The Public and Private Marginal Product of Capital

Matt Lowe  
University of Oxford  
matt.lowe@economics.ox.ac.uk

Chris Papageorgiou  
International Monetary Fund  
cpapageorgiou@imf.org

Fidel Perez-Sebastian  
University of Alicante  
fidel.perez@ua.es

February 2012  
Preliminary and Incomplete Comments Welcome

Abstract

Why doesn’t more capital flow to developing countries as predicted by the neoclassical model? Is the explanation simply that the cross-country marginal product of capital (MPK) is equalized, and if so, why? We revisit these issues by unpacking MPK measurement into public and private, since there is good reason to believe that the process of MPK determination is enormously different across the two sectors, especially in developing countries. We do so by calculating MPK schedules across the two sectors, for a large sample of advanced and developing countries. The key findings are as follows: the overall MPK across countries is slightly upward sloping. Disaggregation suggests that the positive slope is driven by the public sector – we find the public MPK to be upward sloping but the private MPK to be flat. We interpret this as the result of overinvestment and inefficiencies within the governments of developing countries – put another way, governments are not profit-maximizers in the

*The authors thank Andy Berg, Francesco Caselli, Paul Collier, Raphael Espinoza, Alvar Kangur, Aart Kraay, Era Dabla-Norris, Cathy Pattillo and seminar participants at the IMF, Royal Holloway University of London, and the Conference on Economic Development and Growth 2011 at the University of Barcelona, for valuable comments and suggestions. The views expressed in this study are the sole responsibility of the authors and should not be attributed to the International Monetary Fund, its Executive Board, or its management.
context of investment. Based on this result, we advance new explanations for
the Lucas paradox related to the behavior of the public sector.

**JEL Classification:** O41, O47

**Keywords:** Marginal productivity of capital, private and public capital, capital
prices, return to investment, government investment inefficiencies, capital flows,
financial aid.
1 Introduction

The Lucas paradox is a focal point for many big picture issues in development; whether the efficacy of aid, the extent of international capital market frictions, or the importance of institutions and complementary factors. Lucas (1990) stated the paradox as follows: if capital-labor ratios are so low in poor countries and thus returns high as the standard one-sector growth model predicts, why doesn’t more capital flow from rich to poor countries?

The paradox pre-supposes a downward-sloping financial return to investment in the cross-section of nations. Intuitively, there are only a small set of possible explanations: (i) either the return has been mis-measured, and it is not actually downward-sloping; (ii) the return is downward-sloping but capital movement is restricted by capital market imperfections; or (iii) investors in some way defy standard theories of profit maximization (e.g. there is some home bias which permits returns differentials to persist). Lucas himself posited that the explanation could be that of failing to account for complementary factors to physical capital, such as human capital, resulting in an overstating of the MPK. Lucas placed little credence on the argument of capital market frictions.

The MPK is the most common measure employed to approximate the return to investment, in an attempt to resolve the paradox. Unfortunately estimating the MPK is no easy task. Several approaches exist: among them, comparison of interest rates across countries, production function estimation, and calibration. Caselli and Feyrer (2007) (CF from hereafter) argue that these approaches have collectively failed at producing reliable and comparable estimates of the cross-country MPK. In a persuasive, yet provocative, contribution to the literature, they present the case for direct MPK estimation using easily accessible macroeconomic data. Their approach assumes competitive markets and imposes no restrictions on production functions

\[ \text{MPK} = \frac{P_Y Y}{P_K K} \]

1 See Banerjee and Duflo (2005) for an extensive review.

2 According to CF, the comparison of interest rates in developing countries is problematic because markets are heavily distorted, identification in production function estimation presents many caveats, and calibration exercises rely on specific forms of the production function.

3 Taylor (1998) measures the MPK similarly for Argentina, and Bai et al. (2006) use a similar approach to measure the return to capital (both in aggregate and by sector/region) in China, though they use current price data to measure \( P_Y Y / P_K K \) rather than real data followed by a price adjustment as in CF.
other than that of constant returns to scale.

CF’s main contribution is that they derive a MPK measure that is more suitable for the purpose of international credit flows. The standard MPK derived from the one-sector growth model is not a good measure of capital returns because it provides output per unit of physical capital invested. Multisector models imply that the financial return to investment is better proxied as output per unit of output invested. By making two reasonable adjustments to a naïve measure of the MPK, Lucas Paradox resolved, CF find that the cross-country MPK is roughly flat.

In this paper, we give a more nuanced analysis along one dimension: that of sectoral returns, distinguishing between the public and private MPK. The private and public distinction is important for a host of reasons. Foremost amongst these is that the theory behind MPK determination is likely to differ significantly between the two sectors. There is much literature elsewhere with results that hinge on the contrasting behavioral idiosyncrasies of public and private agents (e.g., Becker (1957), Fama (1980), Besley and Burgess (2002), Robinson and Torvik (2005)). The empirical evidence in Keefer and Knack (2007) is also consistent with the notion of governments as non-maximizers. To be consistent with MPK equalization, public investment should be highest where the returns are highest. Keefer and Knack find instead that public investment is dramatically higher in countries with low-quality governance and limited political checks and balances. Their interpretation of this result is that governments use public investment as a means for rent-seeking. If the public sector maximizes a wholly different objective function to the private sector, capital allocation and the resultant MPK should be determined differently.

Second, the private and public sector tend to make different types of investment. The public sector tends to invest where markets fail: where social returns exceed private returns, where the capital is non-rivalrous and non-excludable and where high fixed costs make a natural monopoly a strong possibility. In short, public and pri-

---

4This within-country heterogeneity in returns is, for example, a key theme in Banerjee and Duflo (2005).
5Robinson and Torvik (2005), for example, aim to explain why governments don’t act like profit maximizers when it comes to investing. In particular, the model explains the political motivation behind the construction of white elephants. Politicians construct these inefficient projects when they find it difficult to make credible promises to political supporters. The general point of this and other political economy models is that governments are driven more by an electoral motive than by a profit motive.
vate capital should be considered imperfectly substitutable in a country’s production function. In this sense, the overall MPK is misleading, whilst the private and public MPK are more informative.

Third, following Pritchett (2000), the separation between public and private capital is warranted in light of public investment inefficiency. As Pritchett emphasizes, there is no plausible behavioral model by which we would expect public investment to be efficient in the same way that might be expected of private investment. Caselli (2005) echoes this sentiment and argues for the future separation of public and private investment, when appropriate data comes available, in the context of development accounting. In this paper, we use data that makes this separation possible.

Pritchett (2000) and Caselli (2005), among others, thought that data issues would make it near impossible to convincingly estimate the private and public MPK. One main contribution of this paper is to break the impasse and carry out this exercise for the first time. To do this, we employ improved data on the sectoral share of investment from IMF’s World Economic Outlook (WEO). This data permits us to estimate the private and public MPK for a broad sample of advanced and developing countries.

We find that the overall MPK is slightly upward sloping, with this result driven by public sector investment behavior. After disaggregating, we find the private MPK to be flat and the public MPK to be upward-sloping. Our results point to the importance of public sector inefficiencies and political factors (rather than economic returns) driving public investment across countries. This distinct behavior of the public sector promotes new explanations of the Lucas Paradox.

We proceed as follows. Section 2 takes a close look at the primary sources of the data used to disaggregate total capital into its public and private components and discusses the steps followed to calculate the public and private MPK. Section 3 presents and discusses the main results of the paper. Section 4 extends the main analysis in two directions: by incorporating inefficiencies in the measurement of MPK, and by examining the effects of allowing for public and private capital in a two-sector neoclassical growth model. Section 5 concludes.
2 Data

In this section we show in detail the steps followed to construct the public and private marginal product of capital. The relevant price-corrected MPKs are as follows:

\[ MPKP = \beta \frac{P_y Y}{P_k K_p}; \quad MPKG = \gamma \frac{P_y Y}{P_k K_g} \]

where \( \beta \) and \( \gamma \) denote the shares of private and public capital in income, respectively. Our core sample comprises fifty developing and developed countries with public, private and overall MPK data in 2006.\(^6\) We also look at time series data from 1990, with the sample size beginning at fifty-two, but falling to forty-eight across 1990-2009. As in CF, the main constraint on sample size is due to the need for data on the overall capital share from Bernanke and Gurkaynak (2001).

We measure the cross-country private and public MPK using current price local currency data from World Development Indicators (WDI), rather than real data from Penn World Tables (PWT) adjusted for relative price differences as in CF. The use of current price local currency data is preferred here since it side-steps any reliance on PPP adjustments and extrapolated ICP data shown to be quite unstable for non-OECD countries (see Johnson et al. (2011)). In addition, it has been argued elsewhere (e.g. Knowles (2001)) that investment shares are more accurately measured using local price data, rather than data from PWT. In any case, the two approaches yield essentially the same results, as will be seen. The data we require are: income shares of public and private capital (\( \beta, \gamma \)), GDP in current price local currency (\( P_y Y \)), public capital (\( P_k K_g \)) and private capital (\( P_k K_p \)).

Current price local currency data on GDP and investment are taken from WDI. In principle, each capital series could be obtained by using the perpetual inventory method on current price historic investment data, deflated each year by a sector-specific investment deflator. In practice however, only a common investment deflator exists.\(^7\) In applying this common deflator, we are constrained in identifying differences

---

\(^6\) Though we have data for subsequent years until 2010 (sample sizes of 49, 49, 48 and 23 in 2007, 2008, 2009 and 2010, respectively).

\(^7\) This investment deflator is derived from WDI data as \(100^{*}(\text{current price local currency gross fixed capital formation}/\text{constant price local currency gross fixed capital formation})\). Missing constant investment data is set equal to the product of constant price GDP and gross fixed capital formation as a proportion of GDP, for countries with available data.
in the relative price of capital faced by the public and private sector. But for the baseline estimation, the public and private MPK are adjusted by the same price ratio.

With current and constant price investment data, the next step is to split these investment flows into their private and public sector constituents. This split is crucial as it drives the resultant differences in the private and public MPK. To do this disaggregation, we use private investment share data from IMF’s World Economic Outlook (WEO), as was done in Gupta et al. (2011). For the sample of fifty countries in 2006, the mean number of time series observations of the private investment share is thirty-three (ranging from a minimum of twelve to a maximum of forty-nine). Before total investment is disaggregated, the first available observation of the private investment share is extrapolated back to the first year of investment data.

In the absence of any investment data at all prior to 1960, it is necessary to set initial conditions for both the public and private capital stocks. As is common practice (given the notion of a steady-state capital stock), we set the initial condition, $K_{j0}$, to $I_{j0}/(g_j + \delta_{j0})$ where sector is indexed by $j = p, g$. $I_{j0}$ is current price investment in the first year available, $g_j$ is the country- and sector-specific average growth rate of constant price investment over the first twenty years of available data, $\delta_{j0}$ is the relevant depreciation rate for the first year of available investment data, with the pattern of depreciation rates taken from Gupta et al. (2011). Caselli (2005) shows that sufficiently recent capital measures tend to be insensitive to the exact assumptions made on these initial conditions. Armed with disaggregated investment and deflator data, assumptions on initial conditions and a pattern of depreciation rates, we apply the perpetual inventory method to construct current price capital series for each country as follows:

---

8 For seven countries (Austria, Denmark, Spain, Greece, Ireland, Jamaica and Sweden) investment share data was missing from the latest WEO. We opted to take the share data from WEO 2003, using forecasted shares for the years 2004-2008.

9 Since a negative $g_j$ could result in implausibly large or impossibly negative initial conditions, the measure was bounded at zero. For the core sample of fifty countries, this bounding only affected the public capital initial condition for Zambia.

10 The depreciation rates employed in Gupta et al. (2011) are as follows (all 1960-2008): (1) Public sector: 2.5% in LICs, 2.5% rising to 3.4% in MICs and 2.5% rising to 4.3% in Advanced; (2) Private sector: 4.25% in LICs, 4.25% rising to 7.6% in MICs, 4.25% rising to 9.6% in Advanced. We extrapolated the 2008 income- and sector-specific depreciation rates to 2009 and 2010. The underlying empirical and intuitive basis for this pattern of depreciation rates can be found in Arslanalp et al. (2010).
\[ P_{kt}K_{jt} = (1 - \delta_{jt}) \left( \frac{P_{kt}}{P_{kt-1}} \right) P_{kt-1}K_{jt-1} + I_{jt} \] (1)

so that

\[ P_{kt}K_{jt} = (1 - \delta)^t \left( \frac{P_{kt}}{P_{k0}} \right) I_{j0} + \frac{I_{j0}}{g_j + \delta_{j0}} + \sum_{i=1}^{t} (1 - \delta)^{t-i} \left( \frac{P_{kt}}{P_{k0}} \right) I_{ji} \] (2)

The total capital stock is then simply set equal to the sum of the private and public stocks. The capital measures become less sensitive to the initial conditions and investment share extrapolation as \( t \) becomes closer to the present. This suggests that time series results have to be interpreted with greater care than the cross-section results in 2006.

Having constructed public and private capital stocks, the remaining specification choice is to choose income shares \( \beta \) and \( \gamma \). Unlike the share of capital in income, these shares cannot be straightforwardly derived from national accounts data. We proceed instead by taking the overall share of reproducible capital, \( \beta + \gamma \), to be equal to the share data used by CF \( (\alpha_k) \). This share data derives initially from Bernanke and Gurkaynak (2001)\(^{11}\) who compute the overall capital share as one minus the labour share in GDP. CF then make an additional adjustment using wealth data from World Bank (2006) in order to account for natural capital. The result is data on the share of reproducible capital in income, \( \alpha_k \).

Next, we take the composition of this reproducible capital share to be consistent with the results of columns (2) and (3) in Table 6 of Gupta et al. (2011). This approach treats the production function regressions in Gupta et al. (2011) with some confidence. Their approach is to estimate system-GMM panel regressions assuming a Cobb-Douglas production function with skill-adjusted labour, private and public capital as its arguments. Since the estimation is in logs, each coefficient captures the income share of the associated factor input – provided that the identification is credible. Still, we don’t take the absolute coefficients for our measurement since the aim is to maintain full country-specificity of the reproducible capital share in income. In addition to this, we place more credence on shares derived from national accounts

\(^{11}\)Bernanke and Gurkaynak (2001) themselves extended an earlier dataset compiled in Gollin (2002).
as opposed to those derived from regression estimates. With this in mind, we use the results in Gupta et al. (2011) to infer only the relative income shares of public and private capital for income groups, with $\beta/(\beta + \gamma) = 0.63$ in Middle Income and Advanced economies and $\beta/(\beta + \gamma) = 0.48$ in Low Income Countries (LICs) (i.e. the relative income share of public capital is lower in richer countries). This is certainly an imperfect approach to measuring income shares since we fail to identify full heterogeneity in relative shares across countries, and even for the income groups, the regression estimates may be suspect if endogeneity problems persist. However, the results of Gollin (2002) provide at least some support that there is no systematic relationship between income levels and factor shares; that is the parameters of the aggregate production function are broadly similar across countries. If this is the case, assuming away full cross-country heterogeneity in relative public to private capital shares should not affect our estimates substantively.

3 Public and Private MPK Calculations

With the necessary data at hand we turn to calculating each country’s private and public MPK.\textsuperscript{12} Table 1 presents baseline summary statistics for 2006, unless stated otherwise.\textsuperscript{13} It is reassuring that the only difference between our country sample and that of CF is the loss of Jamaica and Trinidad and Tobago due to a lack of updated investment data covering these countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>Ccode</th>
<th>y</th>
<th>Sh#</th>
<th>MPK(96)</th>
<th>MPK</th>
<th>MPKG</th>
<th>MPKP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algeria</td>
<td>DZA</td>
<td>14560.1</td>
<td>21</td>
<td>0.03</td>
<td>0.05</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>Australia</td>
<td>AUS</td>
<td>75219.6</td>
<td>23</td>
<td>0.08</td>
<td>0.09</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td>Austria</td>
<td>AUT</td>
<td>73793.3</td>
<td>40</td>
<td>0.08</td>
<td>0.09</td>
<td>0.20</td>
<td>0.07</td>
</tr>
<tr>
<td>Belgium</td>
<td>BEL</td>
<td>77973.8</td>
<td>16</td>
<td>0.08</td>
<td>0.10</td>
<td>0.25</td>
<td>0.07</td>
</tr>
<tr>
<td>Bolivia</td>
<td>BOL</td>
<td>7933.45</td>
<td>30</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>Botswana</td>
<td>BWA</td>
<td>19477.6</td>
<td>31</td>
<td>0.14</td>
<td>0.16</td>
<td>0.12</td>
<td>0.19</td>
</tr>
</tbody>
</table>

\textsuperscript{12} In future work we will explore the sensitivity of the results to different assumptions regarding public investment efficiency, factor shares and relative prices.

\textsuperscript{13} Ccode refers to each country’s isocode, y is PPP Real GDP Per Worker from PWT 7, Sh# indicates the number of time series observations of sectoral investment shares, MPK(96) refers to CF’s MPK measure for 1996 using PWT data.
<table>
<thead>
<tr>
<th>Country</th>
<th>Code</th>
<th>Sh#</th>
<th>MPK(96)</th>
<th>MPK</th>
<th>MPKG</th>
<th>MPKP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burundi</td>
<td>BDI</td>
<td>702.85</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Canada</td>
<td>CAN</td>
<td>68609.3</td>
<td>32</td>
<td>0.07</td>
<td>0.09</td>
<td>0.17</td>
</tr>
<tr>
<td>Chile</td>
<td>CHL</td>
<td>27516.7</td>
<td>26</td>
<td>0.09</td>
<td>0.11</td>
<td>0.25</td>
</tr>
<tr>
<td>Colombia</td>
<td>COL</td>
<td>17441</td>
<td>42</td>
<td>0.06</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>Congo, Republic of</td>
<td>COG</td>
<td>5138.94</td>
<td>42</td>
<td>0.02</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>CRI</td>
<td>23514.9</td>
<td>20</td>
<td>0.03</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>Cote d'Ivoire</td>
<td>CIV</td>
<td>3352.81</td>
<td>41</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Denmark</td>
<td>DNK</td>
<td>67132</td>
<td>31</td>
<td>0.08</td>
<td>0.11</td>
<td>0.24</td>
</tr>
<tr>
<td>Ecuador</td>
<td>ECU</td>
<td>14134.6</td>
<td>32</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Egypt</td>
<td>EGY</td>
<td>13872.9</td>
<td>42</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>El Salvador</td>
<td>SLV</td>
<td>15734.4</td>
<td>30</td>
<td>0.17</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>Finland</td>
<td>FIN</td>
<td>65269</td>
<td>12</td>
<td>0.08</td>
<td>0.10</td>
<td>0.17</td>
</tr>
<tr>
<td>France</td>
<td>FRA</td>
<td>68105.4</td>
<td>15</td>
<td>0.08</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>Greece</td>
<td>GRC</td>
<td>61528.3</td>
<td>40</td>
<td>0.05</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>Ireland</td>
<td>IRL</td>
<td>75376.3</td>
<td>49</td>
<td>0.11</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>Israel</td>
<td>ISR</td>
<td>60113.8</td>
<td>27</td>
<td>0.11</td>
<td>0.14</td>
<td>0.27</td>
</tr>
<tr>
<td>Italy</td>
<td>ITA</td>
<td>70881.8</td>
<td>23</td>
<td>0.08</td>
<td>0.10</td>
<td>0.18</td>
</tr>
<tr>
<td>Jamaica</td>
<td>JAM</td>
<td>20282.8</td>
<td>40</td>
<td>0.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>JPN</td>
<td>64150.5</td>
<td>31</td>
<td>0.08</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>Jordan</td>
<td>JOR</td>
<td>14078.7</td>
<td>48</td>
<td>0.09</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>Korea, Republic of</td>
<td>KOR</td>
<td>48077.8</td>
<td>41</td>
<td>0.1</td>
<td>0.12</td>
<td>0.19</td>
</tr>
<tr>
<td>Malaysia</td>
<td>MYS</td>
<td>25977.6</td>
<td>42</td>
<td>0.06</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td>Mauritius</td>
<td>MUS</td>
<td>19235.3</td>
<td>38</td>
<td>0.12</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>Mexico</td>
<td>MEX</td>
<td>30136.5</td>
<td>38</td>
<td>0.09</td>
<td>0.14</td>
<td>0.20</td>
</tr>
<tr>
<td>Morocco</td>
<td>MAR</td>
<td>8860.36</td>
<td>41</td>
<td>0.09</td>
<td>0.10</td>
<td>0.19</td>
</tr>
<tr>
<td>Netherlands</td>
<td>NLD</td>
<td>75013.1</td>
<td>31</td>
<td>0.09</td>
<td>0.11</td>
<td>0.16</td>
</tr>
<tr>
<td>New Zealand</td>
<td>NZL</td>
<td>52261.3</td>
<td>26</td>
<td>0.05</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Norway</td>
<td>NOR</td>
<td>94797.3</td>
<td>21</td>
<td>0.08</td>
<td>0.13</td>
<td>0.19</td>
</tr>
<tr>
<td>Panama</td>
<td>PAN</td>
<td>18734.6</td>
<td>42</td>
<td>0.06</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td>Paraguay</td>
<td>PRY</td>
<td>7943.12</td>
<td>42</td>
<td>0.09</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>Peru</td>
<td>PER</td>
<td>13645</td>
<td>42</td>
<td>0.09</td>
<td>0.14</td>
<td>0.21</td>
</tr>
<tr>
<td>Philippines</td>
<td>PHL</td>
<td>6540.46</td>
<td>25</td>
<td>0.09</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>Portugal</td>
<td>PRT</td>
<td>38554.6</td>
<td>16</td>
<td>0.08</td>
<td>0.08</td>
<td>0.14</td>
</tr>
<tr>
<td>Singapore</td>
<td>SGP</td>
<td>84558.7</td>
<td>32</td>
<td>0.14</td>
<td>0.20</td>
<td>0.25</td>
</tr>
<tr>
<td>South Africa</td>
<td>ZAF</td>
<td>19752.7</td>
<td>40</td>
<td>0.08</td>
<td>0.13</td>
<td>0.19</td>
</tr>
<tr>
<td>Spain</td>
<td>ESP</td>
<td>58428.5</td>
<td>40</td>
<td>0.09</td>
<td>0.10</td>
<td>0.17</td>
</tr>
<tr>
<td>Sri Lanka</td>
<td>LKA</td>
<td>8347.63</td>
<td>34</td>
<td>0.06</td>
<td>0.08</td>
<td>0.14</td>
</tr>
<tr>
<td>Sweden</td>
<td>SWE</td>
<td>70425.8</td>
<td>49</td>
<td>0.07</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>Switzerland</td>
<td>CHE</td>
<td>64887.2</td>
<td>31</td>
<td>0.06</td>
<td>0.28</td>
<td>0.30</td>
</tr>
</tbody>
</table>
Next we attempt to reproduce CF’s main results on the overall MPK using our baseline dataset. Let $Y$ and $P_y$ be the quantity and price of final goods; $K$ and $P_k$ the quantity and price of capital goods; $\alpha_w$ the share of reproducible plus natural capital; and $\alpha_k$ the share of reproducible capital. By construction, $\alpha_w > \alpha_k$. CF define the following MPK measures:

$$MPKN = \alpha_w \frac{Y}{K}; \quad MPKL = \alpha_k \frac{Y}{K}; \quad PMPKN = \alpha_w \frac{P_y Y}{P_k K}; \quad PMPKL = \alpha_k \frac{P_y Y}{P_k K}.$$ 

$MPKN$ is the naïve MPK, while $MPKL$ adjusts the income share of capital to exclude natural capital, and $PMPKN$ controls for the relative price of final goods to capital products. Finally, we arrive at the preferred measure, $PMPKL$, which incorporates natural-capital with relative-output-to-capital-price. Table 2 presents summary statistics of CF’s four main MPK measures with increasing sophistication for 1996. CF argue on the basis of the final measure ($PMPKL$) that the MPK is essentially flat.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Mean</th>
<th>Std. Dev</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPKN</td>
<td>0.20</td>
<td>0.10</td>
<td>0.07</td>
<td>0.49</td>
</tr>
<tr>
<td>PMPKN</td>
<td>0.14</td>
<td>0.05</td>
<td>0.08</td>
<td>0.27</td>
</tr>
<tr>
<td>MPKL</td>
<td>0.10</td>
<td>0.06</td>
<td>0.03</td>
<td>0.33</td>
</tr>
<tr>
<td>PMPKL</td>
<td>0.08</td>
<td>0.03</td>
<td>0.01</td>
<td>0.17</td>
</tr>
</tbody>
</table>

CF’s results are also illustrated in Figure 1. Fitted lines are added to obtain a better sense of the implied relationship between the MPK measures and income.
Figure 1: Main CF Results
levels. The top-left panel in Figure 1 shows clearly that the naïve MPK implied by the standard neoclassical one-sector model is downward sloping. As we move to the right or down, each of the two adjustments disproportionately reduces the MPK in developing countries, since developing countries tend to have a higher share of natural capital in income and tend to face a higher relative price for capital goods.\textsuperscript{14} The preferred MPK measure, $PMPKL$ (bottom-right panel), is actually slightly upward sloping (the fitted line is in fact statistically significant at the one percentage level, though its economic significance could be disputed\textsuperscript{15}). CF’s main result then suggests that international capital markets do a good job of allocating capital efficiently across countries; and that, if anything, there are upward financial frictions in capital movements\textsuperscript{16}, that is, efficiency requires that capital flows from poor to rich nations and not the other way round. Provocative as it is, there is, in our view, not much cause to argue with the result presented in CF. Both adjustments are reasonable and make intuitive sense. Whilst the individual country MPK estimates could be challenged on the basis of data, the overall shape of the MPK schedule is quite robust.\textsuperscript{17}

Closer inspection of the charts demonstrates that this result obtains mostly from CF’s adjustments affecting developing, not advanced economies. Focusing our attention on advanced economies, with Portugal (PRT) the poorest of this group, we can see that the naïve MPK measure ($MPKN$) is slightly increasing even without the two adjustments made in CF. The main finding of a non-decreasing MPK obtains from the impact of the adjustments on developing economies. Before the adjustment, there is no clear relationship between the return and income in developing nations, whereas after it a clear upward sloping relationship is unravelled.

The core analysis in the remainder of the paper focuses on shedding light on these intriguing results by turning attention to the distinction between private and public

\textsuperscript{14}Hsieh and Klenow (2003) point to the relatively low productivity in capital goods producing sectors in developing countries as a cause of the high relative price.

\textsuperscript{15}For example, as CF show, the misallocation of capital implies an overall efficiency loss of only 0.1% of global GDP.

\textsuperscript{16}Indeed, the evidence for upward financial frictions has been emphasized in Caselli’s subsequent work.

\textsuperscript{17}Having said that, other adjustments can be suggested which in principle have the ability to overturn the result of a flat MPK. Chirinko and Mallick (2008) draw attention to the role played by adjustment costs, finding that a large MPK differential re-emerges once adjustment costs are accounted for.
Table 3: The Composition of Public Investment

<table>
<thead>
<tr>
<th>Variable</th>
<th>LICs</th>
<th>MICs†</th>
<th>Advanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Public Share in Total Investment (2010)</td>
<td>42.6%</td>
<td>29.4%</td>
<td>16.6%</td>
</tr>
<tr>
<td>Mean Public Investment as % of GDP (2007)</td>
<td>6.0%</td>
<td>6.9%</td>
<td>3.9%</td>
</tr>
</tbody>
</table>

Source: World Economic Outlook, Penn World Tables

*LICs: low income countries
†MICs: middle income countries

Figure 2: The Composition of Investment
capital. Put differently, the analysis will attempt to unpack the results found by CF by examining the role of the public sector in capital allocation. Our key motivation stems from the observed variation of public investment across nations. Specifically, Table 3 and Figure 2 show that the public sector plays a disproportionately large role in investment in developing countries.\(^{18}\) Therefore disaggregating the MPK into private and public may have important implications for the slope of the MPK.

As explained in Section 3, our approach is to measure the cross-country MPK (overall, public and private) using current price data on income and capital along with income share data. Using current price data, we can estimate the MPK schedule for the exact same sample as CF, for the year 1996. This exercise is performed in Figure

\(^{18}\) The relationship is flat when public investment as a percentage of GDP is considered because of two opposing forces: whilst public investment as a proportion of overall investment falls in income, investment as a proportion of GDP rises in income.
Table 4: Current Price Public and Private MPK

<table>
<thead>
<tr>
<th>Measure</th>
<th>Year</th>
<th>Mean</th>
<th>Std. Dev</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPKG</td>
<td>1996</td>
<td>0.116</td>
<td>0.067</td>
<td>0.005</td>
<td>0.302</td>
</tr>
<tr>
<td>MPKP</td>
<td>1996</td>
<td>0.081</td>
<td>0.042</td>
<td>0.015</td>
<td>0.202</td>
</tr>
<tr>
<td>MPKG</td>
<td>2006</td>
<td>0.130</td>
<td>0.069</td>
<td>0.005</td>
<td>0.265</td>
</tr>
<tr>
<td>MPKP</td>
<td>2006</td>
<td>0.085</td>
<td>0.037</td>
<td>0.014</td>
<td>0.193</td>
</tr>
</tbody>
</table>

3, which shows that the current price approach yields essentially the same slightly increasing MPK as CF, and that little has changed over the 10-year period 1996-2006. The advanced economies remain bunched closely around a financial rate of return of ten percent; the developing nations have a similar mean but greater variation, with rates of return from below one to sixteen percent.

In what follows, we focus on the distinction between the private and the public MPK. We can explore the prior that the theory of MPK equalization applies more to one sector than the other, and investigate whether the time-variation of the distinct MPKs differ.

Charts contained in Figure 4 show the public MPK (MPKG), its private counterpart (MPKP), and their ratio (MPKG/MPKP) for 1996 and 2006. Table 4 shows the summary statistics. The charts are noticeably different between the public and the private sector, first and second rows, respectively. The following observations can be made, both from the graphs and the descriptive statistics. First, there is greater variation in the public MPK. This indicates that the most significant loss in world GDP may be due to the misallocation of public capital, not private capital.\(^\text{19}\)

Second, the public MPK is upward sloping whereas the private MPK is flat. The fitted lines for the MPKG measure (first row) are highly significant, whereas the ones for the MPKP measure (second row) are insignificant. In fact, the evidence here suggests that it is the returns in the public sector which make CF’s preferred MPK schedule upward sloping for rich and for developing countries. Once the public and private components of capital are separated, the return to investment relevant for markets, that is, the private MPK, suggest neither downward nor upward rigidities

\(^{19}\)To investigate this formally, it would be necessary to return to the approach of using real PPP-adjusted data in order to construct capital stock data that is comparable across countries. This exercise will be left as a future extension.
Figure 4: Public and Private MPKs
to international capital flows. An upward sloping public MPK, on the other hand, fits with political economy stories such as Robinson and Torvik (2005) and Keefer and Knack (2007) of public MPK determination.

Third, there is an interesting pattern amongst the advanced economies: the private MPKs are extremely similar, the public MPKs are much more dispersed, and the mean public MPK is higher. According to this, the graphs potentially tell another story: private capital is allocated efficiently in advanced economies, but inefficiently in poor countries; public capital is allocated inefficiently everywhere.

Efficiency requires not only that marginal returns are equalized across countries, but also across sectors. The ratio of the public to the private return, appearing in the third row of Figure 4, gives information about how countries deviate from this cross-sector equalization. These results should be interpreted with care, however, as they depend, more than before, on correct assumptions made on income shares. Still, the natural interpretation is that a ratio below one reflects a government that overinvests in public capital, whereas a number above one suggests underinvestment. Few nations show values around one – the degree of dispersion is indeed high. In 2006, 48%\(^{20}\) of developing countries (LICs and MICs) provide a ratio above one, making the case for underinvestment in public capital. The developing country sample is then fairly evenly split between over and underinvestment in public capital, within-country, according to this measure. Governments in advanced economies, in contrast, appear consistently to accumulate too little public capital. Greece is the only exception, and shows signs of overinvestment. Underinvestment in advanced economies may hide a provision of public goods by the private sector, something more difficult to argue for developing economies.

---

\(^{20}\)Thirteen out of twenty-seven.
Figure 5: Mean Value and Dispersion Time series
Figure 5 contains the temporal evolution from 1990 to 2009 of the mean values of the public and private returns to capital (first row), their standard deviation (second row), and the standard deviation of the ratio of public to private MPK (third row). The standard deviation of each of the MPKs reflects efficiency in the distribution of resources across countries, whereas the dispersion of the MPK ratio tells us about differences in the degree of under/overinvestment in public capital. To some extent, this dispersion may reflect the magnitude of divergence in political views towards public participation in investment across countries. Focusing on the first row, the annual means, public and private, have increased since 1990 in the full sample (left chart). The increase is stronger for public capital in all country groups. The rise in the private MPK is, on the other hand, evident in advanced nations (right chart), but not in developing economies (middle panel). The trend in the developed-world private MPK is most likely due to technical change, whereas the divergence between the two MPKs can be due to several factors such as a decrease in the relative inefficiency of the public sector and an increase in the private provision of public capital; further research is needed here.

The annual standard deviation of the MPK is more closely related to the concept of capital misallocation. In particular, a falling variation suggests more efficient allocation of capital worldwide. The second row in Figure 5 shows that private capital is persistently allocated more efficiently across countries than public capital – a result which may reflect the purer profit-maximising incentives of private agents, compared with those in government. The Figure also suggests that private capital has become more efficiently allocated across nations since 1990, whilst public capital has not. This result is driven by developing countries (middle panel); in this group we see that, in 1990, the standard deviation of the private and public MPK was roughly equal, but there is divergence over the subsequent two decades. The left chart suggests the opposite in advanced economies – the standard deviation of the public MPK falls whilst that of the private MPK does not. Though in 2009, the dispersion of the private MPK remains much smaller than that of the public MPK.

The left panel in the third row shows a decrease in the standard deviation of the relative MPK until about 2003, and a rise after that date. It suggests that governments converged in the degree of underinvestment of infrastructure until 2003
and then diverged. For the developing world (middle panel), there is also divergence in underinvestment policies until 1996. Divergence can be due to technologies that require different private-to-public capital ratios, or to government policies; differences in policy may be, for example, the result of different degrees of privatization.

4 Extensions

In this section we introduce two extensions to our baseline analysis. First, we try to correct public capital stocks for possible inefficiencies in public investment by using a recently developed measure by Dabla-Norris et al. (2011). Public investment inefficiencies are notorious in several developing and emerging economies, therefore incorporating them into our analysis is potentially important. Second, in the context of a partial equilibrium multisector neoclassical growth model that incorporates public and private capital, we investigate the effects of allowing for different public and private MPK slopes motivated by the empirical analysis. In addition, the model allows for public investment inefficiency and examines how such a distortion in the public sector may affect the core results of the neoclassical economy.

4.1 Incorporating inefficiencies in public investment and capital formation

Pritchett (2000) and Caselli (2005) correctly argue that in many countries only a fraction of the actual accounting cost of investment passes into the value of the public capital stock. It is then the case that the public capital stock suffers from an upwards bias measurement problem when the perpetual inventory method is applied to past investment flows. In our case that would imply that the MPK schedules could also be biased – perhaps the public MPK is not upward sloping after all. By overstating the public capital stock in developing countries where public investment efficiency is lower, the public MPK is understated. Here we carry out an efficiency adjustment to the public capital stock measures by using a newly constructed measure, the Public Investment Management Index (PIMI), as our inefficiency proxy. This is the approach also taken in Gupta et al. (2011) where the overall PIMI score (averaged across its four sub-components) is normalized to lie between zero and one, and subsequently
Table 5: PIMI-adjusting and Infrastructure

<table>
<thead>
<tr>
<th></th>
<th>Infrastructure Quantity*</th>
<th>Infrastructure Quality*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public Capital/GDP</td>
<td>0.1205</td>
<td>-0.0712</td>
</tr>
<tr>
<td>PIMI-adjusted Public Capital/GDP</td>
<td>0.5596</td>
<td>0.5573</td>
</tr>
<tr>
<td>N</td>
<td>70</td>
<td>47</td>
</tr>
</tbody>
</table>

*Infrastructure Quantity and Infrastructure Quality are from Calderón and Servén (2008).

used as the efficiency parameter in the capital accumulation equation.

Using the Dabla-Norris et al. (2011) data, correlations in Table 5 suggest that the PIMI-adjusted public capital stock is a better proxy for actual public capital than the unadjusted measure. The unadjusted measure of public capital (as % of GDP) is only weakly positively correlated with infrastructure quantity and actually negatively correlated with quality. The PIMI-adjusted measure on the other hand has a fairly strong positive correlation with both. As shown in Dabla-Norris et al. (2011), the PIMI possesses a lot of attractive features and is a good first proxy of public inefficiencies; however, it is also only an ordinal measure. Subsequent results using this approach should therefore be interpreted with care.

To efficiency-adjust the public capital stocks, we alter the method of Gupta et al. (2011) slightly by focusing only on the two components of the PIMI most closely related to investment efficiency – project appraisal and selection. These two components are summed for each country and normalized to lie between zero and one, resulting in a time-invariant efficiency measure $\xi$. The construction of the public capital stock becomes:

$$P_{kt}K_{gt}^{adj} = (1 - \delta_{gt}) \left( \frac{P_{kt}}{P_{kt-1}} \right) P_{kt-1}K_{gt-1}^{adj} + \xi I_{gt}; \quad (3)$$

so that

$$P_{kt}K_{gt}^{adj} = (1 - \delta)^t \left( \frac{P_{kt}}{P_{k0}} \right) \frac{\xi I_{j0}}{g + \delta_{g0}} + \sum_{i=1}^{t} (1 - \delta)^{t-i} \left( \frac{P_{kt}}{P_{k0}} \right) \xi I_{gi}, \quad (4)$$

which (because of time-invariance) implies the straightforward adjustment

$$P_{kt}K_{gt}^{adj} = \xi P_{kt}K_{gt}. \quad (5)$$

21 The other two components are implementation and evaluation.
Our assumptions here differ from Gupta et al. (2011) as in that paper they assume efficiency equal to one prior to 1960 – in a sense, inefficiency only ‘kicks in’ from the 1960s. Furthermore, we assume that efficiency equals one in advanced economies, that is, their public capital stock is not adjusted.\(^\text{22}\) To estimate the public MPK, we employ different estimates of the public capital income share from the regressions in Gupta et al. (2011) which use an efficiency-adjusted measure\(^\text{23}\) – this is their Table 6, columns (5) and (6). The private share in total capital income is assumed to be 0.66 in MICs and advanced economies; 0.68 in LICs.

Figure 6 compares the public MPK measured before (\(MPKG\)) to the PIMI-adjusted measure (\(eMPKG\)). The use of PIMI data causes a drop in the sample size from fifty to thirty-seven countries. The result, for both 1996 and 2006, is a potential resolution of the upward-sloping public MPK. However, these suggestive results demand careful interpretation. Upon introducing the concept of imperfect public investment efficiency, the measure of interest changes. Under the standard assumption of perfect efficiency (i.e. \(\xi = 1\) in the capital accumulation equation), the price-corrected MPK reflects well the returns to investment. Once we consider \(\xi < 1\), there is a disconnect between investment flows and capital accumulation. The correct measure of the returns to public investment is now

\[
MPIG_t = \frac{\delta Y_t}{\delta K_t} \frac{\delta K_t}{\delta I_t} = MPKG_t \cdot \xi_t = \gamma \frac{P_{yt}Y_t}{P_{kt}K_{g,t}^{adj}} \cdot \xi_t,
\]

which we call here \(MPIG\), the marginal product of public investment. It is crucial to notice that \(\frac{\delta K_t}{\delta I_t}\) depends only on efficiency at time \(t\), whereas \(\frac{\delta Y_t}{\delta K_t}\) is a function of the full history of efficiency (though with declining weight as we go further into the past). Under the assumptions of constant efficiency and Cobb-Douglas technology, the \(MPIG\) is in fact identical to the un-adjusted \(MPKG\) (first row, Figure 6) – this comes as the result of an exact offset with \(\xi\) cancelling out. Low historic efficiency may imply a smaller capital stock today and a correspondingly higher \(MPKG\) (as we see in the second row of Figure 6), but this is offset by the low present-day public investment efficiency. The exact offset is delivered with the assumption of Cobb-

\(^{22}\)There is no PIMI data available for advanced economies – but it is notable that the most developed countries in the PIMI sample tend to have high scores.

\(^{23}\)Recall that in this case, they take into account all four components comprising the PIMI when adjusting capital stocks.
Figure 6: Adjusted Public MPK
Douglas, though other technologies with diminishing returns to public capital would
give some kind of offset. The purpose of all this is to show that the result of an upward-
sloping return to public investment across countries is robust to the introduction of inefficiency in public investment, provided the inefficiency is either constant or at least
that present-day efficiency is a close proxy for ‘historic efficiency’. This proposition
that the $MPIG$ may be proxied reasonably by the $MPKG$ has the added benefit
that the reader may remain skeptical of the value of adjusting capital stocks using
the PIMI.

Even if the $MPKG$ schedule is the most useful for understanding actual public
investment returns, the $eMPKG$ schedule may still have a useful interpretation. In
some sense it hints at the returns to public investment possible if public investment
efficiency in developing countries was brought up to the level of advanced economies,
even leaving the greater relative price of capital in developing countries unchanged.
From a policy perspective, there seem to be two implications for bringing public
returns in developing countries in line with those in advanced economies: reform
public investment efficiency, and tackle the higher relative price of capital goods.

Returning to the adjusted public capital results, Figure 7 shows the time series
dispersion exercise repeated with the addition of the efficiency-adjusted public MPK,
only for those countries with PIMI data, including advanced economies. The efficiency
adjustment gives no new meaningful time variation since the PIMI is time-invariant.
The interesting result is that the efficiency adjustment does not overturn the initial observation that the variation in the public MPK is significantly higher than the variation in the private MPK, as one might expect. The efficiency-adjustment only reinforces this conclusion along with the finding that the private MPK variance has fallen since the 1990s whilst the variance of the public MPK has not.\textsuperscript{24}

4.2 A Two-Sector Model with Private and Public Capital

[Work in progress] The results in previous sections leave some open questions. For example, can other public sector inefficiencies affect our MPK measures? And in addition, why does the public MPK schedule slope up? What kind of inefficiencies can be behind it? We now present a simple framework that explore these issues. In particular, it focus on inefficiencies related to public capital prices. The model is close to the standard neoclassical multisector framework in CF, but contains more structure.

4.2.1 Production

Eliminate for simplicity time subscripts. The economy produces two final goods: a consumption product \((Y_c)\), and an investment product or physical capital \((Y_m)\). For that purpose, firms in sector \(j\) employ private capital \((K_{pj})\), public capital \((K_{gj})\) and other factors \((L_j)\) as inputs. The two goods are manufactured by the private sector under constant returns to scale and perfect competition according to

\[
Y_c = A_c K_{pc}^{\beta} K_{gc}^{\gamma} L_c^{1-\beta-\gamma},
\]

\[
Y_m = A_m K_{pm}^{\beta} K_{gm}^{\gamma} L_m^{1-\beta-\gamma}.
\]

Input elasticities are such that \(\beta, \gamma \in (0,1)\) and \(\beta + \gamma < 1\). Employing results in Herrendorf and Valentinyi (2008), we assume that both sectors display the same input intensities. The total factor productivity (TFP) parameters \(A_c\) and \(A_m\) are assumed to grow exogenously at rate \(g\).

\footnote{Whilst the \(\zeta\)-adjusted public MPK is based on a smaller sample (non-Advanced economies without PIMI data are dropped), we would expect this to make the standard deviation smaller, not larger. If anything, the graph understates the difference in variation by using non-comparable samples. Of course, the analysis can (and should) be repeated for the common sample of countries.}
The idea behind expressions (7) and (8) is that the public and private sectors invest systematically in different types of capital. It seems reasonable then that private and public capital be considered complements in the production function. In fact, there is some capital that the public sector is more willing to invest in; historically, for example, the public sector has been instrumental in the provision of health, education and infrastructure investment in many nations.

Even though investment goods in the model are entirely supplied by the private sector, their production can be financed either by private agents to increase firms’ stocks or by the public sector to provide infrastructure to the economy. We also allow physical capital to come from both domestic production and imports ($Y_m^*$). All this implies the following feasibility constraint, and motion equations for capital:

$$ Y_m^* + Y_m = I_p + \xi I_g, \quad (9) $$

$$ \dot{K}_g = \xi I_g - \delta K_g, \quad (10) $$

$$ \dot{K}_p = I_p - \delta K_p. \quad (11) $$

The variables $I_p$ and $I_g$ represent private and public funds available for investment in physical capital, respectively. The capital stock $K_g = K_{gc} + K_{gm}$, and $K_p = K_{pc} + K_{pm}$. The coefficient $\xi \in (0, 1]$ is a measure of government inefficiency in channeling funds to investment in line with Agénor (2010), and $\delta$ is the depreciation rate.

Profit maximization in the two production sectors imply the following FOCs for capital and other inputs in sector $j$:

$$ rP_k = P_c MPK_{pc} - \delta P_k = P_k MPK_{pm} - \delta P_k, \quad (12) $$

$$ \tau P_k = P_c MPK_{gc} - \delta P_k = P_k MPK_{gm} - \delta P_k, \quad (13) $$

and

$$ w = P_c MPL_c = P_k MPL_m. \quad (14) $$

In these conditions, $P_c$ and $P_k$ represent the prices of the consumption good and the investment good, respectively; $MPK_{pj}$ and $MPK_{gj}$ in turn denote the marginal product of private and public capital in sector $j$, respectively. Firms pay an interest rate $r$ to private savers for the capital borrowed, and a rate $w$ to each unit of other
inputs hired. The price $\tau$ is given by taxes and fees that private firms perceived as being paid for the use of public infrastructure, like airport fees for example.

Expressions (12) to (14) can be expanded to obtain

$$r + \delta = \frac{P_c}{P_k} \beta A_c \left( \frac{K_{gc}}{K_{pc}} \right)^\gamma \left( \frac{L_c}{K_{pc}} \right)^{1-\beta-\gamma} = \beta A_m \left( \frac{K_{gm}}{K_{pm}} \right)^\gamma \left( \frac{L_m}{K_{pm}} \right)^{1-\beta-\gamma},$$  \hspace{1cm} (15)

$$\tau + \delta = \frac{P_c}{P_k} \gamma A_c \left( \frac{K_{pc}}{K_{gc}} \right)^\beta \left( \frac{L_c}{K_{gc}} \right)^{1-\beta-\gamma} = \gamma A_m \left( \frac{K_{pm}}{K_{gm}} \right)^\beta \left( \frac{L_m}{K_{gm}} \right)^{1-\beta-\gamma},$$  \hspace{1cm} (16)

$$w = P_c(1-\beta-\gamma) A_c \left( \frac{K_{pc}}{K_{gc}} \right)^\beta \left( \frac{K_{gc}}{L_c} \right)^\gamma = P_k(1-\beta-\gamma) A_m \left( \frac{K_{pm}}{K_{gm}} \right)^\beta \left( \frac{K_{gm}}{L_m} \right)^\gamma.$$  \hspace{1cm} (17)

From these FOCs, it can be shown that the capital-labor ratio needs to be equalized between sectors, the relative price of goods is exclusively pinned down by the relative TFP, and the private to public capital ratio depends on input elasticities and prices. Mathematically,

$$\frac{K_{pc}}{L_c} = \frac{K_{pm}}{L_m} = \frac{K_p}{L},$$  \hspace{1cm} (18)

$$\frac{K_{gc}}{L_c} = \frac{K_{gm}}{L_m} = \frac{K_g}{L},$$  \hspace{1cm} (19)

$$\frac{P_c}{P_k} = \frac{A_m}{A_c},$$  \hspace{1cm} (20)

and

$$\frac{K_{pc}}{K_{gc}} = \frac{K_{pm}}{K_{gm}} = \frac{\beta r + \delta}{\gamma r + \delta},$$  \hspace{1cm} (21)

where

$$L_c + L_m = L,$$  \hspace{1cm} (22)

$$K_{pc} + K_{pm} = K_p, \text{ and } K_{gc} + K_{gm} = K_g.$$  \hspace{1cm} (23)

The total amount of other resources $L$ is assumed to grow at rate $n$. An important implication is that expression (20) suggest that relative output prices are only affected by inefficiencies related to the relative TFP.

### 4.2.2 MPK differentials

Let $P_y Y$ be the economy’s GDP, with the price $P_y$ representing the average price of final goods. Then

$$P_y Y = P_c Y_c + P_k Y_m.$$  \hspace{1cm} (24)
Expressions (7), (8), (18) to (20), (22) and (23) imply that

\[ P_y Y_c = P_c Y_c \frac{L}{L_c}. \]  

(25)

The price-corrected MPK measure constructed by CF \( PMPKL \) is just a weighted average across sectors. In terms of our model,

\[ PMPKL = \frac{P_c}{P_k} \left( \frac{K_p}{K} MPK_{pj} + \frac{K_g}{K} MPK_{gj} \right), \]  

(26)

with \( K = K_p + K_g \). The equality must hold for all \( j \). Focusing on the consumption-goods production activity, we can expand (26) using (18), (22) and (23) to obtain

\[ PMPKL = \frac{K_p}{K} \beta \frac{P_c Y_c}{P_k K_p} + \frac{K_g}{K} \gamma \frac{P_c Y_c}{P_k K_g} = (\beta + \gamma) \frac{P_c Y_c}{P_k K}. \]

Which employing the value of consumption-goods production implicit in (25) implies

\[ PMPKL = (\beta + \gamma) \frac{P_y Y}{P_k K}. \]  

(27)

Expression (27) provides a direct measure of the MPK that can be obtained using the physical capital share in income to approach \( \beta + \gamma \), GDP at domestic prices, and the capital stock at domestic prices. The key prediction of the multisector framework that CF exploit is that the financially-relevant MPK depends on the relative price of final-to-capital goods, \( P_y/P_k \).

However, equality (27) holds only if capital is efficiently allocated domestically. To see this, notice that in our model with private and public capital, the financial return to investment (abstracting for simplicity from capital gains) is given by the price-corrected MPK for the private sector \( MPKP \), \( \gamma + \delta \) in expression (12). In particular, focusing again in the \( c \) sector,

\[ MPKP = \frac{P_c}{P_k} MPK_{pc} = \beta \frac{P_c Y_c}{P_k K_{pc}}. \]  

(28)

Which from (21) to (23) become

\[ MPKP = \beta \frac{P_y Y_c}{P_k K_p} = \left( \beta + \gamma \frac{\tau + \delta}{\tau + \delta} \right) \frac{P_y Y}{P_k K}. \]  

(29)

Comparing \( PMPKL \) and \( MPKP \), they differ because input prices play a role in the latter measure. In fact, when payments to both types of capital are the same, that is, the public sector charges a fee for the use of public infrastructure equal to the market return \( (\tau = r) \), the two measures coincide.
4.2.3 Public investment inefficiency

These two inputs can, however, differ at least for two reasons. First, government’s inefficiency in investment, given by parameter $\xi$. Second, the government may not be driven by profit maximization in public goods provision.

More specifically, we can write

$$\tau = \mu \frac{r}{\xi}. \quad (30)$$

For each unit invested, the public sector needs to collect taxes or borrow debt in an amount equal to $1/\xi$. As a consequence, a price $\tau$ equal to $r/\xi$ would be the profit-maximizing one, that is, the efficiency-pricing one taking $\xi$ as given. But politicians might not be driven by profit maximization. They can charge then a price above or below $r/\xi$. This is captured by coefficient $\mu$ above, with $\mu > 0$.

In the same manner, we can derive from expression (13) the following price-corrected MPK for the public sector:

$$MPKG = \gamma \frac{P_y Y}{P_k K_g} = \left( \beta \frac{\tau + \delta}{r + \delta} + \gamma \right) \frac{P_y Y}{P_k K}. \quad (31)$$

Again, if the price of both physical capital types coincide ($\tau = r$), their returns are equalized. Any discrepancy between measures (29) and (31) will signal inefficiencies either related to pricing or related to investment channeling.

Expressions (29) to (31) imply that a lower degree of inefficiency (higher $\xi$) and a higher degree of subsidization of public goods (smaller $\mu$) will reduce the private-to-public capital ratio and, as a consequence, tend to increase $MPKP$ and decrease $MPKG$. Regarding the shape of $MPKG$ with respect to income, if efficiency pricing dominates, the $MPKG$ should be downward sloping because poorer nations show lower values of the inefficiency parameter $\xi$. A upward sloping $MPKG$, on the other hand, will suggest that political considerations that lead to subsidization dominate. Our results in Section 3 indicate that these political factors are the most important.

The effects described in the last paragraph are partial equilibrium, since the overall capital stock is assumed to remain constant in expressions (29) and (31). Differences between the price of the inputs, and the degree of inefficiency in public investment can affect the private and public capital stocks. To assess the impact of these general equilibrium effects on $MPKP$ and $MPKG$, we focus on the balanced growth path,
assuming that the share of public investment is exogenous. Employing expressions (10), (15), (21) and (30), it is standard to obtain that along the balanced growth path

\[ MPKP = \frac{\xi^\gamma}{(\mu r + \xi \delta)^{1-\beta}} \beta^\beta A_m [\gamma(r + \delta)]^{1-\beta} \left( \frac{\delta + n + g}{I_g} L \right)^{1-\beta-\gamma}, \]  

(32)

and

\[ MPKG = \frac{(\mu r + \xi \delta)^{\beta}}{\xi^{1-\gamma}} \gamma^{1-\beta} A_m \left( \frac{\beta}{r + \delta} \right)^{\beta} \left( \frac{\delta + n + g}{I_g} L \right)^{1-\beta-\gamma}. \]  

(33)

Therefore, identical qualitative effects of \( \mu \) remain as in partial equilibrium. So do the ones of \( \xi \) as long as the depreciation rate \( \delta \) is sufficiently small; otherwise higher levels of inefficiency can increase or decrease the public and private MPK. Overall, a larger \( \xi \) will decrease \( K_g \), which counteracts to some extent the partial equilibrium effect, but will not completely neutralized them. Larger levels of public investment reduce both public and private returns. To summarize, the two-sector model suggests that the overall capital stock does not deliver the right MPK in the presence of public sector peculiarities. Output prices only counteract the impact of inefficiencies related to the relative TFP. General equilibrium effects that operate through the economy’s stock of capital do not either neutralize them. The right measure for the financial return to investment is the one related to private capital. Differences between this one and the MPK of public capital give us an idea of the degree of public sector inefficiencies. Finally, both price-corrected MPKs, private and public, can be upward or downward sloping because of the public sector.

5 Conclusion

In their influential work Caselli and Feyrer (2007) deliver an intriguing result: after appropriately adjusting the share and relative price of capital, overall MPK estimates are shown to be broadly the same across a large group of advanced and developing economies, thus casting doubt on the international capital frictions explanation of Lucas’ Paradox. Motivated by the extensively documented and remarkable differences between public and private sector incentives, especially in developing countries, we have attempted in this paper to unpack the overall MPK into its public and private components.
First, we have used the most recent data from WDI, WEO and PWT 7.0, and consistent with recent independent work by Francesco Caselli, we have shown that the cross country schedule of the total MPK is not flat but rather significantly positively sloped. Second and most important, we have shown that the main reason for the upward sloping overall MPK is the substantial public sector overinvestment and inefficiencies in developing countries. Whilst the private MPK is found to be flat, the public MPK is upward sloping. This counter-intuitive feature of the public MPK can persist even in a context of frictionless capital markets – and it does so because of public sector behavior as non-maximizers.

In addition, we have pursued two extensions. We extended the analysis to incorporate a recently developed index of public investment management inefficiency in our measurement of public capital. While our application of this index is certainly not ideal, modifying public capital for public investment inefficiencies is hugely important as originally argued by Pritchett (2000). Whilst the adjustment suggests a flat public MPK, we show that the measure of interest, the marginal product of investment, is still proxied well by the unadjusted MPK, provided public investment efficiency is roughly constant. We also formalized the importance of relative prices and public sector inefficiencies in the context of a two-sector neoclassical growth model. The model gives structure to our interpretation of the results – that of political factors driving the upward sloping MPK.

These findings lend a new set of explanations to the Lucas Paradox, placing public sector idiosyncrasies center stage. The data supports our intuition – public agents act differently to private agents in the context of investment decisions. The result is a vastly different cross-country variation in the marginal product of capital across the two sectors. Our interpretation has emphasized the possibility of overinvestment by governments in developing countries facing few checks and balances and driven by an electoral motive. In contrast, the story we give is one of underinvestment in advanced economies – perhaps because of greater political pressure for a laissez-faire approach or due to increasing private participation in typically public investment ventures. Still, the interpretation we suggest should be further explored.

Taking these claims seriously, there are important implications relating to the role of foreign aid in building capital stocks. Caselli and Feyrer (2007) gave a skeptical
view on aid, concluding that greater flows of aid will only be displaced by capital outflows, given the flat MPK. Our disaggregation suggests a refinement of this view. Based on our findings, the provision of foreign aid is not growth enhancing but could rather facilitate inefficient overinvestment by the public sector, since aid is less accountable to returns than private sources of finance. However, given imperfect substitutability between private and public capital in the production function, this overinvestment leads not to capital outflows, but inflows of private capital, since the greater stock of public capital raises the returns to private capital. Displacement aside, to raise the returns to aid for investment projects, the results suggest a renewed focus on improving public investment efficiency and tackling the high relative cost of capital goods.

We started out on this research with one particular prior: that with careful measurement, the financial return to public capital would be found to be relatively high in developing countries reflecting the large needs in education and infrastructure, to name a few. We were stunningly wrong. It may be fitting to close with a story of Tanzania’s ability to attract foreign capital. Taking a walk in the busy streets of Dar es Salaam, the capital city, one is impressed by the vibrant private economic activity, entrepreneurship and the many bank branches (local and multinational) scattered across town. One gets the favorable impression that, although at embryonic stages, the private sector operates under close proximity to “market” conditions. A look at public goods (e.g. roads) and the provision of public services (e.g. power generation) signals clear deficiencies. Experts correctly insist on the major progress, including in the public sector, that Tanzania has been through over the last two decades as captured by the country’s seven percent average GDP growth. But by all accounts this progress is not sustainable unless capital starts to flow inwards from abroad. This paper points to public sector frictions rather than financial frictions or complementarities to low human capital or TFP as the key constraint to enhancing the MPK and with it, accelerating international capital inflows.
References


