Sovereign Default and Debt Renegotiation through IFIs

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Abstract

In practice countries rarely simply default on their debt. They usually undergo a renegotiation procedure leading to a restructuring, or opt for financing from an IFI such as the IMF, before declaring an inability to repay their debts. This paper extends the small open economy model of sovereign debt and default of Eaton and Gersovitz (1981) to study the decisions of a country before a default. The model consolidates the studies on debt renegotiation and IFI lending and introduces fiscal policy instruments for the sovereign. Guided by the stylized fact that emerges from the data on recent default episodes the model is able to explain how countries that default alongside the IMF can obtain a larger haircut in a debt renegotiation process.

JEL Classification: E44, E62, F34, F41

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1 Introduction

World capital markets have experienced large scale sovereign defaults on a number of occasions the last one being in Moldova in 2004. However, in practice countries rarely simply default on their debt. They usually undergo a renegotiation procedure leading to a restructuring, or opt for financing from an International Financial Institution (IFI), such as the International Monetary Fund (IMF), before declaring an inability to repay their debts. As a result, although the options available to a defaulting country are evident, this issue of timing conceals the information that exists on each option’s consequence.

This paper extends the small open economy model of sovereign debt and default of Eaton and Gersovitz (1981) to study the decisions of a country after default. The main question of interest is: ‘Whether countries that renegotiate their debt alongside the IMF can obtain a larger haircut.’ The purpose of the framework is to enrich the already available studies on quantitative sovereign default models in order to study the connections between IFI involvement, debt restructuring and fiscal policy. By solving the model numerically it is possible to match the stylized fact that emerges from the data on recent default episodes, namely that a country that renegotiates its debt having previously been involved with the IMF obtains a larger haircut.

The answer to the question of this paper can provide substantial guidance as to the reasons and consequences of each action, and in extension to the ongoing debate on IMF involvement. This is particularly important in the context of the current EMU crisis where in March 2012 Greece just restructured over 50% of its government debt, having at the same time received financing from the IMF and the European Union in May 2009 to resolve its borrowing needs. It is thus important to understand these issues in order to guide the rules that govern the European Stability Mechanism (ESM) and the European Financial Stability Facility (EFSF).

The model can be seen as consolidating the studies on debt renegotiation and IFI lending and enriching them with an element of fiscal policy. At the same time the role of the IFI is extensively developed from the previous considerations in the literature (e.g. Boz (2011)). The story begins with a country issuing debt to private sector lenders in the form of one-period non-contingent bonds. Since there is an absence in international insolvency legislation, sovereigns have an incentive to default on their debt. The decision to default is made by the sovereign by comparing the present value function under honoring its debt obligations and defaulting, where defaulting consists of reputational damage and direct losses to output. However, before the decision to default is made the sovereign is able to request IMF assistance, which comes in the form of a non-repayable grant. In return for financing the IMF requires more prudent fiscal management and an attempt to improve the institutions in the
country. Finally, if and when the country defaults and in turn renegotiates its debt its bargaining power is endogenized on whether the IMF was previously called to intervene, but also on whether the reforms of the country’s institutions were succesful.

The model is structured in a dynamic recursive form with stochastic productivity shocks and renegotiation of the debt contract is undertaken within a (generalized) Nash bargaining setting. Default happens along the equilibrium since the assets traded are incomplete. Thus, in bad states of nature lenders are willing to offer debt contracts that have a higher premium attached resulting in a higher incentive for the sovereign to default. However, since institutional reforms imposed by the IMF may raise the country’s productivity there are two offsetting forces working on affecting the sovereign’s probability of default. From one side is the higher debt accumulation, which accompanies IMF involvement since creditors view this as a guarantee on repayment, and on the other side is the increased labour productivity, which works in decreasing default incentives.

The rest of the paper is organized as follows. In section 2 I present motivating evidence on debt restructuring outcomes from 73 default episodes, but also study the case of the Mexican default of the 1980s. After discussing the relevant literature in section 3, in section 4 I build and analyze the model theoretically. In section 5 I proceed with solving a simpler economy and studying the model’s predictions in this setting. Finally, in section 6 I conclude.

2 Empirical Evidence

This section focuses on documenting the debt renegotiation process and presents some stylized facts that emerge from the data. In addition, I highlight certain features that are emphasized in the Mexican default of the 1980’s and are useful in rationalizing the fiscal policy elements present in the theoretical model of Section 4.

2.1 Debt Renegotiation and IMF Involvement

The main result of interest that emerges is the following:

1. *Haircuts are larger for countries that renegotiate their debts alongside the IMF.*

In practice debt renegotiations occur through what is known as the ‘Paris Club.’ The Paris Club is an unofficial group of lenders where the members are the leaders of the most developed economies and based on voluntary participation they meet approximately 10 times a year to discuss any potential debt restructurings (Weiss, 2012). Although they do not abide to any formal regulations their operations
follow a set of principles. The main principle, which is of importance to this paper is the fact that “debt renegotiations are applied only to countries that [...] have a demonstrated track record of implementing reforms under an IMF program.”\(^1\)

Guided by this observation I collect data on debt recovery rates from 73 default episodes over the period 1980 - 2004. Debt recovery rates are obtained from Benjamin and Wright (2009), who estimate haircuts for all countries that have defaulted over this period. Although arguably, these figures are based on estimations of a theoretical model of delays on sovereign debt renegotiations, for the countries for which data exist the correlation between the Benjamin and Wright debt recovery rates and the World Bank data is 0.9.\(^2\) Figure 1 below plots the haircuts (1 - the debt recovery rate) against the external debt-to-GDP ratio for a subset of the Benjamin and Wright sample.\(^3\) The two panels of Figure 1 are then divided according to whether the IMF was involved during the time horizon from which the country declared a default up until the time it settled the renegotiation agreement, which can range from 0 quarters to 12.6 years. IMF involvement is captured by the presence of an IMF programme during this period of the type that are usually implemented in debt burdened economies (such as Stand-By Agreements, Structural Adjustment Facilities, Extended Fund Facilities). The main result the emerges it that for countries where the IMF was involved (right panel of Figure 1) the haircuts are on average substantially larger, with a mean haircut of 54.5% of defaulted debt. On the other hand, for countries where the IMF was not involved this number stands at 15.4%.

In order to motivate the presence of the IMF and the structure that its involvement takes place, in Figure 2 I present the time series of the logarithm of the ratio of public spending-to-private consumption (blue line) and a measure of the tax rate (red line). The left panel shows these series for a country that achieved a sizeable haircut on its defaulted debt and bargained alongside the IMF (Mexico), whereas the right panel is for a country that achieved a haircut of 0% and did not bargain alongside the IMF (Colombia). The common feature across both panels is that during the default periods (shaded in grey) there was no significant movement in the tax rates of these countries, whereas the big changes occurred in the ratio of public spending-to-private consumption. For Mexico, this involves a peak at around 0.17, whereas for Colombia it seems that this ratio actually reached a lower bound before starting to increase after the default. Armed with this result, I posit that an IMF intervention can be translated in the model as an additional constraint in the sovereign’s optimisation programme, which takes the form as an upper bound on this spending mix.

\(^1\)http://www.clubdeparis.org/sections/composition/principes/cinq-grands-principes
\(^2\)Similarly, Benjamin and Wright also report the correlation with the preferred estimates of Sturenzeeger and Zettelmeyer (forthcoming), which also calculate debt recovery rates. This correlation is 0.86.
\(^3\)I have excluded countries that have defaulted for reasons that cannot be accounted for by the model. For example: Mozambique and Zambia defaulted during periods of war.
Figure 1: Haircuts by IMF Involvement during a Default

![Graph showing haircuts by IMF involvement](image1)

- Mean Haircut (%): IMF=54.5, No IMF=15.4
- Graphs by imf_def

Figure 2: Ratio of Public-to-Private Consumption and Taxes

![Graphs showing ratios of public-to-private consumption and taxes](image2)

- Mexico: hp_gc_sm_1 vs. VAT
- Colombia: hp_gc_sm_1 vs. net tax on products (US$)
Finally, in order to justify IMF financing as a grant rather than a loan I turn to evidence presented in Table 1. Here, it is evident that IMF interest rates are rather low and approach relatively well those of the 3-year US Treasury bond. This occurs at the time where country-specific interest rates, proxied by JP Morgan’s EMBI Global Index are substantially higher. This result, in combination with the fact that terms of repayment on IMF loans are more favourable than compared to those offered by the private sector, suggest that IMF financing can more suitably be rationalized as a grant. Moreover, as stated by Boz (2011), only 14% of countries which received IMF financing in the period prior to 1990 defaulted on this debt, so considering this as a grant becomes even more plausible.

<table>
<thead>
<tr>
<th>Years</th>
<th>IMF</th>
<th>2-year</th>
<th>3-year</th>
<th>5-year</th>
<th>EMBI Global</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990-1999</td>
<td>5.75</td>
<td>5.77</td>
<td>5.97</td>
<td>6.30</td>
<td>14.03</td>
</tr>
<tr>
<td>2000-2010</td>
<td>3.44</td>
<td>3.06</td>
<td>3.29</td>
<td>3.74</td>
<td>8.68</td>
</tr>
</tbody>
</table>

2.2 The evidence from the Mexican default in the 1980s

Mexico defaulted on its debt in the second quarter of 1982 and was considered to be in a state of default until the first quarter of 1990. As is shown in Figure 3, during this period its real GDP, private consumption, and government spending declined dramatically and it is also widely documented that the interest rate spreads between the Mexican Peso and the US Dollar soared.

Figure 4 plots the national accounts against the VAT for Mexico. Although not in scale it can be seen that from approximately midway during the default VAT rates rose dramatically. This can be rationalized by the fact that the government was in need of revenues to finance its declining GDP over that period and was an outcome resulting from its optimizing behavior. This feature is accounted for in the model, by allowing GDP to suffer a truncated loss on the onset of the default.

Figure 5 shows the various measurements of debt as a percentage of the Mexican GDP. Although most studies focus on different debt-to-gdp ratios in their models’ calibration I choose to focus on the blue line (external debt public and publicly guaranteed) as the model considers that the sovereign is borrowing and lending from foreign investors not residing in the domestic economy. Furthermore, I wish to emphasize the red line (service on external debt), which plots the amounts repaid in each

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4These countries are Cambodia, Guyana, Haiti, Honduras, Liberia, Panama, Peru, Sierra Leone, Somalia, Sudan, Vietnam, Zaire, and Zambia.

5Theoretically, it would be possible to consider IMF payments as a loan however this would increase the dimensionality of the state space and I choose to avoid this computational complexity at this stage.
Figure 3: National Accounts and Interest Rates

Figure 4: National Accounts and Consumption Taxes
period on any external debt by the government, and is the measure most appropriate for the numerical simulation presented in section 5

Finally, Figure 6 illustrates the positive comovement of VAT rates against the use of IMF credit as a percentage of the Mexican GDP. As the IMF strengthened its involvement within the Mexican economy VAT rates rose implying that an increase in consumption taxes may not only be a result of the government’s optimization, but rather mandated by the IMF in return for funds. These lending conditionalities are also accounted for in the model of section 4.

3 Literature Review

The studies by EG and Grossman and Van Huyck (1988) serve as the theoretical foundations for the dynamic, stochastic macro models that have prospered in the last decade. They assume that a sovereign maximizes the households’ intertemporal utility resulting from consumption subject to budget constraints, and finances public expenditures and interest on outstanding debt by borrowing. Since then the literature has focused both on the theoretical properties of default models as well as on empirical exercises aimed at matching certain facts to data from emerging market economies. Most

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of these succeeding dynamic models can be seen as focusing each time on a particular modification of the foundations of the Eaton and Gersovitz model.\footnote{Due to space limitations I will only present the modifications of the Eaton and Gersovitz model that are directly relevant to this study. For a full review of this literature, see Staehler (2000).}

Arellano’s (2008) study can be seen as the basis for the quantitative exercises that have followed. She shows that the standard Eaton and Gersovitz model replicates most facts about the Argentine default and further illustrates theoretically how interest rates respond to output fluctuations through endogenous time-varying default probabilities.

Yue (2010) and D’Erasmo (2011) focus specifically on the options available for a sovereign post default. They concentrate on the problem of renegotiating defaulted debt in order to regain access to capital markets. Yue endogenizes the default probability and the debt recovery rate and finds that her model justifies the debt reduction pattern, the volatile and countercyclical bond spreads, and the countercyclical trade balance of the Argentine default. Her study differs to this in that she presents a default mechanism based on the past credit history of the sovereign. This allows her to partially endogenize the sovereign’s probability of exclusion from financial markets. The work by D’Erasmo (2011) builds on Yue (2010) as he fully endogenizes the exclusion period. He does this by introducing political uncertainty and incomplete information on the government’s type. In this way, he is able to better match the problematic debt-to-GDP ratio observed in the data. More specifically, he finds that
it is 2-5 times higher than in previous models, thus explaining 55% of the data. Both of these studies follow from the work of Fernandez and Rosenthal (1990), which look at the strategic interactions between debtor and creditor in the case of default.

Benjamin and Wright also focus on debt renegotiations and are able to endogenize the probability of re-entering financial markets after a default, which has previously left unaccounted for. They obtain this by allowing for multiple stages in the bargaining process between the sovereign and the lender, and thus effectively induce a delay in the restructuring process.

Regarding models that incorporate institutional investors, Boz (2011) touches upon the issue of a government borrowing from two sources of funds: private sector creditors and IFIs, where the latter contracts are assumed to be enforceable and are tied to conditionality terms. She then explains why countries chose to borrow less from IFIs than private sector lenders and shows how borrowing from commercial creditors is procyclical, whereas IFI lending is countercyclical in emerging economies. This paper differs substantially differs from Boz as I do not consider a repayable IMF loan, but also because I do not assume that conditionality is tied to an endogenous discount factor but rather to an endogenous change in the sovereign’s fiscal policy.

Finally, this paper is also reminiscent of the framework of Cuadra, Sanchez and Sapriza (2010), which introduce fiscal instruments in the quantitative sovereign default model to study the links of fiscal variables, interest rate spreads and default risk in Mexico.

By combining the work on debt renegotiation and lending from institutional investors this paper explicitly allows for multiple choices for the sovereign before a default, and also considers the effects of default risk on fiscal instruments. Since so far default models have entirely focused on preserving a single route to arrive to a default, but in practice we observe that several choices are available that lead to this outcome, this paper allows to examine the debt repayment incentives and associated welfare levels of a model that is very accurate in representing several default episodes that have occurred in recent history. This is mostly accomplished by endogenizing an aspect of sovereign default models that was previously considered to be entirely endogenous: the bargaining power during renegotiations, which differs on whether the (very richly modeled) IMF has intervened.

4 Model Environment

Consider a small open economy characterized by an infinitely lived representative agent, the government, who benevolently maximizes the utility of its households and every period receives a stochastic

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8Boz assumes that when a country is indebted towards the IFI the IFI will only provide a loan conditional on a higher discount factor for the sovereign, thus back-loading the utility from intertemporal consumption.
stream of income, affected by a shock in labour productivity. The sovereign borrows from risk-neutral private sector lenders in the form of one-period non-contingent bonds without the commitment to repay the debt. Every two periods the sovereign has the option to request an IMF intervention whereby the IMF offers a grant to the sovereign in return for conditionality measures and an attempt at implementing reforms in the country’s institutions. In the intermittent period, and if in debt, the government chooses whether to honor its obligations, by rolling over its debt and refinancing it by selling bonds to international lenders, or to not repay and default. In the latter case, during the period of a default the sovereign attempts to restructure its debt at a negotiated recovery rate with the lenders. Depending on whether the sovereign previously chose to call for IMF assistance he may now enjoy a higher bargaining power in the negotiations. I consider the case where bargaining alongside the IMF can induce the sovereign to extract a larger surplus during the renegotiation process.

The model can be illustrated schematically in Figure 7.

4.1 Preferences and Endowments

Preferences of the representative household are characterized by:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t, 1 - l_t) \]  

where \( u(c) \) is concave, strictly increasing and twice differentiable, \( c_t \) is private consumption in period \( t \), \( g_t \) is public spending, \( l_t \) is labour and \( \beta \in (0, 1) \) is the discount factor. Output is produced using labour and can be divided between private consumption and public spending. The production technology faces a productivity shock \( A_t \) that is assumed to follow a Markov process with transition function \( \Pi(A'/A) \), which has values over the set \( \mathcal{Y} \).

\[ y_t = A_t F(l_t) \]

The government taxes private consumption and finances its public spending from the revenues from taxation as well as from external borrowing from risk-neutral private sector creditors in the form of one-period non-contingent bonds. The household takes public spending \( g_t \) and taxation \( \tau_t \) as given and makes choices on consumption and labour, subject to the budget constraint:

\[ (1 + \tau_t)c_t = A_t F(l_t) \]
Figure 7: Timing
Optimization of the household’s problem gives rise to the following intratemporal condition, which describes the optimal behavior of the private sector:

\[
\frac{U_c(c, g, 1 - l)}{U_l(c, g, 1 - l)} = \frac{1 + \tau}{AF_l(l)}
\]  \hspace{1cm} (3)

**Government**

The government chooses public spending, taxation and consumption to benevolently maximize the expected discounted utility of the representative household in the economy. Every two periods it makes a choice about whether to request IMF involvement \((I = 1)\), or to act alone \((I = 0)\). Regardless of its choice it observes the productivity shock \(A\) and makes a decision on whether to default on its private debt \((D = 1)\), or not \((D = 0)\). If it chooses to default, then it does not pay its accumulated debt, suffers an output loss, is excluded from private sector borrowing and then has the possibility to negotiate a haircut with the private sector lenders. This will then give the government the ability to borrow again in the following period. The main contribution of the model is the different bargaining power that the sovereign has when renegotiating a recovery rate and this is dependent on whether the sovereign arrives at the bargaining process having been through IMF supervision, or not. If the sovereign has been through IMF supervision it is assumed that its bargaining power will be greater than in the case where it has chosen to default and renegotiate alone. This can be rationalized by the evidence presented in Section 2 and the fact that debt restructurings occurring through the Paris Club almost are almost always conditioned on an IMF intervention.

The sovereign’s optimization problem can be written in recursive dynamic programming form in the following way. The state variables for the government are the level of debt inherited from the previous period, \(d\), and the productivity shock \(A\).

The sovereign then makes the initial decision to request IMF assistance in the following way:

\[
V^0(d, A) = \max_{I \in \{0,1\}} \left\{ IV^I(d, A) + (1 - I) V^{NI}(d, A) \right\}
\]  \hspace{1cm} (4)

I now consider and describe what happens under either of the two options the sovereign has at its disposal. It is instructive to follow Sections 4.2 and 4.3 by simultaneously consulting Figure 7.

**4.2 No IMF Involvement: \((I = 0)\)**

If \(I = 0\) then no agreement with the IMF has been chosen and \(V^0(d, A) = V^{NI}(d, A)\). The following choice the sovereign has to make is then to default or not and this is given by comparing the value
functions of these associated choices:

\[ V^{NI}(d, A) = \max_{D \in \{0, 1\}} \{ DV^D(d, A) + (1 - D)V^C(d, A) \} \]

where now if \( D = 0 \) a decision to repay has been chosen and \( V^{NI}(d, A) = V^C(d, A) \). The value function associated with a repay decision takes the form:

\[ V^C(d, A) = \max_{g, \tau, d'} \left\{ u(c, g, 1 - l) + \beta \sum_{A'} V^1(d', A') \Pi(A'/A) \right\} \]

\[ s.t. \quad g = \tau c - d + q(d', A)d' \]

\[ (1 + \tau)c = AF(l) \]

\[ \frac{U_c(c, g, 1 - l)}{U_l(c, g, 1 - l)} = \frac{1 + \tau}{AF(l)} \]

where \( d \) is the debt due this period, \( d' \) is the debt acquired this period and due next period and \( q(d', y) \) is the market price of the household debt. If \( d' > 0 \) it means the sovereign has entered a debt contract where it promises to deliver \( q(d', y)d' \) units of the consumption good, conditional on not defaulting, in the next period. Notably, when the government optimizes over the welfare of the representative household it chooses the tax rate, public spending as well as its position on the asset, taking into account how the household will respond to these policies. For this reason the optimization is subject to the government’s budget constraint, the household’s budget constraint as well as the household’s intertemporal optimality condition.

If the sovereign chooses no IMF involvement and the repay option \((I = 0), (D = 0)\) then in the next period it still has to face the option of defaulting and renegotiating, or repaying back the newly acquired debt, without having the possibility to call the IMF. This is the same thing as saying that if the IMF has not been chosen in period \( t \) then it cannot be chosen in period \( t + 1 \), but rather in period \( t + 2 \). This consideration reflects the fact that requesting IMF intervention is usually planned in advance. Thus if a country does not choose the IMF route then it faces two periods of potential default before being able to re-opt for IMF assistance. So, the continuation value following a repay decision is:
\[ V^1(d, A) = \max_{D^1 \in \{0, 1\}} \left\{ D^1 V^R(d, A) + (1 - D^1) V^C(d, A) \right\} \]  

(7)

where in turn \( V^R(d, A) \) is the value function denoting the expected life-time utility of defaulting when the debt recovery rate will be settled at \( a(d, A) \), (the fraction of debt to be repaid after renegotiation, to be elaborated on below) and takes into account the impact of debt reduction, as well as the temporary exclusion from private sector borrowing associated with a default. This last feature is reflected through the decreased consumption of the truncated endowment in the current period. However, if a debt recovery rate is then established through a renegotiation mechanism with the lenders, then from next period onward it regains access to financial markets and is again able to borrow having outstanding debt only the transformation \( a(d, A)d \).

\[ V^R(d, A) = \max_{g, \tau, d'} \left\{ u(c, g, 1 - l) + \beta \sum_{A'} V^0(a(d, A)d, A') \Pi(A'/A) \right\} \]  

(8)

s.t. \[ g = \tau c \]

\[ (1 + \tau)c = A^{def} F(l) \]

\[ \frac{U_c(c, g, 1 - l)}{U_l(c, g, 1 - l)} = \frac{1 + \tau}{AF_l(l)} \]

On the other hand, if following a non-IMF intervention a choice of defaulting is made then \( D = 1 \) and the value function associated with a default, \( V^D \) is denoted as:

\[ V^D(d, A) = \max_{g, \tau, d'} \left\{ u(c, g, 1 - l) + \beta \sum_{A'} V^R(d, A') \Pi(A'/A) \right\} \]  

(9)

s.t. \[ g = \tau c \]

\[ (1 + \tau)c = A^{def} F(l) \]

9This assumption is standard in the sovereign debt literature and characterizes the fact that the endowment drops in case of default. This is consistent with evidence presented by Chuhan and Sturzenegger (2003) and Rose (2005). The latter provides support of a significant decrease in bilateral trade flows following the initiation of debt renegotiation by a country (8% a year). In this setup this truncation of output translates to the fact that the same number of hours worked produce less units of goods. Thus, the productivity shock in autarky takes the form \( A^{def} \leq A \) and since the productivity shock follows a Markov chain this persists through time, thus also affecting future periods of the model.
\[
\frac{U_c(c, g, 1 - l)}{U_l(c, g, 1 - l)} = \frac{1 + \tau}{AF_l(l)}
\]

Note that \(V^R(d, A)\) here is the same outcome as defaulting in period \(t + 1\) after having previously chosen to repay in period \(t\). The main difference between defaulting and repaying in period \(t\) is the fact that if the sovereign defaults then in the next period it cannot do anything more than remain in a default (but now with the option of renegotiating). This reflects the more cumbersome process of arriving at a renegotiation without the IMF that is frequently observed in default episodes of recent history. Notably, arriving at a renegotiation following a decision to default in period \(t\) implies that the sovereign is facing persistent output losses in two periods.

### 4.3 IMF Involvement: \((I = 1)\)

If \(I = 1\) then an agreement with IMF has been made and \(V^0(d, A) = V^I(d, A)\). The first and most evident feature of IMF involvement is the fact that the country is placed into a temporary safezone of one period. Contrary to the case where \(I = 0\) and the sovereign is immediately facing a choice of defaulting or repaying in period \(t\), here the sovereign is guaranteed a state of no default for one period. So, the value function associated with IMF assistance is denoted as:

\[
V^I(d, A) = \max_{g, \tau, d'} \left\{ u(c, g, 1 - l) + \beta \sum_{A'} V^S(d', A') \Pi(A'/A) \right\}
\]

\[s.t.\quad g = \tau c - d + q(d', A)d' + \Lambda(d)\]

\[(1 + \tau)c = \hat{A}F(l)\]

\[
\frac{U_c(c, g, 1 - l)}{U_l(c, g, 1 - l)} = \frac{1 + \tau}{AF_l(l)}
\]

\[
\frac{g}{c} \leq G
\]

In addition, the IMF offers a grant \(\Lambda(d)\) to the sovereign, which is dependent upon the accumulated debt. In return for these benefits however, the IMF imposes 2 conditionality terms: more prudent fiscal management, which is reflected as an additional constraint into the sovereign’s optimisation problem in the form of an upper bound on its public spending-to-private consumption ratio, and an
attempt to reform the country’s institutions. The latter is reflected in the productivity process of the sovereign where $\hat{A}_t = \exp\{A_t + \delta(\zeta)\}$. Here, $\delta$ represents institutions and $\zeta$ are reforms, which with some exogenous probability $p$ they are successful and take a value of 1, and with probability $(1-p)$ they are not successful and take a value of 0. So if IMF imposed reforms are successful then they are productivity enhancing and this is reflected in an upward transformation of the productivity function.

In period $t+1$ when the IMF guarantee is over and the country moves out of the safezone then it faces a choice of defaulting and renegotiating $V^{IR}$ or repaying its previously issued debt $V^{IC}(d, A)$:

$$V^S(d, A) = \max_{D^1 \in \{0, 1\}} \{D^1V^{IR}(d, A) + (1 - D^1)V^{IC}(d, A)\}$$ (11)

If $D^1 = 0$ and the sovereign chooses to repay its debt then $V^S(d, A) = V^{IC}(d, A)$ and $V^{IC}(d, A)$ takes the form:

$$V^{IC}(d, A) = \max_{g, \tau, d'} \left\{ u(c, g, 1 - l) + \beta \sum_{A'} V^0(d', A') \Pi(A'/A) \right\}$$ (12)

s.t. $g = \tau c - d + q(d', A)d' + \Lambda(d)$

$$(1 + \tau)c = \hat{A}F(l)$$

$$\frac{U(c, g, 1 - l)}{U(l, c, 1 - l)} = \frac{1 + \tau}{AF(l)}$$

$$\frac{g}{c} \leq \bar{G}$$

whereas if $D^1 = 1$ and the sovereign chooses to default on its debt then $V^S(d, A) = V^{IR}(d, A)$ and $V^{IR}(d, A)$ is denoted as:

$$V^{IR}(d, A) = \max_{g, \tau, d'} \left\{ u(c, g, 1 - l) + \beta \sum_{A'} V^0(a(d, A)d, A') \Pi(A'/A) \right\}$$ (13)

s.t. $g = \tau c + \Lambda(d)$

$$(1 + \tau)c = \hat{A}^{def}F(l)$$
\[ \frac{U_c(c, g, 1 - l)}{U_l(c, g, 1 - l)} = \frac{1 + \tau}{AF_l(l)} \]
\[ \frac{g}{c} \leq \bar{G} \]

Similarly to when the sovereign arrives at a default without having chosen the IMF route, a default entails a truncated loss in output and an exclusion from financial markets until a debt recovery rate is settled with the lenders. Then, in period \( t + 2 \) the sovereign regains access to financial markets and only pays a fraction of its defaulted debt. On the contrary to the no IMF scenario, here the sovereign is still receiving the grant, is further constrained by the upper bound on its spending mix and is also attempting to implement reforms.

To summarize, the sovereign’s policy can be characterized by a default set \( D(d) \subset \Upsilon \). To define the default set I first define:

\[ \hat{V}^D(d, A) = \max \{ V^{IR}(d, A), V^R(d, A), V^D(d, A) \} \]
\[ \hat{V}^C(d, A) = \max \{ V^{IC}(d, A), V^C(d, A) \} \]

and the default set is then:

\[ D(d) = \{ A \in \Upsilon : \hat{V}^D(d, A) \geq \hat{V}^C(d, A) \} \]  \( \text{(14)} \)

### 4.4 Debt Renegotiation Mechanism

In this section I present the simple debt renegotiation mechanism, which takes place following the decision to default. It takes the form of Nash bargaining game as in Yue (2010). This will pinpoint the debt recovery rate, which is an important element of the model that is needed in order to provide the welfare comparison of the sovereign’s choices following default. Under the bargaining agreement, the value of defaulted debt is reduced to a fraction \( a(d, A) \) of the unpaid debt \( d \). Most importantly, the bargaining power during the bargaining mechanism is endogenous and is dependent on whether the country arrives at the renegotiation stage having previously been through IMF involvement, and in addition on whether the IMF imposed reforms (if \( I = 1 \)) have been successful (so if \( \zeta = 1 \)).

The value of a renegotiation agreement to the sovereign is given by the value functions \( V^{IR}(d, A) \) if \( I = 1 \) and \( V^R(d, A) \) if \( I = 0 \). I assume that the renegotiation takes place only once for one default.
event. The outside option of the lenders is walking away with no debt repayment at all, whereas the outside option of the country is permanent financial autarky where this is defined as:

\[
V^\text{aut}(A) = \max_{g, \tau, d'} \left\{ u(c, g, 1 - l) + \beta \sum_{A'} V^\text{aut}(A') \Pi(A'/A) \right\}
\]

\[\text{s.t. } g = \tau c\]

\[(1 + \tau)c = A^\text{def} F(l)\]

\[
\frac{U_c(c, g, 1 - l)}{U_l(c, g, 1 - l)} = \frac{1 + \tau}{AF_l(l)}
\]

For any debt recovery rate \(a\), the sovereign’s surplus in the Nash bargaining game is denoted as \(\Delta^S(a; d, A)\) and it characterizes the difference between the value of accepting the debt recovery rate \(a\) and the value of rejecting it, given the country’s debt level \(d\) and productivity shock \(A\):

\[
\Delta^S(a; d, A) = IV^IR(d, A) + (1 - I)V^IR(d, A) - V^\text{aut}(A)
\]

Similarly, without elaborating further on the actions of the representative risk-neutral lender (this is done below), their surplus is the present value of the recovered debt:

\[
\Delta^L(a; d, A) = \frac{ad}{1 + \tau}
\]

I assume an allocation of bargaining power in that the sovereign has bargaining power with probability \(\theta\) and the representative risk-neutral lender with probability \(1 - \theta\). Note that if the sovereign had all the bargaining power \((\theta = 1)\) then it could get a complete debt reduction through the bargaining process, whereas if the lender had all the bargaining power \((\theta = 0)\) he would be able to expropriate all the surplus of the sovereign and demand a level of debt repayment such that the sovereign is made indifferent between accepting and rejecting. The bargaining power set is defined as \(\theta \in \Theta \subset [0, 1]\). Since in practice we observe renegotiations of private debt occurring through the Paris Club, which in the majority of circumstances require that the restructuring be conditioned on IMF involvement, I postulate that for any renegotiation case where the IMF is involved and reforms are successful then \(\theta \geq 0.5\), otherwise \(\theta < 0.5\). Finally, given the debt level \(d\) and the productivity process \(A\), the debt recovery rate \(a(d, A)\) solves the following bargaining problem:
\[
\begin{align*}
  a(d, A) &= \arg \max_{a \in [0, 1]} \left( (\Delta^S(a; d, A))^\theta, (\Delta^L(a; d, A))^{1-\theta} \right) \\
  \Delta^S(a; d, A) &\geq 0 \\
  \Delta^L(a; d, A) &\geq 0
\end{align*}
\]

4.5 Lenders

Finally, I turn to the characterization of the foreign investors' problem. The risk-neutral lenders take
the debt holding price \(q(d', A)\) as given and choose the amount of debt \(d'\) to maximize their expected
profits:

\[
\pi(d', y) = \begin{cases} 
  q(d', A)d' - \frac{d'}{1+r} & \text{if } d' \leq 0 \\
  q(d', A)d' - \left[ \frac{1-E[\delta(d', A) + \delta(d', A)a(d', A)]}{1+r} \right]d' & \text{if } d' > 0
\end{cases}
\]

(19)

Since we assume that private sector lenders are perfectly competitive the bond price function is
then given by:

\[
q(d', A) = \begin{cases} 
  \frac{1}{1+r} & \text{if } d' \leq 0 \\
  \frac{1-E[\delta(d', A)] + E[\delta(d', A)a(d', A)]}{1+r} & \text{if } d' > 0
\end{cases}
\]

(20)

As it is never in the sovereign's interest to default when its asset position is nonnegative (\(d' \leq 0\)),
this implies that the country is receiving payments from foreign investors, so the sovereign bond
price is equal to the price of a risk-free bond \((1 + r)^{-1}\). By defining the sovereign's gross interest
rate as \(1/q = 1 + r^c\) the country spread is then the difference between the country interest rate
and the risk-free rate \(r^c - r\). Thus, it follows that \(D(d)\) is empty for all \(d \leq 0\). On the other
hand, when the sovereign country is indebted the expected profits of the lender take into account
the expected probability of default \(E\{\delta(d', A)\} = Pr[\bar{V}^D(d, A) > \bar{V}^C(d, A)]\) as well as the expected
recovery rate \(E\{a(d', A)\}\), which is given by the expected fraction of defaulted debt that the investors
can expropriate, conditional on default and a choice of renegotiation.
4.6 Recursive Equilibrium

I now define a recursive equilibrium in this economy.

Definition 1:

The recursive equilibrium in this economy is defined as a set of value functions for the government $V^0$, $V^D$, $V^C$, $V^I$, $V^NI$, $V^S$, $V^R$, and $V^{IR}$; price schedules for private sector debt $q(d', A)$; debt allocations $d^*(d, A)$; policy functions for households’ consumption allocations $c^*$, $c^*_d$, $c^*_r$ and labour allocations $l^*$, $l^*_d$, $l^*_r$; policy functions for the government’s taxation $\tau^*$, $\tau^*_d$, $\tau^*_r$ and public spending $g^*$, $g^*_d$, $g^*_r$; default sets $D^*(d)$; and a recovery rate $a(d, A)$ such that:

1. Given the government policy functions and the debt price functions, the households’ consumption and labour policy functions solve the household’s problem.

2. Given the debt pricing schedules $q^*(d', A)$, the recovery rate $a^*(d, A)$ and the households’ policy functions, the government’s value functions $V^0$, $V^D$, $V^C$, $V^I$, $V^NI$, $V^S$, $V^R$, and $V^{IR}$, debt holdings $d^*(d, A)$, default set $D^*(d)$ and the government’s policy functions solve the sovereign’s optimization problem (4), (5), (6), (7), (8), (9), (10), (11), (12), (13) and (14).

3. Given the private sector pricing schedule $q^*(d', A)$ and value functions $V^R$ and $V^{IR}$ the recovery rate $a^*(d, A)$ solves the debt renegotiation problem (18).

4. Given the recovery rate $a^*(d, A)$, the private sector debt pricing schedule $q^*(d', A)$ satisfies the zero profit condition for the private sector lenders, where the expected default probability and expected recovery rate are consistent with the country’s debt position, output and renegotiation agreement.

5. The goods market clears.

In equilibrium the default probability is given by:

$$E\{\delta^*(d', A)\} = \int_{d' \in D^*(d)} \Pi(A', A)dA'$$

so when $D(d) = \emptyset$, $E\{\delta^*(d', A)\} = 0$ and when $D(d) = \Upsilon$, $E\{\delta^*(d', A)\} = 1$. Finally, the expected recovery rate is determined by:

$$E\{a^*(d', A)\} = \frac{\int_{d' \in D^*(d)} a^*(d', A') \Pi(A', A)dA'}{E\{\delta^*(d', A)\}}$$

21
As a result, default occurs in equilibrium as the assets traded are incomplete and the default probability depends on both the debt acquired this period and the endowment, which is stochastic and differs depending on whether the IMF was chosen and on whether the IMF reforms were successful. Thus, in bad states of nature lenders are willing to offer debt contracts that have a higher premium attached resulting in a higher incentive for the sovereign to default.

5 Quantitative Analysis

5.1 A Simpler Economy without Renegotiation

In this section I solve a simpler economy where the sovereign simply faces a choice between choosing to request IMF assistance, or not, and then has the option to either default or repay its debt. Moreover, there is not a debt renegotiation problem in this economy, and following a default the sovereign may regain access to markets with some exogenous probability $p$, or remain in autarky with probability $(1-p)$. Finally, when the IMF comes in there is no mandate to implement reforms from the part of the sovereign and thus the only conditionality associated with the grant is the upper bound on the spending mix.

5.2 Parameter Values

Since no calibration has been performed all parameter values are borrowed from the relevant studies. These are presented in Table 2. The country matched is Mexico and its default of the 1980’s.

The utility function for the sovereign has the following functional form:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

where $\sigma$ is the coefficient or risk aversion and is set equal to 2 following the consensus in the macroeconomics literature. The output process is linear in labour and given by $y = AF(l) = Al$, where the productivity shock follows an AR(1) process $AR(1): ln(A_t) = \rho ln(A_{t-1}) + \varepsilon_t$ with $\varepsilon_t \sim N(0, \sigma^2)$. Arellano (2008) estimates this from Argentina’s GDP during the period 1980 - 2001 and obtains values of $\rho = 0.85$ and $\sigma^2 = 0.025$. This has been confirmed to be approximately equal to the output series for Mexico. Since it is assumed that output entails a direct cost during financial exclusion the truncated output takes the following form:

\[10\] I am in the process of solving the full economy presented in Section 4 using approximation methods, such as Chebyshev polynomials or interpolation splines.

\[11\] see Arellano (2008) for the determination of re-entering financial markets exogenously following a default.
\[ A_{def} = \begin{cases} \xi E(A) & \text{if } A > E(A) \\ A & \text{if } A < E(A) \end{cases} \tag{24} \]

with \( \xi \in (0,1) \), and where \( \xi \) is calibrated by Arellano (2008) to target the 5.53% debt service-to-GDP in Argentina. Again this is also similar to Mexico and the values calculated by Cuadra, Sanchez and Sapriza (2010). The re-entry probability is taken from Cuadra and Sapriza (2008), which target the standard deviation of trade balance of Mexico from 1980 to 2001, that is a trade balance volatility of 1.75. The risk-free interest rate \( r \) is set to 1%, which is the average quarterly interest rate of a five-year US treasury bond during 1980-2000.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Source</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td>( \gamma )</td>
<td>2</td>
<td>Aguiar and Gopinath (2007)</td>
<td>-</td>
</tr>
<tr>
<td>Discount factor</td>
<td>( \beta )</td>
<td>0.97</td>
<td>Yue (2010)</td>
<td>-</td>
</tr>
<tr>
<td>Labor elasticity</td>
<td>( \psi )</td>
<td>0.455</td>
<td>Mendoza (1991)</td>
<td>-</td>
</tr>
<tr>
<td>Re-entry probability</td>
<td>( p )</td>
<td>0.1</td>
<td>Cuadra and Sapriza (2008)</td>
<td>3 yrs 1980-1999</td>
</tr>
<tr>
<td>Default penalty</td>
<td>( \xi )</td>
<td>0.96</td>
<td>Arellano (2008)</td>
<td>( \sigma(TB) )</td>
</tr>
<tr>
<td>( g ) weight</td>
<td>( \pi )</td>
<td>0.3</td>
<td>CSS (2010)</td>
<td>-</td>
</tr>
<tr>
<td>Risk free interest rate</td>
<td>( r )</td>
<td>0.01</td>
<td>Yue (2010)</td>
<td>-</td>
</tr>
<tr>
<td>Cap on ( g/c )</td>
<td>( G )</td>
<td>0.19</td>
<td>data</td>
<td>-</td>
</tr>
<tr>
<td>Output</td>
<td>( \rho )</td>
<td>0.85</td>
<td>data</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( \sigma_c )</td>
<td>0.006</td>
<td>data</td>
<td>-</td>
</tr>
</tbody>
</table>

5.3 Numerical Solution

Since the decisions to default and renegotiate are taken by the sovereign by comparing the present value functions for each option, a linear analytical closed-form solution cannot be obtained. As a result, we employ non-linear techniques, namely the standard solution method of the discrete space technique to solve the maximization problem. This will allow the calculation of the default probability and the debt recovery rate endogenously. I first discretize the state space for the government’s debt holdings. The the bounds of the debt space are defined to be \( d = [-0.05,0.5] \). These are set to ensure that the limits do not bind in equilibrium. A certain number of evenly spaced grid points are then taken from these spaces as proxies for all possible debt levels. The grid range is verified ex post by comparing the values obtained in the simulations with the business cycle moments of Mexico. Furthermore, a discrete state space has to also be generated for output, as is a matrix of transitional
probabilities. The AR(1) process defined above is then converted into a 12-state Markov chain using the quadrature-based method of Tauchen and Hussey (1991). The limits of the endowment space are large to allow for big deviations from the mean value of the shock. Finally, the quantitative model assumes 12 output states and 50 states for debt holdings. After this standard preparation the model is solved numerically using value function iterations.

5.4 Results

This section summarizes the main equilibrium features of the calibrated model and then attempts to evaluate its quantitative performance in matching business cycle statistics for Mexican data.

![Figure 8: Probability of Default](image1.png)

![Figure 9: Bond Price Schedules](image2.png)

Figures 8 and 9 plot the probability of default and the bond prices schedule for the sovereign’s debt as a function of assets-to-GDP ratios. As is evident in Figure 9 for low levels of debt the bond price is approximately equal to the risk free interest rate, which implies that lenders are not incorporating any default incentives into the risk premia they are charging. This is also reflected in Figure 8 where for low levels of debt the probability of a default is close to 0. At the same time the bond price is an increasing function of the productivity shock, which reflects that default incentives are larger when the economy is hit by an adverse shock.

\[12\] The computation algorithm is presented in the Appendix.

\[13\] I thank Nikolai Staehler for kindly sharing the Matlab codes of his baseline model.
The Green lines in the two figures are the probability of default and bond prices for the benchmark model with the IFI. The Blue lines instead are for the same model but with no IFI present. As can be seen the bond price in the presence on the IFI is greater, which implies that lenders view the IFI as a guarantee on their repayment and thus charge a lower risk premium on the sovereign’s bond prices. However, at the same time, since the higher bond price encourages higher debt accumulation from the part of the sovereign, the default probability also increases in the presence of the IFI. Therefore, for a given level of assets, the default probability increases when the IFI is present in the model. Thus, in equilibrium default happens for large levels of assets when the IFI is present in the model.

Moreover, the value functions in Figure 10 suggest that for a given parametrization of the model, the route of choosing the IMF is always chosen. Brief sensitivity analysis suggests that the main deep parameter that affects the choice of choosing the IMF, or not in the first period is the upper bound on the spending mix, $\bar{G}$. For a minimum value of 0.19, which is the maximum of the series of public spending-to-private consumption in Mexico during 1984Q3 the choice of the IMF is always chosen. Anything beyond that suggests a tradeoff between choosing the IMF or not and this is driven by the state of the productivity shock.

6 Discussion

This paper has attempted to answer the question of whether a country that defaults and then renegotiates its debt alongside the IFI can obtain a larger haircut of its defaulted debt, as opposed to renegotiating without having been through an IFI involvement. Empirical evidence on recovery rates settled during restructuring episodes up until 2004 suggests that this is indeed the case. Furthermore, the Paris Club’s principles of operation directly emphasize that any debt restructuring is only conditioned on a previous IMF involvement.

This question has been studied by extending the framework of the baseline sovereign default model of Eaton and Gersovitz (1981) and Arellano (2008) and by consolidating the studies on potential debt repudiation and multiple types of creditors. I have built a small open economy with incomplete markets characterized by an infinitely lived government, who maximizes the utility of its households and implements fiscal policy decisions. The sovereign is able to issue debt to risk-neutral private sector lenders without any enforcement legislation. In addition, every two periods the sovereign is able to request IMF assistance, where this is financed as a grant, but in return for certain conditionality measures. At the time of a potential default and eventual renegotiation, if the country has previously opted for IMF assistance, it will enjoy a higher bargaining power and hence a higher ability to extract surplus from the lenders. The results of the numerical solution of a simpler economy without
renegotiation find that the presence of an IFI in the model incentivizes the sovereign to accumulate larger levels of debt, as the lenders view the IFI as a guarantee on their repayment and hence charge the sovereign lower risk premia on bonds. However, a larger accumulation of debt also implies a higher default probability, so in the end in the presence of an IFI the sovereign defaults holding lower levels of assets.

Further investigation in the context of this paper should primarily address the numerical solution of the richer economy with renegotiation and a more precise calibration of the model.

References


Appendix

Numerical Algorithm

In order to find the equilibrium policy functions, default decisions and the debt recovery rate the following algorithm will be used:

1. Discretize the state space as described in Section 5.

2. Start with a guess of the debt recovery schedule $a_0(d, A)$.

3. Guess a private sector debt price schedule $q_0(d, A)$. We choose the risk-free interest rate for this: $q_0(d, A) = (1 + r)^{-1}$ for all $d$ and $y$.

4. Use $q_0(d, y)$ to solve the government’s problem recursively using value function iteration taking the conjectured price schedule for private sector debt as given, and obtain the optimal policy functions for consumption $c(d, A)$, debt holdings $d'(d, A)$ and default sets $D(d)$.

5. Given these policy functions and the default set, compute the probability of default $\delta(d', A)$.

6. Given the probability of default, update the price for private sector debt holdings $q_1(d, A)$ using equation (20), given the debt recovery schedule.

7. Use the updated price of private sector debt to repeat steps 3 to 6 until the convergence criterion $\max\{q_0(d, A) - q_1(d, A)\} < \epsilon$ is met.

8. Finally, solve the bargaining problem given the converged private sector debt price schedule and compute the new recovery rate $a_1(d, A)$ for all $d$ and $A$.

9. Use the updated debt recovery schedule to repeat steps 2 to 8 until the convergence criterion $\max\{a_0(d, A) - a_1(d, A)\} < \epsilon$ is met.

10. Compute business cycles statistics from 100 samples of data containing a default. If the model business cycles match the data stop; otherwise adjust parameters and grid, and go to step 2.