Health and Optimal Taxation

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April 2016

Abstract

This paper studies the effects of health on optimal taxation of labour income, capital income and health care spending using a general equilibrium overlapping generations model, where health affects the level of utility, the probability of survival and labour productivity. Our analytical results suggest that health affects optimal taxation via three channels. First, since health is a stock that naturally deteriorates over time, the optimal level of taxation of medical spending is not constant over the life-cycle. Secondly, the productivity-enhancing aspect of health affects labour supply decisions over the life-cycle, where it is optimal for the government to use age-dependent labour income taxes to minimize distortions in the labour market. If the government cannot condition health care spending and labour income taxes on age, then a non-zero capital income tax can be implemented to achieve the optimal allocation. Finally, endogenous longevity affects the level of optimal taxation even in the case the government is not restricted in using uniform tax rate across cohorts in contrast to the results in the literature (Garriga, 2001; Peterman, 2013). Furthermore, we study the effects of endogenous longevity and the ageing of the population on the optimal financing of increasing burdensome social security and health care systems. Our results suggest the optimal capital income tax is optimal to adjust to changes in longevity, even if the ageing of the population is fiscally neutral.

JEL-Classification: E21, H31, H51, I00

Keywords: Overlapping Generations, Optimal Taxation, Health spending, Human Capital

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1 Introduction

Since the seminal papers of Judd (1985) and Chamley (1986) the literature on optimal taxation has focused on whether it is optimal to tax capital income in the absence of non-distortionary taxation. Early studies\(^1\) verified the results of the optimality of zero capital income taxation in the long run using a representative agent framework. As Lucas (1990) notes:

> “When I left graduate school, in 1963, I believed that the single most desirable change in the U.S. tax structure would be the taxation of capital gains as ordinary income. I now believe that neither capital gains nor any of the income from capital should be taxed at all. My earlier view was based on what I viewed as the best available economic analysis, but of course I think my current view is based on better analysis.”

However subsequent studies identified three main specification under which the optimal capital tax rate is different from zero. The first branch of the literature focuses on liquidity constraints and uninsured idiosyncratic risk (Hubbard and Judd, 1986; Aiyagari, 1994) where the agents do not save optimally and capital taxation acts as an instrument to influence savings decisions. The second branch of the literature focuses on the wedge between age-specific optimal labour income tax rates, since labour supply and consumption are generally not constant over the life-cycle. If the government is constrained in using a uniform labour income tax rate, a positive capital income tax can be implemented in order to achieve the second best allocation (Erosa and Gervais, 2002; Garriga, 2001). Finally, Conesa, Kitao and Krueger (2009) incorporating both the above motivations of a non-zero capital income tax, they quantitatively assess the magnitude of the capital income tax rate when the social welfare function takes into account the distribution of wealth which is affected by innate ability and productivity shocks.

With government spending accounting for a significant share of GDP, the optimal tax policy can have a significant effect on growth and welfare. Capital income, labour income and consumption tax rates vary significantly between countries and there are substantial changes over time (Trabandt and Uhlig, 2012), which raises the question of whether tax rates are set optimally. Furthermore, health care spending and the ageing of the population applies additional pressure on government finances, which puts health care and pension financing on the center stage of the public debate.

This paper contributes to the literature by incorporating health care spending in a life cycle model

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\(^1\)See for example Jones, Manuelli and Rossi (1997) and Atkeson, Chari and Kehoe (1999)
and studies the effects of health on optimal taxation in a quantitative overlapping generations model. In our model, health is a stock that depreciates over time and it can be replenished by medical spending, affecting the quality of life of the households, the probability of survival and labour productivity. We abstract from liquidity constraints, idiosyncratic risk and the wealth distribution, hence our analysis is restricted on the second branch of the literature, studying elasticity differentials between cohorts.

First, we solve for the decentralized equilibrium for the UK economy, in order to calibrate the model and assess whether our model can match key moments of interest using the current tax policy. Next, using the same specification, we obtain the second best allocation and the optimal tax policy, taking into account the distortions of capital income, labour income and health care spending taxation on savings, labour supply and the level of health respectively. Finally, we investigate the effects of technological progress in the medical sector that affects longevity on the optimal path of the tax policy, taking into account the ageing of the population. Hence, we can assess what is the optimal financing of increased government spending on health care and social security caused by the ageing of the population, endogeneizing the effects of increased life expectancy on labour supply, saving and health care spending decisions of the households and the subsequent optimal tax policy.

Our results suggest that optimal capital and labour income tax rates are affected by health via three channels; First, by endogeneizing longevity the optimal capital income tax rate is non-zero even if the government can use age-dependent labour income tax rates, in contrast to the results in the literature (Erosa and Gervais, 2002; Garriga, 2001; Peterman, 2015). Secondly, varying labour productivity due to health deterioration over the life-cycle, results in age-dependent optimal labour income tax rates. If the government cannot condition labour income taxes on age, then the government can use capital income taxes to achieve the same effect, which can amplify or diminish the effect of endogenous longevity on capital income tax rates described above. Our simulations suggest that the cumulative effect of endogenous longevity and labour productivity is an optimal capital income tax rate of approximately 7.7%. Lastly, productivity growth in the medical sector which directly or indirectly affects longevity creates an evolutionary path of optimal capital income tax rate. Cohorts’ decisions regarding medical spending are not affected homogeneously, affecting over time the wedge of health care spending elasticities the capital income tax compensates for. Furthermore, higher life expectancy affects the savings decision of the households, since they discount the future less, reducing the distortion of capital income tax. Our simulations suggest that an increase in social security payments due to increased longevity is optimal to be financed through
an increase of the capital income tax rate, in contrast to a fiscally equivalent increase in pension payments due to an increase in the pension replacement rate, where the government reduces the capital income tax rate. Since there is no evidence suggesting a slow down in improvements in medical technology and longevity (Whitehouse, 2007), it is optimal for the capital income tax to adjust for changes in longevity. This result has significant policy implications since the ageing of the population, changes not only the level of the optimal tax rates, in order to finance a more burdensome PAYG pension system, but also the optimal relative tax rates between labour income, health care spending and capital income.

The paper closest to ours is by Peterman (2015) in which he studies the effect of endogenous human capital accumulation over the life-cycle on optimal capital and labour income taxation. In the model, human capital accumulates through learning-by-doing, which reduces the elasticity of labour supply of younger cohorts who face the trade-off between leisure today and labour income earnings in later periods. The effects of health as human capital on optimal taxation follow the results of learning-by-doing, since health is a stock that depreciates over time and instantaneous health care spending affects subsequent periods, however the underlying mechanism is different. In our model, health depreciation affects optimal capital income tax rates through the productivity channel as Peterman (2015), however optimal capital income tax rates are also affected by the inability of the government to discriminate between the cohorts with respect to health care spending taxation. Moreover, the characteristics of health affect households beyond labour productivity, affecting per period utility and the number of periods households can derive utility from, aspects that cannot be studied in a model of pure human capital accumulation. The introduction of endogenous longevity has non-trivial effects as described above both for the level of optimal taxation and the relation between labour income, capital income and health care spending tax rates.

The rest of the paper is organized as follows. In the next section we set up the environment under which we are going to study the effects of health on optimal taxation. In section three, we derive analytically the second best allocation and the optimal capital income, labour income and health care spending tax rates. In section four, we describe the functional forms and we calibrate the model using the decentralized equilibrium and we subsequently solve for the second best allocation quantitatively. We conclude with section five.

2Leroux, Pestieau and Ponthiere (2009) introduce endogenous longevity, where the government subsidises savings. However, their results are driven by myopic households who do not perfectly optimize over their life-cycle, accumulating suboptimal savings. This is not the main source of non-zero capital taxation in our model since agents are rational and they have perfect foresight.
2 The Model

The model is a standard general equilibrium overlapping generations model with $J$ cohorts. Households make decisions with respect to consumption, labour supply and health care spending, with the latter affecting the probability of survival between cohorts, labour productivity and the level of utility. There is a representative firm which produces a single composite good $Y_t$ utilizing capital and labour. Finally, The government runs a balanced budget every period, taxing capital income, labour income, consumption and health care spending and provides a Pay-As-You-Go social security schemes for cohorts $j > J_R$.

2.1 Households

Households survive up to $J$ periods and derive utility from consumption and the level of health and disutility from labour. Their life-time utility, which takes into account the probability of survival is denoted as:

$$U = \sum_{j=1}^{J} \beta^{j-1} P_{j,t}(h_{j,t}(m_{j,t})) u_{j,t}(c_{j,t}, h_{j,t}(m_{j,t}), l_{j,t})$$

(1)

with:

$$P_{j,t}(h_{j,t}(m_{j,t})) = \prod_{i=1}^{j-1} [p(h_{i,t}(m_{i,t}))] p(h_{j,t}(m_{j,t}))$$

(2)

where $j$ denotes age cohort, $c_{j,t}, h_{j,t}(m_{j,t})$ and $m_{j,t}$ are consumption, level of health and aggregate medical spending in period $t$ for cohort $j$ respectively, $\beta$ is the discount rate, while $p_{j,t}(h_{j,t}(m_{j,t}))$ denotes the probability of surviving to age cohort $j$, conditional on surviving until $j - 1$ and $P_{j,t}(h_{j,t}(m_{j,t}))$ the unconditional probability of being alive at cohort $j$.

In addition, as in Feng (2010) we assume the following function for the probability of survival:

$$p_j(h_{j,t}) = 1 - e^{-(a_{p,j}h_{j,t})^b p_{j}}$$

(3)

3Consumption taxation as an instrument is redundant from the perspective of second best optimal government policy. However, consumption taxation accounts for a significant fraction of government revenues and we incorporate a flat, exogenous consumption tax in order to be closer to the actual government policy.
where \( a_{pj} \) and \( b_{pj} \) are scale and curvature parameters respectively, calibrated to fit the age specific probability of survival given a level of health. The latter is described as in Zhao (2014):

\[
h_{j,t} = h_{j-1,t-1}(1 - \delta_j) + Q_j m_{j,t} z_j
\]

(4)

with \( \delta_j \) being the health depreciation parameter and \( Q_j, z_j \) are the scale and curvature parameters for the health production function.

In our model, (4) and (3) endogeneize the level of health and the probability of survival, taking into account the natural deterioration of health over time \( (\delta_j) \), the decreasing returns of medical spending on the level of health (since \( z_j < 1 \)) and the indirect effect of medical spending on the probability of survival through the level of health.

The households face the following budget constraint:

\[
\left(1 - \tau_j^w\right)(l_{j,t} w_t e^{\phi_h j,t} + SS_t) + \left(1 + r\left(1 - \tau_j^a\right)\right) a_{j-1,t-1} = (1 + \tau^c) c_{j,t} + \left(1 + \tau_j^h\right) m_{j,t} + a_{j,t}
\]

(5)

where \( \tau_j^w \), \( \tau_j^h \) and \( \tau_j^a \) denote the cohort specific labour income and health care spending and capital income tax respectively, \( \tau^c \) is the consumption tax, \( w_t \) is the wage rate, \( r_t \) the interest rate and \( a_{j,t} \) denotes savings. In addition households receive pensions \( SS_t \) such that \( SS_t = \chi w_t e^{\phi_h j} \) for \( j \geq J_R \) and zero otherwise\(^4\), with \( J_R \) being the age of eligibility for pension payments, \( w_t e^{\phi_h j} \) is the effective wage and \( \chi \) is the replacement rate.

### 2.2 Firms

There is one representative firm, hiring labour and capital and produces a single composite good according to the production function:

\[
Y_t = AK_t^{\alpha} L_t^{1-\alpha} \quad \forall t,
\]

(6)

where

\[
K_t = \sum_{j=1}^{l-1} \mu_{j,t} k_{j,t} \quad \forall t,
\]

(7)

\(^4\)For the effects of taxable or non taxable pensions see Conesa and Garriga (2009). In a nutshell, taxable pensions affect the optimal level of labour income tax, since pension receipts are completely inelastic affecting the age-specific optimal labour income tax. We choose this formulation since it is closer to the actual government policy in most developed countries.
\[ \hat{L}_t = \sum_{j=1}^{j_R} \mu_{j,t} l_{j,t} e^{\phi h_{j,t}} \forall t, \]  

(8)

with \( \mu_{j,t} = \frac{p(h_{j-1,t-1})}{1+n} \mu_{j-1,t-1} \) denoting the cohort size. In our model, the population of young agents grows at a constant rate \( n \) and they bear children at the end of the first period of life, before the mortality rate is realized.

Assuming perfect competition, the wage and interest rates are equal to their marginal products:

\[ w_t = (1 - a) \frac{Y_t}{L_t} \forall t, \]  

(9)

\[ r_t = a \frac{Y_t}{K_{t-1}} - \delta \forall t, \]  

(10)

### 2.3 The Government

The government runs a balanced budget every period\(^5\), collecting taxes from or subsidizing consumption, labour income, health care expenditure and capital income in order to finance an exogenous sequence of government spending \( G_t \) and pension payments as in:

\[ \tau^c \sum_{j=1}^{I} \mu_{j,t} c_{j,t} + \sum_{j=1}^{I} \mu_{j,t} \tau^h m_{j,t} + \sum_{j=1}^{I} \mu_{j,t} \tau^w l_{j,t} w_t e^{\phi h_{j,t}} + \sum_{j=1}^{I} \mu_{j,t} \tau^a a_{j,t} + B_{j,t} = G + \sum_{j=j_R}^{I} \mu_{j,t} s S_t \forall t \]  

(11)

with \( B_{j,t} = \sum_{j=1}^{I} \mu_{j,t} \left(1 - P_{j-1,t}\right) a_{j,t} \) denoting accidental bequests left by deceased households that are seized by the government\(^6\).

Finally, the resource constraint of the economy is expressed as:

\[ C_t + K_t + M_t + G_t - Y_t (K_{t-1}, L_t) - (1 - \delta) K_{t-1} = 0 \forall t, \]  

(12)

\(^5\)We abstract from government debt because it is beyond the scope of this paper. For a comprehensive examination of debt in the Ramsey problem see Erosa and Gervais (2002) and Conesa, Kitao and Krueger (2009). In a nutshell, under certain conditions the government can accumulate negative debt in order to finance future government spending via interest rate payments or accumulate large positive debt, depending on the relative weight on future generations.

\(^6\)The assumptions regarding bequest motives and the manner bequests are taxed have significant effects on the optimal capital income taxation (Fuster, Imrohoroglu and Imrohoroglu, 2008; Peterman, 2013). If bequests are intentional, the optimal capital income is lower than the optimal capital income for unintended bequest since the latter are completely inelastic. In addition, with accidental bequest, distinguishing between ordinary capital income and bequests (for example introducing an inheritance tax), results in confiscatory tax rates since there is no distortion to the optimal decision of the households. For simplicity, we adopt the latter specification, assuming that bequest are accidental and the government imposes a non-distortionary inheritance tax of 100%.
where \( C_t = \sum_{j=1}^{J} \mu_{j,t} c_{j,t}, \) \( M_t = \sum_{j=1}^{J} \mu_{j,t} m_{j,t} \) and \( G_t = gY_t \) denotes the exogenous sequence of unproductive government spending and \( \delta \) is the depreciation rate.

### 3 Competitive Equilibrium and Second Best

Here we present the formal definitions and the analytical derivations of the competitive equilibrium and the second best optimal fiscal policy of the government. We focus on the decentralized equilibrium for two reasons; first, it is necessary to obtain the optimal decision rules of the household in order to solve for the second best allocation, where the government takes into account not only the budget constraints of the agents but also the first order conditions. Secondly, in the quantitative exercise, the decentralized equilibrium serves as a benchmark case in order to compare the second best allocation. Furthermore, we focus on the second best allocation since it represents a more realistic description of the actual government policy.

#### 3.1 Competitive Equilibrium

In the competitive equilibrium, households take the fiscal policy of the government as given and they maximize their life-time utility subject to the budget constraint. In order for this model economy to clear, we set exogenously the fiscal variables, but the labour income tax is set endogenously in order to balance the government budget. The formal definition is described below:

**Definition 1. (Competitive Equilibrium):** Given fiscal policy \( \pi: \{\tau_h^t\}_{t=0}^{\infty}, \{\tau_a^t\}_{t=0}^{\infty}, \{\tau_h^t\}_{t=0}^{\infty}, \{\tau_a^t\}_{t=0}^{\infty}, \{\chi^t\}_{t=0}^{\infty}, \{g_t\}_{t=0}^{\infty} \), a competitive equilibrium for this economy is the sequence of individual allocations \( \{(c_{j,t}, l_{j,t}, m_{j,t}, a_{j,t})\}_{t=0}^{\infty}, \) production factors \( \{K_t, L_t\}_{t=0}^{\infty} \) and relative prices \( \{r_t, w_t\}_{t=0}^{\infty} \), such that:

1. Households maximize life-time utility (I) subject to their budget constraint (5)
2. Labour and capital are compensated as in (9) and (10) respectively for all \( t \)
3. The government budget constraint (11) is satisfied for all \( t \)
4. The resource constraint (12) holds for all \( t \)

The first order necessary conditions for optimality with respect to consumption, medical spending, labour supply and savings respectively are:
\[ c_{j,t} = \tilde{\beta}_{j,t} u_{c_{j,t}} = \lambda_t (1 + \tau_t^c) \quad \forall t, j \] (13)

\[ m_{j,t} = \tilde{\beta}_{j,t} \tilde{u}_{m_{j,t}} = \lambda_t \left(1 + \tau_t^m\right) - \Phi_{j,t} \quad \forall t, j \] (14)

\[ l_{j,t} = \tilde{\beta}_{j,t} \tilde{u}_{l_{j,t}} = \lambda_t \left(1 + \tau_t^w\right) w_t e^{\phi h_{j,t}} \quad \forall t, j \] (15)

\[ a_{j,t} = \lambda_t = \lambda_{t+1} \left[1 + (1 - \tau_t^a) r_t\right] \quad \forall t, j \] (16)

where:

\[ \tilde{\beta}_{j,t} = \beta^{j-1} P_{j,t} \left(h_{j,t} \left(m_{j,t}\right)\right) , \] (17)

\[ \tilde{u}_{m_{j,t}} = u_{j,m_{j,t}} + \sum_{i=j+1}^{j} P_{i,m_{j,t}} u_i + \sum_{i=j+1}^{j} P_{i} u_{i,m_{j,t}} , \] (18)

\[ \Phi_{j,t} = \sum_{i=j+1}^{j} \lambda_{t+i-j} \left(1 - \tau_{t+i-j}^w\right) \phi_{h_{i,m_{j,t}}} \left(w_{t+i-j} e^{\phi h_{j,t+i-j}}\right) \] (19)

Which becomes after substituting (15) into (19):

\[ \Phi_{j,t} = - \sum_{i=j+1}^{j} \tilde{\beta}_{i,t+i-j} u_{l_{i,t+i-j}} \phi h_{i,m_{j,t}} \] (20)

In a nutshell, \( \tilde{\beta}_{j,t} \) expresses the effective discount rate after taking into account the probability of survival and \( \tilde{u}_{m_{j,t}} \), \( \Phi_{j,t} \) are the effects of the endogenous probability of survival and labour productivity on the optimal decision of the household. Since health is a stock that depreciates over time and households can invest in their level of health through health care spending \( m_{j,t} \), medical decisions in period \( j \) affect the probability of survival, the level of utility and labour productivity of all subsequent periods, which is taken into account in the optimal decision of the households.

The optimal decisions of the households with respect to consumption, health care spending, labour supply and savings described above, are crucial not only for the decentralized equilibrium, which is used as a benchmark, but the second best allocation as well. Households choose the sequence of their decisions taking into account the effective discount \( \tilde{\beta} \) rate, which is endogenously determined in our model. Hence, changes in longevity have a direct effect on the effective discount rate and the spending and saving decision of the households. Furthermore, households choose the optimal level of health care spending considering the effects of health on longevity (18) and effective labour...
supplied (20), while the labour supply decision per se is affected by the age-specific productivity of
the households (15).

The second best allocation derived in the next section, takes into account these optimality condi-
tions of the households and shapes the optimal government policy.

3.2 The Second Best

The government’s problem is to maximize the lifetime utility of all cohorts subject to the agent’s
first order conditions, the agent’s budget constraints and the resource constraint of the economy7.

We rely on the primal approach to obtain the optimal set of tax instruments, where the government
maximizes welfare subject to the implementability constraint, which takes into account the agent’s
budget constraint and first order conditions to substitute for prices and the resource constraint of
the economy (Atkinson and Stiglitz, 1976; Lucas and Stokey, 1983). This ensures that the efficient
allocation chosen by the government can be decentralized satisfying the agent’s budget constraint
and the agent’s first order condition.

In the analytical models below, we choose to study the effects of productivity and probability of
survival separately for the sake of clarity. The two analytical models that are presented in the
following section can distinguish the two channels via which health affects the optimal taxation
chosen by the government.

3.2.1 Endogenous Labour Productivity

First, we focus on the effects of endogenous labour supply treating the probability of survival as
exogenous from the perspective of the household. We denote the exogenous probability of survival
as $\bar{P}_j$. Hence, we substitute (18) and (17) from the household’s maximization problem with the
following expressions8:

\begin{align*}
U_{m_{j,t}} &= u_{m_{j,t}} \quad \forall t, j, \\
\tilde{\beta}_{j,t} &= \beta_{j}^{-1}\bar{P}_{j,t} \quad \forall t, j
\end{align*}

The government’s problem can be expressed as:

7 The government budget is balanced due to Walras’ Law
8 In a nutshell, with exogenous probability of survival, the marginal utility with respect to medical spending, affects only the level of instantaneous utility and the effective discount rate is exogenous.
\[
\max_{\left\{ \{c_{j,t},m_{j,t},l_{j,t}\}_{j=1}^{J} \right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{j=1}^{J} \theta^t \left[ \mu_{j,t} \beta^{j-1} \mathcal{U}\left(c_{j,t},h_{j,t}\left(m_{j,t}\right),l_{j,t}\right) \right]
\]  

(23)

where \(\theta\) is the relative weight of each generation, subject to the budget constraint of the households (5), the households’ first order conditions (13)-(16) and the resource constraint of the economy (12) for all \(t\).

**Proposition 1 (Implementable Allocations).** The second best allocation \(\left\{\{c_{j,t},l_{j,t},m_{j,t},a_{j,t}\}_{j=1}^{J}\right\}_{t=0}^{\infty}\) that maximizes the government’s objective function (23) subject to the implementability constraint:

\[
\sum_{j=1}^{J} \beta^{j-1} P_j \left[ u_{c_{j,t+j-1}} c_{j,t+j-1} + \left( u_{m_{j,t+j-1}} - u_{l_{j,t+j-1}} \right) m_{j,t+j-1} + u_{l_{j,t+j-1}} \right] = 0
\]

(24)

and the resource constraint of the economy (12) can be decentralized given tax policy \(\pi = \left\{\tau^\alpha, \tau^h, \tau^c, \tau^w\right\}_{t=0}^{\infty}\) such that:

\[
\tau^h_{j,t} = \left(1 + \tau^c_t \right) \left( \tilde{\beta}_{j,t} \bar{u}_{m_{j,t}} + \Phi_{j,t} \right) / \tilde{\beta}_{j,t} \bar{u}_{c_{j,t}} - 1 \forall t, j
\]

(25)

\[
\tau^w_{j,t} = \frac{u_{l_{j,t}} \left(1 + \tau^c_t \right)}{w_t \phi_{h_{j,t}} u_{c_{j,t}}} + 1 \forall t, j
\]

(26)

\[
\tau^a_{j,t} = \frac{1}{r_t} \left[ (1 + r) - \frac{\tilde{\beta}_{j,t} u_{c_{j,t}}}{\tilde{\beta}_{j+1,t+1} u_{c_{j+1,t+1}}} \right] \forall t, j
\]

(27)

with \(\tau^c = \tilde{\tau}^c\)

**Proof.** First we need to derive the implementability constraint, which is the household’s budget constraint where we have substituted the tax rates with the first order conditions of the household. Hence, the implementability constraint encompasses both restrictions of the government’s maximization problem\(^9\). Multiplying the first order conditions of the household (13)-(16) with their respective variable we obtain:

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\(^9\)Another merit of the primal approach is that substituting the tax rates with the first order conditions of the agents and using the resource constraint of the economy rather than the government budget constraint, the government doesn’t need to maximize over the tax policy \(\pi\) explicitly. Instead, we can construct the optimal tax rates ex-post, substituting the second best allocation into the first order conditions of the agents, as described below.
\[ \tilde{\beta}_{j,t} u_{c_{j,t}} = \lambda_t (1 + \tau^c_j) c_{j,t} \quad \forall t, j \]  
(28)

\[ \left( \tilde{\beta}_{j,t} \tilde{u}_{m_{j,t}} + \Phi_{j,t} \right) m_{j,t} = \lambda_t (1 + \tau^h_j) m_{j,t} \quad \forall t, j \]  
(29)

\[ \tilde{\beta}_{j,t} u_{l_{j,t}} = -\lambda_t (1 - \tau^u_j) w_t e^{\phi_{h_{j,t}}} l_{j,t} \quad \forall t, j \]  
(30)

\[ \lambda_t a_{j,t} = \lambda_{t+1} \left[ 1 + (1 - \tau^a_j) \right] a_{j,t} \quad \forall t, j \]  
(31)

Adding up (28)-(31) for all \( j \), we obtain:

\[
\sum_{j=1}^{J} \tilde{\beta}_{j,t} u_{c_{j,t}} + \left( \tilde{\beta}_{j,t} \tilde{u}_{m_{j,t}} + \Phi_{j,t} \right) m_{j,t} + \tilde{\beta}_{j,t} u_{l_{j,t}} = \\
= \sum_{j=1}^{J} \lambda_t \left[ (1 + \tau^c_j) c_{j,t} + (1 + \tau^h_j) m_{j,t} - (1 - \tau^u_j) w_t e^{\phi_{h_{j,t}}} l_{j,t} \right] \\
= \sum_{j=1}^{J} \lambda \left( 1 - \tau^w_{j,t} \right) SS_t
\]  
(32)

From equation (32), we observe that we can rewrite the household’s budget constraint (5) with the first order conditions instead of tax rates. However, the right hand side of the (28)-(31) does not include the pension receipts since it is not a household decision. Hence, from the budget constraint of the household:

\[
\sum_{j=1}^{J} \lambda_j \left[ (1 + \tau^c_j) c_{j,t} + (1 + \tau^h_j) m_{j,t} - (1 - \tau^w_{j,t}) w_t e^{\phi_{h_{j,t}}} l_{j,t} \right] = \sum_{j=1}^{J} \lambda \left( 1 - \tau^w_{j,t} \right) SS_t
\]

From equation (15) of the household’s first order conditions, if we re-arrange with respect to the labour income tax:

\[
\sum_{j=1}^{J} \lambda \left( 1 - \tau^w_{j,t} \right) SS_t = -\sum_{j=1}^{J} \tilde{\beta}_{j,t} u_{l_{j,t}} \frac{w_t e^{\phi_{h_{j,t}}}}{SS_t}
\]  
(33)

Substituting (33) into (32) and re-arranging we obtain:

\[
\sum_{j=1}^{J} \beta^{j-1} \left[ U_{c_{j,t+j-1},c_{j,t+j-1}} + \left( U_{m_{j,t+j-1},m_{j,t+j-1}} - U_{l_{j,t+j-1},\Phi_{h_{j,t+j-1}}} \right) m_{j,t+j-1} + U_{l_{j,t+j-1}, l_{j,t+j-1}} \left( l_{j,t+j-1} + \frac{SS_{t+j-1}}{w_t e^{\phi_{h_{j,t+j-1}}}} \right) \right] = 0
\]

Hence, the allocation obtained from solving the government’s maximization described above is the second best allocation and satisfies the first order conditions and the budget constraint of the
household (via the implementability constraint), the resource constraint of the economy and the
government budget constraint.

This allocation can be decentralized if and only if the government sets the tax rate policy \( \pi \) such
that it satisfies the restrictions described above. Thus, we can obtain the optimal tax rates by
re-arranging the first order conditions of the household with respect to the tax rates and substitute
the second best allocation \( \left\{ (c_{j,t}, l_{j,t}, m_{j,t}, a_{j,t})_{j=1}^{\infty} \right\}_{t=0}^{\infty} \) and the exogenous consumption tax \( \tau_c \). It is
trivial to show that by re-arranging equations (28)-(31) we can obtain (25)-(27).

\[
\text{\textit{Proposition 2.} If the government is restricted in using uniform tax rates across cohorts, hence}
cannot use age-dependent taxation, the following additional restrictions need to be implemented
in the government’s maximization problem:

\[
\frac{U_{c_{1,t}}}{U_{c_{2,t+1}}} = \ldots = \frac{U_{c_{j-1,t}}}{U_{c_{j+1,t}}} \quad \forall t, \quad (34)
\]

\[
\frac{U_{c_{1,t}} e^{\phi h_{1,t}}}{U_{l_{1,t}}} = \ldots = \frac{U_{c_{j,t}} e^{\phi h_{j,t}}}{U_{l_{j,t}}} \quad \forall t, j, \quad (35)
\]

\[
\frac{\tilde{\beta}_{1,t} \tilde{u}_{m_{1,t}} - \Phi_{1,t}}{\tilde{\beta}_{1,t} u_{c_{1,t}}} = \ldots = \frac{\tilde{\beta}_{j,t} \tilde{u}_{m_{j,t}} - \Phi_{j,t}}{\tilde{\beta}_{j,t} u_{c_{j,t}}} \quad \forall t, j \quad (36)
\]

\text{\textit{Proof.} It is straightforward to show that for the tax rates to be uniform across cohorts the following}
condition with respect to health care spending, labour and capital tax must hold for any \( j \) and
\( j + 1 \), using the expression of the optimal tax rates (25)-(27):

\[
\frac{(1 + \tau_i^c) \left( \tilde{\beta}_{j,t} \tilde{u}_{m_{j,t}} + \Phi_{j,t} \right)}{\tilde{\beta}_{j,t} u_{c_{j,t}}} - 1 = \frac{(1 + \tau_{i+1}^c) \left( \tilde{\beta}_{j+1,t} \tilde{u}_{m_{j+1,t}} + \Phi_{j+1,t} \right)}{\tilde{\beta}_{j+1,t} u_{c_{j+1,t}}} - 1 \quad (37)
\]
\[
\frac{u_{j,t} (1 + \tau^c_t)}{w_t e^{\phi h_{j,t} u_{c,j,t}}} = 1 = \frac{u_{j+1,t} (1 + \tau^c_t)}{w_t e^{\phi h_{j+1,t} u_{c,j+1,t}}} + 1 \quad (38)
\]

\[
\frac{1}{r_t} \left[ (1 + r_t) - \frac{\tilde{\beta}_{j,t} u_{c,j,t}}{\tilde{\beta}_{j+1,t+1} u_{c,j+1,t+1}} \right] = 1 \frac{1}{r_{t+1}} \left[ (1 + r_{t+1}) - \frac{\tilde{\beta}_{j+1,t+1} u_{c,j+1,t+1}}{\tilde{\beta}_{j+2,t+2} u_{c,j+2,t+2}} \right] \quad (39)
\]

Simplifying the expressions, we obtain the additional restrictions (34)-(36).

Even in the model with exogenous probability of survival, health has significant effects on the Ramsey problem of optimal taxation. Introducing health as a form of human capital that naturally deteriorates over time, changes the elasticity of health care spending over the life cycle, since both the level of health and the effectiveness of health care spending diminishes with age\(^{10}\). The conditions described in Garriga (2001) and Peterman (2013) for zero capital income taxation in life-cycle models seize to hold. In a nutshell, even if there are no incentives to use age dependent labour income taxation, the peculiarity of health dictates different tax rates across cohorts. In the restricted version of the model, the optimal capital income tax is not zero, even if there is only one cohort supplying labor\(^{11}\), or the Frisch elasticity of labour supply is constant over the life cycle.

**Proposition 3.** In the model with endogenous productivity, even if there are two cohorts with only the first supplying labour or the Frisk elasticity is constant over the life-cycle the government has still an incentive to use age-dependent tax rates resulting in a non-zero capital income tax rate when the government is restricted to use uniform tax rates.

**Proof.** We need to show that in the unrestricted version of the model, the optimal capital income tax is indeed zero and the optimal level of health care spending tax is not constant over the life cycle. In the restricted version of the model, the latter results in non-zero capital income tax.

As in Garriga (2001), let \(V(c_{j,t}, h_{j,t}, l_{j,t}, \rho_{t-j})\) be the pseudo-utility function denoted as:

\[
V(c_{j,t}, h_{j,t}, l_{j,t}, \rho_{t-j}) = U(c_{j,t}, h_{j,t}, l_{j,t}) + \rho_{t-j} \Omega_{j,t} \quad (40)
\]

\(^{10}\)Even if we exclude age-specific parameters in the health production function (4), health deteriorates faster as the household ages, resulting in lower effectiveness of instantaneous health care spending for future periods.

\(^{11}\)If the economy consists of only two cohorts, with the first one supplying labour, there is a single labour income tax rate and no incentive to use age-dependent labour income taxation or a non-zero capital income tax in the restricted model.
where $\Omega_{j,t}$ is the implementability constraint and $\rho_{t-j}$ is the Lagrange multiplier of the implementability constraint.

The government’s problem for the second best allocation is to maximize the lifetime utility of a newborn household\(^{12}\):

$$
\sum_{j=1}^{J} \tilde{\beta}_{t+j-1,j} V \left( c_{t+j-1,j}, h_{t+j-1,j} \left( m_{t+j-1,j}, l_{t+j-1,j}, \rho_{t+j-1,j} \right) \right)
$$

subject to the resource constraint of the economy (12). From the Lagrangian, the first order necessary condition with respect to consumption, health care spending, labour supply and savings are the following:

$$
\begin{align*}
[c_{j,t}] & : \tilde{\beta}_{j,t+j-1} V c_{j,t+j-1} + \theta^{t+j-1} \xi_{t+j-1} = 0 \quad \forall t,j \\
[m_{j,t}] & : \tilde{\beta}_{j,t+j-1} V m_{j,t+j-1} + \theta^{t+j-1} \xi_{t+j-1} Y_{m_{j,t+j-1}} = 0 \quad \forall t,j \\
[l_{j,t}] & : \tilde{\beta}_{j,t+j-1} V l_{j,t+j-1} - \theta^{t+j-1} \xi_{t+j-1} Y_{l_{j,t+j-1}} = 0 \quad \forall t,j \\
[a_{j,t}] & : \theta^{t+j-1} \xi_{t+j-1} - \theta^{t+j} \xi_{t+j} (1 + r_t) = 0 \quad \forall t,j
\end{align*}
$$

where $\xi$ is the Lagrange multiplier of the resource constraint.

Forwarding (42) one period and substituting in (45) we obtain:

$$
\frac{\tilde{\beta}_{j,t+j-1} V c_{j,t+j-1}}{\tilde{\beta}_{j+1,t+j} V c_{j+1,t+j}} = (1 + r_t)
$$

Substituting (46) into the optimal capital income tax (27):

$$
\tau_{a,t+j-1} = \frac{\tilde{\beta}_{j,t+j-1} V c_{j,t+j-1}}{\tilde{\beta}_{j+1,t+j} V c_{j+1,t+j}} \left[ \frac{V c_{j,t+j-1}}{V c_{j+1,t+j}} - \frac{u_{c_{j,t+j-1}}}{u_{c_{j+1,t+j}}} \right]
$$

As Garriga (2001) shows, with the standard class of utility functions equation (47) is zero since the government has no incentive to distort consumption choices in the unrestricted version of the government’s problem\(^{13}\).

\(^{12}\)As in Samuelson (1958), Diamond (1965), Conesa, Kitao and Krueger (2009) and Peterman (2013), we choose for simplicity to solve for a single newborn household since the cohort size does not affect our main results when the probability of survival is exogenous.

\(^{13}\)For clarity of exposition, take a two-period overlapping generation model with CRRA utility function and $\gamma$ the coefficient of relative risk aversion. Then:
The next step is to show that health care spending tax rates are not uniform across cohorts, since when the government cannot use age-dependent taxation, the Garriga (2001) and Peterman (2013) results with respect to labour income tax become irrelevant and the optimal capital income tax is not zero.

From the optimal health care spending tax (25), the ratio of the optimal tax rate of cohort $j$ and $j+1$ becomes:

\[
\frac{\tau_{j,t}^h}{\tau_{j+1,t}^h} = \frac{\left(\tilde{\beta}_{j,t} \tilde{u}_{mj,t} + \Phi_{j,t}\right)}{\tilde{\beta}_{j+1,t} \tilde{u}_{mj+1,t} + \Phi_{j+1,t}} - 1
\]

The above ratio cannot be one for two reasons; First, in overlapping generations models the sequence of household’s optimal decisions is not constant over the life-cycle, thus $m_j$ and $m_{j+1}$ are not equal. Secondly, recall from equation (20) of the parameter $\Phi_{j,t}$ that even when the probability of survival is exogenous, the nature of health as a stock that depreciates over time influences the optimal medical spending decisions because health affects labour productivity. Health care spending today, affects the level of labour productivity during the current and all subsequent periods. Households, not only face a shorter life-span as they age, but health depreciates faster with age, diminishing the incentive to invest in health as they get older. This diminishing incentive of the household through the life-cycle affects the optimal health care spending tax significantly.\(^{14}\)

The intuition behind this result is that the incorporation of health as an endogenous variable that determines labour productivity, affects not only the optimal labour income tax rates faced by each cohort if the Frisch elasticity is not constant during the life-cycle (as in Garriga (2001)), but the optimal tax rates of health care spending as well, which is not eliminated with preferences that assume a constant Frisch elasticity (as in Peterman (2013)). Furthermore, even if there are two cohorts, with the second cohort not supplying labour, households still face different elasticities with respect to health care spending through their life-cycle.

Hence, the results of Garriga (2001) and Peterman (2013) hold as long the government can implement an age-dependent tax system, including health care spending. This is not true for the model

\(^{14}\)In our simulations health care spending tax rates are negative (subsidies) and in the three-generations model the young are subsidized at a rate of 82.4%, the middle-aged at 35.6%, while the old at a rate of 16.5%.
with endogenous probability of survival that we study in the next section, where even if the government can discriminate between cohorts, the optimal capital income tax rate is non-zero.

### 3.2.2 Endogenous Probability of Survival

Now we turn to the effects of endogenous probability of survival on the second best allocation, shutting down the endogenous productivity channel in order to make our results more transparent. In a nutshell, we substitute \( \Phi_{j,t} \) (the intertemporal labour productivity incentive), the marginal utility of health and the effective discount respectively with the following expressions:

\[
\Phi_{j,t} = 0 \quad \forall t, j
\]  
(49)

\[
U_{m_{j,t}} = \tilde{u}_{m_{j,t}} \quad \forall t, j
\]  
(50)

\[
\tilde{\beta}_{j,t} = \beta^{j-1} P(h_{j,t}(m_{j,t})) \quad \forall t, j
\]  
(51)

The objective of the government is to maximize (23) subject to the implementability constraint and the modified resource constraint of the economy (without endogenous labour productivity):

\[
C_t + K_t + M_t + G_t - Y_t(K_{t-1}, L_t) - (1 - \delta)K_{t-1} = 0 \quad \forall t,
\]  
(52)

Naturally, the implementability constraint and the optimal tax policy \( \pi \) are expected to be different than the previous model, where labour productivity is endogenous and longevity is treated as exogenous. One crucial difference between the previous analytical model and the model with endogenous probability of survival is that the government can no longer maximize the life-time utility of a single representative household since cohort size now matters\(^{15}\). In addition, endogenous longevity transforms the household problem from a sequence of allocation over a fixed length of life-cycle to a problem where households also choose the number of periods they can derive utility from, fundamentally changing the optimality conditions (Rossen, 1988). Recall from (18), that households make optimal health care spending decisions taking into account the level of future utility:

\[^{15}\text{It is a gross simplification to assume that the social welfare function does not take into account the number of living agents, when the government can directly influence longevity.}\]
\[ \tilde{u}_{m_{j,t}} = u_{j,m_{j,t}} + \sum_{i=j+1}^{j} P_{i,m_{j,t}} u_i + \sum_{i=j+1}^{j} P_{i} u_{i,m_{j,t}} \]

Both peculiarities of endogenous longevity result in non-zero optimal capital income tax rates even in the unrestricted version of the government problem.

**Proposition 4.** With endogenous probability of survival, even if the government can make use of age-dependent taxation, the optimal tax rate on capital income is non-zero.

**Proof.** We follow the same method of obtaining the implementability constraint, multiplying the first order conditions of the household with the respective choice variable and add up the expressions:

\[ \sum_{j=1}^{J} \hat{\beta}_{j,t} \left[ u_{c_{j,t}} c_{j,t} + \tilde{u}_{m_{j,t}} m_{j,t} + u_{l_{j,t}} \left( l_{j,t+1} + \frac{S_{t+j-1}}{w_{t} e^{\phi h_{j,t+j-1}}} \right) \right] = 0 \quad \forall t, j \quad (53) \]

The new optimal tax rates, after re-arranging (13)-(16) and substituting (49)-(51) are the following:

\[ \tau^h_t = \frac{\tilde{u}_{m_{j,t}} (1 + \tau^c_t)}{u_{c_{j,t}}} - 1 \quad \forall t, j \quad (54) \]
\[ \tau^w_t = -\frac{u_{l_{j,t}} (1 + \tau^c_t)}{w_{t} e^{\phi h_{j,t}} u_{c_{j,t}}} + 1 \quad \forall t, j \quad (55) \]
\[ \tau^a_t = \frac{1}{r_t} \left[ (1 + r) - \frac{\hat{\beta}_{j,t} u_{c_{j,t}}}{\hat{\beta}_{j+1,t+1} u_{c_{j+1,t+1}}} \right] \quad \forall t, j \quad (56) \]

The first order conditions of the government’s problem, which will determine the optimal capital income tax rate in equation (56) are the following:

\[ [c_{j,t}] \quad \mu_{j,t} \hat{\beta}_{j,t} V_{c_{j,t}} + \xi_t = 0 \quad \forall t, j \quad (57) \]
\[ [m_{j,t}] \quad \mu_{j,t} \hat{\beta}_{j,t} V_{m_{j,t}} + \xi_t = 0 \quad \forall t, j \quad (58) \]
\[ [l_{j,t}] \quad \mu_{j,t} \hat{\beta}_{j,t} V_{l_{j,t}} - \xi_t Y_{l_{j,t}} = 0 \quad \forall t, j \quad (59) \]
\[ [a_{j,t}] \quad \theta^t \xi_t - \theta^{t+1} \xi_{t+1} (1 + r) = 0 \quad \forall t, j \quad (60) \]

Forwarding (57) one period and substituting into (60) we obtain:
Substituting (61) into (56):

\[
\tau_{j,t}^a = \frac{\tilde{\beta}_{j,t} V_{c,j,t}}{\tilde{\beta}_{j+1,t+1} r_t} \left[ \frac{\mu_{j,t} V_{c,j,t}}{\theta \mu_{j+1,t+1} V_{c,j+1,t+1}} - \frac{u_{c,j,t}}{u_{c,j+1,t+1}} \right]
\]  

(62)

The optimal capital income tax is not zero since:

\[
\frac{\mu_{j,t} V_{c,j,t}}{\theta \mu_{j+1,t+1} V_{c,j+1,t+1}} > \frac{u_{c,j,t}}{u_{c,j+1,t+1}}
\]

This result stems from the two peculiarities of endogenous longevity described above. First, it is straightforward to show that the government weights consumption today and tomorrow differently from households, since (i) the relative weights of the generations matter and (ii) the government discounts future generations with an exogenous discount rate \(\theta\). Secondly, and more importantly, the government’s marginal utility of consumption in the pseudo-utility function is not the same as the marginal utility of consumption of the household, resulting in non-zero capital income tax rates even if the government ignores cohort size and doesn’t discount future generations.

Households choose consumption and leisure in order to maximize instantaneous utility and health care spending in order to maximize instantaneous and future utility. Hence the marginal utility of consumption today, is affected only by today’s consumption. Contrariwise, in the government’s problem the marginal utility of consumption takes into account the first derivative of household’s utility with respect to consumption today and the level of utility of all subsequent periods. To see this, recall the implementability constraint when longevity is endogenous:

\[
\sum_{j=1}^J \tilde{\beta}_{j,t} \left[ u_{c,j,t} c_{j,t} + \tilde{u}_{m,j,t} m_{j,t} + u_{l,j,t} \left( l_{j,t+j-1} + \frac{SS_{t+j-1}}{w e^{\phi h_{j,t+j-1}}} \right) \right] = 0 \quad \forall t, j
\]

As described above:

\[
\tilde{u}_{m,j,t} = u_{j,m,j,t} + \sum_{i=j+1}^J P_{i,m,j,t} u_i + \sum_{i=j+1}^J P_i u_{i,m,j,t}
\]
where $u_i$ is the level of utility of cohort $i$. Hence, the government has an incentive to distort the consumption choices of the household, because they are not chosen optimally from the perspective of the second best allocation. An alternative interpretation of this result is that by explicitly maximizing over the number of periods the household can derive utility from, the elasticity of consumption is different from the model with exogenous longevity, reducing the distortion of the capital income tax. In the absence of lump-sum taxation, the government minimizes the overall distortion of the tax policy $\pi$ and taxing capital becomes optimal, relative to the benchmark case. As in Garriga (2001), one way of affecting the household’s consumption choice is by setting a non-zero capital income tax.

Another significant implication of the wedge between the government’s and household’s optimization with respect to consumption is that changes in longevity affect the level of the optimal capital income tax. When the probability of survival changes, it affects both the relative size of the cohorts and the discount rate of future consumption that the government maximizes over. Considering the technological improvements in the medical sector and the steady increase in life expectancy over the last centuries, with no projected slowdown, this result has significant policy implications for the optimal tax structure. Not only the relative size of capital income and labour income tax rates change with any given level of government spending, but taking into account the effects of endogenous longevity we can investigate what is the optimal financing of increasing social security and health care spending caused by increases in life expectancy, namely the ageing of the population.

Moreover, assuming that the government cannot discriminate between cohorts and is constrained in using a uniform health care spending tax, the following additional restrictions need to be imposed, by the same token as the previous analytical model, which influence the optimal capital income tax:

\[
\frac{u_{c_1,t}}{u_{c_2,t+1}} = ... = \frac{u_{c_{J-1},t}}{c_{J,t+1}} \forall t, \tag{63}
\]

\[
\frac{u_{c_1,t} e^{\alpha h_1,t}}{u_{l_1,t}} = ... = \frac{u_{c_{J},t} e^{\alpha h_J,t}}{u_{l_J,t}} \forall t, j, \tag{64}
\]

\[
\frac{\tilde{\beta}_{1,t} u_{m_1,t}}{\tilde{\beta}_{1,t} u_{c_1,t}} = ... = \frac{\tilde{\beta}_{J,t} u_{m_J,t}}{\tilde{\beta}_{J,t} u_{c_J,t}} \forall t, j \tag{65}
\]
The two analytical models described above have significant policy implication for the design of the optimal tax policy, since health is a significant form on human capital which affects productivity, longevity and the quality of life of the households. Furthermore, a sizeable share of GDP devoted on health care which is projected to increase even further in the future due to the ageing of the population, which raises the question of the optimal level of health care spending, the respective level of government subsidy and the financing mechanism of these expenditures. Hence, the omission of the distortions of tax policy on health care spending, savings and labour supply can have substantial welfare and growth implications.

In the next section, we assess quantitatively the effect of health on optimal taxation in an overlapping generations model which combines the effects of health both on productivity and longevity. As an additional exercise, we study the effects of a change in expected longevity and the optimal tax policy to finance the resulting excess burden of health care spending and social security payments from an ageing population, a pressing issue in developed countries.

4 Quantitative Results

In order to estimate the magnitude of the two effect of health on optimal taxation, we turn to the quantitative version of our model. First, we estimate the decentralized equilibrium using the current tax policy in order to calibrate the model and assess how closely our model fits the data. Then, using the calibrated parameter values, we estimate the second best allocation and the optimal tax policy $\pi$ and compare it with the actual data. Furthermore, we simulate the effects of technological progress in the medical sector, which affects longevity and estimate the optimal response of the government, considering the effects of the ageing of the population on health care spending and social security.

Finally, we estimate the optimal tax policy $\pi$ with and without restrictions with respect to age-dependent taxation in order to estimate the quantitative effect of such restriction on the optimal tax system.

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16In our paper health includes both physical and mental health. Lately, the direct and indirect (through the interaction of mental and physical health) effects of mental health have revealed the singificance of mental illness on labour productivity.
4.1 Calibration

Here, we present the functional forms and the parameter values for the decentralized equilibrium and the second best allocation, calibrated to match relevant aggregates of the UK economy. By focusing in the UK, where health care insurance is mainly publicly provided, free at the point of service and financed by general taxation, we can abstract from complicated insurance schemes without being less realistic and concentrate on aspects that are more relevant for our analysis.\(^{17}\)

4.1.1 Demographics

The maximum number of periods agents can survive is set to \(J = 3\), with each period in the model corresponding to 20 years. Agents enter the economy at the age of 20 and they survive maximum at the age of 80. The eligible age for pension payments in the benchmark model is set to \(J_R = 2\), which means that agents receive pensions after the age of 60, although they can supply labour if they choose so. The yearly population growth is set to \(n = 0.62\) (ONS, 2013b), consistent with the UK data.

4.1.2 Preferences

The instantaneous utility of the agent of cohort \(j\) at time \(t\) is determined by the modified utility function as proposed by Hall and Jones (2007) where we have incorporated labour supply:

\[
    u_{j,t} = b + \frac{c_{j,t}^{1-\gamma}}{1-\gamma} + \psi \frac{h_{j,t}^{1-\sigma}}{1-\sigma} + \nu \frac{(1-l_{j,t})^{1-\eta}}{1-\eta}
\]

where \(b\) is a constant parameter.\(^{18}\) \(\psi\) and \(\nu\) are the relative weights of health and leisure in the utility function and \(\gamma, \sigma\) and \(\eta\) are the coefficients of relative risk aversion of consumption and

\(^{17}\)The level of health care spending tax in our model (which is always negative under all reasonable specifications) is interpreted as the share of overall health care spending that the National Health Service is willing to cover. Since co-payments in the UK do not account for a significant fraction of out-pocket payments, the level of subsidy expresses the share of medical services and pharmaceuticals that the NHS chooses to supply free of charge. In essence, a change in the subsidy level in our model corresponds to changes in the medical coverage of the population, with a subsequent increase in out-of-pocket payments, since households receive treatment in the private sector.

\(^{18}\)As Rossen (1988) points out, in contrast to the standard models with exogenous probability of survival, where only marginal utility is relevant for optimal decisions, when the probability of survival is endogenous the level of utility is significant as well. In a nutshell, agents do not only decide the allocation of consumption between periods, but also the number of periods they derive utility from. However, as Hall and Jones (2007) note, in the standard CRRA utility functions the level of utility is negative for standard values of the coefficient of relative risk aversion. In order to overcome this issue, where it is optimal for the agent to choose less periods of consumption since the sum of per period utility is negative, they add a constant parameter. The intuition of the constant parameter, is simple; the utility that the agent derives simply by being alive, ignoring consumption and the level of health.
health and leisure respectively. We calibrate the parameters, presented in Table 1, drawing directly from the estimated parameter values of Hall and Jones (2007), setting $b = 66.27$, $\gamma = 2$, $\sigma = 1.051$, and $\psi = 2.396$. In the life-time maximization problem we set the agent's discount rate $\beta$ to 1.01. We calibrate the parameters of leisure in order to match the labour supply of roughly one third in the data, setting $\nu = 10.5$ and $\eta = 6$.

4.1.3 Health and Probability of Survival

The health and probability of survival functions are described as in (3) and (4) respectively.

The values of $\delta_j$, $Q_j$ and $z_j$ of the health production function, are age and model specific in order to match the level of health care spending and the probability of survival in the UK economy. We assume that the value of $\delta_j$ increases with age, $\delta_j = [0, 0.7, 0.8]$, in order to reflect the health deteriorating, while $Q_j$ and $z_j$ remain constant to 10 and 0.6 respectively.

The parameter values of the probability of survival (3), $a_{p_j}$ and $b_{p_j}$ are set to 10 and 0.3 respectively for all cohorts. With the given health level of each cohort, we calibrate the parameter values in order to fit the UK data with respect to the survival probability.

4.1.4 Production Technology

The share of capital in our economy is set to $\alpha = 1/3$ (Attanasio, Kitao and Violante, 2010). The productivity enhancing parameter $\phi$ in $e^{\phi h_{jt}}$ is set to 0.1 to match the household’s health care spending profiles. Total factor productivity is set as $A = 1$ for normalization.

4.1.5 Social Security and Government Spending

We assume that the government adjust the income tax rate $\tau^w$, in order to balance its budget and captures both income tax and social insurance contributions. The consumption tax rate, $\tau^c$ is set to 13%, $\tau^a$ is 50.2% and $\chi = 0.326$, which is the average pension replacement rate in the UK OECD (2013), and $g = 0.28$ to match the level of overall government spending as a percentage of GDP. The health care spending tax rate is set to $\tau^h = -0.833$, which in turns suggests that the level of government subsidies of health care expenditure is 83.3%, close to the historical average of the UK economy ONS (2013a).
4.2 Decentralized Equilibrium and Second Best

In Table 1 we provide the UK data for key variables of interest, along with the results of the decentralized equilibrium and the second best for the restricted version of the government problem in order to obtain comparable results with the UK data and the decentralized equilibrium.

Our benchmark model fits the UK data quite well. With a given level of government subsidies on health care, which is set exogenously to reflect the actual UK policy, the medical spending chosen by the households is close to the observed health care spending to the UK (11.73% as opposed to 8.8% in the data), which results to a higher labour income tax of 36.63% as opposed to 24.8% as calculated by Uhlig (2012). This level of medical spending results to a level of health and subsequently probabilities of survival to 40 and 60 years respectively that are very close to the observed data. Thus, the percentage of GDP devoted to pensions payments (7.75%), which is influenced by the population structure and the generosity of the social security system, follows closely the actual data (8.17%)\textsuperscript{19}. Furthermore, the savings rate in the benchmark model reflects the actual savings rate of the economy at 15.41% as opposed to 15.6% in the data. These results and the accurate calibration of the benchmark economy are crucial for the second best allocation since the savings and spending decisions of the household, together with the fiscal implications through the probability of survival and pensions affect the optimal tax policy of the government. Government spending, especially health care spending and pensions, naturally affects the level of the optimal taxation since the government budget is balanced.

\textsuperscript{19}In our model, life expectancy is slightly lower than the actual data due to the fact that we chose a three-generations model, which reduces the number of pensioners slightly.
Table 1: Results

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Decentralized</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labour Income Tax, %</td>
<td>24.80</td>
<td>36.63</td>
<td>34.78</td>
</tr>
<tr>
<td>Capital Income Tax, %</td>
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<tr>
<td>Consumption Tax, %</td>
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<td>13.00</td>
<td>13.00</td>
</tr>
<tr>
<td>Aggregate Health Spending,%GDP</td>
<td>8.80</td>
<td>11.73</td>
<td>3.15</td>
</tr>
<tr>
<td>Savings,%GDP</td>
<td>15.60</td>
<td>15.41</td>
<td>19.26</td>
</tr>
<tr>
<td>Probability of Survival to 40</td>
<td>98.00</td>
<td>98.48</td>
<td>98.45</td>
</tr>
<tr>
<td>Probability of Survival to 60</td>
<td>89.01</td>
<td>91.31</td>
<td>91.16</td>
</tr>
<tr>
<td>Labour Supply Young (Hours/Week)</td>
<td>36.30</td>
<td>41.04</td>
<td>42.58</td>
</tr>
<tr>
<td>Labour Supply Middle Age (Hours/Week)</td>
<td>36.30</td>
<td>25.20</td>
<td>18.79</td>
</tr>
<tr>
<td>Labour Supply Old (Hours/Week)</td>
<td>36.30</td>
<td>9.30</td>
<td>0.35</td>
</tr>
<tr>
<td>Pensions,%GDP</td>
<td>8.17</td>
<td>7.75</td>
<td>7.84</td>
</tr>
<tr>
<td>Government Health Spending,%GDP</td>
<td>7.33</td>
<td>9.77</td>
<td>2.34</td>
</tr>
<tr>
<td>Government Consumption,%GDP</td>
<td>28.00</td>
<td>28.00</td>
<td>28.00</td>
</tr>
<tr>
<td>Total Government Spending,%GDP</td>
<td>43.50</td>
<td>45.52</td>
<td>38.18</td>
</tr>
</tbody>
</table>

Notes: Data are collected from stat.OECD, Labour Force Survey (2013), Human Mortality Database (2014) and Trabandt and Uhlig (2012).

However, the most interesting results stem from the second best allocation, with an optimal health care spending subsidy of 74.31% and capital income tax of 7.69% when the government cannot make use of age dependent taxation. Since this a benchmark model, with no idiosyncratic shocks and the usual humped-shaped labour productivity age profile, the optimal capital income tax is interpreted as an additional motivation to tax capital beyond the previously mentioned factors that are studied in the literature. The level of the optimal capital income tax rate is expected to be significantly lower in this context. This results in significantly higher savings rate in the second best compared to the decentralised equilibrium, which comes from reduced medical spending. Savings are increased by 3.85 percentage points compared to the decentralized equilibrium which, in addition to a significantly lower health care subsidy rate, results to lower health care spending in the second best.

In order to infer what is the effect of uniform tax rates across cohorts we calculate the second best allocation without restricting the government with respect to age dependent tax rates. The results are interesting for all three of the government instruments; labour income tax, capital income tax and health care spending subsidies.

The government finds optimal to tax labour income more heavily for the young than the middle age (35.98% compared to 30.86%), while the most heavily taxed are the old (40.68%). As health and subsequently labour productivity falls with age, the elasticity of labour supply increases, which
results in lower labour income tax rates for older cohorts. However, the old receive an exogenously sustained pensions which is taxed at the same rate as labour income tax. Since pensions are completely inelastic, the government chooses to tax the old heavily reducing the distortionary effect of labour income taxes on the middle age and young, even if this results in lower labour supply for the old.

Health and pensions have the same effect on capital income tax, albeit to a more extreme degree, changing not only the magnitude, but the sign of capital income tax rates. Savings are more inelastic for the young, compared to the middle-age cohort since in order to smooth consumption they need to save from their higher labour income to consume in the following periods. Middle-age households not only receive less labour income, but anticipate an exogenous pension payment the next period and hence savings are very elastic. Hence the government chooses a positive income tax for the young and a subsidy for the middle-aged whose savings are inefficiently low compared to the second best allocation with age-dependent taxation.

Age-specific health care subsidies are straightforward in this model since the government finds optimal to subsidise the young more than older cohorts. A higher level of health care subsidies encourages young cohorts to invest in their level of health, which has life-cycle consequences for labour productivity, quality of life and longevity. As cohort age, this incentive becomes weaker, since their health care spending decisions at the present period affect fewer periods in the future.

Table 2: Age-Dependent Taxation

<table>
<thead>
<tr>
<th>Tax Rate</th>
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<tbody>
<tr>
<td>Labour Income Tax (Young),%</td>
</tr>
<tr>
<td>Labour Income Tax (Middle-Aged),%</td>
</tr>
<tr>
<td>Labour Income Tax (Old),%</td>
</tr>
<tr>
<td>Capital Income Tax (Young),%</td>
</tr>
<tr>
<td>Capital Income Tax (Middle-Aged),%</td>
</tr>
<tr>
<td>Health Spending Tax (Young),%</td>
</tr>
<tr>
<td>Health Spending Tax (Middle-Aged),%</td>
</tr>
<tr>
<td>Health Spending Tax (Old),%</td>
</tr>
</tbody>
</table>

Recall from the previous section that the inability of the government to make use of age-dependent taxation affects not only the levels of each tax rate, but the structure of the fiscal policy as well, since the optimal capital income tax becomes non-zero. The uniform tax rates (Table 1) are not the weighted average of their respective age-dependent counterparts (Table 2). A uniform capital income tax is set optimally taking into account the effects of setting a uniform labour income and
health care spending tax as well, a standard result in the literature. The contribution of this paper is that with endogenous probability of survival the optimal level of capital income tax is not zero, even if we do allow the government to set the tax rates optimally without any further restrictions (Table 2) in contrast to the standard results in the literature (Garriga, 2001; Peterman, 2013).

Hence, the introduction of health in the Ramsey problem of optimal taxation has substantial effects on the optimal level of the capital income tax. In the restricted version of the government’s problem, the optimal capital income tax rate is underestimated in the literature ignoring the effects of health on labour productivity and the health care spending and savings decisions of the households through the effective discount rate. In the unrestricted version of the model, the results are more interesting since the optimal capital income tax is not only non-zero but with different signs between cohorts, which complicates the optimal tax policy when the government can at least partially discriminate between cohorts.20

4.3 Increased Longevity

Next we study the effect of technological progress in the medical sector that affects longevity. In our model this translates to an increase in the probability of survival for each level of health. Hence, in the function of the probability of survival (3) the parameter \( a_p \) increases over time.21

Our results have significant policy implications for the challenging issue of our ageing societies and the financing of social security. Increased longevity does not only change the level of optimal taxation, as expected from increased government spending but the relative tax rates as well. Our results suggest that it is optimal to finance social security of an ageing population through increase capital income tax rates, without significantly affecting the optimal labour income tax.

<table>
<thead>
<tr>
<th>Table 3: Simulation Results</th>
</tr>
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<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Capital Income Tax,%</td>
</tr>
<tr>
<td>Labour Income Tax,%</td>
</tr>
<tr>
<td>Health Spending Tax,%</td>
</tr>
<tr>
<td>Pensions,%GDP</td>
</tr>
</tbody>
</table>

In order to distinguish the effects of changes in longevity on the optimal capital income tax

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20 In practice, health care subsidies are not uniform, at least implicitly, since treatment in the NHS is provided taken into account the cost and benefits of the treatment which vary with age. Another example are government policies that affect labour supply on the extensive and intensive margin for older cohorts.

21 This can be interpreted as either better treatment for life-threatening conditions or new treatments that were previously unavailable.
stemming from a higher burden on social security (which is expected to change the level of capital income tax) to the changes in the relative tax rates we do the following simulation; What is the optimal levels of capital income, labour income and health care spending tax rates after (i) increasing pensions by one percentage point (from 32.6% to 33.6%) and (ii) increasing longevity such that the fiscal impact with respect to pension payments as a percentage of GDP is identical (from $a_{p1} = 1$ to $a_{p2} = 1.15$). In both case the government spends 8.8% of GDP on pension payments, up to 7.84%, however the financing of social security is significantly different (Table 3).

An increase in the pension replacement rate, reduces the optimal capital income tax since agents have an incentive to save less, expecting higher future income from pensions and labour income tax increases. Partially, the increase in the labour income tax rate can be explained by the increase in the replacement ratio itself. Recall that pensions are taxed with the same tax rate as labour income and being completely inelastic, increase the optimal labour income tax rate, which is restricted to be uniform across cohorts. In contrast, with higher probability of survival, agents discount the future less, decreasing the elasticity of savings, increasing the optimal level of the optimal capital income tax rate. However, the labour income tax rate increases slightly since improvements in the medical sector increase the incentive of the government to subsidy health care spending, which raises the level of government spending more than the fiscally equivalent increase in the pension replacement rate. Better medical treatment has a dual effect on the optimal tax policy; first, the government increases health care subsidies and secondly, the elasticity of savings falls, reducing the distortions of capital income tax rates, allowing the government to finance the excess social security and health care burden with higher capital income tax rates.

Hence, the introduction of health and particularly endogenous longevity in the standard life-cycle model sheds light in another aspect of the ageing of the population largely ignored in the literature. Beyond cost containment and labour supply incentives, we can study what is the optimal tax policy for an ageing society not only accounting for greater costs and the corresponding increase in tax rates in order to make social security and health care system sustainable, but the optimal policy with respect to the relative weights of the tax instruments.

5 Conclusions

This paper studied the problem of optimal taxation in a model of health as (i) a quality of life parameter, (ii) quantity of life parameter affecting the number of periods the agents can derive utility and (iii) human capital that affects productivity. In our model the level of health is a stock
that naturally deteriorates over time and agents can invest in their level of health via medical spending, which is taxed (or subsidized) by the government.

Our results suggest that incorporating health in the standard overlapping generation model affects the optimal set of government instruments substantially. Even with preference that set the Frisch elasticity of labour supply constant over the life cycle, capital taxation is not zero if the government cannot condition all taxes on age. This result stems both from health as a form of human capital which affects the optimal level of labour income tax and the elasticity of health care spending per se, which affects the optimal health care tax rate. Hence, we contribute in the literature by considering an additional form of human capital which affects the optimal level of capital income tax, beyond incomplete insurance markets and non constant labour supply elasticity over the life cycle. Moreover, the peculiarity of health as a factor that affects longevity has implication on the optimal path of capital gains taxation. In our model, technological progress in the medical sector that affects longevity dictates an optimal path of capital income taxation instead of an optimal level.

The model could be extended in various directions. First, we focus on presenteism, which affects the labour supply productivity and we abstract from absenteism, namely sick time which significantly impacts the effective labour supply. Although working hours lost due to illness vary between countries, a substantial fraction of potential working hours are lost every year. In addition, considering uncertainty and heterogeneity in an overlapping model with health we can study the welfare implications of optimal taxation taking into account differences in productivity, labour supply in the extensive and intensive margin and probability of survival explicitly extending the framework of Conesa, Kitao and Krueger (2009). Thus, we can study the socio-economic gradient of health observed in developed countries which reflects differences in productivity and longevity and study the optimal response of the government if the objective is to take into account unequal outcomes.

\[ \text{For example, in the UK 2\% of working hours are lost due to illness (ONS, 2014)} \]
References


