Profit sharing in a unionized differentiated duopoly

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Abstract

Profit sharing, with one form or the other, has been in wide use all over the world. Unionization does survive and plays a major role in wage setting in most OECD countries. We have endogenize unions decision of merging, and firms decision of giving profit sharing, in a multistage Cournot unionized duopoly. In equilibrium, unions will always merge, and multiple equilibria arise: profit sharing might be used by none, by one firm, or by both firms.

JEL Classification: J30; J40; J20

Keywords: union oligopoly; bargaining; profit sharing;

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1 Introduction

Profit sharing scheme is a type of remuneration scheme, in which additionally to the fixed wage received by the worker, part of a worker’s salary is based on the profits made by the firm. Profit sharing schemes can be paid in cash, stocks or other, and can be paid annually, semi-annually or be kept by the firm, and be given to workers at the form of a pension. In practice, profit sharing is quite complex, and contains a set of different elements (OECD 1995).

Nevertheless, profit sharing, with one form or the other, has been in wide use all over the world (Dhillon and Petrakis 2001). A survey of the largest 1250 global corporations (Weeden et al. 1998) found that 33% of them offered some short of profit sharing schemes to all employees, and another 11% had plans to put in place a broad-based profit sharing scheme.

Liberal economic reforms and deregulations of 1970’s and 1980’s in USA, UK and elsewhere greatly affected unionization. But, empirical studies (Visser 2006) and OECD statistics (stats.oecd.org) shows that unionization is not dead; is in effect today, but with strong variations from a country to another. Unions might be nation-wide (central grand union of workers) or firm-specific (decentralized dedicated union); in any case they can metamorphose the remuneration scheme bargain agenda, and strongly affect product market competition.

Product market competition, unionization form and remuneration schemes, as we will show, are interdependent. A strong union affects remuneration scheme, which can affect firm’s labor cost, which can affect product prices, consumer’s welfare, and finally competition.

The driving force that moved us to do research over profit sharing is the generally accepted idea that profit sharing can lead to higher firm profits and higher worker earnings, but not always do so. We consider: a) a decentralized environment with two separate and dedicated unions $U_i$ and $U_j$, and two competing firms $F_i$ and $F_j$, or b) a central environment with one grand (nation-wide?) union $U$, and two competing firms $F_i$ and $F_j$. The firms use a homogeneous and single quality labor force supplied by the union(s), as the only input to produce a differentiated product. Firms compete in product market à la Cournot (quantity competition).

Using a Leontief-style production function for both firms (constant returns to scale),
firms also decide for the amount of labor force they will rent from their respective union(s).

In this environment, the choice of horizontal merging of unions, allows the internalization of an institutional choice often found in labor economics: if a central grand union can delegate better in a bargain, in contrast to multiple firm-specific unions. Furthermore, the choice of contract type from firms (fixed wage or profit scheme), internalizes the choice of the optimum remuneration scheme over different states of unions. All these are backed from a competition stage, in which firms compete over quantities or price, forming a "right to manage" framework, where the amount of labor force used by firms, is a firm-specific choice, but at the same time, the total compensation is a matter of bargain between firm and union.

We have chosen the so called "right to manage" delegation model because we believe that it is more realistic than a monopoly wage setting from unions (often used in literature). In our defense, Horn and Wolinsky (1988) criticize those who simply assume that unions are setting wages unilaterally, rather than participating in some bargain process with the firm.

Our contribution to literature is multi-fold.

a) First, we highlight the significance of remuneration schemes and unionization for product market competition, consumer’s welfare and employment.

b) Second, we extent existing literature on Cournot competition by using product differentiation $0 < \gamma < 1$, and central coordinated bargains between one grand union and two separate firms. Both of them are absent from existing literature.

c) Third, we prove that mix situations are possible. With "mix situations" we mean one firm using fixed wage scheme, while th other using profit sharing scheme. And with "possible" we mean that this mix situation is sub perfect pure strategy Nash equilibrium.

The rest of the paper is as follows: second section makes a literature review; third section describes the model; fourth section makes the equilibrium analysis, solves the game and shows the results; fifth section states the results of this research; sixth section makes some conclusions and suggest further research. Finally, follows references.
2 Literature Review

Profit sharing schemes gained a lot of attention due to Professor Weitzman’s work. Weitzman (1983, 1984, 1985, 1987), a pioneer of profit sharing schemes, states that profit sharing makes the cost of labor completely flexible and gives firms the incentive to hire as many workers as are willing to take jobs. This leads to an economy of profit sharing firms with lower levels of unemployment and greater macroeconomic stability.

Literature on the motive of introduction of profit sharing (OECD 1995) suggests that several groups of variables affect the introduction of profit sharing, such as: firm size, internal organizational structure, industrial relations, labor and legal institutions, and the external environment.

Sesil, Kroumova, Blasi et al. (2002), studied 229 US major "New Technology" Firms (pharmaceuticals, semiconductors, software, telecommunications & high-technology manufacturing), all offering broad-based profit sharing plans. Using multivariate analysis with panel data they found that, in contrast with their non-profit sharing counterparts, profit sharing firms’ productivity increased 4%, total shareholder returns increase by 2%, and profit levels jump by about 14%. These gains are after dilution effect is taken into account.

Arriving at similar results, Kruse (1992), uses data from almost 3000 US firms, from 1971 to 1985, and shows that the introduction of a profit sharing scheme is (statistically significant) associated with a productivity increase of 2.8% to 3.5% for manufacturing firms, and 2.5% to 4.2% for non-manufacturing firms. Kruse suggests that only the most profitable and most productive firms introduce profit sharing schemes, in order to align firm and workers interests, and through this alignment to reach new, higher levels of profitability and market share.

In Germany, Kraft and Ugarkovic (2005), using panel data from more than 2000 German firms from 1998 to 2002, show that the introduction of a profit sharing system improves profitability.

Bensaïd and Gary-Bobo (1991) are the first to view profit sharing schemes not just as an internal incentive system, but as a strategic commitment. They prove that the adoption of a profit sharing scheme by a firm and its union shifts market equilibrium and is a Pareto improvement for both parties. They have shown that the choice of a profit sharing scheme by each firm is an equilibrium, and in the case of Cournot competition, the game has the structure of the prisoners’ dilemma. Last, they argue that profit sharing
schemes seems to be credible in short and middle run, but not in the long run, because of institutional and legal influences in contracts.

Sorensen (1992) sets a three stage game, in which two firms producing a single type homogeneous product, and two unions, one for each firm, as a single labor supplier for that firm. In stage one, firms decide the remuneration system (fixed wage versus profit share), in stage two there is a determination of wages and profit share via bargaining among firms and unions, and in stage three there is a Cournot style competition among firms, which determines output, prices, and employment levels. He shows that union’s bargain power is of critical importance in firm’s choice of introducing a profit sharing scheme. Nevertheless, the introduction of a profit sharing scheme seems to increase employment, and decrease prices.

Fung (1989a) examines profit sharing in three different competition regimes: monopoly, perfect competition and Cournot oligopoly. He found that profit sharing can lower wages in all three different types of competition. This will not lower rents extracted from the union, because there will be a positive profit share percentage taken as well. Also, states that the joint rents of the union and the firm will increase with profit sharing. Finally, he shows that, under some restrictions, union’s wage demand will be smaller under profit sharing than under fixed wage regime.

In an article related with the previous, Fung (1989b) sets up a two-stage duopoly game in order to explore the effects of profit sharing in unemployment rates. In first stage unions set wages as Bertrand duopolists (maximizing the economic rent extracted from labor), while in the second stage firms compete à la Cournot. Under the assumptions of his model, Fung finds out that the effects of profit sharing can be decomposed in two parts; first, an industry wide effect in which profit sharing causes a wage reduction, which leads to a lower product price and thus in bigger total quantities produced. This causes employment to rise. Second, a firm specific effect in which firms using profit sharing schemes gain a bigger market share, and have higher employment with lower wage rates. These beneficial effects give to the profit sharing firm a strategic advantage over the non profit sharing counterparts. An argument in this paper is that because in many Japanese industry sectors, profit sharing is common, this might be a possible explanation for the low unemployment rates and economy’s success in this country.

Stewart (1989) proves that while for a monopolist, the introduction of a profit sharing
scheme it is not a Pareto improvement, for a firm in an oligopolistic sector there are incentives for such an introduction. He sets up a two stage n-opoly game, using standard assumptions. In first stage, one of the homogeneous n-firms commit to profit sharing scheme, while in second stage all n-firms compete à la Cournot. Holding workers’ income constant (subsidizing equally a part of fixed wage with a profit share), there is an increase in firm’s profits. This creates incentives to more firms entering a profit sharing scheme, even if entering all n-firms in profit sharing schemes creates profits below the level obtained under the wage system.

Goeddeke (2010) analyzes the emergence of profit sharing schemes, when wages are negotiated in two extremes: a bilateral monopoly way or decentralized way. She is based upon Sorensen (1992) homogeneous product model, but it extends it, by using an n-opoly structure. She founds that in n-opoly, under decentralized bargains, firms have incentives to replace fixed base wage schemes with profit sharing schemes. But when the majority of firms (depends on magnitude of n) adopts profit sharing schemes, it is in the collective interest of both firms and unions to move back to fixed base wages. Also, she shows that the existence of profit sharing scheme creates incentives for firms to bargain independently, and not centrally.

Negotiations between firm and union over the profit share are not common in all industrial countries. Poole (1988), surveys British firms, and shows that the decision about the level (percentage) of profit shares made by the managers in 97.7% of the cases studied, while only 0.7% was made under bargain. This assumption is behind the research made by Pemberton (1991); he assumes that the very existence and extent of profit sharing cannot be a process of bargain between firm and union, but in real world is part of firm’s profit maximization strategy. He proves that in the absence of product subsidies, profit sharing schemes will not be profitable for the firm, but if firms’ facing heterogeneous demand, and have different technologies, there are incentives for firms to apply a profit share.

In France, since 1967, there is a legal obligation for firms employing more than 50 workers, to apply some short of profit share, calculated on a basis of predetermined profit sharing formula. In 2011 this legal obligation was reinforced and became wider, requiring firms to also pay a "social" dividend, if their net profits were higher than last year.

In a recent study (Fang, 2016) there has been a review of empirical studies showing
that profit sharing is beneficial for employees through higher earnings and employment stability, and at the same time is beneficial for employers through higher workspace productivity, and thus higher firm profitability. Profit sharing reduces supervision costs and is against shirking behavior, while at the same time creates a bigger flexibility in wages. The author concludes that labor unions may want to work collaboratively with management to enhance the mutual benefits of profit sharing.

A short literature review over unionization could reveal a great variation: unionization is still strong in some countries, while in other is not. Union density\(^1\) in Visser (2006) and OECD Statistics online\(^2\), shows that during the first decade of 2000, in some countries (e.g. Iceland, Belgium, Finland, Denmark, Norway, Sweden) unionization is way above 50%, while in some other countries (e.g. France, Korea, USA, Japan, Spain, Turkey, Netherlands) unionization is below 20%. Many countries lie in between 20% and 50% (Canada, Italy, Ireland, Greece, Austria, Luxembourg). These charts shows a great variation in unionization, but nevertheless unionization is in effect and with stable percentages during 2000’s.

Labor market institutions show high variability as of the level of wage negotiations. In United States, United Kingdom, Australia, Canada, and Japan, wages are usually bargained on a decentralized firm level between unions and firms. In contrast, in Germany, Austria, Belgium, Greece and Scandinavian countries, bargain take place either on an sector-wide level or even at a national level (Goeddeke 2010).

3 The model

3.1 Market structure and contract types

We consider an industry, consisting of two competing firms, denoted \(F_i\) and \(F_j\) respectively. Each firm has either: a) a separate, dedicated workers union (decentralized version) as its sole supplier of labor, denoted \(U_i\) and \(U_j\) respectively, or b) both firms face the same grand union of workers (central coordinated version), denoted \(U\).

We assume that: i) the two firms \(F_i\) and \(F_j\) use a two-factor Leontief production

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\(^1\)Ratio of employed union members to all employed civilians.
\(^2\)OECD Trade Union Density charts could be found at https://stats.oecd.org
function $q_i = \text{Min}[L_i, K_i]$, where $L_i=\text{Labor}$ and $K_i=\text{Capital}$, ii) produce under constant returns to scale, iii) the amount of capital $K_i$ is fixed in the short run, and large enough not to induce zero marginal product of labor $(L_i << K_i)$. From these three assumptions we get that quantity equals employment: $q_i = L_i$, $i = 1, 2$.

Furthermore, we assume that each firm produces a single type of a differentiated good. Firm $F_i$ faces the following inverse demand function: $P_i = \alpha - q_i - \gamma * q_j$, $i = 1, 2$, where $\gamma \in (0, 1)$ measures the differentiation between the two products.

Total quantity sold by firms equals $Q = q_i + q_j$. Because of the assumption $q_i = L_i$, total quantity $Q$ equals total employment $Q = L = L_i + L_j$. We assume that market is big enough ($\alpha$ is big enough) to consume all quantity produced and sold, and thus no stock is kept by the firms.

We assume a constant marginal cost per product, being the sum of a (non-labor) cost: average cost = marginal cost = $c$ per quantity $q_i$ produced, where $0 < c < \alpha$, and a labor cost (wage rate) $w_i$ per unit of labor $L_i = q_i$. Firms assumed to have no other costs or income, so they have a (gross) profit function equal to:

$$\Pi_i = P_i \cdot q_i - (c + w_i) \cdot q_i = (\alpha - c - w_i - q_i - \gamma q_j) \cdot q_i, \ i = 1, 2 \ & i \neq j \ (1)$$

Firm’s decision makers (management) are assumed to be risk-neutral, thus their utility function equals (net) profits, which are: a) with profit share $\pi_i = (1 - a_i) \cdot \Pi_i, i = 1, 2$, and b) without profit share: $\pi_i = \Pi_i, i = 1, 2$.

Union’s leaders are also assumed to be risk-neutral, thus their utility function can be modeled as: a) with profit share $UW_i^P = [((w_i - w_0) + \frac{a_i \Pi_i}{L_i})L_i] = [(w_i - w_0)q_i + a_i \Pi_i]$, and b) without profit share $UW_i^W = (w_i - w_0)L_i = (w_i - w_0)q_i$ where $w_i$ is the fixed base wage, and $w_0 \geq 0$ is the (exogenous, and assumed constant) unemployment benefit.

We use the union’s utility function of Pemberton (1988), who shows that a rent-maximizing union is equivalent to a ‘managerial union’ with union leaders who are in-

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3 As in Singh and Vives (1984), this demand function comes from a quadratic and strictly concave utility function of a continuous of representative consumers. Quadratic mean that consumers prefers more to less, and strictly concave mean consumers are risk averse.

4 For firms we use $\Pi = \text{for gross profits}$ i.e. profits before profit share (if applicable), and $\pi = \text{net profits}$ i.e. profits after profits share deduction(if applicable). In case of using a fixed base wage as remuneration scheme, then gross and net profits are the same $\pi = \Pi$. For unions we use $UW$: union’s welfare, which is calculated always after the addition of profit share percentage.

5 Unemployment benefits exists all over the world. They protect workers during unemployment against major income losses or accepting jobs below their qualifications. They ought to be below wages, else they could raise unemployment rates.
interested in employment, and union members who are interested in excess wages (i.e. the amount \((w_i - w_0)\)).

### 3.2 Sequence of events

Firms and unions engage in a four stage game with observable actions. Game timing reflects the idea that long run decisions, such as the forming of grand coalitions of workers, may have considerable strategic effects in short run decisions, such as the employment decision made by the firm. This timing implies that some variables (such as employment) are easier to change and are greatly affected by other variables, which may be much more difficult to alter. This timing is standard in literature, and allows us to capture the contract forms’ commitment value.

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**Stage 1** *The merger stage.* This is a union’s decision stage. The unions decide whether they will merge or not horizontally, forming a grand union of workers. We use two different types of bargain: central coordinated and decentralized.

**Stage 2** *The remuneration scheme stage.* This is a firm’s decision stage. Firm \(F_i\) decides whether to offer a fixed wage \(w_i\), or a profit sharing scheme made from a fixed wage plus a profit share percentage \((w_i, F_i)\). This is a take it or leave it offer in a right to manage bargain framework \(^6\). There are four possible outcomes: a) both firms decide to give a profit sharing scheme, b) both firms decide to give fixed wage, and c) & d) a double symmetrical case, where one firm applies a profit sharing scheme, while the other applies a fixed wage scheme.

**Stage 3** *The bargaining stage.* In stage 3, two separate unions \(U_i\ & U_j\), or one grand union \(U\) from one side, and two firms \(F_i\ & F_j\) on the other side, bargain over the remuneration scheme. The solution concept used here is the sub perfect pure strategy

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\(^6\) There is a strong debate between right to manage and Efficient Bargain frameworks. We have chosen right to manage because we feel that no union can enforce a certain amount of employment to a firm; employment has to do with production, and production has to do with competition.
Nash equilibrium of simultaneous generalized Nash Bargaining problems. Union(s) has bargain power equal to $\beta \in (0, 1)$ while firm has bargain power equal to $(1 - \beta)$. We assume that union(s) leadership engage in wage bargaining with firm’s management on behalf of their (same skill/homogeneous) affiliated workers. Firms and union(s) negotiate simultaneously, and it is assumed that at the end they will reach an agreement. The main goal of solving this stage is to find, under all possible remuneration schemes and under all possible forms of centralized/decentralized bargain, Nash equilibria values of wages $w^*_i$ and of profit sharing (if applicable) $a^*_i$ for $i = 1, 2$, as functions of exogenous variables.

**Stage 4** The competition stage. Firms compete in Cournot style competition, where firms maximize profits over quantity. This stage determines employment, since $q_i = L_i$.

### 3.3 Bargaining Framework

We model the bargain between one grand union $U$ or two separate unions $U_1$ and $U_2$, and two separate firms $F_i$ and $F_j$ as to be done simultaneously and separately. This assumption captures the fact that each pair of negotiations ($U \& F_i$ vs $U \& F_j$) has incentives to behave opportunistically and to reach a mutually favorable agreement that enhances firm’s competitive position in expense to the other rival firm.

We obtain a unique equilibrium by imposing pairwise proofness on the equilibrium contracts. That is, we require that the contract between $U$ and $F_i$ is immune to a bilateral deviation of $U$ with the rival firm $F_j$, holding the contract with $F_i$ constant.

We also impose exclusivity in firm-union relations. Each firm supplies it’s labor force from a single dedicated union of workers, while a single union of workers supply it’s labor force to a single firm. If bargain between them fails, then both gets zero profits.

Two additional key assumptions of our model are passive beliefs and non-contingent contracts.

Passive beliefs state that firm $F_i$ will handle any out-of-equilibrium offer from $U$ as a ”tremble”, uncorrelated with any offer from $U$ to rival firm $F_j$. $F_i$ believes that under any offer received from $U$, the pair $U \& F_j$ has reached an equilibrium outcome. Different beliefs will give rise to different equilibria, as stated in literature (McAfee and Schwartz, 1989).

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Footnote: Not many thinks are mentioned in literature about bargain power $\beta$. It is generally accepted being an exogenous variable of the game, mainly affected from legal framework, firm’s internal organization, team spirit, union’s ability to strike, firm’s costs of hiring, training, and firing, unemployment rates and/or difficulties to match firm’s needs with skilled workers, wage discrimination, and many more.
Non-contingent contracts state that any breakdown in bargains between $F_i$ and $U$ will be non-permanent and non-irrevocable, and this is common knowledge (Horn and Wolinsky 1988). This will lead the rival pair $F_j$ and $U$ to bargain in a bilateral monopoly fashion, with $F_j$ employing and selling monopoly quantity, but using the same wage $w_j$ and the same profit share percentage $a_j$ as in duopoly (because of the fact that any breakdown in bargains between $F_i$ and $U$ will be non-permanent).

4 Firms’ competition and bargaining outcomes

4.1 Stage 4: Cournot Competition

In the last stage of the game, firms $F_i$ & $F_j$ engage in quantity (Cournot) competition. Under Cournot competition, firms maximize their profits $\Pi_i$ over quantity $q_i$:

$$\max_{q_i} [\Pi_i(q_i, q_j, w_i)] = \max_{q_i} [(P_i(q_i, q_j) - c - w_i) \cdot q_i] = \max_{q_i} [(\alpha - q_i - \gamma q_j - c - w_i) \cdot q_i]$$

(2)

The first order conditions give rise to the following reaction function:

$$R_i(q_j, w_i) = \frac{1}{2}(\alpha - c - \gamma q_j - w_i)$$

(3)

A decrease in the wage $w_i$ faced by $F_i$ shifts its reaction function upwards, and turns $F_i$ into a more aggressive competitor in product market. Solving the system of reaction functions for $i = 1, 2$ & $j \neq i$, we obtain the equilibrium quantities and ("gross") profits for given levels of wages:

$$q_i(w_i, w_j) = \frac{(2 - \gamma)(\alpha - c) - 2w_i + \gamma w_j}{4 - \gamma^2}, \quad \Pi_i(w_i, w_j) = (q_i(w_i, w_j))^2$$

(4)

4.2 Stage 3: Bargaining Stage

In Stage 3 firms and union(s) bargain over wages and profit shares. Since the bargaining game in case of one grand central union is different from the case of two decentralized unions, we will analyze the two cases separately.
4.2.1 Stage 3a: Decentralized unions

When unions are separate and dedicated, then two pairs \((U_i, F_i)\) & \((U_j, F_j)\) exists. Each pair has to choose between a fixed wage scheme \((w_i)\), or a profit sharing scheme \((w_i, a_i)\).

Due to symmetry, there are three possible subgames:

- **I. Fixed wage case**: both firms use fixed wage schemes,
- **II. Mix case**: one firm use fixed wage, while the other use profit sharing, and
- **III. Profit sharing case**: both firms use profit sharing schemes.

We impose an exclusivity in firm-union relations; each firm gets labor force from its own union, while each union supply only one firm. If bargains fail, then both firm and union gets zero profits.

**I. Fixed wage case** Both firms use fixed wage schemes. Unions are separate and dedicated to respective firms. Firms and unions bargain over the following Nash bargain product:

\[
NBP^DF_i(w_i, w_j) = [\Pi_i(w_i, w_j)]^{1-\beta} \cdot [(w_i - w_0) \cdot q_i(w_i, w_j)]^\beta
\]

where superscript "DF" stands for: D=decentralized bargains, and F=fixed wage remuneration schemes for both firms. Substituting \(\Pi_i(w_i, w_j)\) and \(q_i(w_i, w_j)\) from Equation 4, and maximizing \(NBP^DF_i(w_i, w_j)\) over wage \(w_i\) we get:

\[
\max_{w_i}[NBP^DF_i] \Rightarrow w^{DF}_i(w_j) = \frac{1}{4}(4w_0 + 2\beta(\alpha - c - w_0) - \beta\gamma(\alpha - c - w_j))
\]

Notice that \(\frac{\partial w^{DF}_i(w_j)}{\partial w_j} > 0\), that is: an increase in one firm’s wage, will cause the other firm to increase wages as well. Imposing symmetry in equilibrium: \(w^{DF}_i(w_j) = w^{DF}_j(w_i) = w^{DF}\), we derive the equilibrium wage:

\[
w^{DF} = w_0 + \frac{\beta(2 - \gamma)\tilde{a}}{4 - \beta\gamma}
\]

We have set: \(0 < \alpha - c - w_0 = \tilde{a} < \alpha\). Clearly: \(w^{DF} > w_0\), while \(\frac{\partial w^{DF}}{\partial \beta} > 0\) and \(\frac{\partial w^{DF}}{\partial \gamma} < 0\). Equilibrium wage increases, as union’s bargain power increases, and decreases as product differentiation decreases (\(\gamma \to 1\)). Nash equilibrium quantities, prices, and
"gross" = "net") profits are:

\[ q^{DF} = \frac{2(2 - \beta)\tilde{a}}{(\gamma + 2)(4 - \beta\gamma)}, \quad \Pi^{DF} = \pi^{DF} = (q^{DF})^2, \quad p^{DF} = \alpha - \frac{2(2 - \beta)(\gamma + 1)\tilde{a}}{(\gamma + 2)(4 - \beta\gamma)} \]  \hspace{1cm} (8)

It is \( \frac{\partial q^{DF}}{\partial \beta} < 0 \) and \( \frac{\partial q^{DF}}{\partial \gamma} < 0 \), so equilibrium wage, and equilibrium profits are decreasing as union’s bargain power increase, and as product differentiation decrease. Furthermore, equilibrium price increase as union’s bargain power increase, and decrease as product differentiation decrease. Overall, a strong union will demand higher wages. Firms will bypass this higher labor cost to consumers through higher prices. Lesser product differentiation will enhance product market competition, leading to price cut, and so to lower firm’s profits.

Union welfare and Consumer Surplus are:

\[ UW^{DF}_i = UW^{DF}_j = \frac{2(2 - \beta)\beta(2 - \gamma)\tilde{a}^2}{(\gamma + 2)(4 - \beta\gamma)^2}, \quad CS^{DF} = \frac{8(2 - \beta)^2\tilde{a}^2}{(\gamma + 2)^2(4 - \beta\gamma)^2} \] \hspace{1cm} (9)

Union welfare is increasing with its own bargain power \( \beta \), and decreasing as product differentiation decreases. Consumer surplus \( \left( \frac{1}{2}Q^2 \right) \) is decreasing when union’s bargain power increase, and when product differentiation decrease.

\section*{II. Mix case}

In this mix case scenario one firm use fixed wage scheme, while the other use profit sharing scheme. Unions are separate and dedicated to respective firms. Due to different remuneration schemes, the two bargaining pairs \((U_i, F_i) \& (U_j, F_j)\) bargain over different Nash bargain products:

\[ NBP^{DM}_i(w_i, w_j, a_i) = [(1 - \alpha)\Pi_i(w_i, w_j)]^{1-\beta} \cdot [(w_i - w_0)q_i(w_i, w_j) + \alpha\Pi_i(w_i, w_j)]^\beta \] \hspace{1cm} (10a)

\[ NBP^{DM}_j(w_i, w_j) = [\Pi_j(w_i, w_j)]^{1-\beta} \cdot [(w_i - w_0) \cdot q_j(w_i, w_j)]^\beta \] \hspace{1cm} (10b)

where superscript ”DM” stands for: D=decentralized bargains, and M=mix case. Maximizing \( NBP^{DM}_i(w_i, w_j, a_i) \) over both \( w_i \) and \( a_i \), and maximizing \( NBP^{DM}_j(w_i, w_j) \)
over \( w_j \) only, we get:

\[
\begin{align*}
\max_{w_i, a_i} [NBP_i^{DM}(w_i, w_j, a_i)] & \Rightarrow w_i^{DM}(w_j) \, \& \, a_i^{DM}(w_i, w_j) \quad (11a) \\
\max_{w_j} [NBP_j^{DM}(w_i, w_j)] & \Rightarrow w_j^{DM}(w_i) \quad (11b)
\end{align*}
\]

Due to the existence of profit share percentage \( 0 < a_i < 1 \), pair’s \( U_i \) & \( F_i \) remuneration contract is *bilaterally efficient*, that is, it maximizes the joint surplus of the bargaining pair, given the rival pair’s bargaining outcome. In contrast, a fixed wage remuneration scheme contract is not bilaterally efficient, since it does not include a profit share percentage.

\[
\begin{align*}
\max_{a_i} [NBP_i^{DM}(w_i, w_j, a_i)] & \Rightarrow \frac{\partial NBP_i^{DM}(w_i, w_j, a_i)}{\partial a_i} = 0 \\
& \Rightarrow a_i^*(w_i, w_j) = \frac{\beta \cdot \Pi_i(w_i, w_j) - (1 - \beta) \cdot [(w_i - w_0)q_i(w_i, w_j)]}{\Pi_i(w_i, w_j)} \quad (12)
\end{align*}
\]

The reason because \( a_i \) is divided by \( \Pi_i(w_i, w_j) \) is the fact that \( a_i \) is a percentage of firm’s profits, so it has to be normalized to 0%-100%. The use of \( a_i \) is to split the "pie" of joint profits in two pieces, in accordance with each party’s bargain power. If we set Equation 12 to Equation 11, then we get:

\[
NBP_i^{DM}(w_i, w_j)|_{a_i^*(w_i, w_j)} = (1 - \beta)^{1-\beta} \cdot \beta^3 \cdot JP_i^{DM}(w_i, w_j) \quad (13)
\]

where: \( JP_i^{DM}(w_i, w_j) = \Pi_i(w_i, w_j) + (w_i - w_0) \cdot q_i(w_i, w_j) \) are the joint profits of bargain pair \( F_i \) & \( U_i \). That is, maximizing Nash bargain product is about maximizing joint profits.

Solving the system of these three equations over \( (w_i^{DM}, w_j^{DM}, a_i^{DM}) \) we get:

\[
\begin{align*}
w_i^{DM} & = w_0 - \frac{(2 - \gamma)(\beta \gamma + 4)\gamma^2 \hat{a}}{32 + 3\beta \gamma^4 - 16 \gamma^2}, \quad a_i^{DM} = \beta + \frac{1}{2}(1 - \beta)\gamma^2 \quad (14a) \\
w_j^{DM} & = w_0 + \frac{\beta(2 - \gamma)(\gamma + 2)(4 - \gamma(\gamma + 2))\hat{a}}{32 + 3\beta \gamma^4 - 16 \gamma^2} \quad (14b) \\
q_i^{DM} & = \frac{2(2 - \gamma)(\beta \gamma + 4)\hat{a}}{32 + 3\beta \gamma^4 - 16 \gamma^2}, \quad \pi_i^{DM} = (1 - a_i^{DM}) \cdot (q_i^{DM})^2 \quad (14c) \\
q_j^{DM} & = \frac{2(2 - \beta)(\gamma(\gamma + 2) - 4)\hat{a}}{\beta \gamma^4 - 16 \gamma^2 + 32}, \quad \pi_j^{DM} = (q_j^{DM})^2 \quad (14d)
\end{align*}
\]

We can easily check that: \( w_i^{DM} < w_0 \) while \( w_j^{DM} > w_0 \). The firm \( F_i \) that uses a profit
sharing scheme is able to subsidize fixed wage with profit share percentage, effectively lowering fixed wage below unemployment benefit. Total worker’s compensation (both profit share and fixed wage) is still above unemployment benefit, but the fact that part of fixed labor cost transformed into profit share (variable cost) gave to firm $F_i$ a strategic advantage over firm $F_j$ which use a fixed wage scheme only.

Furthermore, $\frac{\partial w_{DM}^i}{\partial \beta} < 0$ and $\frac{\partial w_{DM}^i}{\partial \gamma} < 0$, while $\frac{\partial w_{DM}^j}{\partial \beta} > 0$ and $\frac{\partial w_{DM}^j}{\partial \gamma} < 0$. That is, while both wages ($w_{DM}^i$ and $w_{DM}^j$) lower with product differentiation (as $\gamma$ goes to 1), wage $w_{DM}^i$ lowers with union’s bargain power $\beta$ while $w_{DM}^j$ is ascending. Consider also that $\frac{\partial a_{DM}^i}{\partial \beta} > 0$, we get the intuition that a stronger union will prefer more profit share and less fixed wage.

Unions’ welfare and consumer surplus are:

$$UW_{DM}^i = \frac{2\beta(2 - \gamma)^2(2 - \gamma^2)(\beta \gamma + 4)^2 \delta^2}{(\beta \gamma^4 - 16 \gamma^2 + 32)^2}$$
$$UW_{DM}^j = \frac{2(2 - \beta)\beta(4 - \gamma^2)(4 - \gamma(\gamma + 2))^2 \delta^2}{(\beta \gamma^4 - 16 \gamma^2 + 32)^2}$$

(15a)

$$CS_{DM} = \frac{8(2\beta(\gamma - 1) - \gamma(\gamma + 4) + 8)^2 \delta^2}{(\beta \gamma^4 - 16 \gamma^2 + 32)^2}$$

(15b)

Both unions $U_i \& U_j$ have their welfare decrease as product differentiation decrease, and $U_i$ has its welfare increase as bargain power increase. For $U_j$, interestingly, things are a bit more complicated: $UW_j > 0$ only for $\beta(\gamma^4 + 32) < 32 - 16(1 - \beta)\gamma^2$. So, there is an area in $(\beta, \gamma)$-plane where $UW_j < 0$. Especially for $\beta = \gamma = 1$ this holds true. In mix case, when product is homogeneous, monopolistic bargain power will harm the union with fixed wage.

**III. Profit sharing case** Both firms use profit sharing schemes. Unions are separate and dedicated to respective firms. Firms and unions bargain over the following Nash bargain product:

$$NBP_{DP}^i(w_i, w_j, a_i) = [(1 - a_i)\Pi_i(w_i, w_j)]^{1-\theta} \cdot [(w_i - w_0)q_i(w_i, w_j) + a_i\Pi_i(w_i, w_j)]^\theta$$

(16)

where superscript ”DP” stands for: D=decentralized bargains, and P=profit sharing remuneration schemes for both firms. Substituting $\Pi_i(w_i, w_j)$ and $q_i(w_i, w_j)$ from Equation 4, and maximizing $NBP_{DP}^i(w_i, w_j, a_i)$ over wage $w_i$, and over profit share percentage $a_i$, we get:
\[
\max_{w_i, a_i} [NB_i^{DP}] \Rightarrow w_i^{DP}(w_j) \quad \& \quad a_i^{DP}(w_i, w_j)
\]

Imposing symmetry in equilibrium: \(w_i^{DP}(w_j) = w_j^{DP}(w_i) = w^{DP}\) and \(a_i^{DP}(w_i, w_j) = a_j^{DP}(w_i, w_j) = a^{DP}\), we derive the equilibrium wage and profit share:

\[
w^{DP} = w_0 - \frac{\gamma^2 \tilde{a}}{4 + \gamma(2 - \gamma)} \quad \& \quad a^{DP} = \beta + \frac{1}{2}(1 - \beta)\gamma^2
\]

Notice that: \(w^{DP} < w_0\) which is in contrast with other versions of the game, while \(\frac{\partial w^{DP}}{\partial \gamma} < 0\) and \(\frac{\partial w^{DP}}{\partial \beta} = 0\). The last inequality is different from other versions of the game, showing that a variation in union’s bargain power will not affect wages. Nash equilibrium quantities, prices, and ("gross" and "net") profits are:

\[
q^{DP} = \frac{2\tilde{a}}{4 + \gamma(2 - \gamma)}, \quad \pi^{DP} = (1 - a^{DP})(q^{DP})^2, \quad P^{DP} = \alpha - \frac{2(1 + \gamma)\tilde{a}}{4 + \gamma(2 - \gamma)}
\]

The interesting conclusion when both firms using profit sharing, is that equilibrium wage, quantity, profits and price are independent of union’s bargain power \(\beta\). This has a straightforward economic intuition: under profit sharing, union and firm maximize joint profits, are to distribute them they use profit share percentage and not wage. Thus, bargain power affects only profit sharing and not any other variable of the game.

A decrease in product differentiation \((\gamma \to 1)\) means lower equilibrium wage, lower equilibrium quantity, profits and price, but higher profit share (in favor of the union).

Union welfare and consumer surplus are:

\[
UW_i^{DP} = UW_j^{DP} = \frac{2\beta(2 - \gamma^2)\tilde{a}^2}{(4 + (2 - \gamma)\gamma)^2}, \quad CS^{DP} = \frac{8\tilde{a}^2}{(4 + (2 - \gamma)\gamma)^2}
\]

Union welfare is decreasing as product differentiation decreases, and increasing as union’s bargain power increases. Interestingly, in this version of the game, consumer surplus is independent of union’s bargain power \(\beta\). Mathematically this happens because equilibrium quantity is independent of \(\beta\).

An economic intuition, though, says that through profit sharing union and firm maximize joint profits, and face consumers rather like a bilateral monopoly, so union’s bargain power has nothing to do with quantity produced or consumer surplus; is part of an internal procedure of splitting joint profits into two parts, weighted by each party’s bargain power.
4.2.2 Stage 3b: Central grand union

We will analyze the case of one grand union of workers $U$, a monopolist of labor to both firms $F_i$ and $F_j$. This single monopolist of labor creates considerable different incentives to firms. To illustrate the difference with decentralized bargains, consider the following example: if in decentralized bargains, a firm-union pair, say $U_i$ & $F_i$, fails to reach an agreement over wages, then they both get a zero profit. In different words, the disagreement payoff of both firms and unions was zero.

Things are different in central bargains. If a grand union of workers $U$ (a monopolist of labor supply to both firms) fail to reach an agreement with one firm, say firm $F_i$, then there is always the other firm $F_j$ to bargain with. This creates the so called ”outside option” (or disagreement payoff) in favor of the grand union. While firm $F_i$ still gets zero profits if an agreement is not set, a grand union has a minimum ”security net” of profits, which holds its profits way above zero.

In Horn and Wolinsky (1988), we encounter a report over different types of outside options. In their paper, they use a ”non-contingent” duopoly outside option: if the grand union $U$ fails to bargain with firm $F_i$ (so $F_i$ gets zero labor force and thus produce zero quantity and have zero profits), then the other firm $F_j$ will fail to notice, and pair $U$ & $F_j$ will bargain over duopoly quantity and prices.

We choose to use a ”non-contingent” monopoly outside option: if the grand union $U$ fails to bargain with firm $F_i$ (again zero quantity and profits for firm $F_i$), then firm $F_j$ will notice that, and thus pair $U$ & $F_j$ will bargain over monopoly quantity and prices.

We have chosen this type of outside option because it is more reasonable to suppose that the failure in negotiations between the pair $U$ & $F_i$ will be noticed by the other firm $F_j$, which will adjust its behavior from duopolist to monopolist, knowing that it is the last and only chance the grand union has to ”sell” its workforce.

Having these in mind, we model outside option as:

$$OutOpt^{CF}_i(w_j) = (w_j - w_0) \cdot q^{Mon}_j(w_j) = (w_j - w_0) \cdot \frac{1}{2} (\alpha - c - w_j)$$ (21)

for fixed wage remuneration schemes, and:
\(\text{OutOpt}^{CP}_i(w_j) = (w_j - w_0)q_j^{Mon}(w_j) + a_j\Pi_j^{Mon}(w_j) = (w_j - w_0)\left(\frac{1}{2}(\alpha-c-w_i) + a_j[\frac{1}{2}(\alpha-c-w_i)]^2\right)\) \hspace{1cm} (22)

for profit sharing schemes. Also, profits of grand union \(U\) are now profits made from supplying labor to both firms, instead of just one firm in decentralized version of the game, so:

\[\text{GUW}^{CF}(w_i, w_j) = (w_i - w_0)q_i(w_i, w_j) + (w_j - w_0)q_j(w_i, w_j)\]

(23)

for fixed wage remuneration schemes, and:

\[\text{GUW}^{CP}(w_i, w_j) = (w_i - w_0)q_i(w_i, w_j) + (w_j - w_0)q_j(w_i, w_j) + a_i\Pi_i(w_i, w_j) + a_j\Pi_j(w_i, w_j)\]

(24)

for profit sharing schemes.

**I. Fixed wage case** Both firms use fixed wage schemes. There is only one grand union for both firms. Firms and grand union bargain over the following Nash bargain product:

\[\text{NBP}^{CF}_i(w_i, w_j) = [\Pi_i(w_i, w_j)]^{1-\beta} \cdot [\text{GUW}^{CF}(w_i, w_j) - \text{OutOpt}^{CF}_i(w_j)]^\beta\]

(25)

where the superscript "CF" denotes: C=central coordinated bargains, and F=fixed wage. We will substitute \(q_i(w_i, w_j)\) and \(\Pi_i(w_i, w_j)\) from Equation 4, and we will maximize \(\text{NBP}^{CF}_i(w_i, w_j)\) over wage \(w_i\). We will do the same for pair \(U & F_j\), and we will impose symmetry in equilibrium \(w^{CF}_i(w_j) = w^{CF}_j(w_i) = w^{CF}\). After all these steps, we will get equilibrium quantities, profits, and wages:

\[w^{CF} = w_0 + \frac{1}{2}\beta\tilde{a}, \quad q^{CF} = \frac{(2 - \beta)\tilde{a}}{2(\gamma + 2)}, \quad \Pi^{CF} = (q^{CF})^2, \quad \Pi^{CF} = \alpha - \frac{(2 - \beta)(\gamma + 1)\tilde{a}}{2(\gamma + 2)}\]

(26)

Wage \(w^{CF}\) is higher than unemployment benefit \(w^{CF} > w_0\), and becomes higher as union’s bargain power rises \(\frac{\partial w^{CF}}{\partial \beta} > 0\). Moreover, quantity falls as union’s bargain power rises \(\frac{\partial q^{CF}}{\partial \beta} < 0\).
Grand Union’s welfare and consumer surplus are:

\[ GUW^C = \frac{(2 - \beta)\beta a^2}{2(\gamma + 2)} \quad CS^C = \frac{(2 - \beta)^2\tilde{a}^2}{2(\gamma + 2)^2} \] (27)

Grand union’s welfare decreases as product differentiation decrease, and increase with grand union’s bargain power. On the other hand, consumer surplus decreases with both product differentiation decrease, and with grand union’s bargain power increase.

**II. Mix case** In this mix case, one firm, say firm \( F_i \), use a profit sharing scheme \( (w_i, a_i) \), while the rival firm \( F_j \) use a fixed wage scheme \( (w_j) \). Both firms face the same grand union of workers \( U \), which has an outside option of not giving labor force to one firm, making the other firm a monopolist in product market. Nash bargain products of these pairs \( (U, F_i) \) & \( (U, F_j) \) are:

\[
NBP^{CM}_i(w_i, w_j, a_i) = [(1 - a_i)\Pi_i(w_i, w_j)]^{1-\beta} \cdot [(w_i - w_0)q_i(w_i, w_j) + (w_j - w_0)q_j(w_i, w_j) + a_i\Pi_i(w_i, w_j) - (w_j - w_0)q_j^{Mon}(w_j)]^\beta
\]

\[
NBP^{CM}_j(w_i, w_j, a_i) = [\Pi_j(w_i, w_j)]^{1-\beta} \cdot [(w_i - w_0)q_i(w_i, w_j) + (w_j - w_0)q_j(w_i, w_j) + a_i\Pi_i(w_i, w_j) - (w_j - w_0)q_j^{Mon}(w_i) - a_i\Pi_j^{Mon}(w_i)]^\beta
\] (28a)

where the superscript ‘CM’ denotes: C=central coordinated bargains, and M=mixed case. Substituting \( q_i(w_i, w_j) \) and \( \Pi_i(w_i, w_j) \) from Equation 4, and solving the bargain problem for pair \( U & F_i \) we get:

\[
\max_{w_i, a_i}[NBP^{CM}_i(w_i, w_j, a_i)] \quad \Rightarrow \frac{\partial NBP^{CM}_i(w_i, w_j, a_i)}{\partial w_i} = 0 \quad \frac{\partial NBP^{CM}_i(w_i, w_j, a_i)}{\partial a_i} = 0
\] (29)

From these two first order conditions we will get two expressions: one for wages \( w^{CM}_i(w_j, a_i) \), and one for profit share \( a^{CM}_i(w_i, w_j) \). We have proven before that the existence of profit share percentage \( 0 < a_i < 1 \) makes the remuneration contract *bilaterally efficient*, that is it maximizes (excess) joint profits of pair \( U & F_i \). Solving the bargain problem of the other pair \( U & F_j \) we get:
\[
\max_{w_j}[NBP^CM_j(w_i,w_j,a_i)] \Rightarrow \frac{\partial NBP^CM_j(w_i,w_j,a_i)}{\partial w_j} = 0 \tag{30}
\]

This first order condition will give us an expression of wage \(w^CM_j(w_i,a_i)\). Solving the system of three equations we derive equilibrium expressions of wages, profits, profit shares, quantities, and prices. We have closed form solutions, which will not state, in order to save some space. What we could state are the following: \(w^CM_j > w_0, \frac{\partial w^CM_j}{\partial \gamma} > 0\), and \(\frac{\partial q^CM_i}{\partial \beta} = 0\).

Grand union’s welfare is a very big expression to state here (expressions available upon request), while consumer surplus is:

\[
CS^CM = \frac{1}{8}(2 - \gamma)^2a_0^2\left(\frac{2(\beta - 2)(\beta(\gamma^2 - 2)\gamma - (\gamma + 2)\gamma^2 + 4)}{(\gamma^2 - 2)((\beta - 1)^2\gamma^4 - 2((\beta - 1)\beta + 2)\gamma^2 + 16)} + \frac{1}{2 - \gamma^2}\right)^2 \tag{31}
\]

Grand union’s welfare is always increasing with grand union’s bargain power \(\beta\), while \(\frac{\partial GUW^CM}{\partial \gamma}\) has no stable sign. On the other hand, consumer surplus is decreasing with both \(\beta\) and \(\gamma\).

**III. Profit sharing case**

Both firms \(F_i\) and \(F_j\) have decided to give profit sharing schemes, while unions have merged into one grand union \(U\). Pair \(U\) & \(F_i\) bargains over the following Nash bargain product:

\[
NBP^CP_i(w_i,w_j,a_i,a_j) = [(1 - a_i)\Pi_i(w_i,w_j)]^{1-\beta} \cdot [(w_i - w_0)q_i(w_i,w_j) + (w_j - w_0)q_j(w_i,w_j) + a_i\Pi_i(w_i,w_j) + a_j\Pi_j(w_i,w_j) - (w_j - w_0)q_j^{Mon}(w_j) - a_j\Pi_j^{Mon}(w_j)]^{\beta} \tag{32}
\]

where superscript ‘CP’ denotes: C=central coordinated bargains, and P=profit sharing scheme for both firms. Maximization of Nash bargain product \(NBP^CP_i(w_i,w_j,a_i,a_j)\) over profit sharing percentage \(a_i\) leads to excess joint profits maximization, where excess joint profits are:

\[
EJP^CP_i(w_i,w_j,a_j) = (w_i - w_0)q_i(w_i,w_j) + (w_j - w_0)q_j(w_i,w_j) + \Pi_i(w_i,w_j) + a_j\Pi_j(w_i,w_j) - (w_j - w_0)q_j^{Mon}(w_j) - a_j\Pi_j^{Mon}(w_j) \tag{33}
\]
So:

$$\max_{w_i}[NBP^C_P(w_i, w_j, a_i, a_j)] \equiv \max_{w_i}[EJP^C_P(w_i, w_j, a_j)] \Rightarrow w^C_P(w_j, a_j) \quad (34)$$

And:

$$\max_{a_i}[NBP^C_P(w_i, w_j, a_i, a_j)] \Rightarrow a^C_P(w_i, w_j, a_j) \quad (35)$$

Imposing symmetry in equilibrium: $w^C_P(w_j, a_j) = w^C_P(w_i, a_i) = w^C_P(w, a)$, and $a^C_P(w_i, w_j, a_j) = a^C_P(w_i, w_j, a_i) = a^C_P(w, w, a)$, and solving the system of equation we get equilibrium expressions:

$$w^C_P = w_0 + \frac{\gamma(\beta(\gamma + 4) - 8)}{2\gamma((\gamma + 2)((\beta - 1)\gamma^2 - 2\beta) + 6\gamma) - 16} + \frac{\beta(\gamma + 4)(\gamma + 4) - \gamma(\gamma + 4) - 4)\tilde{a}}{2\gamma((\gamma + 2)((\beta - 1)\gamma^2 - 2\beta) + 6\gamma) - 16} \quad (36a)$$

$$a^C_P = \frac{4(\beta + 2)}{(\beta - 1)\gamma^2 + 4} - 2 \quad (36b)$$

$$q^C_P = \frac{(\gamma - 2)((\beta - 1)\gamma^2 + 4)\tilde{a}}{2\gamma((\gamma + 2)((\beta - 1)\gamma^2 - 2\beta) + 6\gamma) - 16} \quad (36c)$$

$$\pi^C_P = (1 - a^C_P) \cdot \Pi^C_P = (1 - a^C_P) \cdot (q^C_P)^2 \quad (36d)$$

$$P^C_P = \alpha - \frac{(\gamma - 2)(\gamma + 1)((\beta - 1)\gamma^2 + 4)\tilde{a}}{2\gamma((\gamma + 2)((\beta - 1)\gamma^2 - 2\beta) + 6\gamma) - 16} \quad (36e)$$

Grand Union’s welfare and consumer surplus are:

$$GUW^C_P = \frac{(\gamma - 2)((\beta - 1)\gamma^2 + 4)(\beta(\gamma(\gamma + 4) - 8) - \gamma(\gamma + 4) - 4) - 8) - \gamma^3(\gamma + 2))\tilde{a}^2}{2(\gamma((\gamma + 2)((\beta - 1)\gamma^2 - 2\beta) + 6\gamma) - 8)^2} \quad (37a)$$

$$CS^C_P = \frac{2(\gamma - 2)^2((\beta - 1)\gamma^2 + 4)^2\tilde{a}^2}{2(\gamma((\gamma + 2)((\beta - 1)\gamma^2 - 2\beta) + 6\gamma) - 8)^2} \quad (37b)$$

Grand union welfare is increasing with $\beta$, while $\frac{\partial GUW^C_P}{\partial \gamma}$ has no stable sign. Consumer surplus is decreasing with both $\gamma$ and $\beta$.

**4.3 Stage 2: Remuneration stage**

In this stage, firms separately and simultaneously decide over the remuneration scheme. They have two options: a) to give a fixed wage, or b) to give a profit sharing scheme.
This creates a world with four possible outcomes. Due to imposed symmetry, these four outcomes collapse into three: 1) both firms give fixed wage schemes, 2) both firms give profit sharing schemes, and 3) one firm gives fixed wage while the other gives profit sharing scheme.

The decision over which remuneration scheme to provide is clearly a part of firm’s profit maximization procedure. It is clear by now that this decision is highly dependent on values of product differentiation $\gamma$, and union’s bargain power $\beta$, and it is a different decision when unions are separate than when they are merged. To sum up, we state the following proposition.

![Graphs](a) $(\beta, \gamma)$ plane in decentralized version  
(b) $(\beta, \gamma)$ plane in central version

**Figure 1**

**Proposition 4.1.**  
- When unions are separate (decentralized version), we have two equilibria:
  - Both firms give Fixed wage, when $\beta, \gamma$ are in Area I of Figure 1a,
  - Both firms give Profit Sharing, when $\beta, \gamma$ are in Area II of Figure 1a.
  - Area III of Figure 1a, is a double equilibrium area. Pareto-ranked equilibrium: both firms to give fixed wage.

  In decentralized version, mix case is never an equilibrium.

- When unions merge (central coordinated version), we have three equilibria:
  - Both firms give Fixed wage, when $\beta, \gamma$ are in Area I of Figure 1b,
  - Both firms give Profit Sharing, when $\beta, \gamma$ are in Area II of Figure 1b,
Mix case is an equilibrium, when $\beta, \gamma$ are in Area III of Figure 1b.

Area III of Figure 1b, is a double equilibrium area. Pareto-ranked equilibrium: both firms to give fixed wage.

Proof. Using the expressions we have derived for firm’s profits under fixed wage, and under mix case, it is straightforward to verify that (deviations reasoning):

A) For decentralized unions: (i) $\pi_{DF} > \pi_{DM}^i$ for $\beta, \gamma$ in Area I and III of Figure 1a. (ii) $\pi_{DP} > \pi_{DM}^j$ for $\beta, \gamma$ in Area II and III of Figure 1a. (iii) Following system of inequalities never hold: $\pi_{DF} < \pi_{DM}^i \& \pi_{DP} < \pi_{DM}^j$, so mix case can never be an equilibrium in decentralized version of the game. (iv) In Area III of Figure 1a, where both equilibria exists, we can Pareto-rank them because for $\beta, \gamma$ in this are the following hold: $\pi_{DF} > \pi_{DP}$. So if a firm can choose between two possible equilibria, it will choose the one with the highest profits.

B) For a central union: (i) $\pi_{CF} > \pi_{CM}^i$ for $\beta, \gamma$ in Area I and III of Figure 1b. (ii) $\pi_{CP} > \pi_{CM}^j$ for $\beta, \gamma$ in Area II and III of Figure 1b. (iii) System of inequalities: $\pi_{CF} < \pi_{CM}^i \& \pi_{CP} < \pi_{CM}^j$ hold for $\beta, \gamma$ in Area III of Figure 1b, so mix case can be an equilibrium in central coordinated version of the game. (iv) In Area III of Figure 1b, where both equilibria exists, we can Pareto-rank them because for $\beta, \gamma$ in this are the following hold: $\pi_{CF} > \pi_{CP}$. So if a firm can choose between two possible equilibria, it will choose the one with the highest profits.

4.4 Stage 1: Merger stage

In this stage, unions decide whether to bargain separately, or to merge and form a grand union. The following proposition clarifies this decision.

Proposition 4.2. Unions always have incentives to merge. Central coordinated version is the only equilibrium version for any $\beta \in (0, 1)$ and $\gamma \in (0, 1)$.

Proof. A) When both firms give fixed wage, the following hold under any value of $\beta \in (0, 1)$ and $\gamma \in (0, 1)$: $UW_{DF}^i + UW_{DF}^j < GUW_{CF}$.

B) When both firms give fixed wage, the following hold under any value of $\beta \in (0, 1)$ and $\gamma \in (0, 1)$: $UW_{DP}^i + UW_{DP}^j < GUW_{CP}$.

C) When one firm give fixed wage and the other firm give profit sharing scheme, the following hold under any value of $\beta \in (0, 1)$ and $\gamma \in (0, 1)$: $UW_{DM}^i + UW_{DM}^j < GUW_{CM}$. 

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D) There are spots in $\beta, \gamma$-plane, where firms might choose a different remuneration scheme under decentralized version, or central coordinated version. It is straightforward to prove that these "cross" inequalities hold for any $\beta \in (0, 1)$ and $\gamma \in (0, 1)$: $UW_i^{DF} + UW_j^{DF} < GUW^{CP}$, $UW_i^{DP} + UW_j^{DP} < GUW^{CMP}$, and $UW_i^{DF} + UW_j^{DF} < GUW^{CM}$. □

5 Conclusions

In Introduction and in Literature Review we have seen empirical evidence and stylized facts that support profit sharing as a wide spread remuneration scheme. We have seen that many researchers agree that profit sharing can increase firm’s profits and can increase employment, by altering fixed labor costs into variable costs per quantity produced. Furthermore, we have encounter OECD contemporary statistics that show that unionization still exists, but with great variation from country to country.

In the main body of our paper we have endogenize two decisions: 1) unions decision to merge or not, and 2) firms decision for giving profit sharing or fixed wage. This is something unique in existing literature. We went even further, using a bargain framework, and product differentiation.

With a deviations reasoning, and with Pareto ranking we have proven that unions will always want to merge and form a grand union. Under a grand union, and for different values of $\beta \in (0, 1)$ and $\gamma \in (0, 1)$, all three cases can be equilibria: both firms might choose to give fixed wage, both firms might choose profit sharing, and for a rather small area of $(\beta, \gamma)$-plane one firm could choose fixed wage while the other could choose profit sharing scheme. Firms net profits are always higher when use fixed wage, but a prisoner’s dilemma situation lead a firm to follow the rival and introduce profit sharing scheme (except for the small area of $(\beta, \gamma)$-plane mentioned just before).
6 References


