Information, Misallocation and Aggregate Productivity

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June 29, 2014

Abstract

We propose a theory linking imperfect information to resource misallocation and hence to aggregate productivity and output. In our setup, firms learn from both private sources and imperfectly informative stock market prices. We devise a novel empirical strategy that uses a combination of firm-level production and stock market data to pin down the information structure in the economy. Applying this methodology to data from the US, China, and India reveals substantial losses in productivity and output due to informational frictions - even when only one factor, namely capital, is subject to the friction. Our estimates for these losses range from 7-10% for productivity and 10-14% for output in China and India, and are smaller, though still significant, in the US. Losses are substantially higher when labor decisions are also made under imperfect information. Private learning plays a significant role in mitigating uncertainty and improving aggregate outcomes; learning from financial markets contributes little, even in the US.

*We thank Jaroslav Borovicka, Virgiliu Midrigan, Pete Klenow and Laura Veldkamp for their helpful comments, Andy Atekson, Yongs Shin, Jennifer La’O and Bernard Dumas for their insightful discussions of earlier versions, Cynthia Yang for excellent research assistance, and seminar participants at the NYU Macro lunch, UBC, NY Fed, Brown, the World Bank, Stanford, USC Marshall Finance Brown Bag, Arizona State, the 2013 North American Summer Meeting of the Econometric Society, the 2013 SED Annual Meeting in Seoul, and the 4th Advances in Macro-Finance Tepper-LAEF conference. David gratefully acknowledges financial support from the Center for Applied Financial Economics at USC.

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1 Introduction

In a frictionless environment, the optimal allocation of factor inputs across productive units requires the equalization of marginal products. Deviations from this outcome represent a misallocation of resources and translate into sub-optimal aggregate outcomes, specifically, depressed levels of productivity and output. A recent literature empirically documents the presence of substantial misallocation and points out its potentially important role in accounting for large observed cross-country differences in productivity and income per-capita. With some notable exceptions, however, the literature has remained largely silent about the underlying factors driving this misallocation.

In this paper, we propose just such a theory, linking imperfect information to resource misallocation and hence to aggregate productivity and output. Our point of departure is a standard general equilibrium model of firm dynamics along the lines of Hopenhayn (1992). The key modification here is that firms choose inputs under limited information about their idiosyncratic fundamentals, specifically demand conditions in their own markets. This informational friction leads to a misallocation of factors across firms in an ex-post sense, reducing aggregate productivity and output. The size of this effect depends on the residual uncertainty at the time of the input choice, which, in turn, is a function of the volatility of the fundamental shocks and the quality of the information at the firm level. Our analytical framework enables a sharp characterization of these relationships and yields simple closed-form expressions linking informational parameters at the micro-level to aggregate outcomes.

The second piece of our theoretical framework focuses on the firm’s learning problem. Our flexible information structure assumes that firms have access to a variety of sources. In addition to information generated purely within the firm (which we will refer to as ‘private information’), firms also observe movements in their own stock prices, which aggregate the information of financial market participants about the firm’s future prospects. We capture this aggregation using an explicit model of financial market trading\(^1\) in the noisy rational expectations paradigm of Grossman and Stiglitz (1980). Informed investors and noise traders trade shares of the firm’s stock, generating imperfectly revealing equilibrium prices. To the extent that this information is conditionally orthogonal to the firm’s own signals, stock markets provide firms with valuable information and guide real activity. Note that this does not require investors to know more than firms; only that they are privy to different information that may also be relevant for firm decisions\(^2\).

\(^{1}\)We rely particularly on recent work by Albagli et al. (2011b) for our specific modeling structure.

\(^{2}\)The information content of stock returns and their role in guiding real activity is the focus of an active body of work - both theoretical and empirical - in corporate finance. We will briefly survey this literature later in this section.
The presence of informative asset prices serves two purposes: first, as we describe next, the informational content of observed market prices is at the core of our empirical approach and allows us to identify the severity of otherwise unobservable informational frictions in the economy. Second, we are able to quantitatively evaluate the contribution of financial markets to allocative efficiency through an informational channel, i.e., by providing higher quality information to decision-makers within firms. Our analysis is, to the best of our knowledge, the first to measure and shed light on the aggregate consequences of this channel in a standard macroeconomic framework.

Any attempt at quantifying uncertainty runs into an obvious difficulty - the econometrician seldom observes information directly. In our context, one approach would be to use measures of misallocation (e.g. the dispersion in marginal products), but this would amount to little more than a fitting exercise. Inferring uncertainty purely from production-side moments is also problematic. For example, volatility of investment and its covariance with measures of fundamentals are affected by (and therefore, informative about) firm-level uncertainty, but they are also likely to be influenced by a number of other factors that are not present in our theoretical framework. Without explicitly modeling (and disciplining the magnitudes of) these factors, disentangling the severity of informational frictions from data on inputs and outputs is not a straightforward exercise.

One of the contributions of this paper is a novel empirical strategy that is robust to these concerns. The main insight behind this strategy is that we can draw the structure sharp inferences about uncertainty with only a subset of firm’s information set. Stock market prices allow us to do just that. The first step is to measure the information content of prices and the extent to which input decisions covary with them. The correlation of stock returns with future fundamentals and investment respectively are natural and intuitive candidates. The second, and key, step is to note that for a given level of noise, the extent to which firms decisions comove with signal is a function of the overall quality of their information (from all sources, including those we do not observe). The poorer this quality, the higher is the correlation between investment and stock market returns. Intuitively, the information contained in stock returns plays a bigger role in the firm’s investment decision when the firm is more uncertain. It is worth emphasizing the need to analyze these moments together - the correlation of returns with investment alone does not tell us much about the extent of uncertainty.

For special cases of our model, the intuition in the previous paragraph can be formalized very sharply. When firm-level fundamentals are iid or follow a random walk, we prove that the

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3To give a simple example, the correlation between returns and investment can be high either because firms and investors are both perfectly informed, in which case all firm-level variables are functions of a single fundamental shock, or alternatively, because firms are poorly informed and therefore learn much from market prices.
parameters governing the informational structure in our environment are identified by these two correlations (of stock returns and fundamentals and investment, respectively, as well as the volatility of returns in the latter case). Our use of correlations, as opposed to variances and covariances, makes this strategy more robust to some important perturbations of our baseline model, for example, to the inclusion of additional factors that have a “scaling” effect on the firm’s capital choice. For example, distortions correlated with firm fundamentals that dampen the responsiveness of investment. For the iid case, we show that the scaling effects introduced by such correlated distortions do not affect our estimate of the total uncertainty faced by firms, despite generating dispersion in marginal products. Our identification strategy also seem robust to the presence of correlation between firm and market information. We prove this result to be exact in the iid case: here, our measure of firm uncertainty remains valid no matter for any degree of correlation. In fact, even if the information contained in stock prices is redundant from the perspective of the firm, implying that firms do not learn anything new from them, our strategy uncovers the true extent of uncertainty. This is a reassuring finding because it shows that our estimates of the severity of informational frictions are not sensitive to assumptions about the extent of commonality between firm and market information sets, an object which is difficult to take a strong stand on, given the richness of information flows between firms and investors (financial statements, announcements, regulatory filings etc.).

In our quantitative work, we depart from these polar cases and consider an intermediate level of persistence for fundamentals in line with data. However, we show, numerically, that the informational parameters are still well identified by the same set of moments and that our estimates of uncertainty display a similar robustness to perturbations.

We apply our empirical methodology to data on publicly traded firms from 3 countries - the US, China and India. Our results show substantial uncertainty at the micro level, particularly in China and India. Even in the US, which has the highest degree of learning, our most conservative estimate for the posterior variance of the firm is about 40% of the ex-ante, or prior, uncertainty. The corresponding estimates for the other two countries ranges from about 60-90%. The associated implications for aggregate productivity and output are then quite significant. In China and India, TFP losses (relative to the first best) are in the range of 7-10%, while losses in steady state output (again, relative to the first best) range from 10% to almost 15%. The corresponding values in the US are noticeably smaller but still significant - 4% for productivity and 5% for output. Importantly, these baseline calculations assume that only investment decisions are made under imperfect information and while labor is assumed to adjust perfectly to contemporaneous conditions. In this sense, they are conservative estimates of the total impact of informational frictions. Assuming that the friction affects labor inputs

\[4\text{Given our AR(1) structure, this prior uncertainty is the simply variance of contemporaneous innovations.}\]
to the same degree as capital leads to losses that are substantially higher. For example, in 
this case, the gap between status quo and first best increases to 55-80% in TFP for India and 
China.\(^5\) We interpret this as an upper bound on the total effect of the friction, with reality likely 
falling somewhere in between this and the baseline version.\(^6\) To put these numbers in context, 
we compare them to direct measures of misallocation in our sample and find that informational 
frictions account for anywhere from 20-60% of observed dispersion in the marginal product of 
capital. This fraction goes up once we control for firm-fixed effects.

Our framework also enables us to quantify the sources of learning, particularly the contribution 
of financial markets. Here, we arrive at a striking conclusion - learning from stock prices is 
at best only a small part of total learning at the firm level, even in a relatively well-functioning 
financial market like the US. Thus, the impact of this channel on overall allocative efficiency 
and hence, on aggregate performance of the economy is actually quite limited\(^7\). We show that 
this is primarily due to the high levels of noise in market prices, making them relatively poor 
signals of fundamentals.\(^8\) A counterfactual experiment delivering access to US-quality financial 
markets (in a purely informational sense) to firms in China and India generates only small 
improvements in allocative efficiency\(^9\). In contrast, a significant amount of learning occurs from 
private sources, i.e., those internal to the firm. Moreover, disparities along this dimension, that 
is, in the quality of such information, are the primary drivers of cross-country differences in 
the severity of informational frictions, much more so than access to well-functioning financial 
markets. This finding, in spirit, parallels those in Bloom et al. (2013), who highlight the role 
of manager skill and/or better management practices in explaining cross-country differences in 
performance. Finally, we show that differences in the volatility of firm-level fundamentals also 
play a meaningful role in explaining cross-country differences in the severity of informational 
frictions. Firms in China and India are subject to larger shocks to fundamentals than firms 
in the US, making the inference problem more difficult in those countries even without the 
differences in signal qualities.

Our paper relates to several existing branches of literature. We bear a direct connection

\(^5\) The corresponding numbers for steady state output are 80-100%.

\(^6\) We provide suggestive evidence that this is the case using dispersion in the marginal product of labor 
among US firms.

\(^7\) This is true even under our baseline assumption that the information in stock prices is conditionally 
independent of the firm’s own signals, which is, in a sense, the most optimistic case. Allowing for correlation 
will further reduces the extent of new information in stock prices.

\(^8\) This is related to the concepts of Forecasting Price Efficiency (FPE), namely to what extent do prices reflect 
and predict fundamentals and Revelatory Price Efficiency (RPE), namely to what extent do prices promote real 
efficiency by revealing new information to the firm, as defined in Bond et al. (2012). Our result show that the 
poor RPE of stock prices stems from their poor FPE.

\(^9\) Of course, this does not consider other channels through which informative prices, and more generally, 
well-functioning stock markets may improve efficiency. See also the discussion in Section 4.4.
to recent work on the aggregate implications of misallocated resources, for example, Hsieh and Klenow (2009), Restuccia and Rogerson (2008), Guner et al. (2008), and Bartelsman et al. (2013). Indeed, we can map our measure of informational frictions directly into the measures of misallocation studied in these papers, i.e., into the dispersion in marginal products and the covariance between firm-level fundamentals (productivity, for example) and activity (i.e., factor use or output). We differ from these papers in our explicit modeling of a specific friction as the source of misallocation, a feature we share with Midrigan and Xu (2013), Moll (2014), Buera et al. (2011), and Asker et al. (2012), who study the role of financial frictions and capital adjustment costs, respectively. Our focus on the role of imperfect information is related to that of Jovanovic (2013), who studies an overlapping generations model where informational frictions impede the efficient matching of entrepreneurs and workers.

Our structure of firm learning holds some similarity with Jovanovic (1982) and our linking of financial markets, information transmission, and real outcomes is reminiscent of Greenwood and Jovanovic (1990). The informational role of financial markets has been the subject to much study, dating back at least to Tobin (1982). We discuss a few particularly relevant examples below and refer the reader to Bond et al. (2012) for an excellent survey. One strand of this literature focuses on measuring the information content of stock prices. Durnev et al. (2003) show that firm-specific variation in stock returns, i.e., “price-nonsynchronicity,” is useful in forecasting future earnings and Morck et al. (2000) find that this measure of price informativeness is higher in richer countries. A related body of work closer to our own and recently surveyed by Bond et al. (2012) looks directly at the feedback from stock prices to investment and other decisions. Chen et al. (2007), Luo (2005), and Bakke and Whited (2010) are examples of studies that find evidence of managers learning from markets while making investment decisions. Bai et al. (2013) combines a simple investment model with a noisy rational expectations framework to assess whether US stock markets have become more informative over time. Our analysis complements these papers by placing information aggregation through financial markets into a standard macroeconomic setting, which allows us to make precise statements about the quantitative importance of this channel for information transmission, real activity and aggregate outcomes. Our results on the limited role for stock market information bear some resemblance to the well-known results in Morck et al. (1990), who find a limited incremental role for stock prices in predicting investment, once fundamentals are controlled for. Our focus here is different - we are interested in measuring the contribution of stock market information to aggregate allocative efficiency. In a sense, our analysis provides a structural interpretation of their results. But, more importantly, it allows us to quantify the implications for aggregate outcomes.  

10More precisely, they find very small improvements in $R^2$ when adding stock returns to an investment regression that already includes fundamentals.
The paper is organized as follows. Section 2 describes our model of production and financial market activity under imperfect information. Section 3 spells out our approach to identifying informational frictions using the two analytically tractable special cases, while Section 4 details our numerical analysis and presents our quantitative results. We summarize our findings and discuss directions for future research in Section 5. Details of derivations and data work are provided in the Appendix.

2 The Model

In this section, we develop our model of production and financial market activity under imperfect information. We turn first to the production side of the economy, where we derive sharp relationships linking the extent of micro level uncertainty to aggregate outcomes. Next, we flesh out the information environment, and in particular, lay out a fully specified financial market in which dispersed private information of investors and noise trading interact to generate imperfectly informative price signals.

2.1 Production

We consider a discrete time, infinite-horizon economy, populated by a representative large family endowed with a fixed quantity of labor supplied inelastically. The aggregate labor endowment is denoted by $N$. The household has preferences over consumption of a final good and rents capital to firms. The household side of the economy is deliberately kept simple as it plays a limited role in our analysis.

Technology. A continuum of firms of fixed measure one, indexed by $i$, produce intermediate goods using capital and labor according to

$$Y_{it} = K_{it}^{\alpha_1} N_{it}^{\alpha_2}, \quad \alpha_1 + \alpha_2 \leq 1.$$ 

These intermediate goods are bundled to produce the single final good using a standard CES aggregator

$$Y_t = \left( \int A_{it} Y_{it}^{\frac{\alpha_1}{\theta}} \, dt \right)^{\frac{\theta}{\theta-1}}.$$ 

The term $A_{it}$ represents an idiosyncratic demand shifter and is the only source of fundamental uncertainty in the economy (i.e., we abstract from aggregate risk). We assume that $A_{it}$ follows
an AR(1) process in logs:

\[ a_{it} = (1 - \rho) \bar{a} + \rho a_{i,t-1} + \mu_{it}, \quad \mu_{it} \sim N(0, \sigma_{\mu}^2), \quad (1) \]

where we use lower-case to denote natural logs, a convention we follow throughout, so that, e.g., \( a_{it} = \log A_{it} \). In this specification, \( \bar{a} \) represents the unconditional mean of \( a_{it} \), \( \rho \) the persistence, and \( \mu_{it} \) an i.i.d. innovation with variance \( \sigma_{\mu}^2 \).

**Market structure and revenue.** The final good is produced by a competitive firm under perfect information. This yields a standard demand function for intermediate good \( i \)

\[ Y_{it} = P_{it}^{-\theta} A_{it}^\theta Y_t \quad \Rightarrow \quad P_{it} = \left( \frac{Y_{it}}{Y_t} \right)^{-\frac{1}{\theta}} A_{it}, \]

where \( P_{it} \) denotes the relative price of good \( i \) in terms of the final good, which serves as numeraire.

The elasticity of substitution \( \theta \) indexes the market power of intermediate good producers. Our specification nests various market structures. In the limiting case of \( \theta = \infty \), we have perfect competition, i.e., all firms produce a homogeneous intermediate good. In this case, the survival of heterogenous firms requires decreasing returns to scale in production to limit firm size, that is, \( \hat{\alpha}_1 + \hat{\alpha}_2 < 1 \). When \( \theta < \infty \), we have monopolistic competition, with constant or decreasing returns to scale. No matter the assumption here, however, firm revenue can be expressed as

\[ P_{it} Y_{it} = Y_t^{\frac{1}{\theta}} A_{it}^{\alpha_1} N_{it}^{\alpha_2}, \quad (2) \]

where

\[ \alpha_j = \left( 1 - \frac{1}{\theta} \right) \hat{\alpha}_j \quad j = 1, 2 \]

This framework accommodates two alternative interpretations of the idiosyncratic component \( A_{it} \) as a firm-specific shifter of either demand or productive efficiency. Neither the theory nor our empirical strategy requires us to differentiate between the two, so we will simply refer to \( A_{it} \) as a firm-specific fundamental.

**Input choices under imperfect information.** The key element of our theory is the effect of imperfect information on the firm’s choice of factor inputs, that is, capital and labor. These are modeled as static and otherwise frictionless decisions, i.e., firms rent capital and/or hire labor period-by-period, but with potentially imperfect knowledge of their fundamentals \( A_{it} \). Clearly, the impact of the information friction will depend on whether it affects both inputs or
just one. Rather than take a particular stand on this important issue regarding the fundamental
nature of the production process, we present results for two cases: in case 1, both factors of
production are chosen simultaneously under the same (imperfect) information set; in case 2,
only capital is chosen under imperfect information whereas labor is freely adjusted after the
firm perfectly learns the current state.

**Case 1: Both factors chosen under imperfect information.** In this case, the firm’s profit-
maximization problem is given by

$$\max_{K_{it}, N_{it}} \ Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} [A_{it}] \ K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it} - R_t K_{it},$$

where $\mathbb{E}_{it} [A_{it}]$ denotes the firm’s expectation of fundamentals conditional on its information set
$\mathcal{I}_{it}$, which we make explicit below. Standard optimality and market clearing conditions imply

$$\frac{N_{it}}{K_{it}} = \frac{\alpha_2 R_t}{\alpha_1 W_t} = \frac{N}{K_t},$$

i.e., the capital-labor ratio is constant across firms.

Our empirical analysis uses moments of firm-level investment data and with this in mind,
we use the optimality conditions characterized in (4) to rewrite (3) simply as a capital input
choice problem:

$$\max_{K_{it}} \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} [A_{it}] \ K_{it}^{\alpha_2} - \left( 1 + \frac{\alpha_2}{\alpha_1} \right) R_t K_{it},$$

where

$$\alpha = \alpha_1 + \alpha_2 = (\hat{\alpha}_1 + \hat{\alpha}_2) \left( \frac{\theta - 1}{\theta} \right)$$

Notice that the firm’s expected revenues depend only on the aggregate capital-labor ratio, its
conditional expectation of $A_{it}$, and the chosen level of its capital input. The curvature parameter
$\alpha$ depends both on the returns to scale in production as well as on the elasticity of demand,
and will play an important role in our quantitative analysis below. Solving this problem and
imposing capital market clearing gives the following expressions for the firm’s capital choice
(the labor choice exactly parallels that of capital):

$$K_{it} = \frac{(\mathbb{E}_{it} [A_{it}])^{\frac{1}{1-\alpha}}}{\int (\mathbb{E}_{it} [A_{it}])^{\frac{1}{1-\alpha}} \, di} K_t.$$
Case 2: Only capital chosen under imperfect information. The firm’s problem now is

\[
\max_{K_{it}} \mathbb{E}_{it} \left[ \max_{N_{it}} Y_{t}^{\frac{1}{\beta}} A_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_{t}N_{it} \right] - R_{t}K_{it},
\]

and optimizing over \(N_{it}\) gives

\[
N_{it} = \left( \frac{\alpha_2}{W_{t}} Y_{t}^{\frac{1}{\beta}} A_{it}^{\alpha_1} \right)^{\frac{1}{1-\alpha_2}}.
\]

(7)

Note that in contrast to (4), capital-labor ratios are now functions of the firm’s fundamental \(A_{it}\) and chosen level of capital \(K_{it}\), the former fully observed when making the labor choice and the latter fixed. Imposing labor market clearing and substituting, we can write the firm’s capital choice problem as:

\[
\max_{K_{it}} (1 - \alpha_2) \left( \frac{\alpha_2}{W_{t}} \right)^{\frac{1}{\alpha_2}} Y_{t}^{\frac{1}{\beta}} A_{it}^{\alpha_1} \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] K_{it}^{\tilde{\alpha}} - R_{t}K_{it},
\]

(8)

where

\[
\tilde{A}_{it} = A_{it}^{\frac{1}{1-\alpha_2}}, \quad \tilde{\alpha} = \frac{\alpha_1}{1 - \alpha_2}.
\]

Thus, the firm’s capital choice problem here has the same structure as in case 1 (compare equations (5) and (8)), but with a slightly modified fundamental and overall curvature. This will make the two cases qualitatively very similar, though, as we will see, the quantitative implications will be quite different. We mark with a ~ the transformed objects that are relevant in case 2, a convention we will carry throughout this section. The firm’s input choices can be shown to satisfy

\[
K_{it} = \frac{\left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{1}{\alpha}}}{\int \mathbb{E}_{it} \left[ \tilde{A}_{it} \right]^{\frac{1}{\alpha}} d\tilde{\pi}} K_{it}, \quad N_{it} = \frac{\tilde{A}_{it} \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{\alpha}}}{\int \tilde{A}_{it} \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{\alpha}} d\tilde{\pi}} N_{it}.
\]

(9)

While the capital choice looks similar to case 1, the labor choice now depends on the joint distribution of \(\tilde{A}_{it}\) and \(\mathbb{E}_{it} \left[ \tilde{A}_{it} \right]\). Despite this, the analysis remains quite tractable and we will derive simple expressions for the economic aggregates as functions of the underlying uncertainty.

To complete our characterization of the firm’s problem and therefore of the production-side equilibrium in the economy, we must explicitly spell out the information set \(\mathcal{I}_{it}\) on which the firm relies to form its expectations. We defer this discussion to the following subsection and for now directly make conjectures about firm beliefs, which we will later verify under our information structure. Specifically, we assume the conditional distribution of the fundamental
to be log-normal in both case 1 and 2, i.e.,

\[ a_{it}|I_t \sim N\left( E_{it}[a_{it}], V \right) \]

\[ \tilde{a}_{it}|I_t \sim N\left( E_{it}[\tilde{a}_{it}], \tilde{V} \right) \]

where \( E_{it}[a_{it}] \) and \( V \) denote the posterior mean and variance of \( a_{it} \) in case 1, respectively, and similarly \( E_{it}[\tilde{a}_{it}] \) and \( \tilde{V} \) in case 2. Further, as we will show, the cross-sectional distribution of the posterior mean \( E_{it}[a_{it}] \) is also normal, centered around the true mean \( \bar{a} \) with associated variance \( \sigma_a^2 - V \). Focusing on case 1 for a moment, the variance \( V \) indexes the severity of informational frictions in the economy and will turn out to be a sufficient statistic for misallocation and the associated productivity/output losses. It is straightforward to show that \( V \) is closely related to commonly used measures of allocative efficiency. For example, it maps exactly into the dispersion of the marginal revenue product of capital (in logs), measured along the lines of Hsieh and Klenow (2009), i.e., \( \sigma_{mrpk}^2 = V \). Similarly, it has a negative effect on the covariance between fundamentals and firm activity as examined, for example, in Bartelsman et al. (2013) and Olley and Pakes (1996), i.e., the covariance between \( a_{it} \) and \( k_{it} \) satisfies \( \sigma_{ak} = \frac{\sigma_a^2 - V}{1-\alpha} \). Thus, our measure of informational frictions is easily related to measures of misallocation studied in the literature. An analogous correspondence holds for case 2.

**Aggregation.** We now turn to the aggregate economy, and in particular, measures of total factor productivity (TFP) and output. Given our focus on misallocation, we abstract from aggregate risk and restrict our attention to a stationary equilibrium, in which all aggregate variables remain constant through time. From here on, we will also assume constant returns to scale in production, i.e., \( \hat{\alpha}_1 + \hat{\alpha}_2 = 1 \). Though no essential to our results, this greatly simplifies the expressions.\(^{11}\) Relocating the lengthy but straightforward derivations to Appendix A.1 and A.2, we use (6) and (9) along with the fact that \( Y = \int P_{it}Y_{it}di \) as well as standard properties of the log-normal distribution to derive the following simple representation for aggregate output:

\[ \log Y \equiv y = a + \hat{\alpha}_1 k + \hat{\alpha}_2 n. \]  

(10)

Aggregate TFP, denoted \( a \), is endogenous and is given by

Case 1:

\[ a = a^* - \frac{1}{2} \theta \bar{V} \]  

(11)

Case 2:

\[ a = a^* - \frac{1}{2} \left( \theta \hat{\alpha}_1 + \hat{\alpha}_2 \right) \hat{\alpha}_1 \bar{V} \]  

(12)

\(^{11}\)It is straightforward to relax this assumption and work in the more general case; we do so in our derivations in the Appendix.
where
\[ a^* = \frac{\theta}{\theta - 1} \bar{a} + \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\sigma_a^2}{1 - \alpha} \]
is aggregate TFP under full information, which is identical in the two cases.

These expressions are at the heart of our mechanism and reveal a sharp connection between the micro-level uncertainty summarized by \( V \) (or \( \tilde{V} \)) and aggregate TFP: in both cases, aggregate productivity monotonically decreases in uncertainty, with the magnitude of the effect depending on the elasticity of substitution (i.e., the degree of curvature) \( \theta \). The higher is \( \theta \), that is, the closer we are to perfect competition, the more severe are the losses from misallocated resources. The intuition is easy to see - when goods are highly substitutable, misallocation is particularly costly. In case 1, only \( \theta \) plays a role. In contrast, in case 2, the relative shares of capital and labor in the production function matter. Intuitively, the greater is labor’s share \( \hat{\alpha}_2 \) (and so the lower capital’s share \( \hat{\alpha}_1 \)), the greater the ability of firms to mitigate the effects of imperfect capital choice by adjusting labor. To take the extreme case, as \( \hat{\alpha}_2 \) approaches one and so \( \hat{\alpha}_1 \) zero, the multiplier on \( \tilde{V} \) approaches zero, that is, the informational friction has no effect on aggregate TFP. This flexibility is absent in case 1, in which both inputs are subject to the same friction. Notice that the two cases are equivalent at the opposite extreme, i.e. when \( \hat{\alpha}_1 = 1 \) and \( \hat{\alpha}_2 = 0 \). It is easy to see that for a fixed set of parameters, the coefficient on uncertainty in case 2 is smaller than that in case 1.

Holding the aggregate factor stocks fixed, the effect of informational frictions on aggregate productivity \( a \) is also the effect on aggregate output \( y \). However, the aggregate capital stock in the steady state is not invariant to informational frictions: misallocation reduces incentives for capital accumulation and so the steady state stock of capital decreases with uncertainty. Incorporating this additional effect, we obtain the familiar expression for amplified the impact of allocative inefficiencies on aggregate output:

\[ \frac{dy}{dV} = \frac{da}{dV} \left( \frac{1}{1 - \hat{\alpha}_1} \right) \]  
(13)

\[ \frac{dy}{d\tilde{V}} = \frac{da}{d\tilde{V}} \left( \frac{1}{1 - \hat{\alpha}_1} \right). \]  
(14)

### 2.2 Information

We have shown that \( V \) (or in case 2, by \( \tilde{V} \)), i.e., the variance of the firm’s posterior beliefs, is a sufficient statistic for the impact of informational frictions on resource misallocation and the resulting consequences for aggregate outcomes. We now make explicit the information structure in the economy, that is, the elements of the information set \( \mathcal{I}_t \), which in turn will allow us to characterize \( V \) in terms of the primitives of the economy - specifically, the variances
of fundamentals and signal errors.

The firm’s information set $\mathcal{I}_{it}$ has three elements. The first is the entire history of its fundamental shock realizations, i.e., $\{a_{it-s}\}_{s=1}^{\infty}$. This is a natural assumption to make since ex-post revenues reveal the fundamental perfectly. Second, the firm also observes a noisy private signal of its contemporaneous fundamental

$$s_{it} = a_{it} + e_{it}, \quad e_{it} \sim \mathcal{N}(0, \sigma_e^2)$$

where $e_{it}$ is an i.i.d. mean-zero and normally distributed noise term. The third and last element of the information available to the firm is the price of its own stock, $P_{it}$. The final piece of our theory then is to outline how the stock price is determined and to characterize its informational content.

**The stock market.** Our formulation of the stock market and its informational properties follows the noisy rational expectations paradigm in the spirit of Grossman and Stiglitz (1980). For our specific model structure, we draw heavily from recent work by Albagli et al. (2011a) and Albagli et al. (2011b). For each firm $i$, there is a unit measure of outstanding stock or equity, representing a claim on the firm’s profits. These claims are traded by two groups of agents - imperfectly informed investors and noise traders.\(^{12}\)

There is a unit measure of risk-neutral investors for each stock. Every period, each investor decides whether or not to purchase up to a single unit of equity in firm $i$ at the current market price $P_{it}$. The market is also populated by noise traders who purchase a random quantity $\Phi(z_{it})$ of stock $i$ each period, where $z_{it} \sim \mathcal{N}(0, \sigma_z^2)$ is iid and $\Phi$ denotes the standard normal CDF.

Like firms, investors also observe the entire history of fundamental realizations, and in particular, know $a_{it-1}$ at time $t$. They also see the current stock price $P_{it}$. Finally, each investor $j$ is endowed with an independent noisy private signal about the firm’s contemporaneous fundamental $a_{it}$:

$$s_{ijt} = a_{it} + v_{ijt}, \quad v_{ijt} \sim \mathcal{N}(0, \sigma_v^2)$$

The random variables $v_{ijt}$ and $z_{it}$ are assumed to independent of the fundamental and the

\(^{12}\)There are several possible interpretations of the precise nature of these agents. One is that they are intermediaries investing on behalf of the representative family. Some of them are rational, optimizing investors, while others trade randomly. Because the household cannot distinguish between the two, they co-exist. An alternative is that they are members of the representative large family, again, with some members trading rationally and some randomly. The exact interpretation of these agents in the model is not crucial for our purposes - only that some trade rationally given their information and others randomly, with the net result that prices only imperfectly reflect firm fundamentals.
noise in the firm’s signal\textsuperscript{13}. The total demand of investors for stock $i$ is then given by

$$D(a_{it-1}, a_{it}, P_{it}) = \int d(a_{it-1}, s_{ijt}, P_{it}) \, dF(s_{ijt}|a_{it})$$

where $d(a_{it-1}, s_{ijt}, P_{it}) \in [0, 1]$ is the demand of investor $j$ and $F$ is the conditional distribution of investors’ private signals. The market clearing condition is

$$D(a_{it-1}, a_{it}, P_{it}) + \Phi(z_{it}) = 1.$$ 

The expected payoff to investor $j$ from purchasing the stock is given by

$$E_{ijt}[\Pi_{it}] = \int \left[ \pi(a_{it-1}, a_{it}, P_{it}) + \beta \bar{P}(a_{it}) \right] \, dH(a_{it}|a_{it-1}, s_{ijt}, P_{it}).$$

The term $\pi(a_{it-1}, a_{it}, P_{it})$ denotes the expected current profit of the firm as a function of history\textsuperscript{14}, the current realization of the fundamental $a_{it}$ and the current stock price $P_{it}$. This is a function of the current price $P_{it}$ because it enters the firm’s information set and through that, influences firm decisions\textsuperscript{15}. The distribution $H(a_{it}|a_{it-1}, s_{ijt}, P_{it})$ is the investor $j$’s posterior over $a_{it}$ and $\bar{P}(a_{it})$ is the expected price in period $t+1$, conditional on the current fundamental $a_{it}$. Formally,

$$\bar{P}(a_{it}) = \int \mathcal{P}(a_{it}, a_{it+1}, z_{it+1}) \, dG(a_{it+1}, z_{it+1}|a_{it})$$

is obtained by integrating the price function $\mathcal{P}(\cdot)$ over $(a_{it+1}, z_{it+1})$ (using the conditional distribution $G$). Clearly, optimality implies:

$$d(a_{it-1}, s_{ijt}, P_{it}) = \begin{cases} 
1 & \text{if } E_{ijt}[\Pi_{it}] > P_{it} \\
0 & \text{if } E_{ijt}[\Pi_{it}] < P_{it} \\
1 & \text{if } E_{ijt}[\Pi_{it}] = P_{it}
\end{cases} \quad (15)$$

that is, an investor purchases the maximum quantity allowed (1 share) when the expected payoff (conditional on her information) strictly exceeds the price, does not purchase any shares when the expected payoff is (strictly) less than the price, and is indifferent when the two are equal.

An rational expectations equilibrium is a set of functions for prices $\mathcal{P}(\cdot)$, expected profits $\pi(\cdot)$, investor decision rules $d(\cdot)$ and capital choice $k(\cdot)$ such that, for any history of shocks, (i) $\pi(\cdot)$ is consistent with firm optimization and the market price function $\mathcal{P}(\cdot)$; (ii) $d(\cdot)$ is consistent

\textsuperscript{13}This in turn implies that the noise in the stock price is orthogonal to the firm’s information set. We relax this assumption later in our robustness section.

\textsuperscript{14}Given the assumption of an AR(1) structure for fundamentals and an iid process for $z_{it}$, the most recent realization, $a_{it-1}$, is a sufficient statistic for historical information.

\textsuperscript{15}Appendix A.3 explicitly characterizes $\pi(\cdot)$ in terms of the firm’s problem studied in the previous subsection.
with investor optimality as in (15) above (iv) $k(\cdot)$ is optimal given the firm’s information set and (v) markets for capital and each firm’s stock clear.

We conjecture that, for almost all realizations, equilibrium outcomes have the following 2 properties: (a) trading decisions of investors are characterized by a threshold rule: there is a signal $\hat{s}_{it}$ such that only investors observing signals higher than $\hat{s}_{it}$ choose to buy and (b) the market price $P_{it}$ is an invertible function of $\hat{s}_{it}$.$^{16}$

Aggregating the demand decisions of all investors, market clearing implies

$$1 - \Phi \left( \frac{\hat{s}_{it} - a_{it}}{\sigma_v} \right) + \Phi \left( z_{it} \right) = 1,$$

which leads to a simple characterization of the threshold signal

$$\hat{s}_{it} = a_{it} + \sigma_v z_{it} \ .$$

This defines a monotonic relationship between $P_{it}$ and $\hat{s}_{it}$, implying that observing the stock price is informationally equivalent to observing $\hat{s}_{it}$: in other words, from an informational standpoint, the stock price is simply a signal of firm fundamentals of the form (16). The precision of this signal, $\frac{1}{\sigma_v^2}$, is decreasing in both the variance of the noise in investors’ private signals and the size of the noise trader shock.

This simple expression for price informativeness in (16) is the key payoff of the structure we have imposed on stock market trading: we now have a complete characterization of the firm’s information set and hence the posterior variance $\mathbb{V}$, even without an explicit solution for the price function.$^{17}$ Formally, the firm’s information set is $\mathcal{I}_{it} = (a_{it-1}, s_{it}, \hat{s}_{it})$, which yields a simple expression for $\mathbb{V}$:

$$\mathbb{V} = \left( \frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2 \sigma_z^2} \right)^{-1}$$

The firm’s posterior variance is thus increasing in the noisiness of the two signals ($\sigma_e^2$ and $\sigma_v^2 \sigma_z^2$). In the absence of any learning, $\mathbb{V} = \sigma_\mu^2$, that is, all fundamental uncertainty remains unresolved at the time of the firm’s input choice. At the other extreme, under perfect information, $\mathbb{V} = 0$.

Finally, note that the marginal investor, i.e., the investor whose signal $s_{ijt} = \hat{s}_{it}$, must be exactly indifferent between buying and not buying. It follows then that the price $P_{it}$ must equal

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$^{16}$Given that we have unbounded shocks, there are always histories where this conjecture does not hold. However, these realizations are extremely unlikely - in fact, they do not show up at all in all our simulated sample paths. In this sense, we verify this conjecture in our numerical results.

$^{17}$It is straightforward to show that the conditional and cross-sectional distributions are log-normal under this information set, exactly as conjectured.
her expected payoff from holding the stock:

\[ P_{it} = \int \left[ \pi (a_{it-1}, a_{it}, P_{it}) + \beta \tilde{P} (a_{it}) \right] dH (a_{it}|a_{it-1}, \tilde{s}_{it}, P_{it}) \]

\[ = \int \left[ \pi (a_{it-1}, a_{it}, \tilde{s}_{it}) + \beta \tilde{P} (a_{it}) \right] dH (a_{it}|a_{it-1}, \tilde{s}_{it}, \tilde{s}_{it}) , \]

where, with a slight abuse of notation, we replace \( P_{it} \) with its informational equivalent \( \tilde{s}_{it} \) in the arguments of \( H (\cdot) \). Rewriting this equation in recursive form yields a fixed-point characterization of the price function:

\[ P (a_{-1}, a, z) = \int \pi (a_{-1}, a, a + \sigma_v z) dH (a|a_{-1}, a + \sigma_v z, a + \sigma_v z) \]

\[ + \beta \int \left[ \int P (a, a', z') dG (a', z'|a) \right] dH (a|a_{-1}, a + \sigma_v z, a + \sigma_v z) . \quad (17) \]

In our numerical analysis, we solve this functional equation using an iterative procedure with a discrete grid of shock realizations. We then verify that the invertibility and threshold properties of the equilibrium hold for all points on the grid.

### 3 Identifying Informational Frictions

The main hurdle in quantifying the effects of imperfect information is imposing discipline on the information structure, given that we do not directly observe signals. We overcome this difficulty with a novel empirical strategy that combines moments from firm-level production and stock market data to pin down the informational parameters of our model. In this section, we develop the intuition for that strategy by analyzing two special cases of our model - when firm level shocks are iid and when they follow a random walk. When we return to our general model in the following section, we will verify numerically that the properties of the special cases analyzed here extend to the general model used there.

We first show that the informational parameters of our model are identified by three key observable moments - the correlations of stock returns with both fundamentals and investment, and the volatility of returns.\(^{18}\) In both cases (iid and random walk fundamentals), there is an intuitive analytic mapping between these moments and the informational parameters. We further exploit the tractability of these settings to demonstrate the robustness of our approach to two important modifications of the baseline framework: in the first, we introduce other distortions into the firm’s investment choice problem; in the second, we allow for a more general correlation

\(^{18}\)Note that we can use the structure of the model to directly back out the fundamental \( a_{it} \) from data on revenues and capital.
structure between firm and market information. These variants also serve to showcase some merits of our approach relative to reduced-form, regression-based strategies used extensively in existing work on the relationship between stock markets and investment.

3.1 Identification

Transitory shocks. Consider the case where shocks to fundamentals are i.i.d., i.e., $\rho = 0$ in equation (1). Using a log-linear approximation, the price (around the deterministic case) leads to

$$ p_{it} \equiv \log P_{it} \approx \xi \tilde{E}[a_{it}] + \text{Constant}, $$

where $\xi = \frac{1-\alpha}{1-\alpha}$ and $\tilde{E}[a_{it}]$ is the expectation of $a_{it}$, conditional on the marginal investor’s information set. It is then straightforward to derive

$$ \tilde{E}[a_{it}] = \tilde{E}[u_{it}] = \psi (u_{it} + \sigma_v z_{it}), $$

where

$$ \psi = \frac{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2} + \frac{1}{\sigma_z^2}}. $$

Similarly, capital is a log-linear function of the firm’s expectation of the current innovation

$$ k_{it} = \frac{\tilde{E}[u_{it}]}{1-\alpha} + \text{Constant}, \quad (18) $$

which is a precision-weighted average of its private signal and the information in prices:

$$ \tilde{E}[u_{it}] = \phi_1 (u_{it} + e_{it}) + \phi_2 (u_{it} + \sigma_v z_{it}), $$

where

$$ \phi_1 = \frac{V}{\sigma_e^2}, \quad \phi_2 = \frac{V}{\sigma_v^2 \sigma_z^2}. $$

From here, we derive the following expressions for the two correlations of interest, that between returns and changes in fundamentals, denoted $\rho_{pa}$, and between returns and investment, denoted $\rho_{pk}$:

$$ \rho_{pa} \equiv \text{Corr} (p_{it} - p_{it-1}, a_{it} - a_{it-1}) = \frac{1}{\sqrt{1 + \frac{\sigma_e^2 \sigma_z^2}{\sigma_p^2}}}, \quad (19) $$

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19See Appendix A.4 for derivations. From here on, in a slight abuse of notation, we use $V$ to denote the uncertainty in both case 1 and case 2, where it should be understood that $V$ in case 2 corresponds to $\tilde{V}$ in the theory. We similarly use $a_{it}$ to denote the fundamental in both cases and $\alpha$ the relevant curvature parameter.

20Note that both of the signals in the marginal investor’s information set are equal to $u_{it} + \sigma_v z_{it}$. 
\[ \rho_{pk} \equiv \text{Corr}(p_{it} - p_{it-1}, k_{it} - k_{it-1}) = \frac{1}{\sqrt{1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_{\mu}^2}} \left(1 - \frac{\mathbb{V}}{\sigma_{\mu}^2}\right) \right.} \]  \quad (20)

Equation (19) shows that the higher is \( \frac{\sigma_v^2 \sigma_z^2}{\sigma_{\mu}^2} \), the noise-to-signal ratio in prices, the lower is the correlation of returns with fundamentals. Equation (20) then implies that for a given level of noise in prices, \( \rho_{pk} \) is increasing in the firm’s posterior variance \( \frac{\mathbb{V}}{\sigma_{\mu}^2} \) - investment choices covary more strongly with the signal when firms are more uncertain. Note that we work with \( \frac{\mathbb{V}}{\sigma_{\mu}^2} \) for convenience - combined with \( \sigma_u^2 \) and \( \sigma_v^2 \sigma_z^2 \) (from the expression for \( \rho_{pa} \)), this bears a one-to-one relationship with \( \sigma_e^2 \), the noise in the firm’s private signal.

Notice that a high \( \rho_{pk} \) is not by itself indicative of the degree of uncertainty. Firm choices can be highly correlated with returns either because they both track fundamentals very closely or because firms are uncertain.\(^{21}\) Observing \( \rho_{pa} \) allows us to isolate the effect of the latter. To see this more clearly, substitute for \( \rho_{pa} \) in (20) to derive,

\[ \frac{\rho_{pa}}{\rho_{pk}} = \sqrt{1 - \frac{\mathbb{V}}{\sigma_{\mu}^2}} \implies \frac{\mathbb{V}}{\sigma_{\mu}^2} = 1 - \left(\frac{\rho_{pa}}{\rho_{pk}}\right)^2. \]  \quad (21)

Thus, with iid fundamentals, the severity of informational frictions is pinned down directly by the ratio of the correlations, \( \frac{\rho_{pa}}{\rho_{pk}} \). Under full information, this ratio takes a value of 1. This sharp link between the relationship between the correlations and the severity of informational frictions guides our empirical approach in our quantitative analysis below.\(^{22}\) The more general version of the model precludes an analytical mapping between these correlations and \( \mathbb{V} \), but numerical simulations reveal a very similar positive relationship between the relative correlation and \( \mathbb{V} \).

**Relationship to investment-Q regressions.** This special case also leads to a reduced-form representation of investment as a log-linear function of fundamentals, signal errors and prices:

\[ \Delta k_{it} = \lambda_1 \left(\Delta \mu_{it} + \Delta e_{it}\right) + \lambda_2 \Delta p_{it}, \]  \quad (22)

where \( \Delta \)’s denote changes. This is in the spirit of specifications widely used in the empirical corporate finance literature to examine the role of learning from stock markets. For example, Morck et al. (1990) regress investment growth on stock returns, sales growth and other con-

\(^{21}\)For example, suppose we make prices more informative, i.e., decrease \( \sigma_v^2 \). Then \( \rho_{pk} \) rises even though uncertainty decreases.

\(^{22}\)For completeness, we show in Appendix A.4 that the volatility of returns and their correlation with fundamentals can be used in a final step to separately identify \( \sigma_v^2 \) and \( \sigma_z^2 \).
Our structural model enables us to interpret the coefficients from these reduced-form regressions in terms of the informational primitives of the economy. For example, consider the coefficient on stock returns, $\lambda_2$:

$$\lambda_2 = \frac{1}{(1 - \beta)} \psi \left( \frac{V}{\sigma_v^2} \right).$$

Equation (23) reveals the same intuition as in (20): $\lambda_2$ can be high either because firms are subject to a good deal of uncertainty, i.e., $V$ is large, and so rely heavily on information from markets, or because markets are highly informative, i.e., $\sigma_v^2 \sigma_z^2$ is low.

The regression implied by (22) consistently identifies the coefficients only in the case of orthogonality between the error $e_{it}$ and the regressors $a_{it}$ and $p_{it}$. If the noise terms in the signals of firms and investors are correlated, this orthogonality assumption is violated, leading to endogeneity biases in the regression estimates. A similar issue arises if the choice of capital is affected by additional factors that are correlated with fundamentals. As we will show later in this section, our approach is robust to these concerns.

**Permanent shocks.** Another tractable special case is that of permanent shocks, i.e., $\rho = 1$. We now show that the main insights from the i.i.d. case extend to this case as well. We start by deriving the expression for the ex-dividend price of the stock, which takes a similar form to the i.i.d. case:

$$p_{it} = \frac{1}{1 - \alpha} E_t [a_{it}] + \text{Constant}$$

We derive the following expressions in the Appendix. As in the iid case, they demonstrate a sharp mapping between the three moments - $\rho_{pk}$, $\rho_{pa}$ and $\sigma_p^2$ - and the informational parameters:

$$\frac{V}{\sigma_p^2} = \frac{\rho_{pk} - \rho_{pa}}{\eta}$$

$$\frac{\sigma_v^2 \sigma_z^2}{\sigma_p^2} = \frac{(1 - \eta^2)}{2 \rho_{pa}} + \frac{\eta}{\rho_{pa}} - 1$$

$$\frac{\sigma_z^2 + 1}{\sigma_p^2 + 1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_p^2}} = \frac{1}{\eta}.$$  

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23 Chen et al. (2007) use $Q$ and cash flow growth as their independent variables.

24 Consider, for example, the effects of introducing a correlated distortion $\tau_{it} = \gamma u_{it}$ into the firm’s capital choice in (18) so that $k_{it} = \frac{(1 + \gamma) u_{it}}{1 - \alpha} + \text{Constant}$. The coefficient $\lambda_2$ is given by $\frac{1 + \gamma}{(1 - \beta) \psi V} \sigma_v^2 \sigma_z^2$. In other words, inferring $V$ from $\lambda_2$ requires knowledge of (or at the very least, an adjustment for) $\gamma$. Note that uncorrelated distortions will not have any effect on $\lambda_2$.

25 With permanent shocks and no exit, there is no stationary distribution. Since our goal here is primarily to provide intuition for our empirical strategy, we ignore this complication and interpret this as a limiting case.

26 All derivations are in Appendix A.4.
where \( \eta = \left( \frac{1}{1 - \alpha} \right) \frac{\sigma_p}{\sigma_p}. \) In contrast to the i.i.d. case, however, now all three moments are necessary to infer the extent of uncertainty, yet the intuition is very similar: all else equal, a higher relative correlation \( (\rho_{pk} - \rho_{pa}) \) implies greater uncertainty, a lower \( \rho_{pa} \) higher levels of noise in prices, and higher return volatility, larger noise trader shocks.

### 3.2 Robustness

We now turn to two important exercises aimed at demonstrating the robustness of the identification strategy outlined above. Specifically, we use the iid case to explore the effects of relaxing two assumptions in our baseline setup. The first assumption relates to the information structure and the second to the nature of the firm’s investment choice.

**An alternative information structure.** In our baseline setup, the noise terms in the signals received by firms and investors are assumed to be orthogonal to each other. This is unlikely to hold in a world with a lot of communication between firms and financial market participants. For example, in practice, firms routinely release forecasts and announce their investment plans to investors/analysts. This raises a potential concern - correlation in these noise terms (specifically, \( e_{it}, v_{ijt}, \) and \( z_{it} \)) may generate co-movement between investment and returns and so may bias the inference of \( V. \) However, this turns out not to be the case - the ratio of correlations still leads us to the true \( V. \) More precisely, (21) holds for an arbitrary correlation structure between market and firm information.\(^{27}\) This implies that, with iid shocks, our assessment of firm-level uncertainty is not sensitive to assumptions about the structure of information flows between firms and investors. Note that this only applies to our estimate of \( V \) - our conclusions about the individual variances \( (\sigma_{e}^{2}, \sigma_{v}^{2} \) and \( \sigma_{z}^{2}) \) do change with assumptions about the correlation structure.

**Other distortions.** Our baseline setup also makes a somewhat extreme assumption that informational frictions are the only impediment to marginal product equalization. In reality, investment decisions are likely affected, or “distorted,” by a number of other factors. These may originate, for example, from technological limitations (e.g., adjustment costs), contracting frictions (e.g., financial constraints), or distortionary government policies. All of these can lead to dispersion in marginal products. Quantifying their contribution - whether individually or jointly - to observed cross-country differences is certainly the overall objective of the growing literature on misallocation, but one that is well beyond the scope of this paper. Our more limited goal here is to isolate and quantify the degree of firm-level uncertainty and its particular role

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\(^{27}\)See Appendix A.5 for the derivation.
in generating misallocation. Having said that, by abstracting from these other factors, we raise a potential concern about the validity of our measurement strategy.

To address this concern, we introduce other distortions drawn from a flexible, albeit stylized, class, which allows us to incorporate some essential features of the factors listed above without sacrificing analytical tractability. Specifically, we directly modify the firm’s optimality condition (18):

$$k_{it} = \frac{(1 + \gamma) E[u_{it}] + \varepsilon_{it}}{1 - \alpha} + \text{Constant}$$

This is equivalent to introducing a ‘distortion’ $\tau_{it}$, comprising two terms:

$$\tau_{it} = \gamma u_{it} + \varepsilon_{it} \quad \varepsilon_{it} \sim N(0, \sigma^2_{\varepsilon})$$

The parameter $\gamma$ indexes the extent to which $\tau_{it}$ covaries with the fundamental $u_{it}$. For example, if $\gamma < 0$, $\tau$ is to be interpreted as a factor that discourages (encourages) investment by firms with stronger (weaker) fundamentals. The second term, $\varepsilon_{it}$, represents factors that are uncorrelated with firm-specific fundamentals. The specification above assumes that they are idiosyncratic (though it is straightforward to add a common component) and perfectly observed by the firm (though not by the econometrician). The two parameters $\gamma$ and $\sigma^2_{\varepsilon}$ together pin down the amplitude (measured by the standard deviation) of this distortion and its correlation with firm-level fundamentals. To understand how the presence of $\tau_{it}$ affect our inference strategy, we use equation (21) to estimate $V$ and compare it to the true measure of uncertainty.

Consider first the case with only correlated distortions, i.e., $\sigma^2_{\varepsilon} = 0, \gamma \neq 0$. In this case, we can show that equation (21) is unaffected, that is, the ratio of the two correlations still uncovers the true $V$. This is despite the fact that $\tau_{it}$ does have an effect on more direct measures of misallocation. For example, the dispersion in the marginal revenue product of capital (MRPK) is given by

$$\sigma^2_{\text{mrpk}} = \gamma^2 (\sigma^2_{\mu} - V) + V$$

which is increasing in the absolute value of $\gamma$. Similarly, the covariance between fundamentals and investment is also affected by $\gamma$:

$$\sigma_{ak} = \left(\frac{1 + \gamma}{1 - \alpha}\right) (\sigma^2_{\mu} - V)$$

Thus, a strategy which targets these measures directly (i.e., chooses $V$ to match these moments) would lead to a biased estimate of the severity of informational frictions. In particular, inferring $V$ from $\sigma^2_{\text{mrpk}}$ without adjusting for $\gamma$ overstates the extent of uncertainty, while using $\sigma_{ak}$ can lead to an upward or downward bias, depending on the sign of $\gamma$. Similar concerns also
apply to inferences made directly from the cross-sectional variance of investment or revenue - these moments also confound the effects of uncertainty with other factors, and so using them without taking a stand on the nature of these other distortions could be problematic. Using the relative correlation as outlined above, on the other hand, continues to identify the true level of uncertainty. In this sense, our identification strategy is robust to the presence of correlated distortions which have a scaling effect on capital choice.

Next, we turn to the case where distortions are uncorrelated with fundamentals, i.e., \( \sigma_{\varepsilon}^2 \neq 0, \gamma = 0 \). In this case, we can show

\[
\frac{\mathcal{V}}{\sigma_{\mu}^2} = \left[ 1 - \left( \frac{\rho_{pa}}{\rho_{pk}} \right)^2 \right] + \frac{\sigma_{\varepsilon}^2}{\sigma_{\mu}^2}.
\]

The terms inside the square bracket on the right hand side correspond to our measure of uncertainty using (21). Since the last term is strictly positive, this underestimates the true uncertainty. In other words, factors uncorrelated with fundamentals would tend to make our estimate of \( \mathcal{V} \) more conservative, in that we would infer less uncertainty than is truly the case. This is despite the fact that these distortions exacerbate misallocation, as the following expression shows

\[
\sigma_{mrpk}^2 = \mathcal{V} + \sigma_{\varepsilon}^2.
\]

Again, a strategy of choosing \( \mathcal{V} \) to directly match observed misallocation would lead to an overstatement of the extent of informational frictions.

Note that in both instances (correlated and uncorrelated), if the true \( \mathcal{V} \) were 0 (i.e., firms were perfectly informed), using (21) would not lead us to a positive estimate. In other words, we will not find evidence of informational frictions in a full information economy, even one with marginal product dispersion.

## 4 Quantitative Analysis

Our analytic results in the previous section demonstrated a tight relationship between the three moments \( (\rho_{pk}, \rho_{pa}, \sigma_p^2) \) on which we focus and the three informational parameters in our economy \( (\sigma_e^2, \sigma_v^2, \sigma_z^2) \), and through them, the degree of micro-level uncertainty. With this intuition in hand, we now return to our general model and, following an identification approach quite similar in spirit, proceed to quantitatively infer these parameters using data from 3 countries - the US, China, and India. We use the results to quantify the extent of informational frictions, the

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28 In the random walk case, even in the absence of other distortions, we show in Appendix A.4 that the cross-sectional variance of investment is invariant to uncertainty.

29 Note, however, that the effect of uncertainty on aggregate outcomes does depend on \( \gamma \).
degree of resulting misallocation, and the impact on aggregate outcomes across these countries. Our analysis also sheds light on the role of learning from various channels, and perhaps of particular interest, on the importance of financial markets in improving allocative efficiency by delivering useful information to firms. We make use of the multi-country aspect of our analysis to perform a number of counterfactual experiments assessing the sources of cross-country variation in uncertainty and their respective effects on allocative efficiency.

4.1 Parameterization

We begin by assigning values to the more standard parameters in our model - specifically, those governing the preference and production structure of the economy. Throughout our analysis, we will hold these constant across countries; cross-country differences will come only from the parameters governing the stochastic processes on firm fundamentals and learning. Although a simplification, we feel that this is a natural starting point and allows us to focus on the role of imperfect information in leading to differences in aggregate outcomes across countries.

An important issue here is the choice of period length in the model. The focus of this paper - investment decisions - and our modeling choices push us towards larger period lengths. There is significant empirical evidence of long lags in planning and implementing investment projects, with estimates of the mean duration time between the planning stage and project completion of between 2 and 3 years.\(^{30}\) It seems reasonable then to assume that firms are required to forecast fundamentals over a relatively long horizon (2-3 years) when making large investment decisions and have only limited flexibility to adjust capital choices ex-post in response to additional information. One approach would be to explicitly model these lags as well as other features (such as adjustment costs/irreversibilities) which are likely relevant for investment decisions over short horizons. Largely for tractability, we take a different route and model the capital choice as a static one, but interpret a period in the model as spanning a relatively long horizon, specifically 3 years. This makes the omission of explicit lags/irreversibilities, etc. somewhat less of a concern.\(^{31}\) More importantly, this approach allows us to preserve the analytical tractability on the production side, particularly the simple expressions linking uncertainty to aggregate outcomes.

\(^{30}\)The classic reference here is Mayer (1960), who uses survey data on new industrial plants and additions to existing plants to find a mean gestation lag between the drawing of plans and the completion of construction of about 22 months. More recently, Koeva (2000) finds the average length of time-to-build lags to be about 2 years in most industries, defined as the period between the announcement of new construction and the ensuing date of completion. Given that this excludes the planning period prior to the announcement date, the total gestation lag is likely somewhat longer.

\(^{31}\)Morck et al. (1990) make a similar argument and perform their baseline empirical analysis using 3-year spans. They also point out that the explanatory power of investment growth regressions at shorter horizons (e.g., 1 year) are quite low.
In line with our choice of period length, we set the discount rate $\beta$ equal to 0.9. We assume constant returns to scale in production and set the production parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$ to standard values of 0.33 and 0.67, respectively. Given our CRS assumption, firm scale is then limited only by the curvature in demand, captured by the elasticity of substitution $\theta$. This will be a key parameter in influencing the quantitative impact of informational frictions. The literature contains a wide range of estimates for this parameter. We set our baseline estimate at 6, roughly in the middle of the commonly used range.\textsuperscript{32} Our choice of $\theta$ translates into an $\tilde{\alpha} = 0.62$ in case 2 and $\alpha = 0.83$ in case 1.

Next, we turn to the country-specific parameters. We begin with those governing the process on firm fundamentals $a_{it}$: the persistence $\rho$ and variance of the innovations $\sigma^2_{\mu}$. In both of our cases, we can directly construct the fundamental for each firm (up to an additive constant) as $rev_{it} - \alpha k_{it}$, where $rev_{it}$ denotes the log of revenues, and $\alpha$ depends on whether we are in case 1 or case 2.\textsuperscript{33} We then estimate the parameters of the fundamental process by performing the autoregression implied by (1), additionally including a time fixed-effect in order to isolate the idiosyncratic component of the innovations. The resulting coefficient delivers an estimate for $\rho$ and the variance of the residuals for $\sigma^2_{\mu}$.\textsuperscript{34}

Finally, it remains to pin down the three informational parameters of our model, i.e., the variances of the error terms in firm and investor signals $\sigma^2_e$ and $\sigma^2_v$, and the variance of the noise trader shock $\sigma^2_z$. We follow almost directly the strategy outlined above, i.e., target second moments in the growth rates of firm-level investment, fundamentals, and prices.\textsuperscript{35} The precise moments we use are the cross-sectional correlations of stock returns with investment growth (denoted $\rho_{pi}$) and changes in fundamentals, as well the variance of returns (as above, denoted $\rho_{pa}$ and $\sigma^2_{p}$, respectively). Importantly, stock returns are lagged by one period, so that the correlations reflect the comovement of investment and fundamentals with stock returns over the preceding period. This avoids feedback from investment and fundamentals to returns, the reverse of the relationship in which we are interested. Table 1 summarizes our empirical approach.

We use a simulated method of moments (SMM) approach to assign values to the informational parameters. Formally, we search over the parameter vector $(\sigma^2_e, \sigma^2_v, \sigma^2_z)$ to find the combination that minimizes the (unweighted) sum of squared deviations of the model-implied moments from the target moments. Before reporting the parameter estimates, we provide a

\textsuperscript{32} Additionally, we report our results, at least in case 2, for two other values of $\theta$, namely 4 and 10.

\textsuperscript{33} Substitute either (4) or (7) into (2).

\textsuperscript{34} See Appendix C for details.

\textsuperscript{35} We follow the literature examining the feedback effects of stock prices and use growth rates of investment (in the analytical cases studied earlier, we used investment, i.e. the growth rate of capital). By working with growth rates, we partly cleanse the data of firm fixed-effects, which are a significant component in cross-sectional differences in these variables. See Morck et al. (1990) for a more detailed discussion of these issues.
### Table 1: Parameterization - Summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common</td>
<td>Time period</td>
<td>3 years</td>
</tr>
<tr>
<td></td>
<td>Discount rate</td>
<td>0.90</td>
</tr>
<tr>
<td>(\hat{\alpha}_1)</td>
<td>Capital share</td>
<td>0.33</td>
</tr>
<tr>
<td>(\hat{\alpha}_2)</td>
<td>Labor share</td>
<td>0.67</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Elasticity of substitution</td>
<td>6</td>
</tr>
<tr>
<td>Country-specific</td>
<td>Persistence of fundamentals</td>
<td>Estimates of (1):</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Shocks to fundamentals</td>
<td>(a_{it} = (1 - \rho) \pi + \rho a_{it-1} + \mu_{it})</td>
</tr>
<tr>
<td>(\sigma_{\mu}^2)</td>
<td>Firm private signal</td>
<td>(\rho_{pi})</td>
</tr>
<tr>
<td>(\sigma_{\nu}^2)</td>
<td>Investor private signal</td>
<td>(\rho_{pa})</td>
</tr>
<tr>
<td>(\sigma_{\xi}^2)</td>
<td>Noise trading</td>
<td>(\sigma_p^2)</td>
</tr>
</tbody>
</table>

A numerical analogue of the identification argument in the previous section. This more general version of the model does not yield analytical expressions for the moments of interest, but we show in Figure 1 that the relationships between moments and parameters highlighted above go through almost exactly. The first panel shows a positive relationship between \(V\) and the relative correlation, as suggested by equations (21) and (24).\(^{36}\) Similarly, in the second panel, we see that higher levels of noise in prices are associated with lower \(\rho_{pa}\), exactly as we saw in the analytical cases.\(^{37}\) Finally, the bottom panel shows that, holding fixed the total noise \((\sigma_{\nu}^2 \sigma_{\xi}^2)\), increasing the size of the noise trader shock makes returns more volatile.

### 4.2 Data and Parameter Values

We construct the target moments using data on firm-level production variables and stock returns from Compustat North America for the US and Compustat Global for China and India. We focus on a single cross-section of firms in each country for the year 2012. This is the period with the largest number of observations, particularly so for China and India.\(^{38}\) Note that our data requirements are quite stringent: due to our focus on 3 year horizons and our use of growth

\(^{36}\)Here, we plot the difference in correlations, i.e. \(\rho_{pi} - \rho_{pa}\), but using the ratio yields a very similar picture. All moments in the figure are computed using simulated firm-level data from the model as we vary the corresponding parameter, holding the remaining parameters fixed at their estimated values (for the US).

\(^{37}\)Here, we hold \(V\) and the ratio \(\sigma_{\nu}/\sigma_{\xi}\) fixed, so as to focus on the information content of prices.

\(^{38}\)This is less of an issue for the US, and so as a robustness exercise, we recomputed the moments using a larger sample with more years. While there is some time variation, the results are quite similar to those from the single cross-section.
rates, we require at least 9 consecutive years of data for a firm in order to construct two 3 year periods and include it in our sample, with 2012 representing the final year of the second period. We compute the firm’s capital stock $k_{it}$ in each period as the average of its property, plant and equipment (PP&E) over the relevant 3 years, and investment as the change in the capital stock relative to the preceding 3 year period. To construct our measure of the fundamental $a_{it}$, we compute sales/revenue analogously as the average over the 3 year period and calculate $a_{it} = rev_{it} - \alpha k_{it}$. First-differencing gives investment and changes in fundamentals between the two periods. Stock returns are constructed as the change in the firm’s stock price over each 3 year period, adjusted for splits and dividend distributions. In order to be comparable to the unlevered returns in our model, stock returns in the data need to be adjusted for financial leverage. To do so, we assume that claims to firm profits are sold to investors in the form of debt and equity in a constant proportion (within each country). Observed return variances must then be multiplied by a constant factor in order to make them comparable to returns in the model, where the factor depends on the ratio of debt to total assets (or alternatively, debt to equity). Computing the debt-asset ratio from our sample gives average values of about 0.30 in both the US and India, and about 0.16 in China, with corresponding adjustment factors of about 0.5
and 0.7, respectively, and all values reported below reflect this adjustment. To isolate the firm-specific variation in our data series, we extract a time fixed-effect from each and utilize the residual as the component that is idiosyncratic to the firm. This is equivalent to demeaning each series from the unweighted average in each time period. As mentioned above, the target moments are computed using returns lagged by one period. This avoids the simultaneity problems associated with comparing price movements to contemporaneous investment/revenue numbers. We trim the 2% tails of each series in order to mitigate measurement error. Appendix C provides further details on how we build our sample and construct the variables.

We report the three target moments in the left hand panel of Table 2 both for case 2 in the upper panel and case 1 in the lower. In both cases, the moments exhibit significant cross-country variation. The US and India show similar levels of return volatility and a similar relation between returns and investment growth, but quite different comovements with fundamentals - returns in the US are more highly correlated with future changes in fundamentals. China has a return variance that is almost half that of the other two countries, along with the lowest correlations of returns with investment and fundamentals. As made clear by the analytic results in Section 3, none of these moments is a sufficient statistic to identify the informational role of markets or infer the extent of micro-level uncertainty; rather, it is the joint pattern of the three moments that matters and our explicit modeling of both production and financial market activity is precisely what allows us to tease this out.

In the right hand panel of Table 2, we report the resulting parameter values. The parameters governing the process on fundamentals $\rho$ and $\sigma_\mu$ are inferred from the regression implied by (1) as detailed above and the informational parameters from the target moments and SMM procedure just described. As we would expect from the cross-country variation in the target moments, the parameter estimates also vary markedly across countries. The US has less volatile fundamental shocks, lower levels of noise both in private signals at the firm level (lower $\sigma_e$) and generally in the stock market as well (lower $\sigma_v$ and $\sigma_z$). In the next section, we gauge the detrimental impact of the estimated frictions on productivity and output and in leading to differences in these aggregates across countries.

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39 In brief, letting $d$ denote the debt-asset ratio, the observed return variances must be multiplied by $(1 - d)^2$ to obtain the variance of unlevered returns. We describe the details of the calculations in Appendix C.

40 An alternative is to use CAPM $\beta's$ to remove the aggregate component from individual stock returns. This approach yields very similar results.

41 For example, $\rho_{pi}$ is the correlation between 2006-09 returns and investment growth during 2009-12.

42 $\rho_{pa}$ changes across cases since $\alpha$ affects our estimates of $a$. The remaining two moments change almost imperceptibly due to trimming.
Table 2: Target Moments and Parameter Estimates

<table>
<thead>
<tr>
<th>Case 2</th>
<th>US</th>
<th>China</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.23</td>
<td>0.16</td>
<td>0.25</td>
</tr>
<tr>
<td>China</td>
<td>0.18</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td>India</td>
<td>0.23</td>
<td>0.14</td>
<td>0.23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 1</th>
<th>US</th>
<th>China</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.24</td>
<td>0.15</td>
<td>0.26</td>
</tr>
<tr>
<td>China</td>
<td>0.10</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>India</td>
<td>0.23</td>
<td>0.14</td>
<td>0.22</td>
</tr>
</tbody>
</table>

4.3 Results

We report our baseline results in Table 3. The first three columns present the implied value for \( \psi \) based on the parameter estimates in Table 2, both in absolute terms and as a percentage of the underlying fundamental uncertainty, \( \sigma^2_\mu \), and of the total dispersion in the MRPK in our sample, \( \sigma^2_{mrpk} \). In the last two columns, we compute the implied losses in aggregate TFP and output relative to a full information benchmark. The top panel contains results for case 2, in which only capital is chosen under imperfect information, and the bottom panel the analogous results for case 1, in which both capital and labor are. Case 2 is the more conservative scenario (in the sense that it leads to lower TFP/output losses). We return to this issue in our discussion below and provide some suggestive evidence that reality likely falls in the middle.

Table 3: The Impact of Informational Frictions

<table>
<thead>
<tr>
<th>Case 2 (( \alpha = 0.62 ))</th>
<th>( \psi )</th>
<th>( \frac{\psi}{\sigma^2_\mu} )</th>
<th>( \frac{\psi}{\sigma^2_{mrpk}} )</th>
<th>( a^* - a )</th>
<th>( y^* - y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.08</td>
<td>0.41</td>
<td>0.22</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>China</td>
<td>0.16</td>
<td>0.63</td>
<td>0.34</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>India</td>
<td>0.22</td>
<td>0.77</td>
<td>0.48</td>
<td>0.10</td>
<td>0.14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 1 (( \alpha = 0.83 ))</th>
<th>( \psi )</th>
<th>( \frac{\psi}{\sigma^2_\mu} )</th>
<th>( \frac{\psi}{\sigma^2_{mrpk}} )</th>
<th>( a^* - a )</th>
<th>( y^* - y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.13</td>
<td>0.63</td>
<td>0.35</td>
<td>0.40</td>
<td>0.60</td>
</tr>
<tr>
<td>China</td>
<td>0.18</td>
<td>0.65</td>
<td>0.39</td>
<td>0.55</td>
<td>0.83</td>
</tr>
<tr>
<td>India</td>
<td>0.26</td>
<td>0.86</td>
<td>0.56</td>
<td>0.77</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Turning to the first two columns, there is substantial uncertainty in all cases: as a percent of the fundamental uncertainty \( \sigma^2_\mu \), the residual uncertainty ranges from a low of 41% in the US

\( \sigma^2_{mrpk} \) is computed as the average of within-industry dispersion in each country.
to a high of 77% in India in case 2, and similarly in case 1, although China and the US move much closer together on this score. In other words, firm learning \((\sigma^2 - V)\) eliminates from a high of about 60% of total uncertainty in the US to a low of about 20% in India in case 2, with a generally similar pattern in case 1, although the estimated degree of learning falls in the US and India. As we would expect from the cross-country variation in the parameter estimates in Table 2, the level of uncertainty varies systematically across countries: US firms are the most informed and Indian firms the least, with Chinese firms falling in the middle.

Next, we ask, how much of total MRPK dispersion do informational frictions account for? The answer is a significant portion: as a percentage of the total \(\sigma^2_{mrpk}\), \(V\) represents between about 20% and almost 60%, with the share generally lower in the US than in the two emerging markets. Later, we will decompose observed MRPK into a firm fixed effect and a transitory component and assess the role of informational frictions in explaining the latter. In a sense, this is a more meaningful comparison because informational frictions really cannot speak to fixed effects, which seem to an important component of observed MRPK dispersion.

Finally, the last two columns of Table 3 show that the substantial degree of residual uncertainty implies significant losses in productivity and output. Compared to the full-information benchmark, in case 2, steady state TFP is lower in the US by 4% (more precisely, 0.04 log-points) and 10% in India (the corresponding numbers for output are 5% to 14%). Estimated losses are significantly higher in case 1, ranging from 40% to about 80 in productivity and 60% to over 1 in output. In both cases, the US exhibits the smallest losses, reflecting the fact that US firms exhibit the smallest degree of ex-post uncertainty, and India the largest, with China falling in the middle. The differential impact of informational frictions leads to significant differences between the US and the two emerging markets in productivity and output, ranging from 3% to almost 40% for the former (across the two cases) and from 5% to over 50% for the latter (subtract \(a^* - a\) and \(y^* - y\) for the US from the corresponding values for China and India). These are somewhat modest, however, in comparison to standard measures of cross-country differences in aggregate TFP and income per-capita, but very much in line with other estimates based on particular frictions - for example, Midrigan and Xu (2013). In Section 4.5, we will use the structural parameters of our model to investigate in more detail the differences in informational frictions across countries; specifically, we perform a number of counterfactual experiments exploring the potential gains in the emerging markets from having access to a US-quality information structure.

**Case 1 vs Case 2.** Table 3 shows that under either scenario, informational frictions have a significant detrimental impact on aggregate performance, both when comparing the parameter-

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44In what follows, we adopt the convention of referring to log points as percentages.
ized economies to their full information benchmarks and when comparing the consequences of these frictions in emerging markets to the US. The magnitude of the effects depend to a great extent on the nature of the firm’s input decisions, that is, whether all inputs are chosen subject to the friction or only traditionally quasi-fixed inputs like capital.\textsuperscript{45} The intuition for these differences comes directly from equations (11) and (12): in case 1, given the level of uncertainty $V$, the aggregate consequences depend only on the elasticity of substitution $\theta$; in case 2, the relative shares of capital and labor in production also matter. The greater is labor’s share $\hat{\alpha}_2$, the greater the ability of firms to mitigate the effects of capital misallocation due to the informational friction by reallocating labor in a compensating fashion. Any such flexibility is absent in case 1, in which both inputs are subject to the same friction. Thus, the aggregate “cost” of a given level of uncertainty is larger in case 1 than case 2. To dig a bit deeper into this issue, note from equations (7) and (6) that in case 2, the marginal revenue product of labor (MRPL) is equalized across firms so that there should be no dispersion, while in case 1, dispersion in the MRPL exactly equals that in the MRPK. This is a direct result of our assumptions that in the former, all information is revealed before the labor choice, while in the latter, labor is chosen under the same limited information as capital. This implies a tight link between the cases and the ratio of the dispersion in the MRPL to that in the MRPK ($\sigma_{\text{mrpl}}^2 / \sigma_{\text{mrpk}}^2 = 0$ in case 2 and $= 1$ in case 1), which suggests that the empirical counterpart of this ratio might be useful in assessing how close we are to the two cases. We compute this ratio for the US firms in our sample, using total employment as an admittedly rough measure of labor inputs.\textsuperscript{46} The ratio is equal to 0.57, suggesting that reality is indeed in the middle of the two extremes we have analyzed.\textsuperscript{47} In this sense, we view our results as providing bounds on the adverse consequences of imperfect information for aggregate performance. Certainly, an important direction for future work is a more in-depth analysis of the precise nature of the firm’s input choices.

\textbf{Transitory MRPK dispersion.} Table 3 shows that uncertainty can account for a significant portion of the total MRPK dispersion observed in the data, ranging from 20-60\% across countries and cases. Note, however, that the dispersion induced by uncertainty is short-lived. In the data, on the other hand, there is some evidence of permanent deviations - in other words, measured MRPK deviations seem to comprise both a firm-specific fixed effect (which we

\textsuperscript{45}It is straightforward to extend our methodology to labor moments. The main hurdle is availability of data on labor input, especially in India and China. Even for the US, the coverage and quality of data on labor variables is lower than for capital.

\textsuperscript{46}$\sigma_{\text{mrpl}}^2$ is computed analogously to $\sigma_{\text{mrpk}}^2$.

\textsuperscript{47}Data on the wage bill is available only for a small set of the US firms in our dataset; however, using wage bill to measure labor input for this small set of firms gives a very similar ratio of 0.53. Reliable employment data is not available in the emerging markets.
cannot speak to with our theory) and a transitory component. In a sense, a more appropriate
gauge of the contribution of information frictions would be a comparison to this latter piece.
To address this question, we separate the two components for the US firms in our sample.\textsuperscript{48}
To do so, we restrict the sample to firms with at least 15 years of data and extend the data
as far back as 1985, so that we can construct at least 5 and as many as 10 3-year observations
for each firm. We compute the MRPK for each firm in each period and regress the result on a
firm fixed-effect. The residuals from this regression capture the purely transitory component of
MRPK deviations. We then compute the variance of this object, i.e., the dispersion in the trans-
itory component of MRPK deviations, and ask how much of this dispersion do informational
frictions account for? This exercise reveals, first, that dispersion in the transitory component
is substantial, representing about one-third of the total. Moreover, our estimated $\mathbb{V}$ for the US
accounts for about 60\% in case 2, and just about the entirety in case 1, again pointing to the
empirical relevance of informational frictions.\textsuperscript{49}

The effect of curvature. The impact of informational frictions is sensitive to the degree
of curvature, captured here by the elasticity of substitution $\theta$, set equal to 6 in our baseline
computations. Table 4 reports results for case 2 under two alternative values: $\theta = 4$, which is
on the low end of the commonly used range, and $\theta = 10$, which is on the high end. Changes
in $\theta$ lead to significant changes in the effects of the friction. Both the estimates of $\mathbb{V}$ and their
impact on TFP are lower with a smaller $\theta$, with TFP losses ranging from 2\% in the US to 6\% in
India and higher, but similarly ordered, output losses. In contrast, the higher value of $\theta$ shows
a more severe aggregate impact, ranging from 8\% in the US to 16\% in India for TFP and from
11\% to almost 25\% for output.\textsuperscript{50} The impact of the friction varies with $\theta$ for 2 reasons: first,
as can be seen in (12), for a given $\mathbb{V}$ the aggregate consequences are larger for higher values of
$\theta$ (i.e., higher $\alpha$). Second, the estimated $\mathbb{V}$ itself increases with $\theta$, due to the fact the $\rho_{pa}$ we
measure from the data falls as $\theta$ increases but the other two moments (importantly, $\rho_{pi}$) are
unchanged.

4.4 The Sources of Learning

Decomposing $\mathbb{V}$. Table 3 shows that firm learning can be quite significant and potentially
mitigates a substantial portion of the underlying fundamental uncertainty. Here, we explore

\textsuperscript{48}Sufficient data to perform this analysis are not available in the other two countries.

\textsuperscript{49}We again compute the average of within-industry dispersion.

\textsuperscript{50}The sensitivity of our results to $\theta$ is not particular to our framework, but rather is common when using
this class of model to study the aggregate implications of misallocation. See, for example, Hsieh and Klenow
(2009), who find that gains from marginal product equalization approximately double in China and India when
moving from $\theta = 3$ to $\theta = 5$.  

31
the relative importance of the two sources of learning present in our model, i.e., private sources within the firm versus market prices. We begin by reporting in the left-hand panel of Table 5 the total extent of learning and its aggregate consequences. To do so, we compute the reduction in $V$ both in absolute and percentage terms due to learning from both channels, i.e., $\Delta V = V - \sigma^2_{\mu}$, and the resulting effects on aggregate productivity and output. The table shows that total learning can be quite important and translates into significant improvements in TFP and output: in case 2 ranging from 3% in India to 5% in the US for the former and from 4% to 8% for the latter, with substantially higher gains in case 1.\(^{51}\) Interestingly, the contribution of learning in China in case 1 appears comparable to that in the US. Note that this does not imply that Chinese firms are necessarily as well-informed as US firms: due to the convexity of $V$, it is possible that a noisier signal leads to a greater reduction in uncertainty. Intuitively, if there is simply a greater amount of underlying uncertainty, as is the case in China, even a signal with identical precision is in a sense more valuable.

To break down the sources of learning, it is easier to work with the inverse of $V$, i.e., the total precision of the firm’s information, which lends itself to a simple linear decomposition:

$$\frac{1}{V} = \frac{1}{\sigma^2_{\mu}} + \frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_m r p k}.$$  

We focus on the latter two terms, which capture the contributions of private and market learning to overall precision and compute their relative import as $\frac{1}{\sigma^2_{\mu} + 1/\sigma^2_m r p k}$ and $\frac{1/\sigma^2_{\mu} + 1}{\sigma^2_e + 1/\sigma^2_m r p k}$, respectively. In this way, we calculate the fraction of the total increase in precision due to each source. We report the results in the right hand panel of Table 5. Strikingly, learning is due overwhelming to private sources: at best, markets account for about 10% of the increase in precision in the US and India, and about half of this in China. Clearly, these results point to firm private sources as the dominant channel for learning, with markets making only a small additional contribution. As we will see, the limited informational role of markets will prove to

\(^{51}\)Note the difference between these calculations and those in Table 3. There, we compute losses compared to a full-information benchmark. Here, we are computing gains versus a no-information (about innovations to fundamentals) benchmark.
be a robust finding throughout our analysis.

The role of market information. To explore in greater depth the importance of new information in stock prices, we recompute $V$ under the assumption that firms learn nothing from these prices, i.e., that they contain no information. This simply entails sending the noise in markets, $\sigma_v^2$, to infinity. The change in $V$ is a measure of the contribution of stock market information to firm learning and satisfies

$$
\Delta V = V - \left( \frac{1}{\sigma_{\mu}^2} + \frac{1}{\sigma_{e}^2} \right)^{-1}.
$$

We report this figure in the first columns of Table 6 along with the associated aggregate consequences. Even for the US, which has the highest degree of investment-return correlation, market information reduces uncertainty only modestly, eliminating about 2% of the fundamental uncertainty, with associated output gains as small as 0.3% and at a maximum, 2%. This also serves to reinforce our earlier point - the need to analyze these moments jointly through the lens of a structural model. For China and India, the contribution of market-produced information is even smaller, reducing uncertainty by between 0.5% and 2% and representing a maximum gain in output of about 1%.

Next, we ask whether the limited informational role of markets is due to the level of noise in prices or to the fact that firms already have considerable information about fundamentals, mitigating the incremental contribution of market information. This distinction is related to the concepts of forecasting price efficiency (FPE) versus revelatory price efficiency (RPE) as put
Table 6: The Contribution of Market Information

<table>
<thead>
<tr>
<th></th>
<th>With both sources</th>
<th>With only market learning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta V$</td>
<td>$\Delta \frac{V}{\sigma^2}$</td>
</tr>
<tr>
<td>Case 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>-0.004</td>
<td>-0.021</td>
</tr>
<tr>
<td>China</td>
<td>-0.003</td>
<td>-0.010</td>
</tr>
<tr>
<td>India</td>
<td>-0.005</td>
<td>-0.019</td>
</tr>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>-0.004</td>
<td>-0.021</td>
</tr>
<tr>
<td>China</td>
<td>-0.003</td>
<td>-0.009</td>
</tr>
<tr>
<td>India</td>
<td>-0.001</td>
<td>-0.005</td>
</tr>
</tbody>
</table>

**Notes:** The table shows the reduction in $V$ due to market information and the corresponding gains in the aggregates (hence the pattern of negatives and positives). The left hand panel shows the effects of market learning in the presence of private learning and the right hand panel when markets are the only source of information.

forth in Bond et al. (2012). The former captures the extent to which prices reflect and predict fundamentals, i.e., the absolute level of information in prices; the latter, the extent to which prices promote real efficiency by revealing new information to the firm. In our framework, $\Delta V$ in the first columns of Table 6 is the natural measure of RPE, since it is the marginal impact of the information in prices on uncertainty, given the other sources of firm information. As the table shows, at the estimated parameter values, markets in all 3 countries exhibit relatively low RPE. To measure FPE, we compute the reduction in $V$ from the information in stock prices alone, i.e.,

$$\Delta V = \left(\frac{1}{\sigma^2} + \frac{1}{\sigma^2_2}\right)^{-1} - \sigma^2_2$$

This measures the contribution of market information assuming it is the only source of learning for firms. We report the results in the right hand panel of Table 6. In general, stock markets are rather weak predictors of fundamentals - even as the only source of learning, the information they provide lead to only modest reductions in uncertainty. For the US, which has the most informative prices, market generated information reduces fundamental uncertainty by between 5% and 10%. Compare this to the US values in Table 5, where a total of between 35% and 60% of uncertainty is eliminated. The associated TFP and output effects of markets as a standalone source of information in the US are, at highest, 3% and 5%, respectively, in case 1. The aggregate impact of markets is even more modest for the US in case 2, and is lower in the two emerging markets than in the US in both cases. Even in a forecasting sense then, the efficiency of stock markets is fairly low, suggesting that the limited role of market information is in large part due to its poor quality; in other words, the poor RPE of stock markets can
be largely attributed to their low FPE. Comparing these values to the corresponding ones in the left hand panel shows that the already modest impact of the information in prices is even further diluted by the presence of alternative information available to firms, leading in net to the low levels of RPE shown.

While the limited informational role of markets is a robust pattern across countries and cases, these results should be interpreted somewhat cautiously. Our analysis focuses exclusively on information and decisions over the medium term. It is silent on the role of stock markets in guiding longer-term, strategic decisions (e.g., the decision to enter a new market or acquire another company). In fact, one possible source of ‘noise’ in stock returns is information about the firm’s prospects over a much longer horizon. So long as it is orthogonal to fundamentals over the medium term, the relevant object for the firm’s current decisions, our strategy will still lead us to the right measure of uncertainty and the associated misallocation - our estimates for the specific informational parameters and the informational role of financial markets, however, is likely to be sensitive to the assumptions about the nature of this ‘noise’ term.

Our analysis also abstracts from the contribution of financial market information in mitigating uncertainty about aggregate (or industry) conditions, particularly for outsiders (e.g., potential entrants, creditors, regulators, etc.). Explicitly modeling these features in a fully-fledged general equilibrium environment is a challenging task, but may well be essential for a comprehensive evaluation of the informational role of well-functioning financial markets\textsuperscript{52}.

**The role of private information.** Next, we explore the contribution of learning from information generated within the firm (i.e., firm ‘private information’) by performing experiments analogous to those above: specifically, we calculate the marginal contribution of private information to allocative efficiency, both in the presence of market information and when it is the only source of learning for firms.

We report the results in Table 7. In contrast to the effect of market learning, firm private information plays a much larger role in reducing uncertainty and improving aggregate performance. Turning to the first set of columns, private information eliminates between about 30\% and 50\% of the fundamental uncertainty in the US. The associated aggregate gains are substantial, ranging from 4-20\% for TFP and from 6-30\% for output. The corresponding results are smaller for China and India in case 2 (although only slightly so in China), reflecting a worse quality of firm private information in those countries, and are smaller for India in case 1. Having said that, they are still quite significant across countries and are noticeably larger than the effects of market information. Similar to what we saw in Table 5, the contribution of private learning in China in case 1 is on par with that in US, again because even a less precise

\textsuperscript{52}See section 5 in Bond et al. (2012) for a related discussion.
signal can have more value if the extent of uncertainty is greater.

Table 7: The Contribution of Private Information

<table>
<thead>
<tr>
<th>Case 2</th>
<th>With both sources</th>
<th>With only private learning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta V )</td>
<td>( \Delta V )</td>
</tr>
<tr>
<td></td>
<td>( \frac{\Delta V}{\sigma^2_e} )</td>
<td>( \Delta a )</td>
</tr>
<tr>
<td>US</td>
<td>-0.10</td>
<td>-0.48</td>
</tr>
<tr>
<td>China</td>
<td>-0.09</td>
<td>-0.35</td>
</tr>
<tr>
<td>India</td>
<td>-0.06</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 1</th>
<th>With both sources</th>
<th>With only private learning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta V )</td>
<td>( \Delta V )</td>
</tr>
<tr>
<td></td>
<td>( \frac{\Delta V}{\sigma^2_e} )</td>
<td>( \Delta a )</td>
</tr>
<tr>
<td>US</td>
<td>-0.07</td>
<td>-0.32</td>
</tr>
<tr>
<td>China</td>
<td>-0.09</td>
<td>-0.33</td>
</tr>
<tr>
<td>India</td>
<td>-0.04</td>
<td>-0.13</td>
</tr>
</tbody>
</table>

Notes: The table shows the reduction in \( V \) due to private information and the corresponding gains in the aggregates (hence the pattern of negatives and positives). The left hand panel shows the effects of private learning in the presence of market learning and the right hand panel when private sources are the only source of information.

The right hand panel of Table 7 reports the contribution of firm private information were it the only source of learning. A comparison of the two panels shows that the values are quite similar - the presence of market information does not significantly alter the importance of private information for overall learning.

4.5 Cross-country counterfactuals

Next, we use our estimates to run a couple of instructive counterfactual experiments. Specifically, we assess the potential gains to China and India from having access to US quality information, whether through more informative financial markets or from better firm-level private information. In the first experiment, we compute the change in \( V \) in China and India under the assumption that the informativeness of prices - summarized by \( \sigma^2_e\sigma^2_z \) - is equal to that in the US, leaving all other country-specific parameters fixed. This quantifies the improvements in aggregate performance that would accrue in China and India from having financial markets as informative as that in the US. Second, we perform the same exercise for firm private information, that is, compute the change in \( V \) and aggregate improvements under the assumption that firms in China and India have the same \( \sigma^2_e \) as their US counterparts (again leaving the other country-specific parameters fixed). In a final experiment, we turn away from learning and study the role of fundamentals in leading to a differential impact of informational frictions across countries; to do so, we recompute \( V \) in China and India assuming that firms in these countries face the same fundamental shocks as do US firms.
We report the results from these experiments in Table 8. The top panel shows that delivering US-quality markets to emerging economies results in potentially significant yet modest reductions in uncertainty. As a percent of total fundamental uncertainty, these range from 2% to 7% (across cases) with a corresponding aggregate impact ranging, for example, from 1% to 6% in output. The middle panel shows that access to US-quality private information would have a much larger impact, reducing $\sigma^2$ by as much as 40% of the underlying uncertainty in the most optimistic case, and, excepting China in case 1 which is an outlier here, more than 25% in the other cases. The resulting aggregate gains can be substantial, ranging, for example, from 4% to almost 40% in output. Across the board, the gains from accessing US-quality private information are much larger than from US-quality market information. To the extent that differences in learning lead to cross-country variation in economic aggregates, these disparities appear to be largely due to a lack of high quality firm private information, rather than to a lack of well-functioning (in an informational sense) financial markets in emerging markets. This is yet another instance of a result we have now seen several times: financial markets play only a modest informational role in promoting aggregate efficiency.

Table 8: The Effects of a US Information Structure

<table>
<thead>
<tr>
<th></th>
<th>Case 2</th>
<th></th>
<th>Case 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta V$</td>
<td>$\frac{\Delta V}{\sigma^2}$</td>
<td>$\Delta a$</td>
<td>$\Delta y$</td>
</tr>
<tr>
<td><strong>Market Information</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>-0.01</td>
<td>-0.05</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>India</td>
<td>-0.02</td>
<td>-0.07</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td><strong>Private Information</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>-0.07</td>
<td>-0.26</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>India</td>
<td>-0.12</td>
<td>-0.43</td>
<td>0.05</td>
<td>0.08</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>-0.02</td>
<td>-0.09</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>India</td>
<td>-0.05</td>
<td>-0.19</td>
<td>0.02</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Notes: The table shows the reduction in $V$ and the corresponding gains in the aggregates (hence the pattern of negatives and positives) in India and China from having access to US-quality financial market information (top panel), private information (middle panel) and facing US fundamental shocks (bottom panel).

Finally, we recompute $V$ in China and India under the assumption that firms in those countries face shocks of the same volatility as those in the US, i.e., we place the $\sigma^2_{\mu}$ in the two countries with that of the US. We report the results in the bottom panel of Table 8, which shows that disparities in the volatility of shocks also contribute significantly to differences in the impact of informational frictions. Exposure to a fundamental process such as that in the US reduces $V$ between about 10% and 25% of the underlying uncertainty, leading to TFP and
output gains$^{53}$ that are potentially substantial, ranging from 1-20% and 2-30% respectively. $^{54}$

### 4.6 Robustness

We now turn to two important robustness exercises. In Section 3, we used our special cases to demonstrate the validity of our measurement approach under two variants of the baseline model: first, in the presence of other factors entering the firm’s capital choice decision, and second, with a richer correlation structure between firm and market information. To explore the extent to which that intuition carries over to the general model, in this section we conduct two very similar experiments numerically. In particular, we consider the effects of (1) an alternative information structure in which investors observe a noisy signal of the firm’s private signal (in contrast to the conditional independence of signals in the baseline model), and (2) a specific friction in the firm’s capital choice, namely adjustment costs. In both experiments, our goal is to assess how the presence of these alternative factors affects our measure of the severity of informational frictions; the results from both continue to demonstrate the robustness of our baseline results.

**Correlated information.** Recall that in the special case analyzed in Section 3, our identification strategy proved robust to an arbitrary correlation structure, i.e., no matter how rich or complex the flow of information between firms and markets, our measure of uncertainty remained valid. Here, the lack of an analytical characterization means that we have to resort to numerical experiments to demonstrate a similar robustness. Without direct data on firm/market information, it is not obvious how to discipline the extent of the commonality of information. We take a different route and show results from a rather extreme modification of our baseline information structure - we assume that the signal observed by investors is a noisy signal of the firm’s private signal, i.e.,

\[ s_{ijt} = a_{it} + e_{it} + v_{ijt} \]

Investors then have 2 sources of information - imperfect readings of the firm’s own information and the stock price. Notice that by construction, firms have nothing to learn from market prices. Despite that, investment and returns will comove in part due to the common component in the two information sets. We estimate \( \Psi \) in this modified version of the model using the same procedure and data as above. We report the results in Table 9. The estimates

$^{53}$These numbers correspond to \( \Delta (a - a^*) \) and \( \Delta (y - y^*) \). Recall that \( a^* \), the full information productivity level, is also affected by the size of fundamental shocks, \( \sigma^2_\mu \).

$^{54}$Asker et al. (2012) also highlight the role of different firm-specific shock processes in generating misallocation in a model with capital adjustment costs.
for firm-level uncertainty are very close to those in our base case (which, of course, implies that the associated aggregate implications are also unchanged). Thus, we conclude that our estimates of uncertainty are not particularly sensitive to assumptions about correlation in firm and market information.\(^{55}\)

<table>
<thead>
<tr>
<th></th>
<th>(\sigma_e)</th>
<th>(\sigma_v)</th>
<th>(\sigma_z)</th>
<th>(V)</th>
<th>Baseline (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.37</td>
<td>0.17</td>
<td>8.01</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>China</td>
<td>0.60</td>
<td>0.63</td>
<td>4.65</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>India</td>
<td>0.79</td>
<td>0.45</td>
<td>6.37</td>
<td>0.19</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Notes: The table shows parameter estimates and the resulting extent of uncertainty when investors are assumed to observe a noisy signal of the firm’s private signal. All parameters not displayed retain the same values as the baseline case.

**Capital adjustment costs.** In the special cases analyzed in Section 3, we showed that our measure of uncertainty was robust to the presence of other distortions, both correlated and uncorrelated, affecting the firm’s capital choice decision. That analysis, while reassuring, was done with a stylized representation of these factors, primarily for analytical tractability. To explore this result in the general model, we ask how the presence of a particular friction, capital adjustment costs, affects our assessment of the severity of informational frictions. We focus on a polar case - an economy where fully informed firms are subject to convex costs of adjusting capital. We then take our model with informational frictions to moments generated from this hypothetical economy - our goal is to see whether our empirical strategy will lead us to an incorrect inference about uncertainty.

This experiment requires two modifications to our baseline model. First, following Bloom (2009), Cooper and Haltiwanger (2006), and Asker et al. (2012), we subject firms to quadratic costs of adjusting capital \(\zeta K_{t-1} \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right)^2\).\(^{56}\) Second, we assume that firms have full information \((\sigma^2_e = 0)\) but that stock markets are noisy \((\sigma^2_v \neq 0, \sigma^2_z \neq 0)\). We parameterize and solve the model in general equilibrium.\(^{57}\) We choose values of \((\sigma^2_v, \sigma^2_z, \zeta)\) to match 3 moments in the

\(^{55}\)Our estimates of the individual error variances do change, however.

\(^{56}\)We focus on convex costs and abstract from the fixed-type costs also considered in the literature, as the former are most capable of delivering a configuration of moments that resemble those under informational frictions. Intuitively, convex costs imply that investment decisions respond to innovations in fundamentals only in part contemporaneously and then with a lag. This latter feature resembles the pattern implied by informational frictions.

\(^{57}\)Since the firm’s capital choice problem is now a dynamic one, we can no longer analytically characterize the joint distribution of fundamentals and capital across firms. We have to solve numerically for this distribution (and the associated general equilibrium implications) in steady state. We refer the reader to Appendix B for details.
data - the volatility of returns and their correlation with fundamentals, $\sigma^2_p$ and $\rho_{pa}$, and the cross-sectional variation in firm-level investment. The first two were also used in our baseline analysis, while the last was added to the list to discipline the adjustment cost parameter $\zeta$.\footnote{This is along the lines of the estimation strategy employed in Bloom (2009), Cooper and Haltiwanger (2006), and Asker et al. (2012).}

We then simulate data using this parameterized model and compute the correlation of prices with investment (growth), $\rho_{pi}$. Since $\rho_{pa}$ is directly targeted, this parameter reveals the extent to which adjustment costs generate a large relative correlation, the statistic at the heart of our inference about $V$. Finally, we reestimate the parameters of our model with informational frictions using this simulated value for $\rho_{pi}$, along with the other two moments (namely, $\sigma^2_p$ and $\rho_{pa}$).

We report the results in Table 10.\footnote{We show in Appendix B the full set of target moments and resulting parameter values.} The correlation of returns with investment choices implied by the full-information adjustment cost model is significantly lower than that observed in the data, leading to the substantially lower values for relative correlation reported in the table. In other words, in the absence of firm-level uncertainty, investment decisions covary much less with stock returns even in the presence of adjustment costs. The relative correlation numbers suggest that applying our empirical strategy to these simulated moments would result in substantially lower estimates of $V$. The third column of the table confirms that this is indeed the case. The estimated $V$’s from the simulated data are only about one-third of those in our baseline analysis (compare to Table 3, case 2). The resulting aggregate consequences are also substantially lower: for example, TFP losses are only between 1 and 4%, compared to 4-10% above. These results provide some reassurance that our estimates are not simply picking up the effects of adjustment costs, but are indeed robust measures of firm-level uncertainty.

<table>
<thead>
<tr>
<th></th>
<th>Relative Correlation</th>
<th>Implied Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>US</td>
<td>0.05</td>
<td>-0.03</td>
</tr>
<tr>
<td>China</td>
<td>0.10</td>
<td>-0.08</td>
</tr>
<tr>
<td>India</td>
<td>0.17</td>
<td>-0.02</td>
</tr>
</tbody>
</table>

Notes: The table shows the relative correlation of returns with investment growth and returns with fundamentals ($\rho_{pi} - \rho_{pa}$), both as computed from the data and as predicted by the full-information adjustment cost model, and the implied extent of uncertainty from the latter. $\rho_{pa}$ is a targeted moment and so is the same in the data and model.

While this analysis had the admittedly limited goal of confirming the robustness of our estimates of uncertainty, it also suggests that both types of frictions (adjustment costs and...
imperfect information) might be necessary to match a broader set of empirical moments. Such a unified framework would also allow us to disentangle their effects on aggregate outcomes as well as explore how they interact with each other. At some level, it is not even clear that they should be thought of as two independent frictions: imperfect information leads to delayed responses to fundamentals and in this sense, provide a deeper theory of adjustment costs. Exploring these issues, both theoretically and quantitatively, is an important, if challenging, direction for future research.

5 Conclusion

We have laid out a theory of informational frictions that distort the allocation of factors across heterogeneous firms, leading to reduced aggregate productivity and output. Taking this theory to the data, we find evidence of substantial micro-level uncertainty, particularly in China and India.

There are several promising directions for future research. In our modeling strategy, we have aimed to strike a balance between realism and transparency of the forces at play, as well as of our empirical methodology. In doing so, we have made several admittedly rather extreme assumptions. For example, the investment choice is modeled as a static and otherwise undistorted decision problem. Similarly, the learning process is somewhat stark, with perfect revelation at the end of each period. Persistence in fundamentals means that firms will - and are able to - ‘correct’ their past errors. In other words, investment in the current period includes a component related to the forecast error in the previous period. When shocks are highly persistent, this adjustment makes a substantial contribution to investment volatility. These assumptions limit our ability to match a larger set of micro moments on investment dynamics. Relaxing them is straightforward from a conceptual standpoint - for example, by introducing financial constraints, adjustment costs or a richer stochastic process for fundamentals - however, there are substantial computational/empirical challenges. First, this introduces additional state variables into the firm’s problem and precludes the simple analytical mapping between uncertainty and aggregate outcomes. Second, this requires additional data to infer the additional parameters - particularly on the time series behavior of firm-level investment. This is less of an issue for the US, but in India and China, both availability and quality of the data decline dramatically as

\footnote{Fuchs et al. (2013) provide a microfoundation for adjustment costs based on asymmetric information and adverse selection.}

\footnote{Suppose, for example, that the fundamental }$a_{it}$\textsuperscript{consists of a persistent component and a purely transitory one. Then, even if }$a_{it-1}$\textsuperscript{is observed perfectly ex-post, firms will need to form beliefs about the persistent component (e.g., by using a Kalman filter) while forecasting }$a_{it}$\textsuperscript{. This implies that innovations to the persistent component are revealed slowly.}
Another important direction for future work is towards a deeper theory of differences in information quality. This is valuable both from a conceptual point of view as well as from the perspective of designing policies. For example, much of the cross-country variation seems to stem from differences in the quality of firms’ internal sources and only marginally due to the quality of information from financial markets. These results suggest that policies aimed at improving the quality of firm-level information may be more fruitful in improving aggregate performance than those intended to bring financial markets to a level closer to that in the US. Given that we follow the standard approach of modeling information as exogenous noisy signals, we are left to speculate on the exact form of such policies. In our view, the most likely sources of variation in the quality of firm private information are differences in manager skill and/or information collection/processing within the firm. Under this interpretation, improving manager training and/or accounting and information systems within firms hold great promise of generating substantial gains in aggregate TFP.

References


Appendix

A Detailed Derivations

A.1 Case 1: Both factors chosen under imperfect information

As we show in equation (5) in the text, the firm’s capital choice problem can be written as

$$\max_{K_{it}} \left( \frac{N}{K_t} \right)^{\alpha_2} Y_{it} \hat{\mathbb{E}}_{it} [A_{it}] K_{it}^{\alpha} - \left( 1 + \frac{\alpha_2}{\alpha_1} \right) R_{it} K_{it}$$
and optimality requires
\[
\alpha \left( \alpha_1 \right) \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^\frac{\beta}{\gamma} K_t^{\alpha-1} \mathbb{E}_{it} [A_{it}] = R_t
\]
\[
\Rightarrow \left[ \alpha \left( \frac{\alpha_1}{\alpha_1 + \alpha_2} \right) \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^\frac{\beta}{\gamma} \frac{\mathbb{E}_{it} [A_{it}]}{R_t} \right]^{\frac{1}{1-\alpha}} = K_{it}
\]

Capital market clearing then implies
\[
\int K_{it} di = \left[ \left( \frac{N}{K_t} \right)^{\alpha_2} \left( \frac{\alpha_1}{\alpha_1 + \alpha_2} \right) Y_t^\frac{\beta}{\gamma} \frac{\mathbb{E}_{it} [A_{it}]}{R_t} \right]^{\frac{1}{1-\alpha}} \int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di = K_t
\]
\[
\Rightarrow \left[ \left( \frac{N}{K_t} \right)^{\alpha_2} \left( \frac{\alpha_1}{\alpha_1 + \alpha_2} \right) Y_t^\frac{\beta}{\gamma} \frac{\mathbb{E}_{it} [A_{it}]}{R_t} \right]^{\frac{1}{1-\alpha}} = \frac{K_t}{\int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di}
\]

from which we can solve for
\[
K_{it} = \frac{\left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}}}{\int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di} K_t
\]

From here, it is straightforward to express firm revenue as
\[
P_{it} Y_{it} = K_t^{\alpha_1} N^{\alpha_2} Y_t^\frac{\beta}{\gamma} A_{it} \left( \frac{\left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}}}{\int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di} \right)^\alpha
\]

and noting that aggregate revenue must equal aggregate output, we have
\[
Y_t = \int P_{it} Y_{it} di = K_t^{\alpha_1} N^{\alpha_2} Y_t^\frac{\beta}{\gamma} \int A_{it} \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di \frac{\int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di}{\int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di} \alpha
\]
or in logs,
\[
y_t = \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n + \log \int A_{it} \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di - \alpha \log \int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di
\]

Now, note that under conditional log-normality,
\[
a_{it} | \mathcal{I}_{it} \sim \mathcal{N} \left( \mathbb{E}_{it} [a_{it}], \mathcal{V} \right) \Rightarrow \mathbb{E}_{it} [A_{it}] = \exp \left( \mathbb{E}_{it} a_{it} + \frac{1}{2} \mathcal{V} \right)
\]
The true fundamental $a_{it}$ and its conditional expectation $E_{it}a_{it}$ are also jointly normal, i.e.

$$
\begin{bmatrix}
a_{it} \\
E_{it}a_{it}
\end{bmatrix} \sim \mathcal{N}
\left(
\begin{bmatrix}
\bar{a} \\
\bar{a}
\end{bmatrix},
\begin{bmatrix}
\sigma_a^2 & \sigma_a^2 - \mathbb{V} \\
\sigma_a^2 - \mathbb{V} & \sigma_a^2 - \mathbb{V}
\end{bmatrix}
\right)
$$

We then have that

\[
\log \int A_{it} (E_{it} [A_{it}]) \frac{1}{\alpha} di = \log \int \exp \left( a_{it} + \frac{\alpha}{1 - \alpha} E_{it}a_{it} + \frac{1}{2} \frac{\alpha}{1 - \alpha} \right) di
\]

\[
= \frac{1}{1 - \alpha} \bar{a} + \frac{1}{2} \sigma_a^2 + \frac{1}{2} \left( \frac{\alpha}{1 - \alpha} \right)^2 (\sigma_a^2 - \mathbb{V})
\]

\[
+ \frac{\alpha}{1 - \alpha} (\sigma_a^2 - \mathbb{V}) + \frac{1}{2} \frac{\alpha}{1 - \alpha} \mathbb{V}
\]

\[
= \frac{1}{1 - \alpha} \bar{a} + \frac{1}{2} \sigma_a^2 + \frac{1}{2} \alpha \left( \frac{2 - \alpha}{1 - \alpha} \right)^2 (\sigma_a^2 - \mathbb{V})
\]

\[
+ \frac{1}{2} \frac{\alpha}{1 - \alpha} \mathbb{V}
\]

and

\[
\log \int (E_{it} [A_{it}]) \frac{1}{\alpha} di = \frac{1}{1 - \alpha} \bar{a} + \frac{1}{2} \left( \frac{1}{1 - \alpha} \right)^2 (\sigma_a^2 - \mathbb{V}) + \frac{1}{2} \frac{1}{1 - \alpha} \mathbb{V}
\]

Combining these,

\[
\log \int A_{it} (E_{it} [A_{it}]) \frac{1}{\alpha} di - \alpha \log \int (E_{it} [A_{it}]) \frac{1}{\alpha} di = \bar{a} + \frac{1}{2} \frac{\sigma_a^2}{2 - \alpha} - \frac{1}{2} \frac{\alpha}{1 - \alpha} \mathbb{V}
\]

Substituting and rearranging, we obtain the expressions in (10) and (11) in the text,

\[
y_t = \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n + \bar{a} + \frac{1}{2} \frac{\sigma_a^2}{2} - \frac{1}{2} \frac{\alpha}{1 - \alpha} \mathbb{V}
\]

\[
= \hat{\alpha}_1 k_t + \hat{\alpha}_2 n + \frac{\theta}{\theta - 1} \bar{a} + \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\sigma_a^2}{1 - \alpha} - \frac{1}{2} \theta \mathbb{V}
\]

with aggregate productivity given by

\[
a = \frac{\theta}{\theta - 1} \bar{a} + \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\sigma_a^2}{1 - \alpha} - \frac{1}{2} \theta \mathbb{V}
\]

It now remains to endogenize $K_t$. The rental rate in steady state satisfies

\[
R = \frac{1}{\beta} - 1 + \delta
\]
Then, from the optimality and market clearing conditions, we have from (4) that
\[ \frac{\alpha_1 N}{\alpha_2 K} = \frac{R}{W} \Rightarrow K \propto W \]
i.e., the aggregate capital stock is proportional to the wage. To characterize wages, we return to the firm’s profit maximization problem
\[ \max_{K_{it}, N_{it}} Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} [A_{it}] \left( K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W N_{it} - RK_{it} \right) \]
which, after maximizing over capital, can be written as
\[ \max_{N_{it}} (1 - \alpha_1) \left( \frac{\alpha_1}{R} \right)^{\frac{1-\alpha_1}{1-\alpha_1-\alpha_2}} \left( Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} [A_{it}] N_{it}^{\alpha_2} \right)^{\frac{1}{1-\alpha_1}} - WN_{it} \]
Optimality and labor market clearing imply
\[ \left( \frac{\alpha_2}{W} \right)^{\frac{1-\alpha_1}{1-\alpha_1-\alpha_2}} \left( \frac{\alpha_1}{R} \right)^{\frac{1-\alpha_1}{1-\alpha_1-\alpha_2}} \left( Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha_1-\alpha_2}} = N_{it} \]
\[ \left( \frac{\alpha_2}{W} \right)^{\frac{1-\alpha_1}{1-\alpha_1-\alpha_2}} \left( \frac{\alpha_1}{R} \right)^{\frac{1-\alpha_1}{1-\alpha_1-\alpha_2}} \int \left( Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha_1-\alpha_2}} di = N \]
As before, letting \( \alpha = \alpha_1 + \alpha_2 \), we see that
\[ W \propto \left( \int \mathbb{E}_{it} [A_{it}]^{\frac{1}{\alpha}} di \right)^{\frac{1-\alpha}{1-\alpha_1}} Y_t^{\frac{1}{\theta} \frac{1}{1-\alpha_1}} \]
\[ = \left[ \left( \int \left( \exp \left( \mathbb{E}_{it} a_{it} + \frac{1}{2} V \right) \right)^{\frac{1}{\alpha}} di \right)^{\frac{1}{1-\alpha}} Y_t^{\frac{1}{\theta}} \right]^{\frac{1}{1-\alpha_1}} \]
\[ = \left[ \exp \left( \bar{a} + \frac{1}{2} \left( \frac{\sigma_a^2 - \alpha V}{1 - \alpha} \right) \right) Y_t^{\frac{1}{\theta}} \right]^{\frac{1}{1-\alpha_1}} \]
or in logs,
\[ w \propto \left( \frac{1}{1-\alpha_1} \right) \bar{a} + \frac{1}{1-\alpha_1} \frac{1}{2} \left( \frac{\sigma_a^2 - \alpha V}{1 - \alpha} \right) + \frac{1}{\theta} \frac{1}{1-\alpha_1} y_t \]
Recalling that
\[ K \propto W \Rightarrow \frac{dk}{dV} = \frac{dw}{dV} \]
which, in conjunction with (10) and (11), implies

\[
\frac{dy}{dN} = \hat{\alpha}_1 \left( \frac{dk}{dN} \right) - \frac{1}{2} \theta \\
= \frac{\hat{\alpha}_1}{1 - \alpha_1} \left[ -\frac{1}{2} \frac{\alpha}{1 - \alpha} + \frac{1}{\theta} \frac{dy}{dN} \right] - \frac{1}{2} \theta
\]

and finally, collecting terms and rearranging, and using the fact that \( \hat{\alpha}_1 = \frac{\theta}{\theta - 1} \alpha_1 \), we obtain

\[
\frac{dy}{dN} = -\frac{1}{2} \theta \frac{1}{1 - \hat{\alpha}_1} = \frac{da}{dN} \frac{1}{1 - \hat{\alpha}_1}
\]

### A.2 Case 2: Only capital chosen under imperfect information

The firm’s labor choice problem can be written as

\[
\max_{N_{it}} \quad Y_{it}^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it}
\]

and optimality requires

\[
N_{it} = \left( \frac{\alpha_2}{\bar{W}_t} Y_{it}^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} \right)^{\frac{1}{1 - \alpha_2}}
\]

Labor market clearing then implies

\[
\int N_{it} \, di = \int \left( \frac{\alpha_2}{\bar{W}_t} Y_{it}^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} \right)^{\frac{1}{1 - \alpha_2}} \, di = N
\]

\[
\Rightarrow N_{it} = \frac{\left( A_{it} K_{it}^{\alpha_1} \right)^{\frac{1}{1 - \alpha_2}}}{\int \left( A_{it} K_{it}^{\alpha_1} \right)^{\frac{1}{1 - \alpha_2}} \, di} N
\]

Letting \( \tilde{A}_{it} = A_{it}^{\frac{1}{1 - \alpha_2}} \) and \( \tilde{\alpha} = \frac{\alpha_1}{1 - \alpha_2} \), we have

\[
N_{it} = \frac{\tilde{A}_{it} K_{it}^{\tilde{\alpha}}}{\int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} \, di} N
\]
which implies

\[
W_t = Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_2} \left( \frac{\tilde{A}_{it} K_{it}^{\tilde{\alpha}}}{\int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} \, di} \right)^{\alpha_2-1}
\]

\[
= \frac{\alpha_2}{N^{1-\alpha_2}} Y_t^{\frac{1}{\theta}} \left( \int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} \, di \right)^{1-\alpha_2} A_{it} K_{it}^{\alpha_1} \left( A_{it}^{\frac{1}{1-\alpha_2}} K_{it}^{\frac{\alpha_1}{1-\alpha_2}} \right)^{\alpha_2-1}
\]

\[
= \frac{\alpha_2}{N^{1-\alpha_2}} Y_t^{\frac{1}{\theta}} \left( \int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} \, di \right)^{1-\alpha_2}
\]

From here, it is straightforward to express the firm’s capital choice problem as in (8):

\[
\max_{K_{it}} (1 - \alpha_2) \left( \frac{\alpha_2}{W_t} \right)^{\frac{1}{1-\alpha}} Y_t^{\frac{1}{\theta}} \frac{1}{1-\alpha} \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] K_{it}^{\tilde{\alpha}} - R_t K_{it}
\]

Optimality requires

\[
R_t = (1 - \alpha_2) \left( \frac{\alpha_2}{W_t} \right)^{\frac{1}{1-\alpha}} Y_t^{\frac{1}{\theta}} \frac{1}{1-\alpha} \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \tilde{\alpha} K_{it}^{\tilde{\alpha}-1}
\]

\[
\Rightarrow \quad K_{it} = \left[ (1 - \alpha_2) \tilde{\alpha} \right]^{\frac{1}{1-\alpha}} \left( \frac{\alpha_2}{W_t} \right)^{\frac{1}{1-\alpha}} Y_t^{\frac{1}{\theta(1-\alpha_2)(1-\alpha)}} \mathbb{E}_{it} \left[ \tilde{A}_{it} \right]^{\frac{1}{1-\alpha}}
\]

Capital market clearing then implies

\[
\int K_{it} \, di = \left[ (1 - \alpha_2) \tilde{\alpha} \right]^{\frac{1}{1-\alpha}} \left( \frac{\alpha_2}{W_t} \right)^{\frac{1}{1-\alpha}} Y_t^{\frac{1}{\theta(1-\alpha_2)(1-\alpha)}} \int \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{1}{1-\alpha}} \, di = K_t
\]

\[
\Rightarrow \quad K_{it} = \left( \frac{\mathbb{E}_{it} \left[ \tilde{A}_{it} \right]}{\int \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \, di} \right)^{\frac{1}{1-\alpha}} K_t
\]

and we can rewrite the labor choice as

\[
N_{it} = \frac{\tilde{A}_{it} K_{it}^{\tilde{\alpha}}}{\int A_{it} K_{it}^{\tilde{\alpha}} \, di} N = \frac{\tilde{A}_{it} \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\alpha}}}{\int \tilde{A}_{it} \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\alpha}} \, di} N
\]
Combining the solutions for capital and labor, we can express firm revenue as

\[ P_{it}Y_{it} = Y_t^{\frac{1}{\theta}} \left\{ \frac{\left( E_{it} [\tilde{A}_{it}] \right)^{\frac{1}{1-\alpha}}}{\int \left( E_{it} [\tilde{A}_{it}] \right)^{\frac{1}{1-\alpha}} di} K_t \right\} ^{\alpha_1} \left\{ \frac{\tilde{A}_{it} \left( E_{it} [\tilde{A}_{it}] \right)^{\frac{\alpha}{1-\alpha}}}{\int \tilde{A}_{it} \left( E_{it} [\tilde{A}_{it}] \right)^{\frac{\alpha}{1-\alpha}} di} N \right\} ^{\alpha_2} \]

\[ = Y_t^{\frac{1}{\theta}} K_t^{\alpha_1 N^{\alpha_2}} A_{it} \tilde{A}_{it}^{\alpha_2} \left( E_{it} [\tilde{A}_{it}] \right)^{\frac{\alpha_1 + \alpha_2 \alpha}{1-\alpha}} \left\{ \int \left( E_{it} [\tilde{A}_{it}] \right)^{\frac{1}{1-\alpha}} di \right\} ^{\alpha_1} \left\{ \int \tilde{A}_{it} \left( E_{it} [\tilde{A}_{it}] \right)^{\frac{\alpha}{1-\alpha}} di \right\} ^{\alpha_2} \]

and again using the fact that \( y_t = \log \int P_{it}Y_{it} \, di \), we can write

\[ y_t = \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n - \alpha_1 \log \left( E_{it} [\tilde{A}_{it}] \right)^{\frac{1}{1-\alpha}} + (1 - \alpha_2) \log \left( E_{it} [\tilde{A}_{it}] \right)^{\frac{\alpha}{1-\alpha}} \]

Again, we exploit log-normality to obtain

\[ \log \int \tilde{A}_{it} \left( E_{it} [\tilde{A}_{it}] \right)^{\frac{\alpha}{1-\alpha}} di = \log \int \exp \left( \tilde{a}_{it} + \frac{\tilde{\alpha}}{1 - \tilde{\alpha}} E_{it} \tilde{a}_{it} + \frac{1}{2} \frac{\tilde{\alpha}}{1 - \tilde{\alpha}} \tilde{V} \right) di \]

\[ = \frac{1}{1 - \tilde{\alpha}} \tilde{\alpha} + \frac{1}{2} \sigma_{\tilde{a}}^2 + \frac{1}{2} \tilde{\alpha} \frac{2 - \tilde{\alpha}}{(1 - \tilde{\alpha})^2} \left( \sigma_{\tilde{a}}^2 - \tilde{V} \right) + \frac{1}{2} \frac{\tilde{\alpha}}{1 - \tilde{\alpha}} \tilde{V} \]

and similarly,

\[ \log \int \left( E_{it} [\tilde{A}_{it}] \right)^{\frac{1}{1-\alpha}} di = \log \int \left( \exp \left( E_{it} \tilde{a}_{it} + \frac{1}{2} \tilde{V} \right) \right)^{\frac{1}{1-\alpha}} di \]

\[ = \frac{1}{1 - \tilde{\alpha}} \tilde{\alpha} + \frac{1}{2} \sigma_{\tilde{a}}^2 - \tilde{V} + \frac{1}{2} \frac{1}{1 - \tilde{\alpha}} \tilde{V} \]

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Combining, and using the fact that \( \tilde{a}_{it} = \frac{a_{it}}{1 - \alpha_2} \):

\[
-\alpha_1 \log \int (E_{it} [\tilde{A}_{it}])^{\frac{1}{\alpha}} \, di + (1 - \alpha_2) \log \int \tilde{A}_{it} (E_{it} [\tilde{A}_{it}])^{\frac{\hat{\alpha}}{\alpha}} \, di
\]

\[
= \frac{1 - \alpha_2 - \alpha_1}{1 - \alpha} \tilde{a} + \frac{(1 - \alpha_2)}{2} \sigma_{\tilde{a}}^2 + \frac{1}{2} \left( \frac{(1 - \alpha_2)(2\hat{\alpha} - \tilde{a})}{(1 - \hat{\alpha})^2} \right) \left( \sigma_{\tilde{a}}^2 - \tilde{V} \right)
\]

\[
= \frac{1 - \alpha}{1 - \alpha} \frac{\tilde{a}}{1 - \alpha_2} + \frac{(1 - \alpha_2)}{2} \sigma_{\tilde{a}}^2 + \frac{1}{2} \frac{\alpha_1}{1 - \hat{\alpha}} \left( \sigma_{\tilde{a}}^2 - \tilde{V} \right)
\]

\[
= \tilde{a} + \frac{1}{2} \frac{1}{1 - \alpha} \sigma_{\tilde{a}}^2 - \frac{1}{2} \frac{\alpha_1}{1 - \alpha} \alpha (1 - \alpha_2) \tilde{V}
\]

Substituting and collecting terms, we obtain expressions (10) and (12) in the text,

\[
y_t = \tilde{\alpha}_1 k_t + \tilde{\alpha}_2 n + \frac{\theta}{\theta - 1} \tilde{a} + \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\sigma_{\tilde{a}}^2}{1 - \alpha} - \frac{1}{2} (\theta \tilde{\alpha}_1 + \tilde{\alpha}_2) \tilde{\alpha}_1 \tilde{V}
\]

with aggregate productivity given by

\[
a = \frac{\theta}{\theta - 1} \tilde{a} + \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\sigma_{\tilde{a}}^2}{1 - \alpha} - \frac{1}{2} (\theta \tilde{\alpha}_1 + \tilde{\alpha}_2) \tilde{\alpha}_1 \tilde{V}
\]

To endogenize \( K_t \), we begin by characterizing the steady state wage \( W \) in terms of \( \int \tilde{A}_{it} (E_{it} [\tilde{A}_{it}])^{\frac{\alpha_1}{\alpha}} \, di \) and \( Y^{\frac{1}{\alpha}} \):

\[
W = \frac{\alpha_2}{N^{1 - \alpha_2}} Y^{\frac{1}{\alpha}} \left( \int \tilde{A}_{it} K_{it}^{\frac{\alpha}{\alpha_2}} \, di \right)^{1 - \alpha_2}
\]

\[
= \frac{\alpha_2}{N^{1 - \alpha_2}} Y^{\frac{1}{\alpha}} \left\{ \int \tilde{A}_{it} \left( \frac{1 - \alpha_2}{R} \tilde{a} \left( \frac{\alpha_2}{\tilde{W}} \right)^{\frac{\alpha_2}{1 - \alpha_2}} Y^{\frac{1}{\alpha_2}} \tilde{A}_{it} \right)^{\frac{\alpha}{\alpha_2}} \, di \right\}
\]

\[
= \frac{\alpha_2}{N^{1 - \alpha_2}} Y^{\frac{1}{\alpha}} \left[ \frac{(1 - \alpha_2) \tilde{a}}{R} \right]^{\frac{\alpha_2}{1 - \alpha_2}} \left( \frac{\alpha_2}{\tilde{W}} \right)^{\frac{\alpha_2}{1 - \alpha_2}} \left[ \int \tilde{A}_{it} (E_{it} [\tilde{A}_{it}])^{\frac{\hat{\alpha}}{\alpha}} \, di \right]^{1 - \alpha_2}
\]

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and rearranging,

\[ W = \left( \frac{\alpha_2}{N^{1-\alpha_2}} \right)^{1-\tilde{\alpha}} \left[ \frac{\tilde{\alpha} \tilde{\alpha}_1}{\tilde{\alpha}_2 \, \tilde{\alpha}_1} \right] Y_1 Y_\theta \int \tilde{A}_it \left( \mathbb{E} \tilde{A}_it \right) \frac{\tilde{\alpha}}{1-\alpha_1} \frac{1}{1-\tilde{\alpha}_1} \frac{1}{1-\alpha_1} d\tilde{\alpha}_1 \]

\[ \propto \left\{ \int \tilde{A}_it \left( \mathbb{E} \tilde{A}_it \right) \frac{\tilde{\alpha}}{1-\alpha_1} \frac{1}{1-\tilde{\alpha}_1} \frac{1}{1-\alpha_1} \right\} Y_1 Y_\theta \]

or in logs,

\[ w \propto \left( \frac{1}{1-\alpha_1} \right) \tilde{\alpha} + \frac{1}{2} \left( \frac{1}{1-\alpha_1} \right) \frac{1}{1-\alpha} \sigma^2_a - \frac{1}{2} \left( \frac{1}{1-\alpha_1} \right) \frac{1}{1-\alpha_1} \tilde{\alpha} \left( \frac{1}{1-\alpha_1} \right) \frac{1}{1-\tilde{\alpha}_1} \frac{1}{1-\alpha_1} \tilde{V} + \frac{1}{1-\alpha_1} \frac{1}{1-\alpha_1} \theta_1 y_t \]

As before,

\[ K \propto W \quad \Rightarrow \quad \frac{dk}{d\tilde{\alpha}} = \frac{dw}{d\tilde{\alpha}} = -\frac{1}{2} \left( \frac{\tilde{\alpha} (1-\alpha_2)}{(1-\alpha_1)(1-\tilde{\alpha})} + \frac{1}{\tilde{\alpha}_1} \frac{1}{1-\alpha_1} \frac{dy}{d\tilde{\alpha}_1} \right) \]

and substituting into the derivative of aggregate output,

\[ \frac{dy}{d\tilde{\alpha}} = \tilde{\alpha}_1 \left( \frac{dk}{d\tilde{\alpha}} \right) - \frac{1}{2} \left( \theta \tilde{\alpha}_1 + \tilde{\alpha}_2 \right) \tilde{\alpha}_1 \]

\[ = \tilde{\alpha}_1 \left[ -\frac{1}{2} \left( \frac{\tilde{\alpha} (1-\alpha_2)}{(1-\alpha_1)(1-\tilde{\alpha})} + \frac{1}{\tilde{\alpha}_1} \frac{1}{1-\alpha_1} \frac{dy}{d\tilde{\alpha}_1} \right) - \frac{1}{2} \left( \theta \tilde{\alpha}_1 + \tilde{\alpha}_2 \right) \tilde{\alpha}_1 \right] \]

Finally, collecting terms and rearranging, and using the facts that \( 1-\alpha = (1-\tilde{\alpha}) (1-\alpha_2) \) and \( \left( \frac{\theta}{\tilde{\alpha}_1} \right) \alpha_1 = \tilde{\alpha}_1 \), we obtain

\[ \frac{dy}{d\tilde{\alpha}_1} = -\frac{1}{2} \left( \theta \tilde{\alpha}_1 + \tilde{\alpha}_2 \right) \tilde{\alpha}_1 \frac{1}{1-\tilde{\alpha}_1} = \frac{da}{d\tilde{\alpha}_1} \left( \frac{1}{1-\tilde{\alpha}_1} \right) \]

### A.3 The stock market

Here, we connect expected profits \( \pi (\cdot) \) in the price function to the firm’s problem. For brevity, we only show the derivation for case 1 (case 2 is very similar). The firm’s profit is
= \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^\frac{1}{\alpha} A_{it} K_{it}^\alpha - \left( 1 + \frac{\alpha_2}{\alpha_1} \right) R K_t
\]

= \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^\frac{1}{\alpha} A_{it} \left( \frac{\mathbb{E}_{it} [A_{it}]}{\int (\mathbb{E}_{it} [A_{it}])^{-\frac{1}{\alpha}} di} \right)^\alpha - \left( 1 + \frac{\alpha_2}{\alpha_1} \right) \frac{\mathbb{E}_{it} [A_{it}]}{\int (\mathbb{E}_{it} [A_{it}])^{-\frac{1}{\alpha}} di} R K_t
\]

= \Gamma_1 A_{it} (\mathbb{E}_{it} [A_{it}])^{-\frac{1}{\alpha}} - \Gamma_2 (\mathbb{E}_{it} [A_{it}])^{-\frac{1}{\alpha}}

From the definition of the firm’s information set,

\[ \mathbb{E}_{it} [A_{it}] = \mathbb{E} [A_{it} | a_{it-1}, a_t, a_{it} + e_{it}, a_{it} + \sigma v z_{it}] \]

The profit function \( \pi (\cdot) \) is obtained after integrating out the noise term in the firm’s signal

\[ \pi (a_{it-1}, a_t, a_{it} + \sigma v z_{it}) = \Gamma_1 \int A_{it} (\mathbb{E} [A_{it} | a_{it-1}, a_t, a_{it} + e_{it}, a_{it} + \sigma v z_{it}])^{-\frac{1}{\alpha}} d\Phi \left( \frac{e_{it}}{\sigma e} \right) \]

\[ -\Gamma_2 \int ((\mathbb{E} [A_{it} | a_{it-1}, a_t, a_{it} + \sigma v z_{it}]))^{-\frac{1}{\alpha}} d\Phi \left( \frac{e_{it}}{\sigma e} \right) \]

\section{A.4 Special cases: Identification}

Transitory shocks. A log-linear approximation of prices (around the deterministic case, ignoring constants)\textsuperscript{62}

\[ P \exp (p_{it}) = \mathbb{E}_{it} AK^\alpha \exp (a_t + \alpha k_{it}) - \mathbb{E}_{it} R K \exp (k_{it}) + \beta \mathbb{E}_{it} P_{it+1} \]

\[ \Rightarrow P + P p_{it} \approx AK^\alpha - R K + \beta P + AK^\alpha \mathbb{E}_{it} (a_t + \alpha k_{it}) - R K \mathbb{E}_{it} k_{it} + \beta \mathbb{E}_{it} P_{it+1} \]

\[ P p_{it} \approx AK^\alpha \mathbb{E}_{it} a_{it} + (\alpha AK^\alpha - R K) \frac{\mathbb{E} [E_{a_{it}}]}{1 - \alpha} + \beta \mathbb{E}_{it} P_{it+1} \]

\[ p_{it} = \frac{AK^\alpha}{P} \mathbb{E}_{it} a_{it} + \beta \mathbb{E}_{it} p_{it+1} + \text{Const.} \]

where \( \mathbb{E} \) denotes the marginal investor’s expectations. Guess-and-verify

\[ p_{it} = \xi \mathbb{E}_{it} a_{it} \]

\textsuperscript{62}The aggregate constants multiplying revenues are normalized to 1.
Substituting,
\[
\xi \tilde{E}_{it} a_{it} = \frac{AK^\alpha}{P} \tilde{E}_{it} a_{it} + \beta \xi \tilde{E}_{it} a_{it+1} \quad \Rightarrow \quad \xi = \frac{Y}{P} = \frac{1 - \beta}{1 - \alpha}
\]

Variances (of growth rates):
\[
\sigma^2_p \equiv \text{Var}(p_{it} - p_{it-1}) = 2\psi^2 \xi^2 \left( \sigma^2_\mu + \sigma^2_e \sigma^2_z \right) = 2\psi^2 \xi^2 \left( 1 + \frac{\sigma^2_e \sigma^2_z}{\sigma^2_\mu} \right) \sigma^2_u
\]
\[
\sigma^2_k \equiv \text{Var}(k_{it} - k_{it-1}) = \left[ 2(\phi_1 + \phi_2)^2 \sigma^2_\mu + \phi_3^2 \sigma^2_e + \phi_2^2 \sigma^2_v \sigma^2_z \right] \left( \frac{1}{1 - \alpha} \right)^2
\]
\[
= 2 \left( \frac{1}{\sigma^2_\mu} + \frac{1}{\sigma^2_e + \frac{1}{\sigma^2_z}} \right) \sigma^2_u + \left( \frac{1}{\sigma^2_\mu} + \frac{1}{\sigma^2_e + \frac{1}{\sigma^2_z}} \right) \sigma^2_e + \left( \frac{1}{\sigma^2_\mu} + \frac{1}{\sigma^2_e + \frac{1}{\sigma^2_z}} \right) \sigma^2_v \sigma^2_z \left( \frac{1}{1 - \alpha} \right)^2
\]
\[
\sigma^2_a \equiv \text{Var}(a_{it} - a_{it-1}) = 2\sigma^2_\mu
\]

Covariances (of growth rates):
\[
\text{Cov}(p, k) \equiv \text{Cov}(p_{it} - p_{it-1}, k_{it} - k_{it-1})
\]
\[
= 2\xi \psi \left( \phi_1 + \phi_2 \right) \sigma^2_\mu \left( \frac{1}{1 - \alpha} \right)
\]
\[
= 2\xi \psi \left( \frac{1}{\sigma^2_\mu + \frac{1}{\sigma^2_e + \frac{1}{\sigma^2_z}}} \right) \sigma^2_\mu + \left( \frac{1}{\sigma^2_\mu + \frac{1}{\sigma^2_e + \frac{1}{\sigma^2_z}}} \right) \sigma^2_e \sigma^2_z \left( \frac{1}{1 - \alpha} \right) = 2\xi \psi \sigma^2_\mu \left( \frac{1}{1 - \alpha} \right)
\]
\[
\text{Cov}(p, a) = 2\xi \psi \sigma^2_\mu
\]

The correlations:
\[
\rho_{pa} = \frac{\text{Cov}(p, a)}{\sigma_p \sigma_a} = \frac{2\xi \psi \sigma^2_\mu}{\sqrt{4\psi^2 \xi^2 \left( \sigma^2_\mu + \sigma^2_e \sigma^2_z \right)}} = \frac{1}{\sqrt{1 + \frac{\sigma^2_e \sigma^2_z}{\sigma^2_\mu}}}
\]
\[
\rho_{pk} = \frac{\text{Cov}(p, k)}{\sigma_p \sigma_k} = \frac{2\xi \psi \sigma^2_\mu}{\sqrt{4\psi^2 \xi^2 \left( \sigma^2_\mu + \sigma^2_e \sigma^2_z \right) \left( \sigma^2_\mu - \psi \right)}}
\]
\[
= \frac{1}{\sqrt{\left( 1 + \frac{\sigma^2_e \sigma^2_z}{\sigma^2_\mu} \right) \left( 1 - \frac{\psi}{\sigma^2_\mu} \right)}}
\]
The volatility of returns:

\[
\sigma_p^2 = \xi^2 \left( \frac{\sigma_z^2 + 1}{\sigma_z^2 + \frac{\sigma^2 u}{\rho_p^2}} \right)^2 \left( 1 + \frac{\sigma^2 u}{\sigma^2 \mu} \right) \sigma_u^2
\]

\[
= \xi^2 \left( \frac{\sigma_z^2 + 1}{\sigma_z^2 + \frac{1}{\rho_p^2}} \right)^2 \frac{1}{\rho_p^2} \sigma^2 \mu
\]

**Permanent shocks.** Profits in period \( t \) are

\[
\pi_{it} = HA_{it}K_{it}^\alpha - RK_{it}
\]

where \( H \) is a constant term, summarizing general equilibrium interactions. This can be written as

\[
\pi_{it} = He^\rho a_{it-1} + E_{it}u_{it} + \text{Const} \quad \frac{1}{1 - \alpha} - Re^{\rho a_{it-1} + E_{it}u_{it} + \text{Const}} \quad \frac{1}{1 - \alpha}
\]

\[
= e^{\rho a_{it-1}} \left[ He^{E_{it}u_{it} + \text{Const}} \quad \frac{1}{1 - \alpha} - Re^{E_{it}u_{it} + \text{Const}} \quad \frac{1}{1 - \alpha} \right]
\]

When \( \rho = 1 \), i.e., shocks are permanent, we guess that the ex-dividend component\(^63\) of price function (17) takes the form

\[
\mathcal{P}(a_{-1}, a, z) = e^{\frac{a_{-1}}{1 - \alpha}} \hat{P}(a - a_{-1}, z) = e^{\frac{a}{1 - \alpha}} \hat{P}(u, z)
\]

Direct substitution into equation (17) verifies the guess (the term \( e^{\frac{a_{-1}}{1 - \alpha}} \) multiplies all the terms from both sides). In logs, the price is then,

\[
p_{it-1} = \log \left[ E_{it-1}e^{\frac{a_{it-1}}{1 - \alpha}} \hat{P}(u, z) \right]
\]

\[
= \log \left[ E_{it-1} \left( e^{\frac{a_{it-1}}{1 - \alpha}} \right) \hat{E}_{it-1} \hat{P}(u, z) \right]
\]

\[
= \frac{1}{1 - \alpha} E_{it-1} a_{it-1} + \text{const.}
\]

where the last two lines follow from the fact that \( (u, z) \) is i.i.d. This is the affine representation of the ex-dividend price used in the text.

To derive the moments of interest, note that with permanent shocks

\[
Ea_{it} = a_{it-1} + E\mu_{it}, \quad E\mu_{it} = a_{it-1} + E\mu_{it}
\]

\(^63\)The ex-dividend price in \( t - 1 \) is the marginal investor’s expectation of the price in \( t \).
which leads to
\[ p_{it} = \frac{1}{1-\alpha} a_{it-1} + \frac{1}{1-\alpha} \psi (u_{it} + \sigma_v z_{it}) + \text{Const} \]
\[ (1-\alpha) k_{it} = a_{it-1} + \phi_1 \mu_{it} + \phi_2 e_{it} + \phi_3 \sigma_v z_{it} + \text{Const.} \]

where the coefficients \( \psi, \phi_1 \) and \( \phi_2 \) are the same as in the i.i.d. case. In growth rates,
\[ \Delta a_{it} = \mu_{it} \]
\[ \Delta p_{it} = \frac{1}{1-\alpha} (a_{it-1} - a_{it-2}) + \frac{1}{1-\alpha} \psi (\mu_{it} + \sigma_v z_{it} - \mu_{it-1} - \sigma_v z_{it-1}) \]
\[ = \frac{1}{1-\alpha} (1-\psi) \mu_{it-1} + \frac{1}{1-\alpha} \psi \mu_{it} + \xi \psi \sigma_v (z_{it} - z_{it-1}) \]
\[ (1-\alpha) \Delta k_{it} = (a_{it-1} - a_{it-2}) + \phi_1 (\mu_{it} - \mu_{it-1}) + \phi_2 (e_{it} - e_{it-1}) + \phi_3 \sigma_v (z_{it} - z_{it-1}) \]
\[ = (1-\phi_1) \mu_{it-1} + \phi_1 \mu_{it} + \phi_2 (e_{it} - e_{it-1}) + \phi_3 \sigma_v (z_{it} - z_{it-1}) \]

Second moments (of growth rates):
\[ \sigma_p^2 = \left( \frac{1}{1-\alpha} \right)^2 (1-\psi)^2 \sigma^2 + \left( \frac{1}{1-\alpha} \right)^2 \psi^2 \sigma^2 + 2 \left( \frac{1}{1-\alpha} \right)^2 \psi^2 \sigma_v^2 \sigma_z^2 \]
\[ \sigma_k^2 = \left( \frac{1}{1-\alpha} \right)^2 \sigma_\mu^2 \left[ 1 - 2\psi + 2\psi^2 (1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}) \right] > 0 \]
\[ \sigma_k^2 = \left( \frac{1}{1-\alpha} \right)^2 \left[ (1-\phi_1 - \phi_2)^2 \sigma_\mu^2 + (\phi_1 + \phi_2) \sigma_\mu^2 + 2\phi_1^2 \sigma_e^2 + 2\phi_2^2 \sigma_v^2 \sigma_z^2 \right] \]
\[ \sigma_k^2 = \left( \frac{1}{1-\alpha} \right)^2 \left[ \sigma_\mu^2 - 2 (\sigma_\mu^2 - \mu) + 2\sigma_\mu^2 - 4\mu + 2 \frac{\psi^2}{\sigma_\mu^2} + 2 \frac{\psi^2}{\sigma_e^2} + 2 \frac{\psi^2}{\sigma_v^2 \sigma_z^2} \right] > 0 \]

In other words, the variance of investment is invariant to uncertainty! The intuition is that undoing the effects of past forecast errors is a source of investment volatility. When shocks are permanent, as is the case here, this exactly offsets the contemporaneous dampening that occurs due to imperfect information. In other words, with permanent shocks, uncertainty shifts the timing of the response of capital to fundamentals, but not the magnitude.

Covariances
\[
\text{Cov}(p, a) = \frac{1}{1 - \alpha} \psi \sigma^2 \mu
\]
\[
\text{Cov}(p, k) = \left(\frac{1}{1 - \alpha}\right)^2 [(1 - \psi) (1 - \phi_1) \sigma^2 \mu + \psi \phi_1 \sigma^2 \mu + 2\psi \phi_3 \sigma^2 \sigma^2 \mu]
\]
\[
= \left(\frac{1}{1 - \alpha}\right)^2 [(1 - \psi - \phi_1) \sigma^2 \mu + 2\psi \phi_1 \sigma^2 \mu + 2\psi \phi_3 \sigma^2 \sigma^2 \mu]
\]
\[
= \left(\frac{1}{1 - \alpha}\right)^2 [(1 - \psi - \phi_1) \sigma^2 \mu + 2\psi (\sigma^2 \mu - \mathbb{V}) + 2\psi \mathbb{V}]
\]
\[
= \left(\frac{1}{1 - \alpha}\right)^2 (\mathbb{V} + \psi \sigma^2 \mu)
\]

Directly,
\[
\text{Cov}(p, k) - \left(\frac{1}{1 - \alpha}\right) \text{Cov}(p, a) = \left(\frac{1}{1 - \alpha}\right)^2 \mathbb{V}
\]
\[
\rho_{pk} \sigma_p \sigma_k - \left(\frac{1}{1 - \alpha}\right) \rho_{pa} \sigma_p \sigma_a =
\]
\[
\rho_{pk} \sigma_p \sigma_u \left(\frac{1}{1 - \alpha}\right) - \left(\frac{1}{1 - \alpha}\right) \rho_{pa} \sigma_p \sigma_u =
\]
\[
(\rho_{pk} - \rho_{pa}) \left(\frac{1}{1 - \alpha}\right) \sigma_p \sigma_u =
\]

Re-arranging yields the expression in the text.
\[
\frac{\mathbb{V}}{\sigma^2 \mu} = \frac{(\rho_{pk} - \rho_{pa}) \sigma_p}{\sigma_u} (1 - \alpha)
\]
\[
\rho_{pk} = \frac{\psi + \frac{\mathbb{V}}{\sigma^2 \mu}}{\sqrt{1 - 2\psi + 2\psi^2 \left(1 + \frac{\sigma^2 \sigma^2 \mu}{\sigma^2 \mu}\right)}}
\]
\[
\rho_{pa} = \frac{\psi}{\sqrt{1 - 2\psi + 2\psi^2 \left(1 + \frac{\sigma^2 \sigma^2 \mu}{\sigma^2 \mu}\right)}} = \frac{1}{\sqrt{(\frac{1}{\psi} - 1)^2 + 1 + 2\frac{\sigma^2 \sigma^2 \mu}{\sigma^2 \mu}}}
\]

Then,
\[
\frac{\rho_{pk}}{\rho_{pa}} = 1 + \frac{\mathbb{V}}{\psi \sigma^2 \mu} \quad \Rightarrow \quad \psi = \frac{\mathbb{V}}{\sigma^2 \mu} \frac{\rho_{pk}}{\rho_{pa}} - 1 = \frac{\rho_{pa}}{\eta}
\]
Given $\psi$, we then use the expression for $\rho_{pa}$ to solve for $\frac{\sigma^2_v}{\sigma^2_\mu}$:

$$\rho^2_{pa} = \frac{1}{\left(\eta_{pa} - 1\right)^2 + 1 + 2 \frac{\sigma^2_v}{\sigma^2_\mu}}$$

$$\Rightarrow \frac{\sigma^2_v}{\sigma^2_\mu} = \frac{1}{2\rho^2_{pa}} - \frac{1}{2} \left(\frac{\eta_{pa} - 1}{\rho_{pa}}\right)^2 - \frac{1}{2}$$

$$= \frac{(1 - \eta^2)}{2\rho^2_{pa}} + \frac{\eta_{pa}}{\rho_{pa}} - 1$$

Finally, combining this with the definition of $\psi \equiv \frac{\sigma^2_v}{\sigma^2_\mu + \frac{\sigma^2_v}{\sigma^2_\mu}}$ allows us to disentangle $\sigma^2_v$ and $\sigma^2_\mu$.

### A.5 Correlated signals

Here, we present an alternative, more direct derivation of equation (21), which does not rely on the details of the information structure. Since all shocks are iid, $p_{it}$ is an iid random variable, uncorrelated with past and future realizations of $a_{it}$.

$$Cov(p_{it} - p_{it-1}, a_{it} - a_{it-1}) = Cov(p_{it}, a_{it}) + Cov(p_{it-1}, a_{it-1})$$

$$= 2Cov(p_{it}, a_{it})$$

$$= 2Cov(p_{it}, Ea_{it} + \varpi_{it})$$

$$= 2Cov(p_{it}, Ea_{it}) + 2Cov(p_{it}, \varpi_{it})$$

$$= 2Cov(p_{it}, Ea_{it})$$

where $\varpi_{it}$ denotes the firm’s forecast error, which is uncorrelated with firm information - in particular, with any element in the firm information set. Therefore, $Cov(p_{it}, \varpi_{it}) = 0$. Note that this is true independent of the correlation structure between $p_{it}$ and the other elements of that information set. Now, we use $Cov(p_{it}, k_{it}) = \frac{1}{1-\alpha}Cov(p_{it}, Ea_{it})$ and divide both sides of the equation by $\sigma (p_{it} - p_{it-1}) \sigma (a_{it} - a_{it-1})$ to get
\[
\frac{\text{Cov} (p_{it} - p_{it-1}, a_{it} - a_{it-1})}{\sigma (p_{it} - p_{it-1}) \sigma (a_{it} - a_{it-1})} = \frac{2 \left( 1 - \alpha \right) \text{Cov} (p_{it}, k_{it}) \sigma (k_{it} - k_{it-1})}{\sigma (p_{it} - p_{it-1}) \sigma (a_{it} - a_{it-1}) \sigma (k_{it} - k_{it-1})} = \frac{\text{Cov} (p_{it} - p_{it-1}, k_{it} - k_{it-1}) (1 - \alpha) \sigma (k_{it} - k_{it-1})}{\sigma (p_{it} - p_{it-1}) \sigma (k_{it} - k_{it-1}) (1 - \alpha) \sigma (a_{it} - a_{it-1})} = \rho_{pa} \frac{(1 - \alpha) \sigma (k_{it} - k_{it-1})}{\sigma (a_{it} - a_{it-1})}
\]

\[
\Rightarrow \rho_{pa} = \frac{\rho_{pk} \sqrt{1 - \psi}}{\sigma_{\mu}}
\]

Since

\[
\sigma^2 (k_{it} - k_{it-1}) = \left( \frac{1}{1 - \alpha} \right)^2 2\sigma^2 (\mathbb{E} a_{it}) = \left( \frac{1}{1 - \alpha} \right)^2 2 (\sigma^2_{\mu} - \psi)
\]

\[
= \left( \frac{1}{1 - \alpha} \right)^2 2 \sigma^2_{\mu} \left( 1 - \frac{\psi}{\sigma^2_{\mu}} \right)
\]

\[
= \left( \frac{1}{1 - \alpha} \right)^2 \sigma^2 (a_{it} - a_{it-1}) \left( 1 - \frac{\psi}{\sigma^2_{\mu}} \right)
\]

\[
\Rightarrow \frac{(1 - \alpha) \sigma (k_{it} - k_{it-1})}{\sigma (a_{it} - a_{it-1})} = \sqrt{1 - \psi} \sigma^2_{\mu}
\]

Combining yields (21).

\[
\rho_{pa} = \rho_{pk} \sqrt{1 - \frac{\psi}{\sigma^2_{\mu}}}
\]

### A.6 Other distortions

Since distortions have only a second order effect on profits, the only moment that changes is \( \rho_{pk} \). With correlated distortions, \( k_{it} = \frac{1+\gamma}{1-\alpha} \mathbb{E} a_{it} \), which implies

\[
\rho_{pk} = \frac{\text{Cov} (p, k)}{\sigma_p \sigma_k} = \frac{2 \xi \psi \sigma^2_{\mu} (1 + \gamma)}{\sqrt{4 \psi^2 \xi^2 (\sigma^2_{\mu} + \sigma^2_{\sigma^2}) (\sigma^2_{\mu} - \psi) (1 + \gamma)^2}} = \frac{2 \xi \psi \sigma^2_{\mu}}{\sqrt{4 \psi^2 \xi^2 (\sigma^2_{\mu} + \sigma^2_{\sigma^2}) (\sigma^2_{\mu} - \psi)}}
\]
the same as before. In the uncorrelated case,

\[
\rho_{pk} = \frac{\text{Cov}(p, k)}{\sigma_p \sigma_k} = \frac{2\xi \psi \sigma^2_{\mu}}{\sqrt{4\psi^2 \xi^2 (\sigma^2_{\mu} + \sigma^2_{\psi} \sigma^2_{\varepsilon}) (\sigma^2_{\mu} + \sigma^2_{\varepsilon} - \mathcal{V})}}
\]

\[
= \frac{1}{\sqrt{(1 + \frac{\sigma^2_{\psi} \sigma^2_{\varepsilon}}{\sigma^2_{\mu}}) \left(1 - \frac{\mathcal{V} \sigma^2_{\mu}}{\sigma^2_{\mu} + \sigma^2_{\varepsilon}}\right)}}
\]

Re-arranging yields the expression in the text.

**B Adjustment Costs**

When firms are fully informed but subject to capital adjustment costs, their capital choice is a dynamic decision, and the value function takes the form:

\[
V(\tilde{A}_{it}, K_{it-1}) = \max_{K_{it}, N_{it}} G \tilde{A}_{it} K_{it}^{\tilde{\alpha}} - K_{it} - H(K_{it}, K_{it-1}) + \beta \mathbb{E} V(\tilde{A}_{it+1}, (1 - \delta) K_{it})
\]

where

\[
H(K_{it}, K_{it-1}) = \zeta K_{it-1} \left(\frac{K_{it}}{K_{it-1}} - 1\right)^2
\]

To characterize \(G\) and \(\tilde{A}_{it}\), we begin similarly to case 2 in the text and examine the firm’s labor choice problem

\[
Y^{\frac{1}{\theta}}_{t} A_{it} K^{\alpha_1}_{it} N^{\alpha_2}_{it} - W N_{it}
\]

where \(\alpha\)’s are as defined in (2). Optimizing over \(N_{it}\) and substituting back into the objective gives

\[
P_{it} Y_{it} - W N_{it} = (1 - \alpha_2) \left(\frac{\alpha_2}{W}\right)^{\frac{1}{\alpha_2}} Y_t^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta - \alpha_2}} \tilde{A}_{it} K_{it}^{\tilde{\alpha}}
\]

where \(\tilde{\alpha}\) and \(\tilde{A}_{it}\) are defined as in case 2 in the text. We can then solve for

\[
G = (1 - \alpha_2) \left[\left(\frac{\alpha_2}{W}\right)^{\frac{1}{\alpha_2}} Y_t^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta - \alpha_2}} \int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} di \right]^{\frac{1}{1 - \alpha_2}}
\]

Next, using the firm’s labor optimality condition, labor market clearing implies

\[
\left(\frac{\alpha_2}{W}\right)^{\frac{1}{\alpha_2}} Y_t^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta - \alpha_2}} \int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} di = N
\]
from which we can solve for
\[(\frac{\alpha_2}{W})^{\alpha_2} Y_t^{\frac{1}{\theta}} = \left[ \int \tilde{A}_{it} \tilde{K}_{it}^{\alpha} \, di \right] \gamma \frac{1-\alpha_2}{\theta-1} (1-\theta\alpha_2) N(\frac{\theta}{\theta-1} \alpha_2)\]

and so
\[G = (1 - \alpha_2) \left[ \int \tilde{A}_{it} \tilde{K}_{it}^{\alpha} \, di \right] \gamma \frac{1-\alpha_2}{\theta-1} N(\frac{\theta}{\theta-1} \alpha_2)\]

Note that under full information, the production side of the economy is completely decoupled from stock markets, so we have all we need to solve the model laid out above. Starting with a guess for the general equilibrium term $G$, we solve for the value functions (using a standard iterative procedure and a discretized grid for capital), simulate to obtain the steady state distributions and verify that our initial guess for $G$ is consistent with that distribution. If not, we update the guess and iterate until convergence.

We then solve for stock prices, again using the conjecture that the informed investors follow a threshold rule. Proceeding exactly as in the baseline model, we can then derive the following functional equation for the price function
\[P (a_{it-1}, k_{it-1}, a_{it}, z_{it}) = \int \pi (a_{it}, k_{it-1}) H (a_{it}|a_{it-1}, a_{it} + \sigma_v z_{it}, P_{it}) + \beta \int P (a_{it}, k^* (a_{it}, k_{it-1})) H (a_{it}|a_{it-1}, a_{it} + \sigma_v z_{it}, P_{it}) \]

The model is parameterized as described in the text. Table 11 reports the full set of target moments and parameter estimates. Notice that the first two columns are identical to those in Table 2 (case 2). $\sigma_k^2$ denotes the variance of investment rates, which is used to pin down the size of the adjustment cost $\zeta$.

<table>
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<th>Table 11: Targets and Parameters - Adjustment Cost Model</th>
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<td><strong>Target moments</strong></td>
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C Data

We use annual data on firm-level production variables and stock market returns from Compustat North America (for the US) and Compustat Global (for China and India). For each country, we exclude duplicate observations (firms with multiple observations within a single year), firms not incorporated within that country, and firms not reporting in local currency. We build three year production periods as the average of firm sales and capital stock over non-overlapping 3-year horizons (i.e., $K_{2012} = \frac{K_{2010} + K_{2011} + K_{2012}}{3}$, and analogously for sales). We measure the capital stock using gross property, plant and equipment (PPEGT in Compustat terminology), defined as the “valuation of tangible fixed assets used in the production of revenue.” We then calculate investment as the change in the firm’s capital stock relative to the preceding period. We construct the firm fundamental $a_{it}$ as the log of revenue less the relevant $\alpha$ (which depends on the case) multiplied by the log of the capital stock. Finally, we first-difference the investment and fundamental series to compute investment growth and changes in fundamentals.

Stock returns are constructed as the change in the firm’s stock price over the three year period, adjusted for splits and dividend distributions. We follow the procedure outlined in the Compustat manual. In Compustat terminology, and as in the text, using $p_{it}$ as shorthand for log returns, returns for the US are computed as

$$p_{it} = \log \left( \frac{PRCCM_{it} \cdot TRFM_{it}}{AJEXM_{it}} \right) - \log \left( \frac{PRCCM_{it-1} \cdot TRFM_{it-1}}{AJEXM_{it-1}} \right)$$

where periods denote 3 year spans (i.e., returns for 2012 are calculated as the adjusted change in price between 2009 and 2012), PRCCM is the firm’s stock price, and TRFM and AJEXM adjustment factors needed to translate prices to returns from the Compustat monthly securities file. Data are for the last trading day of the firm’s fiscal year so that the timing lines up with the production variables just described. The calculation is analogous for China and India, with the small caveat that global securities data come daily, so that the Compustat variables are PRCCD, TRFD, and AJEXDI, where “D” denotes days. Again, the data are for the last trading day of the firm’s fiscal year.

To extract the firm-specific variation in our variables, we regress each on a time fixed-effect and work with the residual. This eliminates the component of each series common to all firms in a time period and leaves only the idiosyncratic variation. As described in the text, we limit our sample to a single cross-section, namely 2012, and finally, we trim the 2% tails of each series. It is then straightforward to compute our calibration targets, i.e., $\sigma^2_p$, $\rho_{pi}$, and $\rho_{pa}$. As described in the text, we lag returns by one period, so that, e.g., $\rho_{pi}$ is the correlation of 2006-09 returns with investment growth from 2009-12.
To estimate the parameters governing the evolution of firm fundamentals, i.e., the persistence $\rho$ and variance of the innovations $\sigma^2$, we perform the autoregression implied by (1). Here we use annual observations on $a_{it}$ at a 3-year frequency in order to simplify issues of time aggregation. We estimate the process using our data from 2012 and 2009. We include a time fixed-effect in order to isolate the idiosyncratic component of the innovations in $a_{it}$. Differences in firm fiscal years means that different firms within the same calendar year are reporting data over different time periods, and so the time fixed-effect incorporates both the reporting year and month. The results from this regression deliver an estimate for $\rho$ and $\sigma^2$, from which we trim the 1% tails.

Our leverage adjustment is as follows: we assume that claims to firm profits are sold to investors in the form of both debt and equity in a constant proportion (within each country). This implies that the payoff from an equity claim is $S_{it} = V_{it} - D_{it-1}$, where $V_{it}$ is the value of the unlevered firm and $D_{it-1} = dE_t[V_{it}]$, where $d \in (0, 1)$ represents the share of expected firm value in the hands of debt-holders. In other words, firm value is allocated to investors as a debt claim that pays off a constant fraction of its ex-ante expected value and as a residual claim to equity holders. The change in value of an equity claim is then equal to $\Delta S_{it} = \Delta V_{it}$ and dividing both sides by the ex-ante expected value of the claim (i.e., the price) $\frac{\Delta S_{it}}{S_{it}} = \frac{\Delta V_{it}}{E_t[V_{it}]}$, gives returns as $\frac{\Delta S_{it}}{S_{it}} = \frac{(1-d)^2 \sigma^2_s}{(1-d)\sigma^2_v}$. Taking logs and computing variances shows $\sigma^2_v = (1-d)^2 \sigma^2_s$, i.e., the volatility in (unlevered) firm value is a fraction $(1-d)^2$ of the volatility in (levered) equity returns. To assign values to $d$ in each country, we examine the debt-asset and debt-equity ratios of the set of firms in Compustat over the period 2006-2009. Because these vary to some degree from year to year and depend to some extent on the precise approach taken (i.e., whether we use debt-assets or debt-equity and whether we compute average ratios or totals), we simply take the approximate midpoints of the ranges for each country, which are about 0.30 for the US and India and 0.16 for China, leading to adjustment factors of about 0.5 and 0.7, respectively.