The costs and benefits of symmetry in common-ownership allocation problems

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Abstract

We experimentally compare the “winner’s-bid auction” and “divide-and-choose” mechanisms when two parties who jointly own a good must decide who retains the good and how the other is compensated. Theoretically, both mechanisms provide an equal-income competitive (EIC) allocation, winner’s-bid auction is unbiased, and divide-and-choose favors the proposer. We examine these conjectures experimentally and compare the mechanisms with “ultimatum bargaining.” Contrary to theoretical predictions: (i) divide-and-choose is more efficient and achieves EIC allocations more often; (ii) divide-and-choose, if anything, exhibits a slight chooser’s advantage. Finally, we further our understanding of agents’ behavior in these competing mechanisms and identify possible improvements.

JEL classification: C91, D63, C72.

Keywords: experimental economics; no-envy; divide-and-choose; behavioral mechanism design; NFL ball possession auctions.

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1 Introduction

The distribution of resources that are collectively owned by a group of agents is a problem that naturally arises in a market economy. For instance, in any venture project, gains, losses, tasks, etc., have to be allocated among project participants. These problems pose special challenges for the mechanism designer. In most cases, due to the diversity of preferences, even when all participants have equal rights over resources, it is not evident what distribution should be implemented. Thus, the challenge for economists is to design institutions that achieve normative objectives that parties can agree on. As a result, we make our market economy more successful: one in which each participant has the incentive to cooperate, create value, and abide by the rules that arbitrate the resolution of economic conflicts.

In this paper we study common-ownership allocation problems among two agents when there are indivisibilities. Consider for instance two agents, Ann and Bob, who know each other well, collectively own a company (50% each), and decide to dissolve their partnership. They have to determine who gets the company and how much he or she pays the other for it. Suppose that Ann values the company at $1.2 million, Bob at $1 million, and staying out of the company is normalized to be valued zero for both agents. If all relevant information is known to an arbitrator, the allocations that are supported as competitive equilibria when aggregate income is divided equally among both agents, i.e., equal-income competitive (EIC) allocations, are a normatively compelling target to implement. They are the allocations that a market would obtain if Ann and Bob were price takers and had symmetric ownership of the endowment. These allocations are efficient (Svensson, 1983). Moreover, at each such allocation, no agent prefers the allotment of any other agent to her own. The main purpose of our study is to experimentally evaluate the performance of two prominent mechanisms that obtain EIC allocations in Nash equilibria: the winner’s-bid auction (McAfee, 1992), a simultaneous move mechanism, and the popular divide-and-choose, a sequential move mechanism.

We focus on an equivalent formulation of the single object case described in our example. We consider the allocation of a social endowment of two objects, say \{A, B\}, among two agents, when each agent receives exactly one object, there is the possibility of monetary compensation.

\footnote{We borrow this example from Velez (2012).}

\footnote{In the example, EIC allocations are the outcomes of a market in which the price of the company is between $1 million and $1.2 million and each agent receives half of the aggregate income as endowment. For instance, if price is $1.1 million, both agents have an income of $0.55 million. Then, both agents can either buy the company at $1.1 million, or keep the $0.55 million. Ann prefers to buy the company and Bob to stay outside. Thus, the market clears.

\footnote{Allocations for which no agent prefers the allotment of any other agent to her own are known as “envy-free” allocations. In our model EIC allocations are exactly the set of envy-free allocations (Svensson, 1983). One could also see our study as the experimental investigation of envy-free mechanisms. A long literature on normative economics initiated by Foley (1967) and Varian (1974) identifies no-envy as a central desirable property in the fair division problem (see Thomson, 2006, for a survey).

\footnote{There is a one-to-one correspondence between the two-agent single-object case and the two-agent, two-object case, for receiving no object is actually another indivisible good.}}
and allotments of money should add up to an amount we refer to as the budget (in our example with Ann and Bob, as in our experiments, the budget is zero). Examples are the aforementioned dissolution of a partnership (Cramton et al., 1987; McAfee, 1992), the allocation of rooms and the division of the rent among roommates (Abdulkadiroğlu et al., 2004), and the allocation of ball possession in NFL overtime (Che and Hendershott, 2008). We find our environment more appropriate for our experimental setting, for having two objects allows us to avoid, to some extent, any non-monetary incentives for agents to “win” items.

We assume complete information, i.e., agents know each other’s valuation of resources and this is common knowledge. Even though incomplete information may be more realistic in some environments, our assumption of complete information allows us to evaluate the effect of bounded rationality and reciprocity across different mechanisms without the noise that would be introduced by the asymmetry of information. Indeed, our results confirm that these two issues are relevant and have to be taken into consideration when designing mechanisms for common-ownership distributive situations.

Our first mechanism, the winner’s-bid auction, operates as follows: both agents bid for a predetermined object, say $B$; then, an agent whose bid is maximal receives object $B$ and pays his bid; the other agent receives object $A$ and an amount of money that balances the budget. When the budget is zero, as in our experiments, this is equivalent to the winner of object $B$ paying his bid to the other agent. The Nash equilibrium outcomes of this game are the set of all EIC allocations for true preferences. In our example, the these allocations are those at which Ann gets the company and transfers an amount of money to Bob between $0.5$ million and $0.6$ million.

Our second mechanism, divide-and-choose, operates as follows: an agent is selected at random to propose a pair of amounts of money, one for each object, that balance the budget; let $t_A$ and $t_B$ be the proposer’s report. The other agent gets to choose between the bundles composed of an object and its corresponding amount of money. In our example with Ann and Bob, in the subgame perfect equilibrium of the induced game, if Ann gets to propose, she proposes that the “buyer” of the company transfer $0.5$ million to the other agent; then, Bob accepts staying out of the company and receives $0.5$ million. If Bob is the proposer, he proposes that the buyer of the company transfer $0.6$ million to the other agent; then, Ann gets the company and transfers $0.6$ million to Bob. These are both EIC allocations. However, these are the extremes of the set of EIC allocations. At no EIC allocation would Bob stay out for less than $0.5$ million; symmetrically, Ann would not transfer Bob more than $0.6$ million in order to get the company.

We also test the performance of the benchmark ultimatum bargaining mechanism. Here

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5Since we require budget balance, the winner’s-bid auction is strategically equivalent to any other auction in which an agent with a highest bid receives the company and pays an amount in between the bids. For instance a second price or intermediate price auction.

6See Brams and Taylor (1996) for a historical account of the role of the divide-and-choose mechanism in fair allocation problems.
an agent is selected to propose an allocation of the indivisible goods with their corresponding amounts of money that add up to the budget. Then, the other agent gets to accept or reject the proposal. Rejected proposals produce zero payoff for both agents.\textsuperscript{7}

The winner’s-bid auction is normatively superior to divide-and-choose. On the one hand, both mechanisms provide a form of endstate justice, because their equilibrium outcomes are EIC allocations. On the other hand, one can argue that divide-and-choose lacks procedural justice.\textsuperscript{8} After divider and chooser are selected, the mechanism loses symmetry. Not only do strategy spaces for the agents and timing of their actions differ, but also the proposer gains a considerable advantage, i.e., has the power to enforce her preferred EIC allocation (see our Section 2.3 and also Moulin (2006) for a related discussion). Thus, theory provides a clear ranking among these mechanisms: (i) the winner’s-bid auction should provide an EIC allocation, free of any bias predetermined by the mechanism designer; (ii) the divide-and-choose should provide an EIC allocation that is biased towards the proposer; and (iii) ultimatum bargaining obtains no EIC allocation and is biased toward the proposer. Our study is the first to test these mechanisms in comparable experiments and thus evaluate the extent to which this normative ranking based on equilibrium predictions is realized in an experimental environment.

Contrary to the theoretical predictions, our results support the superiority of divide-and-choose in the laboratory. Divide-and-choose obtains 81% EIC allocations and 85% efficient allocations. The more procedurally fair winner’s-bid auction obtains 41% EIC allocations and 73% efficient allocations. The differences are significant. Symmetry turns out to be a costly feature for the winner’s-bid auction. First, it may induce a coordination problem. More importantly, our results reveal that among boundedly rational players, as in our experiments, it induces a distortion caused by the simultaneity of play—this result is specially clear in the valuations for which the winner’s-bid auction requires no coordination. Our results suggest that this distortion slowly improves over time, but does not improve to the level of divide-and-choose. This observation not only explains the loss of efficiency and competitiveness of the winner’s-bid auction, but also points in a precise direction for its improvement. (We examine such improved mechanisms in Section 5.2.) Ultimatum bargaining, our benchmark mechanism, performs as expected, i.e., worse than the other two mechanisms with a high degree of proposer bias.

Even though divide-and-choose performs better than the winner’s-bid auction, it obtains inefficient allocations with positive probability. Essentially, in the subgame perfect equilibrium of the divide-and-choose game, the chooser is nearly indifferent among the bundles selected by the proposer. Thus, when the proposer selects a proposal that is close to the subgame perfect proposal, the chooser has a little incentive to choose the proposer’s best bundle among the

\textsuperscript{7}Because it is trivial, and not the focus of the paper, the equilibrium solution for ultimatum bargaining is not mentioned here. In our example, Ann will receive the company and either pay slightly under $1.2 million if Bob is the proposer or slightly more than $0 if Ann is the proposer. This analysis requires the company to be forfeited and both players to receive $0 if there is a rejection in bargaining.

\textsuperscript{8}Note that ultimatum bargaining has neither form of justice.
proposed ones. This “retaliation strategy” induces a considerable loss for the proposer and is responsible for the majority of inefficient outcomes. This suggests that the performance of the mechanism may be improved by returning some symmetry without losing the sequentiality of play. We discuss a mechanism that makes such improvement in Section 5.1.

This paper is organized as follows. The remainder of this section summarizes the related literature on this topic. Section 2 introduces the model and mechanisms we study. Section 3 presents our experimental design. Section 4 presents our results. Section 5 discusses our results, the possible improvements to both winner’s-bid auction and divide-and-choose, the relevance of our results for the allocation of ball possession in NFL overtime, and concludes.

Related literature

The study of EIC mechanisms is at the center of normative economics (see Thomson, 2006, for a survey). Several authors (Svensson, 1983; Maskin, 1987; Alkan et al., 1991; Velez, 2011b) establish the existence of EIC allocations under general conditions on preferences for the allocations of indivisible goods and money. It is well known that there is no incentive compatible mechanism that implements EIC allocations in our environment (Alkan et al., 1991; Tadenuma and Thomson, 1995a). (This is in stark contrast to the related problem in which there is no initial ownership, i.e., Ann and Bob do not own the company. There, the popular second price auction would be incentive compatible.) In this environment, and under complete information, a growing literature has investigated the Nash equilibria of the direct revelation mechanisms that select an EIC allocation for each preference report—the winner’s-bid auction is strategically equivalent to one of these mechanisms. The common result is that regardless of mechanism, as long as it selects an EIC allocation for the reported preferences, its Nash equilibrium set is exactly the set of EIC allocations for the true preferences (Tadenuma and Thomson, 1995b; Azacis, 2008; Beviá, 2010; Velez, 2011a; Fujinaka and Wakayama, 2012; Velez, 2012). More recently, the possibilities of an agent to manipulate these mechanisms has been quantified (Andersson et al., 2010, 2012).

An extensive literature started by Cramton et al. (1987) focuses on efficiency issues for the dissolution of a partnership under incomplete information (see Moldovanu, 2002, for a survey). Since we assume complete information, our theoretical benchmark is closer to Tadenuma and Thomson (1995b) and subsequent literature. It is worth noting that McAfee (1992), Morgan (2004), and Che and Hendershott (2008) considerably advance incomplete information theory for the winner’s-bid auction and divide-and-choose. Remarkably, if agents are risk averse, divide-and-choose is predicted to exhibit a chooser’s advantage (McAfee, 1992). We discuss the relation of this prediction and our results in Section 4.2.

There is relatively little experimental work in the area of common-ownership problems and the performance of EIC mechanisms. Guth et al. (1982) study the outcomes of ultimatum bargaining for the division of an amount of money and divide-and-choose for the allocation of a set
of indivisible “chips” that have different value for proposer and chooser. Their results are qualitatively consistent with ours; they observe responders’ willingness to retaliate in both mechanisms. However, their experimental design cannot evaluate proposer bias in divide-and-choose, for valuations in their experiments are equal over all proposers and are equal over all choosers. So it is not clear whether differences in earnings should be attributed to proposer bias or valuation structure. An important difference with our experimental design, is that our experiments are comparable across mechanisms, and thus we are able to evaluate differences in competitiveness and efficiency among the mechanisms we study. Herreiner and Puppe (2009) investigate whether EIC allocations result from an “infinitely many proposals” bargaining mechanism under a time limit. They document that this mechanism achieves few EIC allocations. However, it is difficult to interpret these results, for it is not clear that Nash equilibrium outcomes of this bargaining procedure are EIC allocations. Schneider and Krämer (2004) and Dupuis-Roy and Gosselin (2009, 2011) experimentally evaluate the performance of the divide-and-choose mechanism. However, in their environment, monetary compensation is not possible. Thus, their experiments are silent about the performance of this mechanism in the distributive situations that motivate our study.

2 Model

In this section we introduce our model and the mechanisms that we experimentally study. We will preserve much of the same structure as our motivating examples, but will focus on more generalized environments where two agents bid over two objects. Note that our examples can be thought of as a specific case of our environment where item A is worth zero.

2.1 Environment

Two agents, \{1, 2\}, are collectively endowed with two objects \{A, B\}. These resources are to be distributed with the possibility of side transfers of a divisible good, which we refer to as money. These transfers add up to an amount we refer to as the budget. We normalize the budget to zero. We assume that each agent has to receive exactly one object. An allocation is completely described by a pair \((i_B, t_B)\) where \(i_B \in \{1, 2\}\) is the agent who receives object B and \(t_B\) is the monetary transfer from the agent who receives object B to the agent who receives object A. We impose no restriction on \(t_B\). If \(t_B > 0\), this amount is the compensation paid by agent \(i_B\) to the other agent in order to receive object B instead of object A. Symmetrically, if \(t_B < 0\), this amount is the compensation received by agent \(i_B\), and paid by the other agent, in order to receive object B instead of object A. The set of all allocations is \(Z\) and the generic allocation \(z\). We denote the agents’ allotment at \(z\) by \(z_1\) and \(z_2\), respectively. Agents have quasi-linear preferences, i.e., for each agent \(i \in \{1, 2\}\), there is a valuation function \(v_i : \{A, B\} \rightarrow \mathbb{R}\) such that agent \(i\)’s utility
from allocation $z \equiv (i_B, t_B)$ is $u_i(z) \equiv t_B + v_i(A)$ if $i_B \neq i$ and $u_i(z) \equiv -t_B + v_i(B)$ if $i_B = i$. We normalize agents’ valuations so that $v_1(A) = v_2(A) = 100$. We also consider the possibility that both agents receive no object and no money transfer and denote this option as $d$, or disagreement outcome. We assume that for each agent her utility of $d$ is zero. Let $Z^* \equiv Z \cup \{d\}$.

### 2.2 Equal-income competitive (EIC) allocations

An equal-income competitive (EIC) equilibrium in our environment is a pair $(p, z)$ where $p \equiv (p_A, p_B)$ is a price vector and $z \equiv (z_1, z_2)$ is an allocation at which each agent is maximizing in the budget set with income $I \equiv \frac{1}{2}(p_A + p_B)$, i.e., consuming object $A$ and amount of money $t_A = I - p_A$ or consuming object $B$ and amount of money $t_B = I - p_B$. Since both agents are maximizing in the same budget set at an EIC allocation, then these allocations satisfy that no agent prefer the allotment of the other agent to her own, i.e., are envy-free (Foley, 1967; Varian, 1974). The converse is also true, each envy-free allocation at a profile $u$ is an EIC allocation at that profile (Svensson, 1983). A standard argument shows that each EIC allocation is Pareto efficient.\(^9\) Let $EIC(u)$ be the set of all EIC allocations for $u$.

We now characterize the set of EIC allocations. Let $z \equiv (i_B, t_B) \in EIC(u)$ and $v_{\max}(B)$ and $v_{\min}(B)$ be the maximum and minimum of $\{v_1(B), v_2(B)\}$, respectively. Since each EIC allocation is efficient and $v_1(A) = v_2(A)$, then $i_B$ is an agent with maximal valuation for object $B$. Thus, the difference in agent $i_B$’s utility between her allotment at $z$ with that of the other agent is $2t_B - (v_{\max}(B) - 100)$. Let $i_A$ be the agent who receives object $A$ at $z$. The difference in agent $i_A$’s utility between her allotment at $z$ with that of the other agent is $-2t_B - (100 - v_{\min}(B))$. Since both agents maximize preferences in a common budget set at each EIC allocation, we have that $z \in EIC(u)$ if and only if $i_B \in \text{argmax}_i v_i(B)$ and

$$-\frac{100 - v_{\min}(B)}{2} \leq t_B \leq \frac{v_{\max}(B) - 100}{2}.$$ \quad (1)

One can geometrically represent the ultimatum bargaining set by means of the interval of possible transfers by the agent who receives object $B$ (Figure 1). This set is a subset of the allocations at which each agent receives non-negative utility.

### 2.3 Mechanisms

We consider two types of mechanisms: simultaneous move and two-stage mechanisms.

A **simultaneous move mechanism** is a pair $\langle M_1 \times M_2, \varphi \rangle$, in which $M_1$ and $M_2$ are message spaces and $\varphi : M_1 \times M_2 \rightarrow Z^*$ is an outcome function that selects an outcome for each message profile. For each preference profile $u$, $\langle M_1 \times M_2, \varphi, u \rangle$ is the game in which agents

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\(^9\)Note that $z \in Z$ is efficient for $u$ if there is no other $z' \in Z$ that is weakly preferred to $z$ by both agents and strictly preferred to $z$ by at least one agent.
with preferences $u$ simultaneously report messages from $M_1 \times M_2$ and the outcome is determined by $\varphi$. We denote the set of pure strategy Nash equilibrium outcomes of this game by $\mathcal{O}_{\text{PNE}}(M_1 \times M_2, \varphi, u)$.

A two-stage mechanism is a 4-tuple $\langle p, M_p, M_r, \varphi \rangle$ in which $p \in \{1, 2\}$, $M_p$ and $M_r$ are message spaces, and $\varphi : M_p \times M_r \to \mathbb{Z}^+$ is an outcome function that selects an outcome for each message profile. For each preference profile $u$, $\langle p, M_p, M_r, \varphi, u \rangle$ is the extensive form game defined as follows: agent $p$, who we refer to as the “proposer” selects a message $m_p \in M_p$, which is observed by the other agent, who we denote by $r$ and refer to as the “responder;” then the responder selects a message $m_r \in M_r$ and the outcome is $\varphi(m_p, m_r)$. We denote the set of subgame perfect Nash equilibrium outcomes of this game by $\mathcal{O}_{\text{SPNE}}(p, M_p, M_r, \varphi, u)$.

### 2.3.1 Winner's-bid auction

Our first mechanism resembles a first-price auction for a single object in which agents report bids (possibly negative) for object $B$. A subject’s bid is interpreted as the amount of money that they are willing to transfer to (or receive from) the other agent in order to receive object $B$. Then an agent with the highest bid receives object $B$ and pays her bid, say $b$ (thus her allotment of money is $-b$). The other player receives object $A$ and $b$. In case of a tie, an agent with highest true valuation of object $B$ receives object $B$ and pays her bid.

Assume without loss of generality that $v_1(B) \leq v_2(B)$. Formally, a winner’s-bid auction is the mechanism $\langle \mathbb{N} \times \mathbb{N}, \varphi^{\text{WBA}} \rangle$ where for each $b \equiv (b_1, b_2)$, $\varphi^{\text{WBA}}(b) \equiv (i_B, t_B)$ is the allocation such that $i_B \equiv 2$ if $2 \in \text{argmax}_i b_i$ and $i_B \equiv 1$ otherwise, and $t_B \equiv \max\{b_1, b_2\}$.

The following lemma states that the equilibrium outcomes of this mechanism are exactly the EIC allocations for true preferences. We omit the standard proof.

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9A pure strategy Nash equilibrium of $(M_1 \times M_2, \varphi, u)$ is an action profile $(m_1, m_2)$ such that for each agent $i \in \{1, 2\}$ there is no $m'_i \in M_i$ such that $u(\varphi(m_{-i}, m'_i)) > u(\varphi(m_i, m_{-i}))$. An $\varepsilon$-Nash equilibrium of $(M_1 \times M_2, \varphi, u)$ is an action profile $(m_1, m_2)$ such that for each agent $i \in \{1, 2\}$ there is no $m'_i \in M_i$ such that $u(\varphi(m_{-i}, m'_i)) > u(\varphi(m_i, m_{-i})) + \varepsilon$. A Limit Nash equilibrium outcome of $(M_1 \times M_2, \varphi, u)$ is the limit as $\varepsilon \to 0$ of a sequence of $\varepsilon$-Nash equilibrium outcomes of $(M_1 \times M_2, \varphi, u)$.

10The characterizations of equilibria of our mechanisms are essentially those in Tadenuma and Thomson (1995b) for winner’s-bid auction and Che and Hendershott (2008) for divide-and-choose. The only difference in our environment is our requirement of integer transfers.
Lemma 1. For each $u$, 

$$\text{OPNE}(\mathbb{N} \times \mathbb{N}, \varphi_{WBA}, u) = \text{EIC}(u).$$

The intuition of this result is simple. Suppose that subject 1 is the winner of the auction in equilibrium. Should subject 1 prefer subject 2’s allotment in equilibrium, she would be able to attain that level of welfare by bidding below subject 2. Symmetrically, should subject 2 prefer subject 1’s allotment in equilibrium, she would be able to attain almost that level of welfare by bidding one unit more than subject 1 (Figure 2).

Let us remark that winner’s-bid auction selects an efficient allocation for true preferences when agents’ reports are identical. One can avoid the use of valuation information while maintaining the equilibrium outcomes of the mechanism as follows. Randomly choose the winner of $B$ in case of a tie. Then allow for agents’ voluntary trade of allotments.\(^{12}\) Thus, our mechanism can be seen as a simplified version of a mechanism that does not depend on our knowledge of subjects’ true valuations.

2.3.2 Divide-and-choose

Our second mechanism is a two-stage mechanism that proceeds as follows: an agent, say 1, is selected to propose a payment $t_B$. Then agent 2 chooses between receiving object $B$ and paying $t_B$ for it, or receiving object $A$ and $t_B$. Formally, this is the two-stage mechanism $\langle p, \mathbb{N}, \{A, B\}, \varphi_{DC}\rangle$ in which for each $(m_p, m_r) \in \mathbb{N} \times \{A, B\}$, $\varphi_{DC}(m_p, m_r) = (i_B, t_B)$ where $i_B = p$ if $m_r = A$ and $i_B = r$ otherwise.

The following lemma characterizes the subgame perfect equilibrium outcomes of the divide-and-choose game. We omit the standard proof.

Lemma 2. For each $u$, $\text{OSPNE}(p, \mathbb{R}, \{A, B\}, \varphi_{DC}, u)$ is the set:

1. $\left\{ \left( r, \frac{\max_i v_i(B) - 100}{2} \right), \left( r, \frac{\max_i v_i(B) - 100}{2} - 1 \right) \right\}$, if $\min_i v_i(B) < \max_i v_i(B)$ and $p = \text{argmin}_i v_i(B)$.

\(^{12}\)Our equilibrium prediction for this extended mechanism is subgame perfect equilibria.
2. \( \{(p, \min_i v_i(B) - \frac{100}{2}), (p, \min_i v_i(B) - \frac{100}{2} + 1)\} \), if \( \min_i v_i(B) < \max_i v_i(B) \) and \( p = \arg\max_i v_i(B) \).

3. \( \{(i_B, t_B) : i_B \in \{1, 2\}, t_B = \frac{\max_i v_i(B) - 100}{2}\} \), if \( \min_i v_i(B) = \max_i v_i(B) \).

Intuitively the equilibria of our divide-and-choose game are equivalent to the equilibria of this game for the division of a pie: the proposer selects a division for which the responder is indifferent between both allotments in the proposed division. More precisely, imagine that the proposer is an agent in \( \arg\max_i v_i(B) \). Given any proposal \( t_B \), the responder will choose her preferred bundle. Thus, the proposer can induce the responder to choose any allocation in the interior of the EIC range. If the proposer selects \( t_B \), the responder will select \( B \) and pay \( t_B \). Since the proposer would be better off at any EIC allocation, she would never propose such a \( t_B \). Symmetrically, the proposer never selects \( t_B > \frac{\max_i v_i(B) - 100}{2} \), i.e., to the right of the EIC range. Thus, the proposer will always propose \( t_B \) inside the EIC range. Indeed, she proposes in equilibrium her best outcome from the EIC set if the responder selects \( A \) in that proposal. Otherwise, she proposes to pay the best “interior” EIC payment \( t_B \) and the responder selects \( A \) (Figure 3).

Figure 3: Equilibrium allocations of divide-and-choose mechanism. The equilibrium allocations of the divide-and-choose are EIC allocations, but not all EIC allocations are equilibrium allocations. The equilibria are the EIC allocations that most favor the proposer. The two points on the graph denote whether the proposer has the low or high value on item B.

### 2.3.3 Ultimatum bargaining

Finally, we consider an ultimatum bargaining mechanism. First, one agent is selected as proposer \( p \) and the other as responder \( r \). The proposer reports an allocation \( z \equiv (z_p, z_r) \) such that \( u_i(z_r) \geq 0 \). Then the responder accepts or rejects the proposal. They both receive no object and no transfer of money if the proposal is rejected. Formally, this is the two-stage mechanism \( \langle p, \{p, r\} \times \mathbb{N}, \{\text{Accept, Reject}, \varphi^U\} \rangle \) in which for each message profile \( (m_p, m_r) \in \{\{p, r\} \times \mathbb{N}\} \times \{\text{Accept, Reject}, \varphi^U(m_p, m_r) = m_p \text{ if } m_r = \text{Accept and } \varphi^U(m_p, m_r) = d \text{ otherwise.} \)

One can easily see that at each subgame perfect equilibrium outcome of this mechanism, subject \( p \) proposes an efficient allocation \( z^* \) such that \( u_r(z^*_r) \leq 1 \) and subject \( r \) accepts the proposal (for the true valuations considered in our experiments, these allocations are not EIC).\(^{13}\)

\(^{13}\)A proposal such that \( u_i(z^*_r) = \varepsilon > 0 \) is strictly preferred by subject \( r \) to rejecting it. A proposal where \( u_i(z^*_r) = 0, \)
The equilibrium allocations of the ultimatum bargaining are not EIC allocations. The equilibria are allocations where the proposer captures nearly all (if not all) possible profits. The two points on the graph denote whether the proposer has the low or high value on item B.

Figure 4: Equilibrium allocations of ultimatum bargaining. The equilibrium allocations of the ultimatum bargaining are not EIC allocations. The equilibria are allocations where the proposer captures nearly all (if not all) possible profits. The two points on the graph denote whether the proposer has the low or high value on item B.

Table 1 summarizes the equilibrium properties of the three mechanisms we consider in our study.

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Efficient allocations achieved</th>
<th>Symmetric</th>
<th>Coordination required</th>
</tr>
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<tbody>
<tr>
<td>Winner’s-bid auction</td>
<td>+</td>
<td>+</td>
<td>yes</td>
</tr>
<tr>
<td>Divide-and-choose</td>
<td>+</td>
<td>+</td>
<td>no</td>
</tr>
<tr>
<td>Ultimatum bargaining</td>
<td>+</td>
<td>−</td>
<td>no</td>
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Table 1: Theoretical properties of each mechanism. The winner’s-bid auction is the only mechanism to satisfy three normative theoretical properties. Because of its simultaneity, it is also the only mechanism to require coordination.

3 Experimental Design

This experiment implemented the aforementioned theoretical environment with each of the three mechanisms. Subjects, randomly selected each round into groups of two, chose how to allocate two indivisible items with possible transfer payments. In all possible allocations, each subject received exactly one item. Each period, subjects received points for acquiring an item, equal to their value of that item. Thus, the values for each item are induced values.

The experiment consisted of 50 periods, with 5 different valuations occurring sequentially over 10 consecutive periods each. For any period, for each grouping of subjects, one subject was randomly assigned the high value on item B, the other was assigned the low value on item B. Thus a subject’s value on item B could change for any period, but the valuation structure (the

14 Rather than try to match the motivating example in the introduction and use only one item, we chose to have subjects bid over two items to match the generalized theoretical environment more closely and reduce the possibility that of subjects are motivated by the non-monetary desire to “win” an item (the utility of winning, c.f., Smith, 1989; Parco et al., 2005; Van den Bos et al., 2008; Price and Sheremeta, 2011).

15 In the case of ultimatum bargaining if a proposal was rejected each subject received nothing. However, all ultimatum bargaining proposals required that each subject receive exactly one item.
Player Value of Item A  Value of Item B
Player 1 100 120
Player 2 100 40

Table 2: A sample valuation. In this valuation, one subject valued item A at 100 and item B at 120. As in our theoretical environment, the other subject with whom she is paired had the same value for item A, but valued item B at 80. Each player had an equal chance of receiving the high value on item B for any period. Valuations were common knowledge to both players. This specific valuation structure ("valuation 3") was used for periods 21–30.

high and low values for item B) remained the same for the 10 periods. Subjects always valued item A at 100. In order of appearance, the pairing of subject values for item B were (40, 80), (120, 160), (40, 120), (160, 160), and (0, 40).\(^\text{16}\) Table 2 provides an example valuation, the third valuation used in the experiments. Here, both subjects have the same value for item A, but their values for item B are different: player 1 has the high value for item B (120 points), and player 2 has the low value for item B (40 points).

To avoid incentives associated with repeated play, subjects were randomly re-assigned to each other at the beginning of each period. Subjects were instructed that they were to be randomly rematched each period, and no identifying information (e.g., subject number) was disclosed to a subject about her match in any round. Each period began with each subject seeing the valuation for the period.

3.1 Winner’s-bid auction procedures

In the winner’s-bid auction session, after observing the valuation for the period, subjects simultaneously submitted their bids for item B. The subject with the higher bid received item B, and the subject with the lower bid received item A. In this way, the winner’s-bid auction is a first-price auction to acquire item B over item A. In the case of equal bids, the subject with the higher value of item B received item A.\(^\text{17}\) The subject who received item B, then pays the subject who receives item A the full amount of her bid.\(^\text{18}\)

After submitting a bid, each subject was allowed to submit a possible value for the other

\(^{16}\)The ordering of the valuations was chosen at random, but remained constant across all three mechanisms. The valuations were specifically chosen to allow for two possibilities where both players value item A more than B (40, 80) and (0, 40), two possibilities where both players value item B more than A (120, 160) and (160, 160), and one where one player one player values B more than A and one values A more than B (40, 120). Additionally the fourth valuation (160, 160) was chosen to allow an environment where there is one unique equilibria over all three mechanisms, and coordination should not be an issue.

\(^{17}\)In the case of the fourth valuation where both subjects value item B at 160, one subject was randomly selected to win ties, this was known before bids were submitted (i.e., ex-ante). The tie-breaking rules were chosen to allow for easier analysis of equilibrium.

\(^{18}\)To eliminate the possibility of grossly negative earnings, bids were restricted so that no bid could be lower than the opposite of twice the value of item A (-200, always) and no bid could be higher the twice the maximum value of item B (varies by valuation, i.e, 160, 320, 240, 320, and 80 for each valuation, respectively).
player's bid. The experimental software then displayed the outcome (i.e., who gets which item, what amount is transferred for each player, each players' earnings for that period) that would occur with those two bids as well as a table that showed all possibilities that could happen if the other player's bid were below, equal to, or above the subject's bid (see Figure 5). After a subject viewed these possibilities, she could chose to confirm her bid, or chose an alternate bid. If she chose an alternate bid, the process repeated. The process ended when a subject confirmed her bid.

After both subjects submitted their bids, they were asked to guess what they believed was the other subject bid.\(^{19}\) If they guessed correctly they received a small bonus of 5 points. The value of this bonus was deliberately chosen to be small, so that subjects did not alter their bidding

\(^{19}\) Subject guesses appear to be unrelated any other part of their behavior. The data are not meaningful and not presented here. Subjects are clearly not best responding to their guesses; this result is reminiscent of Costa-Gomes and Weizsäcker (2008).
strategy to receive the bonus. After both subjects submitted their guesses they saw the outcome of their bidding. They learned what the other player bid, which items they both received, the transfer payment between them, their earnings, and their partners’ points earned. Subjects learned if they received a bonus for guessing the other player’s bid correctly, but did not learn if the other player had received the bonus for guessing their bid correctly. After this information was disclosed, a new period began. This process continued for 50 periods.

3.2 Divide-and-choose procedures

In the divide-and-choose session, at the beginning of each period, one subject was randomly selected to be the divider. That subject chose which item should receive a transfer and the amount of that transfer. Transfer payments were limited so that no subject received negative earnings for each period (so, for example in Table 2, the divider could propose whomever receives item A could get a transfer up to 80, the minimum value of B, or whomever receives item B could get a transfer up to 100, the minimum value of A).

After one subject made a proposal, she saw a table of possible outcomes similar to Figure 5 that displayed the two possible outcomes (i.e., who gets which item, what amount is transferred for each player, each players earnings for that period) when the other subject choose to take item A or item B. The subject had the opportunity to confirm her decision or make another one. If she chose to try another proposal, the process repeated until she confirmed a proposal. Once a proposal was confirmed, the other subject viewed the proposal. The display showed her two outcomes—both her own and the other subject’s total earnings if she chose to take item A or item B. The chooser then had the opportunity to choose either item. After that decision was made, both subjects viewed the outcome of the period. They saw what the divider proposed, which item the chooser selected, the items and transfers received by each subject (if applicable) and the points earned by each subject for the period. After this information was disclosed, a new period began. This process continued for 50 periods.

3.3 Ultimatum bargaining procedures

In the ultimatum bargaining session, at the beginning of each period, one subject was randomly selected to be the proposer. That subject chose who received each item, and if any transfer payments should be paid from one subject to the other. Transfer payments were limited so that no subject could receive negative earnings for each period (so, for example in Table 2, player 1 cannot propose player 2 receive item A and pay player 1 a transfer payment of 101 points, because that would result in player 1 receiving negative earnings).

After the proposer made a proposal, she saw a table of possible outcomes similar to Figure 5 that displayed the two possible outcomes (i.e., who gets which item, what amount is transferred for each player, each player’s earnings for that period) when the other subject accepted or re-
jected her proposal. Note that in the case of rejection, the outcome was that both subject received no items and no points for the period. The proposer had the opportunity to confirm or modify the proposal. If she chose to modify, the process repeated until she confirmed a proposal. Once a proposal was confirmed, the other subject, the responder, viewed the proposal. The display showed her two outcomes—what happened if he chose to accept or reject the other subject’s proposal. The responder then had the opportunity to accept or reject the proposal. After that decision was made, both subjects viewed the outcome of the period. They saw the proposer’s proposal, whether the responder accepted or rejected that proposal, the items and transfers received by each subject (if applicable) and the points earned by each subject for the period. After this information was disclosed, a new period began. This process continued for 50 periods.

3.4 End of experiment procedures

Once the 50 periods were complete, subjects completed a survey about their opinions of the other players with whom they had been matched, the mechanism used, and their general feelings of what fairness means. They were also given the opportunity to provide a tip up to $5, which was be doubled and divided among all other subjects. The final survey question told them they were to play one more period at a valuation 10 times greater than before and they voted on the mechanism to be used by all subjects for that round. Subjects in the ultimatum bargaining and divide-and-choose sessions voted between their current mechanism and the winner’s-bid auction; subjects in the winner’s-bid auction session voted between the winner’s-bid auction and the ultimatum bargaining. Since only one mechanism was used per experimental session a brief description was provided of the other mechanism. After all subjects completed the vote, the winning mechanism was implemented for the final period. As carefully explained to the subjects a majority vote was required to change to the new mechanism, meaning the status-quo won all ties. After the high-stakes period subjects again completed more surveys, a demographic survey and a five-factor personality assessment (from John et al., 2008).

All experiments were held at the Economic Research Laboratory (ERL) in the Economics Department at Texas A&M University. Subjects were recruited using ORSEE software (Greiner, 2004) and made their decisions on software programmed in the z-tree language (Fischbacher, 2007). Subjects sat at computer terminals with dividers to make sure their anonymity was preserved. Subjects were 78 Texas A&M undergraduates from a variety of majors, twenty-six subjects took part in the ultimatum bargaining session on October 21, 2010; twenty-four subjects took part in the ultimatum bargaining session on October 22, 2010; twenty-eight subjects took part in the winner’s-bid auction session on October 22, 2010; twenty-eight subjects took

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20The results of this last period are not included in the data analysis.
21The status-quo won in all three sessions, though by a much smaller margin in the ultimatum bargaining session.
22Survey results are not reported in this paper. We did not find any interesting interactions between survey responses and mechanism type or subject behavior. They are available from the authors upon request.
Mechanism: winner’s-bid auction ultimatum bargaining divide-and-choose

<table>
<thead>
<tr>
<th></th>
<th>winner’s-bid auction</th>
<th>ultimatum bargaining</th>
<th>divide-and-choose</th>
</tr>
</thead>
<tbody>
<tr>
<td>efficient outcomes (percent)</td>
<td>437 (0.728)</td>
<td>500 (0.769)</td>
<td>594 (0.849)</td>
</tr>
<tr>
<td>EIC outcomes (percent)</td>
<td>283 (0.472)</td>
<td>134 (0.206)</td>
<td>567 (0.810)</td>
</tr>
<tr>
<td>average profit (in points) (standard error)</td>
<td>99.917 (1.111)</td>
<td>88.644 (1.731)</td>
<td>102.40 (1.128)</td>
</tr>
<tr>
<td>percent of maximum possible profit (standard error)</td>
<td>0.935 (0.005)</td>
<td>0.829 (0.014)</td>
<td>0.962 (0.001)</td>
</tr>
</tbody>
</table>

Table 3: Summary statistics of outcomes for each subject pair by experimental mechanism. Across all measures efficient outcomes, EIC outcomes, average profit and percent of profit realized, the divide-and-choose outperforms the winner’s-bid auction and ultimatum bargaining. The winner’s-bid auction outperforms ultimatum bargaining on three measures (EIC outcomes, average profit and percent of profit realized), but achieves fewer efficient outcomes on average.

part in the divide-and-choose session on April 12, 2012. Experiments lasted about three hours. Subject point values were totaled and converted to cash at the rate of 400 points = $1.00, rounded up to the nearest dollar. Subject earnings ranged from $14.70 to $33.80 ($25.44 average earnings), $16.20 to $31.70 ($27.65 average earnings), $14.20 to $33.80 ($26.40 average), for the ultimatum bargaining, divide-and-choose, and winner’s-bid auction sessions, respectively.

4 Results

4.1 Outcomes achieved by mechanism

Table 3 provides summary statistics for each mechanism’s performance over realized outcomes. Over all categories, efficient outcomes, equal-income competitive (EIC) outcomes, average profit and percent of profit realized, divide-and-choose outperforms the winner’s-bid auction and ultimatum bargaining. The winner’s-bid auction outperforms ultimatum bargaining on most measures, except efficient outcomes where ultimatum bargaining does slightly better. Figures 6(a–c) show these results hold over all valuations and are not dependent on a specific valuation structure. (Recall that each mechanism was tested for 50 periods, divided into five 10-period segments with a different type of valuation structure.) For every valuation, subjects in divide-and-choose achieve a greater percentage of efficient outcomes realized, a greater percentage of EIC outcomes realized, and higher average payoffs realized. The winner’s-bid auction does better than ultimatum bargaining with respect to EIC outcomes realized and average payoffs, but ultimatum bargaining achieves a greater percentage of efficient outcomes realized in all but the (160, 160) valuation. In that valuation, all outcomes are efficient except for a rejection in ultimatum bargaining.

So far, none of our analysis has broached the issue of statistical significance. In settings such
as these, with 24–28 subjects being randomly paired over 50 periods, it is clear we do not have 1200–1400 independent observations. Each subject decision should not be assumed to be independent with her past decision. Similarly each period should not be assumed to be independent from a previous period. To address this issue, we use regression techniques for panel data to determine statistical significance. Unless specified otherwise, we will use random effects for each subject in our randomly matched pair, and one dummy variable for each period in our data (i.e., fixed effects), to control for idiosyncratic properties of certain subjects or periods, respectively. Formally, for binary outcomes we will use the regression model

\[
\text{logit} \left( \Pr \left( y_{ijk} = 1 \mid X_{ijk}, \zeta_{1j}, \zeta_{2k} \right) \right) = \beta X_{ijk} + \gamma_i + \zeta_{1j} + \zeta_{2k} + \epsilon_{ijk},
\]

where \( y_{ijk} \) represents the outcome of the pairing of subject \( j \), and subject \( k \) in period \( i \). The random coefficients \( \zeta_{1j} \sim N(0, \psi_1) \), \( \zeta_{2k} \sim N(0, \psi_2) \) are determined for subject \( j \) and subject \( k \) by their positions in the mechanism. In this case subject \( j \) is in the first position (e.g., higher value on item B, or first mover) and \( k \) is in the second position (e.g., lower value on item B, or second mover); \( \gamma \) is a vector of fixed effects for each period; \( X_{ijk} \) contains other data of interest, such as mechanism type, and \( \beta \) is a vector of coefficients. The residual error term \( \epsilon_{ijk} \sim N(0, \theta) \).
similar model is used for non-binary outcomes

\[ y_{ijk} = \beta X_{ijk} + \gamma_i + \zeta_{1j} + \zeta_{2k} + \epsilon_{ijk}. \]  

Table 4 shows the results of four regressions using the models described in equations 2 and 3. The regression generally confirms the results implied by Table 3. The ultimatum bargaining and divide-and-choose terms are dummy variables that indicate the difference between these mechanisms and the winner's-bid auction (which is the omitted term). The divide-and-choose mechanism achieves significantly more efficient and more EIC outcomes than either other mechanism; the coefficient for divide-and-choose in the efficient outcome and EIC regressions is positive and significant at the 1% level. The winner's-bid auction achieves significantly more EIC outcomes than ultimatum bargaining (note the ultimatum bargaining coefficient is negative and significant at the 1% level in the EIC regression), but does not differ significantly on efficiency (the ultimatum bargaining term in the efficient outcome regression is positive, but not statistically meaningful). Both the winner's-bid auction and divide-and-choose mechanisms achieve significantly higher earnings, both in nominal terms and as a percentage of total earnings than the ultimatum bargaining (The ultimatum bargaining term is slightly below -10 on both average earnings and total percentage regressions, a statistically different result at the 1% level). Those two mechanisms do not significantly differ from each other on these earnings metrics. (The divide-and-choose term in both regressions is not statistically different from 0.)

To summarize our findings so far, the two EIC mechanisms greatly outperform ultimatum bargaining on all of our metrics, with the lone exception being efficiency where winner's-bid auction and ultimatum bargaining perform similarly. Divide-and-choose significantly outperforms the winner's-bid auction in efficient and in EIC outcomes. Divide-and-choose also achieves higher earnings, but they are not significantly different from those of winner's-bid auction. Our figures show these relationships are not the result of a single valuation structure; they are mostly consistent throughout all five valuations.

4.2 Mechanism bias

The previously discussed measures of mechanism performance only observe outcomes for the subject-pair, ignoring the distribution of earnings within the pair. If a mechanism had a systematic bias toward one of the subjects, these measures would not observe it. To address this issue, we examine individual subject earnings for each mechanism controlling for each subject's role within a subject pair. Our theory shows that the winner's-bid auction should be the only mechanism to not have any structural bias because it is symmetric. Both divide-and-choose and ultimatum bargaining favor the first mover (i.e., proposer/divider) in subgame-perfect equilib-
Table 4: Regressions of mechanism outcomes on type of mechanism, controlling for period fixed effects and subject random effects.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Efficient outcome</th>
<th>EIC outcome</th>
<th>Average earnings of pair</th>
<th>Percent of total possible earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimatum bargaining</td>
<td>0.297</td>
<td>−1.657***</td>
<td>−11.152***</td>
<td>−10.472***</td>
</tr>
<tr>
<td></td>
<td>(0.238)</td>
<td>(0.268)</td>
<td>(2.097)</td>
<td>(2.046)</td>
</tr>
<tr>
<td>Divide-and-choose</td>
<td>0.933***</td>
<td>1.932***</td>
<td>2.698</td>
<td>2.932</td>
</tr>
<tr>
<td></td>
<td>(0.243)</td>
<td>(0.263)</td>
<td>(2.063)</td>
<td>(2.012)</td>
</tr>
<tr>
<td>Regression Type?</td>
<td>logit</td>
<td>logit</td>
<td>linear</td>
<td>linear</td>
</tr>
<tr>
<td>Fixed effects?</td>
<td>each period</td>
<td>each period</td>
<td>each period</td>
<td>each period</td>
</tr>
<tr>
<td>First random effects term?</td>
<td>subject with high value</td>
<td>subject with high value</td>
<td>subject with high value</td>
<td>subject with high value</td>
</tr>
<tr>
<td>Second random effects term?</td>
<td>subject with low value</td>
<td>subject with low value</td>
<td>subject with low value</td>
<td>subject with low value</td>
</tr>
</tbody>
</table>

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

a. Winner's-bid auction dummy variable is omitted. There is a significant difference (Wald test, p<0.01) between the coefficients of divide-and-choose and ultimatum bargaining variables in all four regressions.
b. Giving fixed terms for each valuation type (five dummy variables for valuations 1–5), and valuation within period (ten dummy variables for periods 1–10) do not significantly alter results.
c. Sorting random effects by subject favored by mechanism (high value in auction, proposer in ultimatum bargaining/divide-and-choose) does not significantly alter regression results.
Figure 7: First-mover bias in ultimatum bargaining and divide-and-choose separated by whether the proposer had the high or low valuation for item B and by valuation. In each session, over 50 periods, five valuations occurred sequentially for ten periods each. Proposer bias is calculated as (average payoff proposer/average payoff responder)-1. (a, left) In ultimatum bargaining, regardless of whether the propose had a high or low valuation for item B, the mechanism exhibited a pronounced proposer bias. The proposer on average would earn more than twice as many points as the responder. (b, right) In divide-and-choose, no proposer bias is observed. In some valuations, there appears to be evidence of a small chooser bias, but the bias is not of the same magnitude of the bias found in ultimatum bargaining.

respectively) than second movers (see row 2, columns 1 and 2), revealing if anything a second-mover advantage in divide-and-choose.

Figure 8 provides an explanation for this last result. It shows the distribution of EIC proposals from high valuation and low valuation subjects in the divide-and-choose mechanism for all valuations with non-trivial EIC range. Unlike our theoretical predictions (see Figure 3), we see subjects proposing less self-serving, EIC allocations (i.e., those at the boundaries of EIC allocations in the figure). When subjects do propose these allocations, and those close to it, are “rejected” by the chooser with high probability. Here we understand rejection as the selection of the allocation that gives the lower utility to both proposer and chooser from the two allocations proposed (these are non-credible threats that cannot be sustained as off equilibrium strategies in a subgame perfect equilibrium). Thus, the proposer ends up with a lower payoff compared to a chooser with same valuations. This is due to the losses incurred when choosers decide to retaliate and reject their proposals.

Table 5 also provides regressions on “proposed allocations,” that is, how allocations would be decided if the second-mover never rejected a proposal. With ultimatum bargaining, the meaning of rejection is clear. As mentioned above, for divide-and-choose we consider a “rejection” to be any time the chooser chose an option that provided the lower payoff for both
EIC allocations

non-negative utility efficient allocations

Figure 8: Distribution of EIC proposals from subjects with the high value (HV) and low value (LV) on item B in divide-and-choose mechanism. Excludes valuation 4, i.e., (160, 160). Subjects with the high value on item B propose to take less of the surplus in an efficient allocation than subjects with the low value. The high-value-subject offers are accepted more often. Acceptance means the divider chooses the efficient allocation, one that will give both parties more. See figure 3 for the theoretical predictions of the divide-and-choose mechanism.

As expected, these results (third and fourth columns of Table 5) favor the first mover more. First movers propose to take for themselves on average over 60 points (roughly 30%, both p<0.001) more than they give to second-movers in their proposals in ultimatum bargaining (row 1, columns 3 and 4). In divide-and-choose first movers take slightly more, but this result is not statistically significant (row 2, columns 3 and 4). Note that when we restrict these results to only EIC proposals (see Appendix Table A.2) first movers take 5 points (3%) more in their proposals (p<0.01), suggesting off-equilibrium play is responsible for some of the reduction in the first-mover advantage with divide-and-choose.

The last two columns (columns 5 and 6) of Table 5 show regressions on subject earnings restricted to proposals that are not rejected. The first-mover advantage falls between the values for overall results (columns 1 and 2) and proposed allocations (columns 3 and 4). Not surprisingly, this indicates that proposals that are more “equitable” are more likely to be accepted with both mechanisms (see Figure 8). Also, the act of rejection—each player getting 0 in ultimatum bargaining, the chooser taken their lower alternative (and likely causing inefficiency)—reduces the discrepancy between the first and second mover’s earnings. This is not surprising for ultimatum bargaining as a rejection assures both players will earn the same amount. In divide-and-choose this implies that these rejections often cause an outcome with higher earnings for the second player, enough to erode any first-mover advantage.

One can relate the absence of proposer advantage in our experimental results and the chooser’s advantage predicted for divide-and-choose under incomplete information when agents are risk averse (McAfee, 1992). On the one hand, a proposer in our experiments has some uncertainty about what proposals will be “accepted” by the chooser. On the other hand, with both complete

24In the case of equal payoffs, a rejection is when the second player would choose inefficiency (and giving the first mover less) over efficiency (and giving the first mover more).
Figure 9: Externality HV vs. percentage of aggregate payoff captured by HV. HV externality is calculated by \((HV - LV)/(HV + LV)\). In winner’s-bid auction and divide-and-choose, when the subject with the high value of item B has a valuation much greater than the other subject's valuation of item B, the high value subject receives more points. In ultimatum bargaining, the valuation structure makes no difference on the earnings of the subject with the high value on item B, because the proposer, regardless of valuation, receives most of the points.

and incomplete information (with independent private values) the chooser faces no uncertainty. Thus, in both cases, the proposer bears all risk in the allocation process and ends up offering on average more than the sub-game perfect prediction under complete information. In our experiments, this effect is aggravated by the fact that the proposer also absorbs a disproportionate share of the efficiency loss caused by the chooser's rejection.

In Figure 9 and rows 3–5 in Table 5 we analyze whether our mechanisms are payoff monotone with respect to valuations. That is, whether the mechanisms assign higher payoffs to the agent with the high value for object \(B\) (recall that here both agents have equal values for object \(A\)). This monotonicity is not a mechanism bias; instead, it is a desirable property that allows a player to benefit from her externality to the group. Figure 9 shows that high-valuation players achieve a greater share of the surplus than low valuation players in the winner’s-bid auction and divide-and-choose, but not in ultimatum bargaining. This result is confirmed in Table 5. High valuation subjects in the winner’s-bid auction achieve about 20 more points (12%) in earnings than low valuation subjects \((p<0.001)\) (row 3, columns 1 and 2). High valuation subjects in divide-and-choose achieve about 19 more points (9%) in earnings than low valuation subjects \((p<0.001)\) (row 5, columns 1 and 2). There is no statistically meaningful difference for earnings between high and low valuation subjects in ultimatum bargaining (row 4, columns 1 and 2).
### Table 5: Regressions of subject earnings on mechanism, valuation, first mover, and item B value, controlling for period fixed effects and subject random effects.

<table>
<thead>
<tr>
<th>dependent variable:</th>
<th>overall</th>
<th>proposed offers(^b)</th>
<th>accepted offers(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>profit (in points)</td>
<td>share of total profit (percent)(^a)</td>
<td>profit (in points)</td>
</tr>
<tr>
<td>Proposer/first mover in ultimatum bargaining</td>
<td>47.874***</td>
<td>22.516***</td>
<td>62.956***</td>
</tr>
<tr>
<td></td>
<td>(1.615)</td>
<td>(0.585)</td>
<td>(1.040)</td>
</tr>
<tr>
<td>Divider/first mover in divide-and-choose(^c)</td>
<td>-3.620**</td>
<td>-2.057***</td>
<td>1.243</td>
</tr>
<tr>
<td></td>
<td>(1.564)</td>
<td>(0.567)</td>
<td>(1.002)</td>
</tr>
<tr>
<td></td>
<td>(1.694)</td>
<td>(0.613)</td>
<td>(1.649)</td>
</tr>
<tr>
<td>High value on B in ultimatum bargaining</td>
<td>0.522</td>
<td>0.680</td>
<td>-0.122</td>
</tr>
<tr>
<td></td>
<td>(1.707)</td>
<td>(0.618)</td>
<td>(1.111)</td>
</tr>
<tr>
<td></td>
<td>(1.649)</td>
<td>(0.597)</td>
<td>(1.074)</td>
</tr>
<tr>
<td></td>
<td>(2.707)</td>
<td>(0.948)</td>
<td>(1.164)</td>
</tr>
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<td></td>
<td>(2.719)</td>
<td>(0.954)</td>
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</tr>
<tr>
<td>Type of regression?</td>
<td>linear</td>
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<td>linear</td>
</tr>
<tr>
<td>Fixed effects?</td>
<td>each period</td>
<td>each period</td>
<td>each period</td>
</tr>
<tr>
<td>First random effects term?</td>
<td>subject</td>
<td>subject</td>
<td>subject who is first mover</td>
</tr>
<tr>
<td>Second random effects term?</td>
<td>player paired with subject</td>
<td>player paired with subject</td>
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</tr>
<tr>
<td>observations</td>
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<td>2700</td>
</tr>
<tr>
<td>log likelihood</td>
<td>-18710.021</td>
<td>-14746.764</td>
<td>-11742.358</td>
</tr>
</tbody>
</table>

\(^a\) Significant at the 10% level.

\(^b\) Significant at the 5% level.

\(^c\) Significant at the 1% level.

---

a. In divide-and-choose, a rejection is the divider choosing the option that gives him less points, or in a case of equal points, the inefficient outcome. Thus a divider's proposal is what would be implemented given the chooser does not reject. Auction data are not included for these four regressions.

b. Winner's-bid auction observations are not included in these regressions as proposals and rejections are not possible. A rejection in ultimatum bargaining leads to earnings of 0 for each player. We consider this to be a 50% share for each player.

c. This value represents the relative gain over a low valuation/second mover.

d. This value represents the predicted value for a low valuation second mover, relative to period fixed effects.
4.3 Learning and bounded rationality in the winner’s-bid auction

Given the simplicity of the choice of the second-mover in divide-and-choose and ultimatum bargaining, we can conclude that the failure of those mechanisms to achieve an efficient outcome is due to factors beyond profit maximization (and likely are due to social or other regarding preferences; e.g., Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000). However, the complexity of the winner’s-bid auction makes it harder to determine why efficient outcomes are achieved only 72% of the time, and EIC outcomes, those predicted as the only outcomes under Nash-equilibria occur less than half of the time. Two explanations come to mind. Either subjects are boundedly rational and have trouble figuring out their bid, or the multiplicity of equilibria leads to a coordination failure, causing disequilibrium among possible strategies.

In Table 6 we present regression analysis evaluating these two hypotheses. The first column gives the results of a logistic regression of efficient and EIC outcomes on valuation and a variable indicating the period within the valuation (ranging from 1–10). There is evidence to suggest subjects are improving over time, albeit slightly, and achieving more EIC outcomes. (Observing odds ratios, shown in Appendix, Table A.1, each subsequent period within a valuation increases the likelihood of an EIC allocation being obtained by about 2%.) The coefficient for the first valuation is also the lowest of all valuations. This is consistent with the idea that subjects became better at achieving EIC outcomes as the experiment progressed.

The last three regressions show a logistic regression of Nash bids on valuation, whether a subject had the high value on item B, and the period within valuation. The tables indicate that subjects with the high value on item B are much more likely to provide a Nash bid than subjects with the low value. In fact, experienced subjects with the low value on item B are no more likely to provide a Nash bid than inexperienced subjects with the low value on item B. The winner’s-bid structure may be a reason for this deviation. With the winner’s bid structure, as long as the high bid is a Nash bid, an efficient and EIC outcome will be obtained. Given that the subject with the high value on item B proposes a Nash bid, there is no incentive for the low-value subject to bid over that value (or even bid that value). Instead, they may underbid, hoping to reduce issues from coordination.

Unfortunately, this strategy of underbidding by the subject with the low value on item B provides an incentive for subjects with the high value to lower their bids. If they lower those bids below Nash levels but still above the other subject’s bid, this can lead to efficient, but not EIC outcomes. If they bid under the other subject’s bid, this leads to inefficiency. Bid distribution in the winner’s-bid auction when both agents had the same value on item B, i.e., valuation 4, (160, 160), illustrates this phenomenon (Figure 10). Here, at the unique pure strategy Nash equilibrium, both agents bid 30 points to get item B and both subjects achieve a payoff of 130 points. However, if a subject is deviating from equilibrium, it is a best response for the other subject to bid below 30. For instance, a bid of 29 guarantees a payoff of at least 130 points and could...
### Table 6: Regressions of equilibrium outcomes and bids for winner's-bid auction on valuation, period within valuation, and subject's value on item B, controlling for subject random effects.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>EIC Outcome</th>
<th>Nash Bid</th>
<th>Nash Bid</th>
<th>Nash Bid</th>
</tr>
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<tr>
<td>period within</td>
<td>0.086***</td>
<td>0.051**</td>
<td>0.019</td>
<td>0.047</td>
</tr>
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<td>valuation</td>
<td>(0.033)</td>
<td>(0.024)</td>
<td>(0.026)</td>
<td>(0.034)</td>
</tr>
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<td>high value on B</td>
<td>0.064***</td>
<td>0.008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>period within valuation</td>
<td>(0.022)</td>
<td>(0.048)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>valuation 1</td>
<td>-1.318***</td>
<td>-0.868***</td>
<td>-0.649**</td>
<td>-0.846***</td>
</tr>
<tr>
<td>(40-80)</td>
<td>(0.335)</td>
<td>(0.277)</td>
<td>(0.266)</td>
<td>(0.307)</td>
</tr>
<tr>
<td>valuation 2</td>
<td>-0.441</td>
<td>-0.586**</td>
<td>-0.369</td>
<td>-0.564*</td>
</tr>
<tr>
<td>(120-160)</td>
<td>(0.321)</td>
<td>(0.275)</td>
<td>(0.264)</td>
<td>(0.305)</td>
</tr>
<tr>
<td>valuation 3</td>
<td>0.661**</td>
<td>1.365***</td>
<td>1.581***</td>
<td>1.388***</td>
</tr>
<tr>
<td>(80-120)</td>
<td>(0.332)</td>
<td>(0.294)</td>
<td>(0.287)</td>
<td>(0.323)</td>
</tr>
<tr>
<td>valuation 4</td>
<td>-1.103***</td>
<td>-2.067***</td>
<td>-1.849***</td>
<td>-2.045***</td>
</tr>
<tr>
<td>(160-160)</td>
<td>(0.330)</td>
<td>(0.298)</td>
<td>(0.287)</td>
<td>(0.326)</td>
</tr>
<tr>
<td>valuation 5</td>
<td>-0.852***</td>
<td>-0.075</td>
<td>-0.138</td>
<td>-0.054</td>
</tr>
<tr>
<td>(0-40)</td>
<td>(0.324)</td>
<td>(0.274)</td>
<td>(0.264)</td>
<td>(0.303)</td>
</tr>
<tr>
<td>has high value on item B</td>
<td>0.433***</td>
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<td>0.388</td>
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<td>logit</td>
<td>logit</td>
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<td>Fixed effects?</td>
<td>each valuation (shown above)</td>
<td>each valuation (shown above)</td>
<td>each valuation (shown above)</td>
<td>each valuation (shown above)</td>
</tr>
<tr>
<td>First random effects term?</td>
<td>subject with high value on B</td>
<td>subject with high value on B</td>
<td>subject with high value on B</td>
<td>subject with high value on B</td>
</tr>
<tr>
<td>Second random effects term?</td>
<td>subject with low value on B</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>observations</td>
<td>600</td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
</tr>
<tr>
<td>log likelihood</td>
<td>-364.124</td>
<td>-663.093</td>
<td>-663.925</td>
<td>-663.079</td>
</tr>
</tbody>
</table>

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

a. The results do not change significantly when period specific fixed effects are used instead of both valuation fixed effects and the period within valuation variable.
generate a payoff of 131 in case that the other subject bids below 129. Indeed, a subject who best responds to the empirical distribution, will bid below 30. This argument, which can sustain only deviations to the “left” of equilibrium actions, is consistent with a Quantal Response Equilibrium (QRE) model (McKelvey and Palfrey, 1995). Brown and Velez (2013) theoretically and experimentally examine the policy implications of QRE behavior in a winner's-bid auction and alternative simultaneous move mechanisms. We return to this topic in detail in Section 5.2.

5 Extensions to Other Economic Environments and Concluding Remarks

5.1 Sequential mechanisms

Our experimental results support the superiority of divide-and-choose over the winner's-bid auction both in terms of the percentage of equal-income competitive (EIC) allocations and the percentage of efficient allocations realized. The sequentiality of play in divide-and-choose not only avoids coordination problems, but also bypasses bounded rationality issues associated with simultaneous play. However, for divide-and-choose, there is still a positive probability that an inefficient non-EIC allocation results: choosers select an inefficient allocation in order to retaliate for a proposal that is biased towards the proposer (Section 4.2). This suggests that a more symmetric sequential mechanism may perform better than divide-and-choose in an experimental environment. Motivated by this result, Nicolò and Velez (2013) generalize divide-and-choose to three round sequential mechanisms that implement in subgame perfect equilib-
ria an interior EIC allocation. It is an open question whether Nicolò and Velez’s mechanisms deliver a higher percentage of efficient and EIC allocations in an experimental environment.

An additional challenge that is left is the design of a sequential mechanism that implements EIC allocations in subgame perfect equilibria for more than two agents. Even though there are several proposals for the generalization of divide-and-choose for more than two agents, it is not clear whether the equilibria of these mechanisms are EIC (Steinhaus, 1949; Brams and Taylor, 1996; Nicolò and Velez, 2013).

5.2 Alternative symmetric mechanisms

There are infinitely many simultaneous move EIC mechanisms (Tadenuma and Thomson, 1995a; Beviá, 2010; Velez, 2011a; Fujinaka and Wakayama, 2012). In the two-agent, two-object case some of these mechanisms may be framed as auctions as follows.\(^{25}\) Let \(\alpha \in [0, 1]\). Consider the auction mechanism parameterized by \(\alpha\) in which agents \([1, 2]\) report possibly negative bids for object \(B\) and payment is calculated as the \(\alpha\) convex combination of the bids. That is, given bids \([b_1, b_2]\), an agent with the highest bid receives object \(B\) and pays \(\alpha \max(b_1, b_2) + (1-\alpha) \min(b_1, b_2)\). The other player receives this payment and object \(A\).

Among the family of \(\alpha\)-auctions, winner’s-bid auction corresponds to \(\alpha = 1\). An argument similar to that of Lemma 1 shows that all \(\alpha\)-auctions are strategically equivalent. That is, for each \(\alpha\), the set of Nash equilibrium outcomes of the \(\alpha\)-auction mechanism is the same as that for winner’s-bid auction. Thus, the Nash equilibrium prediction does not illuminate us in a choice among the infinitely many auctions available.

Our experimental results suggest that one cannot expect this strategic equivalence to hold in an experimental setting, however. On the one hand, since all \(\alpha\)-auctions have the same equilibrium outcome correspondence, one may expect that coordination issues are similar across auctions. On the other hand, one of our main findings is that bounded rationality is an important source for the deviations from Nash equilibria in the winner’s-bid auction, due to the simultaneity of players actions. Following this lead, Brown and Velez (2013) theoretically and experimentally study the selection of a maximal envy-free \(\alpha\)-auction based on a Quantal Response Equilibrium model.

5.3 Allocating ball possession in NFL overtime

From 1974 to 2012 the overtime rule for regular season games in the National Football League (NFL) stated that if a game ended up in a tie after regulation playing time, a sudden death 15 minute overtime was to be played to determine its winner. Ball possession, i.e., the team that starts in the offense, was determined by a coin toss. A game ended when a team scored. Often, a

\(^{25}\)These auctions were introduced by Che and Hendershott (2008) for the allocation of ball possession in NFL overtime.
team could win the coin toss, take possession, drive and kick a field goal and end the game. The
game would end before the other team had a chance to score.

The system was criticized by fans, sports commentators, and NFL executives (Bialink, 2003; Hack, 2003). The main issue originated from the perception that the team winning the coin toss, had an advantage to win the game: “it’s… long-accepted NFL wisdom that if you get the ball in OT, you’re probably going to win” (Petchesky, 2011). Fans preferred the winner of the game to be determined by the ability of the teams and not by a coin toss. NFL executives realized that an ex-post unbalanced game is of no interest for audiences. As a reaction, new overtime rules for postseason games were established in 2011 (NFL Media, 2011), and then adopted in 2012 for regular season games (NFL Media, 2012). The main change was that “[t]eams now will have the opportunity to possess the ball at least once in the extra period unless the team that receives the overtime kickoff scores a touchdown on its first possession [...] regular season games can end in a tie if the score remains locked after 15 minutes, while a playoff game cannot.” (NFL Media, 2012).

It is still early to determine whether the new rules will solve the problems that induced the change. It is evident that they lowered the importance of the coin toss outcome in the first overtime period, partially recovering symmetry in the tie-breaker. However, by making it more difficult to end the game, the NFL has extended the length of overtime periods, and likely increased the percentage of games that will end in ties (all regular season games that last over 15 minutes become ties). “Tie games, after all, aren’t much fun for the fans or the players, who finish just as unsatisfied as anyone else” (Associated Press, 2012). As a consequence, it may be necessary for the NFL to reconsider tie-breaking rules, and in particular the possibility of symmetrizing the starting position in overtime.

The allocation of ball possession in a football game could be seen as a problem pertaining to the allocation of indivisible goods, i.e., getting ball possession or not, with perfectly divisible compensation. This interpretation was advanced by Che and Hendershott (2008) who consider compensation by means of the yard, from a team’s end zone, at which play starts. More precisely, consider teams $N \equiv \{1, 2\}$ whose probability of winning the game in overtime depends on who initially gets ball possession and the yard line at which play starts. We denote getting ball possession by $B$ and not getting it by $A$. The football field is 100 yards long. In order to make a more transparent comparison with our model, we set the middle of the field as yard line 0. Thus yard lines range from $−50$ to $50$. Starting play at yard $y$, the team who has ball possession is $50 − y$ yards from the other team’s end zone. For each $i \in \{1, 2\}$ and each $y \in [−50, 50]$, let $f_i(y) \in [0, 1]$ be the probability that team $i$ wins the game when receiving ball possession at yard $y$. Analogously, $g_j(y)$ is the probability that team $i$ wins the game when team $j \neq i$ receives ball possession at yard $y$. We assume that both $f_1$ and $f_2$ are strictly increasing functions. Clearly, for $i \neq j$ and $y \in [−50, 50]$, $f_i(y) + g_j(y) = 1$.

An allocation of ball possession is a pair $(i_B, t_B) \in N \times [−50, 50]$ where $i_B \in N$ is the team who gets ball possession and $t_B$ is the yard line at which play starts. Each team wishes to maximize its
probability of winning. Thus, team $i$’s utility of allocation $(i_B, t_B)$ is given by: $u_i(i_B, t_B) \equiv f_i(t_B)$ if $i_B = i$ and $u_i(i_B, t_B) \equiv g_i(t_B)$ otherwise. For general probability functions, these preferences are not quasi-linear. However, they are quasi-linear if probability functions are affine linear functions and anonymous, i.e., there are $\{\alpha, \beta\} \subseteq \mathbb{R}$ such that $\alpha > 0$ and for each $y \in [-50, 50]$, $f_1(y) = f_2(y) = \beta + \alpha t_B$. Under these assumptions, team $i$’s preferences are represented by utility function $u_i(i_B, t_B) \equiv \frac{\beta}{\alpha} + t_B$ if $i_B = i$ and $u_i(i_B, t_B) \equiv \frac{1 - \beta}{\alpha} - t_B$ otherwise. These are exactly the environment and the preferences in our model when agents have identical preferences.\footnote{In our experiments with equal valuations for both agents (valuation 4), transfers range from $-200$ to $320$. This is an inconsequential change for the qualitative interpretation of our results. This is equivalent to teams being able to submit bids on a field divided in $520$ equidistant yard lines.} Thus, results for valuation 4 in our experiments are relevant for the comparison of EIC mechanisms for the allocation of ball possession.

An EIC allocation of ball possession is an attractive solution. There is only one such an allocation under our assumption that the probability functions $f_1$ and $f_2$ are strictly increasing. At this allocation both teams are indifferent to start either on offense or defense, so there is no ball-possession advantage. Thus, not only is the outcome of the game solely determined by the performance of the teams on the field and not by a coin toss, but also the excitement of the game in overtime is preserved.\footnote{This is the so called “non-arbitrariness” introduced by Che and Hendershott (2008).}

The mechanisms that we study, in theory achieve such an allocation in equilibrium under perfect information. They take the following form in this environment. The winner’s-bid auction requires each team to name a yard at which they would be willing to get possession of the ball. A team that reports the largest distance from its opponent’s end zone gets possession at its named position. Our second mechanism, divide-and-choose, randomly selects a team to propose a starting yard line. The other team chooses to take ball possession at that starting yard line or to give it to the other team. These mechanisms are analyzed in both complete and incomplete information environments by Che and Hendershott (2008), who suggest that winner’s-bid auction is superior to divide-and-choose. Their argument is essentially that in an incomplete information environment, divide-and-choose favors the chooser, for the proposer may reveal some information in his proposal.

Our results confirm that even under complete information divide-and-choose gives a slight advantage to the chooser. More importantly, in valuation 4, our relevant profile for NFL auction games, the chooser has an advantage over the proposer, for any mistake by the proposer leads to a higher payoff to the chooser compared with the SPE prediction (Figure 7). However, this chooser advantage is minimal compared with the loss of EIC allocations in the auction induced by the bounded rationality of play in the simultaneous game (Figure 6). Thus, our results indicate that divide-and-choose, or a sequential generalization of it, as in Nicolò and Velez (2013), may be more appropriate in NFL overtime. Our results caution us about the sort of possible bid-
ding games that can ensue when teams expect the other team to be boundedly rational, which can result in largely biased allocation of ball possession.

5.4 Concluding remarks

This paper examines an economic environment where game theory predicts the superiority of a simultaneous auction mechanism, the winner's-bid auction, because it can achieve efficient and EIC outcomes, without a systematic bias for either player. A sequential divide-and-choose is predicted to achieve the same outcomes, but with a significant proposer bias. Contrary to this prediction, divide-and-choose achieves more efficient and EIC outcomes than winner's-bid auction without exhibiting such bias. Issues with subject coordination and the inability of subjects to play Nash bids likely are responsible for the lower performance of the winner's-bid auction compared with divide-and-choose.

One critique of these results is that they are generated by inexperienced subjects unfamiliar with these types of mechanisms, and experienced subjects would fit more closely with the predictions of theory. While section 4.3 provides some support for this critique—efficient and EIC outcomes are more frequent in later periods of each valuation—there is no evidence to suggest subjects with the low value on item B are any more likely to play Nash bids with experience. Realizing this property, a best-responding subject with the high value on B should also reduce her bid over time, making it unlikely for the mechanism to equilibrate. Additionally, as section 4.2 shows, there is little evidence to support the existence of a proposer bias in the divide-and-choose mechanism. This result does not change when we restrict the data to only EIC proposals, proposals which we would expect fully-experienced subjects to make exclusively.

That being said, symmetry or the lack of a bias in a mechanism toward any agent is still a very desirable property in mechanism design. A promising avenue for future research will be to see if any type of symmetric mechanism can achieve similar levels of efficiency and competitiveness as divide-and-choose does in these experiments. We hope our findings may aide future research in this direction.

References


URL www.nflmedia.com/wps/wcm/myconnect/nflmedia/NFL+Media+Site/News/Postseason+Overtime


Petchesky, B., 2011. Is winning the overtime coin toss a blessing or a curse? Slate, December 5, 2011.


A Supplemental Proofs

Proof of Lemma 1. We prove that for each \( u \), \( \theta_{PNE}(\mathbb{N} \times \mathbb{N}, \varphi^{WBA}, u) = EIC(u) \). Let \( z \equiv (i_B, t_B) \in EIC(u) \). Let \( m \equiv (m_1, m_2) \in \mathbb{N} \times \mathbb{N} \) such that \( m_1 = m_2 = t_B \). We claim that \( m \) is a Nash equilibrium of \( (\mathbb{N} \times \mathbb{N}, \varphi^{WBA}, u) \). Suppose without loss of generality that \( v_1(B) \leq v_2(B) \) and \( i_B = 2 \). It is easy to see that \( m_1 \) weakly dominates each \( m'_1 < m_1 \) for subject 1, and \( m_2 \) dominates each \( m'_2 > m_2 \) for subject 2. Now, for each \( m'_1 > m_1 \), \( \varphi^{WBA}(m'_1, m_2) = (1, m_1) \). Since \( m'_1 > t_B \), then \( u_1(z_1) \geq u_1(z_2) > u_1(\varphi^{WBA}(m'_1, m_2)) \). Finally, for each \( m'_2 < m_2 \), \( \varphi^{WBA}(m_1, m'_2) = (1, m_1) \). Thus, \( \varphi^{WBA}(m_1, m'_2) = z_1 \) and \( u_2(z_2) \geq u_2(\varphi^{WBA}(m_1, m'_2)) \).

Suppose now that \( z \equiv (i_B, t_B) \) is a pure strategy equilibrium outcome of \( (\mathbb{N} \times \mathbb{N}, \varphi^{WBA}, u) \). We prove that \( z \in EIC(u) \). Let \( m \equiv (m_1, m_2) \) be the report profile sustaining \( z \) as a Nash equilibrium. Suppose that \( v_1(B) < v_2(B) \). First we prove that \( i_B = 2 \). Suppose by contradiction that \( i_B = 1 \). If \( t_B > \frac{v(B)+100}{2} \), then for each \( m'_1 < m_1 \), \( u_1(z_2) > u_1(z_1) \) and \( \varphi^{WBA}(m'_1, m_2) = z_2 \). Thus, \( t_B \leq \frac{v(B)+100}{2} \). Since for \( m'_2 = t_B + 1 \), \( \varphi^{WBA}(m_1, m'_2) = (2, t_B + 1) \), then \( t_B \geq \frac{v(B)+100}{2} \). This is a contradiction. Thus, \( i_B = 2 \). A similar argument shows that \( \frac{v(B)+100}{2} \leq t_B \leq \frac{v(B)+100}{2} \). \( \square \)
## B Supplemental Tables and Figures

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>EIC Outcome</th>
<th>Nash Bid</th>
<th>Nash Bid</th>
<th>Nash Bid</th>
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<tr>
<td>period within valuation</td>
<td>1.089*** (0.036)</td>
<td>1.052** (0.025)</td>
<td>1.019 (0.027)</td>
<td>1.048 (0.035)</td>
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<td>1.008 (0.049)</td>
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<td>0.268*** (0.09)</td>
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<td>0.523** (0.139)</td>
<td>0.429*** (0.132)</td>
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<tr>
<td>valuation 2 (120-160)</td>
<td>0.643 (0.133)</td>
<td>0.556** (0.183)</td>
<td>0.691 (0.173)</td>
<td>0.569* (0.173)</td>
</tr>
<tr>
<td>valuation 3 (80-120)</td>
<td>1.938** (1.151)</td>
<td>3.917*** (1.395)</td>
<td>4.858*** (1.294)</td>
<td>4.005*** (1.294)</td>
</tr>
<tr>
<td>valuation 4 (160-160)</td>
<td>0.332*** (0.109)</td>
<td>0.127*** (0.038)</td>
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<tr>
<td>valuation 5 (0-40)</td>
<td>0.427*** (0.138)</td>
<td>0.927 (0.254)</td>
<td>1.148 (0.303)</td>
<td>1.475 (0.287)</td>
</tr>
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<td>has high value on item B</td>
<td>1.541*** (0.215)</td>
<td>0.388 (0.441)</td>
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<td>logit</td>
<td>logit</td>
<td>logit</td>
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<td>Fixed effects?</td>
<td>each valuation (shown above)</td>
<td>each valuation (shown above)</td>
<td>each valuation (shown above)</td>
<td>each valuation (shown above)</td>
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<tr>
<td>First random effects term?</td>
<td>subject with high value on B</td>
<td>subject with high value on B</td>
<td>subject with high value on B</td>
<td>subject with high value on B</td>
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<tr>
<td>Second random effects term?</td>
<td>subject with low value on B</td>
<td>no</td>
<td>no</td>
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<td>observations</td>
<td>600</td>
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<td>log likelihood</td>
<td>-364.124</td>
<td>-663.093</td>
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<td>-663.079</td>
</tr>
</tbody>
</table>

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

a. The results do not change significantly when period specific fixed effects are used instead of both valuation fixed effects and the period within valuation variable.

Table A.1: Regressions of equilibrium outcomes and bids for winner's-bid auction on valuation, period within valuation, and subject's value on item B, controlling for subject random effects. Odd rations shown.
### Table A.2: Regressions of subject earnings on mechanism, valuation, first mover, and item B value, controlling for period fixed effects and subject random effects, restricted to only EIC proposals. Winner’s-bid auction data is not included in any regression.

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<tr>
<th>Condition: overall proposed offers</th>
<th>accepted offers</th>
<th>Dependent variable:</th>
<th>share of profit (percent) (in points)</th>
<th>share of profit (percent) (in points)</th>
<th>share of profit (percent) (in points)</th>
<th>share of profit (percent) (in points)</th>
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<td>45.920***</td>
<td>16.300***</td>
<td>40.044***</td>
<td>19.196***</td>
<td>40.759***</td>
<td>19.787***</td>
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<tr>
<td>in divide-and-choose</td>
<td>(2.445)</td>
<td>(0.824)</td>
<td>(1.393)</td>
<td>(0.493)</td>
<td>(1.438)</td>
<td>(0.718)</td>
</tr>
<tr>
<td>Diviser / first mover in divide-and-choose&lt;sup&gt;c&lt;/sup&gt;</td>
<td>-0.434</td>
<td>-0.12</td>
<td>5.008***</td>
<td>3.162***</td>
<td>2.909***</td>
<td>2.019***</td>
</tr>
<tr>
<td>in ultimatum bargaining</td>
<td>(0.89)</td>
<td>(0.308)</td>
<td>(0.547)</td>
<td>(0.27)</td>
<td>(0.525)</td>
<td>(0.261)</td>
</tr>
<tr>
<td>High value on B in ultimatum bargaining</td>
<td>4.621*</td>
<td>2.851***</td>
<td>7.645***</td>
<td>3.856***</td>
<td>6.927***</td>
<td>3.226***</td>
</tr>
<tr>
<td>High value on B in divide-and-choose</td>
<td>(2.521)</td>
<td>(0.859)</td>
<td>(1.489)</td>
<td>(0.737)</td>
<td>(1.464)</td>
<td>(0.729)</td>
</tr>
<tr>
<td>Type of regression?</td>
<td>linear</td>
<td>linear</td>
<td>linear</td>
<td>linear</td>
<td>linear</td>
<td>linear</td>
</tr>
<tr>
<td>Fixed effects?</td>
<td>each period</td>
<td>each period</td>
<td>each period</td>
<td>each period</td>
<td>each period</td>
<td>each period</td>
</tr>
<tr>
<td>First random effects term?</td>
<td>subject</td>
<td>subject</td>
<td>subject who is first mover</td>
<td>subject who is first mover</td>
<td>subject</td>
<td>subject</td>
</tr>
<tr>
<td>Second random effects term?</td>
<td>player paired with subject</td>
<td>player paired with subject</td>
<td>n/a</td>
<td>n/a</td>
<td>player paired with subject</td>
<td>player paired with subject</td>
</tr>
<tr>
<td>Observations</td>
<td>1583</td>
<td>1583</td>
<td>1583</td>
<td>1583</td>
<td>1378</td>
<td>1378</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-6688.473</td>
<td>-4987.897</td>
<td>-5883.587</td>
<td>-4768.459</td>
<td>-4998.727</td>
<td>-4035.462</td>
</tr>
</tbody>
</table>

<sup>a</sup> Significant at the 10% level.
<sup>b</sup> Significant at the 5% level.
<sup>c</sup> Significant at the 1% level.

a. In the divide-and-choose mechanism, a rejection is the divider choosing the option that gives him less points, or in a case of equal points, the inefficient outcome. Thus a divider’s proposal is what would be implemented given the chooser does not reject.
b. A rejection in ultimatum bargaining leads to earnings of 0 for each player. We consider this to be a 50% share for each player.
c. This is the relative gain over a low valuation/second mover.
d. This is the predicted value for a low valuation second mover, relative to period fixed effects.