Do Jumps Matter for Volatility Forecasting? Evidence from Energy Markets*

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Abstract

This paper characterizes the dynamics of jumps and analyzes their importance for volatility forecasting. Using high-frequency data on four prominent commodity markets, we perform a model-free decomposition of realized variance into its continuous and discontinuous components. We find strong evidence of jumps in energy markets between 2008 and 2012. We then investigate the importance of jumps for volatility forecasting. To this end, we estimate and analyze the predictive power of several Heterogenous Autoregressive (HAR) models that explicitly aim to capture the dynamics of jumps. Conducting extensive in-sample and out-of-sample analyses, we establish that explicitly modeling jumps does not significantly improve forecast accuracy. Our results hold for all markets, forecasting horizons and loss functions.

JEL classification: C1, C53, C58, G1, G13.

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I. Introduction

The theory of quadratic variation (QV) posits that the total variation of an asset return can be decomposed into continuous and discontinuous components. The aim of this paper is to advance our understanding of the dynamics of each of these components and investigate their importance for volatility forecasting.

We make three important contributions to the literature. First, we identify and characterize the dynamics of jumps in four leading energy markets, namely crude oil, gasoline, heating oil and natural gas. Using intraday transaction prices, we implement a non-parametric jump detection test to identify jumps. We then rigorously analyze the time-series behaviour of jumps, thus shedding more light on their dynamics. Our analysis shows that jumps are rare events that affect only a small proportion of our sample. Moreover, we find important asymmetries in the intensity of positive and negative jumps, suggesting that it may be important to separately model the dynamics of positive and negative jumps.

Second, we investigate the importance of disentangling continuous volatility from jumps for volatility forecasting. We present and thoroughly assess the predictive ability of several models of the Heterogenous Autoregressive (HAR) family that seek to explicitly capture the dynamics of jumps.[1] We begin by analyzing the in-sample predictive power of all models, which we compare to our benchmark HAR–RV model. To this end, we perform a simple ordinary least squares (OLS) regression of realized volatility on the in-sample forecasts obtained from the various HAR models. We find that all models yield adjusted R^2 that are very close to each other, indicating that the benefits of explicitly modeling jumps are likely to be small. This is true for all forecast horizons, i.e. 1-, 5- and 22-day ahead.

Third, we go beyond the in-sample analysis and rigorously analyze the out-of-sample performance of competing models. We use a rolling window of 600 days to estimate the parameters of forecasting models. Equipped with these, we then forecast the volatility of the next period, which we compare to realized volatility (observed ex-post). We employ six distinct loss functions to analyze the accuracy of competing forecasts. Our results establish that all models yield forecast errors

[1] See Corsi (2009) for an excellent treatment of the HAR model.

that are of the same order of magnitude, indicating that explicitly modeling jumps does not noticeably improve forecast accuracy. We analyze the question of statistical significance by implementing the test of Giacomini and White (2006), hereafter GW. This analysis suggests that explicitly modeling jumps does not significantly improve forecast accuracy. This result generally holds for all horizons, loss functions and markets.

We conduct several robustness checks. First, one may wonder whether our conclusions change depending on whether we forecast variance rather than volatility. Focusing on the task of forecasting realized variance, we repeat our analyses and obtain broadly similar conclusions. Second, we investigate the robustness of our findings to the jump detection methodology. In particular, we draw on recent theoretical results by Andersen et al. (2012), who introduce novel jump–robust estimators of integrated variance. Repeating our analysis with the new estimator does not change our main insights. Third, one may argue that the width of the rolling window may impact our analysis. We consider alternative windows of 400, 800 and 1,000 observations. Our core message is the same: explicitly modeling jumps does not significantly improve volatility forecasts. Finally, we assess the robustness of our results to the estimation methodology. Because our dependent variable, i.e. volatility, varies substantially over time, our OLS estimates may be driven by a highly volatile pocket of data. To address this concern, we repeat our analyses by estimating all models using a weighted least square (WLS) approach (rather than OLS) and reach similar conclusions.

Our study relates to the literature on the econometrics of jumps. Barndorff-Nielsen and Shephard (2004), Barndorff-Nielsen et al. (2004), Barndorff-Nielsen and Shephard (2006), Tauchen and Zhou (2011) and Andersen et al. (2012) propose a number of non-parametric tests to identify jumps. Eraker et al. (2003) and Eraker (2004) rely on tightly parameterized continuous-time models to estimate jumps. Aït-Sahalia et al. (2014) and Maneesoonthorn et al. (2014) model jumps as processes that are self-exciting and explore the implications of this modeling framework for derivatives prices. We contribute to this literature by presenting a thorough and comprehensive model-free study on the dynamics of jumps in four leading energy markets. Furthermore, our finding that explicitly modeling jumps does not significantly improve volatility forecasts may have important implications for continuous-time models that are needed for energy prices. More specifically, our results suggest that models with self-exciting jumps are unlikely to successfully match the dynamics of energy prices.

Our paper also connects with the growing literature that uses intraday data to obtain more accurate volatility forecasts (Andersen and Bollerslev, 1998; Andersen et al., 1999; Koopman et al., 2005; Corsi and Reno, 2009; Corsi et al., 2010; Patton and Sheppard, 2011; Sévi, 2014). The closest paper to ours is that of Andersen et al. (2007), who argue that jumps matter for volatility forecasting. The authors only focus on the in-sample forecasting power and do not investigate the out-of-sample predictive ability of these models. Our paper analyzes not only the in-sample but also the out-of-sample performance of these models, thus significantly advancing our understanding of the role jumps play in volatility forecasting.

The remainder of the paper is organized as follows. Section II introduces our methodology and the dataset. Section III presents our empirical results. Section IV discusses various robustness checks. Finally, Section V concludes.

II. Methodology and Data

This section begins with a brief overview of jump detection tests. We then introduce the competing models. Finally, we present our dataset of intraday transaction prices.

A. Jump Detection Test

Consider the logarithmic price process, p_t, defined on the probability space $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{0\leq t\leq T}, P)$, where $\mathscr{F}_t$ is the information set available up to time t, such that p_t is $\mathscr{F}_t$-measurable and evolves in continuous-time as a jump-diffusion process:

$$dp_t = \mu_t dt + \sigma_t dW_t + \eta_t dN_t \tag{1}$$

where dp_t denotes the change in log price. μ_t is the drift, which is a locally bounded and predictable process of finite variance. dt is an increment of time. σ_t is the instantaneous (or spot) volatility, which is a càdlàg process. W_t refers to the

Brownian motion. η_t is a random variable capturing the jump size. Finally, N_t is a Poisson counting process. If a jump occurs during the increment dt, then $dN_t = 1$. Otherwise, $dN_t = 0$. The probability of a jump during the increment dt is $P\left[dN_t = 1\right] = \lambda_t dt$, where λ_t is the (time-varying) jump intensity.

The quadratic variation, QV_t, of the above return process can be expressed as the sum of a continuous and discontinuous components (Barndorff-Nielsen and Shephard, 2002a, 2003). More formally, we have:

$$QV_t = \underbrace{\int_{t-1}^{t} \sigma_s^2 ds}_{\text{continuous}} + \underbrace{\sum_{t-1 \leq \tau_i \leq t} \eta_{\tau_i}^2}_{\text{discontinuous}} \tag{2}$$

where τ_i are the times of the corresponding jumps (with i = 1,2,...,N_t) and all other variables are as previously defined. The first term on the right hand side of the equality sign is the "integrated variance"; it is the continuous component of the quadratic variation. The second term is the discontinuous component of the quadratic variation.

We now explain in detail how to empirically compute each of the quantities shown in Equation (2). Suppose that on a trading day t we observe $M + 1$ prices at times: $t_0, t_1, ..., t_M$. If p_{t_j} is the logarithmic price at time t_j, then the corresponding return, r_{t_j}, for the j^{th} intraday interval of day t is defined as follows:

$$r_{t_j} = p_{t_j} - p_{t_{j-1}} \tag{3}$$

Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002a,b) propose the "realized variance" estimator, defined as the sum of squared intraday returns:

$$RV_t = \sum_{j=1}^{M} r_{t_j}^2 \tag{4}$$

The authors show that, as the sampling frequency increases ($M \to \infty$), realized variance is a consistent estimator of daily quadratic variation:

$$\lim_{M \to \infty} RV_t \equiv QV_t \tag{5}$$

Barndorff-Nielsen and Shephard (2004, 2006) introduce the "bipower variation" (BPV), which is a consistent estimator of the continuous component of QV_t:[2]

$$BPV_t = \mu_1^{-2} \left(\frac{M}{M-2} \right) \sum_{j=2}^{M} |r_{t_{j-1}}| \cdot |r_{t_j}| \tag{6}$$

where $\mu_1 = \mathbb{E}(|Z|) = \sqrt{2/\pi}$ is the first moment of the absolute value of a standard normal random variable. The term $M/(M-2)$ corresponds to a finite sample correction. BPV_t consistently estimates the continuous sample path of quadratic variation as $M \to \infty$ (Barndorff-Nielsen and Shephard, 2004, 2006; Barndorff-Nielsen et al., 2006):

$$\lim_{M \to \infty} BPV_t \equiv \int_{t-1}^{t} \sigma_s^2 ds \tag{7}$$

where all variables are as previously defined.

Huang and Tauchen (2005), Andersen et al. (2007) and others document that using staggered returns alleviates many of the microstructure biases inherent in high-frequency data. Instead of computing the product of adjacent returns, i.e. $|r_{t_{j-1}}| \cdot |r_{t_j}|$, they consider $|r_{t_{j-(k+1)}}| \cdot |r_{t_j}|$, where k is a positive integer indicating the number of returns to skip. When staggered returns are employed, the realized bipower variation is as follows:

$$BPV_t = \mu_1^{-2} \left(\frac{M}{M-(k+1)} \right) \sum_{j=k+2}^{M} |r_{t_{j-(k+1)}}| \cdot |r_{t_j}| \tag{8}$$

Throughout our study, we work with staggered returns to make our analysis immune to microstructure concerns. We skip 1 return observation, i.e. setting $k = 1$.

Since the quadratic variation is the sum of continuous and discontinuous components (see Equation (2)), one can express the discontinuous component as the difference between the quadratic variation and the continuous component. A direct implication of this is that we can infer the discontinuous component from the

[2]Andersen et al. (2012) point out that because of microstructure effects, the contribution of jumps does not vanish asymptotically, leading to an upward bias in the BPV estimator. We address this issue in our robustness analysis by replacing the BPV estimator with the MedRV estimator based on nearest neighbor truncation (Andersen et al., 2012).

realized variance and the bipower variation:

$$RV_t - BPV_t \xrightarrow{p} \sum_{t-1 \leq \tau_i \leq t} \eta_{\tau_i}^2 \tag{9}$$

This intuition lies at the heart of most jump detection tests. Huang and Tauchen (2005) show that the ratio statistic with a maximum adjustment (Barndorff-Nielsen and Shephard, 2006) has good size and power properties. This result motivates us to use the following estimator:

$$z_{TQ,t} = \Delta^{-1/2} \frac{(RV_t - BPV_t)/RV_t}{\sqrt{\left((\frac{\pi}{2})^2 + \pi - 5\right) max(1, \frac{TQ_t}{BPV_t^2})}} \tag{10}$$

TQ_t above is the realized tripower quarticity:

$$TQ_t = M\left(\frac{M}{M-2(k+1)}\right)\mu_{4/3}^{-3} \sum_{j=3}^{M} |r_{t_{j-2(k+1)}}|^{4/3} \cdot |r_{t_{j-k-1}}|^{4/3} \cdot |r_{t_j}|^{4/3} \tag{11}$$

where $\mu_{4/3} = 2^{2/3}[\Gamma(7/6)/\Gamma(1/2)]$.

Using the test statistic of Equation (10) and a significance level α, which we set equal to 0.1 %, we extract the significant jumps, J_t, as follows:

$$J_t = \mathbb{I}_{\{Z_{TQ,t} > \Phi_{1-\alpha}\}} \cdot (RV_t - BPV_t) \tag{12}$$

where $\mathbb{I}_{\{Z_{TQ,t} > \Phi_{1-\alpha}\}}$ is an indicator function which is equal to 1 when a significant jump occurs and zero otherwise, $\Phi_{1-\alpha}$ is the corresponding critical value from the cumulative standard normal distribution at confidence level $1-\alpha$. Since RV_t is equal to the sum of the continuous component (C_t) plus jumps (J_t), the continuous path of realized variance can be identified as follows:

$$C_t = \mathbb{I}_{\{Z_{TQ,t} \geq \Phi_{1-\alpha}\}} \cdot BPV_t + \mathbb{I}_{\{Z_{TQ,t} < \Phi_{1-\alpha}\}} \cdot RV_t \tag{13}$$

where all variables are as previously defined.

B. Volatility Forecasting Models

1. **HAR-RV**: Our benchmark econometric model is the HAR–RV originally proposed by Corsi (2009) and used in several empirical studies (Andersen et al., 2007; Corsi et al., 2010; Busch et al., 2011; Patton and Sheppard, 2011).[3] Following Patton and Sheppard (2011), we model the volatility realized over the next h days as an affine function of the historical volatility computed over various horizons:[4,5,6]

$$RV_{t:t+h}^{1/2} = \omega + \beta_d RV_t^{1/2} + \beta_w RV_{t-5:t-1}^{1/2} + \beta_m RV_{t-22:t-5}^{1/2} + e_{t+h} \tag{14}$$

As mentioned above, each component in the HAR-RV model is computed over different horizons. Therefore, if RV_t is the realized variance of day t (from time $t-1$ to t), then the *h-day* realized variance is expressed as:

$$RV_{t:t+h} = \frac{252}{h}\left(RV_{t+1} + RV_{t+2} + \ldots + RV_{t+h}\right) \tag{15}$$

[3]Motivated by the Heterogenous ARCH model of Müller et al. (1997) and Dacorogna et al. (1997), the HAR model includes volatility components aggregated at different frequencies. This approach is motivated by the heterogenous trading horizons of market participants. The HAR model is appealing not only from an economic perspective but also from a purely econometric standpoint. Indeed, it is flexible enough to capture the persistence of realized volatility without the complexity that typically characterizes long-memory models.

[4]We focus on the task of forecasting volatility (rather than variance) because volatility plays a key role in modern finance theory. For instance, it is a key variable for option pricing and asset allocation. We also consider the task of forecasting variance. See Section IV. for further results.

[5]The baseline HAR specification we use is that of Patton and Sheppard (2011). It differs from the original model of Corsi (2009) along several dimensions. First, the current specification uses historical estimates of realized volatility that are computed over *non-overlapping* intervals, thus allowing for a clear and unambiguous interpretation of the coefficient estimates (Patton and Sheppard, 2011). Furthermore, computing each of the historical estimates of volatility over non-overlapping intervals decreases the correlation between the regressors, thus mitigating concerns related to statistical issues.

[6]Strictly speaking, the daily realized volatility should be written as $RV_{t-1:t}$. However, to simplify our notation, we write it as RV_t.

In a similar manner, the weekly and monthly components are computed as:

$$RV_{t-5:t-1} = \frac{252}{4}\sum_{i=2}^{5} RV_{t-i+1} \tag{16}$$

$$RV_{t-22:t-5} = \frac{252}{17}\sum_{i=6}^{22} RV_{t-i+1} \tag{17}$$

2. **HAR–J**: Andersen et al. (2007) propose the HAR–J, which is a simple extension of the HAR–RV model that seeks to capture the dynamics of jumps. The main feature of the HAR–J model is that it replaces the most recent realized volatility ($RV_{t-1}^{1/2}$) with two components: $C_{t-1}^{1/2}$ and $J_{t-1}^{1/2}$. Each of these components has its own coefficient estimate:

$$\begin{aligned} RV_{t:t+h}^{1/2} &= \omega + \beta_d {C_t}^{1/2} + \beta_w RV_{t-5:t-1}^{1/2} + \beta_m RV_{t-22:t-5}^{1/2} \\ &\quad + \gamma_J J_t^{1/2} + e_{t+h} \end{aligned} \tag{18}$$

where all variables are as previously defined.

3. **HAR–RJ**: The previous model can be criticized on the grounds that it ignores the sign of jumps. The HAR-RJ addresses this limitation (Tauchen and Zhou, 2011). We identify significant realized jumps as follows:

$$RJ_t = sign(r_t) \cdot \sqrt{J_t} \tag{19}$$

where RJ_t is the realized jump on day t, $sign(\cdot)$ is the sign operator. The HAR–RJ model is then defined as:

$$\begin{aligned} RV_{t:t+h}^{1/2} &= \omega + \beta_d {C_t}^{1/2} + \beta_w RV_{t-5:t-1}^{1/2} + \beta_m RV_{t-22:t-5}^{1/2} \\ &\quad + \gamma_{RJ} RJ_t + e_{t+h} \end{aligned} \tag{20}$$

where all components are as previously defined.[7]

4. **HAR–ARJ**: It may be that positive and negative observations of RJ exert an asymmetric impact on volatility. As a result, it is interesting

[7]Notice that the superscript 1/2 is omitted from RJ_t, since it is already expressed in volatility form (see Equation (19)).

to investigate which of positive and negative jumps is more important for volatility forecasting. We advance in this direction by further decomposing RJ_t into components due to positive and negative jumps:

$$RJ_t^+ = max(RJ_t; 0) \tag{21}$$

$$RJ_t^- = min(RJ_t; 0) \tag{22}$$

The HAR–ARJ specification is employed to test whether the variation from negative jumps has a more pronounced impact on future volatility than that of positive jumps:

$$\begin{aligned} RV_{t:t+h}^{1/2} &= \omega + \beta_d {C_t}^{1/2} + \beta_w {RV_{t-5:t-1}}^{1/2} + \beta_m {RV_{t-22:t-5}}^{1/2} \\ &\quad + \gamma_{RJ^+} RJ_t^+ + \gamma_{RJ^-} RJ_t^- + e_{t+h} \end{aligned} \tag{23}$$

5. **HAR–C–J**: Finally, we consider a more general specification, similar to that of Andersen et al. (2007), which fully decomposes each realized variance component (i.e. daily, weekly, monthly) into its continuous and jump parts:

$$\begin{aligned} RV_{t:t+h}^{1/2} &= \omega + \beta_d C_t^{1/2} + \beta_w C_{t-5:t-1}^{1/2} + \beta_m C_{t-22:t-5}^{1/2} + \gamma_{Jd} J_t^{1/2} \\ &\quad + \gamma_{Jw} J_{t-5:t-1}^{1/2} + \gamma_{Jm} J_{t-22:t-5}^{1/2} + e_{t+h} \end{aligned} \tag{24}$$

C. Data

Our dataset consists of tick-by-tick transaction prices on four energy futures contracts traded at NYMEX, namely WTI crude oil, gasoline, heating oil and natural gas. The data comes from TickData and spans the period from January 2, 2007 to June 29, 2012. Energy futures contracts trade on two venues at the CME: pit and electronic. Trading hours on both platforms have no overlap and collectively span 22:45 hours. Pit trading takes place between 9:30 AM (ET) and 4:15 PM (ET). Electronic trading starts at 4:30 PM (ET), pauses at 5:15 PM (ET) for 45 minutes, resumes at 6:00 PM (ET) and stops the following day at 9:15 AM (ET).

We use both pit and electronic transaction records and process the dataset as follows. First, we discard all transactions with prices lower than or equal to zero. Second, we expunge all trades with time-stamps that are inconsistent with the exchange's trading hours. Third, we retain the futures contract with the highest number of transactions only (usually the first or second nearest contract). Following existing studies, e.g. Lee and Mykland (2008) and Bradley et al. (2014), we sample our data at the 15-min frequency.[8]

Table 1 presents summary statistics for the different (annualized) measures of variance. Columns 2 to 4 relate to realized variance (RV), bipower variation (BPV) and jump variation (JV), i.e. RV-BPV, respectively. Columns 5 to 7 present results for the square root of RV, BPV and JV, respectively.[9] A comparison of $\sqrt{RV}$ across the four energy futures markets reveals that on average natural gas exhibits the highest volatility (44.3 % per year), followed by crude oil (34.4 %), gasoline (34.3 %) and heating oil (30.7 %). To get a feeling of the importance of the smooth (discontinuous) component of variance, one can perform a simple variance decomposition by dividing BPV (JV) by RV. Such analysis reveals that the discontinuous component accounts for up to 20 % of the total variation of gasoline's returns.[10]

III. Empirical Results

This section presents our main results. We begin by characterizing the dynamics of jumps. Next, we compare the predictive ability of these models in an in-sample setting. Finally, we present a comprehensive and rigorous analysis of the performance of these models.

[8]We also analyze the volatility signature plot, which plots realized volatility as a function of sampling frequency (Andersen et al., 1999). The plots support the choice of the 15-min sampling frequency.

[9]It is worth pointing out that the JV is different from the jump component of realized variance, J_t, used in the HAR models. J_t extracts significant jumps based on a formal jump-testing procedure, whereas JV is simply the difference between RV and BPV.

[10]For an extensive study of the distribution properties of realized volatility in futures markets the interested reader can refer to Thomakos and Wang (2003).

A. The Dynamics of Jumps

Table 2 displays summary statistics of significant jumps. We use a conservative significance level of 0.1 %. We observe that the proportion of jump days ($Intensity$) is highest for gasoline (11.5 %), followed by natural gas (10 %), heating oil (8.5 %) and crude oil (6.3 %).

Following Andersen et al. (2007) and Tauchen and Zhou (2011), we compute the geometric average of monthly jump intensity to obtain a smoothed time-series. We define the jump intensity of a specific month as the number of jump days in that month over the total number of trading days in that particular month. Figure 1 reveals important time variations in the intensity of jumps, which peaks between 2008 and 2009. We also observe interesting differences across markets. While the jump intensities of heating oil and gasoline both steadily decline post-2010, the jump intensity of crude oil displays much more variation.

The second and third rows of Table 2 present some evidence of asymmetries in the time-series of jumps. This is particularly visible by looking at the intensities of positive and negative jumps, reported under $Intensity^+$ and $Intensity^-$, respectively. For example, the proportion of positive jumps (4.1 %) is almost twice as high as that of negative jumps in the crude oil market. Another interesting observation is that the average positive jump return ($Mean^+$) is very similar in magnitude to that of negative jumps ($Mean^-$). Remarkably, this pattern holds for all markets.

B. In-sample Analysis

We begin by analyzing the in-sample predictive power of the competing models introduced in the previous section. To this end, we use all daily observations to estimate the models using OLS.[11] We consider 3 forecasting horizons, namely 1-, 5- and 22-day. Tables 3 to 6 report our results. We report in brackets, the Newey–West corrected t-statistics with 5, 10 and 44 lags for the 1, 5 and 22 day forecasting horizons, respectively. We highlight in bold, all significant estimates at the 5 %

[11] One may argue that volatility itself fluctuates significantly over time, raising concerns that the OLS estimation may be driven by a small pocket of data. To address this concern, we also use a weighted least squares (WLS) estimation. See Section IV. for further details.

level. Looking first at the benchmark HAR–RV model, we see that this model predicts realized variance with adjusted R^2 up to 76 % in the crude oil market.

Turning to the HAR–J specification, we observe that the coefficient of the jump component is generally positive and statistically significant. This is true for all forecast horizons. This result indicates that volatility increases following a jump event. The magnitude of the jump component differs across markets. In particular, the coefficient estimate takes the value 0.098, 0.074 and 0.038 in the crude oil, heating oil and natural gas markets, respectively.

Focusing on the HAR–ARJ model, we see that the negative jump component generally dominates its positive counterpart. This is true in terms of both economic magnitude and statistical significance. Interestingly, the negative jump component enters the regression with a negative loading, indicating that negative jumps predict increases in future volatility. These results generally hold across all horizons.

The finding that only the negative jump component of the HAR–ARJ model is statistically significant helps understand why the HAR–RJ typically yields an insignificant jump component. Since positive and negative jumps are mixed together, this blurs the information content of jumps and buries any evidence of predictability.

The last row of each panel reveals that the jump components of the HAR–C–J model are generally significant. In spite of the statistical significance of the jump components exhibited by the more sophisticated models, we can see that there is very little to distinguish between the explanatory power of all models. This is true for all markets and forecast horizons. For instance, in the crude oil market, all models yield adjusted R^2 roughly equal to 82 %. The upshot of this is that the benefits of explicitly modeling the dynamics of jumps are small.

C. Out-of-Sample Analysis

We now turn our attention to the out-of-sample performance of the competing models. To do this, we adopt a simple procedure that allows us to generate forecasts using parameters estimated on a rolling windows basis. Each day, we use the most recent 600 observations to estimate the forecasting models.[12] Equipped with

[12] Section IV. considers other window sizes such as 400, 800 and 1,000. Our main conclusions are robust to the width of the rolling window.

the parameter estimates, we generate out-of-sample volatility forecasts for a given horizon, e.g. 22-day, which we then compare with realized volatility (computed ex-post). We roll our window forward by one day and repeat all the steps above, yielding a time-series of volatility forecasts that are compared with the corresponding realized volatility. We do this for each market, model and forecasting horizon.

We consider the following 6 loss functions: the mean squared error (MSE), the mean squared percentage error ($MSPE$), the mean absolute error (MAE), the mean absolute percentage error ($MAPE$), the logarithmic loss (LL) and the quasi–likelihood loss ($QLIKE$). These loss functions are defined as follows:

$$MSE = \frac{1}{N}\sum_{t=1}^{N}(RV_{t:t+h}^{1/2} - F_{t:t+h}^{1/2})^2 \qquad MSPE = \frac{1}{N}\sum_{t=1}^{N}\left(\frac{RV_{t:t+h}^{1/2} - F_{t:t+h}^{1/2}}{F_{t:t+h}^{1/2}}\right)^2$$

$$MAE = \frac{1}{N}\sum_{t=1}^{N}|RV_{t:t+h}^{1/2} - F_{t:t+h}^{1/2}| \qquad MAPE = \frac{1}{N}\sum_{t=1}^{N}\left|\frac{RV_{t:t+h}^{1/2} - F_{t:t+h}^{1/2}}{F_{t:t+h}^{1/2}}\right|$$

$$LL = \frac{1}{N}\sum_{t=1}^{N}\left[log(RV_{t:t+h}^{1/2}) - log(F_{t:t+h}^{1/2})\right] \qquad QLIKE = \frac{1}{N}\sum_{t=1}^{N}\left[log(F_{t:t+h}^{1/2}) + \frac{RV_{t:t+h}^{1/2}}{F_{t:t+h}^{1/2}}\right]$$

where N is the number of out-of-sample forecasts, $RV_{t:t+h}^{1/2}$ is the ex-post realized volatility and $F_{t:t+h}^{1/2}$ is the volatility forecast from each of the five forecasting models.

Table 7 presents the forecasting errors. Each panel focuses on a specific loss function. While each row corresponds to a specific market, each column represents a specific forecasting model. We present the results for each forecasting horizon.

We observe that the forecast errors of the more complex models are of the same order of magnitude as those of the baseline HAR–RV, indicating that modeling jumps does not noticeably improve forecast accuracy. For instance, the MSEs of natural gas (monthly horizon) vary within a tight range from 0.837 (HAR–ARJ) to 0.847 (HAR–C–J). Clearly, there is very little to distinguish between all competing models. This example also reveals that the most elaborated model, i.e. HAR–C–J, often produces the worst forecast of realized variance, thus strengthening our main conclusion.

Up to this point, we only analyze the magnitudes of the loss functions and do not formally investigate whether the economically small differences are statistically significant. We rigorously address this question by implementing the statistical test of Giacomini and White (2006), which accounts for parameter uncertainty and allows for comparison of nested models.

The GW test is based on the expected difference in forecast errors between two competing models. Let h and $\Delta L_{i,j}$ denote the forecast horizon and the vector of the difference between the loss functions of models i and j, respectively. The null hypothesis of the GW test is:

$$H_0 : E\left[\Delta L_{i,j}\right] = 0 \tag{25}$$

The test follows a chi–squared distribution with one degree of freedom and the null is evaluated on the basis of the following test statistic:

$$GW = P\left(P^{-1}\sum_{t=1}^{T-h}\Delta L_{t+h,i,j}\right)'\hat{V}_h^{-1}\left(P^{-1}\sum_{t=1}^{T-h}\Delta L_{t+h,i,j}\right) \sim \chi^2_1 \tag{26}$$

where P is the total number of out-of-sample forecasts, $\Delta L_{t+h,i,j}$ is the loss difference at time $t+h$ and $\hat{V_h}$ is a heteroskedasticity and autocorrelation consistent (HAC) estimator of the asymptotic variance of $P^{-1}\sum_t \Delta L_{t+h}$. Following Giacomini and White (2006), we employ the Newey-West (1987) estimator with *h-1* lags to account for the serial dependence in multistep-ahead forecasts. Using a significance level α, the null of equal predictive ability is rejected if $|GW| > \chi^2_{1,1-\alpha}$, where $\chi^2_{1,1-\alpha}$ is the critical value from a chi-squared distribution with one degree of freedom.

Tables 8 to 11 summarize our results. The test statistics presented in the table are based on the mean difference between the model [name in row] and the model [name in column]. Hence, a negative test statistics means that the model [name in row] yields more accurate forecasts than the model [name in column]. We highlight in bold statistically significant test statistics at the 5 % significance level.

Comparing our baseline model (HAR–RV) to its more sophisticated rivals, we find very little evidence to suggest that explicitly modeling jumps significantly improves the accuracy of volatility forecasts. To quickly see this, notice that very few entries in the column headed “HAR–RV” are boldfaced, suggesting that the more elaborated models yield forecasts that are not statistically distinguishable from those of the simple and parsimonious HAR–RV. This is true, irrespective of the forecasting horizon, the market and the loss function. Moreover, the most complex model, i.e. HAR–C–J, significantly underperforms all other models (including the benchmark HAR–RV). This is particularly noticeable in the crude oil and gasoline markets, where significantly positive entries are often reported in the last row. This result echoes our core finding: the simpler the model, the better.

In sum, our out-of-sample analysis reveals that models that explicitly seek to capture the dynamics of jumps do not significantly improve the accuracy of volatility forecasts: there is virtually no gain in modeling the dynamics of jumps in energy markets.

IV. Robustness Checks

In this section, we conduct several additional tests to investigate the robustness of our findings. We begin by analyzing whether our main findings hold if we consider the task of predicting variance (rather than volatility). We then explore the robustness of our results with respect to the jump detection procedure by using the nearest neighbor estimator of Andersen et al. (2012). Additionally, we show that our results are robust to the width of the window used to obtain rolling forecasts. Finally, we consider a WLS (rather than OLS) estimation to establish that our findings are not affected by the method of estimation.

A. Volatility vs. Variance Forecasts

Up to this point, our analysis focuses on the task of forecasting volatility. As previously discussed, we focus on volatility instead of variance because of the key role it plays in modern finance. For instance, volatility (not variance) is a key input in option pricing and modern portfolio theories. Nonetheless, one may argue that the jump detection tests identify jumps in variance not in volatility, and this subtle difference may matter for our analysis.

Tables A.1–A.5 of the complementary appendix investigate whether modeling jumps can improve the accuracy of *variance* forecasts. Consistent with our main findings, our results establish that more sophisticated models do not generally outperform the baseline specification. The upshot of this is that our results are the same, irrespective of whether we look at volatility or variance forecasting.

B. Alternative Jump-Robust Estimators

Andersen et al. (2012) point out that the standard multipower variations may be biased in finite samples. The authors then propose jump-robust volatility estimators that use the nearest neighbor truncation. They forcefully show that the "median realized variance estimator" ($MedRV_t$) is more efficient and robust to jumps than its main rivals. As a robustness check, we repeat our analysis replacing BPV with the $MedRV$ variation estimator. This estimator, using staggered (skip–1) returns, is defined as follows:

$$MedRV_t = \frac{\pi}{6-4\sqrt{3}+\pi}\left(\frac{M}{M-2(k+1)}\right)\sum_{j=2k+3}^{M} med\left(|r_{t_{j-2(k+1)}}|, |r_{t_{j-(k+1)}}|, |r_{t_j}|\right)^2 \tag{27}$$

where $med(\cdot)$ stands for the median operator. As in our main analysis, we set k=1 (skip–1 return). The corresponding jump test statistic is as follows:

$$z_{Med,t} = \Delta^{-1/2}\frac{(RV_t - MedRV_t)/RV_t}{\sqrt{0.96\ max(1, \frac{MedRQ_t}{MedRV_t^2})}} \tag{28}$$

The number 0.96 comes from the asymptotic distribution of the $MedRV$ estimator.[13] Notice also that the tripower quarticity in the test statistic of Equation (10) is replaced with the median realized quarticity given by:

$$\begin{aligned} MedRQ_t &= \tfrac{3\pi N}{9\pi+72-52\sqrt{3}}\left(\tfrac{M}{M-2(k+1)}\right) \\ &\cdot \sum_{j=2k+3}^{M} med\left(|r_{t_{j-2(k+1)}}|, |r_{t_{j-(k+1)}}|, |r_{t_j}|\right)^4 \end{aligned} \tag{29}$$

Finally, the decomposition of realized variance into its continuous and jump components is done exactly as in Equations (12) and (13) replacing, BPV with *MedRV*. Tables B.6–B.10 of the online appendix confirm our main findings: specifically accounting for jumps in volatility forecasting does not significantly improve forecasting accuracy.

[13]For further details, we refer the interested reader to Propositions 1–3 in Andersen et al. (2012).

C. Alternative Estimation Periods

Our out-of-sample analysis rests on a rolling window of 600 observations. One may argue that this choice is somewhat arbitrary and wonder what effect, if any, it may have on our results. To investigate this point, we consider windows of 400, 800 and 1,000 observations. Tables C.11 through C.25 of the supplementary appendix clearly show that changing the width of the rolling window has virtually no impact on our main conclusions.

D. Alternative Estimation Methods

Patton and Sheppard (2011) argue that because the dependent variable in the models is volatility, the OLS estimation may put too much weight on highly volatile periods. To address the concern that this may be the driving force behind our results, we estimate each model with WLS (rather than OLS). To be more specific, we first estimate each model using OLS and then employ the inverse of the fitted values as weights for the WLS estimations. Equipped with the parameter estimates, we repeat our main analyses (both in- and out-of-sample) and obtain very similar conclusions (See Tables D.26–D.30) of the appendix.

V. Conclusions

This paper uses high-frequency data on four deep and liquid commodity futures markets, namely crude oil, heating oil, natural gas and gasoline, to identify jumps and analyze their impact on future volatility.

Our analysis establishes that jumps are rare events and their intensity substantially varies over time. We then investigate the importance of jumps for forecasts of realized volatility over horizons ranging from 1 to 22 days. To this end, we estimate and empirically analyze several extensions of the HAR–RV model that explicitly seek to capture the dynamics of jumps. We employ six distinct loss functions and the GW test to carefully assess the predictive ability of these models. Analyzing the magnitude of the error metrics, we find very little to distinguish between the benchmark model and its more complex competitors. Moreover, our rigorous econometric analysis establishes that the differences in forecast errors are not only economically small but also statistically insignificant. Collectively, our results suggest that explicitly modeling jumps does not significantly improve the accuracy of volatility forecasts in energy markets.

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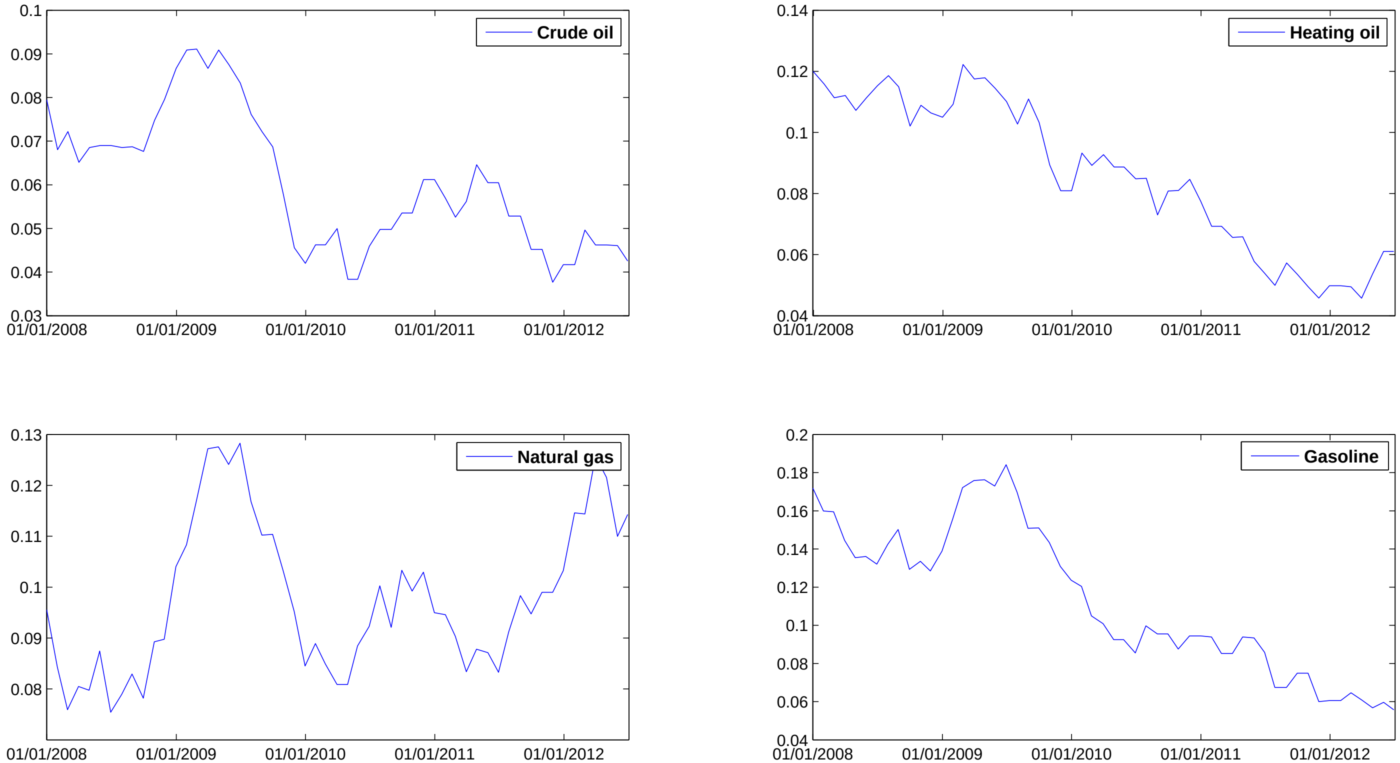

Figure 1: **Time-Varying Jump Intensity**

This figure presents the series of monthly jump intensities for the four energy markets. Monthly jump intensity is the ratio of the number of days associated with jumps in a given month over the total number of days in that particular month. The series of monthly intensities are smoothed by taking a rolling 12-month geometric average.

Table 1: **Summary Statistics for Variation Measures**

This table presents summary statistics of realized variance (RV), bipower variation (BPV), jump variation (JV) and their square root (volatility) counterparts ($\sqrt{RV}$, $\sqrt{BPV}$, $\sqrt{JV}$). JV is computed as: $RV - BPV$. Panels A to D report statistics for crude oil, heating oil, natural gas and gasoline, respectively. The dataset covers the period from January 2, 2007 to June 29, 2012. Daily variance (volatility) series are computed using 15–min returns and are annualized by multiplying by (the square root of) 252. Each panel reports the mean, standard deviation, skewness, kurtosis, minimum and maximum, respectively.

	RV	BPV	JV	$\sqrt{RV}$	$\sqrt{BPV}$	$\sqrt{JV}$
A. Crude oil						
Mean	0.150	0.137	0.017	0.344	0.328	0.086
St. Dev.	0.200	0.185	0.048	0.178	0.171	0.096
Skewness	3.885	3.770	11.118	2.187	2.224	2.526
Kurtosis	23.395	20.217	180.862	8.786	8.875	16.057
Min	0.010	0.008	0.000	0.101	0.091	0.000
Max	2.288	1.655	1.027	1.513	1.286	1.014
B. Heating oil						
Mean	0.114	0.098	0.018	0.307	0.285	0.099
St. Dev.	0.125	0.103	0.038	0.141	0.128	0.090
Skewness	3.230	2.802	6.545	1.732	1.615	1.872
Kurtosis	17.444	12.079	64.123	6.682	5.937	9.522
Min	0.006	0.005	0.000	0.079	0.070	0.000
Max	1.273	0.752	0.540	1.128	0.867	0.735
C. Natural gas						
Mean	0.224	0.189	0.038	0.443	0.408	0.146
St. Dev.	0.204	0.164	0.081	0.167	0.152	0.128
Skewness	4.264	4.227	8.369	1.669	1.433	1.921
Kurtosis	32.985	37.412	104.324	8.576	7.758	11.420
Min	0.013	0.002	0.000	0.116	0.047	0.000
Max	2.295	2.164	1.331	1.515	1.471	1.154
D. Gasoline						
Mean	0.151	0.123	0.030	0.343	0.311	0.123
St. Dev.	0.221	0.169	0.086	0.183	0.160	0.121
Skewness	4.991	4.556	11.844	2.540	2.392	2.886
Kurtosis	37.737	30.949	212.945	11.745	10.714	19.356
Min	0.003	0.001	0.000	0.055	0.023	0.000
Max	2.559	1.831	1.946	1.600	1.353	1.395

Table 2: **Summary Statistics of Significant Daily Jumps**

This table presents summary statistics for the series of significant daily jumps. The first row of the table presents the total number of jump days. We detect statistically significant jumps by using the $z_{TQ,t}$ statistic shown in Equation (10) and a confidence level of 99.9 %. The second and third rows show the total number of jumps positive and negative jumps, respectively. The row labeled "Intensity" shows the jump intensity, that is the ratio of jump days over the total number of days. The next two rows further decompose jump intensity into its positive and negative parts following Tauchen and Zhou (2011). The table also reports the mean ("Mean") and standard deviation ("St.Dev.") of the series of significant jumps, as well as the corresponding statistics for positive and negative jumps.

	Crude oil	Heating oil	Natural gas	Gasoline
#Jumps	90	122	142	163
#Positive	58	65	61	88
#Negative	32	57	81	75
Intensity	0.063	0.085	0.100	0.115
$Intensity^{+}$	0.041	0.045	0.043	0.062
$Intensity^{-}$	0.022	0.040	0.057	0.053
Mean	0.250	0.225	0.337	0.271
$Mean^{+}$	0.254	0.227	0.338	0.281
$Mean^{-}$	-0.243	-0.225	-0.337	-0.260
St. Dev.	0.169	0.134	0.189	0.188
$St.Dev.^{+}$	0.180	0.142	0.224	0.190
$St.Dev.^{-}$	0.149	0.127	0.160	0.186

Table 3: **In-Sample Predictability: Crude Oil**

This table assesses the predictive ability of several models in the crude oil market. Each of the three panels shows results for a different forecasting horizon (1-, 5- and 22-day horizon). The jump components are computed based on Equation (12) using the test statistic of Equation (10) and a significance level of 0.1 %. All regressions are estimated using Newey–West (1987) corrected standard errors with 5, 10 and 44 lags for the 1-, 5- and 22-day horizon, respectively. T-statistics are reported in parentheses. The intercepts are not reported to save space. Significant coefficients at the 5% level are highlighted in bold. The second last column reports the adjusted R^2 of each regression. The last column shows the number of observations. The sample period is from January 2, 2007 to June 29, 2012.

	$\beta_{\mathbf{d}}$	$\beta_{\mathbf{w}}$	$\beta_{\mathbf{m}}$	$\beta_{\mathbf{C_d}}$	$\beta_{\mathbf{C_w}}$	$\beta_{\mathbf{C_m}}$	$\gamma_{\mathbf{J_d}}$	$\gamma_{\mathbf{J_w}}$	$\gamma_{\mathbf{J_m}}$	$\gamma_{\mathbf{RJ}}$	$\gamma_{\mathbf{RJ^+}}$	$\gamma_{\mathbf{RJ^-}}$	$\bar{\mathbf{R}}^{\mathbf{2}}$	**Obs.**
							Panel A: 1-Day Horizon							
HAR–RV	**0.334**	**0.396**	**0.226**	-	-	-	-	-	-	-	-	-	0.758	1392
	(7.115)	(7.56)	(4.561)											
HAR–J	-	**0.397**	**0.230**	**0.334**	-	-	0.053	-	-	-	-	-	0.757	1392
		(7.396)	(4.506)	(6.222)			(0.916)							
HAR–RJ	-	**0.397**	**0.234**	**0.331**	-	-	-	-	-	0.037	-	-	0.757	1392
		(7.401)	(4.362)	(6.055)						(0.78)				
HAR–ARJ	-	**0.396**	**0.231**	**0.334**	-	-	-	-	-	-	0.069	-0.022	0.757	1392
		(7.377)	(4.48)	(6.174)							(0.925)	(-0.359)		
HAR–C–J	-	-	-	**0.326**	**0.409**	**0.220**	0.053	0.041	0.018	-	-	-	0.759	1392
				(6.164)	(7.531)	(4.194)	(0.935)	(1.905)	(1.361)					
							Panel B: 5-Day Horizon							
HAR–RV	**0.264**	**0.431**	**0.257**	-	-	-	-	-	-	-	-	-	0.852	1388
	(7.896)	(8.683)	(5.446)											
HAR–J	-	**0.433**	**0.258**	**0.261**	-	-	**0.080**	-	-	-	-	-	0.851	1388
		(8.635)	(5.416)	(6.846)			(2.178)							
HAR–RJ	-	**0.437**	**0.263**	**0.256**	-	-	-	-	-	-0.031	-	-	0.850	1388
		(8.58)	(5.275)	(6.737)						(-1.207)				
HAR–ARJ	-	**0.435**	**0.256**	**0.261**	-	-	-	-	-	-	0.039	**-0.160**	0.852	1388
		(8.638)	(5.384)	(6.946)							(1.103)	(-2.61)		
HAR–C–J	-	-	-	**0.250**	**0.446**	**0.245**	**0.082**	**0.038**	0.030	-	-	-	0.854	1388
				(6.78)	(8.682)	(4.905)	(2.133)	(2.011)	(1.665)					
							Panel C: 22-Day Horizon							
HAR–RV	**0.233**	**0.395**	**0.270**	-	-	-	-	-	-	-	-	-	0.817	1371
	(7.501)	(4.604)	(3.559)											
HAR–J	-	**0.399**	**0.271**	**0.227**	-	-	**0.098**	-	-	-	-	-	0.817	1371
		(4.577)	(3.587)	(7.03)			(4.027)							
HAR–RJ	-	**0.403**	**0.278**	**0.222**	-	-	-	-	-	-0.010	-	-	0.815	1371
		(4.538)	(3.597)	(7.092)						(-0.567)				
HAR–ARJ	-	**0.400**	**0.270**	**0.228**	-	-	-	-	-	-	**0.068**	**-0.154**	0.817	1371
		(4.554)	(3.544)	(7.054)							(3.156)	(-3.365)		
HAR–C–J	-	-	-	**0.213**	**0.401**	**0.239**	**0.097**	**0.071**	**0.064**	-	-	-	0.825	1371
				(7.461)	(4.662)	(3.131)	(4.459)	(2.382)	(1.671)					

Table 4: **In-Sample Predictability: Heating Oil**

This table assesses the predictive ability of several models in the heating oil market. Each of the three panels shows results for a different forecasting horizon (1-, 5- and 22-day horizon). The jump components are computed based on Equation (12) using the test statistic of Equation (10) and a significance level of 0.1 %. All regressions are estimated using Newey–West (1987) corrected standard errors with 5, 10 and 44 lags for the 1-, 5- and 22-day horizon, respectively. T-statistics are reported in parentheses. The intercepts are not reported to save space. Significant coefficients at the 5% level are highlighted in bold. The second last column reports the adjusted R^2 of each regression. The last column shows the number of observations. The sample period is from January 2, 2007 to June 29, 2012.

	$\beta_{\mathbf{d}}$	$\beta_{\mathbf{w}}$	$\beta_{\mathbf{m}}$	$\beta_{\mathbf{C_d}}$	$\beta_{\mathbf{C_w}}$	$\beta_{\mathbf{C_m}}$	$\gamma_{\mathbf{J_d}}$	$\gamma_{\mathbf{J_w}}$	$\gamma_{\mathbf{J_m}}$	$\gamma_{\mathbf{RJ}}$	$\gamma_{\mathbf{RJ^+}}$	$\gamma_{\mathbf{RJ^-}}$	$\mathbf{\bar{R}^2}$	**Obs.**
							Panel A: 1-Day Horizon							
HAR–RV	**0.296**	**0.394**	**0.265**	-	-	-	-	-	-	-	-	-	0.711	1395
	(5.952)	(7.608)	(5.558)											
HAR–J	-	**0.396**	**0.267**	**0.294**	-	-	**0.097**	-	-	-	-	-	0.710	1395
		(7.265)	(5.492)	(5.137)			(2.59)							
HAR–RJ	-	**0.406**	**0.276**	**0.283**	-	-	-	-	-	0.018	-	-	0.708	1395
		(7.359)	(5.422)	(4.825)						(0.443)				
HAR–ARJ	-	**0.396**	**0.267**	**0.294**	-	-	-	-	-	-	**0.107**	-0.086	0.710	1395
		(7.285)	(5.508)	(5.117)							(2.151)	(-1.462)		
HAR–C–J	-	-	-	**0.291**	**0.399**	**0.250**	**0.096**	0.039	0.027	-	-	-	0.710	1395
				(5.168)	(7.768)	(5.057)	(2.591)	(1.866)	(1.758)					
							Panel B: 5-Day Horizon							
HAR–RV	**0.240**	**0.377**	**0.331**	-	-	-	-	-	-	-	-	-	0.823	1391
	(8.999)	(7.694)	(7.46)											
HAR–J	-	**0.377**	**0.332**	**0.239**	-	-	**0.085**	-	-	-	-	-	0.823	1391
		(7.508)	(7.478)	(7.616)			(3.444)							
HAR–RJ	-	**0.385**	**0.341**	**0.230**	-	-	-	-	-	-0.020	-	-	0.821	1391
		(7.558)	(7.513)	(7.04)						(-0.719)				
HAR–ARJ	-	**0.376**	**0.332**	**0.240**	-	-	-	-	-	-	**0.061**	**-0.114**	0.823	1391
		(7.549)	(7.534)	(7.729)							(2.184)	(-2.887)		
HAR–C–J	-	-	-	**0.233**	**0.395**	**0.303**	**0.083**	0.026	0.033	-	-		0.825	1391
				(7.558)	(7.822)	(6.496)	(3.261)	(1.464)	(1.823)					
							Panel C: 22-Day Horizon							
HAR–RV	**0.180**	**0.304**	**0.425**	-	-	-	-	-	-	-	-	-	0.821	1374
	(9.561)	(5.443)	(6.593)											
HAR–J	-	**0.306**	**0.425**	**0.178**	-	-	**0.074**	-	-	-	-	-	0.821	1374
		(5.474)	(6.577)	(8.98)			(4.167)							
HAR–RJ	-	**0.312**	**0.433**	**0.170**	-	-	-	-	-	-0.034	-	-	0.819	1374
		(5.364)	(6.531)	(8.806)						(-0.966)				
HAR–ARJ	-	**0.304**	**0.426**	**0.179**	-	-	-	-	-	-	0.038	**-0.118**	0.821	1374
		(5.433)	(6.566)	(9.123)							(1.521)	(-2.543)		
HAR–C–J	-	-	-	**0.173**	**0.307**	**0.405**	**0.072**	**0.041**	0.035	-	-	-	0.823	1374
				(8.705)	(5.096)	(5.769)	(4.196)	(2.597)	(1.245)					

Table 5: **In-Sample Predictability: Natural Gas**

This table assesses the predictive ability of several models in the natural gas market. Each of the three panels shows results for a different forecasting horizon (1-, 5- and 22-day horizon). The jump components are computed based on Equation (12) using the test statistic of Equation (10) and a significance level of 0.1 %. All regressions are estimated using Newey–West (1987) corrected standard errors with 5, 10 and 44 lags for the 1-, 5- and 22-day horizon, respectively. T-statistics are reported in parentheses. The intercepts are not reported to save space. Significant coefficients at the 5% level are highlighted in bold. The second last column reports the adjusted R^2 *of each regression. The last column shows the number of observations. The sample period is from January 2, 2007 to June 29, 2012.*

	$\beta_{\mathbf{d}}$	$\beta_{\mathbf{w}}$	$\beta_{\mathbf{m}}$	$\beta_{\mathbf{C_d}}$	$\beta_{\mathbf{C_w}}$	$\beta_{\mathbf{C_m}}$	$\gamma_{\mathbf{J_d}}$	$\gamma_{\mathbf{J_w}}$	$\gamma_{\mathbf{J_m}}$	$\gamma_{\mathbf{RJ}}$	$\gamma_{\mathbf{RJ^+}}$	$\gamma_{\mathbf{RJ^-}}$	$\mathbf{\bar{R}^2}$	**Obs.**
							Panel A: 1-Day Horizon							
HAR–RV	**0.253**	**0.438**	**0.174**	-	-	-	-	-	-	-	-	-	0.444	1396
	(6.97)	(9.44)	(3.771)											
HAR–J	-	**0.419**	**0.169**	**0.284**	-	-	**0.053**	-	-	-	-	-	0.447	1396
		(8.988)	(3.686)	(6.494)			(1.99)							
HAR–RJ	-	**0.418**	**0.171**	**0.286**	-	-	-	-	-	-0.002	-	-	0.446	1396
		(8.923)	(3.67)	(6.444)						(-0.088)				
HAR–ARJ	-	**0.419**	**0.169**	**0.284**	-	-	-	-	-	-	0.054	-0.052	0.447	1396
		(9.021)	(3.682)	(6.417)							(1.571)	(-1.353)		
HAR–C–J	-	-	-	**0.269**	**0.439**	**0.181**	**0.054**	**0.057**	-0.009	-	-	-	0.451	1396
				(6.334)	(8.434)	(3.632)	(2.029)	(2.998)	(-0.873)					
							Panel B: 5-Day Horizon							
HAR–RV	**0.237**	**0.400**	**0.200**	-	-	-	-	-	-	-	-	-	0.602	1392
	(7.202)	(9.344)	(3.947)											
HAR–J	-	**0.390**	**0.198**	**0.252**	-	-	**0.079**	-	-	-	-	-	0.604	1392
		(8.971)	(3.927)	(6.669)			(3.005)							
HAR–RJ	-	**0.388**	**0.201**	**0.256**	-	-	-	-	-	-0.015	-	-	0.600	1392
		(8.854)	(3.88)	(6.575)						(-0.64)				
HAR–ARJ	-	**0.390**	**0.198**	**0.253**	-	-	-	-	-	-	0.068	**-0.089**	0.604	1392
		(9.006)	(3.917)	(6.514)							(1.85)	(-2.39)		
HAR–C–J	-	-	-	**0.236**	**0.423**	**0.198**	**0.080**	**0.040**	-0.004	-	-	-	0.611	1392
				(6.664)	(7.902)	(3.655)	(2.993)	(2.001)	(-0.282)					
							Panel C: 22-Day Horizon							
HAR–RV	**0.170**	**0.291**	**0.266**	-	-	-	-	-	-	-	-	-	0.540	1375
	(8.527)	(8.189)	(3.075)											
HAR–J	-	**0.278**	**0.263**	**0.191**	-	-	**0.038**	-	-	-	-	-	0.542	1375
		(7.383)	(3.033)	(7.796)			(2.11)							
HAR–RJ	-	**0.276**	**0.264**	**0.195**	-	-	-	-	-	**-0.043**	-	-	0.543	1375
		(7.21)	(3.019)	(7.526)						(-2.218)				
HAR–ARJ	-	**0.276**	**0.262**	**0.194**	-	-	-	-	-	-	-0.006	**-0.076**	0.543	1375
		(7.333)	(3.034)	(7.626)							(-0.249)	(-2.414)		
HAR–C–J	-	-	-	**0.177**	**0.315**	**0.245**	**0.038**	0.022	0.011	-	-	-	0.547	1375
				(7.825)	(7.015)	(2.998)	(2.129)	(1.421)	(0.419)					

Table 6: **In-Sample Predictability: Gasoline**

This table assesses the predictive ability of several models in the gasoline market. Each of the three panels shows results for a different forecasting horizon (1-, 5- and 22-day horizon). The jump components are computed based on Equation (12) using the test statistic of Equation (10) and a significance level of 0.1 %. All regressions are estimated using Newey–West (1987) corrected standard errors with 5, 10 and 44 lags for the 1-, 5- and 22-day horizon, respectively. T-statistics are reported in parentheses. The intercepts are not reported to save space. Significant coefficients at the 5% level are highlighted in bold. The second last column reports the adjusted R^2 of each regression. The last column shows the number of observations. The sample period is from January 2, 2007 to June 29, 2012.

	$\beta_{\mathbf{d}}$	$\beta_{\mathbf{w}}$	$\beta_{\mathbf{m}}$	$\beta_{\mathbf{C_d}}$	$\beta_{\mathbf{C_w}}$	$\beta_{\mathbf{C_m}}$	$\gamma_{\mathbf{J_d}}$	$\gamma_{\mathbf{J_w}}$	$\gamma_{\mathbf{J_m}}$	$\gamma_{\mathbf{RJ}}$	$\gamma_{\mathbf{RJ^+}}$	$\gamma_{\mathbf{RJ^-}}$	$\mathbf{\bar{R}^2}$	**Obs.**
							Panel A: 1-Day Horizon							
HAR–RV	**0.210**	**0.475**	**0.265**	-	-	-	-	-	-	-	-	-	0.730	1394
	(3.463)	(7.898)	(4.687)											
HAR–J	-	**0.476**	**0.267**	**0.222**	-	-	0.021	-	-	-	-	-	0.731	1394
		(7.665)	(4.645)	(3.361)			(0.431)							
HAR–RJ	-	**0.490**	**0.266**	**0.213**	-	-	-	-	-	-0.049	-	-	0.732	1394
		(7.687)	(4.686)	(3.216)						(-1.533)				
HAR–ARJ	-	**0.484**	**0.261**	**0.220**	-	-	-	-	-	-	-0.023	-0.080	0.732	1394
		(7.629)	(4.532)	(3.362)							(-0.433)	(-1.166)		
HAR–C–J	-	-	-	**0.204**	**0.491**	**0.332**	0.023	0.032	-0.011	-	-	-	0.734	1394
				(2.979)	(6.821)	(4.893)	(0.474)	(1.453)	(-0.674)					
							Panel B: 5-Day Horizon							
HAR–RV	**0.215**	**0.458**	**0.285**	-	-	-	-	-	-	-	-	-	0.858	1390
	(7.984)	(8.558)	(5.616)											
HAR–J	-	**0.459**	**0.286**	**0.227**	-	-	0.034	-	-	-	-	-	0.859	1390
		(8.495)	(5.466)	(8.145)			(1.222)							
HAR–RJ	-	**0.473**	**0.289**	**0.216**	-	-	-	-	-	-0.031	-	-	0.859	1390
		(8.576)	(5.446)	(7.475)						(-1.522)				
HAR–ARJ	-	**0.464**	**0.282**	**0.226**	-	-	-	-	-	-	0.004	**-0.073**	0.859	1390
		(8.437)	(5.344)	(8.183)							(0.105)	(-2.231)		
HAR–C–J	-	-	-	**0.206**	**0.493**	**0.323**	0.038	0.020	0.000	-	-	-	0.864	1390
				(7.05)	(8.918)	(6.165)	(1.479)	(1.141)	(0.005)					
							Panel C: 22-Day Horizon							
HAR–RV	**0.198**	**0.420**	**0.296**	-	-	-	-	-	-	-	-	-	0.835	1373
	(6.142)	(4.621)	(3.081)											
HAR–J	-	**0.421**	**0.297**	**0.209**	-	-	0.029	-	-	-	-	-	0.836	1373
		(4.611)	(3.049)	(6.113)			(1.195)							
HAR–RJ	-	**0.432**	**0.300**	**0.200**	-	-	-	-	-	-0.020	-	-	0.836	1373
		(4.683)	(3.018)	(5.627)						(-1.054)				
HAR–ARJ	-	**0.424**	**0.295**	**0.208**	-	-	-	-	-	-	0.009	**-0.056**	0.836	1373
		(4.602)	(3.018)	(6.079)							(0.281)	(-1.869)		
HAR–C–J	-	-	-	**0.191**	**0.444**	**0.329**	0.032	0.025	0.005	-	-	-	0.840	1373
				(6.784)	(4.815)	(3.895)	(1.346)	(1.324)	(0.133)					

Table 7: **Forecasting Errors**

This table presents out-of-sample forecasting errors for the five volatility models considered. Each panel focuses on a specific loss function. MSE is the mean squared error, $MSPE$ is the mean squared percentage error, MAE is the mean absolute error, $MAPE$ is the mean absolute percentage error, LL is the logarithmic loss and $QLIKE$ is the quasi likelihood loss function. We consider three forecast horizons, namely 1, 5 and 22 days. Out-of-sample forecasts are obtained using a rolling window of 600 observations. In order to facilitate the presentation of our results, we multiply each loss function by 100.

	1-Day					5-Day					22-Day				
A. MSE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	0.535	0.533	0.530	0.533	0.538	0.376	0.375	0.373	0.374	0.381	0.401	0.401	0.400	0.403	0.419
Heating oil	0.399	0.400	0.396	0.400	0.400	0.247	0.247	0.247	0.247	0.248	0.244	0.245	0.247	0.245	0.249
Natural gas	1.494	1.487	1.491	1.493	1.483	0.842	0.836	0.849	0.841	0.848	0.841	0.838	0.838	0.837	0.847
Gasoline	0.484	0.483	0.484	0.483	0.499	0.297	0.299	0.299	0.299	0.314	0.289	0.291	0.293	0.292	0.309
B. MSPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	7.339	7.342	7.275	7.351	7.511	3.540	3.543	3.480	3.504	3.718	4.320	4.348	4.303	4.354	4.721
Heating oil	7.488	7.513	7.422	7.502	7.501	3.525	3.532	3.511	3.529	3.502	4.054	4.060	4.072	4.063	4.067
Natural gas	8.250	8.209	8.241	8.253	8.204	3.079	3.044	3.104	3.087	3.065	3.128	3.111	3.130	3.128	3.138
Gasoline	7.740	7.733	7.766	7.744	7.950	3.663	3.668	3.682	3.677	3.830	4.197	4.217	4.227	4.226	4.379
C. MAE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	5.150	5.165	5.144	5.169	5.214	4.206	4.205	4.196	4.199	4.303	5.012	5.010	5.005	5.011	5.119
Heating oil	4.505	4.511	4.491	4.511	4.510	3.649	3.650	3.653	3.644	3.650	4.102	4.103	4.117	4.100	4.147
Natural gas	8.721	8.715	8.739	8.731	8.721	6.229	6.212	6.290	6.224	6.287	6.632	6.629	6.631	6.616	6.589
Gasoline	5.142	5.148	5.134	5.132	5.233	4.003	4.015	4.011	4.004	4.136	4.360	4.376	4.390	4.379	4.547
D. MAPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	18.872	18.921	18.839	18.935	19.157	14.226	14.220	14.161	14.179	14.676	17.047	17.050	17.011	17.039	17.626
Heating oil	18.976	18.991	18.906	18.988	19.001	14.474	14.472	14.458	14.448	14.432	16.410	16.412	16.441	16.386	16.497
Natural gas	21.150	21.136	21.205	21.179	21.141	13.646	13.603	13.789	13.641	13.718	14.174	14.162	14.181	14.147	14.059
Gasoline	19.832	19.847	19.812	19.797	20.134	14.616	14.632	14.615	14.591	15.004	16.239	16.273	16.321	16.290	16.776
E. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	5.633	5.634	5.597	5.645	5.708	3.593	3.577	3.549	3.551	3.676	4.238	4.239	4.228	4.242	4.456
Heating oil	5.610	5.615	5.561	5.614	5.626	3.321	3.323	3.318	3.321	3.315	3.539	3.541	3.560	3.546	3.544
Natural gas	6.615	6.597	6.621	6.629	6.562	3.104	3.079	3.132	3.102	3.080	3.250	3.237	3.244	3.241	3.239
Gasoline	5.914	5.901	5.915	5.909	6.072	3.459	3.475	3.482	3.476	3.641	3.658	3.680	3.696	3.691	3.851
F. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	-25.534	-25.536	-25.552	-25.530	-25.507	-23.233	-23.245	-23.255	-23.258	-23.209	-21.189	-21.192	-21.193	-21.191	-21.103
Heating oil	-37.243	-37.242	-37.266	-37.242	-37.236	-35.058	-35.058	-35.059	-35.059	-35.059	-33.582	-33.581	-33.570	-33.577	-33.580
Natural gas	14.556	14.548	14.557	14.563	14.523	16.996	16.984	17.010	16.994	16.980	18.177	18.170	18.172	18.171	18.167
Gasoline	-29.255	-29.263	-29.255	-29.258	-29.171	-27.032	-27.021	-27.019	-27.022	-26.931	-25.457	-25.444	-25.435	-25.438	-25.355

Table 8: **Out-of-Sample Forecast Comparisons for Crude Oil**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for crude oil volatility. Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1-Day Horizon			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
A. SE				
HAR–RV	–			
HAR–J	-0.60	–		
HAR–RJ	-2.75	-2.25	–	
HAR–ARJ	-0.35	0.53	**4.98**	–
HAR–C–J	0.38	**4.30**	**6.30**	3.04
B. SPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	0.00	–		
HAR–RJ	-1.35	-3.36	–	
HAR–ARJ	0.05	0.24	**7.43**	–
HAR–C–J	**4.33**	**6.73**	**8.96**	**5.53**
C. AE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	0.53	–		
HAR–RJ	-0.12	-2.97	–	
HAR–ARJ	0.79	0.71	**5.33**	–
HAR–C–J	**5.00**	**6.44**	**8.97**	**5.17**
D. APE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	0.34	–		
HAR–RJ	-0.26	-3.05	–	
HAR–ARJ	0.55	0.65	**4.93**	–
HAR–C–J	**6.42**	**10.22**	**13.10**	**8.49**
E. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	0.00	–		
HAR–RJ	-1.36	-2.42	–	
HAR–ARJ	0.08	1.45	**4.58**	–
HAR–C–J	2.15	**5.81**	**7.99**	3.70
F. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	-0.01	–		
HAR–RJ	-1.34	-1.72	–	
HAR–ARJ	0.03	1.79	3.31	–
HAR–C–J	1.07	3.78	**5.37**	2.09

	5-Day Horizon			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	-0.31	–		
HAR–RJ	-1.61	-0.70	–	
HAR–ARJ	-1.11	-0.90	0.10	–
HAR–C–J	0.58	1.05	1.67	1.50
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	0.01	–		
HAR–RJ	-1.98	-2.54	–	
HAR–ARJ	-0.79	-2.90	0.80	–
HAR–C–J	3.79	3.76	**6.73**	**5.28**
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	0.00	–		
HAR–RJ	-0.46	-0.45	–	
HAR–ARJ	-0.21	-0.35	0.06	–
HAR–C–J	3.76	**4.37**	**4.74**	**4.70**
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	-0.01	–		
HAR–RJ	-1.04	-1.10	–	
HAR–ARJ	-0.62	-1.11	0.14	–
HAR–C–J	**5.90**	**6.80**	**8.00**	**7.67**
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	-0.57	–		
HAR–RJ	-2.54	-1.42	–	
HAR–ARJ	-2.39	-3.26	0.01	–
HAR–C–J	1.48	2.35	3.59	3.64
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	-1.33	–		
HAR–RJ	-2.86	-0.84	–	
HAR–ARJ	-3.31	-3.12	-0.07	–
HAR–C–J	0.48	1.29	1.88	2.22

	22-Day Horizon			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	0.06	–		
HAR–RJ	-0.17	-0.23	–	
HAR–ARJ	0.40	0.59	0.57	–
HAR–C–J	1.01	1.06	1.09	0.95
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	1.98	–		
HAR–RJ	-0.61	-1.90	–	
HAR–ARJ	1.05	0.05	1.62	–
HAR–C–J	2.76	2.54	3.00	2.59
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	-0.03	–		
HAR–RJ	-0.34	-0.09	–	
HAR–ARJ	-0.01	0.02	0.11	–
HAR–C–J	0.57	0.65	0.64	0.63
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	0.00	–		
HAR–RJ	-0.75	-0.49	–	
HAR–ARJ	-0.02	-0.12	0.25	–
HAR–C–J	1.33	1.41	1.50	1.45
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	0.00	–		
HAR–RJ	-0.39	-0.26	–	
HAR–ARJ	0.03	0.06	0.34	–
HAR–C–J	1.44	1.56	1.57	1.54
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–			
HAR–J	-0.16	–		
HAR–RJ	-0.28	-0.01	–	
HAR–ARJ	-0.03	0.08	0.05	–
HAR–C–J	0.95	1.14	1.05	1.11

Table 9: **Out-of-Sample Forecast Comparisons for Heating Oil**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for heating oil volatility. Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1-Day Horizon				*5-Day Horizon*				*22-Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
A. SE												
HAR–RV	–				–				–			
HAR–J	0.11	–			0.44	–			0.84	–		
HAR–RJ	-2.23	-2.65	–		0.23	0.01	–		2.02	1.07	–	
HAR–ARJ	0.08	0.00	2.73	–	0.19	-0.01	-0.02	–	0.34	0.06	-1.06	–
HAR–C–J	0.34	0.20	2.61	0.20	0.29	0.08	0.04	0.10	0.61	0.51	0.16	0.38
B. SPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.27	–			0.13	–			0.32	–		
HAR–RJ	-1.50	-2.94	–		-0.26	-0.54	–		0.77	0.30	–	
HAR–ARJ	0.07	-0.33	2.66	–	0.03	-0.05	0.33	–	0.18	0.03	-0.24	–
HAR–C–J	0.03	-0.04	0.93	0.00	-0.29	-0.49	-0.04	-0.34	0.03	0.01	0.00	0.00
C. AE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.14	–			0.02	–			0.10	–		
HAR–RJ	-0.87	-1.55	–		0.09	0.04	–		1.20	0.92	–	
HAR–ARJ	0.12	-0.01	1.54	–	-0.25	-0.77	-0.45	–	-0.02	-0.10	-2.50	–
HAR–C–J	0.07	-0.01	0.99	-0.01	0.00	0.00	-0.02	0.06	0.65	0.63	0.31	0.70
D. APE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.04	–			0.00	–			0.01	–		
HAR–RJ	-1.06	-1.56	–		-0.08	-0.07	–		0.34	0.29	–	
HAR–ARJ	0.02	-0.04	1.57	–	-0.40	-0.81	-0.05	–	-0.25	-0.37	-2.07	–
HAR–C–J	0.07	0.04	1.34	0.07	-0.19	-0.19	-0.07	-0.03	0.21	0.21	0.09	0.33
E. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.03	–			0.01	–			0.06	–		
HAR–RJ	-2.63	-3.37	–		-0.03	-0.06	–		0.96	0.66	–	
HAR–ARJ	0.02	-0.01	3.53	–	0.00	-0.06	0.03	–	0.24	0.15	-0.76	–
HAR–C–J	0.18	0.25	3.09	0.27	-0.04	-0.07	-0.01	-0.03	0.01	0.00	-0.07	0.00
F. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.00	–			0.00	–			0.02	–		
HAR–RJ	-2.93	-3.26	–		0.00	0.00	–		0.96	0.72	–	
HAR–ARJ	0.01	0.01	3.53	–	-0.01	-0.02	0.00	–	0.35	0.26	-0.77	–
HAR–C–J	0.16	0.31	2.95	0.25	0.00	0.00	0.00	0.00	0.00	0.00	-0.11	-0.01

Table 10: **Out-of-Sample Forecast Comparisons for Natural Gas**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of the forecasting models for natural gas volatility. Three forecast horizons are considered: daily, weekly and monthly. Entries correspond to test statistics from comparing the mean difference between the forecast errors of model [name in row] and those of the model [name in column]. We report in the lower triangular matrix the Giacomini and White test statistic. The statistic is asymptotically distributed as a chi-squared random variable with 1 degree of freedom. Panels 1 to 6 contain results for the different loss functions. Significant mean differences (rejection of the null) at the 5% level are highlighted in bold. Out-of-sample forecasts are generated using a rolling sample of 600 observations.

	1-Day Horizon					*5-Day Horizon*					*22-Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
A. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-1.19	–			HAR–J	-1.57	–			HAR–J	-0.47	–		
HAR–RJ	-0.08	0.66	–		HAR–RJ	1.01	**7.76**	–		HAR–RJ	-0.33	-0.03	–	
HAR–ARJ	-0.03	3.55	0.18	–	HAR–ARJ	-0.03	2.45	-2.09	–	HAR–ARJ	-0.57	-0.10	-0.15	–
HAR–C–J	-1.11	-0.23	-0.79	-1.39	HAR–C–J	0.13	0.56	0.00	0.18	HAR–C–J	0.16	0.33	0.33	0.37
B. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.39	–			HAR–J	-1.47	–			HAR–J	-0.52	–		
HAR–RJ	-0.01	0.37	–		HAR–RJ	0.50	**10.53**	–		HAR–RJ	0.01	0.45	–	
HAR–ARJ	0.00	1.36	0.06	–	HAR–ARJ	0.19	1.79	-0.23	–	HAR–ARJ	0.00	0.23	-0.05	–
HAR–C–J	-0.31	-0.01	-0.32	-0.69	HAR–C–J	-0.05	0.23	-0.78	-0.13	HAR–C–J	0.01	0.10	0.01	0.01
C. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.07	–			HAR–J	-1.14	–			HAR–J	-0.03	–		
HAR–RJ	0.26	1.53	–		HAR–RJ	3.11	**9.21**	–		HAR–RJ	0.00	0.02	–	
HAR–ARJ	0.16	1.41	-0.23	–	HAR–ARJ	-0.09	1.10	**-6.01**	–	HAR–ARJ	-0.47	-0.51	-1.80	–
HAR–C–J	0.00	0.06	-0.36	-0.14	HAR–C–J	1.26	2.86	-0.01	1.75	HAR–C–J	-0.21	-0.19	-0.21	-0.09
D. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.05	–			HAR–J	-1.10	–			HAR–J	-0.09	–		
HAR–RJ	0.35	1.78	–		HAR–RJ	3.16	**10.06**	–		HAR–RJ	0.01	0.15	–	
HAR–ARJ	0.20	1.07	-0.35	–	HAR–ARJ	-0.02	1.09	**-5.61**	–	HAR–ARJ	-0.27	-0.09	-1.61	–
HAR–C–J	-0.01	0.01	-0.71	-0.30	HAR–C–J	0.49	1.99	-0.56	0.66	HAR–C–J	-0.26	-0.20	-0.27	-0.14
E. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.30	–			HAR–J	-2.19	–			HAR–J	-0.76	–		
HAR–RJ	0.02	0.86	–		HAR–RJ	1.01	**8.91**	–		HAR–RJ	-0.09	0.17	–	
HAR–ARJ	0.19	2.13	0.13	–	HAR–ARJ	-0.03	2.06	-1.58	–	HAR–ARJ	-0.25	0.04	-0.17	–
HAR–C–J	-1.51	-1.69	-2.61	-3.61	HAR–C–J	-0.32	0.00	-2.27	-0.31	HAR–C–J	-0.03	0.00	-0.01	0.00
F. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.28	–			HAR–J	-2.66	–			HAR–J	-0.92	–		
HAR–RJ	0.00	0.60	–		HAR–RJ	0.99	**7.52**	–		HAR–RJ	-0.24	0.07	–	
HAR–ARJ	0.19	2.25	0.29	–	HAR–ARJ	-0.18	2.21	-2.02	–	HAR–ARJ	-0.49	0.01	-0.16	–
HAR–C–J	-2.36	-3.01	-3.32	**-4.94**	HAR–C–J	-0.64	-0.07	-2.97	-0.55	HAR–C–J	-0.12	-0.01	-0.03	-0.02

Table 11: **Out-of-Sample Forecast Comparisons for Gasoline**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for gasoline volatility. Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 5 % significance level.

	1-Day Horizon					*5-Day Horizon*					*22-Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
A. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.64	–			HAR–J	1.74	–			HAR–J	**4.15**	–		
HAR–RJ	-0.11	0.55	–		HAR–RJ	1.31	0.09	–		HAR–RJ	**5.62**	2.00	–	
HAR–ARJ	-0.32	0.09	-0.64	–	HAR–ARJ	0.71	-0.01	-1.16	–	HAR–ARJ	**4.99**	0.62	-1.00	–
HAR–C–J	**9.28**	**13.57**	**12.06**	**13.10**	HAR–C–J	**13.14**	**12.94**	**10.73**	**11.09**	HAR–C–J	**4.78**	**4.40**	3.84	**3.92**
B. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.03	–			HAR–J	0.06	–			HAR–J	1.46	–		
HAR–RJ	0.49	0.77	–		HAR–RJ	0.28	0.20	–		HAR–RJ	1.83	0.36	–	
HAR–ARJ	0.01	0.15	-1.24	–	HAR–ARJ	0.16	0.07	-0.15	–	HAR–ARJ	1.86	0.34	-0.02	–
HAR–C–J	3.69	**4.77**	3.39	**4.35**	HAR–C–J	**7.70**	**9.79**	**6.13**	**5.68**	HAR–C–J	2.81	2.79	2.41	2.44
C. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.11	–			HAR–J	0.98	–			HAR–J	1.44	–		
HAR–RJ	-0.28	-1.32	–		HAR–RJ	0.15	-0.05	–		HAR–RJ	2.64	1.49	–	
HAR–ARJ	-0.34	-1.62	-0.09	–	HAR–ARJ	0.00	-0.44	-2.11	–	HAR–ARJ	1.43	0.09	-3.65	–
HAR–C–J	**6.85**	**7.92**	**9.66**	**10.11**	HAR–C–J	**8.82**	**8.42**	**8.42**	**9.05**	HAR–C–J	**4.35**	**4.33**	**3.92**	**4.27**
D. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.05	–			HAR–J	0.13	–			HAR–J	0.55	–		
HAR–RJ	-0.11	-0.58	–		HAR–RJ	0.00	-0.07	–		HAR–RJ	1.60	1.11	–	
HAR–ARJ	-0.25	-1.14	-0.39	–	HAR–ARJ	-0.09	-0.34	-1.61	–	HAR–ARJ	0.77	0.15	-2.62	–
HAR–C–J	**5.40**	**6.69**	**7.57**	**8.45**	HAR–C–J	**6.06**	**6.62**	**6.46**	**6.88**	HAR–C–J	3.18	3.40	2.99	3.26
E. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.29	–			HAR–J	0.88	–			HAR–J	2.40	–		
HAR–RJ	0.00	0.64	–		HAR–RJ	0.77	0.09	–		HAR–RJ	3.47	1.44	–	
HAR–ARJ	-0.04	0.20	-0.54	–	HAR–ARJ	0.47	0.00	-0.56	–	HAR–ARJ	3.14	0.77	-0.75	–
HAR–C–J	**8.45**	**12.33**	**9.73**	**10.90**	HAR–C–J	**11.78**	**13.47**	**10.33**	**10.20**	HAR–C–J	3.79	3.62	3.18	3.31
F. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.56	–			HAR–J	1.34	–			HAR–J	2.77	–		
HAR–RJ	0.00	0.86	–		HAR–RJ	1.02	0.03	–		HAR–RJ	**3.89**	1.91	–	
HAR–ARJ	-0.07	0.35	-0.53	–	HAR–ARJ	0.67	-0.01	-0.72	–	HAR–ARJ	3.55	0.96	-1.17	–
HAR–C–J	**9.60**	**13.84**	**11.12**	**12.11**	HAR–C–J	**12.40**	**14.14**	**11.40**	**11.49**	HAR–C–J	**3.88**	3.61	3.20	3.35

Appendix to

“Do Jumps Matter for Volatility Forecasting? Evidence from Energy Markets”

Not Intended for Publication!

Will be Provided as Online Appendix

A. Variance Forecasting

Table A.1: **Variance Forecasting Errors**

This table presents out-of-sample forecasting errors for the five variance forecasting models considered. Each panel focuses on a specific loss function. MSE is the mean squared error, MSPE is the mean squared percentage error, MAE is the mean absolute error, MAPE is the mean absolute percentage error, LL is the logarithmic loss, and QLIKE is the quasi likelihood loss function. We consider three forecast horizons, namely 1, 5, and 22 days. Out-of-sample forecasts are obtained using a rolling window of 600 observations. In order to facilitate the presentation of our results, we multiply each loss function by 100.

	1–Day					5–Day					22–Day				
A. MSE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	0.362	0.364	0.363	0.365	0.366	0.222	0.223	0.222	0.222	0.224	0.183	0.185	0.184	0.186	0.187
Heating oil	0.180	0.179	0.178	0.179	0.181	0.098	0.098	0.098	0.098	0.099	0.088	0.088	0.089	0.088	0.095
Natural gas	2.605	2.549	2.551	2.555	2.575	1.565	1.530	1.539	1.534	1.619	1.312	1.300	1.294	1.296	1.316
Gasoline	0.245	0.240	0.239	0.241	0.272	0.136	0.136	0.137	0.137	0.149	0.123	0.122	0.122	0.122	0.138
B. MSPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	65.340	66.748	65.670	67.147	70.838	21.986	22.605	21.751	22.198	24.286	24.924	25.556	24.833	25.538	27.643
Heating oil	69.661	69.234	68.625	68.489	68.372	22.767	22.754	22.352	22.737	22.262	25.579	25.564	25.374	25.549	24.967
Natural gas	75.707	74.884	75.266	75.298	77.036	19.246	18.281	18.886	18.886	18.968	20.182	19.784	20.029	19.996	19.695
Gasoline	75.069	72.879	73.976	72.955	72.952	24.044	23.466	23.470	23.601	23.155	27.841	27.000	26.916	26.980	25.962
C. MAE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	3.491	3.522	3.514	3.532	3.563	2.897	2.908	2.897	2.905	2.957	3.377	3.384	3.383	3.383	3.446
Heating oil	2.671	2.669	2.661	2.667	2.675	2.188	2.188	2.186	2.180	2.196	2.479	2.480	2.485	2.476	2.533
Natural gas	9.034	9.022	9.017	9.027	9.111	6.870	6.854	6.894	6.858	7.055	7.693	7.684	7.669	7.673	7.568
Gasoline	3.344	3.304	3.294	3.302	3.484	2.609	2.590	2.592	2.597	2.686	2.924	2.878	2.876	2.876	2.977
D. MAPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	48.033	48.543	48.355	48.736	49.842	33.995	34.205	33.900	34.072	35.290	40.636	40.840	40.628	40.752	42.349
Heating oil	48.758	48.549	48.284	48.474	48.833	35.316	35.276	35.061	35.131	35.040	40.351	40.324	40.230	40.217	40.008
Natural gas	54.133	54.115	54.178	54.200	54.794	32.929	32.852	33.150	32.974	33.474	36.294	36.209	36.234	36.218	35.344
Gasoline	52.172	51.123	51.248	50.970	50.638	36.004	35.395	35.360	35.405	35.286	41.377	40.471	40.433	40.425	39.805
E. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	24.778	25.105	24.937	25.291	25.698	15.885	15.952	15.816	15.854	16.449	18.441	18.532	18.452	18.521	18.966
Heating oil	24.626	24.471	24.252	24.424	24.646	15.172	15.147	15.050	15.107	15.058	16.150	16.138	16.118	16.121	16.068
Natural gas	29.166	29.036	29.116	29.138	29.306	14.236	14.045	14.251	14.147	14.265	16.029	15.884	15.912	15.900	15.629
Gasoline	27.004	26.360	26.373	26.343	32.301	15.690	15.564	15.557	15.631	17.703	16.833	16.534	16.510	16.544	18.722
F. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	-145.047	-144.948	-145.016	-144.846	-144.836	-141.645	-141.678	-141.685	-141.713	-141.566	-137.461	-137.462	-137.443	-137.457	-137.467
Heating oil	-168.706	-168.762	-168.859	-168.778	-168.702	-165.888	-165.901	-165.923	-165.914	-165.896	-163.174	-163.179	-163.168	-163.178	-163.143
Natural gas	-63.339	-63.427	-63.397	-63.385	-63.379	-61.758	-61.855	-61.758	-61.817	-61.813	-58.635	-58.712	-58.707	-58.713	-58.863
Gasoline	-151.813	-152.086	-152.121	-152.060	-146.202	-149.751	-149.720	-149.738	-149.682	-147.868	-146.850	-146.926	-146.936	-146.916	-144.998

Table A.2: **Comparisons of Out-of-Sample Variance Forecasts for Crude Oil**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for crude oil variance. Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample variance forecasts. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					5–Day Horizon					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.09	–			HAR–J	0.42	–			HAR–J	1.20	–		
HAR–RJ	0.06	-0.09	–		HAR–RJ	0.01	-0.36	–		HAR–RJ	0.25	-0.22	–	
HAR–ARJ	0.27	0.87	1.58	–	HAR–ARJ	0.24	-0.09	0.18	–	HAR–ARJ	1.26	0.44	0.45	–
HAR–C–J	0.72	**4.43**	2.81	0.18	HAR–C–J	0.85	0.54	0.89	0.61	HAR–C–J	0.93	0.53	0.58	0.16
2. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	**5.49**	–			HAR–J	1.16	–			HAR–J	2.70	–		
HAR–RJ	0.13	-1.29	–		HAR–RJ	-0.81	-2.09	–		HAR–RJ	-0.61	-2.77	–	
HAR–ARJ	3.33	0.34	**5.24**	–	HAR–ARJ	0.20	-2.94	1.16	–	HAR–ARJ	2.37	0.00	2.30	–
HAR–C–J	**15.53**	**10.29**	**9.30**	**6.40**	HAR–C–J	**11.74**	**7.84**	**10.95**	**10.23**	HAR–C–J	**9.67**	**10.09**	**9.26**	**9.29**
3. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	**5.03**	–			HAR–J	1.21	–			HAR–J	0.65	–		
HAR–RJ	2.79	-0.49	–		HAR–RJ	0.00	-0.70	–		HAR–RJ	0.47	0.00	–	
HAR–ARJ	**5.06**	1.26	3.74	–	HAR–ARJ	0.46	-0.12	0.28	–	HAR–ARJ	0.26	-0.02	0.00	–
HAR–C–J	**19.90**	**23.31**	**11.73**	**6.40**	HAR–C–J	**7.33**	**6.75**	**6.95**	**6.24**	HAR–C–J	2.66	2.73	1.92	2.56
4. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	**5.02**	–			HAR–J	1.37	–			HAR–J	2.32	–		
HAR–RJ	2.50	-0.93	–		HAR–RJ	-0.59	-2.20	–		HAR–RJ	-0.01	-1.43	–	
HAR–ARJ	**5.51**	1.90	**5.10**	–	HAR–ARJ	0.20	-1.40	0.97	–	HAR–ARJ	0.61	-0.72	0.44	–
HAR–C–J	**40.73**	**51.31**	**31.29**	**22.58**	HAR–C–J	**14.58**	**11.96**	**15.40**	**13.21**	HAR–C–J	**9.03**	**9.17**	**8.17**	**9.10**
5. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	2.49	–			HAR–J	0.28	–			HAR–J	0.80	–		
HAR–RJ	1.22	-0.92	–		HAR–RJ	-0.68	-0.93	–		HAR–RJ	0.02	-0.40	–	
HAR–ARJ	3.23	2.61	2.75	–	HAR–ARJ	-0.07	-2.29	0.10	–	HAR–ARJ	0.48	-0.03	0.27	–
HAR–C–J	**19.12**	**21.76**	**16.63**	**4.57**	HAR–C–J	**6.40**	**5.91**	**7.34**	**7.67**	HAR–C–J	1.66	1.49	1.40	1.39
6. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.63	–			HAR–J	-0.33	–			HAR–J	0.00	–		
HAR–RJ	0.21	-0.44	–		HAR–RJ	-0.89	-0.02	–		HAR–RJ	0.18	0.10	–	
HAR–ARJ	1.27	2.23	1.24	–	HAR–ARJ	-1.34	-1.76	-0.28	–	HAR–ARJ	0.01	0.02	-0.06	–
HAR–C–J	**4.12**	3.34	**4.40**	0.01	HAR–C–J	0.40	1.25	1.11	2.00	HAR–C–J	0.00	0.00	-0.01	0.00

Table A.3: **Comparisons of Out-of-Sample Variance Forecasts for Heating Oil**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for heating oil variance. Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample variance forecasts. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					5–Day Horizon					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.96	–			HAR–J	0.39	–			HAR–J	0.51	–		
HAR–RJ	-3.01	-1.87	–		HAR–RJ	0.00	-0.04	–		HAR–RJ	0.44	0.33	–	
HAR–ARJ	-1.53	-0.80	1.16	–	HAR–ARJ	0.16	0.04	0.08	–	HAR–ARJ	0.07	0.01	-0.10	–
HAR–C–J	0.19	1.76	3.34	2.61	HAR–C–J	1.14	0.88	0.99	0.47	HAR–C–J	1.36	1.32	1.26	1.14
2. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-1.13	–			HAR–J	0.00	–			HAR–J	-0.04	–		
HAR–RJ	-3.62	-1.80	–		HAR–RJ	**-3.94**	-2.43	–		HAR–RJ	-2.14	-2.22	–	
HAR–ARJ	-2.30	-1.52	-0.03	–	HAR–ARJ	-0.01	-0.01	1.18	–	HAR–ARJ	-0.03	-0.01	1.05	–
HAR–C–J	-0.34	-0.17	-0.01	-0.01	HAR–C–J	-1.80	-2.21	-0.06	-1.74	HAR–C–J	-0.65	-0.64	-0.34	-0.57
3. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.08	–			HAR–J	-0.01	–			HAR–J	0.05	–		
HAR–RJ	-1.45	-1.22	–		HAR–RJ	-0.09	-0.07	–		HAR–RJ	0.41	0.43	–	
HAR–ARJ	-0.26	-0.55	0.80	–	HAR–ARJ	-1.26	-2.03	-0.30	–	HAR–ARJ	-0.18	-0.36	-1.50	–
HAR–C–J	0.12	0.92	2.24	1.57	HAR–C–J	0.18	0.22	0.41	0.83	HAR–C–J	0.72	0.72	0.69	0.82
4. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.98	–			HAR–J	-0.14	–			HAR–J	-0.26	–		
HAR–RJ	**-6.17**	-2.94	–		HAR–RJ	-3.09	-2.28	–		HAR–RJ	-0.76	-0.58	–	
HAR–ARJ	-1.55	-0.85	2.01	–	HAR–ARJ	-2.00	-3.67	0.27	–	HAR–ARJ	-1.48	-1.45	-0.02	–
HAR–C–J	0.08	3.74	**6.76**	**6.56**	HAR–C–J	-1.02	-0.90	-0.01	-0.13	HAR–C–J	-0.19	-0.17	-0.10	-0.08
5. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-1.17	–			HAR–J	-0.13	–			HAR–J	-0.15	–		
HAR–RJ	**-8.66**	**-4.06**	–		HAR–RJ	-1.95	-1.22	–		HAR–RJ	-0.13	-0.06	–	
HAR–ARJ	-1.64	-0.65	3.28	–	HAR–ARJ	-0.61	-0.69	0.44	–	HAR–ARJ	-0.18	-0.08	0.00	–
HAR–C–J	0.01	**3.93**	**8.30**	**5.82**	HAR–C–J	-0.53	-0.39	0.00	-0.12	HAR–C–J	-0.04	-0.03	-0.02	-0.02
6. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.84	–			HAR–J	-0.18	–			HAR–J	-0.13	–		
HAR–RJ	**-10.47**	**-4.13**	–		HAR–RJ	-0.59	-0.22	–		HAR–RJ	0.02	0.05	–	
HAR–ARJ	-1.17	-0.52	3.14	–	HAR–ARJ	-0.55	-0.34	0.05	–	HAR–ARJ	-0.02	0.00	-0.12	–
HAR–C–J	0.00	1.72	**5.22**	2.33	HAR–C–J	-0.01	0.01	0.13	0.07	HAR–C–J	0.02	0.03	0.02	0.03

Table A.4: **Comparisons of Out-of-Sample Variance Forecasts for Natural Gas**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for natural gas variance. Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample variance forecasts. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					5–Day Horizon					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-1.67	–			HAR–J	-1.14	–			HAR–J	-1.37	–		
HAR–RJ	-1.59	0.23	–		HAR–RJ	-0.72	**4.57**	–		HAR–RJ	-2.26	-0.96	–	
HAR–ARJ	-1.35	1.86	1.93	–	HAR–ARJ	-1.03	0.47	-2.36	–	HAR–ARJ	-1.94	-0.89	0.55	–
HAR–C–J	-0.39	2.00	1.66	1.15	HAR–C–J	0.35	1.32	1.05	1.19	HAR–C–J	0.01	0.22	0.34	0.31
2. SPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.16	–			HAR–J	-0.83	–			HAR–J	-0.70	–		
HAR–RJ	-0.05	0.75	–		HAR–RJ	-0.28	2.12	–		HAR–RJ	-0.58	0.51	–	
HAR–ARJ	-0.05	0.70	0.07	–	HAR–ARJ	-0.44	1.16	0.00	–	HAR–ARJ	-0.77	0.44	-0.86	–
HAR–C–J	0.29	**7.36**	**4.06**	3.41	HAR–C–J	-0.07	**4.71**	0.03	0.02	HAR–C–J	-0.25	-0.01	-0.12	-0.10
3. AE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.05	–			HAR–J	-0.10	–			HAR–J	-0.07	–		
HAR–RJ	-0.12	-0.14	–		HAR–RJ	0.21	**9.05**	–		HAR–RJ	-0.60	-0.78	–	
HAR–ARJ	-0.02	0.18	1.05	–	HAR–ARJ	-0.07	0.07	**-9.29**	–	HAR–ARJ	-0.44	-0.67	0.37	–
HAR–C–J	1.62	**7.95**	**7.62**	**6.31**	HAR–C–J	2.83	**4.83**	2.99	**4.24**	HAR–C–J	-0.93	-0.91	-0.67	-0.73
4. APE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.00	–			HAR–J	-0.06	–			HAR–J	-0.17	–		
HAR–RJ	0.02	0.27	–		HAR–RJ	0.68	**6.96**	–		HAR–RJ	-0.17	0.04	–	
HAR–ARJ	0.04	0.40	0.26	–	HAR–ARJ	0.04	0.74	**-9.25**	–	HAR–ARJ	-0.29	0.01	-0.43	–
HAR–C–J	2.49	**28.21**	**12.42**	**10.63**	HAR–C–J	1.72	**6.80**	1.45	3.11	HAR–C–J	-1.34	-1.08	-1.14	-1.10
5. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.22	–			HAR–J	-0.78	–			HAR–J	-1.10	–		
HAR–RJ	-0.04	0.81	–		HAR–RJ	0.01	**6.66**	–		HAR–RJ	-1.63	0.10	–	
HAR–ARJ	-0.01	1.12	0.53	–	HAR–ARJ	-0.36	1.07	**-5.49**	–	HAR–ARJ	-1.97	0.04	-0.67	–
HAR–C–J	0.24	**9.30**	2.31	1.66	HAR–C–J	0.01	2.19	0.01	0.42	HAR–C–J	-0.91	-0.35	-0.45	-0.41
6. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.53	–			HAR–J	-1.35	–			HAR–J	-1.82	–		
HAR–RJ	-0.26	0.81	–		HAR–RJ	0.00	**8.55**	–		HAR–RJ	-2.46	0.02	–	
HAR–ARJ	-0.16	1.50	0.82	–	HAR–ARJ	-0.76	1.39	**-6.00**	–	HAR–ARJ	-2.76	0.00	-0.70	–
HAR–C–J	-0.11	0.74	0.08	0.01	HAR–C–J	-0.18	0.28	-0.40	0.00	HAR–C–J	-1.75	-0.78	-0.85	-0.78

Table A.5: **Comparisons of Out-of-Sample Variance Forecasts for Gasoline**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for gasoline variance. Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample variance forecasts. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon				5–Day Horizon				*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE												
HAR–RV	–				–				–			
HAR–J	**-7.72**	–			0.01	–			-0.61	–		
HAR–RJ	**-11.07**	-1.08	–		0.20	0.33	–		-0.62	0.00	–	
HAR–ARJ	**-5.11**	3.63	**4.49**	–	0.97	1.91	**4.05**	–	-0.26	0.51	2.67	–
HAR–C–J	**21.28**	**28.95**	**27.28**	**27.62**	**6.50**	**7.04**	**6.41**	**5.86**	0.96	1.25	1.20	1.18
2. SPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	**-8.95**	–			**-5.02**	–			**-5.56**	–		
HAR–RJ	-1.07	1.07	–		-3.12	0.00	–		**-5.82**	-0.40	–	
HAR–ARJ	**-10.24**	0.05	-1.08	–	-1.37	0.16	1.22	–	**-4.51**	-0.03	1.24	–
HAR–C–J	-0.25	0.00	-0.05	0.00	-0.80	-0.14	-0.14	-0.27	-0.84	-0.35	-0.29	-0.34
3. AE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	**-6.46**	–			-2.29	–			**-3.92**	–		
HAR–RJ	**-11.96**	-1.03	–		-1.25	0.04	–		-3.80	-0.14	–	
HAR–ARJ	**-8.05**	-0.11	0.64	–	-0.58	0.44	2.91	–	-3.31	-0.11	0.00	–
HAR–C–J	**6.85**	**12.65**	**12.75**	**12.78**	1.41	2.54	2.44	2.22	0.10	0.39	0.41	0.42
4. APE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	**-18.42**	–			**-10.17**	–			**-7.77**	–		
HAR–RJ	**-20.90**	0.55	–		**-6.79**	-0.05	–		**-7.21**	-0.16	–	
HAR–ARJ	**-20.66**	-1.53	-2.44	–	**-5.17**	0.00	0.88	–	**-6.40**	-0.20	-0.04	–
HAR–C–J	-2.52	-0.30	-0.43	-0.14	-0.63	-0.02	-0.01	-0.02	-0.52	-0.12	-0.11	-0.11
5. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	**-15.88**	–			-0.88	–			-2.59	–		
HAR–RJ	**-19.39**	0.02	–		-0.82	-0.01	–		-2.66	-0.19	–	
HAR–ARJ	**-13.75**	-0.05	-0.07	–	-0.12	0.38	2.77	–	-1.84	0.04	1.95	–
HAR–C–J	**19.18**	**25.13**	**24.07**	**25.73**	**4.57**	**6.02**	**6.12**	**5.80**	0.71	1.06	1.09	1.07
6. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	**-9.88**	–			0.11	–			-0.76	–		
HAR–RJ	**-12.82**	-0.68	–		0.02	-0.14	–		-0.91	-0.23	–	
HAR–ARJ	**-7.96**	0.46	1.36	–	0.41	0.74	2.36	–	-0.45	0.19	2.60	–
HAR–C–J	**30.48**	**33.29**	**32.77**	**33.65**	**7.42**	**8.33**	**8.54**	**8.30**	1.47	1.70	1.73	1.71

B. The MedRV Estimator

Table B.6: **Volatility Forecasting Errors (MedRV Estimator)**

This table presents out-of-sample forecasting errors for the five volatility models considered. Jumps are detected based on the MedRV estimator of Andersen et al. (2012). Each panel focuses on a specific loss function. MSE is the mean squared error, $MSPE$ is the mean squared percentage error, MAE is the mean absolute error, $MAPE$ is the mean absolute percentage error, LL is the logarithmic loss, and $QLIKE$ is the quasi likelihood loss function. We consider three forecast horizons, namely 1, 5, and 22 days. Out-of-sample forecasts are obtained using a rolling window of 600 observations. In order to facilitate the presentation of our results, we multiply each loss function by 100.

	1–Day					5–Day					22–Day				
A. MSE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	0.536	0.533	0.535	0.533	0.535	0.376	0.374	0.373	0.372	0.380	0.402	0.401	0.401	0.401	0.410
Heating oil	0.399	0.400	0.403	0.401	0.404	0.247	0.248	0.250	0.248	0.254	0.244	0.245	0.246	0.245	0.253
Natural gas	1.493	1.477	1.483	1.481	1.475	0.842	0.833	0.847	0.839	0.834	0.843	0.837	0.841	0.840	0.853
Gasoline	0.484	0.483	0.488	0.484	0.490	0.297	0.297	0.301	0.299	0.304	0.289	0.289	0.290	0.290	0.296
B. MSPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	7.341	7.276	7.314	7.290	7.260	3.539	3.487	3.464	3.466	3.425	4.324	4.306	4.314	4.322	4.341
Heating oil	7.488	7.494	7.563	7.520	7.595	3.525	3.541	3.563	3.554	3.618	4.054	4.060	4.058	4.062	4.163
Natural gas	8.243	8.252	8.278	8.245	8.216	3.081	3.030	3.091	3.057	2.999	3.136	3.100	3.124	3.116	3.158
Gasoline	7.740	7.740	7.781	7.696	7.850	3.663	3.642	3.689	3.684	3.764	4.197	4.190	4.208	4.200	4.313
C. MAE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	5.149	5.131	5.158	5.141	5.111	4.204	4.184	4.187	4.175	4.157	5.020	5.013	5.011	5.013	5.068
Heating oil	4.505	4.517	4.535	4.521	4.526	3.649	3.655	3.678	3.654	3.699	4.102	4.108	4.109	4.104	4.178
Natural gas	8.723	8.691	8.702	8.692	8.696	6.234	6.216	6.313	6.250	6.211	6.648	6.605	6.614	6.617	6.717
Gasoline	5.142	5.144	5.147	5.122	5.180	4.003	4.006	4.033	4.015	4.061	4.360	4.363	4.372	4.366	4.477
D. MAPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	18.866	18.761	18.838	18.789	18.626	14.217	14.117	14.105	14.079	13.937	17.069	17.029	17.021	17.036	17.166
Heating oil	18.976	19.018	19.105	19.035	19.072	14.474	14.490	14.564	14.489	14.644	16.410	16.420	16.409	16.400	16.643
Natural gas	21.149	21.105	21.128	21.094	21.102	13.655	13.618	13.840	13.691	13.575	14.202	14.095	14.118	14.119	14.335
Gasoline	19.832	19.835	19.824	19.725	20.030	14.616	14.607	14.711	14.651	14.851	16.239	16.242	16.276	16.253	16.621
E. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	5.639	5.586	5.607	5.593	5.570	3.591	3.560	3.546	3.540	3.589	4.238	4.227	4.228	4.233	4.372
Heating oil	5.610	5.619	5.658	5.634	5.668	3.321	3.332	3.353	3.334	3.401	3.539	3.545	3.545	3.544	3.634
Natural gas	6.612	6.585	6.605	6.591	6.555	3.106	3.066	3.121	3.088	3.045	3.260	3.222	3.238	3.235	3.262
Gasoline	5.914	5.907	5.944	5.894	6.001	3.459	3.460	3.500	3.483	3.553	3.658	3.662	3.674	3.668	3.739
F. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	-25.479	-25.504	-25.494	-25.502	-25.504	-23.176	-23.190	-23.196	-23.200	-23.151	-21.154	-21.158	-21.159	-21.157	-21.061
Heating oil	-37.243	-37.237	-37.218	-37.231	-37.217	-35.058	-35.053	-35.043	-35.053	-35.017	-33.582	-33.579	-33.579	-33.579	-33.535
Natural gas	14.593	14.574	14.585	14.580	14.559	17.036	17.016	17.044	17.026	17.005	18.229	18.208	18.215	18.215	18.224
Gasoline	-29.255	-29.259	-29.235	-29.260	-29.210	-27.032	-27.028	-27.008	-27.018	-26.983	-25.457	-25.453	-25.447	-25.450	-25.420

Table B.7: **Comparisons of Out-of-Sample Volatility Forecasts for Crude Oil (MedRV Estimator)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for crude oil volatility. Jump components are constructed based on the MedRV estimator of Andersen et al. (2012). Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon				5–Day Horizon				22–Day Horizon			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE												
HAR–RV	–				–				–			
HAR–J	-2.59	–			-1.33	–			-0.11	–		
HAR–RJ	-0.05	0.88	–		-1.32	-0.09	–		-0.16	-0.03	–	
HAR–ARJ	-1.41	0.11	-0.90	–	-3.43	-1.09	-0.58	–	-0.29	-0.03	0.01	–
HAR–C–J	-0.04	0.55	0.00	0.30	0.25	0.59	0.66	0.88	0.60	0.76	0.72	0.76
2. SPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-1.90	–			-1.75	–			-0.67	–		
HAR–RJ	-0.18	0.79	–		-2.02	-0.42	–		-0.13	0.09	–	
HAR–ARJ	-0.87	0.28	-0.49	–	-2.13	-0.54	0.01	–	-0.01	0.61	0.29	–
HAR–C–J	-0.70	-0.04	-0.43	-0.13	-1.21	-0.53	-0.21	-0.24	0.02	0.12	0.06	0.03
3. AE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-1.56	–			-2.20	–			-0.36	–		
HAR–RJ	0.15	1.65	–		-0.69	0.02	–		-0.33	-0.04	–	
HAR–ARJ	-0.23	0.55	-1.38	–	-2.54	-0.46	-0.97	–	-0.24	0.00	0.11	–
HAR–C–J	-2.79	-1.38	-3.05	-1.98	-1.33	-0.55	-0.55	-0.22	0.44	0.72	0.71	0.69
4. APE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-1.88	–			-2.54	–			-0.70	–		
HAR–RJ	-0.07	0.99	–		-1.42	-0.03	–		-0.55	-0.03	–	
HAR–ARJ	-0.66	0.22	-1.05	–	-2.32	-0.40	-0.49	–	-0.32	0.03	0.29	–
HAR–C–J	**-4.60**	-3.15	**-4.07**	-3.13	-2.76	-1.66	-1.30	-0.97	0.15	0.38	0.39	0.34
5. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-2.90	–			-1.33	–			-0.40	–		
HAR–RJ	-0.61	0.57	–		-1.47	-0.27	–		-0.25	0.00	–	
HAR–ARJ	-1.73	0.11	-0.53	–	-2.00	-0.67	-0.16	–	-0.12	0.12	0.23	–
HAR–C–J	-2.11	-0.21	-0.77	-0.33	0.00	0.16	0.33	0.41	0.97	1.27	1.19	1.13
6. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-3.22	–			-1.06	–			-0.21	–		
HAR–RJ	-0.65	0.61	–		-1.19	-0.20	–		-0.25	-0.02	–	
HAR–ARJ	-2.15	0.07	-0.70	–	-1.85	-0.67	-0.31	–	-0.14	0.01	0.20	–
HAR–C–J	-1.23	0.00	-0.22	-0.01	0.30	0.80	0.98	1.12	1.36	1.61	1.58	1.53

Table B.8: **Comparisons of Out-of-Sample Volatility Forecasts for Heating Oil (MedRV Estimator)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for heating oil volatility. Jump components are constructed based on the MedRV estimator of Andersen et al. (2012). Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					5–Day Horizon					22–Day Horizon			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.34	–			HAR–J	1.55	–			HAR–J	0.67	–		
HAR–RJ	1.89	2.71	–		HAR–RJ	3.58	2.39	–		HAR–RJ	0.54	0.11	–	
HAR–ARJ	2.87	**5.42**	-0.39	–	HAR–ARJ	1.36	0.39	-0.76	–	HAR–ARJ	0.57	0.09	-0.02	–
HAR–C–J	**7.30**	**8.04**	0.24	2.37	HAR–C–J	**8.10**	**7.50**	3.36	**6.68**	HAR–C–J	**5.22**	**5.50**	**5.48**	**5.03**
2. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.06	–			HAR–J	0.79	–			HAR–J	0.18	–		
HAR–RJ	2.09	3.31	–		HAR–RJ	2.17	0.86	–		HAR–RJ	0.04	-0.02	–	
HAR–ARJ	1.16	2.96	-0.94	–	HAR–ARJ	0.90	0.69	-0.05	–	HAR–ARJ	0.16	0.05	0.04	–
HAR–C–J	**7.11**	**11.29**	0.46	**5.63**	HAR–C–J	**5.02**	**6.62**	1.82	**4.96**	HAR–C–J	**4.55**	**5.48**	**5.27**	**5.31**
3. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	1.61	–			HAR–J	0.64	–			HAR–J	0.57	–		
HAR–RJ	2.40	1.83	–		HAR–RJ	3.43	**4.06**	–		HAR–RJ	0.27	0.02	–	
HAR–ARJ	2.62	0.95	-1.06	–	HAR–ARJ	0.40	-0.06	**-4.12**	–	HAR–ARJ	0.05	-0.98	-0.42	–
HAR–C–J	2.32	0.84	-0.27	0.17	HAR–C–J	**5.50**	**5.97**	1.26	**6.21**	HAR–C–J	3.47	3.77	**4.33**	**4.21**
4. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	1.06	–			HAR–J	0.30	–			HAR–J	0.10	–		
HAR–RJ	2.65	2.86	–		HAR–RJ	2.12	2.67	–		HAR–RJ	0.00	-0.10	–	
HAR–ARJ	1.75	0.71	-1.86	–	HAR–ARJ	0.20	0.00	-2.51	–	HAR–ARJ	-0.05	-1.37	-0.08	–
HAR–C–J	2.79	1.92	-0.26	0.74	HAR–C–J	**4.33**	**5.18**	1.27	**5.22**	HAR–C–J	2.86	3.43	**4.23**	**4.08**
5. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.33	–			HAR–J	0.87	–			HAR–J	0.43	–		
HAR–RJ	2.18	3.38	–		HAR–RJ	2.54	2.08	–		HAR–RJ	0.19	0.00	–	
HAR–ARJ	1.58	3.14	-1.14	–	HAR–ARJ	0.72	0.10	-0.99	–	HAR–ARJ	0.19	-0.03	0.00	–
HAR–C–J	**6.14**	**8.16**	0.13	3.33	HAR–C–J	**7.09**	**7.27**	3.07	**6.70**	HAR–C–J	**4.64**	**5.04**	**4.86**	**5.12**
6. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.53	–			HAR–J	0.93	–			HAR–J	0.58	–		
HAR–RJ	2.13	2.99	–		HAR–RJ	2.37	1.96	–		HAR–RJ	0.29	0.00	–	
HAR–ARJ	1.70	3.11	-1.25	–	HAR–ARJ	0.63	0.00	-1.30	–	HAR–ARJ	0.24	-0.07	-0.02	–
HAR–C–J	**5.44**	**6.13**	0.01	2.33	HAR–C–J	**7.10**	**6.43**	3.26	**6.18**	HAR–C–J	**4.33**	**4.52**	**4.31**	**4.63**

Table B.9: **Comparisons of Out-of-Sample Volatility Forecasts for Natural Gas (MedRV Etimator)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for natural gas volatility. Jump components are constructed based on the MedRV estimator of Andersen et al. (2012). Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					*5–Day Horizon*					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-1.65	–			HAR–J	-2.01	–			HAR–J	-1.69	–		
HAR–RJ	-0.50	1.17	–		HAR–RJ	0.25	**7.51**	–		HAR–RJ	-0.13	2.40	–	
HAR–ARJ	-0.87	0.74	-0.45	–	HAR–ARJ	-0.30	**4.87**	-2.59	–	HAR–ARJ	-0.49	**5.22**	-0.15	–
HAR–C–J	-1.77	-0.15	-0.99	-0.62	HAR–C–J	-0.29	0.03	-1.20	-0.16	HAR–C–J	0.41	1.85	1.20	1.19
2. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.01	–			HAR–J	-1.12	–			HAR–J	-1.11	–		
HAR–RJ	0.10	0.66	–		HAR–RJ	0.03	**8.81**	–		HAR–RJ	-0.13	**6.55**	–	
HAR–ARJ	0.00	-0.16	-1.67	–	HAR–ARJ	-0.32	**6.59**	-1.66	–	HAR–ARJ	-0.57	2.53	-0.86	–
HAR–C–J	-0.07	-0.49	-0.98	-0.30	HAR–C–J	-0.95	-0.43	-3.79	-1.28	HAR–C–J	0.09	1.27	0.40	0.58
3. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.57	–			HAR–J	-0.46	–			HAR–J	-3.23	–		
HAR–RJ	-0.15	0.34	–		HAR–RJ	3.30	**14.23**	–		HAR–RJ	-1.12	0.60	–	
HAR–ARJ	-0.48	0.00	-0.46	–	HAR–ARJ	0.38	**8.85**	**-6.20**	–	HAR–ARJ	-1.73	2.76	0.06	–
HAR–C–J	-0.32	0.06	-0.04	0.02	HAR–C–J	-0.21	-0.02	**-6.36**	-1.02	HAR–C–J	0.92	3.16	2.76	2.44
4. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.16	–			HAR–J	-0.29	–			HAR–J	-3.14	–		
HAR–RJ	-0.02	0.20	–		HAR–RJ	2.82	**14.25**	–		HAR–RJ	-1.28	0.88	–	
HAR–ARJ	-0.22	-0.09	-0.63	–	HAR–ARJ	0.30	**9.66**	**-6.25**	–	HAR–ARJ	-2.12	1.63	0.00	–
HAR–C–J	-0.14	0.00	-0.10	0.01	HAR–C–J	-0.42	-0.27	**-7.71**	-1.68	HAR–C–J	0.68	2.73	2.18	2.09
5. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.30	–			HAR–J	-2.05	–			HAR–J	-2.81	–		
HAR–RJ	-0.01	0.63	–		HAR–RJ	0.12	**7.05**	–		HAR–RJ	-0.86	3.78	–	
HAR–ARJ	-0.15	0.20	-0.53	–	HAR–ARJ	-0.55	**7.03**	-2.32	–	HAR–ARJ	-1.83	3.48	-0.17	–
HAR–C–J	-0.93	-0.99	-1.53	-1.12	HAR–C–J	-1.41	-0.42	-3.71	-1.36	HAR–C–J	0.00	0.80	0.27	0.33
6. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.61	–			HAR–J	-2.72	–			HAR–J	-3.68	–		
HAR–RJ	-0.05	0.68	–		HAR–RJ	0.15	**5.62**	–		HAR–RJ	-1.34	2.74	–	
HAR–ARJ	-0.23	0.52	-0.32	–	HAR–ARJ	-0.78	**6.60**	-2.32	–	HAR–ARJ	-2.49	3.83	-0.04	–
HAR–C–J	-1.48	-1.17	-1.75	-1.55	HAR–C–J	-1.76	-0.45	-3.21	-1.37	HAR–C–J	-0.03	0.47	0.12	0.14

Table B.10: **Comparisons of Out-of-Sample Volatility Forecasts for Gasoline (MedRV Estimator)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for gasoline volatility. Jump components are constructed based on the MedRV estimator of Andersen et al. (2012). Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					*5–Day Horizon*					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.53	–			HAR–J	0.06	–			HAR–J	0.08	–		
HAR–RJ	0.86	1.37	–		HAR–RJ	1.90	2.81	–		HAR–RJ	0.77	3.02	–	
HAR–ARJ	-0.13	0.01	-3.21	–	HAR–ARJ	1.34	1.52	-0.97	–	HAR–ARJ	0.88	1.37	-0.23	–
HAR–C–J	1.16	1.73	0.09	1.53	HAR–C–J	1.48	1.55	0.41	0.90	HAR–C–J	0.26	0.26	0.19	0.21
2. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.00	–			HAR–J	-0.95	–			HAR–J	-0.13	–		
HAR–RJ	0.88	1.16	–		HAR–RJ	0.77	**8.75**	–		HAR–RJ	0.28	**4.94**	–	
HAR–ARJ	-2.56	-2.48	**-8.32**	–	HAR–ARJ	0.46	1.49	-0.02	–	HAR–ARJ	0.04	0.88	-0.48	–
HAR–C–J	1.39	1.47	0.44	2.60	HAR–C–J	1.14	1.84	0.71	0.79	HAR–C–J	0.25	0.31	0.23	0.26
3. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.03	–			HAR–J	0.07	–			HAR–J	0.07	–		
HAR–RJ	0.06	0.03	–		HAR–RJ	2.68	**5.24**	–		HAR–RJ	0.46	1.75	–	
HAR–ARJ	-2.01	-3.06	**-5.20**	–	HAR–ARJ	0.80	0.99	-2.51	–	HAR–ARJ	0.25	0.36	-0.56	–
HAR–C–J	1.13	1.15	0.78	2.76	HAR–C–J	1.14	1.11	0.28	0.77	HAR–C–J	0.58	0.60	0.52	0.57
4. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.01	–			HAR–J	-0.05	–			HAR–J	0.01	–		
HAR–RJ	-0.01	-0.05	–		HAR–RJ	2.52	**7.34**	–		HAR–RJ	0.37	2.10	–	
HAR–ARJ	**-4.15**	**-5.96**	**-8.28**	–	HAR–ARJ	0.59	1.23	-1.62	–	HAR–ARJ	0.12	0.36	-0.63	–
HAR–C–J	2.15	2.39	2.32	**5.22**	HAR–C–J	1.34	1.54	0.49	1.03	HAR–C–J	0.44	0.47	0.39	0.44
5. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.24	–			HAR–J	0.00	–			HAR–J	0.08	–		
HAR–RJ	0.61	1.08	–		HAR–RJ	2.16	**4.52**	–		HAR–RJ	0.74	3.28	–	
HAR–ARJ	-0.99	-0.41	**-4.60**	–	HAR–ARJ	1.41	1.47	-0.55	–	HAR–ARJ	0.59	0.81	-0.44	–
HAR–C–J	2.16	2.82	0.80	3.36	HAR–C–J	1.82	2.04	0.71	1.18	HAR–C–J	0.21	0.20	0.15	0.17
6. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.37	–			HAR–J	0.21	–			HAR–J	0.27	–		
HAR–RJ	0.74	1.24	–		HAR–RJ	2.27	3.28	–		HAR–RJ	0.92	2.65	–	
HAR–ARJ	-0.28	-0.01	-3.13	–	HAR–ARJ	1.72	1.34	-0.89	–	HAR–ARJ	0.91	0.76	-0.41	–
HAR–C–J	1.95	2.67	0.57	2.76	HAR–C–J	1.92	1.91	0.67	1.23	HAR–C–J	0.18	0.16	0.11	0.13

C. Alternative Estimation Windows

Table C.11: **Volatility Forecasting Errors (Rolling Window of 400 Observations)**

This table presents out-of-sample forecasting errors for the five volatility models considered. Each panel focuses on a specific loss function. MSE is the mean squared error, $MSPE$ is the mean squared percentage error, MAE is the mean absolute error, $MAPE$ is the mean absolute percentage error, LL is the logarithmic loss, and $QLIKE$ is the quasi likelihood loss function. We consider three forecast horizons, namely 1, 5, and 22 days. We use a trailing window of 400 observations to estimate the parameters of the forecasting models. In order to facilitate the presentation of our results, we multiply each loss function by 100.

	1–Day					5–Day					22–Day				
A. MSE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	0.896	0.906	0.907	0.915	0.918	0.504	0.504	0.508	0.504	0.514	0.787	0.787	0.798	0.788	0.783
Heating oil	0.629	0.634	0.639	0.640	0.644	0.347	0.348	0.352	0.348	0.360	0.430	0.430	0.435	0.430	0.448
Natural gas	1.799	1.802	1.807	1.809	1.819	0.938	0.945	0.956	0.954	0.988	0.934	0.938	0.931	0.938	0.954
Gasoline	1.033	1.041	1.032	1.054	1.033	0.502	0.501	0.497	0.500	0.474	0.966	0.954	0.956	0.956	0.883
B. MSPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	6.965	6.963	6.939	6.976	7.160	3.204	3.188	3.160	3.154	3.374	4.424	4.430	4.444	4.445	4.624
Heating oil	7.251	7.277	7.257	7.307	7.566	3.110	3.107	3.109	3.110	3.301	3.664	3.675	3.678	3.673	4.030
Natural gas	8.733	8.763	8.786	8.764	8.839	3.315	3.340	3.380	3.375	3.424	3.653	3.674	3.648	3.666	3.675
Gasoline	7.425	7.462	7.411	7.473	7.472	3.261	3.272	3.245	3.255	3.393	4.651	4.645	4.648	4.652	4.824
C. MAE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	6.455	6.471	6.458	6.485	6.525	4.942	4.956	4.958	4.949	5.054	6.615	6.598	6.646	6.601	6.574
Heating oil	5.549	5.565	5.579	5.581	5.628	4.248	4.256	4.266	4.249	4.335	5.112	5.114	5.145	5.113	5.234
Natural gas	9.464	9.486	9.517	9.515	9.565	6.797	6.836	6.890	6.858	6.949	7.294	7.322	7.292	7.299	7.327
Gasoline	6.701	6.729	6.692	6.753	6.746	4.924	4.927	4.905	4.906	4.940	6.705	6.701	6.709	6.706	6.847
D. MAPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	18.772	18.784	18.751	18.805	18.960	13.736	13.758	13.709	13.704	14.156	17.563	17.509	17.591	17.510	17.566
Heating oil	18.894	18.935	18.939	18.978	19.204	13.669	13.675	13.666	13.644	14.033	16.053	16.061	16.104	16.034	16.595
Natural gas	21.650	21.702	21.771	21.737	21.853	14.220	14.297	14.426	14.333	14.461	15.100	15.155	15.100	15.104	15.086
Gasoline	19.627	19.722	19.599	19.704	19.858	13.927	13.939	13.871	13.867	14.208	17.490	17.484	17.500	17.492	18.156
E. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	5.482	5.494	5.481	5.520	5.562	3.280	3.263	3.253	3.243	3.367	4.493	4.480	4.516	4.500	4.637
Heating oil	5.527	5.538	5.541	5.567	5.617	2.989	2.988	2.994	2.990	3.098	3.396	3.402	3.415	3.405	3.594
Natural gas	6.989	7.008	7.028	7.028	7.048	3.314	3.338	3.375	3.359	3.456	3.576	3.595	3.568	3.594	3.631
Gasoline	5.854	5.884	5.847	5.897	5.993	3.163	3.175	3.139	3.146	3.301	4.405	4.406	4.413	4.418	4.667
F. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	-9.666	-9.657	-9.662	-9.639	-9.633	-7.335	-7.345	-7.347	-7.353	-7.305	-4.814	-4.824	-4.801	-4.811	-4.742
Heating oil	-22.177	-22.175	-22.168	-22.158	-22.153	-19.981	-19.982	-19.977	-19.980	-19.939	-18.060	-18.057	-18.049	-18.054	-17.983
Natural gas	20.136	20.145	20.154	20.158	20.167	22.764	22.776	22.795	22.785	22.847	24.306	24.316	24.301	24.316	24.340
Gasoline	-12.406	-12.393	-12.410	-12.385	-12.318	-10.137	-10.130	-10.152	-10.148	-10.061	-7.730	-7.728	-7.724	-7.721	-7.580

Table C.12: **Out-of-Sample Volatility Forecast Comparisons for Crude Oil (Rolling Window of 400 Observations)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for crude oil volatility. Each day, we use a trailing window of 400 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					*5–Day Horizon*					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	1.48	–			HAR–J	0.08	–			HAR–J	0.00	–		
HAR–RJ	1.25	0.05	–		HAR–RJ	0.99	1.19	–		HAR–RJ	2.88	3.44	–	
HAR–ARJ	2.06	1.21	1.08	–	HAR–ARJ	0.00	-0.08	-1.59	–	HAR–ARJ	0.03	0.05	-2.37	–
HAR–C–J	3.45	3.40	1.60	0.07	HAR–C–J	0.75	0.78	0.26	0.92	HAR–C–J	-0.01	-0.01	-0.18	-0.02
2. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.00	–			HAR–J	-0.25	–			HAR–J	0.04	–		
HAR–RJ	-0.17	-0.24	–		HAR–RJ	-0.89	-1.37	–		HAR–RJ	0.63	0.38	–	
HAR–ARJ	0.01	0.19	0.60	–	HAR–ARJ	-1.23	-2.60	-0.25	–	HAR–ARJ	0.30	0.43	0.00	–
HAR–C–J	3.15	**7.16**	**5.41**	**5.23**	HAR–C–J	2.26	3.35	**4.04**	**4.43**	HAR–C–J	0.32	0.32	0.25	0.28
3. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.40	–			HAR–J	0.78	–			HAR–J	-1.35	–		
HAR–RJ	0.01	-0.40	–		HAR–RJ	0.51	0.01	–		HAR–RJ	1.45	**4.24**	–	
HAR–ARJ	1.00	1.56	1.70	–	HAR–ARJ	0.09	-0.34	-0.33	–	HAR–ARJ	-0.60	0.06	-2.98	–
HAR–C–J	3.21	3.45	3.19	1.65	HAR–C–J	2.64	2.33	1.96	2.67	HAR–C–J	-0.06	-0.02	-0.18	-0.03
4. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.02	–			HAR–J	0.15	–			HAR–J	-1.18	–		
HAR–RJ	-0.08	-0.26	–		HAR–RJ	-0.14	-0.87	–		HAR–RJ	0.27	2.44	–	
HAR–ARJ	0.09	0.47	0.61	–	HAR–ARJ	-0.20	-1.54	-0.02	–	HAR–ARJ	-0.75	0.00	-1.69	–
HAR–C–J	1.82	3.27	3.00	2.32	HAR–C–J	3.27	3.47	**3.96**	**4.25**	HAR–C–J	0.00	0.01	0.00	0.01
5. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.05	–			HAR–J	-0.58	–			HAR–J	-0.36	–		
HAR–RJ	0.00	-0.13	–		HAR–RJ	-0.67	-0.24	–		HAR–RJ	0.86	2.96	–	
HAR–ARJ	0.33	1.13	0.96	–	HAR–ARJ	-1.35	-1.23	-0.70	–	HAR–ARJ	0.06	1.66	-0.43	–
HAR–C–J	1.43	2.78	2.12	0.79	HAR–C–J	1.06	1.85	2.01	2.47	HAR–C–J	0.37	0.46	0.26	0.36
6. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.10	–			HAR–J	-0.85	–			HAR–J	-0.66	–		
HAR–RJ	0.03	-0.06	–		HAR–RJ	-0.52	-0.02	–		HAR–RJ	0.91	**3.99**	–	
HAR–ARJ	0.48	1.27	0.89	–	HAR–ARJ	-1.30	-0.66	-0.96	–	HAR–ARJ	0.03	2.44	-0.88	–
HAR–C–J	0.91	1.35	0.97	0.06	HAR–C–J	0.51	1.13	1.11	1.51	HAR–C–J	0.44	0.59	0.29	0.43

Table C.13: **Out-of-Sample Volatility Forecast Comparisons for Heating Oil (Rolling Window of 400 Observations)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for heating oil volatility. Each day, we use a trailing window of 400 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					*5–Day Horizon*					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	1.21	–			HAR–J	0.67	–			HAR–J	0.00	–		
HAR–RJ	1.53	0.32	–		HAR–RJ	2.72	2.39	–		HAR–RJ	1.96	**6.04**	–	
HAR–ARJ	1.99	2.22	0.01	–	HAR–ARJ	0.57	0.04	-1.99	–	HAR–ARJ	0.03	0.07	-3.31	–
HAR–C–J	3.43	1.74	0.28	0.13	HAR–C–J	1.16	0.95	0.34	0.94	HAR–C–J	0.11	0.12	0.06	0.11
2. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.21	–			HAR–J	-0.02	–			HAR–J	0.58	–		
HAR–RJ	0.01	-0.14	–		HAR–RJ	0.00	0.01	–		HAR–RJ	0.54	0.02	–	
HAR–ARJ	0.72	1.18	0.72	–	HAR–ARJ	0.00	0.03	0.00	–	HAR–ARJ	0.12	-0.01	-0.06	–
HAR–C–J	**4.52**	**4.46**	**5.24**	3.50	HAR–C–J	**7.96**	**9.76**	**8.34**	**8.98**	HAR–C–J	2.27	2.26	2.14	2.26
3. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.89	–			HAR–J	0.29	–			HAR–J	0.03	–		
HAR–RJ	1.69	0.35	–		HAR–RJ	0.80	0.48	–		HAR–RJ	2.37	3.64	–	
HAR–ARJ	2.20	3.00	0.01	–	HAR–ARJ	0.00	-0.62	-1.42	–	HAR–ARJ	0.00	-0.01	**-6.98**	–
HAR–C–J	**6.50**	**5.89**	2.00	2.82	HAR–C–J	2.38	2.20	1.54	2.74	HAR–C–J	0.33	0.34	0.19	0.35
4. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.30	–			HAR–J	0.01	–			HAR–J	0.08	–		
HAR–RJ	0.39	0.00	–		HAR–RJ	0.00	-0.03	–		HAR–RJ	0.90	0.68	–	
HAR–ARJ	0.92	1.30	0.34	–	HAR–ARJ	-0.20	-0.88	-0.30	–	HAR–ARJ	-0.14	-0.44	**-4.10**	–
HAR–C–J	**7.97**	**11.97**	**6.83**	**6.72**	HAR–C–J	**5.85**	**6.59**	**6.47**	**8.02**	HAR–C–J	1.42	1.46	1.21	1.60
5. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.09	–			HAR–J	-0.01	–			HAR–J	0.20	–		
HAR–RJ	0.18	0.02	–		HAR–RJ	0.04	0.14	–		HAR–RJ	0.98	0.62	–	
HAR–ARJ	0.85	2.88	0.58	–	HAR–ARJ	0.00	0.04	-0.06	–	HAR–ARJ	0.19	0.05	-0.69	–
HAR–C–J	2.89	**4.21**	2.44	1.36	HAR–C–J	**4.55**	**5.66**	**4.51**	**5.38**	HAR–C–J	1.18	1.18	0.99	1.12
6. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.02	–			HAR–J	-0.02	–			HAR–J	0.13	–		
HAR–RJ	0.24	0.19	–		HAR–RJ	0.07	0.29	–		HAR–RJ	1.10	1.03	–	
HAR–ARJ	0.72	3.69	0.37	–	HAR–ARJ	0.01	0.09	-0.12	–	HAR–ARJ	0.27	0.18	-0.98	–
HAR–C–J	0.87	1.38	0.42	0.05	HAR–C–J	2.83	3.70	2.69	3.43	HAR–C–J	0.77	0.77	0.59	0.70

Table C.14: **Out-of-Sample Volatility Forecast Comparisons for Natural Gas (Rolling Window of 400 Observations)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for natural gas volatility. Each day, we use a trailing window of 400 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

1–Day Horizon

	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE				
HAR–RV	–			
HAR–J	0.06	–		
HAR–RJ	0.38	0.91	–	
HAR–ARJ	0.89	3.45	0.16	–
HAR–C–J	1.98	2.23	0.83	0.61
2. SPE				
HAR–RV	–			
HAR–J	0.36	–		
HAR–RJ	0.40	0.24	–	
HAR–ARJ	0.25	0.00	-0.46	–
HAR–C–J	1.67	1.42	0.50	1.23
3. AE				
HAR–RV	–			
HAR–J	0.77	–		
HAR–RJ	1.72	1.83	–	
HAR–ARJ	2.16	1.62	-0.01	–
HAR–C–J	**5.84**	**5.39**	1.55	1.57
4. APE				
HAR–RV	–			
HAR–J	0.88	–		
HAR–RJ	1.88	1.69	–	
HAR–ARJ	1.48	0.51	-0.59	–
HAR–C–J	**4.42**	3.43	0.77	1.58
5. LL				
HAR–RV	–			
HAR–J	0.38	–		
HAR–RJ	0.73	0.72	–	
HAR–ARJ	1.13	1.31	0.00	–
HAR–C–J	1.43	0.94	0.18	0.21
6. QLIKE				
HAR–RV	–			
HAR–J	0.27	–		
HAR–RJ	0.49	0.47	–	
HAR–ARJ	1.12	2.09	0.19	–
HAR–C–J	1.29	0.80	0.26	0.14

5–Day Horizon

	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE				
HAR–RV	–			
HAR–J	1.69	–		
HAR–RJ	**4.63**	2.59	–	
HAR–ARJ	2.36	1.20	-0.01	–
HAR–C–J	**4.40**	**4.26**	2.34	2.55
2. SPE				
HAR–RV	–			
HAR–J	2.14	–		
HAR–RJ	**4.22**	3.18	–	
HAR–ARJ	3.11	1.34	-0.02	–
HAR–C–J	2.30	1.70	0.47	0.50
3. AE				
HAR–RV	–			
HAR–J	**3.85**	–		
HAR–RJ	**7.07**	**4.31**	–	
HAR–ARJ	3.00	0.73	-0.75	–
HAR–C–J	**4.31**	3.02	0.77	1.83
4. APE				
HAR–RV	–			
HAR–J	**4.25**	–		
HAR–RJ	**7.93**	**5.49**	–	
HAR–ARJ	3.42	0.62	-1.84	–
HAR–C–J	2.85	1.59	0.07	0.90
5. LL				
HAR–RV	–			
HAR–J	1.97	–		
HAR–RJ	**4.58**	3.02	–	
HAR–ARJ	2.52	1.13	-0.28	–
HAR–C–J	3.54	3.12	1.37	2.17
6. QLIKE				
HAR–RV	–			
HAR–J	1.63	–		
HAR–RJ	**4.19**	2.56	–	
HAR–ARJ	2.07	0.95	-0.35	–
HAR–C–J	3.60	3.31	1.60	2.65

22–Day Horizon

	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE				
HAR–RV	–			
HAR–J	1.02	–		
HAR–RJ	-0.28	-3.18	–	
HAR–ARJ	0.34	-0.02	1.37	–
HAR–C–J	0.79	0.56	0.99	0.58
2. SPE				
HAR–RV	–			
HAR–J	1.67	–		
HAR–RJ	-0.03	-1.63	–	
HAR–ARJ	0.26	-0.13	0.75	–
HAR–C–J	0.05	0.00	0.07	0.01
3. AE				
HAR–RV	–			
HAR–J	1.56	–		
HAR–RJ	0.00	-2.38	–	
HAR–ARJ	0.03	-2.72	0.09	–
HAR–C–J	0.06	0.00	0.07	0.05
4. APE				
HAR–RV	–			
HAR–J	1.51	–		
HAR–RJ	0.00	-1.72	–	
HAR–ARJ	0.01	-2.48	0.01	–
HAR–C–J	0.00	-0.06	0.00	0.00
5. LL				
HAR–RV	–			
HAR–J	1.11	–		
HAR–RJ	-0.13	-1.97	–	
HAR–ARJ	0.39	-0.01	0.93	–
HAR–C–J	0.27	0.14	0.34	0.16
6. QLIKE				
HAR–RV	–			
HAR–J	0.87	–		
HAR–RJ	-0.24	-1.89	–	
HAR–ARJ	0.38	0.00	0.92	–
HAR–C–J	0.35	0.22	0.45	0.24

Table C.15: **Out-of-Sample Forecast Comparisons for Gasoline (Rolling Window of 400 Observations)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for gasoline volatility. Each day, we use a trailing window of 400 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon				*5–Day Horizon*				*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE												
HAR–RV	–				–				–			
HAR–J	0.30	–			-0.01	–			-0.75	–		
HAR–RJ	-0.01	-0.37	–		-0.31	-1.99	–		-0.33	0.14	–	
HAR–ARJ	1.56	0.73	3.40	–	-0.09	-0.30	1.84	–	-0.45	0.20	-0.02	–
HAR–C–J	0.00	-0.13	0.00	-0.67	-0.58	-0.58	-0.42	-0.52	-0.36	-0.31	-0.32	-0.33
2. SPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.45	–			0.14	–			-0.04	–		
HAR–RJ	-0.07	-0.74	–		-0.15	-0.69	–		-0.01	0.01	–	
HAR–ARJ	0.70	0.08	2.23	–	-0.02	-0.20	0.57	–	0.00	0.13	0.14	–
HAR–C–J	0.16	0.01	0.28	0.00	1.55	1.59	2.16	1.75	0.30	0.36	0.34	0.33
3. AE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.90	–			0.01	–			-0.02	–		
HAR–RJ	-0.06	-1.34	–		-0.26	-1.42	–		0.02	0.28	–	
HAR–ARJ	2.56	0.56	**6.01**	–	-0.30	-1.36	0.02	–	0.00	0.15	-0.14	–
HAR–C–J	0.49	0.09	0.73	-0.01	0.02	0.01	0.10	0.09	0.19	0.23	0.20	0.21
4. APE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	1.61	–			0.04	–			-0.01	–		
HAR–RJ	-0.14	-3.17	–		-0.43	-1.02	–		0.02	0.10	–	
HAR–ARJ	0.78	-0.07	**4.17**	–	-0.54	-1.03	-0.04	–	0.00	0.03	-0.18	–
HAR–C–J	2.53	1.22	3.48	1.24	1.59	1.74	2.49	2.48	1.25	1.45	1.38	1.41
5. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.83	–			0.34	–			0.00	–		
HAR–RJ	-0.03	-1.44	–		-0.54	-1.59	–		0.08	0.16	–	
HAR–ARJ	1.30	0.22	**4.93**	–	-0.27	-0.91	0.74	–	0.28	0.70	0.40	–
HAR–C–J	**4.10**	3.39	**4.98**	2.25	2.81	2.76	**3.94**	3.46	1.44	1.62	1.54	1.48
6. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.63	–			0.41	–			0.02	–		
HAR–RJ	-0.04	-1.33	–		-0.78	-1.79	–		0.18	0.23	–	
HAR–ARJ	1.26	0.33	**5.10**	–	-0.48	-1.21	0.87	–	0.54	1.04	0.56	–
HAR–C–J	**6.07**	**5.79**	**7.32**	**4.11**	3.37	3.32	**4.61**	**4.15**	2.26	2.51	2.41	2.31

Table C.16: **Volatility Forecasting Errors (Rolling Window of 800 Observations)**

This table presents out-of-sample forecasting errors for the five volatility models considered. Each panel focuses on a specific loss function. MSE is the mean squared error, $MSPE$ is the mean squared percentage error, MAE is the mean absolute error, $MAPE$ is the mean absolute percentage error, LL is the logarithmic loss, and $QLIKE$ is the quasi likelihood loss function. We consider three forecast horizons, namely 1, 5, and 22 days. We use a trailing window of 800 observations to estimate the parameters of the forecasting models. In order to facilitate the presentation of our results, we multiply each loss function by 100.

	1–Day					5–Day					22–Day				
A. MSE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	0.560	0.560	0.558	0.560	0.563	0.444	0.445	0.443	0.443	0.447	0.507	0.509	0.506	0.510	0.484
Heating oil	0.370	0.371	0.369	0.371	0.369	0.254	0.254	0.256	0.255	0.252	0.236	0.236	0.239	0.237	0.234
Natural gas	1.192	1.192	1.197	1.195	1.180	0.586	0.587	0.598	0.586	0.578	0.605	0.605	0.606	0.605	0.627
Gasoline	0.476	0.472	0.469	0.471	0.490	0.329	0.329	0.330	0.328	0.347	0.337	0.341	0.343	0.342	0.361
B. MSPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	8.060	8.063	8.022	8.077	8.082	4.101	4.109	4.044	4.063	4.118	5.055	5.079	5.033	5.072	4.930
Heating oil	8.105	8.115	8.095	8.118	7.913	3.630	3.630	3.647	3.635	3.540	3.704	3.706	3.740	3.713	3.702
Natural gas	8.762	8.735	8.742	8.754	8.602	3.151	3.142	3.174	3.145	3.087	3.176	3.169	3.181	3.183	3.273
Gasoline	7.929	7.879	7.830	7.868	8.194	3.913	3.897	3.902	3.884	4.072	4.429	4.463	4.479	4.471	4.754
C. MAE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	5.162	5.178	5.172	5.185	5.212	4.397	4.401	4.399	4.397	4.460	5.549	5.559	5.548	5.552	5.441
Heating oil	4.272	4.276	4.268	4.273	4.250	3.523	3.523	3.544	3.518	3.489	3.944	3.945	3.965	3.944	3.939
Natural gas	7.816	7.845	7.896	7.866	7.810	5.468	5.483	5.577	5.479	5.460	5.678	5.678	5.701	5.695	5.702
Gasoline	4.979	4.964	4.937	4.950	5.073	4.053	4.056	4.050	4.046	4.166	4.471	4.490	4.500	4.499	4.627
D. MAPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	19.761	19.819	19.787	19.845	19.956	15.202	15.208	15.179	15.170	15.454	18.741	18.774	18.722	18.735	18.538
Heating oil	19.426	19.427	19.395	19.415	19.281	14.578	14.571	14.636	14.551	14.407	16.101	16.106	16.174	16.092	16.126
Natural gas	21.575	21.622	21.724	21.667	21.489	13.850	13.865	14.058	13.857	13.812	14.036	14.025	14.070	14.060	14.051
Gasoline	20.117	20.050	19.950	19.991	20.459	15.168	15.153	15.123	15.111	15.517	16.706	16.755	16.788	16.781	17.291
E. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	6.081	6.080	6.067	6.093	6.128	4.128	4.133	4.096	4.103	4.184	4.961	4.980	4.949	4.979	4.870
Heating oil	5.923	5.922	5.899	5.920	5.884	3.504	3.504	3.519	3.503	3.479	3.420	3.423	3.442	3.427	3.442
Natural gas	6.757	6.755	6.775	6.770	6.674	3.034	3.031	3.069	3.027	2.987	3.191	3.187	3.186	3.180	3.270
Gasoline	6.148	6.109	6.076	6.095	6.310	3.776	3.780	3.786	3.768	3.976	4.018	4.056	4.072	4.067	4.335
F. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	-30.492	-30.493	-30.497	-30.487	-30.464	-27.819	-27.817	-27.833	-27.831	-27.786	-25.104	-25.095	-25.110	-25.094	-25.148
Heating oil	-45.451	-45.452	-45.463	-45.453	-45.454	-43.154	-43.153	-43.147	-43.154	-43.155	-41.344	-41.343	-41.335	-41.340	-41.329
Natural gas	4.401	4.400	4.410	4.407	4.364	6.898	6.897	6.917	6.894	6.875	8.423	8.421	8.418	8.415	8.464
Gasoline	-35.432	-35.453	-35.469	-35.459	-35.348	-33.085	-33.079	-33.076	-33.086	-32.965	-30.905	-30.884	-30.875	-30.878	-30.736

Table C.17: **Out-of-Sample Volatility Forecast Comparisons for Crude Oil (Rolling Window of 800 Observations**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for crude oil volatility. Each day, we use a trailing window of 800 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					*5–Day Horizon*					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.04	–			HAR–J	2.90	–			HAR–J	1.93	–		
HAR–RJ	-0.36	-0.60	–		HAR–RJ	-0.04	-0.57	–		HAR–RJ	-0.10	-0.64	–	
HAR–ARJ	0.02	0.80	2.83	–	HAR–ARJ	0.00	-1.16	0.08	–	HAR–ARJ	1.44	0.34	1.20	–
HAR–C–J	1.11	2.51	3.07	1.39	HAR–C–J	0.27	0.11	0.31	0.27	HAR–C–J	-0.73	-0.87	-0.66	-0.91
2. SPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.01	–			HAR–J	0.21	–			HAR–J	2.59	–		
HAR–RJ	-0.31	-0.91	–		HAR–RJ	-1.07	-1.70	–		HAR–RJ	-0.24	-0.93	–	
HAR–ARJ	0.10	0.37	3.63	–	HAR–ARJ	-1.05	-2.97	0.37	–	HAR–ARJ	0.44	-0.15	1.00	–
HAR–C–J	0.11	0.15	0.80	0.01	HAR–C–J	0.04	0.01	0.58	0.34	HAR–C–J	-0.23	-0.34	-0.17	-0.31
3. AE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.86	–			HAR–J	0.23	–			HAR–J	1.30	–		
HAR–RJ	0.51	-0.36	–		HAR–RJ	0.02	-0.01	–		HAR–RJ	-0.01	-0.46	–	
HAR–ARJ	1.55	1.96	1.92	–	HAR–ARJ	0.00	-0.22	-0.04	–	HAR–ARJ	0.04	-0.79	0.07	–
HAR–C–J	**5.44**	**5.99**	**5.92**	3.56	HAR–C–J	2.21	2.04	1.81	2.30	HAR–C–J	-0.41	-0.51	-0.38	-0.44
4. APE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.53	–			HAR–J	0.02	–			HAR–J	1.01	–		
HAR–RJ	0.17	-0.58	–		HAR–RJ	-0.11	-0.19	–		HAR–RJ	-0.13	-0.79	–	
HAR–ARJ	1.05	1.85	2.10	–	HAR–ARJ	-0.41	-0.94	-0.04	–	HAR–ARJ	-0.02	-1.91	0.05	–
HAR–C–J	**4.38**	**6.44**	**6.74**	3.73	HAR–C–J	2.49	2.58	2.70	3.29	HAR–C–J	-0.14	-0.19	-0.11	-0.13
5. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.00	–			HAR–J	0.13	–			HAR–J	2.72	–		
HAR–RJ	-0.18	-0.45	–		HAR–RJ	-1.05	-1.68	–		HAR–RJ	-0.25	-1.31	–	
HAR–ARJ	0.15	1.71	2.09	–	HAR–ARJ	-1.29	-3.72	0.11	–	HAR–ARJ	1.07	0.00	1.37	–
HAR–C–J	1.44	**4.21**	**4.14**	1.87	HAR–C–J	0.62	0.55	1.41	1.32	HAR–C–J	-0.11	-0.16	-0.08	-0.16
6. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.00	–			HAR–J	0.12	–			HAR–J	2.58	–		
HAR–RJ	-0.16	-0.25	–		HAR–RJ	-1.11	-1.76	–		HAR–RJ	-0.33	-1.47	–	
HAR–ARJ	0.10	1.76	1.35	–	HAR–ARJ	-1.26	-3.70	0.09	–	HAR–ARJ	1.29	0.02	1.59	–
HAR–C–J	2.13	**6.56**	**5.50**	3.51	HAR–C–J	0.71	0.65	1.37	1.31	HAR–C–J	-0.08	-0.12	-0.06	-0.12

Table C.18: **Out-of-Sample Volatility Forecast Comparisons for Heating Oil (Rolling Window of 800 Observations)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for heating oil volatility. Each day, we use a trailing window of 800 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					*5–Day Horizon*					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	1.17	–			HAR–J	1.07	–			HAR–J	0.04	–		
HAR–RJ	-0.12	-0.58	–		HAR–RJ	3.25	2.06	–		HAR–RJ	2.76	2.27	–	
HAR–ARJ	1.14	0.14	0.60	–	HAR–ARJ	0.42	0.06	-1.31	–	HAR–ARJ	0.38	0.42	-1.23	–
HAR–C–J	-0.11	-0.56	0.00	-0.61	HAR–C–J	-0.21	-0.36	-1.12	-0.40	HAR–C–J	-0.13	-0.14	-0.49	-0.24
2. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.12	–			HAR–J	0.00	–			HAR–J	0.11	–		
HAR–RJ	-0.02	-0.10	–		HAR–RJ	0.32	0.34	–		HAR–RJ	2.37	1.97	–	
HAR–ARJ	0.16	0.19	0.12	–	HAR–ARJ	0.03	0.05	-0.10	–	HAR–ARJ	0.15	0.13	-1.02	–
HAR–C–J	**-3.87**	**-4.08**	-1.77	**-4.16**	HAR–C–J	-1.68	-1.65	-1.78	-1.55	HAR–C–J	0.00	0.00	-0.08	-0.01
3. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.20	–			HAR–J	0.01	–			HAR–J	0.07	–		
HAR–RJ	-0.07	-0.36	–		HAR–RJ	2.40	2.44	–		HAR–RJ	2.07	1.67	–	
HAR–ARJ	0.04	-0.69	0.20	–	HAR–ARJ	-0.43	-0.80	**-4.26**	–	HAR–ARJ	0.00	-0.03	-2.51	–
HAR–C–J	-1.65	-2.94	-0.76	-2.39	HAR–C–J	-1.12	-1.14	-2.48	-0.79	HAR–C–J	-0.01	-0.01	-0.14	-0.01
4. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.00	–			HAR–J	-0.07	–			HAR–J	0.06	–		
HAR–RJ	-0.16	-0.25	–		HAR–RJ	0.92	1.33	–		HAR–RJ	1.45	1.19	–	
HAR–ARJ	-0.04	-0.83	0.10	–	HAR–ARJ	-0.58	-0.65	-2.58	–	HAR–ARJ	-0.06	-0.18	-2.29	–
HAR–C–J	-3.07	**-4.51**	-1.37	-3.63	HAR–C–J	-1.40	-1.29	-2.17	-0.96	HAR–C–J	0.01	0.00	-0.02	0.01
5. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.00	–			HAR–J	0.00	–			HAR–J	0.23	–		
HAR–RJ	-0.51	-0.55	–		HAR–RJ	0.45	0.47	–		HAR–RJ	1.04	0.73	–	
HAR–ARJ	-0.02	-0.13	0.47	–	HAR–ARJ	0.00	0.00	-0.52	–	HAR–ARJ	0.25	0.14	-0.54	–
HAR–C–J	-1.10	-1.36	-0.10	-1.22	HAR–C–J	-0.24	-0.25	-0.50	-0.23	HAR–C–J	0.06	0.05	0.00	0.03
6. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.00	–			HAR–J	0.02	–			HAR–J	0.31	–		
HAR–RJ	-0.58	-0.67	–		HAR–RJ	0.41	0.36	–		HAR–RJ	0.63	0.39	–	
HAR–ARJ	-0.04	-0.29	0.53	–	HAR–ARJ	0.00	0.00	-0.47	–	HAR–ARJ	0.39	0.21	-0.22	–
HAR–C–J	-0.02	-0.01	0.18	0.00	HAR–C–J	0.00	-0.01	-0.10	-0.01	HAR–C–J	0.13	0.11	0.02	0.07

Table C.19: **Out-of-Sample Volatility Forecast Comparisons for Natural Gas (Rolling Window of 800 Observations)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for natural gas volatility. Each day, we use a trailing window of 800 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon				*5–Day Horizon*				*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE												
HAR–RV	–				–				–			
HAR–J	0.00	–			0.12	–			0.00	–		
HAR–RJ	0.36	1.49	–		2.58	3.54	–		0.06	0.09	–	
HAR–ARJ	0.21	1.41	-0.48	–	-0.01	-0.32	-3.41	–	0.00	0.00	-0.30	–
HAR–C–J	-2.82	**-6.91**	**-7.65**	**-8.23**	-2.16	**-4.26**	**-7.84**	-2.85	1.33	1.44	1.37	1.43
2. SPE												
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-0.14	–			-0.22	–			-0.13	–		
HAR–RJ	-0.02	0.02	–		0.21	0.96	–		0.03	0.20	–	
HAR–ARJ	-0.01	0.72	0.04	–	-0.14	0.07	-0.51	–	0.06	0.15	0.01	–
HAR–C–J	-3.46	**-7.25**	-3.29	**-8.20**	-1.60	-2.22	**-4.89**	-1.78	0.86	1.06	0.85	0.75
3. AE												
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	1.44	–			0.92	–			0.00	–		
HAR–RJ	3.60	**4.44**	–		**6.03**	**8.52**	–		0.38	0.75	–	
HAR–ARJ	3.62	2.71	-2.54	–	0.31	-0.19	**-10.00**	–	0.32	0.54	-0.22	–
HAR–C–J	-0.03	-2.73	**-7.05**	**-5.33**	-0.07	-0.84	**-9.20**	-0.50	0.07	0.07	0.00	0.01
4. APE												
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.43	–			0.17	–			-0.07	–		
HAR–RJ	1.65	3.28	–		**4.38**	**7.27**	–		0.15	0.46	–	
HAR–ARJ	1.71	2.32	-1.14	–	0.03	-0.15	**-7.54**	–	0.11	0.32	-0.08	–
HAR–C–J	-0.96	**-5.13**	**-7.90**	**-7.57**	-0.21	-0.66	**-7.58**	-0.39	0.00	0.01	-0.01	0.00
5. LL												
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.00	–			-0.05	–			-0.12	–		
HAR–RJ	0.10	0.70	–		1.07	2.35	–		-0.05	0.00	–	
HAR–ARJ	0.13	1.33	-0.05	–	-0.33	-0.23	-2.17	–	-0.21	-0.07	-0.22	–
HAR–C–J	-3.59	**-8.84**	**-7.47**	**-10.17**	-2.21	-3.39	**-6.95**	-2.22	1.10	1.24	1.17	1.25
6. QLIKE												
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.00	–			-0.04	–			-0.13	–		
HAR–RJ	0.11	0.71	–		1.35	2.59	–		-0.18	-0.10	–	
HAR–ARJ	0.11	1.23	-0.07	–	-0.54	-0.58	-2.60	–	-0.54	-0.35	-0.44	–
HAR–C–J	-3.20	**-7.97**	**-7.13**	**-9.10**	-2.65	**-4.00**	**-6.94**	-2.52	1.23	1.33	1.30	1.44

Table C.20: **Out-of-Sample Volatility Forecast Comparisons for Gasoline (Rolling Window of 800 Observations)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for gasoline volatility. Each day, we use a trailing window of 800 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon				*5–Day Horizon*				*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE												
HAR–RV	–				–				–			
HAR–J	-2.57	–			0.21	–			2.83	–		
HAR–RJ	**-6.58**	**-5.56**	–		0.36	0.35	–		2.69	1.68	–	
HAR–ARJ	-2.93	-0.47	**8.12**	–	-0.01	-0.54	**-3.88**	–	2.87	1.20	-0.84	–
HAR–C–J	**9.84**	**14.22**	**16.07**	**14.80**	**7.10**	**7.40**	**6.86**	**7.25**	**6.24**	**5.53**	**4.69**	**4.80**
2. SPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-1.13	–			-0.36	–			1.09	–		
HAR–RJ	**-3.93**	-1.52	–		-0.06	0.04	–		0.92	0.46	–	
HAR–ARJ	-1.43	-0.08	**4.30**	–	-0.53	-0.34	-1.75	–	0.84	0.16	-0.63	–
HAR–C–J	3.65	**5.46**	**5.88**	**5.46**	**5.07**	**7.17**	**5.70**	**6.77**	**7.71**	**6.58**	**5.31**	**5.58**
3. AE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-0.85	–			0.04	–			2.18	–		
HAR–RJ	**-6.30**	**-6.35**	–		-0.04	-0.33	–		2.40	1.10	–	
HAR–ARJ	-2.75	-1.92	**6.35**	–	-0.27	-0.98	-0.30	–	2.95	0.90	-0.11	–
HAR–C–J	**9.44**	**16.20**	**20.20**	**17.54**	**7.94**	**8.66**	**9.43**	**9.37**	**4.82**	**4.28**	3.69	3.63
4. APE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-1.10	–			-0.13	–			1.21	–		
HAR–RJ	**-5.58**	**-5.35**	–		-0.39	-0.46	–		1.50	0.69	–	
HAR–ARJ	-2.89	-1.86	3.39	–	-0.94	-1.03	-0.21	–	1.65	0.44	-0.13	–
HAR–C–J	**6.93**	**13.21**	**16.95**	**15.05**	**4.99**	**6.28**	**7.06**	**7.10**	**4.98**	**4.68**	**4.01**	**3.97**
5. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-2.34	–			0.06	–			3.05	–		
HAR–RJ	**-5.80**	-3.37	–		0.11	0.11	–		3.00	1.34	–	
HAR–ARJ	-3.05	-0.60	**4.06**	–	-0.12	-0.58	-2.75	–	3.31	0.69	-0.48	–
HAR–C–J	**7.34**	**12.28**	**14.31**	**13.23**	**5.86**	**6.90**	**6.41**	**6.68**	**5.87**	**5.52**	**5.03**	**4.99**
6. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-3.03	–			0.32	–			3.47	–		
HAR–RJ	**-6.21**	-3.34	–		0.38	0.14	–		3.68	1.74	–	
HAR–ARJ	**-3.93**	-0.63	3.55	–	-0.01	-0.71	-2.71	–	**4.26**	0.94	-0.36	–
HAR–C–J	**7.29**	**11.17**	**12.57**	**12.11**	**4.83**	**5.40**	**5.13**	**5.22**	**4.89**	**4.64**	**4.32**	**4.23**

Table C.21: **Volatility Forecasting Errors (Rolling Window of 1,000 Observations)**

This table presents out-of-sample forecasting errors for the five volatility models considered. Each panel focuses on a specific loss function. MSE is the mean squared error, $MSPE$ is the mean squared percentage error, MAE is the mean absolute error, $MAPE$ is the mean absolute percentage error, LL is the logarithmic loss, and $QLIKE$ is the quasi likelihood loss function. We consider three forecast horizons, namely 1, 5, and 22 days. We use a trailing window of 1,000 observations to estimate the parameters of the forecasting models. In order to facilitate the presentation of our results, we multiply each loss function by 100.

	1–Day					5–Day					22–Day				
A. MSE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	0.643	0.643	0.645	0.644	0.641	0.566	0.567	0.565	0.565	0.555	0.574	0.574	0.574	0.575	0.502
Heating oil	0.396	0.397	0.398	0.398	0.395	0.305	0.306	0.308	0.307	0.305	0.246	0.246	0.248	0.248	0.244
Natural gas	1.137	1.134	1.142	1.136	1.117	0.623	0.620	0.636	0.621	0.606	0.698	0.696	0.700	0.698	0.705
Gasoline	0.525	0.522	0.520	0.520	0.544	0.399	0.399	0.400	0.397	0.422	0.373	0.378	0.379	0.378	0.402
B. MSPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	8.274	8.277	8.307	8.300	8.226	4.886	4.898	4.821	4.848	4.754	5.407	5.409	5.383	5.403	4.768
Heating oil	8.537	8.566	8.614	8.575	8.461	4.245	4.258	4.268	4.287	4.213	3.810	3.813	3.856	3.854	3.808
Natural gas	8.427	8.372	8.409	8.386	8.152	3.251	3.209	3.254	3.224	3.104	3.463	3.443	3.475	3.477	3.483
Gasoline	8.211	8.189	8.122	8.124	8.479	4.599	4.572	4.572	4.539	4.771	4.715	4.735	4.743	4.734	5.022
C. MAE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	5.395	5.406	5.412	5.407	5.393	4.979	4.987	4.985	4.975	4.925	5.921	5.926	5.931	5.926	5.444
Heating oil	4.290	4.295	4.296	4.296	4.266	3.833	3.839	3.847	3.844	3.821	3.947	3.949	3.969	3.953	3.906
Natural gas	7.355	7.381	7.440	7.392	7.336	5.401	5.405	5.508	5.401	5.369	5.926	5.914	5.941	5.941	5.910
Gasoline	5.101	5.113	5.076	5.081	5.256	4.438	4.437	4.432	4.428	4.507	4.599	4.619	4.630	4.624	4.772
D. MAPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	20.289	20.314	20.342	20.326	20.291	16.898	16.928	16.878	16.864	16.767	19.454	19.479	19.475	19.471	18.051
Heating oil	19.903	19.920	19.918	19.924	19.809	15.962	15.984	15.988	16.010	15.924	16.187	16.195	16.293	16.219	16.022
Natural gas	21.091	21.108	21.212	21.130	20.913	14.007	13.991	14.184	13.986	13.886	14.745	14.700	14.750	14.764	14.675
Gasoline	20.548	20.585	20.420	20.442	21.033	16.540	16.509	16.479	16.467	16.653	16.871	16.912	16.942	16.922	17.429
E. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	6.536	6.542	6.562	6.554	6.525	5.115	5.124	5.081	5.093	5.016	5.447	5.451	5.438	5.454	4.843
Heating oil	6.311	6.326	6.338	6.330	6.289	4.190	4.202	4.210	4.217	4.176	3.580	3.583	3.608	3.609	3.571
Natural gas	6.503	6.482	6.510	6.492	6.366	3.163	3.138	3.187	3.143	3.057	3.569	3.554	3.567	3.562	3.585
Gasoline	6.578	6.558	6.523	6.529	6.789	4.505	4.512	4.517	4.484	4.746	4.345	4.379	4.389	4.383	4.675
F. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	-30.881	-30.877	-30.867	-30.872	-30.883	-27.714	-27.709	-27.729	-27.723	-27.764	-24.256	-24.254	-24.261	-24.251	-24.570
Heating oil	-48.993	-48.985	-48.980	-48.983	-48.999	-46.298	-46.292	-46.289	-46.285	-46.302	-43.954	-43.953	-43.943	-43.941	-43.960
Natural gas	0.950	0.943	0.956	0.947	0.891	3.273	3.261	3.287	3.262	3.222	4.334	4.326	4.329	4.326	4.342
Gasoline	-36.721	-36.733	-36.747	-36.743	-36.598	-34.135	-34.126	-34.123	-34.141	-33.986	-31.441	-31.421	-31.415	-31.418	-31.260

Table C.22: **Out-of-Sample Volatility Forecast Comparisons for Crude Oil (Rolling Window of 1,000 Observations)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for crude oil volatility. Each day, we use a trailing window of 1,000 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					*5–Day Horizon*					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.04	–			HAR–J	2.44	–			HAR–J	-0.07	–		
HAR–RJ	0.41	3.28	–		HAR–RJ	-0.01	-0.25	–		HAR–RJ	-0.02	-0.01	–	
HAR–ARJ	0.27	0.62	-0.92	–	HAR–ARJ	-0.03	-1.13	0.00	–	HAR–ARJ	0.11	0.24	0.08	–
HAR–C–J	-0.31	-0.76	-1.62	-1.10	HAR–C–J	-1.18	-1.48	-0.96	-1.08	HAR–C–J	**-5.45**	**-5.53**	**-5.17**	**-5.52**
2. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.00	–			HAR–J	0.84	–			HAR–J	0.05	–		
HAR–RJ	0.34	3.53	–		HAR–RJ	-0.83	-1.41	–		HAR–RJ	-0.16	-0.16	–	
HAR–ARJ	0.40	2.07	-0.16	–	HAR–ARJ	-0.80	-2.29	0.43	–	HAR–ARJ	-0.04	-0.09	0.16	–
HAR–C–J	-0.38	-0.60	-1.41	-1.22	HAR–C–J	-1.15	-1.34	-0.24	-0.52	HAR–C–J	-3.70	-3.77	-3.47	-3.74
3. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.57	–			HAR–J	**5.58**	–			HAR–J	1.46	–		
HAR–RJ	0.95	1.37	–		HAR–RJ	0.10	-0.01	–		HAR–RJ	0.28	0.04	–	
HAR–ARJ	0.74	0.07	-1.54	–	HAR–ARJ	-0.12	-1.49	-0.46	–	HAR–ARJ	0.54	0.00	-0.04	–
HAR–C–J	-0.02	-0.60	-1.14	-0.67	HAR–C–J	-1.08	-1.41	-1.07	-0.86	HAR–C–J	**-4.47**	**-4.65**	**-4.28**	**-4.67**
4. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.19	–			HAR–J	**4.28**	–			HAR–J	1.83	–		
HAR–RJ	0.54	1.44	–		HAR–RJ	-0.05	-0.38	–		HAR–RJ	0.10	0.00	–	
HAR–ARJ	0.43	0.43	-0.77	–	HAR–ARJ	-0.47	-2.09	-0.06	–	HAR–ARJ	0.29	-0.10	0.00	–
HAR–C–J	0.00	-0.12	-0.53	-0.26	HAR–C–J	-0.45	-0.67	-0.25	-0.22	HAR–C–J	-3.14	-3.31	-3.01	-3.29
5. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.04	–			HAR–J	1.41	–			HAR–J	0.31	–		
HAR–RJ	0.53	2.40	–		HAR–RJ	-0.74	-1.37	–		HAR–RJ	-0.10	-0.15	–	
HAR–ARJ	0.44	1.29	-0.68	–	HAR–ARJ	-0.86	-2.71	0.20	–	HAR–ARJ	0.22	0.06	0.25	–
HAR–C–J	-0.07	-0.30	-1.27	-0.78	HAR–C–J	-1.07	-1.27	-0.40	-0.60	HAR–C–J	-3.39	-3.46	-3.25	-3.44
6. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.10	–			HAR–J	1.35	–			HAR–J	0.37	–		
HAR–RJ	0.64	2.02	–		HAR–RJ	-0.78	-1.46	–		HAR–RJ	-0.15	-0.22	–	
HAR–ARJ	0.50	0.91	-0.99	–	HAR–ARJ	-0.76	-2.59	0.19	–	HAR–ARJ	0.40	0.19	0.39	–
HAR–C–J	-0.01	-0.16	-0.97	-0.48	HAR–C–J	-0.85	-1.00	-0.38	-0.53	HAR–C–J	-2.93	-2.99	-2.80	-2.98

Table C.23: **Out-of-Sample Volatility Forecast Comparisons for Heating Oil (Rolling Window of 1,000 Observations)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for heating oil volatility. Each day, we use a trailing window of 1,000 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon				*5–Day Horizon*				*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE												
HAR–RV	–				–				–			
HAR–J	**7.44**	–			**7.40**	–			0.19	–		
HAR–RJ	0.83	0.14	–		2.54	1.16	–		1.90	1.50	–	
HAR–ARJ	**9.11**	2.79	-0.06	–	3.20	1.35	-0.09	–	1.96	2.03	-0.03	–
HAR–C–J	-0.18	-1.29	-1.00	-1.59	0.00	-0.06	-0.47	-0.33	-0.06	-0.07	-0.40	-0.46
2. SPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	3.31	–			**6.31**	–			0.46	–		
HAR–RJ	0.77	0.34	–		0.47	0.08	–		3.75	2.99	–	
HAR–ARJ	**4.63**	3.76	-0.21	–	1.87	1.02	0.12	–	2.00	2.12	-0.01	–
HAR–C–J	-0.86	-1.39	-1.11	-1.63	-0.19	-0.38	-0.39	-0.77	0.00	0.00	-0.19	-0.21
3. AE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	2.91	–			**4.21**	–			0.28	–		
HAR–RJ	0.16	0.01	–		0.88	0.29	–		1.95	1.39	–	
HAR–ARJ	3.42	0.56	0.00	–	2.38	0.56	-0.04	–	0.42	0.25	-0.97	–
HAR–C–J	-2.08	-3.01	-1.59	-3.11	-0.13	-0.28	-0.49	-0.43	-0.44	-0.50	-0.93	-0.66
4. APE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	1.10	–			3.53	–			0.22	–		
HAR–RJ	0.03	0.00	–		0.12	0.00	–		2.42	1.81	–	
HAR–ARJ	1.58	1.17	0.01	–	1.78	0.60	0.08	–	0.47	0.36	-1.17	–
HAR–C–J	-1.20	-1.62	-0.77	-1.74	-0.06	-0.14	-0.12	-0.28	-0.34	-0.38	-0.81	-0.54
5. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	**4.37**	–			**6.62**	–			0.30	–		
HAR–RJ	0.51	0.11	–		0.53	0.08	–		1.41	1.02	–	
HAR–ARJ	**6.21**	3.51	-0.05	–	3.22	1.16	0.06	–	2.62	2.64	0.00	–
HAR–C–J	-0.31	-0.87	-0.72	-1.07	-0.06	-0.19	-0.25	-0.45	-0.01	-0.02	-0.16	-0.20
6. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	**4.59**	–			**6.27**	–			0.24	–		
HAR–RJ	0.54	0.09	–		0.40	0.05	–		0.77	0.53	–	
HAR–ARJ	**6.37**	2.90	-0.04	–	**3.94**	1.22	0.08	–	2.90	2.71	0.03	–
HAR–C–J	-0.09	-0.51	-0.49	-0.63	-0.03	-0.13	-0.16	-0.34	-0.01	-0.02	-0.12	-0.19

Table C.24: **Out-of-Sample Volatility Forecast Comparisons for Natural Gas (Rolling Window of 1,000 Observations)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for natural gas volatility. Each day, we use a trailing window of 1,000 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					*5–Day Horizon*					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.13	–			HAR–J	-0.94	–			HAR–J	-0.49	–		
HAR–RJ	0.31	2.74	–		HAR–RJ	1.50	3.35	–		HAR–RJ	0.27	1.11	–	
HAR–ARJ	-0.01	1.25	-1.64	–	HAR–ARJ	-1.00	0.24	-2.62	–	HAR–ARJ	0.02	0.33	-0.44	–
HAR–C–J	**-4.28**	**-7.04**	**-9.58**	**-7.96**	HAR–C–J	**-4.20**	**-5.20**	**-8.32**	**-4.62**	HAR–C–J	0.35	0.62	0.19	0.32
2. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.33	–			HAR–J	-1.53	–			HAR–J	-0.62	–		
HAR–RJ	-0.02	1.00	–		HAR–RJ	0.00	1.87	–		HAR–RJ	0.17	1.23	–	
HAR–ARJ	-0.22	0.61	-0.31	–	HAR–ARJ	-1.70	0.66	-0.42	–	HAR–ARJ	0.25	0.67	0.01	–
HAR–C–J	**-6.03**	**-8.08**	**-7.71**	**-8.75**	HAR–C–J	-2.76	-2.82	**-6.17**	-2.69	HAR–C–J	0.07	0.35	0.01	0.01
3. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.78	–			HAR–J	0.06	–			HAR–J	-0.66	–		
HAR–RJ	2.68	**4.14**	–		HAR–RJ	**3.93**	**6.49**	–		HAR–RJ	0.14	0.68	–	
HAR–ARJ	1.65	1.57	-3.41	–	HAR–ARJ	0.00	-0.18	**-7.00**	–	HAR–ARJ	0.27	1.04	0.00	–
HAR–C–J	-0.23	-2.42	**-6.23**	-3.45	HAR–C–J	-0.56	-1.19	**-8.62**	-0.89	HAR–C–J	-0.07	0.00	-0.27	-0.27
4. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.03	–			HAR–J	-0.13	–			HAR–J	-1.02	–		
HAR–RJ	0.74	2.80	–		HAR–RJ	2.15	**5.24**	–		HAR–RJ	0.00	0.42	–	
HAR–ARJ	0.19	0.83	-1.82	–	HAR–ARJ	-0.25	-0.04	**-5.08**	–	HAR–ARJ	0.07	0.76	0.11	–
HAR–C–J	-2.28	**-5.87**	**-7.94**	**-6.63**	HAR–C–J	-1.01	-1.24	**-6.70**	-0.98	HAR–C–J	-0.18	-0.02	-0.20	-0.28
5. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.19	–			HAR–J	-1.76	–			HAR–J	-0.95	–		
HAR–RJ	0.01	1.15	–		HAR–RJ	0.29	2.55	–		HAR–RJ	-0.01	0.37	–	
HAR–ARJ	-0.07	0.69	-0.46	–	HAR–ARJ	-2.26	0.23	-1.54	–	HAR–ARJ	-0.09	0.10	-0.12	–
HAR–C–J	**-5.17**	**-9.54**	**-9.47**	**-10.09**	HAR–C–J	**-3.91**	**-4.09**	**-7.96**	-3.63	HAR–C–J	0.09	0.37	0.11	0.16
6. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.11	–			HAR–J	-2.01	–			HAR–J	-1.14	–		
HAR–RJ	0.03	1.00	–		HAR–RJ	0.43	2.39	–		HAR–RJ	-0.11	0.11	–	
HAR–ARJ	-0.02	0.64	-0.44	–	HAR–ARJ	-2.66	0.08	-1.74	–	HAR–ARJ	-0.36	0.00	-0.22	–
HAR–C–J	**-4.30**	**-9.16**	**-9.19**	**-9.59**	HAR–C–J	**-4.61**	**-4.81**	**-7.65**	**-4.23**	HAR–C–J	0.10	0.41	0.20	0.29

Table C.25: **Out-of-Sample Volatility Forecast Comparisons for Gasoline (Rolling Window of 1,000 Observations)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for gasoline volatility. Each day, we use a trailing window of 1,000 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon				*5–Day Horizon*				*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE												
HAR–RV	–				–				–			
HAR–J	-1.26	–			0.01	–			1.59	–		
HAR–RJ	-2.48	-3.40	–		0.13	0.37	–		1.42	0.82	–	
HAR–ARJ	-2.17	-2.51	**5.86**	–	-0.34	-2.15	**-4.92**	–	1.48	0.36	-0.61	–
HAR–C–J	**10.07**	**12.76**	**14.38**	**13.98**	**6.14**	**6.74**	**6.37**	**7.01**	**6.62**	**7.07**	**6.72**	**6.52**
2. SPE												
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-0.32	–			-0.50	–			0.22	–		
HAR–RJ	-2.29	**-6.52**	–		-0.17	0.00	–		0.17	0.08	–	
HAR–ARJ	-2.23	**-6.19**	0.18	–	-1.28	-2.94	-3.73	–	0.11	0.00	-0.42	–
HAR–C–J	1.75	2.18	3.09	2.95	2.94	**4.46**	**3.88**	**5.21**	**5.39**	**5.55**	**5.04**	**5.13**
3. AE												
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.46	–			0.00	–			1.55	–		
HAR–RJ	-1.42	**-8.24**	–		-0.05	-0.16	–		1.85	1.72	–	
HAR–ARJ	-0.88	**-7.14**	**5.54**	–	-0.32	-0.96	-0.18	–	1.69	0.46	-0.83	–
HAR–C–J	**15.97**	**18.05**	**23.53**	**22.67**	1.61	2.08	2.48	2.40	**6.17**	**6.25**	**5.82**	**5.81**
4. APE												
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	0.25	–			-0.33	–			0.44	–		
HAR–RJ	-2.09	**-8.73**	–		-0.46	-0.41	–		0.64	0.82	–	
HAR–ARJ	-1.47	**-8.17**	**4.57**	–	-1.19	-1.35	-0.11	–	0.42	0.10	-0.70	–
HAR–C–J	**7.89**	**8.98**	**14.12**	**13.42**	0.26	0.55	0.82	0.84	**4.22**	**4.72**	**4.52**	**4.54**
5. LL												
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-0.54	–			0.07	–			1.55	–		
HAR–RJ	-2.27	**-3.98**	–		0.11	0.10	–		1.32	0.54	–	
HAR–ARJ	-1.81	-2.91	**4.96**	–	-0.49	-2.89	**-4.78**	–	1.38	0.11	-0.36	–
HAR–C–J	**6.44**	**8.64**	**10.85**	**10.32**	**4.87**	**5.69**	**5.46**	**6.13**	**5.97**	**6.26**	**6.27**	**6.03**
6. QLIKE												
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				–				–			
HAR–J	-0.76	–			0.41	–			2.20	–		
HAR–RJ	-2.17	-2.73	–		0.46	0.16	–		1.98	0.75	–	
HAR–ARJ	-1.61	-1.56	**6.88**	–	-0.17	-2.42	**-4.27**	–	2.29	0.19	-0.26	–
HAR–C–J	**7.85**	**9.83**	**11.63**	**11.15**	**4.59**	**5.01**	**4.84**	**5.19**	**5.55**	**5.72**	**5.74**	**5.48**

D. Weighted Least Squares Estimation

Table D.26: **Volatility Forecasting Errors (Weighted Least Squares Estimation)**

This table presents out-of-sample forecasting errors for the five volatility models considered. Each panel focuses on a specific loss function. MSE is the mean squared error, $MSPE$ is the mean squared percentage error, MAE is the mean absolute error, $MAPE$ is the mean absolute percentage error, LL is the logarithmic loss, and $QLIKE$ is the quasi likelihood loss function. We consider three forecast horizons, namely 1, 5, and 22 days. Out-of-sample forecasts are obtained using a rolling window of 600 observations. The models are estimated via weighted least squares using as weights the inverse of the fitted values from OLS estimation. In order to facilitate the presentation of our results, we multiply each loss function by 100.

	1–Day					5–Day					22–Day				
A. MSE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	0.533	0.529	0.527	0.529	0.534	0.373	0.371	0.369	0.369	0.375	0.399	0.399	0.398	0.400	0.418
Heating oil	0.400	0.401	0.397	0.401	0.403	0.245	0.245	0.246	0.245	0.249	0.243	0.243	0.245	0.244	0.252
Natural gas	1.484	1.478	1.484	1.485	1.479	0.832	0.829	0.841	0.834	0.844	0.825	0.824	0.824	0.823	0.842
Gasoline	0.478	0.477	0.477	0.477	0.487	0.292	0.293	0.293	0.292	0.306	0.286	0.288	0.289	0.288	0.302
B. MSPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	7.300	7.270	7.218	7.278	7.396	3.497	3.481	3.422	3.439	3.603	4.319	4.330	4.294	4.334	4.663
Heating oil	7.474	7.513	7.425	7.515	7.540	3.473	3.477	3.466	3.480	3.489	4.023	4.028	4.043	4.034	4.088
Natural gas	8.189	8.158	8.186	8.199	8.146	3.017	2.993	3.044	3.030	3.011	3.009	2.998	3.014	3.013	3.053
Gasoline	7.589	7.601	7.646	7.616	7.819	3.575	3.582	3.592	3.587	3.724	4.155	4.176	4.191	4.185	4.332
C. MAE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	5.136	5.144	5.127	5.148	5.177	4.190	4.186	4.177	4.179	4.256	5.022	5.014	5.011	5.014	5.107
Heating oil	4.506	4.514	4.496	4.517	4.522	3.633	3.634	3.639	3.632	3.663	4.092	4.094	4.106	4.092	4.184
Natural gas	8.701	8.698	8.717	8.717	8.723	6.190	6.178	6.250	6.191	6.274	6.526	6.526	6.527	6.514	6.525
Gasoline	5.100	5.106	5.102	5.096	5.170	3.973	3.983	3.980	3.968	4.091	4.339	4.356	4.375	4.360	4.498
D. MAPE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	18.814	18.836	18.770	18.851	18.987	14.170	14.149	14.093	14.101	14.468	17.103	17.082	17.051	17.068	17.544
Heating oil	18.954	18.992	18.911	19.001	19.036	14.376	14.376	14.374	14.367	14.438	16.369	16.369	16.398	16.355	16.607
Natural gas	21.102	21.094	21.147	21.139	21.128	13.516	13.480	13.649	13.517	13.629	13.878	13.872	13.887	13.855	13.846
Gasoline	19.634	19.669	19.676	19.647	19.947	14.489	14.515	14.499	14.463	14.869	16.161	16.207	16.273	16.226	16.646
E. LL	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	5.606	5.590	5.560	5.600	5.645	3.552	3.527	3.499	3.500	3.604	4.195	4.188	4.179	4.190	4.430
Heating oil	5.605	5.617	5.568	5.624	5.635	3.281	3.281	3.281	3.282	3.300	3.493	3.494	3.513	3.501	3.540
Natural gas	6.582	6.566	6.587	6.596	6.536	3.063	3.046	3.092	3.067	3.045	3.168	3.158	3.165	3.163	3.177
Gasoline	5.827	5.822	5.838	5.831	5.932	3.382	3.395	3.395	3.389	3.519	3.603	3.618	3.634	3.626	3.746
F. QLIKE	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–C–J
Crude oil	-25.547	-25.556	-25.570	-25.550	-25.533	-23.256	-23.272	-23.282	-23.285	-23.241	-21.220	-21.226	-21.227	-21.225	-21.112
Heating oil	-37.244	-37.241	-37.262	-37.237	-37.235	-35.079	-35.079	-35.078	-35.078	-35.068	-33.609	-33.608	-33.598	-33.604	-33.588
Natural gas	14.539	14.531	14.540	14.546	14.512	16.978	16.969	16.992	16.978	16.964	18.141	18.135	18.137	18.136	18.139
Gasoline	-29.295	-29.300	-29.294	-29.295	-29.252	-27.072	-27.064	-27.066	-27.069	-27.000	-25.488	-25.481	-25.473	-25.477	-25.420

Table D.27: **Out-of-Sample Volatility Forecast Comparisons for Crude Oil (Weighted Least Squares Estimation)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for crude oil volatility. Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. The models are estimated via weighted least squares using as weights the inverse of the fitted values from OLS estimation. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					*5–Day Horizon*					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-1.42	–			HAR–J	-1.42	–			HAR–J	-0.09	–		
HAR–RJ	-3.82	-1.78	–		HAR–RJ	-1.97	-0.60	–		HAR–RJ	-0.36	-0.08	–	
HAR–ARJ	-1.05	1.30	3.73	–	HAR–ARJ	-1.96	-0.69	0.06	–	HAR–ARJ	0.07	0.52	0.38	–
HAR–C–J	0.01	3.18	**4.82**	2.36	HAR–C–J	0.17	0.72	1.19	1.08	HAR–C–J	1.20	1.41	1.39	1.30
2. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.30	–			HAR–J	-0.46	–			HAR–J	0.32	–		
HAR–RJ	-2.10	-2.79	–		HAR–RJ	-2.25	-2.48	–		HAR–RJ	-0.84	-1.76	–	
HAR–ARJ	-0.16	0.69	**4.74**	–	HAR–ARJ	-1.80	-2.51	0.67	–	HAR–ARJ	0.19	0.04	1.32	–
HAR–C–J	1.26	3.52	**5.34**	3.06	HAR–C–J	1.37	2.13	**4.34**	3.45	HAR–C–J	2.30	2.34	2.72	2.40
3. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.16	–			HAR–J	-0.13	–			HAR–J	-0.40	–		
HAR–RJ	-0.30	-2.36	–		HAR–RJ	-0.58	-0.34	–		HAR–RJ	-0.62	-0.02	–	
HAR–ARJ	0.35	1.37	**4.64**	–	HAR–ARJ	-0.54	-0.44	0.01	–	HAR–ARJ	-0.28	0.00	0.02	–
HAR–C–J	2.20	3.11	**5.06**	2.39	HAR–C–J	1.91	2.50	2.81	2.88	HAR–C–J	0.45	0.58	0.56	0.58
4. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.07	–			HAR–J	-0.19	–			HAR–J	-0.23	–		
HAR–RJ	-0.45	-2.50	–		HAR–RJ	-1.17	-0.95	–		HAR–RJ	-1.23	-0.33	–	
HAR–ARJ	0.21	1.44	**4.61**	–	HAR–ARJ	-1.19	-1.36	0.03	–	HAR–ARJ	-0.44	-0.20	0.10	–
HAR–C–J	2.42	**4.13**	**6.45**	3.29	HAR–C–J	2.73	3.60	**4.56**	**4.53**	HAR–C–J	0.95	1.13	1.20	1.19
5. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.16	–			HAR–J	-1.71	–			HAR–J	-0.18	–		
HAR–RJ	-1.94	-2.40	–		HAR–RJ	-2.88	-1.51	–		HAR–RJ	-0.75	-0.21	–	
HAR–ARJ	-0.02	2.62	**4.60**	–	HAR–ARJ	-3.47	-2.80	0.00	–	HAR–ARJ	-0.04	0.03	0.26	–
HAR–C–J	0.58	3.06	**5.19**	1.99	HAR–C–J	0.61	1.54	2.66	2.69	HAR–C–J	1.85	2.15	2.15	2.17
6. QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.17	–			HAR–J	-2.45	–			HAR–J	-0.54	–		
HAR–RJ	-1.69	-1.80	–		HAR–RJ	-3.25	-1.01	–		HAR–RJ	-0.62	-0.01	–	
HAR–ARJ	-0.02	3.24	3.73	–	HAR–ARJ	**-4.32**	-2.61	-0.07	–	HAR–ARJ	-0.22	0.04	0.03	–
HAR–C–J	0.30	2.30	**3.94**	1.27	HAR–C–J	0.20	0.97	1.60	1.84	HAR–C–J	1.60	1.98	1.86	1.99

Table D.28: **Out-of-Sample Volatility Forecast Comparisons for Heating Oil (Weighted Least Squares Estimation)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for heating oil volatility. Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. The models are estimated via weighted least squares using as weights the inverse of the fitted values from OLS estimation. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon				*5–Day Horizon*				*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE												
HAR–RV	–				–				–			
HAR–J	0.23	–			0.31	–			1.04	–		
HAR–RJ	-1.59	-2.21	–		0.53	0.19	–		2.53	1.45	–	
HAR–ARJ	0.58	1.30	2.92	–	0.48	0.09	-0.09	–	0.64	0.15	-1.19	–
HAR–C–J	1.58	1.99	**3.88**	0.71	2.93	2.42	1.61	1.92	1.97	1.86	1.25	1.54
2. SPE												
HAR–RV	–				–				–			
HAR–J	0.83	–			0.07	–			0.27	–		
HAR–RJ	-0.92	-3.09	–		-0.08	-0.24	–		1.16	0.62	–	
HAR–ARJ	0.78	0.04	3.39	–	0.14	0.06	0.29	–	0.39	0.16	-0.27	–
HAR–C–J	1.41	0.58	3.49	0.44	0.17	0.10	0.32	0.04	0.80	0.72	0.39	0.50
3. AE												
HAR–RV	–				–				–			
HAR–J	0.37	–			0.04	–			0.09	–		
HAR–RJ	-0.39	-1.21	–		0.27	0.17	–		1.16	0.95	–	
HAR–ARJ	0.63	0.51	1.70	–	-0.01	-0.12	-0.41	–	0.00	-0.02	-1.96	–
HAR–C–J	0.93	0.72	2.04	0.24	1.49	1.54	1.12	1.61	1.86	1.88	1.48	1.88
4. APE												
HAR–RV	–				–				–			
HAR–J	0.28	–			0.00	–			0.00	–		
HAR–RJ	-0.43	-1.43	–		0.00	0.00	–		0.32	0.32	–	
HAR–ARJ	0.42	0.25	1.93	–	-0.07	-0.14	-0.03	–	-0.11	-0.16	-1.47	–
HAR–C–J	1.00	1.39	2.74	0.72	0.42	0.51	0.51	0.59	1.18	1.24	0.97	1.32
5. LL												
HAR–RV	–				–				–			
HAR–J	0.18	–			0.00	–			0.03	–		
HAR–RJ	-1.65	-3.05	–		0.00	0.00	–		1.13	0.86	–	
HAR–ARJ	0.37	0.70	**4.04**	–	0.01	0.03	0.01	–	0.38	0.29	-0.76	–
HAR–C–J	0.81	1.13	**4.21**	0.37	0.47	0.52	0.39	0.38	0.66	0.65	0.20	0.40
6. QLIKE												
HAR–RV	–				–				–			
HAR–J	0.06	–			-0.02	–			0.00	–		
HAR–RJ	-1.79	-2.71	–		0.01	0.03	–		1.04	0.85	–	
HAR–ARJ	0.24	1.12	**4.00**	–	0.00	0.05	-0.01	–	0.45	0.40	-0.76	–
HAR–C–J	0.39	0.64	3.31	0.07	0.62	0.77	0.37	0.54	0.58	0.59	0.13	0.32

Table D.29: **Out-of-Sample Volatility Forecast Comparisons for Natural Gas (Weighted Least Squares Estimation)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for natural gas volatility. Each day, we use a trailing window of 600 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. The models are estimated via weighted least squares using as weights the inverse of the fitted values from OLS estimation. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon				*5–Day Horizon*				*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE												
HAR–RV	–				–				–			
HAR–J	-1.03	–			-0.97	–			-0.14	–		
HAR–RJ	0.00	0.84	–		1.87	**6.45**	–		-0.07	0.00	–	
HAR–ARJ	0.02	**4.70**	0.10	–	0.33	3.00	-1.67	–	-0.20	-0.04	-0.11	–
HAR–C–J	-0.24	0.01	-0.22	-0.44	0.37	0.76	0.03	0.30	0.69	0.89	0.87	0.86
2. SPE												
HAR–RV	–				–				–			
HAR–J	-0.31	–			-1.20	–			-0.32	–		
HAR–RJ	0.00	0.29	–		0.69	**7.27**	–		0.05	0.44	–	
HAR–ARJ	0.04	1.66	0.09	–	0.79	2.14	-0.14	–	0.04	0.25	-0.01	–
HAR–C–J	-0.28	-0.05	-0.33	-0.71	-0.01	0.11	-0.40	-0.08	0.33	0.54	0.24	0.24
3. AE												
HAR–RV	–				–				–			
HAR–J	-0.01	–			-0.89	–			0.00	–		
HAR–RJ	0.22	0.87	–		3.45	**7.72**	–		0.00	0.01	–	
HAR–ARJ	0.49	1.79	0.00	–	0.01	1.46	**-4.29**	–	-0.34	-0.57	-1.41	–
HAR–C–J	0.39	0.75	0.03	0.04	2.16	3.56	0.22	2.37	0.00	0.00	0.00	0.02
4. APE												
HAR–RV	–				–				–			
HAR–J	-0.02	–			-1.15	–			-0.03	–		
HAR–RJ	0.28	1.16	–		3.29	**8.47**	–		0.02	0.10	–	
HAR–ARJ	0.45	1.41	-0.03	–	0.00	1.40	**-4.13**	–	-0.20	-0.13	-1.32	–
HAR–C–J	0.09	0.25	-0.06	-0.02	0.94	2.24	-0.03	1.02	-0.03	-0.02	-0.04	0.00
5. LL												
HAR–RV	–				–				–			
HAR–J	-0.30	–			-1.61	–			-0.50	–		
HAR–RJ	0.02	0.77	–		1.29	**6.75**	–		-0.03	0.18	–	
HAR–ARJ	0.28	2.58	0.21	–	0.15	2.46	-1.07	–	-0.08	0.07	-0.06	–
HAR–C–J	-1.08	-0.90	-1.76	-2.59	-0.15	0.00	-1.46	-0.24	0.03	0.12	0.05	0.06
6. QLIKE												
HAR–RV	–				–				–			
HAR–J	-0.34	–			-1.87	–			-0.64	–		
HAR–RJ	0.00	0.55	–		1.27	**5.83**	–		-0.13	0.08	–	
HAR–ARJ	0.22	2.69	0.40	–	0.02	2.59	-1.42	–	-0.26	0.02	-0.05	–
HAR–C–J	-1.67	-1.51	-2.10	-3.32	-0.36	-0.07	-2.09	-0.45	0.00	0.02	0.00	0.01

Table D.30: **Out-of-Sample Volatility Forecast Comparisons for Gasoline (Weighted Least Squares Estimation)**

This table presents test statistics from pairwise comparisons of equal predictive accuracy of forecasting models for gasoline volatility. Each day, we use a trailing window of 1,000 observations to estimate the parameters of the HAR models. Equipped with these estimates, we then make out-of-sample forecasts of volatility. The models are estimated via weighted least squares using as weights the inverse of the fitted values from OLS estimation. We consider three forecasting horizons: daily, weekly and monthly. We report the test statistics from comparing the mean difference between the forecast errors of the model [name in row] and those of the model [name in column]. The Giacomini and White (2006) test-statistic is distributed as a chi-squared random variable with 1 degree of freedom. We highlight in bold all the significant test statistics based on the 95 % confidence level.

	1–Day Horizon					*5–Day Horizon*					*22–Day Horizon*			
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
1. SE														
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.55	–			HAR–J	1.53	–			HAR–J	**4.14**	–		
HAR–RJ	-0.02	0.60	–		HAR–RJ	0.53	0.00	–		HAR–RJ	**5.49**	2.46	–	
HAR–ARJ	-0.23	0.04	-0.37	–	HAR–ARJ	0.12	-0.19	-1.20	–	HAR–ARJ	**5.08**	0.54	-1.53	–
HAR–C–J	**6.20**	**10.26**	**9.47**	**9.33**	HAR–C–J	**12.84**	**13.64**	**10.62**	**10.93**	HAR–C–J	**6.85**	**6.67**	**5.65**	**5.94**
2. SPE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.09	–			HAR–J	0.16	–			HAR–J	2.29	–		
HAR–RJ	1.50	2.92	–		HAR–RJ	0.22	0.11	–		HAR–RJ	2.80	0.64	–	
HAR–ARJ	0.45	0.37	-1.67	–	HAR–ARJ	0.10	0.02	-0.16	–	HAR–ARJ	2.91	0.35	-0.29	–
HAR–C–J	**7.33**	**8.85**	**8.01**	**7.41**	HAR–C–J	**9.38**	**12.21**	**6.59**	**5.83**	HAR–C–J	**5.42**	**5.59**	**4.32**	**4.73**
3. AE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.19	–			HAR–J	1.24	–			HAR–J	2.62	–		
HAR–RJ	0.02	-0.11	–		HAR–RJ	0.12	-0.04	–		HAR–RJ	**5.00**	2.75	–	
HAR–ARJ	-0.06	-0.68	-0.51	–	HAR–ARJ	-0.04	-0.74	-2.64	–	HAR–ARJ	2.85	0.17	**-3.85**	–
HAR–C–J	**7.53**	**9.34**	**8.89**	**9.81**	HAR–C–J	**11.18**	**11.29**	**10.26**	**11.63**	HAR–C–J	**6.82**	**6.75**	**5.42**	**6.28**
4. APE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	0.38	–			HAR–J	0.49	–			HAR–J	1.64	–		
HAR–RJ	0.43	0.02	–		HAR–RJ	0.01	-0.05	–		HAR–RJ	**4.13**	2.12	–	
HAR–ARJ	0.04	-0.23	-1.08	–	HAR–ARJ	-0.11	-0.51	-2.00	–	HAR–ARJ	2.22	0.24	-3.10	–
HAR–C–J	**11.34**	**14.62**	**11.33**	**12.86**	HAR–C–J	**9.73**	**10.67**	**8.93**	**9.84**	HAR–C–J	**6.05**	**6.27**	**4.64**	**5.47**
5. LL														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.06	–			HAR–J	0.80	–			HAR–J	2.29	–		
HAR–RJ	0.21	1.06	–		HAR–RJ	0.26	0.00	–		HAR–RJ	**4.14**	1.55	–	
HAR–ARJ	0.03	0.31	-0.47	–	HAR–ARJ	0.08	-0.05	-0.49	–	HAR–ARJ	3.52	0.54	-1.10	–
HAR–C–J	**7.84**	**12.37**	**8.46**	**8.61**	HAR–C–J	**12.05**	**14.56**	**9.29**	**8.91**	HAR–C–J	**6.13**	**6.20**	**4.71**	**5.30**
6 QLIKE														
	HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ		HAR–RV	HAR–J	HAR–RJ	HAR–ARJ
HAR–RV	–				HAR–RV	–				HAR–RV	–			
HAR–J	-0.36	–			HAR–J	1.06	–			HAR–J	2.33	–		
HAR–RJ	0.01	0.70	–		HAR–RJ	0.21	-0.04	–		HAR–RJ	**4.32**	1.81	–	
HAR–ARJ	0.00	0.34	-0.15	–	HAR–ARJ	0.05	-0.19	-0.63	–	HAR–ARJ	3.59	0.56	-1.37	–
HAR–C–J	**5.63**	**9.77**	**6.60**	**6.53**	HAR–C–J	**11.89**	**14.39**	**9.38**	**9.28**	HAR–C–J	**5.96**	**5.86**	**4.49**	**5.12**