Corruption and environmental policy: An alternative perspective *

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Abstract

We construct an overlapping generations model comprising of two distinct groups of agents, citizens and politicians. Each agent lives through two periods; childhood and adulthood. She makes choices only as an adult, based on her utility that depends on her own consumption and the human capital and environmental quality endowed to her offspring. Citizens decide upon the proportion of their income that declare to the tax authorities, balancing between their own consumption and their offsprings’s well being. Politicians on the other hand can peculate a part of the tax revenue allocated to education and environmental protection with the rates of peculation for each activity exogenously given. Politicians decide upon the allocation of the tax revenue between the two activities balancing a similar trade-off to that of citizens. In this context, two self-fulfilling stable equilibria can emerge, one with high tax evasion and high allocation to the more rent-seeking activity and one with low tax evasion and low allocation to the more rent-seeking activity. This outcome accords well with existing empirical evidence and outlines that environmental policies may fail in corrupt countries if they are meant to increase rent seeking instead of protecting the environment.

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1 Introduction

Corruption in its various forms is a long-lasting phenomenon prevalent in most contemporary societies. There are abundant examples not only in developing countries such as China, India, and Philippines and in transition economies such as Russia, but also in developed countries.\(^1\) Its detrimental effects have been extensively analyzed and cover a wide range of social and economic aspects. Corruption has been studied extensively by many social scientists including political scientists, sociologists, anthropologists and law scholars. A few economists have contributed to the study of corruption in the past but recently this field attracts growing interest.\(^2\) A number of different analytic approaches to corruption have been developed examining among other issues, the degree of benevolence of the policy maker, the role of institutions in determining the level of corruption and the effect of corruption on growth.

An interesting question arises with regard to the relationship between corruption and tax evasion. The literature indicates that corruption has a significant negative impact on the levels of tax revenue, that is, corruption of politicians significantly erodes the tax base and corrodes the morale of taxpayers.\(^3\) The effect of taxation and tax revenues on corruption although less clear, is evident since, for example, a higher tax rate can potentially induce more corruption. Furthermore, a higher level of tax evasion could induce a higher level of corruption.

Another important empirical finding is that the level of corruption depends on the activity that the politicians choose to finance. For some types of public spending embezzlement of public funds is easier and/or better concealed relative to other types. Thus, politicians and/or bureaucrats prefer to shift resources to areas in which a higher rate of embezzlement is possible, such as high-technology goods produced in oligopolistic markets. Studies

\(^1\) Extensive documentation of instances of corruption can be found in the four volumes of "The Politics of Corruption", edited by Robert Williams and associates.

\(^2\) For a recent survey of the economics literature on corruption see Aidt (2003).

\(^3\) Tanzi and Davoodi (2000) investigated the relationship between levels of corruption (measured by corruption perception indices) and GDP in a sample of 97 countries and found that higher corruption is consistent with lower revenues of all types of taxes, especially from income taxes. Litina and Palivos (2010) have analytically explored the relationship between corruption and tax evasion in an economy with a single public good.
have shown that corruption enhances the portion of military spending, public services and order, fuel and energy, relative to education. Furthermore, Tanzi and Davoodi (2000) show that natural resource abundance is an important determinant of corruption, and Hessami (2010) suggests that the perceived level of corruption increases the shares of spending on environmental protection.

In the present paper we examine the interaction between taxpayers and politicians when both have the option to behave in a corrupt manner. Specifically taxpayers have the option to evade part of their income, while politicians have the option to embezzle part of the tax revenue. The extend of embezzlement on the part of the politicians depends on the allocation between activities. They can allocate part of the total tax revenue on environmental protection / abatement activities and the rest on public education. Each activity implies a rent seeking rate that in our model is fixed and exogenous. Which of the two activities involves more rent seeking is open to various criteria, i.e. on whether each activity is human capital intensive, thereby involving transparent expenses, or technology intensive, thereby involving less transparent expenditure. For instance if spending on education is associated with teachers’ wages while environmental protection involves high technology investment on renewable energy and/or abatement, then evidently environmental protection is considered to be a more rent seeking activity.

What we observe in our model is strategic interactions between the two groups of agents. Whenever taxpayers observe that politicians direct the money to the more rent seeking activity, they react by increasing their evasion rate. On the contrary whenever they observe that the politicians are not corrupt and thus direct more money to the less rent seeking activity, they respond by increasing their compliance rates. Multiple equilibria may arise, one with low compliance rates and extensive rent seeking, and one with high compliance rates and low rent seeking. This interaction that builds a sense of reciprocity between taxpayers and politicians has been reported in the literature. Furthermore, a number of studies have shown that corruption

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6 For example, Alm, Mc Clelandand, and Schulze (1992) and Alm, Jackson and McKee (1992) report that the introduction of a public good in exchange for the taxes paid increases compliance rates.
seems to be contagious, or as Andvig and Moene (1990) put it "corruption may corrupt". Whenever taxpayers feel that politicians are corrupt or that their burden is not fair compared to others they choose to become more corrupt as well.\footnote{Spicer and Becker (1980) and Fortin, Lacroix and Villeval (2006) conducted lab experiments and found that taxpayers tend to evade more taxes if they believe that their tax burden is not fair. Scholtz and Lubell (1998) have conducted a survey study and have found that the higher the trust in government the lower the likelihood of non-compliance.}

With respect to environmental policy our paper yields the following results. If environmental protection involves transparent and effective expenditures, then there is a reciprocal response on the part of the taxpayers who praise politicians by increasing their compliance and consequently the overall tax revenue and thus environmental quality. If on the other hand, environmental protection is a means of raising rent seeking funds, that is, it involves less transparent expenditure, taxpayers punish the government by evading more taxes. This reduces total tax revenue and thus public expenditure on environmental protection / abatement, thereby reducing environmental quality.

This result accords well with real world evidence. We often observe corrupt countries choosing through non-transparent processes highly specialized abatement policies and/or investment in high technology renewable energy, as a means of increasing the rent seeking rates associated with them. In contrast, in less corrupt countries, more transparent processes are chosen and lower shares of public funds are allocated to the high rent-seeking activities. As a result, corrupt countries often face a lower environmental quality due to reduced overall tax revenue that leads to reduced public spending on environmental protection.

Furthermore, given that two stable equilibria can arise, one associated with high and the other with low corruption, we discuss how a higher level public authority could enforce the low corruption equilibrium by change the tax level and / or each activity’s rate of embezzlement. In our model the politician takes the tax and the embezzlement rates are exogenous and he chooses only the distribution of public funds between the two activities. In this sense she is a lower level politician, that is a bureaucrat. If the bureaucrat is corrupt but the higher level politician is not, the latter can influence
the level of corruption by selecting the appropriate levels of tax and/or the embezzlement rates.

Although the economic literature on environmental policy and corruption is limited, there are some important contributions. Two strands of literature have been developed attempting to analyze this topic. The first strand focuses on the interaction between corruption and natural resources, for which a negative relationship has been shown. The channel through which this negative effect occurs is that natural resources enhance rent-seeking activities and therefore may lower economic performance. Some recent papers examine the effect of institutions on the interaction between corruption and natural resources. Mehlum et al (2006) claim that natural resources can enhance economic growth if institutions are producer friendly and vice versa. Hodler (2006) relates ethnically fractionalized societies with natural resources and poor institutions, Collier and Hoefller (2009) examine the effect of democracy on growth on societies with rich natural resources, while Bhattacharyya and Hodler (2010) demonstrate that natural resources can feed corruption and examine how this effect depends on the quality of democratic institutions.

The second strand abstracts from natural resources and mainly focuses on environmental policy. This literature is rather limited and focuses on the effect that bureaucracy has on environmental policy. Pashigian (1985) explained how locational competition among regions with different growth rates affects the stringency of regulations in these regions. Cropper et al. (1992) and Helland (1998) report the effect of environmental interests, of political and budget considerations on US Environmental Protection Agency (US EPA) regulations. Lopez and Mitra (2000) examine the effect of corruption and rent seeking on the relationship between pollution and growth and the shape of the environmental Kuznets curve, while Fredriksson and Millimet (2000) and Fredriksson et al. (2003) examine the effect of corruption and rent seeking on US FDI, on the pollution haven hypothesis and on environmental policy stringency.

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8See for example Leite and Widemann (2002), Sala-i-Martin and Subramanian (2003) and Isham et al. (2005).
The present paper complements existing literature by giving an account of how the formation of public funds between environmental protection and education is affected by the interaction between citizens and politicians. Once the allocation of budget has been decided, other forms of corruption, namely lobbying activities, come into play that affect environmental stringency. At the same time, these activities and the prospect of rent seeking feed back into our model affecting the decisions of tax payer.

Section 2 of the paper introduces the benchmark model. We resort to a simple framework that allow us to obtain analytical results and discuss the intuition regarding the interaction between taxpayers and politicians. More realistic assumptions are introduced in section 3. These assumptions increase the complexity of our model and for this reason we resort to numerical solution, which is illustrated Section 4. We prove that the main results of the benchmark model hold in the more realistic framework. Section 5 concludes the paper.

2 The benchmark model

2.1 The economy

We assume an overlapping generations’ economy producing a single good. In each period \( t \) a generation of individuals of measure one is born. Each individual has a single parent and each individual lives through two periods: childhood and adulthood. In the first period of their lives, young agents enter the public education system where they acquire human capital. Following the literature\(^{10}\) we assume that the learning technology is given by

\[
\hat{h}_t = vH_{t-1} + AE_{t-1},
\]

where \( t \) denotes time, \( h_t \) the level of human capital acquired by an individual born at \( t - 1 \), \( H_{t-1} \) the average stock of human capital at time \( t - 1 \) and \( E_{t-1} \) the public spending on education in the same period. According to this human capital accumulation process, a young agent born in period \( t - 1 \), can pick up a fraction \( v \in [0, 1] \) of the existing (average) level of human capital \( H_{t-1} \) without any cost, simply by observing what the previous generation

\(^{10}\)See for example De Gregorio and Kim (2000) and Ceroni (2001).
The enhancement of an agent’s human capital even further is possible only with the use of resources. In this paper we consider only public education and hence the level of public spending enters the learning technology. The parameter \( A > 0 \) measures the efficiency of the public education system.

For simplicity, we assume that young agents do not make any decisions. Even their consumption is included into their parents’ consumption. In the second period of their lives, adult agents decide their consumption level, which also includes their offsprings’ consumption. Adult agents also care about their offsprings’ future prospects, social status and well being. We formalize their concerns by assuming that adult agents’ utility depends on the level of human capital and the quality of environment they hand over to the next generation.\(^1\)\(^2\) Formally, individuals born at \( t - 1 \), during their adulthood, at period \( t \), maximize the following utility function,

\[
u_t = c_t (h_{t+1} + Q_{t+1}),
\]

where \( c_t \) denotes adults’ levels of consumption, \( h_{t+1} \) their offspring’s human capital and \( Q_{t+1} \) the environmental quality enjoyed by their offsprings.\(^3\) Note that the presence of the offspring’s human capital level and environmental quality in parental utility function results in an agent’s vested interest in public education and public abatement. For simplicity we assume that marginal utility adults derive from improving their offsprings’ human capital and environmental quality is the same between the two activities.

Environmental quality is given by

\[
Q_t = Q_0 H_{t-1} - \psi H_{t-1} + \beta \Pi_{t-1}, \quad Q_0 > \psi,
\]

where \( Q_0 \) denotes the initial state of environmental quality and \( \psi \) and \( \beta \) are technological parameters denoting the environmental damage caused by production and the efficiency of public spending on abatement \( \Pi_{t-1} \), respectively.

\(^{11}\)The term \( 1 - v \) can be taken to capture the depreciation rate of the stock of knowledge.

\(^{12}\)We could alternatively assume that they care about their children’s health status, being enhanced by a cleaner environment or their future prospects as well, assuming that environmental quality enters both the utility function and the production function in the form of natural resources (we will illustrate such a case in the following section).

\(^{13}\)We could introduce a parameter measuring the strength of the altruistic motive, however this would further complicate our analysis without gaining intuition.
Equation (3) can be rewritten as \( Q_t = (Q_0 - \psi) H_{t-1} + \beta \Pi_{t-1} \), implying that environmental quality depends linearly on the average stock of human capital at \( t \).\(^{14}\) This specification derives from the assumption that the only productive factor is human capital. Despite being quite restrictive, this formulation allows us to derive analytical solutions and provide intuitive results. Furthermore, the main results derived from this model hold also in a richer and more realistic formulation which will be introduced latter and illustrated through numerical simulations.

At the end of his childhood period each individual chooses, via a random process, one of the two following types of occupations: productive citizens in the private sector or politicians. Individual preferences are independent of occupation. For simplicity we assume that there is a continuum of agents within each group that is normalized to unity. The subscripts \( c \) and \( p \) are used to denote variables that are related to citizens and politicians, respectively.

**Citizen**

The citizen assumes production of a single good consumed by both groups. Using the appropriate normalization of units, we assume that the individual’s output and income \( y_{ct} \) equals the level of human capital.\(^{15}\)

\[
y_{ct} = h_t. \tag{4}
\]

Note that human capital is the only productive factor, implying that environmental quality \( Q_t \) supports individuals’ lives and increases their utility, but it does not contribute to production. The citizen’s income is taxed at the rate \( \tau \), assumed to be exogenous and time invariant. The citizen decides upon the fraction \( z_t \) of her income that declares to the tax authority. For simplicity, we assume that the citizen’s declaration is never audited by the authority; consequently, tax evasion does not involve any risk. Although in this setting tax is a voluntary contribution, citizens always declare a positive fraction of their income, as we will verify latter, because they care about their offsprings’ education and environmental quality.

**Politicians**

\(^{14}\) Given that public spending on abatement \( \Pi_{t-1} \) is financed by tax revenues which are collected from individuals’ income derived from their human capital, as it will be explained latter.

\(^{15}\) Since all agents have the same level of human capital we omit the subscript \( i = c, p \) from the level of human capital \( h_t \).
The politician’s role lies in allocating public funds between public education (a fraction $1 - \phi$ of the total tax revenue) and public abatement (a fraction $\phi$). The politician receives a fixed income, which for simplicity and without loss of generality we assume zero and she has the option to embezzle part of the total tax revenue. More specifically, she can embezzle a rate $(1 - \omega_q)$ of the funds directed to public abatement, and a rate $(1 - \omega_h)$ of the funds directed to public education. We assume that both $\omega_q$ and $\omega_h$ are exogenously given, strictly positive and less than one. Their relative values, that is, whether $\omega_q \geq \omega_h$, depends mainly on the choice of abatement actions, since education involves mainly transparent transactions, such as wages and standard equipment, and thus, it is associated with low rates of rent seeking. On the other hand, rent seeking rates on abatement can vary significantly depending on the choice of abatement technology. If public spending is directed toward actions relying on wages and standard equipment, such as reforestation, then rent seeking activities are rather limited. If however, technology-intensive abatement methods are chosen, then the rent seeking margin is much larger. The relevant literature suggests that the more technology-intensive is an activity, the higher corruption it usually involves (Tanzi and Davoodi, 1997).

The politician observes the values of $\omega_q$ and $\omega_h$ before allocating the available public funds between the two activities. For simplicity we assume that the politician is never investigated and hence peculation does not involve any risk. Still, given the politician’s vested interest in public education and environmental quality, $1 - \phi$ may not be equal to 1 (when $\omega_q < \omega_h$), or equal to 0 (when $\omega_q > \omega_h$) as it is verified below.

Total tax revenue collected in period $t$ is $R_t = z_t \tau h_t$. A fraction $(1 - \phi)z_t \tau h_t$ of this is earmarked for public education. Nevertheless, the politician peculates a fraction $1 - \omega_h$ of this sum. Hence, the actual amount spent on education $E_t$ is

$$E_t = (1 - \omega_h)z_t \tau h_t .$$ (5)

The remaining fraction $\phi z_t \tau h_t$ of the tax revenue is spend on preserving environmental quality. After the intervention of the politician, only a fraction

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16 The literature shows that the rate of rent-seeking in education is low but can vary across countries (Reinikka and Svensson, 2005) depending on the overall level of corruption and the expenses involved.
\( \omega_q \) of this amount is actually spend on abatement activities,

\[
\Pi_t = \varphi_t \omega_q z_t \tau h_t .
\]  

(6)

Evidently, adults’ decisions at time \( t \) regarding \( z \) and \( \varphi \) affect public spending on education and abatement and consequently the human capital and environmental quality enjoyed by their offsprings. Given the assumptions that adults care about their offsprings’ well being and that tax evasion and peculation are risk free, the only factor determining each type of agent’s choice is the other type of agent’s decision. That is, the citizen’s choice of tax evasion depends on the politician’s choice of peculation and vice versa. We assume that both individuals make their choices simultaneously. In what follows we examine the strategic interaction between the two types of individuals.

2.2 Individual optimization

**Citizen**

As mentioned above, in period \( t \) an adult citizen’s gross income is \( h_t \). A fraction \( z_t \) of this is declared to the tax authorities and an amount \( \tau z_t h_t \) is paid as income tax. Hence, each citizen’s disposable income is \( (1 - \tau) z_t h_t + (1 - z_t) h_t = (1 - z_t \tau) h_t \). Each citizen solves the following optimization problem,

\[
\max_{c_{ct}, z_t} c_{ct} [h_{t+1} + Q_{t+1}] ,
\]  

(7)

subject to

\[
c_{ct} = (1 - z_t \tau) h_t ,
\]

(8)

\[
c_{ct} \geq 0, \ 1 \geq z_t \geq 0 ,
\]

where \( h, Q, E \) and \( \Pi \) are determined by equations (1), (3), (5) and (6), taking \( \varphi_t, Q_0 \) and \( H_t \) as given.

Maximization of the above yields the citizen’s best response function:

\[
z_t = f(\varphi_t) = \frac{(A \omega h - \varphi_t \Omega_T) - \Psi}{2 \tau (A \omega h - \varphi_t \Omega_T)} ,
\]

(9)

where \( \Omega_T = A \omega h - \beta \omega_q \) and \( \Psi = Q_0 - \psi + v \). Note that since \( \varphi \leq 1, A \omega h - \varphi_t \Omega_T > 0 \), securing that concavity always holds. Inspection of equation
(9) reveals that an interior solution \((0 < z < 1)\) exists iff \((2\tau - 1)(A\omega_h - \varphi_t\Omega_T) < \Psi < A\omega_h - \varphi\Omega_T\). On the other hand, a corner solution \(z_t = \min\{0, \frac{\Psi}{-2\tau A\omega_h}\} (z_t = 1)\) will emerge if \(\Psi \geq (A\omega_h - \varphi_1\Omega_T) (\Psi \leq (2\tau - 1)(A\omega_h - \varphi_1\Omega_T))\), that is, if the rate of human capital transferred freely to the next generation, \(v\), is sufficiently high (low), the rate of degradation of environmental quality, \(\psi\), sufficiently low (high) and the initial state of the environment, \(Q_0\), high (low). Capturing a large (small) percentage of the existing human capital freely implies that parents have a weak (strong) incentive to invest in education and thus declare none (all) of their income to the tax authorities. Respectively, starting off with an environmental quality that is rather high (low) implies that parents have a weak (strong) incentive to invest in abatement and thus declare none (all) of their income to the tax authorities. Reversely when environmental damage from production, \(\psi\), is limited (extensive) then parents choose to evade all (none) of their income for abatement.

Whenever an interior solution emerges, the tax evasion rate \((1 - z_t)\) is negatively affected by the efficiency of the education system \((\Lambda)\) and the abatement technology \((\beta)\), and positively affected by the rates of rent seeking \((1 - \omega_h), (1 - \omega_q)\) and the tax rate \(\tau\). Noticeably, for sufficiently high \(\tau (\tau > \frac{1}{2})\) the tax evasion rate is never zero, since \(z_t < 1\).\(^{17}\) For a high level of tax, citizens will always choose to evade some fraction of their income; below this level they might choose to declare all their income to the tax authority.\(^{18}\)

**Politician**

Assuming zero income from other sources, the politician derives income only through the embezzlement of public funds directed to education and abatement\(^{19}\). That is, her income is, \(\varphi_t(1 - \omega_h)\tau z_t H_t + (1 - \omega_q)(1 - \omega_h)\tau z_t H_t\). The politician’s optimization problem is,

\[
\max_{c_{pt}, \varphi_t} c_{pt}[h_{t+1}, +Q_{t+1}].
\]

\(^{17}\)From (9), \(z_t < 1 \implies (1 - 2\tau)(A\omega_h - \varphi_1\Omega_T) < \Psi\), for which a sufficient condition is \(\tau > \frac{1}{2}\), given that both \(\Psi\) and \((A\omega_h - \varphi_1\Omega_T)\) are positive.

\(^{18}\)Citizens might choose to evade taxes even at lower tax rates. \(\tau > \frac{1}{2}\) is a sufficient not a necessary condition for \(z < 1\).

\(^{19}\)Assuming a wage for the politician would only alter the scale of our results, not their qualitative characteristics.
subject to

\[ c_{pt} = [\varphi_t (1 - \omega_q) + (1 - \phi_t)(1 - \omega_h)] \tau z_t H_t, \]  
\[ c_{pt} \geq 0, \ 1 \geq \varphi_t \geq 0, \]  

where \( h, Q, E \) and \( \Pi \) are determined by equations (1), (3), (5) and (6), taking \( z_t, Q_0 \) and \( H_t \) as given.

Straightforward maximization yields the politicians’ best response function,

\[ \varphi_t = g(z_t) = \frac{\Psi \Omega - \tau z_t (1 - \omega_h) \Omega_T}{2 \tau z_t \Omega_T} = -\frac{(1 - \omega_h)}{2 \Omega} + \frac{\Psi}{2 \tau z_t \Omega_T}, \]  

where \( \Omega = \omega_h - \omega_q \) and \( \Omega_T = A \omega_h - \beta \omega_q \). Notice first that for concavity to hold we must have \( \Omega_T \Omega > 0 \). Furthermore, an interior solution \((0 < \varphi < 1)\) exists iff \( \tau z_t (1 - \omega_h) \frac{\Omega_T}{\Omega_T} < \Psi < \tau z_t [2 (1 - \omega_q) - (1 - \omega_h)] \frac{\Omega_T}{\Omega_T} \). On the other hand, corner solutions, leading to the commitment of the total public revenue to a single policy \((\varphi_t = \min \{0, -\frac{(1 - \omega_h)}{2 \Omega}\} \) and \( \varphi_t = 1 \), emerge depending mainly on the values of \( \omega_h, \omega_q \). The politician will allocate the total revenue to abatement if the rate of rent seeking on abatement is not too high relative to that on education, that is, if \( \frac{1 - \omega_q}{1 - \omega_h} < \frac{1}{2} \). Thus, for interior solutions, we assume \( \frac{1 - \omega_q}{1 - \omega_h} > \frac{1}{2} \).

Not surprisingly, \( \varphi_t \) is decreasing in \( A \), and increasing in \( \beta \), while it depends on \( \tau \). The effect of the tax rate on the allocation of public revenues depends on the sign of \( \Omega_T \), since \( \Omega_T \geq 0 \Rightarrow \frac{\partial \varphi_t}{\partial \tau} = -\frac{1}{2 \tau z_t \Omega_T} \leq 0 \). If \( \Omega_T > 0 \Rightarrow A \omega_h > \beta \omega_q \), that is, public revenues directed to education are more effective, both due to better technology and lower rate of rent seeking, then the politician allocates less revenue to abatement as the tax rate increases. She does so in order to maximize the effectiveness of public spending\(^{21} \) and to maximize her own income by minimizing citizens’ tax evasion.\(^{22} \)

**Strategic Interactions**

We now turn to examine the *strategic interaction* between the two types of individuals. Notice that from the citizen’s reaction function, equation (9), we

\[ \frac{1 - \omega_q}{1 - \omega_h} < \frac{1}{2}, \text{ then } \tau z_t [2 (1 - \omega_q) - (1 - \omega_h)] \frac{\Omega_T}{\Pi_T} < 0, \text{ given that } \frac{\Omega_T}{\Pi_T} > 0, \text{ and therefore } \varphi = 1. \]

\(^{20}\)Recall that the politician also cares about her offspring’s well being.

\(^{22}\)Citizens are willing to pay higher taxes when they observe the politician to direct a higher share of the tax revenue to the most productive activity.

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\[ \Psi \Omega - \tau z_t (1 - \omega_h) \Omega_T = -\frac{(1 - \omega_h)}{2 \Omega} + \frac{\Psi}{2 \tau z_t \Omega_T}, \]
get, \( \frac{\partial z_t}{\partial \varphi_l} = \frac{-\psi \Omega_T}{2r(A \omega_h - \varphi \Omega_T)^2} \), with \( \frac{\partial^2 z_t}{\partial (\varphi_l)^2} = \frac{-\psi \Omega_T^2}{2r(A \omega_h - \varphi \Omega_T)^4} < 0 \) since \( A \omega_h - \varphi \Omega_T > 0 \).

Similarly, the slope of the politician’s reaction function is, \( \frac{\partial z_t}{\partial \varphi_l} = \frac{-\psi}{2r \zeta_T^2 \Omega_T} \), with \( \frac{\partial^2 \varphi_l}{\partial (\varphi_l)^2} = \frac{\psi}{4r \zeta_T^4 \Omega_T} \). Therefore, the the sign of reaction functions’ slope depends on the sign of the term \( \Omega_T \). More precisely, we can have the following two cases:

i) \( \Omega_T < 0 \implies \frac{\partial z_t}{\partial \varphi_l} > 0, \frac{\partial \varphi_l}{\partial z_t} > 0 \) i.e. Strategic Complements

ii) \( \Omega_T > 0 \implies \frac{\partial z_t}{\partial \varphi_l} < 0, \frac{\partial \varphi_l}{\partial z_t} < 0 \) i.e. Strategic Substitutes

Case (i) refers to situations in which public spending on education is less productive relative to abatement, either due to relatively higher rates of rent seeking or due to worse transformation technology. In this case, the reaction functions of the two types of individuals are positively sloped, and in this sense their choice variables are strategic complements. Citizens’ utility increases as a higher share of public revenue is directed to the improvement of environmental quality, which is the more productive activity.\(^{23}\) Hence, if they observe that the politician indeed directs higher amounts to abatement, they choose to treat in a similar manner and they evade less \( (z_t) \) increases. Conversely, if politicians become more corrupt, by choosing to direct more funds to the non-productive activity (in this case to public education), then citizens “punish” them by evading more taxes. Put differently, citizens reciprocate to the "ethi cal" behavior of the politicians and vice versa.

Interestingly in case (ii) while both groups’ strategies are strategic substitutes, the outcome is the same. Analytically, if the more productive activity is public education but politicians choose to invest a higher share of public funds on abatement (increase \( \varphi_l \)), citizens punish them by declaring less income and vice versa. Therefore in terms of the outcome both types of interaction are symmetric. For this reason we choose to present only one of the two cases, that of strategic complements.

In the case of strategic complements, both types of individuals’ reaction functions are increasing in a decreasing rate, that is, \( \frac{\partial z_t}{\partial \varphi_l} > 0, \frac{\partial^2 z_t}{\partial (\varphi_l)^2} < 0 \) and \( \frac{\partial z_t}{\partial z_t} > 0, \frac{\partial^2 \varphi_l}{\partial (\varphi_l)^2} < 0 \). From (9) we have that \( z_t|_{\varphi_l=0} = f'(\varphi_l = 0) = \frac{A \omega_h - \varphi}{2r \omega_h} \), which is the citizen reaction function’s vertical intercept. For this to be less

\(^{23}\)Note that we have assumed that marginal utility of human capital and environmental quality are the same.
than unity requires that \( \tau > \frac{1}{2} - \frac{\psi}{2A\omega_h} \). Note that the politician tends to direct all revenues to public education (in this case the more rent-seeking activity), as citizens declare a small part of their income, that is, from (12) we have \( \lim_{z_t \to 0} \varphi_t = -\infty \). From (12) we also derive that \( \varphi_t = 0 \implies z_t = \frac{\psi\Omega}{\tau(1-\omega_h)\Omega_T}. \) For this to be less than unity requires that \( \tau > \frac{\psi\Omega}{(1-\omega_h)\Omega_T} \). Figure 1 illustrates citizen’s \((R_c)\) and the politician’s \((R_p)\) reaction functions in the case that both possible intersections occur within \((0,1)\). Three equilibria may occur denoted by the points \(E_h, E_{ns}, \) and \(E_l\). Using best reply dynamics we observe that \(E_h\) and \(E_l\) are stable equilibria whereas \(E_{ns}\) is an unstable equilibrium. \(E_h\) denotes the high corruption equilibrium (since \(z_t = \min\{0, \frac{1}{2\tau} - \frac{\psi_n}{2\tau\omega_R}\}\) and \(\varphi_t = 0\) implies high tax evasion and that the total tax revenue is directed to the less effective activity\(^{24}\)) whereas \(E_l\) denotes the low corruption equilibrium where citizens declare part of their income \((z_t > 0)\) and a positive part of the tax revenue is directed to the more effective activity \((\varphi_t > 0)\).

### 2.3 Equilibrium

The interaction between the two types of individuals can be described as a coordination game in which there are strategic complementarities (substitutabilities).\(^{25}\) Games of strategic complementarity (substitutability) are those in which the best response of any player is increasing (decreasing) in the actions of the rival, as is the case for \(z_t\) and \(\varphi_t\). Strategic complementarity is a necessary condition for the existence of multiple equilibria in symmetric coordination games.\(^{26}\) The resulting equilibria are not driven by fundamentals. Instead, they are self-fulfilling and critically depend on the expectation of one group concerning the behavior of the other. Nevertheless, the game that we analyze here is not symmetric. Moreover the choice space is bounded and this necessitates the consideration of corner solutions. In fact, as we show below, this game does not share many of the properties of games with strategic complementarities. Consider first the following definition of equilibrium:

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\(^{24}\)Recall that we examine the case of strategic complements, therefore \(A\omega_h < \beta\omega_q\) which implies that abatement is more effective than education spending.

\(^{25}\)See, for example, Cooper and John (1988) and Vives (2005).

\(^{26}\)Notice however that also in games with strategic substitutability multiple equilibria may occur as well (Randon, 2009).
Figure 1: Reaction functions
Definition 1 A Nash equilibrium in this economy consists of sequences \( \{c_{it}\}_{t=0}^{\infty}, \{z_{it}\}_{t=0}^{\infty}, \{h_{it}\}_{t=0}^{\infty}, \{E_{it}\}_{t=0}^{\infty}, \{Q_{it}\}_{t=0}^{\infty}, \{\Pi_{it}\}_{t=0}^{\infty}, \ i = c, p, \) such that, given an initial average stock of human capital \( H_{-1} > 0 \) and an average level of environmental quality \( Q_{-1} > 0 \), in every period \( t \),

1. Private citizens choose \( z_{it} \) to maximize their utility, taking \( \varphi_{it} \) as given.
2. Politicians choose \( \mu_{it} \) to maximize their utility, taking \( z_{it} \) as given.
3. The sequences \( \{h_{it}\}_{t=0}^{\infty}, \{y_{it}\}_{t=0}^{\infty}, \{E_{it}\}_{t=0}^{\infty}, \{Q_{it}\}_{t=0}^{\infty}, \{\Pi_{it}\}_{t=0}^{\infty}, \} \) are determined according to \( (1), (3), (4), (5), (6), (8), \) and \( (11) \).
4. \( h_{it} = H_{t} \).

Each group’s individual optimization problem is well defined since its utility function is strictly concave and the budget constraint linear with respect to the relevant decision variable, \( z_{it} \) or \( \varphi_{it} \). In Proposition 1 below, we prove the existence of a pair \( (z_{it}, \varphi_{it}) \) that satisfies Definition 1 in every period. Given the existence of the equilibrium pair \( (z_{it}, \varphi_{it}) \), we can easily establish the equilibrium values of the remaining variables, following Definition 1.

Proposition 1 An equilibrium pair \( (z_{it}, \varphi_{it}) \) exists.

Proof. We must establish the existence of a pair \( (z_{it}, \varphi_{it}) \) that satisfies equations \( (9) \) and \( (12) \) simultaneously. For an arbitrary time period \( t \), let \( z_{it} = f(\varphi_{it}) \) denote the solution to the citizen’s problem, as described by equation \( (9) \); for each value of the allocation rate \( \varphi_{it} \) there exists a unique value of the tax evasion rate \( z_{it} \). Similarly, let \( \varphi_{it} = g(z_{it}) \) denote the solution to each politician’s problem, as described by equation \( (12) \). Note that both of these functions are continuous (see equations \( (9) \) and \( (12) \)). Thus, the composite function \( g \circ f \) from \([0,1]\) to \([0,1]\) is continuous and, by Brower’s fixed point theorem, has a fixed point. ■

Solving for the equilibrium values of individuals’ choice variables, yields,

\[
\begin{align*}
\varphi_{1}^* &= 0 \\
\varphi_{2}^* &= 4A\omega_{H} \Omega + \Xi - \sqrt{\Xi - 8A\omega_{P}} \\
\varphi_{3}^* &= 4A\omega_{H} \Omega + \Xi + \sqrt{\Xi - 8A\omega_{P}} \\
\end{align*}
\]

\[
\begin{align*}
z_{1}^* &= \min\{0, \frac{1}{2\tau} - \frac{\Psi_{n}}{2\tau \Delta \omega_{P}}\} \\
z_{2}^* &= \frac{\sqrt{\Xi - \sqrt{\Xi - 8A\omega_{P}}} - \Xi}{4\tau \sqrt{\Xi}} \\
z_{3}^* &= \frac{\sqrt{\Xi + \sqrt{\Xi - 8A\omega_{P}}} - \Xi}{4\tau \sqrt{\Xi}}
\end{align*}
\]
where, \( \Xi = \omega_h \omega_q (A - \beta) - \Omega_T \). For the non-zero equilibrium values of \( z \) to be real number in \((0, 1)\), we need \( \Xi \geq 0 \), which implies that
\[
\frac{\beta}{A} > \frac{(1 - \omega_q)/\omega_q}{(1 - \omega_h)/\omega_h}.
\]
That is, for the citizen to declare any positive amount of his income to the tax authority, the ratio of technological efficiency of abatement to education should exceed the ratio of the rates of embezzlement. Depending on the parameter values, sufficient conditions for the existence of a unique or multiple equilibria can be established. We call an equilibrium interior (corner) if it lies in the interior (on the boundary) of the unit square. Figure 1 presents an example in which there are three equilibria: the corner one \( E_h (z_1^*, \varphi_1^*) \) and two that their elements are less than one. Note that \( E_{ns} (z_2^*, \varphi_2^*) \) is not stable while the \( E_l (z_3^*, \varphi_3^*) \) is stable.

**Policy implications**

Even though the aim of the paper is not to discuss policy implications, our analysis yields some results worth noting. In our framework we have assumed that the tax rate, \( \tau \), and the rates of embezzlement \( \omega_h \) and \( \omega_q \) are exogenously given. However, a regulator\(^{27}\) can change both the tax rate through new regulation and the embezzlement rates through institutional changes. Straightforward manipulations of (9) indicate that in order to preclude \( z_t = 0 \), i.e. the full corruption corner solution \( E_h (z_1^*, \varphi_1^*) \) in Figure 1, a policy maker must either contain rent seeking on education spending,\(^{28}\) or contain rent seeking on abatement activities.\(^{29}\) As far as tax policy is concerned, the condition \( \tau < \frac{1}{2} \) is necessary for a nil-corruption \((z_t = 1)\) equilibrium to be feasible. Can the policy maker ensure that the no corruption equilibrium is always chosen? Sufficiently low tax rates and high \( \omega_h \), \( \omega_q \) cannot ensure \( z_t = 1 \) since other variables, i.e. \( A, \beta, \) and \( \Psi \), play an important role as well. Therefore, the main concern of the regulator should be to eliminate the corner solution \( z_t = \min \{0, \frac{1}{2\tau} - \frac{\psi_h}{2\tau A \omega_h}\} \).

Assuming strategic complementarity, i.e. \( \Omega_T < 0 \), equation (12) implies that the regulator should try to preclude that \( \varphi_t = 0 \), i.e. that the total tax revenue is directed to the less effective activity, that is, education spending

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\(^{27}\)The regulator could either be the same as our politician, or an authority at a higher level of decision making relative to our politician. That is, our setting can be interpreted as a situation in which a regulator sets \( \tau, \omega_h \) and \( \omega_q \) and a bureaucrat (our politician) chooses only \( \varphi \).

\(^{28}\)Setting \( \varphi = 0 \) yields that a necessary condition for \( z_t > 0 \) implies \( A \omega_h > \Psi \).

\(^{29}\)Setting \( \varphi = 1 \) yields that a necessary condition for \( z_t > 0 \) implies \( \beta \omega_q > \Psi \).
(since $\Omega_T < 0 \Rightarrow A\omega_h < \varphi \Omega_T$). In this case the regulator should make the appropriate institutional changes that will decrease the value of $\omega_q$ relative to $\omega_h$.30

Dynamics

Due to the set-up of the model, neither the reaction functions nor the equilibrium values for $z^*$ and $\varphi^*$ depend on the evolution of $h_t$ and therefore an equilibrium always exists. However we must study the evolution of human capital and to find the appropriate conditions that ensure the stability of the difference equation (1). The difference equation is homogeneous and therefore its solution is given by:

$$h_t = b'th_0$$

where $b = A(1-\varphi_t)\omega_hz_t + v$. Following Chiang (1984) for stability to hold we require that $b < 1$. Additionally the fact that $b > 0$ indicates non-oscillatory convergence to the steady state.

If equation (1) approaches a steady state then the same holds for equation (3) since it can be rewritten as

$$Q_{t+1} = (Q_o - \psi + v\varphi(\omega_qz_t)h_t)$$

and therefore the solution is given by,

$$Q_t = (Q_o - \psi + v\varphi(\omega_qz_t))b'th_0.$$  

3 A natural resources model: A numerical illustration

In order to reinforce the results of the previous section, we develop a more realistic framework in this section. We will show, using numerical simulations since analytical solutions cannot be derived, that the main properties of the basic model hold in this more elaborate setting. The main improvement in the model concerns the treatment of the environment. We assume that the environment, either as environmental quality affecting citizens’ productive capacity, and/or as natural resources, contribute to production. This

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30Note from eq. (12) that for $\varphi_t > 0$ (for $z_t = 0$) we must assume that $\omega_h > \omega_q$. Even for $z_t = 1$, the condition $\omega_h > \omega_q$ is not necessary but sufficient for $\varphi_t > 0$.  

18
assumption adds realism to our model but it significantly complicates the analysis. For this reason, we will provide analytical results only for the reaction functions and then we will resort to numerical analysis in order to define a range of values for which the model returns plausible results.

3.1 The economy

The learning technology in the public education system is quite similar as in the basic model and given by,

\[ h_t = v + AE_{t-1} \]  

(13)

The difference with (3) lies in that we assume that individuals acquire, without effort, a minimum level of human capital instead of a fraction \( v \in [0, 1] \) of the previous period’s accumulated human capital.\(^{31}\)

The first significant difference with the previous model is introduced in the equation of motion for the environment, which now is,

\[ Q_t = Q_{t-1} - 2\psi H_{t-1}Q_{t-1} + \beta\Pi_{t-1} \]  

(14)

where \( Q_{t-1} \) denotes the state of the environment in the previous period and \( \psi \) the extent of environmental damage, or the rate of the natural resource depletion, caused by production as in the basic model. We assume that \( 1 - 2\psi H_{t-1} > 0 \), that is, production cannot deplete the environment / natural resource. This formulation is rather common in the literature and more plausible than our previous specification.\(^{32}\)

Production uses both human capital and the environment / natural resources as inputs. That is, we assume that citizens’ output \( y_{ct} \) is,\(^{33}\)

\[ y_{ct} = h_t Q_t. \]  

(15)

Evidently at the aggregate level there are increasing returns to scale, which is not the case at the individual level since the quantity of each factor of production is predetermined by the the previous period’s choices. We assume that

---

\(^{31}\)This assumption is also standard in the literature and does not affect our results.

\(^{32}\)See for example, John and Pecchenino (1994) and Economides and Philippopoulos (2008).

\(^{33}\)Since all agents have the same level of human capital and the natural resource is commonly owned, we omit the subscript \( i = c, p \) from both variables.
citizens receive income for both factors of production. Therefore, citizens’ income is the sum of the two inputs’ marginal products. That is, citizens’ income $Y_{ct}$, by Euler’s theorem, is 

$$Y_{ct} = \frac{\partial y}{\partial h_t} h_t + \frac{\partial y}{\partial Q_t} Q_t = h_t Q_t + h_t Q_t = 2 h_t Q_t.$$ 

Both types of individuals make the same choices by maximizing their utility function described by equation (2) as in the basic model. The citizen chooses the fraction $z$ of his income to declare to the tax authority and the politician the fraction $\varphi$ of the total tax revenue to allocate to abatement / preservation of natural resources.

In this model the total tax revenue collected within a period $t$ is $R_t = 2 z_t \tau h_t Q_t$. As in the previous model a fraction $(1 - \phi)2 z_t \tau h_t Q_t$ of the total tax revenue is earmarked for public education. Since the politician peculates a fraction $1 - \omega_h$ of $(1 - \phi)R_t$, the actual amount spent on education $E_t$ is

$$E_t = (1 - \varphi_t)\omega_h 2 z_t \tau h_t Q_t.$$ (16)

The remaining fraction $\phi 2 z_t \tau h_t Q_t$ of the collected revenue is earmarked for public abatement. The politician peculates a fraction $1 - \omega_q$ of this sum, leaving $\Pi_t$ to be spent on abatement

$$\Pi_t = \varphi_t \omega_q z_t \tau 2 h_t Q_t.$$ (17)

Individual optimization decisions regarding $z_t$ and $\varphi_t$ affect the sum and the allocation of public spending between education and abatement and consequently the human capital and the state of the environment / natural resources enjoyed by the next generation.

### 3.2 Individual optimization

**Citizen**

Citizens declare a fraction $z_t$ of their income $Y_{ct}$ to the tax authority and an amount $\tau z_t h_t$ is paid as income tax. Hence, citizens’ disposable income is 

$$(1 - \tau) z_t 2 h_t Q_t + (1 - z_t) 2 h_t Q_t = (1 - z_t \tau) 2 h_t Q_t.$$ 

The individual optimization problem solved by each citizen born in period $t - 1$ is,

$$\max_{c_{t+1}, z_t} c_{ct} [h_{t+1}, Q_{t+1}]$$ (18)
subject to
\begin{align}
  c_t &= (1 - z_t)2h_tQ_t, \quad (19) \\
  c_t &\geq 0, \quad 1 \geq z_t \geq 0,
\end{align}
where \( h, Q, E \) and \( \Pi \) are determined by equations (13), (14), (16) and (17), taking \( \varphi_t, H_t \) and \( Q_t \) as given.

Maximization yields the citizens’ best response function,
\[ z_t = f(\varphi_t) = \frac{2H_tQ_t(A\omega_h - \varphi_t\Omega_T) - \Psi_n}{2\tau2H_tQ_t(A\omega_h - \varphi_t\Omega_T)}, \quad (20) \]
where \( \Psi_n = Q_t + v - \psi 2H_tQ_t \). Concavity holds since \( A\omega_h - \varphi_t\Omega_T > 0 \).

Citizens’ reaction function in (20) has similar characteristics as the one in the basic model (equation (9)). The slope of the citizen’s reaction function is,
\[ \frac{\partial z_t}{\partial \varphi_t} = \frac{-\Psi_n\Omega_T}{2\tau2H_tQ_t(A\omega_h - \varphi_t\Omega_T)} < 0 \] since \( A\omega_h - \varphi_t\Omega_T > 0 \). Therefore, as in the basic model, the sign of citizen reaction function’s slope depends on the sign of the term \( \Omega_T \). The intercept of citizen’s reaction function is,
\[ z_{t|\varphi_t=0} = f(\varphi_t = 0) = \frac{2H_tQ_t(A\omega_h - \Psi_n)}{2\tau2H_tQ_t(A\omega_h)}. \] For this to be less than unity requires that \( \tau > \frac{1}{2} - \frac{\Psi_n}{2\tau2H_tQ_tA\omega_h} \).

The main difference from the basic model is that in (20) the strategy of the citizen, \( z_t \), depends not only the strategy of the politician, \( \varphi_t \), but on the realized values of \( H_t \) and \( Q_t \). The values of these terms evolve over time until the economy approaches a steady state (in case it exists) and therefore the optimal strategy differs among generations. However notice that at time \( t \) the values of \( H_t \) and \( Q_t \) have been already determined by the previous generation and therefore each generation treats them as exogenous.

Inspection of equation (20) reveals that an interior solution \((0 < z < 1)\) exists iff \((2\tau - 1)2H_tQ_t(A\omega_h - \varphi_t\Omega_T) < \Psi_n < 2H_tQ_t(A\omega_h - \varphi_t\Omega_T) \). A corner solution \( z_t = \min\{0; \frac{2H_tQ_tA\omega_h - \Psi_n}{2\tau2H_tQ_tA\omega_h}\} \) \((z_t = 1)\) will emerge if the rate of human capital transferred freely to the next generation, \( v \), is sufficiently high (low), the rate of degradation of environmental quality, \( \psi \), sufficiently low (high) and the rent reeking rates, \((1 - \omega_h)\) and \((1 - \omega_q)\), sufficiently high. As in the basic model, for sufficiently high \( \tau \) \((\tau < \frac{1}{2})\), the tax evasion rate is never zero, since \( z < 1 \).

Whenever an interior solution emerges, the tax evasion rate \((1 - z_t)\) is negatively affected by the efficiency of the education system \((A)\) and the
abatement technology ($\beta$), and positively affected by the rates of rent seeking $(1 - \omega_h)$ and $(1 - \omega_q)$ and the tax rate $\tau$.

**Politician**

The politician’s income is derived solely from peculation of tax revenues and is,  
$$[\varphi_t(1 - \omega_q) + (1 - \phi_t)(1 - \omega_h)]\tau z_t 2h_tQ_t.$$ 

The politician’s optimization problem is,

$$\max_{c_{pt},d_t} c_{pt}[h_{t+1}+Q_{t+1}]$$ \hspace{1cm} (21)

subject to

$$c_{pt} = [\varphi_t(1 - \omega_q) + (1 - \phi_t)(1 - \omega_h)]\tau z_t 2h_tQ_t$$ \hspace{1cm} (22)

where $h, Q, E$ and $\Pi$ are determined by equations equations (13), (14) (16) and (17), taking $\varphi_t, H_t$ and $Q_t$ as given.

Maximization of the politician’s best response function yields,

$$\varphi_t = g(z_t) = \frac{\Omega \Psi_n - \tau z_t X 2H_tQ_t}{2\tau z_t \Omega \Omega 2H_tQ_t} = -\frac{X}{2\Omega T \Omega} + \frac{\Psi}{2\tau z_t \Omega \Omega 2H_tQ_t},$$ \hspace{1cm} (23)

where $X = (1 - \omega_h) \Omega T - A \omega_h \Omega$. Similar to the benchmark model, for concavity to hold we must have $\Omega T \Omega > 0$. The slope of the politician’s reaction function is,  
$$\frac{\partial \varphi_t}{\partial z_t} = \frac{-\Psi}{2\tau z_t^2 2H_tQ_t \Omega_T},$$

with $\frac{\partial^2 \varphi_t}{(\partial z_t)^2} = \frac{\Psi}{4\tau z_t^2 2H_tQ_t \Omega_T}$. The sign of reaction functions’ slope depends on the sign of the term $\Omega T$.

For interior solutions ($0 < \varphi < 1$) it is required that $\tau z_t 2H_tQ_t \frac{X}{\Omega} < \Psi_n < \tau z_t 2H_tQ_t \frac{4\Omega T \Omega}{\Omega}$ and $\frac{\partial \varphi_t}{\partial z_t} > 0$. On the other hand, corner solutions of directing revenue to a unique policy ($\varphi_t = 0$ and $\varphi_t = 1$, respectively) emerge depending on the values of $\omega_h, \omega_q$. Moreover, $\varphi_t$ is decreasing in $A$, and increasing in $\beta$. Similarly to the benchmark case the effect of tax rate on the allocation of revenue depends on the sign of $A \omega_h - \beta \omega_q$ and the same goes for the effects of $Q_0, \psi$ and $v$. Specifically $A \omega_h - \beta \omega_q \geq 0 \implies \frac{\partial \varphi_t}{\partial \tau} \leq 0, \frac{\partial \varphi_t}{\partial \psi} \leq 0, \frac{\partial \varphi_t}{\partial Q_0} \geq 0$, and $\frac{\partial \varphi_t}{\partial v} \geq 0$. Overall we observe that despite the fact that our setting is more complex and realistic, in terms of reaction functions the two models make the same predictions. As was the case with the citizen reaction function, the politician’s reaction function also depends on the realized values of $H_t$ and $Q_t$ which are predetermined by the previous generation and therefore each generation of politicians treats them as exogenous.
Strategic Interactions

Strategic interaction in this setting are similar to the benchmark case. As we show above, the sign of both reaction functions’ slope depends on the sign of the term $\Omega_T$. Analytically

\[ i) \quad \Omega_T < 0 \implies \frac{\partial z_t}{\partial \varphi_t} > 0, \quad \frac{\partial \varphi_t}{\partial z_t} > 0 \quad \text{i.e. Strategic Complements} \]

\[ ii) \quad \Omega_T > 0 \implies \frac{\partial z_t}{\partial \varphi_t} < 0, \quad \frac{\partial \varphi_t}{\partial z_t} < 0 \quad \text{i.e. Strategic Substitutes} \]

The same discussion regarding the reaction functions, as in the previous section applies here as well. Even though the values of $H_t$ and $Q_t$ affect the magnitude of both the intercept and the slope of both reaction functions, they do not affect the direction of the interactions.

3.3 Equilibrium

The definition of equilibrium remains the same in both models. However, the existence of equilibrium is more complicated in this model. Each group’s individual optimization problem is well defined since its utility function is strictly concave and the budget constraint linear with respect to the relevant decision variable, $z_t$ or $\varphi_t$. In Proposition 2 below, we prove the existence of a pair $(z_t, \varphi_t)$ that satisfies Definition 1 in every period, for given values of $H_t$ and $Q_t$. Given the existence of the equilibrium pair $(z_t, \varphi_t)$, we can easily establish the equilibrium values of $H_t$ and $Q_t$ and subsequently of the remaining variables, following Definition 1.

Proposition 2 An equilibrium pair $(z_t, \varphi_t)$ exists for given values of $H_t$ and $Q_t$.

Proof. We must establish the existence of a pair $(z_t, \varphi_t)$ that satisfies equations (20) and (23) simultaneously. For an arbitrary time period $t$, let $z_t = f(\varphi_t, h_t, Q_t)$ denote the solution to each citizen’s problem, as described by equation (20); for each value of the allocation rate $\varphi_t$ there exists a unique value of the evasion rate $z_t$. Similarly, let $\varphi_t = g(z_t, h_t, Q_t)$ denote the solution to each politician’s problem, as described by equation (23). Note that both of these functions are continuous (see equations (20) and (23)). Thus, the
composite function $g \circ f$ from $[0, 1]$ to $[0, 1]$ is continuous and, by Brower’s fixed point theorem, has a fixed point. 

Solving for the equilibrium values of the model we obtain,

$$
\begin{align*}
z_1^* &= \min \{0, \frac{2H_t Q_t A \omega_h \omega_n}{2H_t Q_t A \omega_h} \} \\
z_2^* &= f^2(h_t, Q_t) \\
z_3^* &= f^3(h_t, Q_t) \\
\varphi_1^* &= 0 \\
\varphi_2^* &= g^2(h_t, Q_t) \\
\varphi_3^* &= g^3(h_t, Q_t)
\end{align*}
$$

Therefore in terms of strategies there always exists an equilibrium for given values of $h_t$ and $Q_t$. Since however there is a law of motion describing how these two variables evolve, there will be different equilibrium values in each period for $z_t$ and $\varphi_t$ unless the system approaches a steady state. The dynamics of the model are analyzed in the following subsection.

### 3.4 Dynamic behavior of the system of difference equations

As noted above the stable solutions of the model (if all three are valid) are $(z_1^*, \varphi_1^*)$ and $(z_3^*, \varphi_3^*)$ using best reply dynamics. Since the set $(z_1^*, \varphi_1^*)$ represents a trivial equilibrium of full corruption we will focus on the low-corruption equilibrium $(z_3^*, \varphi_3^*)$. Replacing the equilibrium values for $(z_3^*, \varphi_3^*)$ from equation (24) into equations (13), (14) we obtain the following system of two autonomous non-linear first order difference equations

$$
\begin{align*}
h_{t+1} &= F(h_t, Q_t) \\
Q_{t+1} &= G(h_t, Q_t)
\end{align*}
$$

where $Q_0$ and $h_0$ denote the initial values for $h_t$ and $Q_t$ and are exogenously given. The dynamics of the system are too complex to be analytically studied. Still though we can describe analytically and numerically the kind of solution that is desirable in order for our model to be meaningful.

In order to approximate the dynamics of our benchmark model, i.e. a set of equilibrium values for $(z_t, \varphi_t)$ that remain unchanged in every period, our system of difference equations must reach a steady state. Therefore we first assume that the dynamic system has steady-state equilibrium $(\bar{h}, \bar{Q})$.

---

34We omit analytical expression due to their complexity.
Namely, $\in (\bar{h}, \bar{Q})$ such that

$$
\begin{align*}
\bar{h} &= F(\bar{h}, \bar{Q}), \\
\bar{Q} &= G(\bar{h}, \bar{Q}).
\end{align*}
$$

A Taylor expansion of the system around the steady state values $(\bar{h}, \bar{Q})$, yields:

$$
\begin{align*}
    h_{t+1} &= F(h_t, Q_t) \\
    &= F(\bar{h}) + F_h(\bar{h}, \bar{Q})(h_t - \bar{h}) + F_Q(\bar{h}, \bar{Q})(Q_t - \bar{Q}) + R_1 + R_2, \\
    Q_{t+1} &= G(h_t, Q_t) \\
    &= G(\bar{Q}) + G_h(\bar{h}, \bar{Q})(h_t - \bar{h}) + G_Q(\bar{h}, \bar{Q})(Q_t - \bar{Q}) + R_1 + R_2,
\end{align*}
$$

where $F_h(\bar{h}, \bar{Q})$ and $G_h(\bar{h}, \bar{Q})$ are the partial derivatives of the functions $F(h_t, Q_t)$ and $G(h_t, Q_t)$ evaluated at $(\bar{h}, \bar{Q})$ and $R_1$ and $R_2$ are the error terms which are very small in the neighborhood of $(\bar{h}, \bar{Q})$ and have little influence on the behavior of the system. Thus, the non-linear system is been approximated, locally (around the steady-state equilibrium) by the linear system:

$$
\begin{bmatrix}
    h_{t+1} \\
    Q_{t+1}
\end{bmatrix} =
\begin{bmatrix}
    F(\bar{h}) & F_Q(\bar{h}, \bar{Q}) \\
    G(\bar{Q}) & G_Q(\bar{h}, \bar{Q})
\end{bmatrix}
\begin{bmatrix}
    h_t - \bar{h} \\
    Q_t - \bar{Q}
\end{bmatrix},
$$

where,

$$
J(\bar{h}, \bar{Q}) =
\begin{bmatrix}
    F_h(\bar{h}, \bar{Q}) & F_Q(\bar{h}, \bar{Q}) \\
    G_h(\bar{h}, \bar{Q}) & G_Q(\bar{h}, \bar{Q})
\end{bmatrix},
$$

is the Jacobian matrix evaluated at the steady-state equilibrium.

If all eigenvalues of $J(\bar{h}, \bar{Q})$ have moduli strictly less than 1, $(\bar{h}, \bar{Q})$ is asymptotically stable (a sink). If at least one eigenvalue of $J(\bar{h}, \bar{Q})$ has modulus greater than 1, then $(\bar{h}, \bar{Q})$ is unstable (a source). If the eigenvalues of $J(\bar{h}, \bar{Q})$ are all inside the unit circle, but at least one is on the boundary (has modulus 1), then $(\bar{h}, \bar{Q})$ may be stable, asymptotically stable or unstable. Therefore we take the following steps:

i) Test whether our system approaches the steady state $(\bar{h}, \bar{Q})$.

ii) For this steady state to be a feasible solution, the dynamics of the system must satisfy the limitations of the model, namely concavity and the
implied values \( \tilde{\varepsilon} \leq 1, \tilde{\varphi} \leq 1 \). Second the dynamics of the system must be characterized by stability, i.e. the eigenvalues must be inside the unit circle.

If the above restrictions hold, then we are fully able to describe the behavior of the equilibrium values of \((z^*, \varphi^*)\) in every period of the model up to the steady state. Why is it important to have a stable steady state? Because if the system is unstable, then \( h_t \) and \( Q_t \) grow without limits and taking into account that \( \frac{\partial z^*}{\partial h_t} > 0, \frac{\partial z^*}{\partial q_t} > 0 \) this implies that as \( h_t \) and \( Q_t \) grow without bound the same will hold for \( z_t \) which will eventually become equal to unity.

4 Numerical approximations

The model in Section 3 closely follows the benchmark model up to the point were we obtain the reaction functions. However due to occurring system of non-linear difference equations it quickly becomes rather complicated. Therefore we will make some numerical calculations illustrating our results. The same illustrative method is applied to the benchmark model as well to highlight the similarities between the two models. In what follows we will shortly describe the procedure followed to obtain our equilibrium values in the natural resources model.

In our model there is a wide range of parameters that can vary, namely \( A, \beta, \tau, \omega_h, \omega_q, v, \) and \( \psi \). As became evident from the above analysis of the model the most significant term that drives most of our results was the equation \( A\omega_h - \beta\omega_q \) which defines the kind of strategic interactions between the two groups of agents. Therefore we assume a wide range of parameters for the variables \( A, \beta, \omega_h, \omega_q \) and we also let \( \tau \) vary in order to derive some comparative statics results with respect to policy implications.

Evidently the model is a rough approximation of reality therefore it is hard to clarify what values of the parameters can be considered as "realistic". Still though with respect to tax evasion there is some evidence that in the Western developed countries the rates of tax evasion are estimated around 5%-25% of potential tax revenue (Feige, 1989, Pyle, 1989, Thomas, 1992) while for developing countries higher rates may appear (Tanzi and Shome, 1994). For the year 1988 in the US, the TCMP has estimated that only a 53% has paid its taxes correctly. Of course non compliance does not apply for all these cases, since a 7% has overpaid its taxes while a part of the remaining
40% has underpaid due to errors that result from the complicated procedure involved. According to Fanzoni (1998) the federal income tax gap of the US had been estimated for 1998 at 17%.

Concerning the values of $\omega_h$ and $\omega_q$ there is much evidence that different allocations of public budget are associated with different rent-seeking rates. Mauro (1998) finds evidence that public expenditure on high-technology goods is associated with higher rent-seeking due to low detectability and the same goes for military expenditure. On the other hand education and health sectors involve more transparent expenditure and are thus associated with lower rent-seeking rates. The literature on rent-seeking estimates the social waste in each sector as imposed by bribery and estimated that increases in perceived corruption by one unit (in the BI Index) reduce spending on certain allocations by 0.1-0.5% (0.4% for health, 0.05 for environmental protection according to Hessami, 2010; 0.6% for education spending according to Mauro, 1998). Even though this is a different measure, attempting to counterpart these rates with $\omega_h$ and $\omega_q$, would imply a rate of 0.5%-0.25% depending on how corrupt a country is according to the BI Index. Tax rates vary between 0.25-0.55.

As far as the parameters $A, \beta$ are concerned it is hard to pin down which range of values would be plausible and we will therefore focus on their between ratio as implied by our model, namely $A > 0, \beta > 0$ and $A\omega_h < \beta\omega_q$ for strategic complementarity which is the case we analyze. For the value of $v$ there is also no evidence, still though it is easy to assume that it will take rather small values, well below unity (e.g. Ceroni (2001) takes values of $v$ as low as $v = 0.2$).

Having pinned down the range of parameter values we also imposed the restrictions mentioned earlier, namely concavity, strategic complementarity and an asymptotically stable steady state. A number of feasible steady states occur implying different kinds of equilibria. Below in Figure 2 we illustrate a numerical example of the natural resources economy:

Figure 2 closely resembles Figure 1, illustrating the similarities between the two models. Interestingly the stable equilibrium denoted $E_1$ is given by $(z^*, \varphi^*) = (0.82, 0.25)$. It is hard to comment on the value of $\varphi^*$ due to the duality of the model with respect to the allocation of budget, still though we
Figure 2: Numerical Reaction Functions: The parameter values are $A = 3$, $\delta = 8$, $\omega_h = 0.7$, $\omega_q = 0.85$, $\tau = 0.35$, $\nu = 0.025$, $\psi = 0.5$.

Figure 3: Numerical Reaction Functions: The parameter values are $A = 3$, $\delta = 8$, $\omega_h = 0.7$, $\omega_q = 0.85$, $\tau = 0.35$, $\nu = 0.025$, $\psi = 0.5$, $Q_0 = 1.85$
observe that $z^*$ can take quite realistic values.

Plotting the benchmark economy when using the same parameter values in Figure 3 we observe that we obtain quite similar results. The stable equilibrium denoted $E_1$ is given by $(z^*, \varphi^*) = (0.86, 0.29)$ therefore indicating that the two models are very close even with respect to the occurring equilibria.

5 Conclusions

The detrimental effects of corruption are evident and prevalent in most societies. The problem aggravates when corruption feeds itself or to put it differently when "corruption corrupts". In a simple setting with two public goods, public education and environmental protection, and two groups of agents, politicians and citizens, we find that when one group is corrupt it urges the other group to behave in a corrupt manner as well. This reduces not only the total tax revenue, due to increased tax evasion on the part of citizens, but also reduces the part of the budget allocated to the rent seeking activity. Is this good or bad news for environmental quality? The answer depends on whether environmental policy is a pretext for corrupt politicians to increase their rent seeking or whether abatement policy practically aims at the improvement of environmental quality and involves transparent expenditures.

Given that corruption is a significant problem in many developing but also, to a different extent, in some developed countries, we believe that further study in this field is appropriate. There are a number of extensions that could enrich our framework and yield further and more policy relevant results. For example, it could be quite interesting to assume two sources of tax revenues, income tax and some environmental tax, and examine whether setting different tax levels could decrease corruption.
References


