NON WALRASIAN DECENTRALIZATION OF THE CORE

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Running Title: Decentralization of the Core Through Nash Equilibrium.

Abstract: We show that in large finite economies, core allocations can be approximately decentralized as Nash (rather than Walras) equilibrium. We argue that this exercise is an essential complement to asymptotic core equivalence results, because it implies that in some approximate sense individual attempts to manipulate the decentralizing prices cannot be beneficial, which fits precisely the interpretation of asymptotic core convergence, namely the emergence of price taking.

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1. Introduction

The study of the asymptotic limit of the core along sequences of economies with increasing numbers of individuals, what has been termed 'core equivalence' in the literature, has provided a foundation to the proposition that in large markets consisting of relatively small participants individuals tend to exhibit price taking behavior. With few exceptions core convergence results proceed through the identification of prices which 'decentralize' core outcomes, in the sense that core outcomes approximate (in some appropriate sense) the Walrasian demands of individuals at the decentralizing prices, the approximation getting finer the larger the number of individuals. The deduction from these results is that the effort of individuals to exercise market power by coordinating within coalitions, results in outcomes which are not much different from those corresponding to price taking behavior at some appropriate prices. Hence, the conclusion that price taking is a reasonable approximation of the true behavior of individuals.

Nevertheless, the parsimony of the core with respect to the description of price formation does not permit a clear insight to the strategic possibilities and incentives of individuals to manipulate the prices which decentralize core allocations. In core convergence results the decentralizing prices appear 'magically' (to use the expression of a leader in the field) and not through some explicit mechanism, so it is impossible to discern the extent of individuals' influence on decentralizing prices and its potential benefit. The intended contribution of the present paper is to tighten this loose end, by showing that in large finite economies core allocations along with the associated decentralizing prices are (in some approximate sense) Nash equilibria of strategic market games, of the type originating in [16] and in [17]. We are not looking for alternative equivalence theorems, but rather for an alternative game theoretic interpretation of the decentralizing prices of existing core equivalence results. We believe that the decentralization of core outcomes by means of Nash equilibrium of Shapley-Shubik type strategic market games has several conceptual advantages. Those games feature an explicit price formation mechanism and rules of trade, which put some flesh on the strategic possibilities of individuals to influence market prices and also facilitate the modeling of the emergence of decentralizing prices and core outcomes. Furthermore, the Nash equilibria of these games encapsulate the idea that individuals cannot influence prices in their favor. Therefore, approximate decentralization of core allocations as Nash equilibria of such games signifies that individuals cannot obtain any substantial benefit from manipulating the decentralizing prices, which fact is a *bona fide* justification of the price taking hypothesis. Conversely, a failure of core allocations along with the associated decentralizing prices to satisfy (at least approximately) some Nash property, would cast serious doubts to the merit of the argument in favor of price taking. For, such a failure would imply that individuals could realize substantial benefits by manipulating the prices which decentralize core allocations. Hence, we argue that decentralization of core allocations as Nash equilibria, where price taking in finite economies is not assumed, makes a lot of sense and strengthens the conclusion of core equivalence theorems, because it accurately captures the spirit of their interpretation.

In view of this discussion we proceed in this paper to extend core equivalence results to the non Walrasian framework. The strategic market games of the Shapley-Shubik type are certainly not the only ones available in the literature, that could be used for this purpose. Nevertheless, Shapley-Shubik strategic market games have a Cournotian flavor which we find irresistible, because then we can claim that our study reconciles two classic theories of competition: the theory of the core, which is associated with Edgeworth, and the theory of Shapley-Shubik strategic market games which is associated with Cournot.

There is a long list, in fact too long to cite here in any complete sense, of results on the asymptotic convergence to Walrasian equilibria, both for the core and for Nash equilibria of Shapley-Shubik strategic market games. A comprehensive and quite systematic survey of core convergence results along with references can be found in [2]. A number of authors, [5], [13], [10], [4], [12], [9], among many others, have pursued the issue of asymptotic convergence of strategic market game outcomes to Walrasian ones. Arguably, in view of these results some topological proximity between core and Nash allocations holds true. However, though relevant, this proximity does not capture the issue raised above, because it does not mean that core allocations and the associated decentralizing prices are approximate Nash equilibria[1]. Furthermore,

[1]Careful reflection makes clear that core allocations need not even be implementable via a standard strategic market game.

although the convergence of Nash equilibria implies proximity to some core allocations (notably the Walrasian ones), the proximity of *every* core allocation to a Nash equilibrium is a different matter.

The key result of the present paper is theorem (1), which asserts that in large finite economies any core allocation can be obtained as a Nash equilibrium of a strategic game with 'small' transfers, defined on a 'nearby' economy, the approximation to the original market game and economy becoming finer as the number of individuals becomes larger. Theorems (2) and (3) feature the desired conceptual conclusion concerning the interpretation of core equivalence: the larger the number of individuals, the smaller becomes the potential benefit of any deviation from the strategies which implement a core allocation and the associated decentralizing prices. Finally (4) relates Nash equilibria to the core property, thus completing the relationship between the two sets.

We are not aware of any study of the asymptotic relationship of core and Nash outcomes to one another. Nontheless, the results that we show in this paper complement very well several existing ones. Let us mention here two of them which fit nicely with ours. In [13], it is shown that in large finite economies Nash equilibrium allocations are approximate Pareto optima. Theorem (4) in this paper demonstrates that a Nash equilibrium of a strategic market game is an approximate core allocation. In [14] it has been demonstrated that a Nash equilibrium of a strategic market game is an approximate Walras allocation for an economy which is 'near' the original one. Corollary (1) below shows the converse: a Walrasian equilibrium for a given economy is a Nash equilibrium for an economy which is 'near' the original one, in a similar sense as in [14]. In a different paper [10] it is shown that given any sequence of economies converging to a regular economy, the competitive equilibria of the latter can be approximated by a sequence of Nash equilibria drawn along the sequence. Our result shows that for any Walrasian equilibrium of a given economy (not necessarily regular) there is a sequence of economies and associated Nash equilibria so that the sequence of economies converges to the original economy and the sequence of Nash equilibria converges to the chosen Walrasian allocation.

Finally, besides the conceptual content of our results, the study of this issue will link together the asymptotic convergence results of the core and Nash equilibria of strategic market games and thereby, allow us to make inferences about the asymptotic limits of one concept, from the asymptotic behavior of the other. For example it would open the path for the diffusion of asymptotic properties of the core to the theory of strategic market games where they do not have a counterpart, such as the rate of convergence of the core.

We proceed to develop the context and the results. Some further comments follow in the last section.

2. The model

Let H be a finite set of agents. There are L commodity types in the economy and the consumption set of each agent is identified with $\Re^L_+$. Each individual $h \in H$ is characterized by a preference relation $\succeq_h \subset \Re^L_+ \times \Re^L_+$ and an initial endowment $e_h \in \Re^L_+$. We use the following assumptions:

Assumption 1. *Preferences are C^2, convex and strictly monotone.*

Assumption 2. *Indifference surfaces through the endowment do not intersect the axes.*

Denote by $\mathcal{P}^2_{cm}$ the set of preferences that satisfy (1) and by $\mathcal{P}^2_{cmb}$ the set of preferences that satisfy (1) and (2), both endowed with the topology of closed convergence. Let $T \subset \mathcal{P}^2_{cm} \times \Re^L_{++}$ be compact. An economy is defined as a mapping $\mathcal{E} : H \to T$.

In the sequel we will need a way to express 'proximity' between economies defined on the same set H of consumers. For this purpose we will use a metric defined on economies, which is a variation of the one used in [14]. Given two preference relations $\succeq$ and $\succeq'$ let $d(\succeq, \succeq')$ metrize the topology of closed convergence. Recall that when $u : \Re^L_+ \to \Re$ and $v : \Re^L_+ \to \Re$ represent $\succeq$ and $\succeq'$ respectively, then if $\mid u(\cdot) - v(\cdot) \mid \to 0$ uniformly on compact sets then $d(\succeq, \succeq') \to 0$. Given two economies $\mathcal{E} : H \to T$ and $\mathcal{E}' : H \to T'$, following [14] we define now

$$\sigma(\mathcal{E}, \mathcal{E}') = \frac{1}{\#H} \sum_{h \in H} [d(\succeq_h, \succeq'_h) + 2 \frac{\| e_h - e'_h \|}{\sum_{h \in H} e_h + e'_h}] \tag{1}$$

The set of *feasible* allocations of a given economy $\mathcal{E}$, is

$$F = \left\{ x \in \Re^{LH}_+ : \sum_{h \in H} x_h = \sum_{h \in H} e_h \right\}$$

The standard definition of the core of an economy $\mathcal{E}$ is as follows:

Definition 1. *The core is the set $\mathcal{C}(\mathcal{E}) \subset F$ such that:*

$$\nexists S \subset H \text{ and } y \in \Re_{+}^{LS} \text{ s.t. } \sum_{h \in S} y_h = \sum_{h \in S} e_h \text{ and } y_h \succ_h x_h, \forall h \in S.$$

The above definition identifies allocations with the property that no group of individuals can redistribute their endowments in a way that is unanimously preferred by all its members. The theory of the core postulates that an equilibrium outcome should be characterized by this property. Notice that the procedure via which alternative redistributions of endowments (and core outcomes) are conducted, is left unspecified. The idea of 'decentralization' of the core is that, under some qualifications, in large finite economies the process by which core outcomes are attained can be effectively approximated by a system of Walrasian markets.

It will be useful for our purposes to have an approximate version of the core property, which requires that a blocking allocation be 'substantially' preferred by all members of a coalition. In order to model this idea it will be convenient to introduce some notation. Given $\epsilon > 0$ and $y \in \Re_{+}^{L}$ denote $y \ominus \epsilon \mathbf{1}_L = (\max\{y^i - \epsilon, 0\})_{i=1}^{L}$. We can now state the following definition that we will use in the sequel.[2]

Definition 2. *Given $\epsilon > 0$ the ϵ-core (or approximate core) is the set $\mathcal{C}^{\epsilon}(\mathcal{E}) \subset F$ such that:*

$$\nexists S \subset H \text{ and } y \in \Re_{+}^{LS} \text{ s.t. } \sum_{h \in S} y_h = \sum_{h \in S} e_h \text{ and } y_h \ominus \epsilon \mathbf{1}_L \succ_h x_h, \ \forall h \in S.$$

Obviously $\mathcal{C}(\mathcal{E}) \subset \mathcal{C}^{\epsilon}(\mathcal{E})$. In fact, a core allocation is one that belongs to every ϵ-core.

We now turn to describe a strategic market game, which proposes an explicit model of how exchange in the economy takes place. The version of the strategic market game below has been studied in [13] and in [12].

2.1. **Trade using inside money.** Trade in the economy is organized via a system of trading posts where individuals offer commodities for sale and place bids for purchases of commodities. Bids are placed in terms of a unit of account. The strategy set of each agent is $S_h = \{(b_h, q_h) \in \Re_{+}^{2L} : q_h^i \le e_h^i, \ i = 1, 2, \ldots, L\}$. Given a strategy profile $(b, q) \in \prod_{h \in H} S_h$ let $B^i = \sum_{h \in H} b_h^i$ and $Q^i = \sum_{h \in H} q_h^i$ denote

[2] Several alternative definitions of approximate cores have appeared in the literature. The one we use here is a variant of that in [18] or [19]. For several equivalence results between approximate cores and approximate Walras equilibria see [8].

aggregate bids and offers for each $i = 1, 2, \ldots, L$. Also for each agent h denote $B^i_{-h} = \sum_{k \neq h} b^i_k$, $Q^i_{-h} = \sum_{k \neq h} q^i_k$. For a given a strategy profile, consumption of each commodity $i = 1, 2, \ldots, L$ by each consumer $h \in H$ is determined as follows:

$$x^i_h\,(b, q) = \begin{cases} e^i_h - q^i_h + \frac{b^i_h}{B^i} Q^i & \text{if } \sum_{i=1}^{L} \frac{B^i}{Q^i} q^i_h \geq \sum_{i=1}^{L} b^i_h \\ e^i_h - q^i_h & \text{otherwise} \end{cases} \tag{2}$$

where it is postulated that whenever the term $0/0$ appears in the expressions above it is defined to equal zero. When $B^i Q^i \neq 0$ the fraction $\pi^i(b, q) = \frac{B^i}{Q^i}$ has a natural interpretation as the (average) market clearing 'price'. The relation $\sum_{i=1}^{L} \pi^i(b, q) q^i_h \geq \sum_{i=1}^{L} b^i_h$ is a 'bookkeeping' restriction which ensures that units of account remain at zero net supply (inside money). The interpretation of this allocation mechanism is that commodities (money) is distributed among non bankrupt consumers in proportion to their bids (offers), while the purchases of bankrupt consumers are confiscated.

An equilibrium is defined as a strategy profile $(b, q) \in \prod_{h \in H} S_h$ that forms a Nash equilibrium in the ensuing game with strategic outcome function given by (2). Let $\mathbf{N}(\mathcal{E}) \subset \prod_{h \in H} S_h$ denote the set of Nash equilibrium strategy profiles of the strategic market game and $\mathcal{N}(\mathcal{E}) \subset \Re^{LH}_{+}$ the set of consumption allocations corresponding to the elements of $\mathbf{N}(\mathcal{E})$.

2.2. **Games with transfers.** Just as we often do in exchange economies we may be interested in outcomes which can be sustained as some kind of equilibrium, when some income transfers among individuals are allowed. This can be done just as well in the present context. Let us develop a variant of the original market game which captures this idea. Let $(p, x) \in \Delta^L \times F$ be a price vector and an (feasible) allocation. Consider a strategic market game, which is defined exactly as in the previous section except that the allocation rule is now as follows:

$$\hat{x}^i_h\,(b, q; p, x) = \begin{cases} e^i_h - q^i_h + \frac{b^i_h}{B^i} Q^i & \text{if } \sum_{i=1}^{L} \left(b^i_h - \frac{B^i}{Q^i} q^i_h \right) \leq p\,(x_h - e_h) \\ e^i_h - q^i_h & \text{otherwise} \end{cases} \tag{3}$$

This modified allocation rule differs from the original one, only in that individuals are required to satisfy a different budget constraint, which involves credits/liabilities imposed on individuals determined by the pre specified allocation x along with the vector p. By virtue of the fact that the pre specified allocation x is feasible, those lump sum transfers cancel out on the aggregate so the unit of account remains in zero net supply (money is still 'inside'). Of course, if $p\,(x_h - e_h) = 0$ for all $h \in H$, then (3) is the same as the original one. Moreover, for arbitrarily small transfers the budget restrictions imposed on individuals are arbitrarily close to those imposed by the original allocation rule.

DEFINITION 3. *A transfer market game is the strategic market game induced by the strategic outcome function defined in (3).*

We denote by $\mathbf{N}_{(p,x)}(\mathcal{E})$ the set of Nash equilibrium strategy profiles of the corresponding (p, x)-transfer strategic market game and by $\mathcal{N}_{(p,x)}(\mathcal{E})$ the set of corresponding consumption allocations.

The following notation and familiar facts will be useful in the sequel. Fix $(b_{-h}, q_{-h}) \in \prod_{k \neq h} S_k$ and let[3] $g(y) = \sum_{i=1}^{L} \frac{B^i_{-h}(y^i - e^i_h)}{Q^i_{-h} - y^i + e^i_h}$. The set of allocations which an individual $h \in H$ can achieve via the strategic outcome function (3) is given[4] by the convex set

$$\gamma_h = \{y \in \Re^L_+ : g(y) \leq p\,(x_h - e_h)\,, y \leq Q_{-h} + e_h\}$$

i.e., $(b_h, q_h) \in S_h \Rightarrow \hat{x}_h\,(b, q; p, x) \in \gamma_h$. Conversely, $x_h \in \gamma_h \Rightarrow \exists (b_h, q_h) \in S_h$ *s.t.* $x_h = x_h\,(b, q)$. Therefore, due to the bankruptcy rule imposed by (3), at an equilibrium with nonzero bids and offers we have that $\bar{x} \in \mathcal{N}_{(p,x)}(\mathcal{E})$ if and only if:

$$\begin{aligned} &(i) \quad \bar{x} = \hat{x}(\bar{b}, \bar{q}; p, x), \text{ for some } (\bar{b}, \bar{q}) \in \prod_{h \in H} S_h \\ &(ii) \quad \forall h \in H,\ \gamma_h \cap \{y \in \Re^L_+ : y \succ_h \bar{x}_h\} = \emptyset \end{aligned} \tag{4}$$

Finally, in the sequel we will also refer to approximate Nash equilibria, which are defined along the lines of [15] as follows:

DEFINITION 4. *For a given $\epsilon > 0$ we say that $\bar{x} \in \gamma_h$ is an ϵ-Nash equilibrium allocation if:*

$$\begin{aligned} &(i) \quad \bar{x} = \hat{x}\,(b, q; p, x)\,, \textit{ for some } (b, q) \in \prod_{h \in H} S_h \\ &(ii) \quad \forall h \in H,\ \gamma_h \cap \{y \in \Re^L_+ : y \ominus \epsilon \mathbf{1}_L \succ_h x_h\} = \emptyset \end{aligned} \tag{5}$$

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[3] In order to save on notation we omit the dependency on (b_{-h}, q_{-h}) and (p, x). In the results the values of those variables will be fixed so no confusion should arise.

[4] This is obtained by a straightforward manipulation of (3); see [13], [12] or [10].

In accordance with our previous notation let $\mathbf{N}^{\epsilon}_{(p,x)}(\mathcal{E}) \subset \prod_{h\in H} S_h$ denote the set of approximate Nash equilibrium strategy profiles of the strategic market game and $\mathcal{N}^{\epsilon}_{(p,x)}(\mathcal{E}) \subset \Re^{LH}_{+}$ the corresponding set of consumption allocations. Certainly, $\mathcal{N}(\mathcal{E}) \subset \mathcal{N}^{\epsilon}(\mathcal{E})$. In fact, $x \in \mathcal{N}(\mathcal{E})$ if and only if $x \in \mathcal{N}^{\epsilon}(\mathcal{E})$ for all $\epsilon > 0$.

The interpretation of an ϵ-Nash equilibrium is that individuals optimize 'up to ϵ'. This definition expresses the idea that the actual best responses do not provide 'substantial' improvements over a given allocation. Technically, this is represented by the proximity between the sets involved in (4) and (5). The continuity of preferences suggests that any $y \in \gamma_h$ is not 'far' better than x_h.

We are ready now to proceed with the results of this paper.

3. Results

We begin with the first result of this paper which associates core allocations of a given economy with the Nash equilibria of a nearby economy.

Theorem 1. *Let $\mathcal{E} : H \to T$, where $T \subset \mathcal{P}^2_{cmb} \times [\frac{1}{s}, s]^L$ is compact, be an economy and $\hat{x} \in \mathcal{C}(\mathcal{E})$. For every $\epsilon > 0$ there is N such that if $\#H > N$, there is an economy $\mathcal{E}' : H \to \mathcal{P}^2_{cm} \times [\frac{1}{s}, s]^L$, where $\sigma(\mathcal{E}, \mathcal{E}') < \epsilon$, a vector $\hat{p} \in \Delta^L$ and $(\hat{b}, \hat{q}) \in \mathbf{N}_{(\hat{p},\hat{x})}(\mathcal{E}')$ such that $\hat{x} = \hat{x}(\hat{b}, \hat{q}; \hat{p}, \hat{x})$, $\pi(\hat{b}, \hat{q}) = \hat{p}$ and $|\hat{p}(\hat{x}_h - e_h)| < \epsilon$.*

Proof:

Since core allocations are Pareto optima, by the second welfare theorem there is a $\hat{p} \in \Delta^L$, where $\hat{p} \gg 0$ such that for all $h \in H$:

$$x \succ_h \hat{x}_h \Rightarrow \hat{p}x > \hat{p}\hat{x}_h \tag{6}$$

Given the vector $\hat{p}$ we can define:

$$\eta^i_h = \frac{\hat{p}^i \left(\hat{x}^i_h - e^i_h\right)}{\sum_{k\neq h} \hat{x}^i_h} \tag{7}$$

Let us now fix some constants. Since T is compact, core allocations are uniformly bounded (see [7] p.193 Proposition 2), i.e., there is $r > s$ such that $x \in \mathcal{C}(\mathcal{E}) \Rightarrow x_h \in [\frac{1}{r}, r]^L$, $\forall h \in H$. Therefore,

$$
\begin{aligned}
|\eta_h^i| &= \frac{|\hat{p}^i(\hat{x}_h^i - e_h^i)|}{\sum_{k\neq h}\hat{x}_h^i} \\
&\leq \frac{|\hat{x}_h^i - e_h^i|}{\sum_{k\neq h}\hat{x}_h^i} \\
&\leq \frac{\hat{x}_h^i + e_h^i}{\sum_{k\neq h}\hat{x}_h^i} \\
&\leq \frac{1}{\#H-1}\left(\frac{2r}{1/r}\right) \\
&= \frac{2r^2}{\#H-1} \qquad (8)
\end{aligned}
$$

Define the set $C_h = \{x \in \Re_+^L : (\hat{p}+\eta_h)x \leq (\hat{p}+\eta_h)\hat{x}\}$. The compactness of T implies that there is $\delta > 0$ such that if $(\succeq, e) \in T$, $x \leq \mathbf{1}_L r$ and $x \succeq e$, then $D_{\succeq}(x) \gg \delta\mathbf{1}_L$, where $D_{\succeq}(\cdot)$ denotes the (normalised) gradient vector of $\succeq$. Since $\hat{p} = D_{\succeq_h}(\hat{x}_h)$, for each $h \in H$ we have that $\hat{p} \gg \delta\mathbf{1}_L$. Now, by choosing $N_1 > 1 + \frac{4r^2}{\delta}$ we have that for all $\#H > N_1$, $|\eta_h| < \frac{\delta}{2}\mathbf{1}_L$, for each $h \in H$. Hence, $\hat{p} + \eta_h \gg \frac{\delta}{2}\mathbf{1}_L$. It follows that $C_h \subseteq \{x \in \Re_+^L : (\frac{\delta}{2}\mathbf{1}_L)x \leq (1 + \frac{\delta}{2}L)r\}$, i.e., if we set $\bar{r} = \frac{2+\delta L}{\delta}r$ we have $C_h \subset [0, \bar{r}]^L$. This last set will do for our purpose.

We construct a new economy $\mathcal{E}' : H \to \mathcal{P}_{cm}^2 \times [\frac{1}{s}, s]^L$, defined as $\mathcal{E}'(h) = (\succeq_h', e_h)$, where for each $h \in H$, $\succeq_h'$ is the preference relation defined as follows:

$$x \succeq_h' y \Leftrightarrow \min\{\hat{p}z : z \succeq_h x\} + \eta_h x \geq \min\{\hat{p}z : z \succeq_h y\} + \eta_h y \qquad (9)$$

Note that given our choice of constants, $\succeq_h'$ is monotonic for each $h \in H$.

We first identify strategies so that the core allocation $\hat{x}$ is an outcome of the $(\hat{p}, \hat{x})$-transfer game defined on the economy $\mathcal{E}'$. Let $(\hat{b}, \hat{q}) \in \prod_{h\in H} S_h$ be the strategy profile defined as follows: $\hat{b}_h^i = \hat{p}^i\hat{x}_h^i$, $\hat{q}_h^i = e_h^i$, for each $i = 1, 2, \ldots, L$. In this case:

$$\pi^i(\hat{b}, \hat{q}) = \frac{\hat{B}^i}{\hat{Q}^i} = \frac{\sum_{h\in H}\hat{p}^i\hat{x}_h^i}{\sum_{h\in H}e_h^i} = \hat{p}^i\frac{\sum_{h\in H}\hat{x}_h^i}{\sum_{h\in H}e_h^i} = \hat{p}^i \qquad (10)$$

Therefore, $\pi(\hat{b}, \hat{q}) = \hat{p}$. Furthermore, for each $h \in H$:

$$\sum_{i=1}^{L}(\hat{b}_h^i - \frac{\hat{B}^i}{\hat{Q}^i}\hat{q}_h^i) = \sum_{i=1}^{L}\hat{p}^i(\hat{x}_h^i - e_h^i) \qquad (11)$$

Hence, it follows that since no individual is bankrupt so,

$$\hat{x}_h^i(\hat{b},\hat{q};\hat{p},\hat{x}) = e_h^i - \hat{q}_h^i + \hat{b}_h^i \frac{\hat{Q}^i}{\hat{B}^i} = e_h^i - e_h^i + \frac{\hat{p}^i \hat{x}_h^i}{\hat{p}^i} = \hat{x}_h^i$$

i.e., $\hat{x}_h(\hat{b},\hat{q};\hat{p},\hat{x}) = \hat{x}_h$, $\forall h \in H$.

We now verify that for the above specified strategies $\hat{x}$ is a Nash equilibrium.

Let us fix $h \in H$. Clearly $\hat{x}_h \in \gamma_h$, because $\hat{x}_h \leq \hat{Q}_{-h} + e_h^i$ and also

$$g(\hat{x}_h) = \sum_{i=1}^{L} \frac{\hat{B}_{-h}^i(\hat{x}_h^i - e_h^i)}{\hat{Q}_{-h}^i - \hat{x}_h^i + e_h^i} = \sum_{i=1}^{L} \hat{p}^i(\hat{x}_h^i - e_h^i) \tag{12}$$

In particular, (12) implies that $\hat{x}_h$ lies on the boundary of the set γ_h which is convex. Hence, there is a $p_h \in \Re_+^L$, specifically $p_h = Dg(\hat{x}_h)$ (where $Dg(\cdot)$ denotes the gradient of the function $g(\cdot)$) such that $z \in \gamma_h \Rightarrow p_h z \leq p_h \hat{x}_h$. Notice that

$$p_h = Dg(\hat{x}_h) = \left(\frac{\hat{B}_{-h}^i \hat{Q}_{-h}^i}{(\hat{Q}_{-h}^i - \hat{x}_h^i + e_h^i)^2} \right)_{i=1}^{L}$$

Substituting for the strategies we further have that $p_h = \hat{p} + \eta_h \geq 0$.

Therefore, we deduce that

$$z \in \gamma_h \Rightarrow (\hat{p} + \eta_h) z \leq (\hat{p} + \eta_h)\hat{x}_h \tag{13}$$

Now, suppose that $y \succ_h' \hat{x}_h$ for some $h \in H$. By definition of $\succ_h'$ we have:

$$\begin{aligned} (\hat{p} + \eta_h) y &\geq \min\{\hat{p}z : z \succeq_h y\} + \eta_h y \\ &> \min\{\hat{p}z : z \succeq_h \hat{x}_h\} + \eta_h \hat{x}_h \\ &= (\hat{p} + \eta_h)\hat{x}_h \end{aligned} \tag{14}$$

where the last equality follows from (6). Therefore, it follows from (13) that $y \notin \gamma_h$. Since y is arbitrary, we conclude that $y \succ_h' \hat{x}_h \Rightarrow y \notin \gamma_h$, which implies

$$\gamma_h \cap \{y \in \Re_+^L : y \succ_h' \hat{x}_h\} = \emptyset \tag{15}$$

Thus, $(\hat{b},\hat{q}) \in \mathbf{N}_{(\hat{p},\hat{x})}(\mathcal{E}')$, $\hat{x}(\hat{b},\hat{q}) = \hat{x}$ and $\pi(\hat{b},\hat{q}) = \hat{p}$ as desired.

We now show that when the number of individuals is sufficiently large, the transfers necessary in the market game that implements the core allocation are arbitrarily small and the economy $\mathcal{E}'$, is arbitrarily close to the original economy $\mathcal{E}$.

By proposition 7.4.3 in [11] (see also [6]) we have that since $\hat{p} \in \Delta^L$ supports $\hat{x}$ which is in the core, there is a constant $M > 0$ so that $|\hat{p}(\hat{x}_h - e_h)| \leq \frac{M}{\#H}$. Therefore, by choosing $N_2 > \frac{M}{\epsilon}$ we have that for each $h \in H$, $|\hat{p}(\hat{x}_h - e_h)| < \epsilon$ for all $\#H > N_2$ as needed.

Finally to demonstrate the proximity of the economies, we have that since $|\eta_h x| < \frac{2r^2\bar{r}L}{\#H-1}$ for all $x \in [0,\bar{r}]^L$ we have that if $(\succeq, e) \in T$ then $\min\{\hat{p}z : z \succeq x\} + \eta_h x \rightarrow \min\{\hat{p}z : z \succeq x\}$ uniformly on $[0,\bar{r}]^L$, so $d(\succeq, \succeq') \rightarrow 0$. Therefore, there is $N > N_1$ such that when $\#H > N$ then $d(\succeq, \succeq') < \epsilon$. In this case we have $\sigma(\mathcal{E}, \mathcal{E}') < \epsilon$ as well. □

Walrasian allocations are in the core so the preceding theorem applies to them as well. Since Walrasian allocations are of particular interest we state below the relevant version of our result. Notice that in the case of Walrasian allocations no modification of the market game is necessary, because Walrasian allocations can be supported without any income transfers.

Corollary 1. *Let $\mathcal{E} : H \rightarrow T$ be an economy and $(\bar{p}, \bar{x}) \in \Delta^L \times \Re_+^{LH}$ be a Walrasian equilibrium of this economy. For every $\epsilon > 0$ there is N such that if $\#H > N$, there is an economy $\mathcal{E}'' : H \rightarrow \mathcal{P}_{cm}^2 \times [\frac{1}{s}, s]^L$, where $\sigma(\mathcal{E}, \mathcal{E}'') < \epsilon$, and a Nash equilibrium for the market game defined on this economy, $(\bar{b}, \bar{q}) \in \mathbf{N}(\mathcal{E}'')$ such that $x(\bar{b}, \bar{q}) = \bar{x}$ and $\pi(\bar{b}, \bar{q}) = \bar{p}$.*

In [14] it is shown that Nash equilibrium allocations of a given economy are approximate Walras for an economy which is near the original one. The last corollary is a converse of that result. It states that Walrasian allocations for a given economy are Nash equilibrium allocations for a nearby economy.

Finally, corollary (1) implies that given an atomless economy and any competitive equilibrium (not necessarily regular), there is a sequence of economies and associated Nash equilibria which approximate the given atomless economy and competitive equilibrium. A result to this effect for regular economies is shown in [10]. The difference is that here we have more flexibility in constructing the appropriate sequence of economies, whereas [10] considers a fixed sequence of economies and associated limit.

In the above result we used an approximation of the original economy, in order to associate core allocations with (exact) Nash equilibria. The next result does not involve a perturbation of the original economy, at the cost of relaxing the notion of Nash equilibrium. In this result core allocations are implemented as approximate Nash equilibria (see [15]).

THEOREM 2. *Let $\mathcal{E} : H \to T$ be an economy where $T \subset \mathcal{P}^2_{cmb} \times [\frac{1}{s}, s]^L$ is compact, and $\hat{x} \in \mathcal{C}(\mathcal{E})$. Given $\epsilon > 0$ there is a number N such that if $\#H > N$, there is $\hat{p} \in \Delta^L$ and $(\hat{b}, \hat{q}) \in \mathbf{N}^{\epsilon}_{(\hat{p},\hat{x})}(\mathcal{E})$ such that $\hat{x} = \hat{x}(\hat{b}, \hat{q}; \hat{p}, \hat{x})$ and $\pi(\hat{b}, \hat{q}) = \hat{p}$.*

Proof: Let $\hat{p} \in \Delta^L$, $(\eta_h)_{h\in H}$, r, $\bar{r}$, $\delta > 0$ and $(\hat{b}, \hat{q}) \in \prod_{h\in H} S_h$ be as in the proof of theorem (1). Consider $\epsilon > 0$ and choose $N > 1 + \frac{4r^2\bar{r}L}{\delta\epsilon}$, so that if $\#H > N$ then for each $h \in H$ we have $|\eta_h x| < \frac{\delta\epsilon}{2}$ for all $x \in [0, \bar{r}]^L$.

Suppose by way of contradiction that $\hat{x}_h \notin \mathcal{N}^{\epsilon}_{(\hat{p},\hat{x})}(\mathcal{E})$. Then for some $h \in H$ there must be: $y \in \gamma_h \cap \{z \in \Re^L_+ : z \ominus \epsilon\mathbf{1}_L \succ_h \hat{x}_h\} \neq \emptyset$. Since $y \ominus \epsilon\mathbf{1}_L \succ_h \hat{x}_h$, there must be some $i = 1, 2, \ldots, L$, say $i = 1$, so that $y^1 > \epsilon$. In this case we have

$$\begin{aligned} \hat{p}(y \ominus \epsilon\mathbf{1}_L) &= \sum_{i=1}^{L} \hat{p}^i \max\{y^i - \epsilon, 0\} \\ &\leq \hat{p}y - \hat{p}^1\epsilon \\ &< \hat{p}y - \delta\epsilon \end{aligned} \tag{16}$$

Furthermore, since $y \ominus \epsilon\mathbf{1}_L \succ_h \hat{x}_h$ we have that

$$\min\{\hat{p}z : z \succeq_h y \ominus \epsilon\mathbf{1}_L\} > \min\{\hat{p}z : z \succeq_h \hat{x}_h\} = \hat{p}\hat{x}_h \tag{17}$$

Therefore, $\hat{p}(y - \delta\epsilon) > \hat{p}(y \ominus \epsilon\mathbf{1}_L) \geq \min\{\hat{p}z : z \succeq_h y \ominus \epsilon\mathbf{1}_L\} > \hat{p}\hat{x}_h$ so

$$\hat{p}y > \hat{p}\hat{x}_h + \delta\epsilon > \hat{p}\hat{x}_h + \eta_h\hat{x}_h - \eta_h y$$

But then $(\hat{p} + \eta_h)y > (\hat{p} + \eta_h)\hat{x}_h$, which contradicts $y \in \gamma_h$. $\square$

Remark 1. *Notice the nice conceptual implication of theorem (2): deviations from the strategies which implement the core allocation, cannot not provide substantial improvements in payoff (improvements can be only up to ϵ). In particular, the best response of each individual can, at best, be marginally more advantageous than the strategy which implements the core allocation. Therefore, there can be only meager benefits from manipulating the price that supports a core allocation. Of course, the same observations hold for Walrasian allocations since they are elements of the core.*

In the preceding theorems we used the supporting prices of core allocations. By appealing to a theorem in [1] we can obtain a result which is akin to the one appearing in that paper. The assumptions on characteristics can be somewhat relaxed in this case thanks to the generality of the core equivalence theorem that we appeal to. In fact, we can consider the set $\mathcal{P}_{mb}$ of preferences which are weakly monotonic and satisfy assumption (2).

Theorem 3. *Let $\mathcal{E} : H \to T$ be an economy where $T \subset \mathcal{P}_{mb} \times [\frac{1}{s}, s]^L$ and $\hat{x} \in \mathcal{C}(\mathcal{E})$. There is $\hat{p} \in \Delta^L$, $(\hat{b}, \hat{q}) \in \prod_{h\in H} S_h$ and $K > 0$ such that:*

(i) $\hat{x} = \hat{x}(\hat{b}, \hat{q}; \hat{p}, \hat{x})$ and $\pi(\hat{b}, \hat{q}) = \hat{p}$.

(ii) $\sum_{h\in H} |\hat{p}(\hat{x}_h - e_h)| < K$

(iii) $\sum_{h\in H} |\min\{\hat{p}z : z \succeq_h y_h\} - \min\{\hat{p}z : z \succeq_h \hat{x}_h\}| < K$

for all $y \in \prod_{h\in H}(\gamma_h \cap \{z \in \Re^L_+ : z \succeq_h \hat{x}_h\})$

Proof: Let $\hat{p} \in \Delta^L$ be the price vector associated with $\hat{x}$ in Anderson's theorem, i.e.,

$$\sum_{h\in H} |\hat{p}\hat{x}_h - \min\{\hat{p}z : z \succeq_h \hat{x}_h\}| \leq M \tag{18}$$

and

$$\sum_{h\in H} |\hat{p}(\hat{x}_h - e_h)| \leq M \tag{19}$$

where $M = 2sL^{3/2}$. By continuity it can be assumed that $\hat{p} \gg 0$. Consider $R > 0$ such that $\mathcal{C}(\mathcal{E}) \subseteq [\frac{1}{R}, R]^L$. Let $(\eta_h)_{h\in H}$ and $(\hat{b}, \hat{q}) \in \prod_{h\in H} S_h$ be as in the proof of theorem (1). For this profile of strategies we have that claim (i) of the theorem is true. Finally, take $\bar{R} > 0$ such that $\gamma_h \subset [0, \bar{R}]^L$ for each $h \in H$.

Fix an $h \in H$ and let $y_h \in \gamma_h$. Since $|\eta_h x| \leq \parallel \eta_h \parallel\parallel x \parallel < \frac{R(R+s)\bar{R}L}{\#H-1}$ for all $x \in [0, \bar{R}]^L$, we have $\eta_h(\hat{x}_h - y_h) \leq \frac{2R(R+s)\bar{R}L}{\#H-1}$.

Suppose that for some $h \in H$ we have $\min\{\hat{p}z : z \succeq_h y_h\} > \hat{p}\hat{x}_h + \frac{2R(R+s)\bar{R}L}{\#H-1}$.

It follows that $\min\{\hat{p}z : z \succeq_h y_h\} > \hat{p}\hat{x}_h + \eta_h(\hat{x}_h - y_h)$.

However, in this case we have $\hat{p}y_h \geq \min\{\hat{p}z : z \succeq_h y_h\} > \hat{p}\hat{x}_h + \eta_h(\hat{x}_h - y_h)$ which implies $(\hat{p} + \eta_h)y_h > (\hat{p} + \eta_h)\hat{x}_h$ contradicting $y_h \in \gamma_h$. Therefore,

$$y_h \in \gamma_h \Rightarrow \min\{\hat{p}z : z \succeq_h y_h\} \leq \hat{p}\hat{x}_h + \frac{2R(R+s)\bar{R}L}{\#H-1}$$

Hence, whenever $y_h \in \gamma_h \cap \{z \in \Re^L_+ : z \succeq_h \hat{x}_h\}$ we have for each $h \in H$

$$\begin{aligned} 0 &\leq \min\{\hat{p}z : z \succeq_h y_h\} - \min\{\hat{p}z : z \succeq_h \hat{x}_h\} \\ &\leq \hat{p}\hat{x}_h - \min\{\hat{p}z : z \succeq_h \hat{x}_h\} + \frac{2R(R+s)\bar{R}L}{\#H-1} \end{aligned} \tag{20}$$

Summing up over all individuals and using (18) along with $\#H \geq 2$ we have that for all $y \in \prod_{h\in H}(\gamma_h \cap \{z \in \Re^L_+ : z \succeq_h \hat{x}_h\})$:

$$\text{(21)} \qquad \sum_{h\in H} |\min\{\hat{p}z : z \succeq_h y\} - \min\{\hat{p}z : z \succeq_h \hat{x}_h\}| \leq M + 4R(R+s)\bar{R}L$$

Taking $K = M + 4R(R+s)\bar{R}L$ proves claim (iii) of the theorem. Finally by (19) and $M < K$ we have $\sum_{h\in H} |\hat{p}(\hat{x}_h - e_h)| < K$ which proves claim (ii) of the theorem. $\square$

The interpretation of theorem (3) is that, if the sets of core and attainable market game allocations remain bounded as the number of individuals becomes large the following are true: the average transfer needed to implement a core allocation via a market game converges to zero. Also the average payoff gains over the core allocation, which are attainable via the market game, converge to zero. The last conclusion is especially true for the 'best response' bundle of each individual, to the strategies which implement the core allocation, as the following corollary states.

Corollary 2. *Let $\mathcal{E} : H \to T$ be an economy, where $T \subset \mathcal{P}_{mb} \times [\frac{1}{s}, s]^L$, and $\hat{x} \in \mathcal{C}(\mathcal{E})$. There is $\hat{p} \in \Delta^L$, $(\hat{b}, \hat{q}) \in \prod_{h\in H} S_h$ and $K > 0$ such that:*

(i) $\hat{x} = \hat{x}(\hat{b}, \hat{q}; \hat{p}, \hat{x})$ and $\pi(\hat{b}, \hat{q}) = \hat{p}$.

(ii) Letting $\bar{y} \in \prod_{h\in H} \gamma_h$ be such that $\gamma_h \cap \{z \in \Re^L_+ : z \succ_h \bar{y}_h\} = \emptyset$ for each $h \in H$, we have $\sum_{h\in H} |\min\{\hat{p}z : z \succeq_h \bar{y}_h\} - \min\{\hat{p}z : z \succeq_h \hat{x}_h\}| < K$.

Since Walrasian equilibrium allocations are in the core, the last corollary applies to them as well. We state this fact as a separate conclusion since it has obvious implications regarding the relationship of Walras allocations to Nash equilibria.

Corollary 3. *Let $\mathcal{E} : H \to T$ be an economy, where $T \subset \mathcal{P}_{mb} \times [\frac{1}{s}, s]^L$, and $(\bar{p}, \bar{x}) \in \Delta^L \times \Re^{LH}_+$ be a Walrasian equilibrium of this economy. There is $(\bar{b}, \bar{q}) \in \prod_{h\in H} S_h$ and $K > 0$ such that:*

(i) $\bar{x} = x(\bar{b}, \bar{q})$ and $\pi(\bar{b}, \bar{q}) = \bar{p}$.

(ii) Letting $\bar{y} \in \prod_{h\in H} \gamma_h$ be such that $\gamma_h \cap \{z \in \Re^L_+ : z \succ_h \bar{y}_h\} = \emptyset$ for each $h \in H$, we have $\sum_{h\in H} |\min\{\bar{p}z : z \succeq_h \bar{y}_h\} - \bar{p}\bar{x}_h| < K$.

Notice that no claim of a Nash property is mentioned in this corollary. Nevertheless, it roughly states that the strategies which implement the Walrasian allocation via the strategic market game are asymptotically best responses to each other, i.e., the Walrasian allocation is asymptotically a Nash equilibrium.

For theorem (3) we have appealed to [1] because it is the most basic approximation theorem for the core that we are aware of. Another interesting result to extend to the non Walrasian framework along the same lines is the one appearing in [3]. An extension of that result would establish that in large finite economies given a core allocation there is a Nash equilibrium of a strategic market game with small transfers, with the property that almost all individuals are indifferent between their core and Nash equilibrium allocations. It should be emphasized that in all cases the uniform boundedness of core allocations, which in turn is due to the compactness of characteristics, is crucial for the proximity between core and Nash properties of equilibrium allocations. This fact is not merely a technical observation. It has some economic content because it suggests that lack of individual ability to exercise market power in a market game, does not imply nor it is implied by inability of coalitions to do so in the core. This is the case only when individual characteristics are not too diverse. Finally, with stronger conditions on the characteristics and the sequences of economies the approximation conclusions in claims (ii) and (iii) in theorem (3) can be strengthened to maximum rather than average deviations by appealing to stronger core convergence theorems (for example [11], proposition 7.4.9, p.285).

We finally turn to a result in the converse direction relating Nash equilibria to the core property.

THEOREM 4. *Consider a sequence of economies* $\{\mathcal{E}_n\}_{n\in N}$*, where* $\mathcal{E}_n : H_n \to T$*,* $\#H_n \to \infty$ *and* $T \subset \mathcal{P}_{cm} \times [0,s]^L$ *is compact. Let* $(b_n, q_n) \in \mathbf{N}(\mathcal{E}_n)$ *and suppose that for* n *large enough* $\pi_n = \pi(b_n, q_n) \gg \alpha \mathbf{1}_L$ *for some* $\alpha > 0$*, so that the corresponding* $x_n \in \mathcal{N}(\mathcal{E}_n)$ *is fully active. Then given any* $\epsilon > 0$ *we have* $x_n \in \mathcal{C}^\epsilon(\mathcal{E}_n)$ *for all* n *large enough.*

Proof: Let us fix $x_n \in \mathcal{N}(\mathcal{E}_n)$ and recall a few useful facts.

By the strict monotonicity of preferences, for each $h \in H_n$, $x_{n,h}$ lies on the boundary of the convex set $\gamma_{n,h}$ and since preferences are also convex, the vector $p_{n,h} = Dg(x_{n,h}) = (\pi^i \frac{Q^i_{n,-h}}{Q^i_{n,-h} - x^i_{n,h} + e^i_h})_{i=1}^L$ has the following separating property:

$$\forall z \in \gamma_{n,h},\ p_{n,h} z \leq p_{n,h} x_{n,h} \text{ and } \forall z \succ_h x_{n,h},\ p_{n,h} z > p_{n,h} x_{n,h}$$

Since T is compact by proposition (1) in [9] (see also [10], proposition 1, p.189), $\mathcal{N}(\mathcal{E}_n)$ is uniformly bounded so for some $c > 0$ we have $\| x_{n,h} - e_h \| \leq c1_L$, for each $h \in H_n$.

Furthermore, since $\pi_n \gg \alpha \mathbf{1}_L$ then for n large enough $\frac{1}{\# H_n} Q_n \gg \beta \mathbf{1}_L$ for some $\beta > 0$. Thus, by passing to a subsequence if necessary we may assume that there is $\xi > 0$ so that $\#\{h \in H_n : q_h^i \geq \xi\} \geq \# H_n \xi$ for all $i = 1, 2, \ldots, L$. In this case, we have that for all $h \in H_n$, $Q_{n,-h}^i = Q_n^i - q_{n,h}^i \geq Q_n^i - s \geq \# H_n \xi^2 - s$.

Therefore, $\| p_{n,h} - \pi_n \| \leq L \max\{\frac{x_{n,h}^i - e_h^i}{Q_{n,-h}^i - x_{n,h}^i + e_h^i} : i = 1, 2, \ldots, L\} \leq \frac{Lc}{\# H_n \xi^2 - s - c}$, for each $h \in H_n$, for n large enough. Hence, given any $\theta > 0$ by choosing $\# H_n > \frac{Lc + (s+c)\theta}{\theta \xi^2}$ we can ensure that: $\| p_{n,h} - \pi_n \| \leq \theta$, $\forall h \in H_n$. Take $\theta = \frac{\alpha \epsilon}{\epsilon + 2Ls}$ and choose N so that if $\# H_n > N$ then $\| p_{n,h} - \pi_n \| < \theta$ for each $h \in H_n$.

We can now start the proof of our theorem. Suppose $x_n \notin \mathcal{C}^\epsilon(\mathcal{E}_n)$. Since $x_n \in F_n$, the following must be true:

$$\text{(22)} \quad \exists S \subset H_n \text{ and } y \in \Re_+^{LS} \text{ so that } \sum_{h \in S} y_h = \sum_{h \in S} e_h \text{ and } y_h \ominus \epsilon \mathbf{1}_L \succ_h x_{n,h}, \forall h \in S.$$

Fix any $h \in S$. By the separating property of the vector $p_{n,h}$ we have that $p_{n,h}(y_h \ominus \epsilon \mathbf{1}_L) > p_{n,h} x_{n,h}$. Furthermore, since $e_h \in \gamma_{n,h}$ we have $p_{n,h} e_h \leq p_{n,h} \hat{x}_h$ so we conclude that $p_{n,h}(y_h \ominus \epsilon \mathbf{1}_L) > p_{n,h} e_h$.

Since $y_h \ominus \epsilon \mathbf{1}_L \succ_h \hat{x}_h$, by renumbering if necessary we may assume $y_h^1 > \epsilon$. Hence, we have $p_{n,h}(y_h \ominus \epsilon \mathbf{1}_L) = \sum_{i=1}^L p_{n,h}^i \max\{y_h^i - \epsilon, 0\} \leq p_{n,h} y - p_{n,h}^1 \epsilon$ so, it follows that $p_{n,h} y_h > p_{n,h} e_h + p_{n,h}^1 \epsilon$. In this case $(\pi_n + \theta \mathbf{1}_L) y_h > (\pi_n - \theta \mathbf{1}_L) e_h + (\pi_n^1 - \theta)\epsilon$. Therefore, it follows that:

$$\begin{aligned}
(\pi_n + \theta \mathbf{1}_L) y_h &> (\pi_n - \theta \mathbf{1}_L) e_h + (\pi_n^1 - \theta)\epsilon \\
&> (\pi_n - \theta \mathbf{1}_L) e_h + (\alpha - \theta)\epsilon \\
&= \pi_n e_h + \alpha\epsilon - \theta(\epsilon + \mathbf{1}_L e_h) \\
&= \pi_n e_h + \theta(\epsilon + 2Ls) - \theta(\epsilon + \mathbf{1}_L e_h) \\
&= \pi_n e_h + \theta(2Ls - \mathbf{1}_L e_h) \qquad (23)
\end{aligned}$$

where the fourth equality is due to the choice of θ. Since $e_h \leq \mathbf{1}_L s$ we have that $Ls = \mathbf{1}_L \mathbf{1}_L s \geq \mathbf{1}_L e_h$. Therefore, from (23) it further follows that

$$\begin{aligned}
(\pi_n + \theta \mathbf{1}_L) y_h &> \pi_n e_h + \theta(2Ls - \mathbf{1}_L e_h) \\
&\geq \pi_n e_h + \theta \mathbf{1}_L e_h \\
&= (\pi_n + \theta \mathbf{1}_L) e_h \qquad (24)
\end{aligned}$$

Since (24) is true for all $h \in S$, by summing up we obtain:

$$(\pi_n + \theta \mathbf{1}_L) \sum_{h \in S} y_h > (\pi_n + \theta \mathbf{1}_L) \sum_{h \in S} e_h \tag{25}$$

However, $\sum_{h \in S} y_h = \sum_{h \in S} e_h$ and since $\pi_n + \theta \mathbf{1}_L \neq 0$ we have that (25) implies

$$(\pi_n + \theta \mathbf{1}_L) \sum_{h \in S} e_h = (\pi_n + \theta \mathbf{1}_L) \sum_{h \in S} y_h > (\pi_n + \theta \mathbf{1}_L) \sum_{h \in S} e_h$$

which is a contradiction. □

4. Concluding Remarks

We have attempted here to articulate some results that relate the core to Nash equilibria of strategic market games by extending core equivalence results to the non Walrasian context. In this way the strategic prices and income transfers which decentralize a core allocation as an approximate Nash equilibrium are exactly the same as those which decentralize it as a Walrasian equilibrium. In other words the Walrasian prices which decentralize the core have some approximate Nash property: unilateral deviations from the profile of strategies which implements those prices and associated core allocations cannot provide substantial benefits. We conclude then that in large economies individuals cannot obtain substantial benefits from manipulating the prices that implement a core allocation, which complements and confirms the argument in favor of price taking in such environments.

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