Monetary Policy Rules and the Equity Premium

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May 20, 2014

Abstract

We study monetary policy’s effects on the equity premium using a segmented stock market model. We find that monetary policy is important in affecting the stock and bond prices, and thus, the equity premium. Specifically, the optimal monetary policy is risk sharing and countercyclical with respect to the dividend shock. Under that policy the risky asset is not that risky, and the return on equity is low. On the other hand, inflation targeting policy is risky, contributing to high equity return. At the same time, the inflation targeting policy produces lower return on the nominal bond, than what the optimal monetary policy does. This is because the inflation targeting policy insures against inflation, although the optimal policy does not. These two effects imply high equity premium under the inflation targeting policy compared to the optimal. Our calibration exercise finds equity premium of almost 7% under the optimal policy and 1.5% under the 2% inflation targeting policy, using a coefficient of relative risk aversion equal to 2. Overall, the model suggests that optimal monetary policy corrects the segmentation friction, and minimizes the equity premium.

JEL classification: E44; E52; G12.
Keywords: equity premium, monetary policy, segmented financial markets.

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1 Introduction

Monetary policy’s interference in the financial markets is almost by definition profound, given that monetary policy transmits in the economy through the financial system. However, monetary policy’s response to financial markets advances has been an issue over which economists have not reached a consensus; there are many conflicting ideas about if and how monetary policy should react to financial market changes (Cecchetti et al., 2001; Gilchrist and Leahy, 2002). One reason for the disagreement is the lack of consensus over the way monetary policy affects financial variables. We present a model that studies how monetary policy influences the equity premium, a variable of great interest for financial markets. We find that the equity premium is highly affected by monetary policy, an aspect that previous literature has ignored.

Specifically, we use a segmented financial markets monetary model to study the equity premium produced by the optimal monetary policy rule, and compare it with that produced by other common monetary policy rules. In our model, the optimal monetary policy has risk sharing considerations, redistributing the financial risk that the traders are subject to, among all agents in the economy. Through its distributional effects, the optimal monetary policy discourages the segmentation effect. As a result, the equity premium produced under the optimal monetary policy resembles that produced by the representative agent model, and thus, it is minimal. On the contrary, the equity premium produced by the constant money supply and inflation targeting rules are usually higher. In a quantitative exercise we find that the two per cent inflation targeting policy produces almost four and a half times higher equity premium than what the optimal policy does.

The equity premium, i.e., the difference between the return of the risky, over the risk-free assets, has been the subject of study in financial economics for long time, especially after Mehra and Prescott (1985) indicated the so called equity premium puzzle. The puzzle refers to the failure of the classical growth model with standard preferences, to account for the high equity premium observed in the data. Economists have gone a long way altering the basic assumptions of the model in order to match the observed data. Various approaches have been explored for this purpose, including changing the preference assumptions (e.g., Epstein and Zin, 1989; Constantinides, 1990), considering market incompleteness (e.g.,
Constantinides and Duffie, 1996; Brav et al., 2002), or introducing trading costs (e.g., Constantinides et al., 2002). An aspect that has been previously ignored and we study in this paper, is the potential influence of policy. We focus on monetary policy given the direct effects that it has in the financial markets.

We account for monetary policy’s effects using a segmented financial markets model where monetary policy has real effects through distributional considerations (Grossman and Weiss, 1983; Rotemberg, 1984; Lucas, 1990; Fuerst, 1992; Alvarez et al., 2001; Williamson, 2005; Williamson, 2006, Zervou, 2013). In this model, a monetary policy expansion positively affects financial markets participants, but hurts non-participants. On the contrary, a monetary policy tightening negatively affects financial market participants but benefits non-participants. In addition, this model offers a policy prescription through the study of welfare maximizing, optimal monetary policy. We then compare the equity premium produced under the optimal policy, versus other, usually cited monetary policy rules such as constant money supply, inflation targeting, and a rule that suggests that monetary policy does not intervene in agent’s endowment allocations.

We find that monetary policy plays a significant role in affecting the equity premium. Specifically, the optimal monetary policy tends to produce minimal equity premium. This is because the optimal monetary policy in our model shares the financial income risk only the stock traders hold, among all agents in the economy. Such a policy makes all agents consume the same amount, and the consumption of the stock holders is not more correlated with dividends, relative to the non-stock holders. Then the equity premium is small, similar to the one calculated for the representative agent model, as for example found by Mehra and Prescott (1985). Analyzing the effects on stocks and bonds separately, we find that the optimal policy implies low equity returns, given that it shares the dividend risk; it implies high bond returns, as it does not stabilize inflation. On the contrary, the inflation targeting policy implies high equity returns, as it does not share risk, but implies low bond returns, as it stabilizes inflation. The result is high equity premium under the inflation targeting policy and low equity premium under the optimal policy. In our calibration exercise we find 1.55% premium under the optimal policy, although 6.96% under the inflation targeting.

Financial market segmentation has been documented (Mankiw and Zeldes, 1991; Guiso
et al., 2002; Vissing-Jørgensen, 2002) and used before for the study of the equity premium, for differentiating preference parameters, i.e., risk aversion and elasticity of intertemporal substitution, between the financial market participants and non-participants (Vissing-Jørgensen, 2002; Brav et al., 2002). Our model however, uses this feature in order to study monetary policy’s role in sustaining the equity premium, and how both the return on the risky and the risk-free rates change depending on monetary policy’s considerations.

Lastly, previous work has emphasized using empirical tools, the potential influence that the equity premium has in monetary policy decisions (Cecchetti et al., 2001). Our model focuses on the exactly reverse effects, exploring how asset prices and the equity premium are affected by various monetary policy rules.

The rest of the paper is organized as follows. Section 2 introduces the model economy and Section 3 studies the equilibrium. Section 4 derives the equity premium as a function of monetary policy consideration. Section 5 introduces a quantitative exercise in order to study quantitatively the equity premium produced by the optimal and the two per cent inflation targeting monetary policy rules. Section 7 concludes.

2 The model economy

We consider an infinite horizon economy in discrete time, populated by a continuum of households that are categorized into two types. \( \lambda \in (0, 1) \) fraction consists by agents who participate in the bond and stock markets and are called traders (T), and \( 1 - \lambda \) fraction consists by agents who do not participate into financial markets and are called non-traders (N). All households have identical preferences and maximize:

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c^t_i),
\]

where \( \beta \in (0, 1) \) is the discount factor and \( c^t_i \geq 0 \) is consumption at time \( t \) by consumer of type \( i \in \{T, N\} \). We assume that \( u'(.) > 0 \) and \( u''(.) < 0 \).

Agents of type \( i = T \), i.e., traders, begin period \( t \) with money holding \( m^T_t \), one period bonds \( b_t \), and stock \( z_t \). In addition, they receive the monetary transfer \( \tau_t \) from the monetary
authority. Also, at this point, traders realize the real total dividend \( \varepsilon_t \) which is random and is described below:

\[
\varepsilon_t = \bar{\varepsilon} + \eta_t,
\]

where \( \bar{\varepsilon} > 0 \) denotes the mean dividend and \( \eta_t \in [-\bar{\varepsilon}, \infty) \) is an iid shock with mean zero and variance \( \sigma^2_\varepsilon \).

In each time period \( t = 0, \ldots, \infty \), financial markets open before the goods market does. Without loss of generality, we assume that households cannot consume their own endowments and dividends but need cash in order to finance their consumption in the goods market; thus households face cash-in-advance constraints in the goods market. While the financial markets are open, using their money holdings \( m^T_t \), the money from selling \( z_t \) stocks, the returns from holding \( b_t \) bonds and the monetary transfer \( \tau_t \), traders adjust their money holdings, bonds and stock.

When entering the goods market, traders buy goods using the money holdings left after the financial markets close; non-traders use the money carried from the previous period. The cash-in-advance constraints for traders and non-traders are:

\[
m^T_t + q_t z_t + b_t + \tau_t \geq p_t c^T_t + q_t z_{t+1} + s_t b_{t+1}, \tag{2}
\]

\[
m^N_t \geq p_t c^N_t. \tag{3}
\]

Here, \( p_t, q_t \) and \( s_t \) respectively, denote the goods, stock and bond price at time \( t \).

In addition, traders receive real endowment \( y^T \) and real dividend \( \varepsilon_t \), although non-traders receive real endowment \( y^N \). Since agents cannot consume their non-storable endowment and dividends, they sell them in the goods market; they hold the cash received for staring the next period. The budget constraints for traders and non-traders are as follows:

\[
m^T_t + q_t z_t + b_t + \tau_t + p_t y^T + p_t y^T T \geq m^T_{t+1} + p_t c^T_t + q_t z_{t+1} + s_t b_{t+1}, \tag{4}
\]

\[
m^N_t + p_t y^N \geq m^N_{t+1} + p_t c^N_t. \tag{5}
\]

\footnote{Monetary transfers are directed only to the traders, as it is usually assumed in the segmented markets literature (see for example mine and Alvarez et al., 2001). This assumption captures the fact that open market operations affect directly financial markets and their participants.}
The maximization problem of each household is subject to constraints (2) and (4) for the traders, and (3) and (5) for the non-traders. For positive bond returns and given that there is no uncertainty during period \( t \), the cash-in-advance constraints for traders bind. The budget constraints bind as usually. Then:

\[ p_t \varepsilon_{t+1} + p_t y^T = m^T_{t+1}, \]  

for traders, and

\[ p_t y^N = m^N_{t+1}, \]

for non-traders. Solving the traders’ maximization problem, we get the intertemporal optimal conditions:

\[ \beta \mathbb{E}_t \frac{u'(c^T_{t+1})}{p_{t+1}} = \frac{u'(c^T_t)}{p_t} s_t, \]

\[ \beta \mathbb{E}_t \frac{u'(c^T_{t+1})}{p_{t+1}} (q_{t+1} + p_t \varepsilon_t) = \frac{u'(c^T_t)}{p_t} q_t. \]

The above equations describe the pricing of nominal bonds and stock, respectively. Solving for the real bond and stock price, \( \hat{s}_t \) and \( \hat{q}_t \) respectively, these equations imply:

\[ \hat{s}_t = \beta \mathbb{E}_t \frac{u'(c^T_{t+1})}{u'(c^T_t)} \frac{1}{p_{t+1}}, \]

\[ \hat{q}_t = \beta \mathbb{E}_t \frac{u'(c^T_{t+1})}{u'(c^T_t)} (\hat{q}_{t+1} + \frac{p_t \varepsilon_t}{p_{t+1}}). \]

### 3 Equilibrium

The total output in period \( t \) equals \( y_t \equiv \varepsilon_t + \lambda y^T + (1 - \lambda) y^N \). Then the mean income is:

\[ \bar{y} = \bar{\varepsilon} + \lambda y^T + (1 - \lambda) y^N. \]

The economy’s resource constraint is as follows:

\[ \varepsilon_t + \lambda y^T + (1 - \lambda) y^N = \lambda c^T_t + (1 - \lambda) c^N_t, \]
which in combination with equation (12), implies the goods market clearing condition:

$$\bar{y} + \varepsilon_t - \bar{\varepsilon} = \lambda c^T_t + (1 - \lambda)c^N_t.$$  \hspace{1cm} (13)

Since only traders can participate in the stock and bond market in each period $t$, the clearing condition for stock market is:

$$\lambda z_{t+1} = 1 \Rightarrow z_{t+1} = \frac{1}{\lambda}. \hspace{1cm} (14)$$

The bond market clearing condition is:

$$\lambda b_t = 0. \hspace{1cm} (15)$$

Each trader receives money transfer $\tau_t$ from the monetary authority. The total money supply $\bar{M}_t$ in period $t$ is:

$$\bar{M}_t = \lambda \tau_t + \bar{M}_{t-1} \quad \text{or equivalently}, \quad \bar{M}_t = \bar{M}_{t-1}(1 + \mu_t). \hspace{1cm} (16)$$

where, $\mu_t \in [-1, \infty)$ denotes the money growth rate from time $t - 1$ to time $t$. Negative $\mu_t$ implies that the monetary authority tightens and receives a lump-tax from the traders.

Since the total money demand is $\lambda m^T_{t+1} + (1 - \lambda)m^N_{t+1}$, the money market clearing condition is:

$$\lambda m^T_{t+1} + (1 - \lambda)m^N_{t+1} = \bar{M}_t. \hspace{1cm} (17)$$

Given the participating fraction $\lambda$, endowments $y^T$ and $y^N$, the dividend process $\{\varepsilon_t\}$, the monetary transfer $\{\tau_t\}$ and the initial conditions $\{M_0, b_0, z_0, m^N_0, m^T_0\}$, an equilibrium is a collection of $\{c^T_t, c^N_t, z_{t+1}, b_{t+1}m^T_{t+1}, m^N_{t+1}, p_t, b_t, q_t, s_t\}$ such that: i. traders optimize w.r.t $\{c^T_t, m^T_{t+1}, b_{t+1}, z_{t+1}\}$ in order to maximize their utility, subject to the budget constraint and cash-in-advance constraint, taking the price $\{p_t, s_t, q_t\}$ and the policy processes as given; non-trader optimize w.r.t $\{c^N_t, m^N_{t+1}\}$ in order to maximize their utility, subject to the budget constraint and cash-in-advance constraint, taking the price $\{p_t\}$ and the policy processes as given; ii. Goods market, bond
market, stock market and money market clear.

Using the equilibrium conditions (13), (14), (15), (17), the money supply equation (16) and the cash-in-advance constraints (2), (3) holding with equality, we derive the goods price, which is given by the quantity equation:

\[ p_t = \frac{\bar{M}_t}{\bar{y} + \bar{\epsilon} - \bar{\epsilon}}. \]  

(18)

In order for total output to be independent from the financial market participation rate \( \lambda \), we assume that non-traders’ fixed endowment \( y^N \) equals to the traders’ mean income; that is \( y^T + \frac{\bar{\epsilon}}{\lambda} = y^N \). Combining the non-traders binding cash-in-advance constraint (3) with equation (7) and the goods price (18), we find that the non-traders consumption is given as follows:

\[ c^N_t = p_{t-1} \frac{\bar{y} + \bar{\epsilon} - \bar{\epsilon}}{\bar{y} + \bar{\epsilon}_{t-1} - \bar{\epsilon}} \frac{\bar{y}}{1 + \mu_t}. \]  

(19)

Together with goods market clearing condition, given by (13), the traders’ consumption can be written as follows:

\[ c^T_t = \frac{\bar{y} + \bar{\epsilon} - \bar{\epsilon}}{1 + \mu_t} \frac{(\bar{\epsilon}_{t-1} - \bar{\epsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)}{(\bar{y} + \bar{\epsilon}_{t-1} - \bar{\epsilon})(1 + \mu_t)}. \]  

(20)

From the above equations (19) and (20) we see the distributional effects that monetary policy has in our segmented financial markets model. Specifically, monetary policy affects directly only the financial market participants; however, it affects indirectly, through price adjustments, both financial market participants and non-participants. During an expansion, monetary policy creates an inflation tax for all households, but distributes monetary transfers only to the traders. As an effect, traders’ consumption increases and non-traders’ consumption decreases. On the contrary, a monetary policy tightening, makes consumption cheaper for both types of agents decreasing the price level. However, only the traders get taxed and thus their consumption decreases, although non-traders’ consumption increases.
4 The equity premium

Defining the gross nominal interest rate as \( r_{t+1} \equiv \frac{1}{s_t} \), the gross nominal return of the risky asset as \( R_{t+1} \equiv \frac{q_{t+1} + pt_{t+1}}{qt} \), the gross real interest rate as \( \hat{r}_{t+1} \equiv \frac{1}{pt_{t+1}} \), and the gross real return of the risky asset as \( \hat{R}_{t+1} \equiv \frac{\hat{q}_{t+1} + pt_{t+1}}{qt} \), and substituting in the bond and stock pricing equations (8), (9), (10) and (11), respectively, we have:

\[
\begin{align*}
\beta E_t \frac{u'(c^T_{t+j})}{pt_{t+1}} r_{t+1} & = \frac{u'(c^T_t)}{pt}, \\
\beta E_t \frac{u'(c^T_{t+1})}{pt_{t+1}} R_{t+1} & = \frac{u'(c^T_t)}{qt}, \\
E_t u'(c^T_{t+1}) \hat{r}_{t+1} & = u'(c^T_t), \\
\beta E_t u'(c^T_{t+1}) \hat{R}_{t+1} & = u'(c^T_t). 
\end{align*}
\]

(21)

The expression for the stock price, given by equation (11), can be rewritten replacing the value of \( \hat{q}_{t+1} \) recursively, as follows:

\[
u'(c^T_t) \hat{q}_t = \sum_{j=1}^{\infty} E_t \beta^j \frac{u'(c^T_{t+j})}{pt_{t+j}} pt_{t+j-1} \varepsilon_{t+j-1}.
\]

(22)

The real expected equity premium is defined as \( E_t \Pi_{t+1} \equiv E_t(\hat{R}_{t+1} - \hat{r}_{t+1}) = E_t \left[ \frac{pt}{pt_{t+1}} (R_{t+1} - r_{t+1}) \right] = E_t \frac{pt}{pt_{t+1}} \Pi_{t+1} \), where \( \Pi_{t+1} \) is the nominal equity premium defined as \( \Pi_{t+1} \equiv R_{t+1} - r_{t+1} \). Here, the operator \( E_t(.) \) denotes the conditional expectation based on the realized information until time \( t \), when the shocks of the current period are already known.

By noting that for any random variable \( x, y \), it is true that \( E_t xy = E_t x E_t y + \text{Cov}_t(x, y) \), where \( \text{Cov}_t(.) \) is the conditional covariance, and applying this formula in equation (21) we have for the gross nominal stock return that \( \beta E_t \frac{u'(c^T_{t+1})}{pt_{t+1}} E_t R_{t+1} + \beta \text{Cov}_t \left( \frac{u'(c^T_{t+1})}{pt_{t+1}}, R_{t+1} \right) = \frac{u'(c^T_t)}{pt} \). Doing the same for the gross nominal bond return we have that the nominal equity premium is as follows:

\[
E_t \Pi_{t+1} = -\frac{\text{Cov}_t \left( \frac{u'(c^T_{t+1})}{pt_{t+1}} \frac{R_{t+1}}{pt_{t+1}} \right)}{E_t \frac{u'(c^T_{t+1})}{pt_{t+1}}},
\]

(23)
given that \( r_{t+1} \) is known at time \( t \). As usually (see Mehra and Prescott, 2003; Ljungqvist...
and Sargent 2004) the equity premium equation above reveal that the expected equity premium depends on the covariance of the asset returns with the marginal utility of consumption. Assets with returns that positively correlate with consumption (and thus negatively correlate with marginal utility) are assets that have high premium. These assets pay off when consumption is high and do not pay off when consumption is low. They are not very attractive as they do not smooth consumption, and thus traders ask for a high premium in order to hold them. On the other hand, assets with returns that negatively correlate with consumption can be used as a hedge, are very attractive and have lower equity premium. Normally, an increase in the nominal return of a risky asset leads through the income effect to decreasing marginal utility of consumption in the next period, which means that the sign of the covariance between the asset returns and the marginal utility of consumption is negative.

Similarly, we can compute the real equity premium as:

\[
E_t \hat{\Pi}_{t+1} = - \frac{\text{Cov}_t \left( u' \left( c_{t+1}^T \right), \hat{R}_{t+1} \right)}{E_t u' \left( c_{t+1}^T \right)} + \frac{\text{Cov}_t \left( u' \left( c_{t+1}^T \right), \hat{r}_{t+1} \right)}{E_t u' \left( c_{t+1}^T \right)}. \tag{24}
\]

Usually we assume that there is an indexed bond, so \( \text{Cov}_t \left( u' \left( c_{t+1}^T \right), \hat{r}_{t+1} \right) = 0 \). However, in our model there are no indexed bonds, but only one period nominal bonds. Our analysis is susceptible to inflation premium (for a discussion about the inflation premium see Labadie, 1989). We follow this approach in order to show that monetary policy affects the real equity premium beyond the inflation premium.

We are using two alternative representations of the real equity premium, which are useful for our later calculations. These are the following:

\[
E_t \hat{\Pi}_{t+1} = - \frac{\text{Cov}_t \left( u' \left( c_{t+1}^T \right), \hat{q}_{t+1} \right)}{\hat{q}_t E_t u' \left( c_{t+1}^T \right)} + \frac{\text{Cov}_t \left( u' \left( c_{t+1}^T \right), \frac{1}{p_{t+1}} \right)}{E_t u' \left( c_{t+1}^T \right)} \frac{E_t u' \left( c_{t+1}^T \right) \hat{q}_{t+1}}{\hat{q}_t E_t u' \left( c_{t+1}^T \right) \frac{1}{p_{t+1}}}, \tag{25}
\]

and

\[
E_t \hat{\Pi}_{t+1} = \frac{E_t \hat{q}_{t+1}}{\hat{q}_t} - \frac{E_t \frac{1}{p_{t+1}} E_t u' \left( c_{t+1}^T \right) \hat{q}_{t+1}}{\hat{q}_t E_t u' \left( c_{t+1}^T \right) \frac{1}{p_{t+1}}}, \tag{26}
\]

An increase in real stock price increases traders consumption because of the income effect,
causing \( \text{Cov}_t(u'(c_{t+1}^T), \hat{q}_{t+1}) \) to be negative. Accordingly, the first part of the right hand side of equation (25) is positive. Moreover, an increase in the goods price level, as a result of the substitution effect, decreases consumption, makes \( \text{Cov}_t(u'(c_{t+1}^T), \frac{1}{p_{t+1}}) \) negative, and causes the second part of the right hand side of equation (25) to be negative. Therefore, the value of the equity premium depends on the relative magnitude of the substitution and income effects. Monetary policy can affect the magnitude of these effects, as we will see in the next section.

4.1 Equity Premium under Different Policy Assumptions

In order to compute explicitly the equity premium, we use the logarithmic utility function, \( u(c_i^t) = \ln(c_i^t) \), for \( i = T, N \). From equation (22) we calculate the real stock price, \( \hat{q}_t \equiv \frac{q_t}{p_t} \), as follows:

\[
\hat{q}_t = \sum_{j=1}^\infty \mathbb{E}_t \beta^j \frac{c_T^T}{c_{t+j}^T} p_{t+j-1} \hat{c}_{t+j-1},
\]

or,

\[
\hat{q}_t = \mathbb{E}_t \sum_{j=1}^\infty \beta^j \frac{c_T^T}{c_{t+j}^T} \frac{y_t}{y_{t+j}} \frac{1}{1 + \pi_{t+j}} \hat{\varepsilon}_{t+j-1},
\]

where \( \tilde{x}_t = \frac{x_t}{y_t} \).

The real stock price depends on the stochastic discount factor and the payoff. In our model, with limited stock market participation, the stochastic discount factor changes because of two reasons: First, it changes because of changes in the fraction of total consumption consumed by traders, i.e., the segmentation effect (\( \tilde{x}_t \)). In addition, it changes because of changes in aggregate consumption, i.e., the typical representative agent effect (\( \frac{c_T^T}{c_{t+j}^T} \)). The payoff \( \frac{1}{1 + \pi_{t+j}} \hat{\varepsilon}_{t+j-1} \) depends on the stream of dividends and on inflation. Inflation affects the real payoff because the dividends are received and sold in the current period, but they are used to buy consumption goods a period after. Thus, an increase in inflation rate decreases the real value of the payoff.

Rewriting the expression for the nominal gross return of the risky asset, we have:

\[
R_{t+1} = \frac{\hat{q}_{t+1}(1 + \pi_{t+1}) + \varepsilon_t}{\hat{q}_t},
\]

\( ^2 \)We have completed the same exercise for a constant relative risk aversion utility representation. The analytical expressions are more complicated, but our main results remain the same.
where $\pi_{t+1} = \frac{p_{t+1}}{p_t} - 1$ denotes the inflation rate.

We now use different monetary policy rules and compare them with respect to the equity premium they produce.

4.1.1 Constant Money Supply Policy

We start with the zero money growth policy, which we obtain by setting $\mu_t = 0$ for every period $t$:

$$c_{t+1}^{\mu=0} = \frac{(\bar{y} + \varepsilon_t - \bar{\varepsilon})(\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon})}{\lambda(\bar{y} + \varepsilon_t - \bar{\varepsilon})},$$

$$p_{t+1}^{\mu=0} = \frac{M_t}{\bar{y} + \varepsilon_t - \bar{\varepsilon}},$$

$$1 + \pi_{t+1}^{\mu=0} = \frac{\bar{y} + \varepsilon_t - \bar{\varepsilon}}{\bar{y} + \varepsilon_t - \bar{\varepsilon}},$$

so it is true that:

$$\frac{u'(c_{t+1}^{\mu=0})}{p_{t+1}^{\mu=0}} = \frac{\lambda}{M_t} \left[ 1 + \frac{(1 - \lambda)\bar{y}}{\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon}} \right].$$

The above equations show that in the case of constant money supply policy an increase in future dividend increases future consumption and decreases future prices so that the nominal value of the future marginal utility of consumption does not change with future dividend. In addition, the constant money supply monetary policy does not distort future consumption or prices. Only predetermined variables affect future nominal marginal utility of consumption, which makes it a predetermined variable itself.

Combining the above equations with equation (27), we calculate the expression of real stock price as follows:

$$\hat{q}_{t+1}^{\mu=0} = c_{t+1}^{\mu=0} \bar{c}_{t+1} E_t \sum_{j=1}^{\infty} \beta^j \frac{\lambda \varepsilon_{t+j-1}}{\lambda \bar{y} + \varepsilon_{t+j-1} - \bar{\varepsilon}}.$$

From the equation of marginal utility above we see that the value of marginal utility of consumption at time $t + 1$ is a given number which is not related to any random variable. Therefore, $\text{Cov}_t \left( \frac{u'(c_{t+1}^{\mu=0})}{p_{t+1}^{\mu=0}}, p_{t+1}^{\mu=0} \right) = 0$. Then, the value of the expected nominal equity premium, using equation (23) is:

$$E_t \Pi_{t+1}^{\mu=0} = 0.$$
Remark 1. The Nominal Equity Premium:
When agents have a logarithmic utility function and monetary policy follows a constant money supply rule, the expected nominal equity premium is zero. This is true without making any assumptions for the dividend shocks.

We are also computing the real equity premium. Linearizing the expression of real stock price (29) and bond price (10) around the mean total dividend, we have:

\[
\hat{q}_{t,\mu=0} \approx \beta \lambda \bar{y} \left[ \frac{\varepsilon_t}{\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon}} + \frac{\beta}{1 - \beta} \left( \frac{\bar{\varepsilon}}{\lambda \bar{y}} + \frac{(\bar{\varepsilon} - \lambda \bar{y}) \sigma_e^2}{\lambda^2 \bar{y}^3} \right) \right],
\]

\[
E_t \hat{q}_{t+1,\mu=0} \approx \frac{\beta (\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon})}{\lambda(\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon})} \left[ \frac{\varepsilon_{t+1}}{1 - \beta} + \frac{(\bar{\varepsilon} - \lambda \bar{y}) \sigma_e^2}{\lambda^2 \bar{y}^2} \left( \frac{1}{1 - \beta} - \lambda \right) \right],
\]

\[
E_t u'(c_t) \frac{P_{t+1,\mu=0}}{P_{t+1}} = \frac{\lambda(\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon})}{M_t(\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon})},
\]

\[
E_t \frac{1}{P_{t+1,\mu=0}} = \frac{\bar{y}}{M_t},
\]

\[
E_t u'(c_{t+1}) \hat{q}_{t+1,\mu=0} = \frac{\beta \lambda}{1 - \beta} \frac{e_{t+1}}{\lambda \bar{y} + \varepsilon_{t+1} - \bar{\varepsilon}} + \frac{\beta \lambda}{1 - \beta} \left[ \frac{\varepsilon_t}{\lambda \bar{y}} + \frac{(\bar{\varepsilon} - \lambda \bar{y}) \sigma_e^2}{\lambda^2 \bar{y}^3} \right].
\]

Remark 2. The Real Equity Premium:
Substituting in equation for the real equity premium (25) the expressions above, we have:

\[
E_t \Pi_{t+1,\mu=0} \approx \frac{\beta (\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon})(\lambda \bar{y} - \bar{\varepsilon}) \sigma_e^2}{\hat{q}_{t,\mu=0} \lambda^2 \bar{y}^2 (\bar{y} + \varepsilon_t - \bar{\varepsilon})},
\]

which is an increasing function of the risk volatility \( \sigma_e^2 \).

4.1.2 Optimal Monetary Policy
In this section, we consider a monetary authority that maximizes total welfare by choosing the money supply growth rate \( \mu_t \), to solve \( \max_{\mu_t} E_0 \sum_{t=0}^{\infty} \beta^t (\lambda u(c_t^T) + (1 - \lambda) u(c_t^N)) \). As we assume that monetary authority assigns equal weight to each agent, the first order conditions combined with the equilibrium consumption equations (19) and (20) imply that all agents in the economy should have the same marginal utility of consumption, which
means that $c_t^{T*} = c_t^{N*}$. The optimal money growth rule is:

$$1 + \mu_t^* = \frac{\bar{y}}{\bar{y} + \varepsilon_{t-1} - \bar{\varepsilon}}. \quad (32)$$

This policy rule, similarly to Zervou (2013), reveals the distributional role of monetary policy: low dividend shocks decrease traders’ consumption and command expansionary monetary policy. The expansion supplement traders’ consumption who are hit by the shock but increases prices and hurts the non-traders. On the contrary, after a high dividend shock traders’ consumption increases. Optimal monetary policy tightens, takes away part of the extra dividend traders’ received, and benefits non-traders through lower prices. In this way, monetary policy perfectly shares the financial income risk that only traders face, among all agents in the economy.

The optimal traders’ consumption, goods price and inflation rate is:

$$c_t^{T*} = \bar{y} + \varepsilon_{t+1} - \bar{\varepsilon},$$

$$p_t^{*} = \frac{\bar{M}_t \bar{y}}{(\bar{y} + \varepsilon_{t+1} - \bar{\varepsilon})(\bar{y} + \varepsilon_{t} - \bar{\varepsilon})},$$

$$1 + \pi_t^{*} = \frac{\bar{y}}{\bar{y} + \varepsilon_{t+1} - \bar{\varepsilon}}.$$

Then, we get:

$$\frac{u'(c_t^{*}_{t+1})}{p_t^{*}} = \frac{1}{\bar{M}_t^{*}} = \frac{\bar{y} + \varepsilon_{t} - \bar{\varepsilon}}{\bar{M}_t \bar{y}}. \quad (33)$$

We see that increases in future dividends increase future consumption and decreases future prices. In addition, under optimal monetary policy, only current dividends matter although the effect of the previous period’s shocks do not matter for inflation.

We compute the stock price under the assumption that monetary policy is conducted optimally. Substituting in the expression for real stock price (27) the above equations for consumption and prices, and the optimal policy rule (32), we have the value of the real
stock price under optimal monetary policy:

\[ \hat{q}_t^* = c_t^* E_t \sum_{j=1}^{\infty} \beta^j \frac{\varepsilon_{t+j-1}}{y} \]

\[ = \frac{\beta c_t^*}{y} \left[ \varepsilon_t + \frac{\beta \bar{\bar{\varepsilon}}}{1 - \beta} \right]. \quad (34) \]

When monetary policy operates optimally, the segmentation effect of the real stochastic discount factor disappears. That’s because under the optimal monetary policy the dividend shock is shared among financial market participants and non-participants and there is no variation of the relative consumption of the traders. Every period, traders and non-traders consume an equal part of total output and the discount factor is only affected by the change in total consumption, similarly to the representative agent model. Traders’ consumption under the optimal monetary policy equals current output; then, an increase in current output increases the demand for stock, increasing real stock price. The stream of dividend increases payoff which increases also the real stock price. So with optimal monetary policy the segmented markets effect disappears and only current output and future dividend stream affect the real stock price.

Given a logarithmic utility function the value of marginal utility of consumption in period \( t + 1 \), \( \frac{u'(c_{t+1}^*)}{p_{t+1}} \), is a function of \( \varepsilon_t \) and \( \mu_{t+1}^* \). For the optimal monetary policy rule the money growth rate \( \mu_{t+1}^* \) just depends on previous information, such as \( \varepsilon_t, ..., \varepsilon_0 \) (see equation (33)). Then \( E_t \frac{u'(c_{t+1}^*)}{p_{t+1}} = \frac{u'(c_{t+1}^*)}{p_{t+1}} \), which is a predetermined variable. Therefore, \( \text{Cov}_t(\frac{u'(c_{t+1}^*)}{p_{t+1}}, R_{t+1}^*) = 0 \), which indicates that the value of the expected equity premium is zero.

**Remark 3. The Nominal Equity Premium:**

Under optimal monetary policy and using the logarithmic utility function, the value of expected nominal equity equity premium is \( E_t \Pi_{t+1}^* = 0 \).

We are also calculating the real equity premium. Based on the real stock price under
optimal policy equation (34), we know that:

$$E_t \hat{q}_{t+1}^* = \frac{\beta \varepsilon}{1 - \beta} + \frac{\beta \sigma_x^2}{y},$$
$$E_t u'(c_{t+1}^*) \hat{q}_{t+1}^* = \frac{\beta \varepsilon}{(1 - \beta)y}.$$  

Remark 4. The Real Equity Premium:

Substituting the above expressions in the real equity premium equation (25), we have that:

$$E_t \hat{\Pi}_{t+1} = \frac{\beta \sigma_x^2}{\hat{q}_t y} = \frac{(1 - \beta) \sigma_x^2}{(\bar{y} + \varepsilon_t - \bar{\varepsilon})[(1 - \beta)\varepsilon_t + \beta \bar{\varepsilon}]}.$$  

4.1.3 Inflation Targeting Policy

We now consider an inflation targeting monetary policy rule. The goods price level is defined as:

$$p_{t+1} = \frac{M_{t+1}}{y + \varepsilon_{t+1} - \varepsilon},$$
and thus the inflation rate is calculated as follows:

$$\pi_{t+1} = \frac{p_{t+1}}{p_t} - 1 = \frac{(1 + \mu_{t+1})(\bar{y} + \varepsilon_t - \bar{\varepsilon})}{\bar{y} + \varepsilon_{t+1} - \bar{\varepsilon}} - 1.$$  

For inflation target $\pi_{t+1} = \bar{\pi}$, the corresponding monetary policy is:

$$1 + \mu_{t+1}^\pi = (\bar{\pi} + 1)\frac{\bar{y} + \varepsilon_{t+1} - \bar{\varepsilon}}{\bar{y} + \varepsilon_t - \bar{\varepsilon}}.$$  

For the inflation targeting policy, we have:

$$\frac{u'(c_{t+1}^*)}{p_{t+1}} = \frac{\lambda(\bar{y} + \varepsilon_t - \bar{\varepsilon})}{M_t[(\lambda - 1)\bar{y} + (1 + \mu_{t+1}^\pi)(\bar{y} + \varepsilon_t - \bar{\varepsilon})]}$$
$$= \frac{\lambda(\bar{y} + \varepsilon_t - \bar{\varepsilon})}{M_t[(\lambda - 1)\bar{y} + (1 + \bar{\pi})(\bar{y} + \varepsilon_{t+1} - \bar{\varepsilon})]},$$  
and thus:

$$\frac{c_{t+j+1}^T p_{t+j-1}}{c_{t+j}^T p_{t+j}} = \frac{(\lambda - 1)\bar{y} + (1 + \bar{\pi})(\bar{y} + \varepsilon_t - \bar{\varepsilon})}{[(\lambda - 1)\bar{y} + (1 + \bar{\pi})(\bar{y} + \varepsilon_{t+j} - \bar{\varepsilon})](1 + \bar{\pi})}.$$  

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We assume that the series of dividends \( \{ \varepsilon_t \}_{t=0}^{\infty} \) are i.i.d. Defining \( f(\varepsilon_{t+j}) = \frac{1}{(\lambda-1)\hat{y}+(1+\bar{\pi})(\hat{y}+\varepsilon_{t+j}-\bar{\varepsilon})} \), then for all \( j \geq 1 \), we have:

\[
E_t(f(\varepsilon_{t+j})) = A,
\]

which is a constant. Therefore, combining with the reals stock price equations (27), and equation (38) above, we calculate the value of real stock price as:

\[
\hat{q}^*_t = \sum_{j=1}^{\infty} E_t \beta^j \frac{(\lambda-1)\hat{y} + (1+\bar{\pi})(\hat{y} + \varepsilon_{t+j} - \bar{\varepsilon})\varepsilon_{t+j-1}f(\varepsilon_{t+j})}{(1+\bar{\pi})} \]

\[
= \frac{\beta A(\lambda-1)\hat{y} + (1+\bar{\pi})(\hat{y} + \varepsilon_t - \bar{\varepsilon})}{(1+\bar{\pi})}[\varepsilon_t + \frac{\beta \bar{\varepsilon}}{1-\beta}],
\]

which is a function of \( \varepsilon_t \). Letting \( \hat{q}^*_t = g(\varepsilon_t) \), we rewrite the expression of the nominal stock return as below:

\[
1 + R_{t+1} = \frac{g(\varepsilon_{t+1})(1+\bar{\pi}) + \varepsilon_t}{g(\varepsilon_t)}.
\]

Together with equation (37), the covariance between the marginal utility from an extra unit of money, and the stock return is:

\[
\text{Cov}_t\left( \frac{u'(c^T_{t+1})}{p_{t+1}}, R_{t+1} \right) = \text{Cov}_t\left( \frac{\lambda(\bar{y} + \varepsilon_t - \bar{\varepsilon})f(\varepsilon_{t+1})}{M_t}, \frac{g(\varepsilon_{t+1})(1+\bar{\pi}) + \varepsilon_t}{g(\varepsilon_t)} \right) \]

\[
= \frac{\lambda(\bar{y} + \varepsilon_t - \bar{\varepsilon})(1+\bar{\pi})}{M_t g(\varepsilon_t)} \text{Cov}_t\left( f(\varepsilon_{t+1}), g(\varepsilon_{t+1}) \right).
\]

Using the expression for the nominal equity premium (23) and linearizing around the mean dividend, we have the nominal equity premium equation for the inflation targeting policy, under log utility:

\[
\Pi^*_t \approx (1+\bar{\pi})(\lambda + \bar{\pi})\bar{y}\bar{\varepsilon} + (1-\beta)(1+\bar{\pi})\sigma^2_\varepsilon - \frac{\bar{y}^3(\lambda + \bar{\pi})^3}{\lambda^2(\lambda + \bar{\pi})^2(1+\bar{\pi})^2}\sigma^2_\varepsilon.
\]

(40)

**Remark 5.** The Nominal Equity Premium:

From equation (40), we see that the nominal equity premium under inflation targeting monetary policy is an increasing function of the variance \( \sigma^2_\varepsilon \).
Remark 6. The Real Equity Premium:

Since $E_t \hat{\Pi}_{t+1} = E_t \frac{E_{t+1}}{p_{t+1}} \Pi_{t+1}$, under the inflation targeting policy, we have that:

$$E_t \hat{\Pi}_{t+1} = E_t \frac{\Pi_{t+1}^\beta}{1 + \pi},$$

$$\simeq \frac{\lambda \bar{y} \varepsilon + (1 - \beta)(1 + \bar{\pi})\sigma^2 - \frac{\varepsilon \bar{y}^3 (\lambda + \bar{\pi})^3}{y^3 (\lambda + \bar{\pi})^2 + (1 + \bar{\pi})^2 \sigma^2}}{[(\lambda - 1)\bar{y} + (1 + \bar{\pi})(\bar{y} + \varepsilon_t - \bar{\varepsilon})][(1 - \beta)\varepsilon_t + \beta \bar{\varepsilon}]}.$$ (41)

4.1.4 Endowment Sustaining Monetary Policy

We are computing equity premium for the policy that does not intervene in agent’s endowment allocations. Under that policy, the non-traders consume always their endowment, and the traders consume their endowment and dividends. That is:

$$c_{t}^{N,E} = y^N, \quad c_{t}^{T,E} = y^T + \frac{\varepsilon_t}{\lambda}.$$

Using the equilibrium consumption equations for non-traders and traders, (19) and (20), and given that $y^T + \frac{\varepsilon_t}{\lambda} = y^N$, we get that the policy that let agents consume their endowments, and dividends for the case of traders, is the zero inflation targeting policy. Thus, similarly with the inflation targeting monetary policy rule (36), we have that:

$$1 + \mu_{t+1}^E = \frac{\bar{y} + \varepsilon_{t+1} - \bar{\varepsilon}}{\bar{y} + \varepsilon_t - \bar{\varepsilon}}.$$

Then the equity premium is as calculated in Section 4.1.3, having inflation target of $\bar{\pi} = 0$.

4.2 Comparing Equity Premia

Comparing across the real equity premia produced by different policy rules is not straightforward. Before we proceed with our quantitative analysis, we use an example. Specifically, we let $\bar{y} = 1$, $y^T = 0.9$, inflation target is 2 per cent, $\lambda = 35$ per cent (parameter values are summarized in Table 1) and we leave the variance of the dividend shock free.$^3$

Figure 1 shows how the equity premia produced by various policies change with the dividend’s variance, and how they compare to each other. There we see that the equity

$^3$ $\lambda = 35$ per cent is approximately the percentage of the US population that Vissing-Jørgensen (2002) classifies as bondholders.
premium increases with the dividend variance, under any monetary policy rule. However, the change is steeper for the inflation targeting policy.

In general, the equity premium is much higher under the inflation targeting policy, than what it is under the optimal and constant money supply policy. On the contrary, the constant money supply policy does not produce much higher real equity premium than what the optimal policy does, for this parametrization.\(^4\)

5 Quantitative Analysis

In this section we quantify the differences in the real equity premium produced between an economy that follows a 2\% inflation targeting policy and an economy that follows optimal monetary policy. We choose the 2\% inflation target as an alternative to the optimal policy, because it is often mentioned in policy talks, in academic work and was recently formally stated as the inflation target of the Federal Reserve.\(^5,6\) To quantify the dividend shock we use the mean and variance of the total dividend income in the US.\(^7\) We estimate an AR(1) process for the de-trended part of the dividend income from which we use the persistence coefficient.\(^8\) We calibrate total endowment to match the 1997 average labor share of 66.1\% from Ríos-Rull and Santaeulàlia-Llopis (2010). We take from Walentin (2010) the estimates of the shareholders’ share of labor income, being \(\frac{\lambda y^T}{\lambda y^T + (1-\lambda)y_N} \equiv \eta^T = 45.1\%\) in 1997, and also the stock market participation rate for the same year, \(\lambda = 27.3\%\). We use the following equations in our quantitative analysis, where \(\hat{R}_{t+1}\) is the real gross return on stock, \(R_{t+1}\) is the nominal gross return on stock, \(\hat{r}_{t+1}\) is the real gross return on bonds and \(r_{t+1}\) is the

\(^{4}\)Similar results in terms of direction, are obtained when we use a constant relative risk aversion utility function with coefficient equal to two.

\(^{5}\)Leigh (2008) estimates the implicit inflation target which although unstable, is on average around 2\% for the period we examine.


\(^{7}\)We define dividend income as the sum of Rental income of persons with CCAdj, Corporate profits with IVA and CCAdj, Net interest and miscellaneous payments and Current surplus of government enterprises from the Bureau of Economic Analysis, which we convert to real per capita values using CPI and Civilian population from FRED. We use quarterly, per capita data from the first quarter of 1960 until the second quarter of 2012.

\(^{8}\)For deriving the cyclical component we de-trend the data with HP filtering before estimating the AR(1) process.
nominal gross return on bonds:

\[ y_t = l + \varepsilon_t, \]
\[ y^N_t = \frac{1 - \eta^T_t}{1 - \lambda} l, \]
\[ y^T_t = \frac{\eta^T_t}{\lambda} l, \]
\[ c^N_t = \frac{y^N_t}{1 + \bar{\pi}_t}, \]
\[ c^T_t = y^T_t + \frac{\varepsilon_t}{\lambda} + \frac{1 - \lambda}{\lambda} (y^N_t - c^N_t), \]
\[ 1 + \mu_t = (1 + \bar{\pi}) \frac{y_t}{y_{t-1}}, \]
\[ 1 + \mu^*_t = \frac{y^N_t}{y_t}, \]
\[ 1 + \pi^*_t = (1 + \mu^*_t) \frac{y_{t-1}}{y_t}, \]
\[ \hat{q}^\pi_t = \beta E_t \left[ \left( \frac{c^T_t}{c^T_{t+1}} \right)^\alpha \left( \frac{\varepsilon_t}{1 + \bar{\pi}_t} \right) \right], \]
\[ \hat{q}^*_t = \beta E_t \left[ \left( \frac{y_t}{y_{t-1}} \right)^\alpha \left( \frac{\varepsilon_t}{1 + \pi^*_t} \right) \right], \]
\[ E_t \hat{R}^\pi_{t+1} = E_t \left[ \frac{\hat{q}^\pi_{t+1} + \varepsilon_t}{\hat{q}^\pi_t} \right], \]
\[ E_t \hat{R}^*_t = E_t \left[ \frac{\hat{q}^*_t + \varepsilon_t}{\hat{q}^*_t} \right], \]
\[ E_t R^\pi_{t+1} = E_t \hat{R}^\pi_{t+1} (1 + \bar{\pi}), \]
\[ E_t R^*_t = E_t \hat{R}^*_t (1 + \bar{\pi}), \]
\[ s^\pi_{t+1} = \beta E_t \left[ \frac{1}{1 + \bar{\pi}} \left( \frac{c^T_t}{c^T_{t+1}} \right)^\alpha \right], \]
\[ s^*_t = \beta E_t \left[ \frac{1}{1 + \pi^*_t} \left( \frac{y_t}{y_{t-1}} \right)^\alpha \right]. \]
\begin{align*}
E_t r^\pi_{t+1} &= \frac{1}{s_t^\pi}, \\
E_t r^*_{t+1} &= \frac{1}{s_t^*,1}, \\
E_t \hat{r}^\pi_{t+1} &= \frac{1}{(1+\pi)s_t^\pi}, \\
E_t \hat{r}^*_{t+1} &= \frac{E_t \frac{1}{s_t^*+\rho}}{E_t \frac{1}{1+\pi s_t^*+\rho}}, \\
\hat{\Pi}^\pi_{t+1} &= E_t[\hat{R}^\pi_{t+1} - \hat{r}^\pi_{t+1}], \\
\hat{\Pi}^*_{t+1} &= E_t[\hat{R}^*_{t+1} - \hat{r}^*_{t+1}], \\
\varepsilon_t &= a^\varepsilon + \rho^\varepsilon \varepsilon_{t-1} + \zeta^\varepsilon.
\end{align*}

The estimated process for the de-trended dividend income implies $\rho^\varepsilon = 0.8$. The mean of the dividend series data is $\bar{\varepsilon} = $3.671 and the standard deviation $\sigma^\varepsilon = 0.758$. Then, we derive that $a^\varepsilon = 0.734$ and $\sigma^\zeta = 0.4548$ so to match the mean and standard deviation of the dividend series in the data. The only shock in our economy is the shock to the dividends, $\zeta^\varepsilon$, which has mean zero and standard deviation $\sigma^\zeta$. We let $\beta = 0.99$. The relative risk aversion rate is set to $\alpha = 2$. Also, given that $y_t = l + \varepsilon_t$ and $l = 0.661y_t$, then $\bar{y} = 0.661\bar{y} + 3.671$, and thus $\bar{y} = 10.83$ and $l = 7.16$.

For calculating equity premia we use various maturities bonds, i.e., we use one, four, ten and twenty periods bonds. For example, for the bonds that mature after four quarters, we use the adjusted set of equations below, derived from the four-period intertemporal
optimal condition, which we use instead of the one period Euler equation (8).

\[
\begin{align*}
s_t^\pi,4 & = \beta^4 E_t \left[ \frac{1}{(1 + \pi)^4} \left( \frac{C^T}{c^T_{t+4}} \right)^\alpha \right], \\
E_t r_t^\pi,4 & = \frac{1}{(s_t^\pi,4)^{\frac{1}{2}}}, \\
E_t \tilde{r}_t^\pi,4 & = \frac{1}{(1 + \pi)(s_t^\pi,4)^{\frac{1}{2}}}, \\
s_t^*,4 & = \beta^4 E_t \left[ \frac{1}{\prod_{i=1}^{4} (1 + \pi^*_{t+i})} \left( \frac{y_t}{y_{t+4}} \right)^\alpha \right], \\
E_t r_t^*,4 & = \frac{1}{(s_t^*,4)^{\frac{1}{2}}}, \\
E_t \tilde{r}_t^*,4 & = \frac{E_t \frac{1}{\prod_{i=1}^{4} (1 + \pi^*_{t+i}) s_t^*,4^{\frac{1}{4}}}}{s_t^*,4}. 
\end{align*}
\]

Similarly we do for the bonds with ten and twenty periods maturity.

The results for our quantitative exercise at the steady state are shown in the second column of Tables 2 and 3. At the steady state, under the inflation targeting policy, the real stock and real bond returns are equal to each other, and thus there is no premium. This is true across any maturity of the bonds. Similarly, there is no premium under the optimal monetary policy either. Equity premium develops when there is uncertainty about the stream of future dividends in the economy.

The results for the model economy after we introduce uncertainty are summarized in the third column of Tables 2 and 3. From Table 2 we see that the real stock returns increase under both policies, compared to the steady state. That’s because uncertainty introduces risk for the financial market participants, who ask for higher returns in order to keep the stocks. Notice that the returns are much higher under the inflation targeting policy than under the optimal policy. This is because the optimal policy shares the risk, although the inflation targeting one leaves the financial market participants exposed. Thus, the stock returns under the inflation targeting policy are higher.

On the other hand, we see from Table 2 that the real bond returns decrease under both policies, compared to the steady state. This is because one-period bonds are the safe assets; thus their demand increases after uncertainty is introduced. Note that in our model we have nominal bonds that expire and return one unit of money. In order to buy
goods, traders care about prices. However, in the inflation targeting world, agents do not have uncertainty about next period’s prices. Also, having optimal monetary policy reacting to dividends with a period lag, agents’s uncertainty about a period ahead prices is resolved after observing current dividend, in the optimal policy world. Also, notice that the decrease is much deeper for the bond returns under the inflation targeting policy, compared to the optimal policy. This is because traders are more exposed to risk under the inflation targeting policy compared to the optimal one, and thus their demand for the safe asset is higher.

Given that uncertainty increases stock returns and decreases bond returns, the equity premium becomes positive under both policies. The mean equity premium under the inflation targeting policy is 6.96% yearly although it is only 1.55% yearly under the optimal policy. The large difference that the inflation targeting policy generates in the equity premium, compared to the optimal policy, originates to the fact that the inflation targeting policy exposes financial market participants in higher risk, compared to the optimal policy. Then, two things happen: First, the financial market participants ask for higher stock returns under the inflation targeting policy compared to the optimal one. Second, they value the riskless asset more in the world of inflation targeting policy than what they do under the optimal policy. The return of the riskless asset is smaller under the inflation targeting policy compared to the optimal one.

We repeat the same exercise for 0% and 10% inflation targets; we find minor changes in the premium produced compared to the 2% inflation target. The fact that the 0% inflation target does not significantly change the premium, signifies the importance of the segmentation friction. That is, the 0% inflation targeting policy which would be the optimal one in the New Keynesian model, does not minimize the equity premium. On the contrary, the risk-sharing policy which is optimal considering segmentation effects, produces lower equity premium.

In addition, the fact that the 10% inflation targeting policy does not achieve higher (or lower) premium than what the 2% inflation targeting policy does, shows that the equity premium is not really affected by the specific choice of target. It is the policy consideration that affects the premium. The optimal monetary policy shares the risk across agents and
implies low equity premium, unlike the inflation targeting policy that leaves financial market participants exposed to financial income risk, encourages segmentation effects and implies high equity premium.

Comparing our findings with the data, we see that the equity premium produced under the inflation targeting policy is closer to the data than what it is the premium produced under the optimal policy. Mehra and Prescott (2008) reports that in the data, the average equity premium for the US for the period 1889 to 2005 is 6.36%. Similarly high equity premia are reported in developed countries worldwide by Dimson et al. (2009) (see Table 4). Also, Mehra and Prescott (2008) documents that the equity premium has been increasing over time, from 4.5% from 1900 to 1950, to 7.42 from 1951 to 2005. This is due to the diminishing return on the riskless asset, which they find to decrease from 2.95% that it was on average from 1900 to 1950, to 1.11 from 1951 to 2005.

Table 3 shows results for longer maturity bonds and the real term premium. Unlike the returns for the safe one-period bond, the returns of longer maturity bonds are higher when uncertainty is introduced, compared to the steady state. That’s because the longer maturity bonds are not as safe; thus, they are not in large demand, as the one-period bonds are. Traders are very uncertain about future prices that they would have to pay to buy goods with the cash they receive after the bond matures. Thus, the term premium between the one-period safe bond and any other period bond is large.

Figures 2-5 present the model’s response after a 1% dividend shock. Figure 2 shows that a negative dividend shock of 13.6% decreases output, and thus agents’ consumption under the optimal policy, by more than 4%. It decreases traders’ consumption by almost 7% under the inflation targeting policy. Although traders’ consumption decreases under both policies, the optimal monetary policy implies smaller decrease. This is because the optimal policy smoothes the dividend shock across traders and non-traders. On the contrary, the 2% inflation targeting policy directs all the dividend volatility towards the traders. As a result, traders consumption is more responsive to dividend shocks under the inflation targeting policy compared to the optimal policy.

Our analysis suggests that the two policies respond differently to the negative dividend shock. Specifically, the negative dividend shock decreases current total output, increasing
in turns the price level; the inflation targeting policy tightens by 4.34% in order to keep inflation at the 2% target. However, under the optimal monetary policy money growth increases in response to the negative dividend shock, in order to redistribute the dividend shock among traders and non-traders. A 13.6% decrease in per capita dividend income increases by more than 4% the money growth. Optimal monetary policy increases temporarily inflation, transferring money to traders who suffer the low dividend shock. As we see from Figure 2 the inflation targeting policy becomes contractionary in order to keep inflation to its target, although optimal monetary policy becomes expansionary in order to redistribute the dividend loss.

From Figure 2 we see that the real stock price decreases under both policies. Lower dividend decreases the payoffs and thus decreases real stock price. Also, current consumption decreases more than future consumption and thus the real stochastic discount factor decreases as well, decreasing the real stock price. As percentage change, the real stock price is more responsive to the dividend shock when the inflation targeting policy is used versus the optimal policy. This is because stocks’ demand decreases more under the inflation targeting policy where traders are not compensated for the dividend loss, than what it does under the optimal policy, where traders are partially compensated.

Real stock returns increase after the shock, as we see from Figure 3. This is expected given that the negative dividend shock makes stock a less attractive asset. The real stock return for inflation targeting is more responsive compared to the optimal policy. This is because when there is a negative dividend shock, the traders need a higher increase in stock returns in the inflation targeting world where policy does not share their loss, compared to the risk-sharing, optimal monetary policy world, in order to hold the stock.

We now look at the bonds returns for various maturities.

In addition, as we see from Figures 3 and 4, the effect on the real bond return is stronger under the inflation targeting policy, given that consumption under the inflation targeting policy decreases more than under the optimal policy, after the dividend shock hits.

Calculating the impulse response function for the real equity premium, we find that the premium of the stock return over the one-period bond return is minimal, under both policies. That means that the equity premium is affected by long-run uncertainty, and
its’ interaction with policy decisions, as we discussed above; however, it is not affected by short-term, transitional changes in dividends.

Figures 4 and 5 show that the equity premium over longer maturity bonds increases after the dividend shock, indicating that the stock becomes a less desirable asset. The effect is stronger for the inflation targeting policy, given that the dividend shock affects the traders more severely under the inflation targeting policy than under the optimal policy. Also, the increase in the premium increases with bonds’ maturity, indicating that the bonds of longer maturity are strongly preferred to the stock, when the dividend shock hits. This is because the dividend shock decreases severely the demand for stocks; it decreases bonds’ demand too, but the effect decreases with maturity.

We also calculate impulse response functions for the real term premium, in Figure 5. There we see that the term premium decreases with the dividend; that is, bond returns increase less the longer the maturity. Also note, that the decrease in the term premia is deeper under the inflation targeting policy versus the optimal policy. This is because under the inflation targeting policy the traders are hit severely by the dividend shock, and increase abruptly their demand for the safe, short term bonds, decreasing the term premium. However, the optimal monetary policy shares the dividend risk and generates smoother reaction.

6 Empirical Evidence

Given our conclusion that the inflation targeting policy produces higher equity premium mostly because it implies lower short term bond return, we are looking into the data in search of such evidence.

The countries that adopted inflation targeting early in the 1990s were New Zealand (1990), Canada (1991), UK (1992), Australia (1993) and Sweden (1993). As we see from Figure 6 the short term rate decreases after inflation targeting is adopted. The same trend we see for the developed countries that adopted inflation targeting later, i.e., Iceland, Korea and Norway. Figure 7 shows that short term rates decrease after adoption, indicating that inflation targeting insures investors against inflation uncertainty, and thus are willing to

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9In the parenthesis the year of adoption is indicated

26
accept lower rates.

Finally, we compare the early inflation targeting countries with European non-targeting countries, during the 1980s and the 1990s. Figure 8 shows that the short term rates decrease for the early inflation targeting countries (New Zealand, Canada, UK, Australia and Sweden) during the decade of adoption; i.e., short term rates are lower in the 1990s than what they are in the 1980s. On the contrary, the short term rates for the non-targeting countries increases during the 1990s.

7 Concluding Remarks

We study an economy where agents have identical preferences but some of them do not participate in the financial markets. Agents who participate in financial markets are trading stock and are subject to financial income risk. In such an economy monetary policy’s actions affect the equity premium. We find that the optimal monetary policy minimizes the equity premium compared to other policy rules that emphasize other objectives that the central banks might have, as for example, keeping inflation to its target, or letting agents consume their endowments, and for the case of traders, their dividends. This is because the optimal monetary policy’s objective is to share the risk that the financial market participants are subject to, among all the agents in the economy, discouraging in this way the segmentation effect. Given risk sharing, the return on equity is low under the optimal policy. On the other hand, it is high under inflation targeting, as that policy is not concerned with sharing financial income risk. However, bond returns are higher under the optimal policy, compared to what they are under inflation targeting. This is because the inflation targeting policy removes price uncertainty, and thus makes the nominal bond safe. As a result, the optimal monetary policy minimizes equity premium.

\[10^\text{Note that some European countries like Germany, although non-targeting, are very focused on inflation and thus cannot be included in the non-targeting sample. The same argument applies for the US at various samples.}\]
Appendix

Tables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean income</td>
<td>$\bar{y}$</td>
<td>1</td>
</tr>
<tr>
<td>Trader’s endowment</td>
<td>$y^T$</td>
<td>0.9</td>
</tr>
<tr>
<td>Mean Dividend</td>
<td>$\varepsilon$</td>
<td>0.035</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Participation fraction</td>
<td>$\lambda$</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 1: Parameter Values for Example

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Steady State</th>
<th>Simulation Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net real bond return, optimal policy</td>
<td>$E_t\bar{r}^<em>_{t+1} \bar{r}^</em>_{t+1} - 1$</td>
<td>1.01%</td>
<td>1.0384%</td>
</tr>
<tr>
<td>Net real bond return, 2% inflation target</td>
<td>$E_t\bar{r}^\pi_{t+1} - 1$</td>
<td>1.01%</td>
<td>0.2136%</td>
</tr>
<tr>
<td>Net real stock return, optimal policy</td>
<td>$E_t\bar{R}^*_{t+1} - 1$</td>
<td>1.01%</td>
<td>1.4276%</td>
</tr>
<tr>
<td>Net real stock return, 2% inflation target</td>
<td>$E_t\bar{R}^\pi_{t+1} - 1$</td>
<td>1.01%</td>
<td>1.9542%</td>
</tr>
<tr>
<td>Real premium, optimal monetary policy</td>
<td>$E_t\bar{\Pi}^*_{t+1}$</td>
<td>0</td>
<td>0.3892%</td>
</tr>
<tr>
<td>Real premium, 2% inflation target</td>
<td>$E_t\bar{\Pi}^\pi_{t+1}$</td>
<td>0</td>
<td>1.7406%</td>
</tr>
<tr>
<td>Real premium diff. targeting &amp; optimal policy</td>
<td>$E_t[\bar{\Pi}^\pi_{t+1} - \bar{\Pi}^*_{t+1}]$</td>
<td>0</td>
<td>1.3514%</td>
</tr>
</tbody>
</table>

Table 2: One-Quarter Bond Quantitative Exercise
Table 3: Different Maturity Bonds Quantitative Exercise
Note: Term premium is over the real return of the 1-period bond.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Steady State</th>
<th>Simulation Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net real 4-p bond return, optimal policy</td>
<td>$E_t \hat{r}_{t+1}^{4,4}$ - 1</td>
<td>1.01%</td>
<td>1.1101%</td>
</tr>
<tr>
<td>Net real 4-p bond return, 2% inflation target</td>
<td>$E_t \hat{r}_{t+1}^{\pi,4}$ - 1</td>
<td>1.01%</td>
<td>1.3014%</td>
</tr>
<tr>
<td>Term premium 4-p bond, optimal policy</td>
<td>$E_t \hat{T}_{t+1}^{4,4}$</td>
<td>0</td>
<td>0.0717%</td>
</tr>
<tr>
<td>Term premium 4-p bond, 2% inflation target</td>
<td>$E_t \hat{T}_{t+1}^{\pi,4}$</td>
<td>0</td>
<td>1.0878%</td>
</tr>
<tr>
<td>Net real 10-p bond return, optimal policy</td>
<td>$E_t \hat{r}_{t+1}^{4,10}$ - 1</td>
<td>1.01%</td>
<td>1.0543%</td>
</tr>
<tr>
<td>Net real 10-p bond return, 2% inflation target</td>
<td>$E_t \hat{r}_{t+1}^{\pi,10}$ - 1</td>
<td>1.01%</td>
<td>1.1415%</td>
</tr>
<tr>
<td>Term premium 10-p bond, optimal policy</td>
<td>$E_t \hat{T}_{t+1}^{4,10}$</td>
<td>0</td>
<td>0.0160%</td>
</tr>
<tr>
<td>Term premium 10-p bond, 2% inflation target</td>
<td>$E_t \hat{T}_{t+1}^{\pi,10}$</td>
<td>0</td>
<td>0.9279%</td>
</tr>
<tr>
<td>Net real 20-p bond return, optimal policy</td>
<td>$E_t \hat{r}_{t+1}^{4,20}$ - 1</td>
<td>1.01%</td>
<td>1.0305%</td>
</tr>
<tr>
<td>Net real 20-p bond return, 2% inflation target</td>
<td>$E_t \hat{r}_{t+1}^{\pi,20}$ - 1</td>
<td>1.01%</td>
<td>1.0724%</td>
</tr>
<tr>
<td>Term premium 20-p bond, optimal policy</td>
<td>$E_t \hat{T}_{t+1}^{4,20}$</td>
<td>0</td>
<td>-0.0078%</td>
</tr>
<tr>
<td>Term premium 20-p bond, inflation target</td>
<td>$E_t \hat{T}_{t+1}^{\pi,20}$</td>
<td>0</td>
<td>0.8588%</td>
</tr>
</tbody>
</table>

Table 4: Arithmetic means of yearly % real returns, real equity premium and inflation for 1900-2000.
Table taken from Dimson et al. (2009).
Figures

Figure 1: Real equity premium for different monetary policy rules, as a function of dividend volatility. Parameters values are from Table 1. Blue curve denotes real equity premium under constant money growth; green curve denotes real equity premium under optimal monetary policy; red curve denotes real equity premium under 2% inflation targeting; light green curve denotes real equity premium under 0% inflation targeting.
Figure 2: Percentage deviations from steady state after a 1% dividend shock.

Figure 3: Deviations from steady state after a 1% dividend shock.
Figure 4: Deviations from steady state after a 1% dividend shock.

Figure 5: Deviations from steady state after a 1% dividend shock.
Figure 6: Short term rate averages for early IT countries (in the parenthesis is indicated the year of adoption): New Zealand (1990), Canada (1991), UK (1992), Australia (1993) and Sweden (1993). The averages are across countries, from 1980 to the adoption year, and from the year after adoption to 2006. The data are taken from OECD.

Figure 7: Short term rate averages for chosen late IT countries: Korea, Norway, Iceland, which all adopted IT in 2001. The averages are across countries. For the period before adoption the data are from 1981 for Norway, 1988 for Iceland and 1991 for Korea, to 2001. For the period after adoption the data are from 2002 to 2006. The data are taken from OECD.
Figure 8: Short term rate averages for early IT (New Zealand, Canada, UK, Australia, Sweden) and non-IT countries (France, Italy, Portugal, Spain) for the periods 1980-1990 and 1991-2000.
References


Notes For the Referees

Notes: Log Utility and Inflation Targeting Policy

In the main text we have that, together with equation (37), the covariance between the marginal utility from an extra unit of money, and the stock return is:

\[
\text{Cov}_t\left(\frac{u'(c_{t+1}^T)}{p_{t+1}}, R_{t+1}\right) = \text{Cov}_t\left(\frac{\lambda(y + \varepsilon_t - \bar{\varepsilon})f(\varepsilon_{t+1})}{M_t}, \frac{g(\varepsilon_t)(1 + \bar{\pi}) + \varepsilon_t}{g(\varepsilon_t)}\right)
\]

\[
= \frac{\lambda(y + \varepsilon_t - \bar{\varepsilon})(1 + \bar{\pi})}{M_t g(\varepsilon_t) \text{Cov}_t(f(\varepsilon_{t+1}), g(\varepsilon_{t+1})).}
\]

Thus, the expected nominal equity premium for the inflation targeting policy becomes:

\[
\Pi_{t+1}^\pi = \frac{(1 + \bar{\pi})\text{Cov}_t(f(\varepsilon_{t+1}), g(\varepsilon_{t+1})))}{g(\varepsilon_t)E_t(f(\varepsilon_{t+1}))} - \frac{1 + \bar{\pi}}{g(\varepsilon_t)}[\frac{E_t[f(\varepsilon_{t+1})g(\varepsilon_{t+1})]}{A} - E_t(g(\varepsilon_{t+1}))].
\]

(42)

In addition:

\[
E_t[f(\varepsilon_{t+1})g(\varepsilon_{t+1})] = \frac{\beta A\bar{\varepsilon}}{(1 - \beta)(1 + \bar{\pi})},
\]

\[
E_t(g(\varepsilon_{t+1})) = \frac{\beta A[(\lambda + \bar{\pi})\bar{\varepsilon} + (1 - \beta)(1 + \bar{\pi})\sigma^2]}{(1 - \beta)(1 + \bar{\pi})}.
\]

Here, \(\sigma^2\) denotes the variance of \(\varepsilon\). After substituting the expressions for \(E_t[f(\varepsilon_{t+1})g(\varepsilon_{t+1})]\) and \(E_t(g(\varepsilon_{t+1}))\), we find the following expression for the value of expected equity premium:

\[
\Pi_{t+1}^\pi = \frac{\beta A[(\lambda + \bar{\pi})\bar{\varepsilon} + (1 - \beta)(1 + \bar{\pi})\sigma^2] - \beta \bar{\varepsilon}}{(1 - \beta)q_t^\pi}
\]

\[
= \frac{(1 + \bar{\pi})[\lambda + \bar{\pi})\bar{\varepsilon} + (1 - \beta)(1 + \bar{\pi})\sigma^2 - \bar{\varepsilon}]}{[\lambda - \bar{\pi})\bar{\varepsilon} + (1 + \pi)(\bar{\varepsilon} + \varepsilon_t - \bar{\varepsilon})][(1 - \beta)\varepsilon_t + \beta \bar{\varepsilon}].
\]

(43)

Linearizing \(A = E_t(f(\varepsilon_{t+j}))\) around the mean dividend, we get:

\[
A \simeq \frac{1}{\bar{y}(\lambda + \bar{\pi})} + \frac{(1 + \bar{\pi})^2\sigma^2}{\bar{y}^3(\lambda + \bar{\pi})^3}.
\]

(44)
Replacing the expression $A$ in equation (43) with equation (44), we have the nominal equity premium equation for the inflation targeting policy, under log utility:

$$\Pi_{t+1}^\pi \simeq \frac{(1 + \bar{\pi})[(\lambda + \bar{\pi})\bar{y}\bar{\varepsilon} + (1 - \beta)(1 + \bar{\pi})\sigma_\xi^2 - \frac{\bar{y}\bar{\varepsilon}^3(\lambda + \bar{\pi})^2}{y^2(\lambda + \bar{\pi})^2(1 + \bar{\pi})^2\sigma_\xi^2}]}{[(\lambda - 1)\bar{y} + (1 + \bar{\pi})(\bar{y} + \varepsilon_t - \bar{\varepsilon})][(1 - \beta)\varepsilon_t + \beta\bar{\varepsilon}]}.$$  

**Risk Aversion**

We use the more general constant relative risk aversion utility function $u(c_t) = \frac{c_t^{1-a}}{1-a}$ to calculate the equity premium. Combing with equation (22) and solving for the recursive form of the real stock price, we have the following expression for the real stock price:

$$\tilde{q}_t = \sum_{j=1}^{\infty} E_t\beta^j \left(\frac{c_t^T}{c_{t+j}}\right)^\alpha \frac{P_{t+j-1}}{P_{t+j}} \varepsilon_{t+j-1}. \quad (45)$$

We now substitute into the above equation various monetary policy rules assumptions and calculate the implied equity premia.

**Constant Money Supply Policy**

If the monetary growth rate $\mu_t$ is zero for every period $t$, combing with equation (20), for $j > 1$, the term inside the expectation in equation (45) becomes:

$$E_t(\frac{c_t^T}{c_{t+j}})^\alpha \frac{P_{t+j-1}}{P_{t+j}} \varepsilon_{t+j-1} = (\lambda c_t^T)^\alpha E_t(\frac{\bar{y} + \varepsilon_{t+j-1} - \bar{\varepsilon}}{\lambda \bar{y} + \varepsilon_{t+j-1} - \bar{\varepsilon}})^{\alpha-1} \varepsilon_{t+j-1} (\bar{y} + \varepsilon_{t+j} - \bar{\varepsilon})^{1-\alpha}$$

Assume that the series of dividends $\{\varepsilon_t\}_{t=0}^{\infty}$ are i.i.d, then:

$$E_t(\frac{\bar{y} + \varepsilon_{t+j-1} - \bar{\varepsilon}}{\lambda \bar{y} + \varepsilon_{t+j-1} - \bar{\varepsilon}})^{\alpha-1} \varepsilon_{t+j-1} (\bar{y} + \varepsilon_{t+j} - \bar{\varepsilon})^{1-\alpha} = E_t(\frac{\bar{y} + \varepsilon_{t+j-1} - \bar{\varepsilon}}{\lambda \bar{y} + \varepsilon_{t+j-1} - \bar{\varepsilon}})^{\alpha-1} \varepsilon_{t+j-1} E_t(\bar{y} + \varepsilon_{t+j} - \bar{\varepsilon})^{1-\alpha} = B_1 B_2.$$  

Here, defining $f_1(\varepsilon_{t+j-1}) = (\frac{\bar{y} + \varepsilon_{t+j-1} - \bar{\varepsilon}}{\lambda \bar{y} + \varepsilon_{t+j-1} - \bar{\varepsilon}})^{\alpha-1} \varepsilon_{t+j-1}$, then $B_1 = E_t f_1(\varepsilon_{t+j-1})$ and $f_2(\varepsilon_{t+j}) = (\bar{y} + \varepsilon_{t+j} - \bar{\varepsilon})^{1-\alpha}$, $B_2 = E_t f_2(\varepsilon_{t+j})$ are constant for all $j > 1$. The expression of the stock
price is as follows:

\[
\hat{q}_t^{\mu=0} = \beta (\lambda c_t^T)^{\alpha} \frac{(\bar{y} + \varepsilon_t - \bar{\varepsilon})^{\alpha-1} \varepsilon_t E_t(\bar{y} + \varepsilon_{t+1} - \bar{\varepsilon})^{1-\alpha}}{(\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon})^{\alpha}} + \frac{\beta^2 (\lambda c_t^T)^{\alpha} B_1 B_2}{1 - \beta}
\]

\[
= \beta (\lambda c_t^T)^{\alpha} B_2 \left[ \frac{(\bar{y} + \varepsilon_t - \bar{\varepsilon})^{\alpha-1} \varepsilon_t}{(\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon})^{\alpha}} + \frac{\beta B_1}{1 - \beta} \right] 
\]

\[
= \beta (\lambda c_t^T)^{\alpha} B_2 \left[ f_1(\varepsilon_t) + \frac{\beta B_1}{1 - \beta} \right].
\]  

(46)

Remark 7. The real stock price \( \hat{q}_t^{\mu=0} \) depends on the present dividend \( \varepsilon_t \) and is an increasing function of it.

Also,

\[
u'(c_{t+1}^T) = \frac{1}{(c_{t+1}^T)^{\alpha} p_{t+1}} = \frac{\lambda^\alpha}{M} \left[ \frac{\bar{y} + \varepsilon_t - \bar{\varepsilon}}{\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon}} \right]^{\alpha} (\bar{y} + \varepsilon_{t+1} - \bar{\varepsilon})^{1-\alpha}
\]

\[
= \frac{\lambda^\alpha}{M} \left[ \frac{\bar{y} + \varepsilon_t - \bar{\varepsilon}}{\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon}} \right]^{\alpha} f_2(\varepsilon_{t+1}).
\]

Then, we get \( E_t \frac{u'(c_{t+1}^T)}{p_{t+1}} = \frac{\lambda^\alpha}{M} \left[ \frac{\bar{y} + \varepsilon_t - \bar{\varepsilon}}{\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon}} \right]^{\alpha} B_2 \), which in combination with equations (23) and (46), gives us the value of equity premium as:

\[
\Pi_{t+1}^{\mu=0} = - \frac{\text{Cov}_t \left( \frac{u'(c_{t+1}^T)}{p_{t+1}}, \frac{\hat{q}_t^{\mu=0} (1 + \Pi_{t+1}) + \varepsilon_t}{\hat{q}_t^{\mu=0}} \right)}{E_t \frac{u'(c_{t+1}^T)}{p_{t+1}}} = - \frac{\text{Cov}_t \left( \frac{u'(c_{t+1}^T)}{p_{t+1}}, \frac{\hat{q}_t^{\mu=0}}{\hat{q}_t^{\mu=0}} \right)}{E_t \frac{u'(c_{t+1}^T)}{p_{t+1}}} \left[ f_1(\varepsilon_{t+1}) + \frac{\beta B_1}{1 - \beta} \right]
\]

\[
= - \frac{\beta \varepsilon_t \text{Cov}_t \left( \frac{1}{(c_{t+1}^T)^{\alpha} p_{t+1}}, \frac{c_{t+1}^T}{p_{t+1}} \right) p_{t+1} \left[ f_1(\varepsilon_{t+1}) + \frac{\beta B_1}{1 - \beta} \right]}{\hat{q}_t^{\mu=0} f_1(\varepsilon_t)}
\]

\[
= - \frac{\beta \varepsilon_t \text{Cov}_t \left( f_2(\varepsilon_{t+1}), \frac{1}{f_2(\varepsilon_{t+1})} \right) \left[ f_1(\varepsilon_{t+1}) + \frac{\beta B_1}{1 - \beta} \right]}{\hat{q}_t^{\mu=0} f_1(\varepsilon_t)}
\]

\[
= - \frac{\beta \varepsilon_t \left[ B_1 - (1 - \beta) B_2 E \frac{f_1(\varepsilon_{t+1})}{f_2(\varepsilon_{t+1})} - \beta B_1 B_2 E \frac{1}{f_2(\varepsilon_{t+1})} \right]}{(1 - \beta)\hat{q}_t^{\mu=0} f_1(\varepsilon_t)}
\]

(47)

Remark 8. For relative risk aversion rate \( \alpha \) greater than one, \( f_2(\varepsilon_{t+1}) \) is a decreasing function of \( \varepsilon_{t+1} \), while \( f_1(\varepsilon_{t+1}) \) is an increasing function of \( \varepsilon_{t+1} \). Thus, the covariance between them is negative, which means that the value of nominal equity premium is positive.
Linearizing the functions $f_1(.)$ and $f_2(.)$ around the mean dividend $\bar{\varepsilon}$, we get:

$$B_1 \simeq \frac{\bar{\varepsilon}}{\lambda \bar{y}} + \frac{[\alpha \bar{\varepsilon}(1 - \lambda)(1 + \alpha + \lambda (3 - \alpha)) + 2\lambda^2 \bar{\varepsilon} + 2\lambda \bar{y}(\lambda (\alpha - 1) - \alpha)]\sigma^2_\varepsilon}{2\lambda^2 + \alpha \bar{y}^3}$$

$$B_2 \simeq \frac{\bar{y}^{1-\alpha}}{2\bar{y}^{1+\alpha}} + \frac{\alpha(\alpha - 1)\sigma^2_\varepsilon}{2\bar{y}^{1+\alpha}}$$

$$E\frac{f_1(\varepsilon_{t+1})}{f_2(\varepsilon_{t+1})} \simeq \frac{\bar{\varepsilon} - \bar{\varepsilon}^2}{\lambda \alpha} + \frac{\bar{y}^{\alpha - 3}\sigma^2_\varepsilon}{\lambda \alpha} [2(\alpha - 1) - \frac{\alpha - 1}{\lambda} + \frac{\bar{\varepsilon}}{\bar{y}}((\alpha - 1)(2\alpha - 3)\bar{y} + \frac{\alpha(\alpha + 1)}{2\lambda^2} - \frac{2\alpha(\alpha - 1)}{\lambda})]$$

$$E\frac{1}{f_2(\varepsilon_{t+1})} \simeq \frac{\bar{y}^{\alpha - 1}}{2} + \frac{(\alpha - 1)(2\alpha - 3)\bar{y}^{\alpha - 3}\sigma^2_\varepsilon}{2}$$

Replacing the value of related elements in equation (47), we have the expression of $\Pi_{t+1}^{\mu = 0}$. We calculate real equity premium under the constant money supply policy, using equation (25). To do so, we first calculate the following expressions:

$$E_t u'(c_{t+1}^{T, \mu = 0}) = \frac{\lambda^\alpha (\bar{y} + \varepsilon_t - \bar{\varepsilon})^\alpha}{M_t(\bar{y} + \varepsilon_t - \bar{\varepsilon})^\alpha} E_t(\bar{y} + \varepsilon_t - \bar{\varepsilon})^{1-\alpha}$$

$$= \frac{\lambda^\alpha (\bar{y} + \varepsilon_t - \bar{\varepsilon})^\alpha}{M_t(\bar{y} + \varepsilon_t - \bar{\varepsilon})^\alpha} B_2$$

$$E_t \frac{1}{P_{t+1}^{\mu = 0}} = \frac{\bar{y}}{M_t}$$

$$E_t u'(c_{t+1}^{T, \mu = 0})^2_{t+1} = \beta^\lambda \alpha B_2 [E_t f_1(\varepsilon_{t+1}) + \frac{\beta B_1}{1-\beta}]$$

$$= \frac{\beta^\lambda \alpha B_1 B_2}{1 - \beta}$$

**Remark 9. The Real Equity Premium**

Replacing the value of related elements in equation (25), the expression of real equity premium is:

$$\Pi_{t+1}^{\mu = 0} = \frac{1}{q_t^{\mu = 0}}[E_t q_t^{\mu = 0} - \frac{\beta B_1 \bar{y}(\bar{y} + \varepsilon_t - \bar{\varepsilon})^\alpha}{(1 - \beta)(\bar{y} + \varepsilon_t - \bar{\varepsilon})^\alpha}],$$

where

$$E_t q_t^{\mu = 0} \simeq \beta B_2 \frac{(\bar{y} + \varepsilon_t - \bar{\varepsilon})^\alpha}{\lambda^\alpha} \frac{\beta B_1 (\bar{y} + \varepsilon_t - \bar{\varepsilon})^\alpha}{1 - \beta} + \frac{\bar{\varepsilon}^\alpha - 1}{\lambda^\alpha} + \frac{\bar{y}^{\alpha - 3}\sigma^2_\varepsilon}{2\lambda^2} \varepsilon(\alpha^2 - 2\lambda^2 + \alpha(-6\lambda^2 + 2\lambda + 1) + 2\lambda^2) + 2\lambda \bar{y}(\alpha(2\lambda - 1) - \lambda)]^{2\lambda^2 + 2}. $$
Optimal Monetary Policy

When the monetary authority acts optimally and aims to maximize the total welfare, the optimal money growth rate follows equation (32). Combing with equations (32) and (45) and assuming that the series of dividends \(\{\varepsilon_t\}_{t=0}^{\infty}\) are \(i.i.d\), we find the value of real stock price:

\[
\hat{q}_t^* = \frac{(c^T_t)^\alpha}{\bar{y}} \sum_{j=1}^{\infty} E_t \beta^j (\bar{y} + \varepsilon_{t+j} - \bar{\varepsilon})^{1-\alpha} \varepsilon_{t+j-1}
\]

\[
= \frac{(c^T_t)^\alpha}{\bar{y}} \sum_{j=1}^{\infty} E_t \beta^j f_2(\varepsilon_{t+j}) \varepsilon_{t+j-1}
\]

\[
= \frac{\beta B_2 (c^T_t)^\alpha}{\bar{y}} [\varepsilon_t + \frac{\beta \bar{\varepsilon}}{1 - \beta}].
\]

Remark 10. From the above equation, we see that the real stock price is an increasing function of current dividend \(\varepsilon_t\).

Using the Jensen’s Inequality, we get \(B_2 = E_t (\bar{y} + \varepsilon_{t+1} - \bar{\varepsilon})^{1-\alpha} \geq \bar{y}^{1-\alpha}\), then the stock price is:

\[
\hat{q}_t^* \geq \beta\left(\frac{\bar{y} + \varepsilon_t - \bar{\varepsilon}}{\bar{y}}\right)^\alpha [\varepsilon_t + \frac{\beta \bar{\varepsilon}}{1 - \beta}] = \beta\left(\frac{1}{1 + \mu_{t+1}^{opt}}\right)^\alpha [\varepsilon_t + \frac{\beta \bar{\varepsilon}}{1 - \beta}].
\]

The lower boundary of the stock price is a decreasing function of mean income \(\bar{y}\). When the monetary policy is conducted optimally, the money growth rate is an increasing function of mean income level so as the inflation level, which will lower the real stock price.

Since \(\frac{u'(c^T_{t+1})}{p_{t+1}} = \frac{(\bar{y} + \varepsilon_{t+1} - \bar{\varepsilon})^{1-\alpha}}{M_{t+1}} = f_2(\varepsilon_{t+1})\frac{M_{t+1}}{M_{t+1}}\), then \(E_t \frac{u'(c^T_{t+1})}{p_{t+1}} = \frac{B_2}{M_{t+1}}\), which together with equation (23), gives us the expression of nominal equity premium:

\[
\Pi_{t+1}^* = -\frac{\text{Cov}_t\left(\frac{u'(c^T_{t+1})}{p_{t+1}}, \hat{q}_{t+1}(1+\Pi_{t+1}^{opt}) + \varepsilon_t\right)}{E_t \frac{u'(c^T_{t+1})}{p_{t+1}}} = -\frac{\text{Cov}_t\left(\frac{u'(c^T_{t+1})}{p_{t+1}}, \hat{q}_{t+1}P_{t+1}\right)}{p_t \hat{q}_t^* E_t \frac{u'(c^T_{t+1})}{p_{t+1}}} = \frac{\beta}{\hat{q}_t^*} \text{Cov}_t(f_2(\varepsilon_{t+1}), \frac{1}{f_2(\varepsilon_{t+1})}(\varepsilon_{t+1} + \frac{\beta \bar{\varepsilon}}{1 - \beta}))
\]

\[
= \frac{\beta}{\hat{q}_t^*} [B_2 E_t \frac{1}{f_2(\varepsilon_{t+1})}(\varepsilon_{t+1} + \frac{\beta \bar{\varepsilon}}{1 - \beta}) - \bar{\varepsilon}].
\]

Remark 11. Since \(\text{Cov}_t(f_2(\varepsilon_{t+1}), \varepsilon_{t+1} + \frac{\beta \bar{\varepsilon}}{1 - \beta})\) is positive, the equity premium is always positive, and is an increasing function of mean income \(\bar{y}\).
Combing equations (48) and (50), we have:

\[
\bar{q}_t^* \simeq \frac{\beta}{\bar{y}^\alpha} \left[ 1 + \frac{\alpha(\alpha - 1)\sigma^2}{2\bar{y}^2} \right] [\bar{y} + \epsilon_t - \bar{\epsilon}]^{\alpha} [\epsilon_t + \frac{\beta\bar{\epsilon}}{1 - \beta}]_t.
\]  

(52)

From equation (52) we know that the real stock price is an increasing function of the relatively risk aversion rate \( \alpha \) under optimal monetary policy.

Linearizing equation (51) around the mean dividend, we get the nominal equity premium:

\[
\Pi_t^* \simeq \frac{\beta(\alpha - 1)\sigma^2}{\bar{q}_t^opt} \left[ 1 + \frac{(\alpha - 1)\bar{\epsilon}}{(1 - \beta)\bar{y}} + \frac{\alpha(\alpha - 1)\sigma^2}{2\bar{y}^2} \left( 1 + \frac{(\alpha - 2)\bar{\epsilon}}{2(1 - \beta)\bar{y}} \right) \right].
\]  

(53)

Also, we have that under the optimal policy:

\[
E_t u'(c_{t+1}^*) = E_t \left( \bar{y} + \epsilon_{t+1} - \bar{\epsilon} \right)^{1 - \alpha} = \frac{\bar{y} + \epsilon_t - \bar{\epsilon}}{M_t\bar{y}} B_2,
\]

\[
E_t \frac{1}{p_t^*} = \frac{\bar{y} + \epsilon_t - \bar{\epsilon}}{M_t}.
\]

\[
E_t u'(c_{t+1}^*) = \frac{\beta B_2 \bar{\epsilon}}{(1 - \beta)\bar{y}}.
\]

**Remark 12. The Real Equity Premium**

Replacing the value of the related elements in equation (25), the expression of real equity premium under the optimal monetary policy is:

\[
\hat{\Pi}_t^* = \frac{1}{p_t^*} \left[ E_t \hat{q}_t^* - \frac{\beta\bar{\epsilon}}{1 - \beta} \right],
\]  

(54)

here,

\[
E_t \hat{q}_t^* \simeq \beta B_2 \bar{\epsilon} \left[ \frac{\bar{y}^2}{1 - \beta} + (\bar{y} + \frac{(\alpha - 1)\bar{\epsilon}}{2(1 - \beta)\bar{y}}) \alpha\sigma^2 \right].
\]

**Inflation Targeting Policy**

When monetary authorities target inflation, equation (36) reveals the way monetary policy operates. Trader’s consumption level can be written as follows:

\[
c_{t+1}^T = \frac{(1 + \pi)(\bar{y} + \epsilon_{t+1} - \bar{\epsilon}) + \bar{y}(\lambda - 1)}{\lambda(1 + \pi)} = \frac{1}{\lambda(1 + \pi)f(\epsilon_{t+1})},
\]
which is an increasing function of \( \varepsilon_{t+j} \). \( f(\varepsilon_{t+j}) = \frac{1}{(\lambda-1)\hat{y}+(1+\bar{\pi})(\hat{y}+\varepsilon_{t+j}-\hat{\varepsilon})} \) is as defined in the previous section. For \( j > 1 \), we have:

\[
E_t\left(\frac{c_t^T}{c_t^t}\right)^{\alpha} \frac{p_{t+j}^{-1}}{p_{t+j}} \varepsilon_{t+j-1} = \frac{(c_t^T)^{\alpha}{\varepsilon}_{t+j-1}}{1+\pi} E_t\left(\frac{1}{(c_t^t)^{\alpha}}\right)^{\alpha} = \varepsilon\frac{\bar{\Pi}_t}{(1+\pi)}f(\varepsilon_{t+j})^\alpha.
\]

Assuming that the series of dividends \( \{\varepsilon_t\}_{t=0}^\infty \) are i.i.d, then \( E_t f(\varepsilon_{t+j})^\alpha = C \) is constant for all \( j \geq 1 \). Therefore, we have the expression of real stock price given below:

\[
\hat{q}_t^\pi = \frac{\beta C}{(1+\pi)f(\varepsilon_{t})^\alpha} [\varepsilon_t + \frac{\beta \bar{\varepsilon}}{1-\beta}]. \quad (55)
\]

Combining with equation (23), we calculate the nominal equity premium as follows:

\[
\Pi_{t+1}^\pi = -\frac{\text{Cov}(u'(c_t^t)_{p_{t+1}}, \hat{q}_t^\pi(1+\bar{\pi})+\varepsilon_{t+1})}{E_t u'(c_t^t)_{p_{t+1}}} = -\frac{\text{Cov}(u'(c_t^t)_{p_{t+1}}, \hat{q}_t^\pi(1+\bar{\pi}))}{E_t u'(c_t^t)_{p_{t+1}} ^\pi} \frac{1}{E_t u'(c_t^t)_{p_{t+1}} ^\pi} \frac{1}{f(\varepsilon_{t+1})^\alpha} (\varepsilon_{t+1} + \frac{\beta \bar{\varepsilon}}{1-\beta})
\]

\[
= \frac{\beta}{\hat{q}_t^\pi} [C E_t \frac{1}{f(\varepsilon_{t+1})^\alpha} (\varepsilon_{t+1} + \frac{\beta \bar{\varepsilon}}{1-\beta}) - \frac{\bar{\varepsilon}}{1-\beta}]. \quad (56)
\]

Linearizing the function \( f(\cdot) \) around the mean dividend, we have:

\[
C \simeq \frac{1}{[\lambda+\bar{\pi}]\hat{y}^\alpha} + \frac{\alpha(\alpha+1)(1+\bar{\pi})\sigma_\varepsilon^2}{2[(\lambda+\bar{\pi})\hat{y}]^{\alpha+2}},
\]

\[
E_t \bar{\Pi}_{t+1} = E_t \left[ \frac{\varepsilon_{t+1} \bar{\varepsilon} (\lambda + \bar{\pi}) + \frac{\alpha(\alpha+1)(1+\bar{\pi})\sigma_\varepsilon^2}{2[(\lambda+\bar{\pi})\hat{y}]^{\alpha+2}} \right] [\varepsilon_t + \frac{\beta \bar{\varepsilon}}{1-\beta}]. \quad (57)
\]

Replacing the expression of \( C \) in the equation (55), we get:

\[
\hat{q}_t^\pi \simeq \frac{\beta(\lambda-1)\hat{y} + (1+\bar{\pi})(\hat{y} + \varepsilon_t - \hat{\varepsilon})^\alpha}{(1+\bar{\pi})[\lambda+\bar{\pi}]\hat{y}^\alpha} [1 + \frac{\alpha(\alpha+1)(1+\bar{\pi})\sigma_\varepsilon^2}{2[(\lambda+\bar{\pi})\hat{y}]^{\alpha+2}}] [\varepsilon_t + \frac{\beta \bar{\varepsilon}}{1-\beta}]. \quad (58)
\]

We get the expression of \( \Pi_{t+1}^\pi \) by combing the equation (55) and (57).

**Remark 13. The Real Equity Premium**

*Since \( E_t\Pi_{t+1} = E_t \left[ \frac{\bar{p}_{t+1}}{p_{t+1}} \Pi_{t+1} \right], \) under the inflation targeting policy, the real equity premium*
is as follows:

\[
E_t \dot{\Pi}_{t+1}^x = E_t \frac{\Pi_{t+1}^x}{1 + \bar{\pi}} \\
= \frac{\beta}{(1 + \bar{\pi}) q_t} CE_t \frac{1}{f(\varepsilon_{t+1})^\alpha} (\varepsilon_{t+1} + \frac{\beta \bar{\varepsilon}}{1 - \beta}) - \bar{\varepsilon},
\]

(59)