon)(m-U_m)g(m)>0,$$

which implies

$$\left[G(\hat{z})-G(m)\right]-(1-\varepsilon)(m-U_m)g(m)>\frac{\varepsilon\left[G(\hat{z})-G(m)\right]}{\eta}>0.$$

Therefore, if $\frac{dU_m}{dm}>0$, (71) holds, so $\frac{d\varepsilon}{dm}>0$. ■

5.4 The Social Planner's Problem

The objective of the social planner is the maximization of social welfare (equation [32]) with respect to the policy variable under his control, that is, the minimum wage. The social planner's decisions impact on the individual behaviour of agents in the labor market, given by the equilibrium equations (21), (22), (29), and (30). These reactions of agents have to be factored into the planner's maximization problem. Therefore, the planner's problem can be expressed as follows:

$$\max_m \left\{SW_m = ub+(1-u)z^e-\theta uc\right\}$$

subject to the equilibrium equations (21), (22), (29), and (30). Substituting u from (22), the planner's problem takes the following form

$$\max_m \left\{SW_m = \frac{\delta b+\theta q(\theta)[1-G(m)]z^e-\delta c\theta}{\delta+\theta q(\theta)[1-G(m)]}\right\}$$

subject to equations (21), (29), and (30).

First, take the total differential of social welfare:

$$\frac{dSW_m}{dm} = \frac{\partial SW_m}{\partial \theta}\frac{d\theta}{dm} + \frac{\partial SW_m}{\partial m},$$

which implies

$$\frac{dSW_m}{dm} = -u_m \frac{q(\theta)(1-G(m))(\eta-\varepsilon)[z^e - b + c\theta]}{\delta + \lambda_w[1-G(m)]}\frac{d\theta}{dm} - \frac{\lambda_w g(m)(m-U_m)}{\delta + \lambda_w[1-G(m)]}. \tag{72}$$

From equation (60), substitute $\frac{d\theta}{dm}$ into equation (72) and simplify using equation (30):

$$\frac{dSW_m}{dm} = A\frac{dU_m}{dm}, \tag{73}$$

where

$$A = \frac{u_m\left(q(\theta)(1-G(m))(1-\varepsilon)^2(z^e - b + \theta c)^2\right)}{(\delta + \lambda_w(1-G(m)))c[(1-\varepsilon)(z^e-b) - \theta\varepsilon c]}. \tag{74}$$

The first order condition of the planner's problem is

$$\frac{dSW_m}{dm} = A\frac{dU_m}{dm} = 0.$$

Inspection of (74) suggests that $A > 0$, which means

$$\frac{dSW_m}{dm} \stackrel{sgn}{=} \frac{dU_m}{dm}.$$

Therefore, the planner's problem reduces to the maximization of the value of search in the decentralized equilibrium with respect to the minimum wage.

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