Endogenous Commitment and Nash Equilibria in Competitive Markets with Adverse Selection

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Abstract

In this paper, we provide a unified framework for analyzing competitive markets with adverse selection. The key feature of our model is that whether the firms (uninformed) are committed to the contracts they offer or not is determined endogenously. Due to endogenous commitment, our model always has a unique Bayes-Nash equilibrium without using any refinement to restrict beliefs off-the-equilibrium path. We show that both the non-existence problem in the two-stage screening games and the multiplicity of equilibria in the three-stage screening and the signalling games are due to exogenous full commitment (non-existence) and exogenous lack of commitment (multiplicity).

Key Words: Adverse selection, Endogenous Commitment

JEL classification: D82, G22

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1. Introduction

In his celebrated paper, Akerlof (1970) showed that the introduction of asymmetric information (hidden types) into the standard Walrasian framework may lead to inefficiently low trading or even to a market collapse. Following this, then, striking result, a number of papers have appeared putting forward ways to mitigate this adverse selection problem. Spence (1973) suggested that a signal (education) whose cost is inversely related to the worker's ability could be used to credibly convey information about the worker's productivity to potential employers. Rothschild and Stiglizt (1976) argued that insurers can distinguish high risks from low risks by offering contracts specifying both the price and the quantity (non-linear pricing). The low risks choose less than full insurance to prevent the high risks from mimicking them. For some parameter values, the utility cost of underinsurance for the low risks is more than offset by the lower per unit premium they are charged for the coverage they purchase and a separating equilibrium arises.

Although the suggested solutions work towards mitigating the adverse selection problem, both signaling (Spence (1973)) and screening (Rothschild and Stiglizt (1976)) models have some undesirable properties.[1] Signaling models have many equilibria which are, in general, inefficient.[2] Many authors have developed refinements of the Bayes-Nash equilibrium which restrict beliefs off-the-equilibrium path and reduce the set of equilibria.[3] However, these refinements, in general, do not lead to a unique equilibrium and cannot eliminate Pareto-dominated equilibria. Furthermore, different refinement criteria lead to different sets of equilibria and there is no agreement on which of these criteria is the most appropriate.[4]

In the screening model of Rothschild and Stiglizt (1976) the key issue is the possibility of non-existence of a Nash equilibrium. More specifically, if some feasible pooling allocations Pareto-dominate the Rothschild-Stiglitz separating one, then no Nash equilibrium exists in their model.

[1] For an extensive survey of signalling and screening models see Riley (2001).
[2] Even in the sense of Weak Interim Efficiency (see Maskin and Tirole (1992) for details).
[3] For a discussion of the different refinements see Fudenberg and Tirole (1991) and Bolton and Dewatripont (2005).
[4] In applied theory, the most widely used refinement is the "intuitive criterion" of Cho and Kreps (1987).

In this paper, we provide a unified framework for analyzing competitive markets with adverse selection. For expositional purposes, we consider the insurance example of Rothschild and Stiglitz (1976) and Wilson (1977) with two types of insurees: low risks (good type), high risks (bad type). The key feature of our model is that whether the firms (uninformed insurers) are committed to the contracts they offer or not is determined endogenously.[5] That is, the contracts offered at Stage 1 are triples specifying: price, quantity and whether the firm is committed to the contract or not.[6] Our approach remedies both the non-existence problem in the Rothschild-Stiglitz screening model and the multiplicity of equilibria issue arising in signalling models and in the three-stage screening game of Hellwig (1987).[7] Because of the endogeneity of commitment, our model always (for all parameter values) has a unique Bayes-Nash equilibrium even though we do not use any refinement to restrict beliefs off-the-equilibrium path.

In particular, we show that when some feasible pooling allocations Pareto-dominate the Rothschild-Stiglitz separating allocation then the Wilson pooling contract *without commitment* is the unique equilibrium.[8] Otherwise, the Rothschild-Stiglitz separating allocation *with or without commitment* is the unique equilibrium.[9] Therefore, the unique equilibrium of our game, depending on parameter values, is either separating or pooling. Furthermore, this equilibrium is weakly interim efficient.[10]

Intuitively, the ability of the supplier of the contract (uninformed) to commit to a contract makes the decision of the insurees of whether to take it independent of their

[5] It should be stressed here that the endogeneity of commitment refers to the commitment within the game and until an equilibrium has been achieved. Once an equilibrium has been reached and the two parties have signed the contract, both parties are fully committed to it.
[6] More specifically, we consider a three-stage game where at Stage 1 firms offer contracts and specify which of them they are committed to and which not; at Stage 2, insurees apply for (at most) one contract from one firm; and at Stage 3, the firms decide which of the contracts they offered at Stage 1 without commitment will be withdrawn and which not. Of course, after Stage 3, provided the firms has not withdrawn the contract and the contract is signed by both parties, the firm becomes committed to it.
[7] The difference between our game and Hellwig (1987) is that in Hellwig, at Stage 1, the firms cannot choose whether to commit to (some of) the contracts they offer or not. They are exogenously imposed to offer contracts without commitment.
[8] The Wilson pooling contract is the zero-profit pooling contract which maximizes the expected utility of low risks (good type).
[9] Strictly speaking, in the knife-edge case where the (low risks) good type are indifferent between the Rothschild-Stiglitz separating and the Wilson pooling allocation, then both are equilibria. This knife-edge case disappears if firms are allowed to offer more than one (menus of) contracts.
[10] Because we restrict firms to have zero profits on a contract-by-contract basis, in equilibrium, the appropriate concept of efficiency is Weak Interim Efficiency.

beliefs about who else takes it. As a result, if some feasible pooling allocations Pareto-dominate the Rothschild-Stiglitz separating one, it is possible for an insurer to offer a pooling contract with commitment that profitably attracts both types. Also, for any zero-profit pooling contract, but the Wilson one, a firm can find a deviant contract with commitment which profitably attracts either the low risks or both types profitably. Finally, the ability of a firm to offer a contract without commitment (on-the-equilibrium path) acts a threat for a potential entrant and supports the Wilson pooling contract as an equilibrium.

Therefore, our analysis makes clear that both the non-existence problem in the Rothschild-Stiglitz screening model and the multiplicity of equilibria in the signalling game and the three-stage game of Hellwig are due to the arbitrary restrictions imposed on the insurers' strategy (contract) space. The informational friction (hidden types) does not lead to these problems if an appropriately designed game is used. More specifically, we show that i) the non-existence problem in the Rothschild-Stiglitz model is due to the exogenously imposed commitment to the contracts offered at Stage 1. ii) The multiplicity of equilibria in signalling games and the three-stage game of Hellwig arises because the contracts (signals) available to informed at Stage 2 are without commitment and so their choice depends on their beliefs about the inferences the uninformed will make. Because off-the-equilibrium path these beliefs are arbitrary, one can find beliefs which support many zero-profit allocations as equilibria.

The above results can be better understood if one observes that our model has as special cases i) the Rothschild-Stiglitz model, ii) the Hellwig's three-stage game and iii) the signalling games. The Rothschild-Stiglitz model obtains if the firms are exogenously committed to the contracts they offer at Stage 1. The Hellwig game corresponds to the case where the firms cannot commit to the contracts they offer at Stage 1. The two-stage signalling model can be thought of as the case where the firms, at Stage 1, offer all possible contracts without commitment even if they expect that some of them are loss-making.

In sum, by endogenizing commitment, we make several interesting points: First, we provide an approach to modelling competition in markets under adverse selection which has as special cases all existing approaches. Second, we uncover the driving force of both the non-existence problem in the Rothschild-Stiglitz screening model and the multiplicity of equilibria in the signalling game and the three-stage game of Hellwig. We make clear that these two problems stem from arbitrary restrictions imposed on the firms' strategy (contract) space and are not due to the informational friction (hidden types). Third, our game has always a unique Nash equilibrium even though we do not use any refinement to restrict beliefs off-the-equilibrium path. That is, our model offers the highest possible predictive ability (unique equilibrium) while avoiding the difficulties associated with the equilibrium refinement criteria. This feature is very important for applied theory papers where usually the uniqueness of the equilibrium is crucial for the model's empirical relevance and testability. Fourth, the equilibrium is incentive efficient. Finally, on-the-equilibrium path, firms optimally choose not to commit to the contracts they offer at Stage 1. This prediction is consistent with the widely observed fact that banks, insurance companies and many non-financial firms keep the right to reject applications for (some of) the contracts they offer.

2. Related Literature

A number of solutions have been suggested for the non-existence problem. First, Wilson (1977) proposes a non-Nash-type equilibrium concept (**Anticipatory Equilibrium**) under which an equilibrium always exists. If some feasible (no loss-making) pooling allocations Pareto-dominate the Rothschild-Stiglitz separating allocation, the zero-profit pooling allocation which maximizes the expected utility of the low risk is the unique "Wilson equilibrium". The key in Wilson's solution is that a firm may withdraw a contract if the introduction of a new contract by another firm makes the original contract loss-making. Anticipating this reaction, the new entrant will introduce his contract only if the potential withdrawal of the original contract does not render his contract loss-making. Riley (1979) puts forward another non-Nash-type equilibrium concept (**Reactive Equilibrium**). Instead of withdrawing a contract as in Wilson, in Riley a firm can always add a contract. Riley shows that an equilibrium always (for all parameter values) exists and it is separating.[11]

[11] Engers and Fernandez (1987) provide game-theoretic foundations to Riley equilibrium.

Subsequently, Hellwig (1987) proposed a three-stage game which provides game-theoretic foundations to Wilson pooling equilibrium.[12] [13] He argues that if some feasible pooling allocations Pareto-dominate the Rothschild-Stiglitz separating one, the Wilson pooling contract is the unique "reasonable" equilibrium.[14] Otherwise, the Rothschild-Stiglitz separating allocation is the unique equilibrium. However, as we show in this paper, in Hellwig's game the Rothschild-Stiglitz separating allocation is always (for all parameter values) a "reasonable" equilibrium. Thus, if some feasible pooling allocations Pareto-dominate the separating allocation of Rothschild and Stiglitz, then both the Wilson pooling contract and the Rothschild-Stiglitz separating one are "reasonable" equilibria in Hellwig's game.[15]

Although the application of the "intuitive criterion" reduces the set of equilibria considerably, it does not guarantee the uniqueness of the equilibrium or that the equilibrium allocation is not Pareto-dominated by another feasible allocation.[16] In the two-stage signalling game the Rothschild-Stiglitz separating allocation is the unique "reasonable" equilibrium even if it is Pareto-dominated by some feasible pooling allocations. In Hellwig's three-stage game, exactly because insurers cannot commit to the contracts they offer at Stage 1, both the separating allocation of Rothschild and Stiglitz and the pooling allocation of Wilson are "reasonable" Nash equilibria when the latter Pareto-dominates the former. This observation is important for papers in Macroeconomics and other fields of Economics where the uniqueness of the equilibrium (and so the predictive power of the model) is crucial for their empirical relevance.[17]

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[12] Hellwig (1987) formalizes the argument of Grossman (1979).

[13] More specifically, at Stage 1 firms offer contracts (without committing to them); at Stage 2, insurees apply for (at most) one contract from one firm; and at Stage 3, the firms decide which of the contracts they offered at Stage 1 will be withdrawn and which not.

[14] It is "reasonable" in the sense that it survives the "intuitive criterion" of Cho and Kreps (1987).

[15] Another way to restore the existence of equilibrium would be to use mixed strategies. However, in this case, there are multiple equilibria (see Dasgupta and Maskin (1986a,b) for details).

[16] The issues of the existence of equilibrium and its welfare properties in economies with adverse selection have also been studied in the context of the Walrasian mechanism. It has been shown that the equilibrium, if it exists, is inefficient. Some key papers in this literature are: Prescott and Townsend (1984), Gale (1992, 1996), Dubey and Geanakoplos (2002), and Dubey, Geanakoplos and Shubik (2005).

[17] In fact, many authors have used the game of Hellwig (1987) and have based their results on the uniqueness of the Wilson pooling equilibrium when it Pareto-dominates the Rothschild-Stiglitz separating one (see, for example, Martin (2009) and Reichlin and Siconolfi (2004)). Our paper shows

3. The Model

We consider the basic framework introduced by Rothschild and Stiglitz (1976). There is a continuum of individuals (insurees) and a single consumption good. There are two possible states of nature: good and bad. In the good state there is no loss whereas in the bad state the individual suffers a gross loss of d. Before the realization of the state of nature all individuals have the same wealth level, W. Also, all individuals have the same twice continuously differentiable utility function U with $U' > 0$ and $U'' < 0$, that is individuals are risk averse. Individuals differ only with respect to the probability of having the bad state (accident), p. There are two types of individuals: high risks (H-type) and low risks (L-type) with $1 > p_H > p_L > 0$. Let $\lambda \in (0,1)$ be the fraction of the low risks in the economy.

In this environment, the expected utility of an agent i is given by:

$$EU(W, d, \alpha_1, \alpha_2) = p_i U(W - d - \alpha_1 + \alpha_2) + (1 - p_i) U(W - \alpha_1), \quad i = H, L \qquad (1)$$

where W: insuree's initial wealth

d : gross loss

α_1: insurance premium

$\alpha_2 - \alpha_1$: net payout in the event of loss

α_2 : coverage (gross payout in the event of loss)

where $W - \alpha_1$, $W - d - \alpha_1 + \alpha_2$ are the wealth levels in the good and the bad state respectively.

There are (at least) two risk neutral insurance companies involved in Bertrand competition. Insurance companies cannot observe the true accident probabilities. They, however, know the utility function of the insurees and the proportion of the Hs and Ls in the population. The insurance contract (α_1, α_2) specifies the premium α_1 and the coverage α_2. As a result, the expected profit of an insurer offering such a contract is:

$$\pi_i = \alpha_1 - p_i \alpha_2, \quad i = H, L \qquad (2)$$

that their results are valid only if the commitment to the contract is part of the equilibrium (it is chosen endogenously).

Equilibrium

Insurance companies and insurees play the following three-stage screening game:

Stage 1: At least the two insurance companies simultaneously make offers of contracts. Each insurance company may offer only one contract (but any number of copies). The insurers also specify whether they are committed or not to the contract they offer.

Stage 2: Insurees apply for (at most) one of the contracts offered from one insurance company. If an insuree's most preferred contract is offered by more than one insurance company, he takes each insurer's contract with equal probability.

Stage 3: After observing the contracts offered by his rivals and those chosen by the insurees, the insurers who did not commit to their contract at Stage 1 decide whether to withdraw or not. If a contract is withdrawn, the insurees who have chosen it go to their endowment.

We only consider pure-strategy Bayesian-Nash equilibria. A set of contracts is an equilibrium if the following conditions are satisfied:

- No contract in this set makes negative expected profits.[18]
- No other set of contracts introduced alongside those already in the market would increase an insurer's expected profits.

Although in the Rothschild-Stiglitz paper there is no formal game, their results can be derived if we assume that all insurers who offer contracts at Stage 1 are exogenously committed to them. In this case, our game ends at Stage 2. The game in Hellwig (1987) is a special case of our game which obtains if we assume that insurers cannot commit to the contracts they offer at Stage 1. Finally, the two-stage signalling game can be thought of as the case where insurers, at Stage 1, make available all possible contracts without commitment although they know that some of them are loss-making.

[18] This implies that, in equilibrium, insurers are making zero profits on a contract-by-contract basis. As a result, the definition of optimality we use is *Weak Interim Efficiency* (see Maskin and Tirole (1992) for more details).

We begin by establishing the possible equilibria of our game. Our game always has a unique Bayesian-Nash equilibrium even though we do not use any refinement to restrict beliefs off-the-equilibrium path. The Wilson pooling allocation is the unique equilibrium when it Pareto-dominates the Rothschild-Stiglitz separating one. Otherwise, the latter is the unique equilibrium.

Lemma 1: If the Rothschild-Stiglitz separating allocation is not Pareto dominated by any feasible (no loss-making) pooling allocation, then it is the unique equilibrium (with or without commitment).

Proof: In this case, graphically, the indifference curve of low risks through their Rothschild-Stiglitz separating contract passes above the pooling zero-profit line (PZP) (see Figure 1). We first show that the Rothschild-Stiglitz separating allocation (C_H^{RS}, C_L^{RS}) is an equilibrium. Bertrand competition implies that in any separating equilibrium each contract must lie on the corresponding zero-profit line.

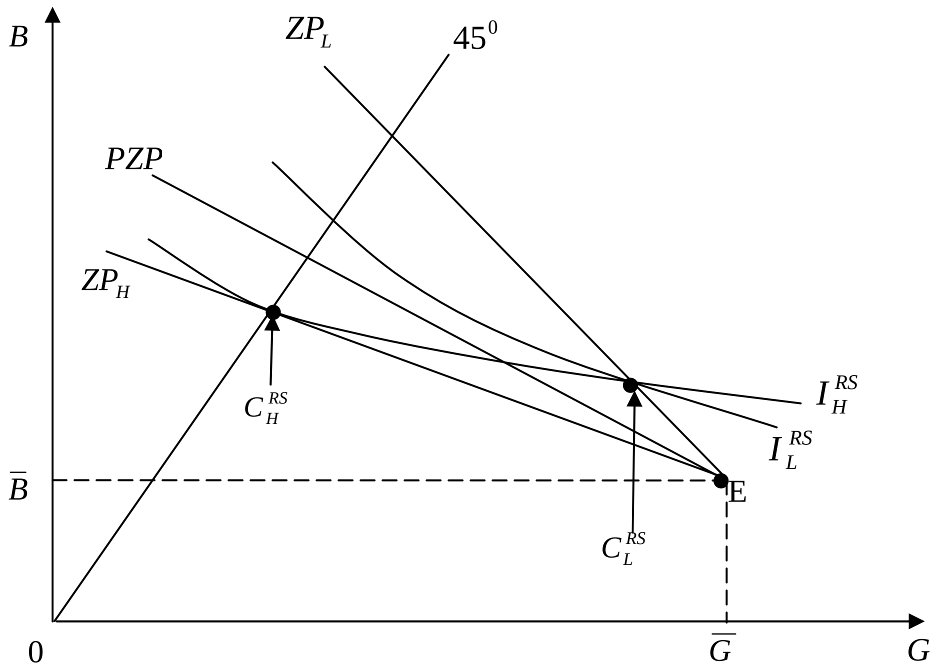

Figure 1

Given that (C_H^{RS}, C_L^{RS}) is offered, there is no other zero-profit contract which is preferred to C_H^{RS} by the high risks. Also, there is no contract below the zero-profit line of low risks, ZP_L, which is preferred by the low risks and not by the high risks. Finally, there is no pooling contract which lies below the pooling zero-profit line and is preferred by the low risks to their separating one, C_L^{RS}. Thus, there is no profitable deviation and the Rothschild-Stiglitz separating allocation, C_H^{RS}, C_L^{RS}, is an equilibrium.

We also have to show that no other separating allocation can be an equilibrium. In any separating equilibrium the incentive compatibility constraint of the low risks is not binding (the low risks strictly prefer their own contract to the high risks' contract). Competition, then, implies that no zero-profit contract on ZP_H, but C_H^{RS} (full insurance) can be an equilibrium contract for the high risks. Given that, competition and incentive compatibility imply that the low risks will be offered C_L^{RS}. Thus, only (C_H^{RS}, C_L^{RS}) can be an equilibrium. Therefore, (C_H^{RS}, C_L^{RS}), is the unique equilibrium. *Q.E.D.*

Lemma 2: If some feasible pooling contracts Pareto-dominate the Rothschild-Stiglitz separating allocation then the latter allocation cannot be an equilibrium.

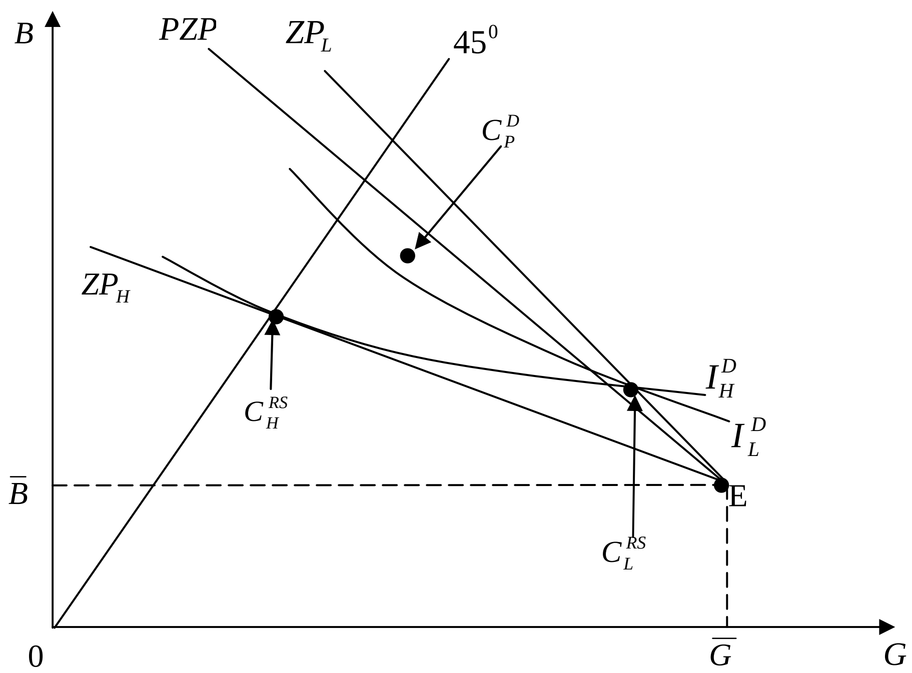

Figure 2

Proof: In this case, the indifference curve of low risks through their Rothschild-Stiglitz separating contract cuts the pooling zero-profit line (PZP) (see Figure 2). Suppose that at Stage 1 the Rothschild-Stiglitz separating allocation, (C_H^{RS}, C_L^{RS}), is offered. Given that at Stage 1 insurers can commit to a contract if they wish, an insurer can offer the deviant pooling contract C_P^D, along with the commitment that he will fulfil his promise at Stage 3 (he will not withdraw it at Stage 3). The deviant pooling contract implies higher utility for both types and because the insurer is committed to it, both types will take it. Also, because this contract lies below the pooling zero-profit line (PZP) it implies a strictly positive profit for the deviant insurer. Therefore, the Rothschild-Stiglitz separating allocation cannot be an equilibrium. *Q.E.D.*

Notice that if the insurer could not commit to the deviant contract, C_P^D, the Rothschild-Stiglitz separating allocation would certainly be an equilibrium for some beliefs. It would also be an equilibrium even if we used the "intuitive criterion" to restrict beliefs off-the-equilibrium path (see Proposition 3 below for more details). The ability of the insurer to commit to the deviant contract makes the decision of the insurees of whether to choose it over the Rothschild-Stiglitz separating allocation independent of their beliefs about who else takes it. Hence, the deviant contract is taken by both types and it is profitable as it lies below the pooling zero-profit line (PZP).

Lemma 3: If some feasible pooling contracts Pareto-dominate the Rothschild-Stiglitz separating allocation, then the Wilson pooling contract *without commitment* is an equilibrium which also passes the "intuitive criterion" (see Figure 3).

Proof: Suppose that at Stage 1 the Wilson pooling contract without commitment, C_P^W, is offered. The potentially profitable contracts are those between the indifference curves of the low and high risk through C_P^W to the right of C_P^W (preferred by the low risks but not the high risks to C_P^W) and below the zero-profit line corresponding to the low risks, ZP_L.

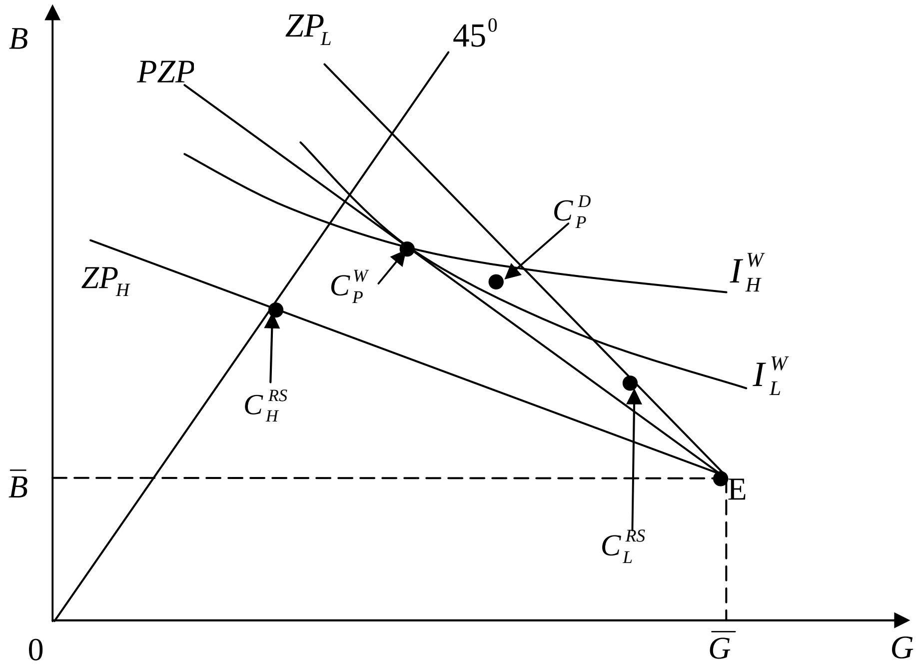

Figure 3

Consider, for example, an insurer offering at Stage 1 the deviant pooling contract C_P^D with or without being committed to it. If the insurer believes that the probability that this contract will be taken by the high risks is equal or greater than the ex ante probability, then this deviant contract is loss-making. Because this belief cannot be ruled out, the Wilson pooling contract is an equilibrium. We will now show that it is also a "reasonable" equilibrium. The reasoning is as follows: Given that the conjectured equilibrium contract C_P^W is offered, the deviant contract, C_P^D, will reasonably attract only the low risks. This, in turn, implies that the pooling contract C_P^W is taken only by the high risks and so it becomes loss-making. As a result, at Stage 3, any application for that contract will be rejected. Anticipating that, the high risks will also choose the deviant contract, C_P^D, at Stage 2, and hence C_P^D becomes also loss-making (since C_P^D lies above the pooling zero-profit line PZP). So, there is no profitable deviation from C_P^W. Thus, C_P^W is a "reasonable" equilibrium. *Q.E.D.*

Lemma 4: If some feasible pooling contracts Pareto-dominate the Rothschild-Stiglitz separating allocation, then the Wilson pooling contract *without commitment* is the unique pooling equilibrium.

Proof: Because of Bertrand competition any equilibrium pooling contract must lie on the pooling zero-profit line, PZP**.** Consider now any point on PZP**,** different from C_P^W, for example, C_P. Through such a point passes an indifference curve of the low risk that cuts PZP (the only tangency point is C_P^W). That is, there is a non-empty set of points between PZP and the indifference curve of the low risk (see Figure 4). Consider now an insurer offering at Stage 1 the deviant pooling contract C_P^D (in Figure 4) with commitment. Given that the deviant insurer is committed to C_P^D, the low risks will certainly take it at Stage 2 regardless of whether the insurer offering C_P is committed to it or not. As a result, the conjectured equilibrium contract C_P becomes loss-making and if it is offered without commitment it will be withdrawn at Stage 3. Thus, if C_P is offered with commitment the high risks will stay there and the deviant contract, C_P^D, is clearly profit-making. If C_P is offered without commitment, at Stage 2 the high risks, anticipating its withdrawal at Stage 3, will take the deviant contract. However, because the deviant contract C_P^D lies below the pooling zero-profit (PZP) is still profitable. So, C_P is not an equilibrium contract.

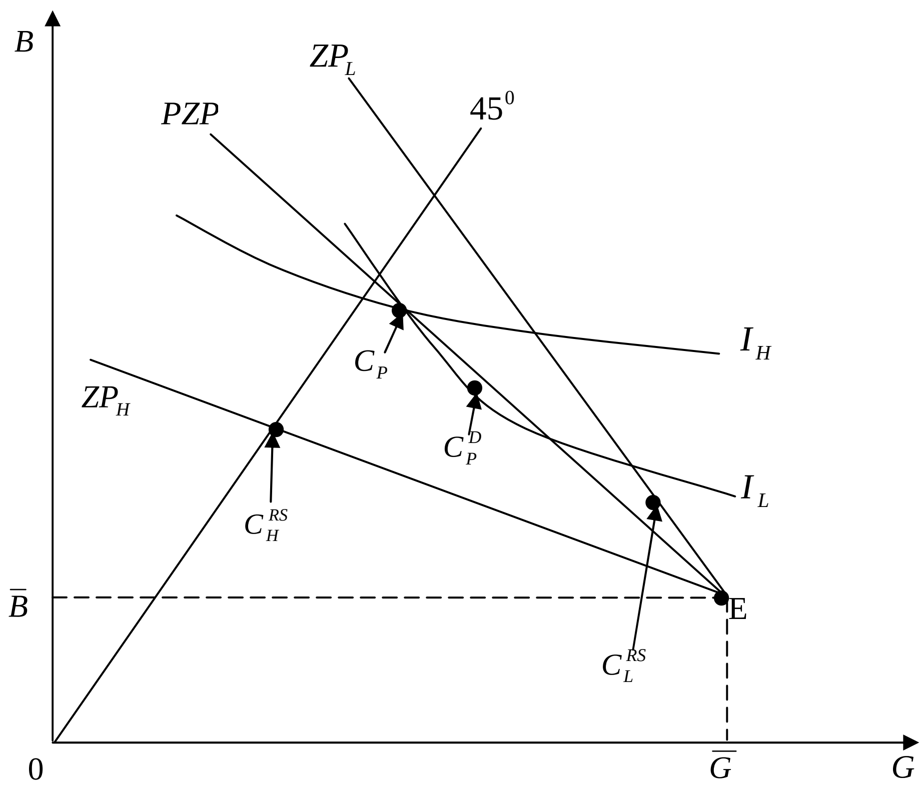

Figure 4

Now we will show that the Wilson pooling contract with commitment at Stage 1 is not an equilibrium. Consider again the Wilson pooling contract C_P^W and the deviant contract C_P^D in Figure 3. Given C_P^W with commitment, the deviant contract will attract only the low risks and so C_P^W will become loss-making. Thus, no rational insurer will offer it at Stage 1. Therefore, the Wilson pooling contract *without commitment* is the unique pooling equilibrium. *Q.E.D.*

That is, the ability of an insurer to offer a contract without commitment, on-the-equilibrium path, acts a threat for a potential entrant and supports the Wilson pooling contract as an equilibrium. Also, the uniqueness of the Wilson pooling equilibrium is due to the endogeneity of the commitment to the contracts offered and not to some refinement which restricts the beliefs off-the-equilibrium path. The ability of the insurer to commit to the deviant contract (off-the-equilibrium path) makes the decision of the insurees of whether to choose it over the conjectured equilibrium pooling contract, C_P, independent of their beliefs about who else takes it. Hence, the deviant contract is taken only by the low risks (if C_P is offered with commitment) or by both types (if C_P is offered without commitment). In either case, it is profitable as it lies below the pooling zero-profit line (PZP).

Lemma 5: If some feasible pooling contracts Pareto-dominate the Rothschild-Stiglitz separating allocation, then no separating equilibrium exists.

Proof: Bertrand competition implies that in any separating equilibrium allocation each contract must lie on the corresponding zero-profit line. As a result, the best contract the high risks can have is C_H^{RS}. Incentive compatibility then implies that the low risks cannot be offered a contract better than C_L^{RS}. Thus, there cannot exist a separating equilibrium but the Rothschild-Stiglitz one. By Lemma 2, under these conditions, the Rothschild-Stiglitz separating allocation is not an equilibrium. Therefore, no separating equilibrium exists. *Q.E.D*

Proposition 1 below summarizes the results of our model.

Proposition 1: i) If the Rothschild-Stiglitz separating allocation is not Pareto-dominated by any feasible pooling allocation, then it is the unique equilibrium. ii) If, on the other hand, it is Pareto-dominated by some feasible pooling allocations, then the Wilson pooling contract *without commitment* is the unique pooling equilibrium.

Proof: The proof for part (i) is given in Lemma 1. Under the conditions of part (ii), by Lemmas 2 and 5 no separating equilibrium exists. By Lemma 4, the Wilson pooling contract *without commitment* is the unique pooling equilibrium. Therefore, the Wilson pooling contract *without commitment* is the unique equilibrium. *Q.E.D*

4. Special Cases: Screening Models

Now, we will show how the model of Rothschild and Stiglitz (1976) and the three-stage game of Hellwig (1987) can be obtained as special cases of our model.

4.1. The Rothschild-Stiglitz Screening Model

The Rothschild-Stiglitz model arises if we assume that insurers are exogenously committed to the contracts they offer at Stage 1. Proposition 2 summarizes the results:

Proposition 2: Suppose that the insurers are always (exogenously) committed to the contracts they offer at Stage 1. Then, i) if the Rothschild-Stiglitz separating allocation is not Pareto-dominated by any feasible pooling allocation, it is the unique equilibrium. ii) If the Rothschild-Stiglitz separating allocation is Pareto-dominated by some feasible pooling allocations, no Nash equilibrium exists.

Proof: The proof for part (i) is given in Lemma 1. Under the conditions of part (ii), by Lemmas 2 and 5 no separating equilibrium exists. By Lemma 4, no pooling contract *with commitment* can be an equilibrium. Therefore, no equilibrium exists.

4.2. The Hellwig Three-Stage Screening Game

Finally, the Hellwig model is the case where the insurers cannot commit to the contracts they offer at Stage 1 but, in equilibrium, they do not offer contracts which are expected to be withdrawn at Stage 3. The equilibria of this game when no refinement is used to restrict beliefs off-the-equilibrium path are discussed in Hellwig (1987). Here, we establish the “reasonable” equilibria of this three-stage game.

Proposition 3: Suppose that, at Stage 1, insurers cannot commit to the contracts they offer. Then, i) if the Rothschild-Stiglitz separating allocation is not Pareto-dominated by any feasible pooling allocation, it is the unique “reasonable” equilibrium. ii) If the Rothschild-Stiglitz separating allocation is Pareto-dominated by some feasible pooling allocations, then both the Rothschild-Stiglitz separating allocation and the Wilson pooling allocation survive the “intuitive criterion” (see Figure 5).

Proof: The proof for part (i) is similar to that in Lemma 1. Regarding part (ii), the proof why the Wilson pooling equilibrium survives the “intuitive criterion” is given in Lemma 3. Here, we have to show that the Rothschild-Stiglitz separating allocation survives the “intuitive criterion”. Consider an insurer offering the deviant pooling contract C_P^D at Stage 1. Because there is no commitment to this deviant contract, an insuree will take it only if he believes that the insurer will not withdraw it at Stage 3.

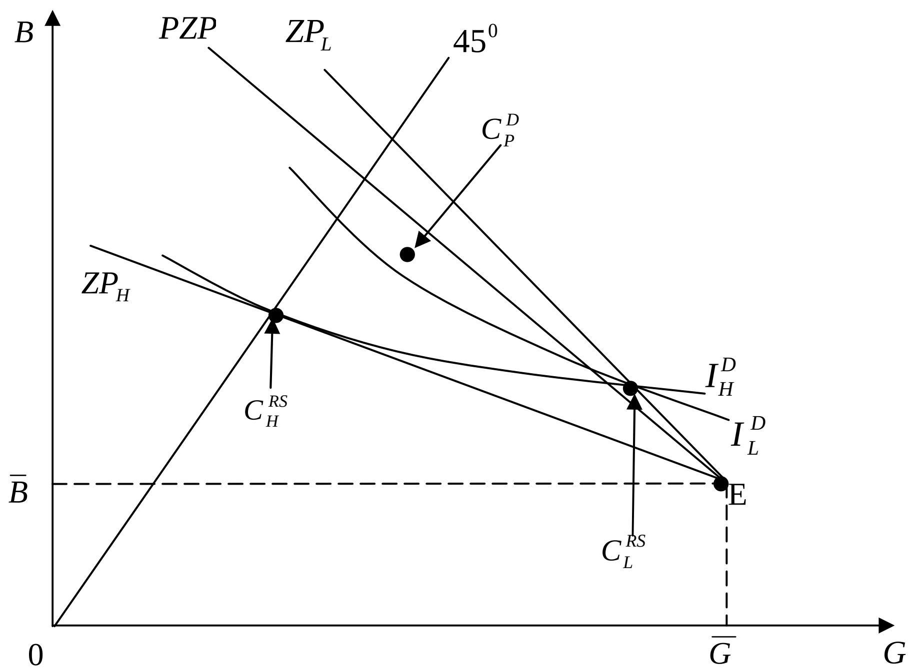

Figure 5

The insurer, in turn, will not withdraw it if he believes that the contract has been chosen by a sufficiently high fraction of low risks so that it is profitable. However, because this deviant contract makes both types of insurees better off compare to the Rothschild-Stiglitz separating allocation, the "intuitive criterion" has no bite and so the off-the-equilibrium path beliefs of the insurer cannot be restricted. Therefore, the deviant contract may not be profitable and so the Rothschild-Stiglitz separating equilibrium cannot be ruled out. *Q.E.D*

In sum, the exogenous commitment to the contracts by the insurers gives rise to the non-existence problem in Rothschild-Stiglitz whereas the lack of commitment in the three-stage game of Hellwig leads to multiple equilibria even if the "intuitive criterion" is used to restrict beliefs off-the-equilibrium path.

5. Screening vs Signalling

As we have already mentioned, the two-stage signalling game can be thought of as the case where insurers, at Stage 1, make available all possible contracts *without commitment* although they know that some of them are loss-making. If beliefs are unrestricted, Maskin and Tirole (1992) show that both the two-stage signalling game and the three-stage game of Hellwig, in general, have multiple equilibria and the two sets of equilibria coincide. If the "intuitive criterion" is used to restrict beliefs off-the-equilibrium path, Maskin and Tirole (1992) show that the unique equilibrium in the signalling game is the Rothschild-Stiglitz separating allocation. In Section 3, we showed that if the Rothschild-Stiglitz separating allocation is Pareto-dominated by some feasible pooling allocations, then in Hellwig's three-stage game both the former and the Wilson pooling contract survive the "intuitive criterion".

Thus, in this case, the two sets of equilibria are different. In Hellwig's game the Wilson pooling contract survives the "intuitive criterion" because the firms (uninformed), at Stage 1, do not offer contracts which are anticipated to be withdrawn at Stage 3. In contrast, in the signalling game, no pooling contract can survive the "intuitive criterion". When the informed choose their contract (signal) all contracts are available and the low risks (good type) can credibly communicate their type by choosing a contract which makes them but not the high risks better off relative to the pooling contract.

In our three-stage screening game, the equilibrium is always unique and so the set of equilibria is different from that of the signalling game even if beliefs are unrestricted. The difference of our results from Maskin and Tirole (1992) stems from the ability of the supplier of the contract (uninformed) to choose whether to commit or not to a contract at Stage 1. The commitment to the contract makes the decision of the insurees of whether to take it independent of their beliefs about who else takes it. Whenever it is feasible for the firms (uninformed) to profitably attract either the low risks or both types they do so by exploiting the power of commitment to the contract. This is the key to the uniqueness of the equilibrium. Also, the ability of a firm to offer a contract without commitment acts a threat for a potential entrant and supports the Wilson pooling contract as an equilibrium.

6. Conclusion

In this paper, we provide a unified framework for analyzing competitive markets with adverse selection. Our approach remedies both the non-existence problem in the Rothschild-Stiglitz screening model and the multiplicity of equilibria issue arising in signalling models and in the three-stage game of Hellwig. The novel feature of our model is that the firms (uninformed) can decide whether to commit to a contract or not. That is, the firms can use the commitment to the contract as an extra tool (along with price and quantity) in their attempt to maximize their profits.

We show that when some feasible pooling allocations Pareto-dominate the Rothschild-Stiglitz separating allocation then the Wilson pooling contract *without commitment* is the unique equilibrium. Otherwise, the Rothschild-Stiglitz separating allocation *with or without commitment* is the unique equilibrium. Therefore, our game has always a unique equilibrium even though we do not use any refinement to restrict beliefs off-the-equilibrium path. Furthermore, this equilibrium is weakly interim efficient.

Therefore, by endogenizing commitment, we make several interesting points: First, we provide an approach to modelling competition in markets under adverse selection which has as special cases all existing approaches. Second, we uncover the driving force of both the non-existence problem in the Rothschild-Stiglitz screening model and the multiplicity of equilibria in the signalling game and the three-stage game of Hellwig. We make clear that these two problems stem from arbitrary restrictions imposed on the firms' strategy (contract) space and are not due to the informational friction (hidden types). The informational friction (hidden types) does not lead to these problems if an appropriately designed game is used. Moreover, our solution to these problems is both simple and fully consistent with the profit-maximizing behaviour of firms.

Third, our game has always a unique Nash equilibrium even though we do not use any refinement to restrict beliefs off-the-equilibrium path. That is, our model offers the highest possible predictive ability (unique equilibrium) while avoiding the difficulties associated with the equilibrium refinement criteria. This feature is very important for applied theory papers where, in most cases, the uniqueness of the equilibrium is crucial for the model's empirical relevance and testability. Fourth, the equilibrium is incentive efficient. Fifth, on-the-equilibrium path, firms optimally choose not to commit to the contracts they offer at Stage 1. This prediction is consistent with the widely observed fact that banks, insurance companies and many non-financial firms keep the right to reject applications for (some of) the contracts they offer.

Finally, as we show elsewhere,[19] if firms are allowed to offer menus of contracts (rather than a single contract) our game has a unique Bayes-Nash equilibrium which is always second-best (interim) efficient.

[19] In Diasakos and Koufopoulos (2008) we consider the case where the single-crossing condition holds and in Koufopoulos (2008) I consider the case where the single-crossing fails and the indifference curves of different types may be tangent.

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