INTERNATIONAL ENVIRONMENTAL AGREEMENTS: HOW OPTIMAL IS THE OPTIMAL SHARING SCHEME?

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Abstract

In this paper we examine the conditions under which the countries are willing to form coalitions or sign agreements, such as international environmental agreements, in order to deal effectively with environmental issues. We assume that the resignation of a single country or of a group of countries from a coalition does not lead the cooperation to a complete breakdown. Furthermore, our analysis allows the existence of multiple noncrossing coalitions. We compare the optimal sharing scheme with one that distributes the benefits from the cooperation to the members of each coalition according to the Nash bargaining solution. We find that in the case when the payoffs of the Nash bargaining solution are sufficient for the full cooperation to prevail, the optimal sharing scheme might not be able to support the formation of the grand coalition. We further show that all the countries form a single coalition if and only if the benefits from the cooperation are high enough.

JEL Classification: C71, C72, D62, Q50

Keywords: multilateral externalities, multiple coalitions, δ—core.

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1 Introduction

In the last few decades we have witnessed countries forming coalitions and signing agreements to deal with environmental externalities that arise from the economic activity. These problems include among others the climate change and the conservation of the biological diversity and require some form of international coordination, as they cannot be tackled effectively when the countries act separately. The coordination between countries usually takes the form of international environmental agreements (IEAs). Since the international law does not recognize any legal entities with the power to enforce international agreements on sovereign countries (Endres (1997)), such agreements must provide enough incentives for the countries to join voluntarily (Barrett (1994)). The Kyoto Protocol and the Convention on Biological Diversity are examples of agreements of this type. In this paper we examine the conditions under which the countries are willing to sign such agreements and their implications on welfare.

By signing the Montreal Protocol on Substances that Deplete the Ozone Layer, its parties intended to protect the ozone layer by phasing out and ultimately eliminating the emissions of substances that deplete it. They also acknowledged that some countries might require a special provision of financial resources in order to develop the capacity to comply with the control measures. For this reason the signatories established a financial mechanism, funded by the developed or industrialized country members of the treaty, that directs aid flows to the developing country members. Further, the disbursement of resources is determined unanimously by a committee with equal representation of the contributing and the benefitting parties. This mechanism has transferred more than US$3 billion to assist the developing countries in ozone depleting substances mitigation (Molina et al. (2009)). Among others, Benedick (1998) argues that the increased participation in the Montreal Protocol was due to its transfer scheme.

China’s project to eliminate the production of the ozone depleting substances by the year 2030 is potentially the largest project that funds the financial mechanism of the Montreal Protocol. This project generates positive externalities as it is expected to prevent the emission of more than 4.3 million metric tons of HCFCs, or 300,000 tons in terms of its ozone depletion potential.\(^1\) Nonetheless, the environmental externalities might be negative. The conversion of tropical forests

\(^1\)See Multilateral Fund for the Implementation of the Montreal Protocol (2013, April 22).
to commercial land in Indonesia and Brazil,\textsuperscript{2} generates negative externalities as it increases the concentration of carbon dioxide in the atmosphere.\textsuperscript{3}

Further, a project may have positive externalities for some countries and negative externalities for some other countries. The Inga I and II hydropower dams in the Basin of the Congo River in the Democratic Republic of the Congo have positive externalities for most countries because they generate electricity without air quality impact. Yet, the dams have a detrimental effect on Angola, as they pose a threat to the freshwater biodiversity in the lower Congo River (Norlander (2009, April 20)). Similarly, the recent enlargement of the Suez Canal has beneficial effects for most countries. This particular project allows the ships from Asia to bypass the route around the Cape of Good Hope, significantly shorten their travel distance and consequently reduce the air pollution that they cause. However, marine experts are concerned with the migration of species from the Red Sea to the Mediterranean and warn about the ecological consequences of the project on the countries with coastlines on the Mediterranean Sea.\textsuperscript{4}

Here we examine how countries may form coalitions, such as IEAs, to solve such problems. We assume that each country has a project with environmental impact that generates both a local benefit for that country and also (positive or negative) externalities for the other countries. A country that has not signed such agreements decides to undertake its project by comparing the project’s local benefit with its cost. Although the externalities of the project occur and affect the other countries, they are not taken into consideration. However, the countries that form coalitions take into consideration the intra-coalition externalities of their projects as well and, as in the Montreal Protocol, they decide unanimously on the projects that they will fund.\textsuperscript{5} The undertaken projects may be different in the two cases.

Intuitively, the degree of international cooperation depends crucially on the distribution of the gains from that cooperation between the countries. We consider two benefit sharing rules: the Nash bargaining solution originally introduced by Nash (1950) and the optimal sharing scheme suggested by Carraro et al. (2006), Eyckmans and Finus (2009), Fuentes-Albero and Rubio (2010),

\textsuperscript{2} The cause of tropical deforestation is to create farmland for palm tree plantation in Indonesia and for soybean, sugar cane, maize production and pasture in Brazil.

\textsuperscript{3} Each acre of tropical forest stores about 180 metric tons of carbon. Worldwide, the tropical forests hold 460-575 billion metric tons of carbon. See Urquhart et al. (1999).

\textsuperscript{4} See Galil et al. (2015) and “Monitor: No way for fish” (2015, May 28).

\textsuperscript{5} In our model unanimity is a result rather than an assumption. See the section 4.4.
McGinty (2007), Nagashima et al. (2009) and Weikard (2009). Empirical evidence supports that very often individuals propose and accept an equal split of the gains (Ostrom (1998)) and for this reason we assume that each coalition chooses tax rates so as to divide those gains equally among its members. However, as the benefits from the cooperation are not defined in the same manner for the two sharing schemes, the payoffs that the countries obtain may not coincide. In addition, we find that under both of the benefit sharing rules some countries might be subsidized when they participate in a coalition. In the example mentioned above, China will receive up to US$385 million of funding until 2030.

Moreover, as international agreements entail significant benefits to the countries involved, it is common for the resignation of a single country or even of a group of countries from such an agreement not to necessarily lead the cooperation to a complete breakdown. For instance, the recent Brexit referendum result to leave the EU and the subsequent initiation of the withdraw process does not appear to lead to the dissolution of the EU. Likewise, the UN did not dissolve following the withdrawal of Indonesia, neither did the Andean Community after the withdrawal of Venezuela and Chile. As far as IEAs are concerned, neither the failure of the US to ratify the Kyoto Protocol nor the withdrawal of Canada from it led the remaining parties to renounce the agreement, even though the US accounted for 36% of the controlled emissions (Dessai 2001). Similarly, when the United Arab Emirates denounced CITES the agreement did not shut down. For this reason, in our analysis we apply the notion of the core under the δ-stability concept introduced by Hart and Kurz (1983) and assume that whenever one or more members leave a coalition, the remaining members stick together. Following Zhao (2016), we refer to this version of the core as the δ-core.

In the spirit of Eyckmans and Finus (2009), we obtain general theoretical results and provide some numerical examples in order to facilitate their interpretation. Specifically, we find that all the countries will participate in a single coalition if and only if the benefits from the full cooperation are sufficiently high. No country or group of countries has incentives to deviate from this coalition and free ride, as in such case a number of positive externalities may not be realized. Although this result is also present in McGinty (2007), here it is verified in a setting with $n$ countries and arbitrary

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6 This assumption has been used in other settings as well. See for example Burbidge et al. (1997).

7 See “Come Together: An EU Summit Shows Unity in the Face of Brexit” (2017, April 30).

8 For a review on the use of the core and its refinements on games with coalition formation when externalities are present see Marini (2009) or Zhao (2016).
levels of local benefits, costs and externalities.\textsuperscript{9} Further, we examine the welfare implications of these agreements and show that it is possible to have welfare losses when the grand coalition fails to emerge.

As far as the sharing schemes are concerned, we show that the adoption of the optimal sharing scheme does not guarantee the formation of the grand coalition. Even though Nagashima et al. (2009) come to the same conclusion, in this setup we further find that in such case the payoffs of the Nash bargaining solution sharing scheme might be sufficient for the full cooperation to prevail. However, this is in contrast to the predictions of Carraro et al. (2006). They show that the optimal transfer scheme leads to better results than, among other schemes, the Nash bargaining solution.

\section{Related literature}

The models that study the formation of IEAs incorporate tools either from the non-cooperative or the cooperative game-theoretic literature, although some papers combine elements from both approaches.\textsuperscript{10} The equilibrium concept that is mainly applied in the non-cooperative models is the notion of internal and external stability, originally introduced by d’Aspremont et al. (1983) in the study of stable cartels in a price leadership model. The main body of this strand of the literature predicts that when the difference between the benefits under full cooperation and no cooperation is large, the number of signatories of a self-enforcing IEA is small as the countries have high incentives to free ride. This result is verified in a variety of specifications: In static models with symmetric countries (Barrett (1994), Carraro and Siniscalco (1993), Diamantoudi and Sartzetakis (2006), Hoel (1992), Rubio and Ulph (2006)), asymmetric countries (Barrett (1997), Botteon and Carraro (1998)) or even when IEAs are examined as repeated games (Barrett (1994), Finus and Rundshagen (1998), Rubio and Ulph (2007)). However, some authors exploit the diversity that arises when the countries are asymmetric and examine the conditions under which self-financing transfer schemes may increase the number of signatories or even induce the full cooperation of the countries (Barrett (2001), Carraro et al. (2006), Colmer (2011), Eyckmans and Finus (2009), McGinty (2007), Nagashima et al. (2009), Petrakis and Xepapadeas (1996)).

\textsuperscript{9}Rather than making particular assumptions on the form or the curvature of the benefit and cost functions, we consider a more general setup where local benefits, costs and externalities are completely arbitrary.

\textsuperscript{10}See Finus (2001 and 2008) for an extensive review.
On the other hand, the equilibrium concept that is mainly applied in the cooperative models is the concept of the core. Both in static models (Chander and Tulkens (1995 and 1997), Mäler (1989)) and in dynamic models, this part of the literature concludes that all the countries will participate in a single coalition so long as a suitable transfer scheme is established. The core concept was initially used by Mäler (1989). The author applies the notion of the $\alpha$-core, originally proposed by Aumann (1967), and finds that (a special case of) this concept of the core results in a set of outcomes that is too large. Chander and Tulkens (1995 and 1997) claim that there is no reason why the complementary coalition should behave in the war-like fashion of the $\alpha$-core and use the $\gamma$-core instead. They assume that the deviating countries expect the non-deviating members to break up into singletons and that every singleton adopts the Nash strategy. Chander and Tulkens (1995 and 1997) find that for a transfer scheme similar to the ratio equilibrium met in Kaneko (1977) and Mas-Colell and Silvester (1989), the full cooperation can prevail and Pareto efficiency can be achieved. In their model, the threat that sustains the grand coalition is the “Nash behavior” of the non-deviating countries.

Nevertheless, we have witnessed that the withdrawal of one or more countries from such agreements does not bring the international cooperation to an end, as the $\gamma$-game implies. Thus, the use of the $\delta$-core is more appropriate. We find that in comparison to the $\gamma$-core, the conditions for the $\delta$-core are more restrictive. However, this does not indicate that when an economy has the payoffs of the grand coalition in the $\delta$-core they also are in the $\gamma$-core. These results are consistent with both of the sharing schemes that we consider. To the best of our knowledge, the application of the $\delta$-core has yet to be explored in the context of IEAs.

Further, in the papers that apply the core concept all countries participate in a single coalition. However, a number of IEAs deal with regional, rather than global, environmental issues that may not concern all the countries. For instance, the UNEP Regional Seas Programmes address the

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12 Aumann (1967) presumes that when a coalition deviates, the rest of the countries choose strategies that cause the highest damage possible to the deviating coalition. Mäler (1989) assumes that the deviating coalitions can be hurt by up to infinite amounts of pollutants emitted by the countries outside the coalition (hence, individual pollution levels are unbounded from above).

13 For an alternative justification of multiple coalitions, see Eyckmans and Finus (2006).
protection of marine and coastal environment at a regional level: only the countries in the Wider Caribbean Region have ratified the Cartagena Convention, while only those with coastlines in the Mediterranean Sea have signed the Barcelona Convention etc. The fact that the $\delta$ – core can accommodate for the coexistence of such multiple non-crossing coalitions is another reason why we apply this particular notion of the core.$^{14}$

The rest of the paper is organised as follows: In Section 3 we describe the model. We focus on the $\gamma$ and $\delta$ – stable coalition structures in Section 4. In Section 5 we consider the welfare implications. Finally, we conclude in Section 6.

3 Preliminaries

We consider a set $N$ of $1, 2, \ldots, n$ countries, each populated with citizens of mass 1. Each $i \in N$ has a discrete project that we call it project $i$, with fixed cost $c_i \in \mathbb{R}^+$. As we want to be as general as possible, we assume that if the project $i$ is funded it generates a local benefit for country $i$ and externalities of any magnitude for the other countries. We denote $b_i \in \mathbb{R}$ the local benefit of the project $i$ and $e_{ji} \in \mathbb{R}$ the externality of the project $i$ to country $j$, $\forall j \in N$, with $e_{ii} = 0$.

Information about local benefits, costs and externalities is common knowledge.

We first examine country $i$ when it is not associated with other countries. We assume that it acts rationally. Country $i$ decides whether it should fund its project or not, by comparing the

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project’s local benefit versus its cost. If it finds that it is profitable to fund it, it obtains the local benefit of project \( i \) and covers the cost by using its own resources. Also, country \( i \) is fully exposed to the (positive or negative) externalities that reach \( i \) and are generated by projects from the other countries or other coalitions of countries. As \( i \) does not contribute to their funding, it free rides on these externalities.

On the other hand, when the countries form coalitions, they coordinate on the projects they fund. To do that, they also take into consideration the intra-coalition externalities that these projects generate, i.e. the externalities that reach the members of the coalition from the projects the coalition funds. In such case, a project is funded if its total benefit, that includes its local benefit and the intra-coalition externalities, exceeds its cost. The set of the projects that are funded in the two cases may not be the same since a coalition might not fund a project that an unassociated country would fund and vice versa. In both cases, the countries fully free ride on the externalities that are generated from the projects that are funded by the other countries or coalitions. Further, by choosing to participate in a coalition, the countries agree to share the total cost by using a predetermined rule that we specify below.\(^{15}\) Choices about coalition partners are taken simultaneously by all countries and each country can participate in only one coalition.

Formally, a coalition \( C \) is a nonempty subset of \( N \), with \( |C| \) the number of its members. Every single membered coalition is called a singleton. As it is possible to have many coalitions, we define a coalition structure \( \pi \) to be a partition on \( N \) with elements the various coalitions that may be formed,

\[
\pi = \{C_1, C_2, \ldots, C_h\},
\]

where \( h \) is the number of coalitions in \( \pi \). We restrict our attention to coalition structures that are noncrossing and cover \( N \) fully. We denote by \( \hat{\pi} \) the coalition structure in which the countries are not associated,

\[
\hat{\pi} = \{\{1\}, \{2\}, \ldots, \{n\}\}.
\]

The coalition structure in which all the countries participate in the same coalition, i.e. the grand coalition, is denoted by

\[
G = \{1, 2, \ldots, n\}.
\]

\(^{15}\) We assume that all other costs, e.g. coalition formation costs or costs of entry, are zero.
As a coalition $C$ chooses which projects to fund in order to maximize the utility of its members, it may not undertake all the available projects. For this reason, we denote $F(\pi, C) \subseteq C$ the set of projects that are funded by $C$. Also, $F(\pi, N \setminus C) \subseteq N \setminus C$ is the set of projects that are funded by coalitions other than $C$, and $F(\pi)$ is the set of funded projects by every coalition in $\pi$, with $F(\pi) = F(\pi, C) \cup F(\pi, N \setminus C)$. $F(G)$ is the set of the funded projects of the grand coalition.

### 3.1 Utility function

Under the coalition structure $\pi$, a country $i$ which participates in the coalition $C$, obtains utility level equal to $u_i(\pi, C, t_i)$ as presented below. In the first case, the project $i$ is funded by $C$ while in the second it is not:

$$
\begin{align*}
    u_i(\pi, C, t_i) &= \begin{cases} 
    b_i + \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, N \setminus C)} e_{ij} - t_i \times A & \text{if } i \in F(\pi, C), \\
    \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, N \setminus C)} e_{ij} & \text{if } i \notin F(\pi, C),
    \end{cases}
\end{align*}
$$

where $b_i$ is the local benefit of project $i$ and occurs only if the project is funded. The country $i$ is fully exposed to both the externalities $\sum_{j \in F(\pi, C)} e_{ij}$ that are generated by the projects the coalition $C$ funds and the externalities $\sum_{j \in F(\pi, N \setminus C)} e_{ij}$ that are generated by the projects that the other countries or coalitions in $\pi$ fund. We denote $A$ the cost of the projects that are funded by the coalition $C$, i.e. $A = \sum_{j \in F(\pi, C)} c_j$, and $t_i \in \mathbb{R}$ the tax rate the coalition $C$ charges to country $i$. In case $t_i \geq 0$, the term $t_i \times A$ is the contribution that the country $i$ makes to its coalition. But if $t_i < 0$, then $t_i \times A$ corresponds to the subsidy that $i$ receives from $C$. Since all the tax revenues of $C$ are directed for project funding, $A$ is fully covered as long as $\sum_{i \in C} t_i = 1$.

In the special case where $C$ is a singleton, $\sum_{j \in F(\pi, C)} e_{ij} = 0$ because $i$ has no co-members. If in such case the project $i$ is funded, the country $i$ covers the cost $A = c_i$ on its own and $t_i = 1$. But if the country $i$ chooses not to fund the project $t_i = 0$. For notational simplicity, we write $u_i(\pi, \{i\})$ instead of $u_i(\pi, \{i\}, t_i)$ whenever $i$ is a singleton.

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16In fact, $G$ is both a coalition and coalition structure.
4 The coalition formation game

4.1 Timing of the game

The game proceeds in the following manner:

\[
\begin{array}{c|c|c}
\text{Stage 1} & \text{Stage 2} & u_i(\pi, C, t_i) \\
\hline
\pi & F(\pi) & t_i \\
\end{array}
\]

In the first stage, the countries choose simultaneously whether to join a coalition or not. As a result we have the formation of the coalition structure \( \pi \). In the successive stage, each coalition decides on which of the projects that are available it should fund, in order to maximize the utility of its members. Each country contributes or receives \( t_i \times A \) from the coalition it belongs. The tax rate \( t_i \) is based on scheme that determines the division of the gains from cooperation. We consider two such sharing rules: Nash bargaining solution and the optimal sharing scheme. Each scheme divides the gains from the cooperation equally among the members of the coalition. However, those gains may not coincide for the two schemes. The projects chosen in the second stage are then funded and utilities are realized. In what follows, we describe in detail the game mentioned above.

4.2 Sharing schemes

We start our analysis of the game backwards and present the rules under which the coalitions determine the tax rates for their members. We assume that the coalitions have already been formed and each coalition has decided on which projects to undertake. Hence, the coalition structure \( \pi \) and the corresponding set of funded projects \( F(\pi) \) are known.

Each coalition chooses the tax rates so as to divide its surplus evenly among its members. To this extent, what matters in every coalition is the surplus which is generated by the coalition and not the possible externalities that each member receives. However, the surplus is not defined in the same manner for both of the schemes. Under the Nash bargaining solution as surplus we consider the payoff that the members accomplish by forming the coalition minus the payoff that they receive when they act alone, taking as given the rest of the coalition structure. But under the optimal sharing scheme, the surplus is the payoff that the members accomplish by forming the coalition
minus the sum of the payoffs that each member could have obtained by deviating from the coalition and becoming a singleton, namely its free rider payoff or outside option payoff, while the rest of the coalition structure remains unchanged.

To formalize our analysis, let a coalition $C \in \pi$ with $C = \{i_1, \ldots, i_t\}$. We denote the worth of coalition $C$ as

$$w(\pi, C) = \sum_{j \in F(\pi, C)} (b_j + \sum_{i \in C} e_{ij} - c_j) + \sum_{j \in F(\pi, N \setminus C)} \sum_{i \in C} e_{ij}.$$  

(2)

$w(\pi, C)$ can be seen as the aggregate amount of the payoff that the members of $C$ receive and consists of two terms.

The first term includes the local benefits, costs and externalities that are generated by the projects that are funded by the coalition $C$. Notice that only the externalities to the members of $C$ enter the coalition’s worth, as the term $\sum_{j \in F(\pi, C)} \sum_{i \in C} e_{ij}$ suggests, and not the externalities towards the members of the other coalitions.

The second term includes the externalities from the projects that are funded by the other coalitions and affect the countries in $C$. As each country is allowed to belong to one coalition only and the members of $C$ do not contribute to the funding of the projects of the other countries, this term corresponds to the externalities $C$’s members free ride on.

4.2.1 The Nash bargaining solution sharing scheme

We denote $\hat{\pi}_C$ the coalition structure in which all the members of $C$ are singletons while the other countries are still organised in coalitions according to $\pi$,

$$\hat{\pi}_C = \{C_1, \ldots, \{i_1\}, \ldots, \{i_t\}, \ldots, C_h\}.$$

Stated differently, $\hat{\pi}_C$ is a coalition structure in which all the coalitions of $\pi$ are formed but $i_1, \ldots, i_t$ fail to agree on the formation of $C$ (or any other subcoalition).\textsuperscript{17} We denote $F(\hat{\pi}_C, C) \subseteq C$ the set of projects that are funded by the countries $i_1, \ldots, i_t$ when they are not associated and $F(\hat{\pi}_C, N \setminus C)$ the set of projects that the other coalitions fund.

\textsuperscript{17}The coalition $C$ does not actually dismantle, but we suppose that its members disagree to facilitate our analysis.
We let $d_i(\pi_C, C)$ be the utility level of $i \in C$ under the coalition structure $\pi_C$. In the first case, the project $i$ is funded by the country $i$ while in the second it is not,

$$d_i(\pi_C, C) = \begin{cases} 
    b_i + \sum_{j \in F(\pi_C, C)} e_{ij} + \sum_{j \in F(\pi_C, \mathcal{N}\setminus C)} e_{ij} - c_i & \text{if } i \in F(\pi_C, C), \\
    \sum_{j \in F(\pi_C, C)} e_{ij} + \sum_{j \in F(\pi_C, \mathcal{N}\setminus C)} e_{ij} & \text{if } i \notin F(\pi_C, C). 
\end{cases} \tag{3}$$

Notice that (3) is a special case of (1) where the countries $i_1, \ldots, i_i$ are not associated, those that fund their projects face tax rates equal to 1 and those that do not face tax rates equal to 0. The term $\sum_{j \in F(\pi_C, \mathcal{N}\setminus C)} e_{ij}$ in (3) suggests that $d_i(\pi_C, C)$ depends on the coalitions that the rest of the countries form. However, since at this stage of the game the complementary coalitions are given, this term is fixed. In the cooperative game literature, the utility vector $(d_{i_1}(\pi_C, C), \ldots, d_{i_i}(\pi_C, C))$ is referred as the disagreement point (Burbidge et al. (1997)).

We define the surplus of the coalition $C$ as the difference of the worth of $C$ and the aggregate utility that the members of $C$ achieve under $\pi_C$:

$$s^*(C) = w(\pi, C) - \sum_{i \in C} d_i(\pi_C, C). \tag{4}$$

From the point of view of the members of $C$, (4) represents the gains from their cooperation. The fact that all the other countries form the same coalitions in $\pi$ and $\pi_C$ implies that they fund the same projects as well. That is:

$$F(\pi_C, \mathcal{N}\setminus C) = F(\pi, \mathcal{N}\setminus C).$$

Hence, they generate the same externalities each member of $C$:

$$\sum_{j \in F(\pi_C, C)} e_{ij} = \sum_{j \in F(\pi, \mathcal{N}\setminus C)} e_{ij}.$$ 

We use the latter along with (3) and simplify (4):

$$s^*(C) = \sum_{j \in F(\pi, C)} (b_j + \sum_{i \in C} e_{ij} - c_j) - \sum_{j \in F(\pi_C, C)} (b_j + \sum_{i \in C} e_{ij} - c_j). \tag{5}$$
Thus, the surplus consists solely of the benefits that are generated by the projects that the countries $i_1, \ldots, i_l$ fund in $\pi$ over and above $\pi_C$.

A coalition $C$ distributes the surplus evenly among its members by adopting the appropriate tax rates. Formally, for every $i \in C$ the tax rate $t^*_i$ is such that:

$$d_i(\pi_C, C) + \frac{s^s(C)}{|C|} = u_i(\pi, C, t^*_i).$$  \tag{6}

The current sharing scheme implies that the coalition partners set the tax rates in such a way to give each member the payoff it obtains when it is alone and then split the surplus equally. Notice that the LHS of (6) corresponds to the payoff level of the symmetric Nash bargaining solution on the coalition’s worth $w(\pi, C)$, given the disagreement point $(d_i(\pi_C, C), \ldots, d_i(\pi_C, C))$.\footnote{See Muthoo (1999).}

### 4.2.2 Optimal sharing scheme

We denote $\pi_{C \setminus \{i\}}$ the coalition structure in which the member $i$ of $C$ breaks away from its coalition and becomes a singleton, while all the other countries are still organised in coalitions according to $\pi$,

$$\pi_{C \setminus \{i\}} = \{C_1, \ldots, \{i_1, \ldots, i_l\}, \{i\}, \ldots, C_h\}.\footnote{The country $i$ does not actually leave the coalition $C$, but we suppose that it does in order to facilitate our analysis.}

In other words, $\pi_{C \setminus \{i\}}$ is a coalition structure in which all the coalitions of $\pi$ are formed but $i$ does not participate in the coalition $C$ nor in any other coalition.\footnote{The country $i$ does not actually leave the coalition $C$, but we suppose that it does in order to facilitate our analysis.}

We define the surplus of the coalition $C$ as the difference of the worth of $C$ and the sum of the utilities that each member could have achieved by defecting from $C$ to become a singleton:

$$s^o(C) = w(\pi, C) - \sum_{i \in C} u_i(\pi_{C \setminus \{i\}}, \{i\})$$  \tag{7}

From the point of view of the members of $C$, (7) represents the residual of the worth of $C$ left after each member gets its outside option payoff. The fact that all the other countries form the
same coalitions in $\pi$ and $\pi_{C \setminus \{i\}}$ implies that they fund the same projects as well. That is:

$$F(\pi_{C \setminus \{i\}}, N \setminus C) = F(\pi, N \setminus C).$$

Hence, they generate the same externalities for the deviating member $i$:

$$\sum_{j \in F(\pi_{C \setminus \{i\}}, N \setminus C)} e_{ij} = \sum_{j \in F(\pi, N \setminus C)} e_{ij}.$$

We use the latter along with (1) and simplify (7):

$$s^0(C) = \sum_{j \in F(\pi, C)} (b_j + \sum_{i \in C} e_{ij} - c_j) - \sum_{i \in C} \sum_{j \in F(\pi_{C \setminus \{i\}}, C)} (b_j + e_{ij} - c_j).$$

Thus, the surplus consists solely of the benefits that are generated by the projects that the countries $i_1, \ldots, i, \ldots, i_i$ fund in $\pi$ over and above $\pi_{C \setminus \{i\}}, \forall i \in C$. Moreover, as $s^*(C)$ and $s^0(C)$ are defined in a different way for the two sharing schemes, it is possible that they may not be the same.

In order for $C$ to have sufficient resources to guarantee each one of its members at least its outside option payoff, the literature assumes that,\textsuperscript{20}

$$\sum_{i \in C} u_i(\pi, C, t^0_i) \geq \sum_{i \in C} u_i(\pi_{C \setminus \{i\}}, \{i\}).$$

In the current setting, this assumption along with (1), (2) and (7) imply that the surplus of $C$ is

$$s^0(C) \geq 0. \quad (9)$$

Further, we assume that the coalition $C$ distributes the surplus evenly among its members by adopting the appropriate tax rates. Formally, for every $i \in C$ the tax rate $t^0_i$ is such that:

$$u_i(\pi_{C \setminus \{i\}}, \{i\}) + \frac{s^0(C)}{|C|} = u_i(\pi, C, t^0_i). \quad (10)$$

Here, the coalition partners set the tax rates in a way that gives each member its outside option and then split the residual of $C$’s worth equally. This sharing scheme gives priority to creating incentives for individual countries to join the coalition.

4.3 Tax rates

We can directly get the tax rates that correspond to the Nash bargaining solution sharing scheme after we replace (1) in (6) and solve for $t^*_i$. Recall that (1) distinguishes between cases where, under $\pi$, the project $i$ is funded and where it is not funded. We have that:

$$t^*_i = \begin{cases} 
\frac{1}{|A|} [b_i + \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, N \setminus C)} e_{ij} - (d_i(\pi_C, C) + \frac{s^*(C)}{|C|})] & \text{if } i \in F(\pi, C), \\
\frac{1}{|A|} \left[ \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, N \setminus C)} e_{ij} - (d_i(\pi_C, C) + \frac{s^*(C)}{|C|}) \right] & \text{if } i \notin F(\pi, C). 
\end{cases} \quad (11)$$

By working in a similar manner with (1) and (10), we obtain the tax rates of the optimal sharing scheme:

$$t^0_i = \begin{cases} 
\frac{1}{|A|} [b_i + \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, N \setminus C)} e_{ij} - (u_i(\pi_{C \setminus \{i\} \cup \{i\}}, \{i\}) + \frac{s^*(C)}{|C|})] & \text{if } i \in F(\pi, C), \\
\frac{1}{|A|} \left[ \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, N \setminus C)} e_{ij} - (u_i(\pi_{C \setminus \{i\} \cup \{i\}}, \{i\}) + \frac{s^*(C)}{|C|}) \right] & \text{if } i \notin F(\pi, C). 
\end{cases} \quad (12)$$

Both of the sharing schemes assign tax rates that are determined endogenously and allows a unique, rational and feasible payoff to every coalition member. Further, the tax rates exhibit a series of interesting properties. First, from (6) and (10) we have that the payoff that the sharing rules allocates to $i$ is strictly increasing in $d_i(\pi_C, C)$ and $u_i(\pi_{C \setminus \{i\} \cup \{i\}}, \{i\})$ respectively. For this reason, $t^*_i$ is negatively related with $d_i(\pi_C, C)$ in (11). The same holds for $t^0_i$ and $u_i(\pi_{C \setminus \{i\} \cup \{i\}}, \{i\})$ in (12) as well. Thus, higher the level of utility that the country $i$ obtains in $\pi_C$ or $u_i(\pi_{C \setminus \{i\} \cup \{i\}}, \{i\})$, the lower is its tax rate. The countries with a relatively better position when they act alone, contribute less to the expenses of the coalition, as an incentive to join in it.

On the other hand, expanding the coalition by including countries that cause little or no change in the surplus, increases the tax rates of the existing members. The slightly greater surplus is split among more countries. Every country gets a smaller share and thus obtains a lower utility level. As a result, the coalition charges higher taxes to the existing members.
The following Lemma introduces the next feature of the sharing schemes:

**Lemma 1** The rules proposed assign tax rates that allow the coalition to cover its expenses fully.

The proof is in Appendix B.

Put differently, if the members of the coalition pay tax rates according to (11) or (12), then
\[ \sum_{i \in C} t_i^* = \sum_{i \in C} t_i^p = 1. \]
By Lemma 1, the contributions made to the coalition are sufficient to cover the costs of the projects that are chosen for funding, regardless of the number of projects that the coalition funds. This feature is significant in the second stage of the game, where each coalition decides on which projects to fund. Also, notice that the externalities that are generated by the other coalitions do not affect the tax rates. If we replace \( d_i(\hat{\pi}_C, C) \) from (3) and \( u_i(\pi_{C \setminus \{i\}}, \{i\}) \) from (1) in (11) and (12) respectively, these externalities are cancelled out.

Furthermore, the participation of some countries in \( C \) may be taxed, while of others might be subsidized. If a member of a coalition has a project with positive and high externalities but with negative net local benefit \( (b_i < c_i) \), the other members might have incentives to subsidize the participation of this country in the coalition (see the examples 1 and 2 in Appendix A). This is in accordance with the financial mechanism established by the Montreal Protocol. Also, there are cases where the tax rates are identical across all members, e.g. when the members are symmetric.

### 4.4 Choosing projects

As the coalition structure \( \pi \) has been formed in the first stage of the game, in the second stage all the countries belong to some coalition. Here, we examine the decision that each coalition makes on the projects it should fund. Among the projects that the members of a coalition \( C \) may adopt, we focus on the profitable ones. That is, the projects which offer to the coalition greater total benefit (local benefit and intra-coalition externalities) than their cost. Formally,

**Definition 1** Consider a country \( i \) that is a member of the coalition \( C \). The project of the country \( i \) is profitable for \( C \) when
\[
\begin{align*}
    b_i + \sum_{j \in C} e_{ji} &> c_i.
\end{align*}
\]
Put differently, a project $i$ is profitable for the coalition $C$ if its local benefit and the sum of its externalities to the members of the coalition exceed its cost. In the special case where $C$ is a singleton, project $i$ is evaluated only for its local benefit versus its cost, so (13) reduces to $b_i > c_i$. As Lemma 1 implies that the coalition covers its expenses fully, regardless of the number of projects it funds, we have that

**Corollary 1** The project of the country $i$ that is a member of the coalition $C$ is funded if and only if it is profitable $C$.

In other words, any project with positive net benefit for $C$ should be funded. To make this more obvious, notice that a profitable project $i$ increases the worth of the coalition $C$ in (2) as well as the surpluses in (4) and (7). As the number of countries in the coalition is fixed, this project increases the payoff of every member of the coalition in (6) and (10). For this reason, the decision to fund such a project is unanimous.

Furthermore, from (5), we have that the surplus of the coalition $C$ is the net benefit generated by the projects funded in $\pi$ over and above $\pi_C$. As the projects that are funded due to the formation of the coalition are profitable for $C$, its surplus is

$$s^*(C) \geq 0.$$  \hspace{1cm} (14)

Then, (9) and (14) suggest that if at least one project is funded due to the formation of the coalition $C$, its surplus is positive. But, if no project is funded due to its formation, the surplus is zero. Another implication of (9) and (14) is that this coalition formation game is superadditive.\footnote{See Bloch (1997).}

### 4.5 Stable coalition structures

Following Hart and Kurz (1983), we do not discuss the process according to which the individual countries form coalitions. Any such process must eventually lead to a coalition structure from which no country or group of countries can improve upon their position by deviating. In particular, our focus is on coalition structures that have the core property. Formally,

**Definition 2** Consider a coalition structure $\pi$ and a group of the countries that, in order to obtain higher payoffs, defect from their current coalition(s) to form the coalition $C$. Due to this deviation
emerges some other coalition structure. The coalition structure π is core stable when such a coalition C does not exist.

In this present setting, the payoffs that the deviating countries obtain depend on the reaction of the non-deviating countries. We adopt the δ – core as a notion of coalition stability and assume that when one or more countries leave a coalition, the remaining members continue to cooperate. Further, we compare this notion of stability to the notion of the γ – core used by Chander and Tulkens (1995 and 1997). In contrast to the δ – core, under the γ – core notion of coalition stability whenever one or more countries leave a coalition they expect a complete breakdown in the cooperation of the non-deviating members.

In order to be compatible with the notion of the coalition structure core we consider one deviation at a time, either unilateral or multilateral. To be more specific, a unilateral deviation involves a single country that breaks away from its current coalition to become a singleton, while a multilateral deviation involves a subset of the countries that leave their current coalition(s) to form another coalition.

As the analysis becomes quickly very complex when coalition stability is studied in this way, we do not study the stability of the fragmented coalition structures but we derive the condition under which the payoffs of grand coalition have the γ – core and δ – core property. This will be enough to draw some insights on the strategic factors affecting international environmental agreements that address issues with global impact and require the full cooperation of the countries, such as the climate change. We find that in comparison to the γ – core, the conditions an economy has to satisfy so that its payoffs are in the δ – core are more restrictive. Moreover, we show that this does not ensure that when the payoffs of the full cooperation of the countries have the δ – core property, then they also have the γ – core one. With regard to the sharing rules, we find that the optimal scheme cannot ensure the stability of the grand coalition. Furthermore, we find that in such case the payoffs of the Nash bargaining solution sharing rule might be sufficient for the full cooperation of the countries to prevail.

In order to facilitate the interpretation of the above, we provide some examples in Appendix A. For the sake of simplicity, in the examples we consider an economy with n = 4 countries. This

\[22\text{See Greenberg (1994) and Bloch (1997).}\]
is the smallest economy that can accommodate for the coexistence of multiple coalitions.

We begin our analysis with the $\gamma - core$ notion of coalition stability:

**Proposition 1** In an economy with $N = \{1, 2, \ldots, n\}$ countries that adopt the Nash bargaining solution sharing scheme, the payoffs of the grand coalition consist the $\gamma - core$ of the game if and only if $\forall C \subseteq G$ for which,

$$\frac{s^*(G)}{n} < \frac{s^*(C)}{|C|}. \quad (15)$$

The proof is in Appendix B.

**Proposition 2** In an economy with $N = \{1, 2, \ldots, n\}$ countries that adopt the optimal sharing scheme, the payoffs of the grand coalition consist the $\gamma - core$ of the game if and only if $\forall C \subseteq G$ for which,

$$\frac{s^o(G)}{n} < \frac{s^o(C)}{|C|}. \quad (16)$$

The proof is in Appendix B.

As a deviating country expects a complete breakdown in the cooperation of the non-deviating countries, a unilateral deviation from the grand coalition leads to the fully fragmented coalition structure $\pi$. Propositions 1 and 2 suggest that no unilateral deviation by the country $i$ may improve upon its payoff and by (15), (16) this occurs when $s^*(G)$, $s^o(G)$ are either positive or zero. Nevertheless, as the latter is implied by (14) and (9), we have that no unilateral deviation is profitable under the $\gamma - core$ notion of coalition stability.

Further, a multilateral deviation involves a group of countries that break away from the grand coalition and form a finer coalition $C$, while the non-deviating countries break up into singletons. According to (15) and (16), no such deviation may improve upon the payoff of the deviating countries when the share of the surplus of $G$ is higher or equal to that of $C$.

As far as the $\delta - core$ notion of coalition stability is concerned, we have that:

**Proposition 3** In an economy with $N = \{1, 2, \ldots, n\}$ countries that adopt the Nash bargaining solution sharing scheme, the payoffs of the grand coalition consist the $\delta - core$ of the game $(N, u)$ if and

\[23\text{Notice that when } C = \{i\} \text{ (that is } i \text{ is a singleton) from (4) and (7) we get } s^\ast(\{i\}) = s^o(\{i\}) = 0.\]
only if \( \emptyset C \subseteq G \) for which \( \forall i \in C \),

\[
\frac{s^*(G)}{n} < \frac{s^*(C)}{|C|} + \sum_{j \in F(\emptyset, G \setminus C)} e_{ij} - \sum_{j \in F(\emptyset, G \setminus C)} e_{ij}.
\]

(17)

The proof is in Appendix B.

Under this notion of coalition stability, a unilateral deviation by the country \( i \) from the grand coalition does not lead to \( \hat{\pi} \). Since \( i \) expects the non-deviating countries to stay together, this deviation leads to the fragmented coalition structure \( \{G \setminus \{i\}, \{i\}\} \). Proposition 3 implies that no country \( i \) may increase its payoff by deviating from \( G \) to become a singleton. By (17), this occurs when the share of the surplus that every \( i \) receives when it participates in \( G \) is higher or equal to the externalities on which it free rides in \( \hat{\pi} \).

Moreover, a multilateral deviation involves a group of countries that break away from the grand coalition in order to form a finer coalition \( C \) and leads to the fragmented coalition structure \( \{G \setminus C, C\} \). Proposition 3 requires that no such deviation may improve upon the payoff for each one of the deviating countries. According to (17), this occurs when for at least one deviating country the share of the surplus that it gets in \( G \) is greater or equal to that of \( C \) plus the externalities on which it free rides in \( \hat{\pi} \). These are the externalities that generate the projects that are funded due to the fact that the non-deviating countries (that is the countries that consist \( G \setminus C \) continue to cooperate.

**Proposition 4** In an economy with \( N = \{1, 2, \ldots n\} \) countries that adopt the optimal sharing scheme, the payoffs of the grand coalition consist the \( \delta - \) core of the game \( (N, u) \) if and only if \( \emptyset C \subseteq G \) for which \( \forall i \in C \),

\[
\frac{s^\delta(G)}{n} < \frac{s^\delta(C)}{|C|} - \left( \sum_{j \in F(\pi_G \setminus \{i\})} e_{ij} - \sum_{j \in F(\pi_C \setminus \{i\})} e_{ij} \right).
\]

(18)

The proof is in Appendix B.

(18) implies that no country \( i \) increases its payoff by defecting from \( G \) to become a singleton when the share of the surplus that it receives when it participates in \( G \) is not lower than 0.\(^{24}\) Since

\[^{24}\text{To make this more clear, notice that in the case of a unilateral deviation:}

\[u_i(G, G, t_c^*) < u_i(G \setminus \{i\}, \{i\}, \{i\}).\]
from (9) we have that $s^0(G) \geq 0$, no unilateral deviation is profitable for this sharing scheme and we do not consider them in the remainder.

Moreover, Proposition 4 requires that no group of countries may improve upon their payoffs by leaving the grand coalition for a finer coalition $C$ in the coalition structure $\{G \setminus C, C\}$. This occurs when for at least one deviating country the share of the surplus that it gets in $G$ is higher or equal to that of $C$ minus the externalities that $i$ forgoes when, along with the rest of the countries in $C$, it defects from $G$. In other words, the difference $\sum_{j \in F(\pi_{C \setminus \{i\}})} e_{ij} - \sum_{j \in F(\pi_{G \setminus \{i\}})} e_{ij}$ consists of the externalities that generate the projects that become profitable when the members of $C \setminus \{i\}$ cooperate with the members of $G \setminus C$.

What is evident from the Propositions 3 and 4 is that in comparison to the $\gamma - core$, the conditions an economy has to satisfy so that its payoffs are in the $\delta - core$ are more demanding. This is because the conditions of the $\delta - core$ notion of coalition stability take into account that every deviating country or every group of deviating countries free-ride on the externalities that are generated due to the cooperation of the non-deviating countries. It is also noteworthy that the potential gains from the full cooperation should be sufficiently high in order to support the participation of all the countries in the grand coalition. If not, either a single country is better off when it is not associated with other countries or a subgroup of the countries can improve their position if they form a finer coalition.

In addition, notice that we are not certain about the sign of the differences $\sum_{j \in F(\tilde{\pi}_C, G \setminus C)} e_{ij} - \sum_{j \in F(\tilde{\pi}_C, G \setminus C)} e_{ij}$ and $\sum_{j \in F(\pi_{C \setminus \{i\}})} e_{ij} - \sum_{j \in F(\pi_{C \setminus \{i\}})} e_{ij}$ in (17) and (18) respectively. Consequently, we are unable to argue that when the conditions of the Propositions 3 and 4 are satisfied, the Propositions 1 and 2 also hold. Put differently, if an economy has the payoffs of the grand coalition in the $\delta - core$, this does not indicate that they are in the $\gamma - core$ as well. Further, the fact that a sharing scheme produces payoffs with the $\gamma - core$ and/or $\delta - core$ property does not imply that the payoffs of the other sharing scheme will exhibit the same property. A direct implication of the above is that in the case where the optimal sharing scheme fails to distribute the gains from the cooperation in a way that sustains the grand coalition, the Nash bargaining solution sharing scheme

After we replace $u_i(G, G, t^{ij}_i)$ from (10) and $u_i(\{G \setminus \{i\}, \{i\}, \{i\})$ from (1) in the above, we have

$$\sum_{j \in F(\pi_{C \setminus \{i\}})} e_{ij} + \frac{s^0(G)}{n} < \sum_{j \in F(\pi_{G \setminus \{i\}})} e_{ij} + 0 \Leftrightarrow \frac{s^0(G)}{n} < 0.$$
succeeds. However, the opposite might also occur.

To make this more clear, we present some examples in Appendix A. In the examples 1 and 2 we consider an economy where the payoffs of the grand coalition are in the $\gamma - core$ and the $\delta - core$ of the game for both of the sharing schemes. In the example 3, the Nash bargaining solution sharing scheme generates payoffs for the grand coalition that have the $\gamma - core$ property but not the $\delta - core$ one, while the opposite occurs in the example 4. Despite this, in both of those the examples the payoffs of the optimal sharing scheme that are in the $\gamma - core$ as well as the $\delta - core$ of the game $(N, u)$. In the example 5, the optimal sharing scheme produces payoffs have only the $\gamma - core$ property, while in the example 6 they only have the $\delta - core$ property. The payoffs of the Nash bargaining solution sharing scheme in the examples 5 and 6 however exhibit the $\gamma - core$ as well as the $\delta - core$ property.

5 Welfare loss

In this section, we examine the loss in the aggregate welfare when the grand coalition does not prevail. We show that the grand coalition may not occur even if the welfare loss is positive.

When all the countries participate in a single coalition there is no loss in welfare. The project of the country $i$ is evaluated for the externalities that it generates for all the other countries. However, when the countries do not form the grand coalition the project of the country $i$ is evaluated only for the externalities that it generates for the other countries of the coalition. Even though the externalities that the project $i$ generates for the rest of the countries occur, these are not taken into account. In this case, we may have welfare losses.

When the countries do not have incentives to cooperate and we have $\hat{\pi}$, the countries suffer a positive welfare loss:

**Corollary 2** In an economy with $N = \{1, 2, \ldots, n\}$ countries, if the fully fragmented coalition structure $\hat{\pi} = \{\{1\}, \{2\}, \ldots, \{n\}\}$ prevails, the countries suffer a welfare loss equal to:

$$W(G) - W(\hat{\pi}) = s^*(G).$$

Corollary 10 is a direct implication of (4).
Recall however that, from the Propositions 1 and 3, no unilateral or multilateral deviations from the grand coalition is profitable if and only if its surplus is sufficiently high. This is the case in the examples 1 and 2. But, since this does hold in the examples 7 and 8, the grand coalition does not prevail even though the countries gain from their cooperation. Then, the Corollary 9 implies that grand coalition forms only when the full cooperation of the countries generates high enough welfare gains in comparison to no cooperation. This however is contrary to the predictions of the bulk of the literature on IEAs that use tools from the non-cooperative game-theoretic approach models.

6 Conclusion

In this paper we study the formation of coalitions, such as IEAs, using the $\delta$ – core in an economy where environmental spillovers are at work. Each country in the economy has a project with a fixed cost that generates a local benefit and externalities of different magnitudes. Since the countries act rationally, some of them may decide not to associate. On the other hand, other countries may cooperate and form coalitions. Each coalition of countries decides endogenously on which of the available projects it should fund, in order to maximize the utility of its members. The participants of each coalition share equally the gains from cooperation by adopting either the Nash bargaining solution or the optimal sharing scheme. However, those gains may not coincide for the two sharing schemes and thus the countries might obtain different payoffs. Moreover, the current setting can accommodate for multilateral deviations and multiple coalitions.

We show that the formation of the grand coalition can be supported as long as the benefits that it generates are sufficiently high. If this is not the case, we might have more than one coalitions. Given that the standard result of the non-cooperative coalition formation literature is that large coalitions are only stable when welfare gains from cooperation are small, the result we obtain from the model at hand is quite surprising. Further, we compare the conditions for the $\gamma$ – core notion of coalition stability with the conditions for $\delta$ – core and find that the latter are more demanding because they take into account that every deviating country or every group of deviating countries free-ride on the externalities that are generated due to the cooperation of the non-deviating countries. However, this does not ensure that when the payoffs of the full cooperation of the countries have the $\delta$ – core
property, then they also have the $\gamma - core$ one. With regard to the sharing rules, we find that the optimal scheme cannot ensure the stability of the grand coalition. However, we show that in such occasion the payoffs of the Nash bargaining solution sharing scheme might be sufficient for the full cooperation of the countries to prevail. In addition, we full characterize the stability of the various coalition structures in a simple economy with $n = 4$ countries and examine the conditions under which the payoffs not only of the grand coalition, but also of the other coalition structures, are in the $\delta - core$. We also consider the welfare losses that the economy may suffer when the grand coalition does not form.

For future research, there are a number of issues that come to our mind. To begin with, some authors conclude that political or other constraints result in an uneven division of the gains from cooperation. For instance, Fleurbaey et al. note:

"Some actors, including governments, make use of negotiation power and/or lobbying activities to influence policy decisions at multiple scales, and by doing so, affect the design and the subsequent allocation and distribution of benefits and costs resulting from such decisions."

Taking this into account, the current analysis could be extenteded to cover the case when some countries have the ability to influence outcomes or exercise power over the way the surplus is shared. Further, wide uncertainties surround the consequences of economic activity on the environment as well as the cross-country distribution of the damage that it causes. Also, in the context of climate change, an assumption commonly made is that marginal damages are strictly decreasing in the global emission level. The current model’s assumption that the local benefit and the externalities of each project are known and independent of which other projects are realized could be relaxed towards these directions. What is more, much scope remains for additional research that utilizes observed data and moves as much as possible from numerical to general theoretical results on the formation of coalitions, as in Botteon and Carraro (2001) and Weikard (2009).

Moreover, it would be interesting to study the strategic interaction of the countries under the $\delta - stability$ concept in a dynamic setting, for example when decisions about cooperation and coalition formation are taken in multiple periods. In this particular case, special consideration should be given to the fact that delaying mitigation action means that greater effort would be needed to achieve the same target, and would ultimately be more expensive than responding more
drastically. Finally, the greatest scope for research lies in exploring the impact of different allocation rules on the gains from participation in an IEA and establishing a sharing scheme that splits the expenses and surplus from the coordination in such a way, that the full cooperation of the countries always prevails.

Appendix A

Example 1 We consider a simple economy with $N = \{1, 2, 3, 4\}$ countries, where the projects have the following local benefits, externalities and costs,

<table>
<thead>
<tr>
<th>Country</th>
<th>$b_i$</th>
<th>$c_i$</th>
<th>$e_{1i}$</th>
<th>$e_{2i}$</th>
<th>$e_{3i}$</th>
<th>$e_{4i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.9</td>
<td>0</td>
<td>0.11</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1.1</td>
<td>0.1</td>
<td>0</td>
<td>0.11</td>
<td>-0.1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.1</td>
<td>-0.1</td>
<td>-0.1</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
<td>1.1</td>
<td>1.14</td>
<td>0.08</td>
<td>1.2</td>
<td>0</td>
</tr>
</tbody>
</table>

Notice that the project 1 has local benefit $b_1 = 1$, cost $c_1 = 0.9$ and generates externalities $e_{21} = 0.11$, $e_{31} = 0.2$ and $e_{41} = 0.1$ for the countries 2, 3 and 4 respectively. Similarly, we have the parameters for the projects 2, 3 and 4.

From (13), we have that the projects 1, 2 and 4 are profitable for the grand coalition,

\[
b_1 + e_{21} + e_{31} + e_{41} > c_1 \Rightarrow 1 + 0.11 + 0.2 + 0.1 > 0.9,\]

\[
b_2 + e_{12} + e_{32} + e_{42} > c_2 \Rightarrow 1 + 0.1 + 0.11 - 0.1 > 1.1,\]

\[
b_4 + e_{14} + e_{24} + e_{34} > c_4 \Rightarrow -1 + 1.14 + 0.08 + 1.2 > 1.1,\]

but the project 3 is not,

\[
b_3 + e_{13} + e_{23} + e_{43} < c_3 \Rightarrow 1 - 0.1 - 0.1 + 0.2 < 1.1.\]
Hence, the set of projects that the grand coalition funds is $F(G) = \{1, 2, 4\}$.

Although the project 4 is profitable for $G$ and 3 is not (because of its negative externalities to the countries 1 and 2), this is not the case when the countries 3 and 4 form a coalition since

$$b_3 + e_{43} > c_3 \Rightarrow 1 + 0.2 > 1.1,$$

and

$$b_4 + e_{34} < c_4 \Rightarrow -1 + 1.2 < 1.1.$$

Thus, the set of the projects that are funded by the coalition $\{3, 4\}$ includes only the project 3.

We also get that the coalition $\{1, 2\}$ funds both of the projects 1 and 2. We then have that under fragmented coalition structure $\{\{1, 2\}, \{3, 4\}\}$ the coalition $\{1, 2\}$ funds

$$F(\{\{1, 2\}, \{3, 4\}\}; \{1, 2\}) = \{1, 2\},$$

while the coalition $\{3, 4\}$ funds

$$F(\{\{1, 2\}, \{3, 4\}\}; \{3, 4\}) = \{3\}.$$

Similarly, we obtain the projects that are funded under the coalition structure $\{\{2, 3, 4\}, \{1\}\}$:

$$F(\{\{2, 3, 4\}, \{1\}\}; \{2, 3, 4\}) = \emptyset, \quad F(\{\{2, 3, 4\}, \{1\}\}; \{1\}) = \{1\}.$$

Also, when the countries are separate only the project 1 is profitable, as its local benefit exceeds its cost:

$$F(\pi, \{1\}) = \{1\}, \quad F(\pi, \{2\}) = F(\pi, \{3\}) = F(\pi, \{4\}) = \emptyset.$$

We calculate the surplus that generates grand coalition for the Nash bargaining solution sharing scheme below and consider the corresponding surplus for optimal sharing scheme in the following example. Recall that only the projects 1, 2 and 4 are profitable for the grand coalition and that countries 2, 3 and 4 do not fund their projects when they are separate. We first consider the Nash bargaining solution sharing scheme. From (4) we have that,

$$s^*(G) = \sum_{j \in \{1, 2, 4\}} (b_j + \sum_{i \in G} e_{ij} - c_j) - \sum_{i \in G} (b_1 + e_{21} + e_{31} + e_{41} - c_1) = 0.33.$$
We further have that,

\[ s^*(\{1, 2\}) = 0, \quad s^*(\{3, 4\}) = 0.1 \quad \text{and} \quad s^*(\{2, 3, 4\}) = 0. \]

The payoffs of the grand coalition have the \( \delta \)–core property when no country or group of countries may improve upon their position by deviating. A unilateral deviation from the grand coalition, say by the country 1, leads to the fragmented coalition structure \( \{\{2, 3, 4\}, \{1\}\} \) and from (??) it does not improve upon 1’s position when the share of the surplus of the grand coalition is not lower than the externalities on which 1 free rides in \( \{\{2, 3, 4\}, \{1\}\} \) over and above \( \hat{\pi} \). Since the coalition \{2, 3, 4\} funds no projects, we have that

\[
\frac{s^*(G)}{4} > \sum_{j \in F(\{2, 3, 4\})} e_{1j} - \sum_{j \in F(\hat{\pi})} e_{1j}.
\]

This implies that 1 has no incentives to deviate from the grand coalition.

A multilateral deviation from the grand coalition that involves two countries, say the countries 3 and 4, leads to the fragmented coalition structure \( \pi = \{\{1, 2\}, \{3, 4\}\} \) and from (??) it does not improve upon their payoffs when for at least one of them the share of the surplus of \( G \) is not lower than that of \{3, 4\} plus the externalities on which it free rides over and above \( \hat{\pi} \). As the coalition \{1, 2\} funds both of the projects 1 and 2, we have that

\[
3 : \frac{s^*(G)}{4} < \frac{s^*(\{3, 4\})}{2} + \sum_{j \in F(\hat{\pi}, \{1, 2\})} e_{3j} - \sum_{j \in F(\hat{\pi}, \{1, 2\})} e_{3j},
\]

and

\[
4 : \frac{s^*(G)}{4} > \frac{s^*(\{3, 4\})}{2} + \sum_{j \in F(\hat{\pi}, \{1, 2\})} e_{4j} - \sum_{j \in F(\hat{\pi}, \{1, 2\})} e_{4j}.
\]

This deviation is profitable for 3, but not for 4. Likewise, a multilateral deviation from the grand coalition by the countries 1 and 2 is not profitable neither for 1 nor for 2.
A multilateral deviation from the grand coalition that involves three countries, say the countries \(2, 3\) and \(4\), leads again to the fragmented coalition structure \(\{\{2, 3, 4\}, \{1\}\}\). From (??), this deviation does not increase the payoff of \(2, 3\) and \(4\) since the share of the surplus of \(G\) is higher than that of \(\{2, 3, 4\}\):

\[
\frac{s^*(G)}{4} > \frac{s^*(\{2, 3, 4\})}{2}.
\]

By working in a similar manner for the remaining deviations we can confirm that no unilateral or multilateral deviation from \(G\) is profitable. The payoffs of the grand coalition consist the \(\delta\)–core of the game \((N, u)\). To verify this we present the payoffs of the various coalition structures in the following table. We only present the payoffs of the coalition structures that involve deviations compatible with the coalition structure core:

<table>
<thead>
<tr>
<th>Coalition Structure</th>
<th>(u_1)</th>
<th>(u_2)</th>
<th>(u_3)</th>
<th>(u_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>({1, 2, 3}, {4})</td>
<td>0.1367</td>
<td>0.1467</td>
<td>0.2367</td>
<td>0</td>
</tr>
<tr>
<td>({1, 2, 4}, {3})</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>({1, 3, 4}, {2})</td>
<td>0.18</td>
<td>0.19</td>
<td>0.28</td>
<td>0.18</td>
</tr>
<tr>
<td>({2, 3, 4}, {1})</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>({1, 2}, {3, 4})</td>
<td>0</td>
<td>0.01</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>({1, 3}, {2, 4})</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>({1, 4}, {2, 3})</td>
<td>0.2</td>
<td>0.115</td>
<td>0.205</td>
<td>0</td>
</tr>
<tr>
<td>(G)</td>
<td>0.1825</td>
<td>0.1925</td>
<td>0.2825</td>
<td>0.1825</td>
</tr>
</tbody>
</table>

According to (11), the tax rates are

\[
(t^*_1, t^*_2, t^*_3, t^*_4) = (0.6637, 0.3217, 0.3959, -0.3813).
\]

Notice that even though the project \(4\) has a negative local benefit, its externalities are so high that the rest of the countries subsidize the participation of the country \(4\) in the grand coalition.

Further, notice that in this economy the conditions of the Proposition 1 are satisfied and thus payoffs
of the grand coalition are in the $\gamma$—core of the game $(N,u)$ too:

The payoffs of the grand coalition consist the $\gamma$—core of $(N,u)$.

<table>
<thead>
<tr>
<th>coalition</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${1,2},{3},{4}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${1,3},{2},{4}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${1,4},{2},{3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${2,3},{1},{4}$</td>
<td>0.2</td>
<td>0.115</td>
<td>0.205</td>
<td>0</td>
</tr>
<tr>
<td>${2,4},{1},{3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${3,4},{1},{2}$</td>
<td>0</td>
<td>0.01</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>${1,2,3},{4}$</td>
<td>0.1367</td>
<td>0.1467</td>
<td>0.2367</td>
<td>0</td>
</tr>
<tr>
<td>${1,2,4},{3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${1,3,4},{2}$</td>
<td>0.18</td>
<td>0.19</td>
<td>0.28</td>
<td>0.18</td>
</tr>
<tr>
<td>${2,3,4},{1}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

$G = \begin{bmatrix} 0.1825 & 0.1925 & 0.2825 & 0.1825 \end{bmatrix}$

**Example 2** We consider the economy of the example 1 and use (7) to calculate the surplus of the grand coalition,

\[
s^\gamma(G) = \sum_{j \in \{1,2,4\}} \left( b_j + \sum_{i \in G} e_{ij} - c_j \right)
\]

\[
\begin{align*}
1's \text{ utility when it defects from } G &= -\left( b_1 - c_1 \right) + 0 + \left( 0 + e_{21} + e_{24} \right) \\
2's \text{ utility when it defects from } G &= \left( 0 + e_{21} + e_{24} \right)
\end{align*}
\]
\[ 3's \text{ utility when it defects from } G = \left( \begin{array}{c} 0 \\ \epsilon_{31} \end{array} \right) + \left( \begin{array}{c} 0 \\ \epsilon_{41} + \epsilon_{42} \end{array} \right) = 0.35 \]

\[ 4's \text{ utility when it defects from } G = \left( \begin{array}{c} 0 \\ \epsilon_{31} \end{array} \right) + \left( \begin{array}{c} 0 \\ \epsilon_{41} + \epsilon_{42} \end{array} \right) \]

We further have that,

\[ s^0(\{1, 2\}) = 0, s^0(\{3, 4\}) = 0.1 \text{ and } s^0(\{2, 3, 4\}) = 0.3. \]

The payoffs of the grand coalition have the \( \delta \)–core property when no country or group of countries may improve upon their position by deviating. Since (9) and (10) guarantee that no country can improve upon its position by unilaterally deviating, we only consider the multilateral deviations. A multilateral deviation from the grand coalition that involves two countries, say the countries 3 and 4, leads to the fragmented coalition structure \( \pi = \{\{1, 2\}, \{3, 4\}\} \) and from (18) (or (??)) it does not improve upon their payoffs when for at least one of them the share of the surplus of \( G \) is greater or equal to that of \( \{3, 4\} \) minus the externalities that it forgoes when 3 and 4 defect from \( G \). As the coalition \( \{1, 2\} \) funds both of the projects 1 and 2, we have that

\[ \frac{0.35}{4} < \frac{0.1}{2} - \left( \sum_{j \in F(\pi_{G \setminus \{3\}})} \epsilon_{3j} \right) - \left( \sum_{j \in F(\pi_{(3, 4) \setminus \{3\}})} \epsilon_{3j} \right), \]

and

\[ \frac{0.35}{4} > \frac{0.1}{2} - \left( \sum_{j \in F(\pi_{G \setminus \{4\}})} \epsilon_{4j} \right) - \left( \sum_{j \in F(\pi_{(3, 4) \setminus \{4\}})} \epsilon_{4j} \right). \]

This deviation is profitable for 3, but not for 4. Likewise, a multilateral deviation from the grand coalition by the countries 1 and 2 is not profitable neither for 1 nor for 2

A multilateral deviation from the grand coalition that involves three countries, say the countries 2, 3 and 4, leads again to the fragmented coalition structure \( \{\{2, 3, 4\}, \{1\}\} \). From (18) (or (??)), this
deviation does not increase the payoff of 2, 3 and 4 since the share of the surplus of $G$ is higher than that of $\{2, 3, 4\}$:

$$\frac{s^o(G)}{4} > \frac{s^o(\{2, 3, 4\})}{2}.$$

By working in a similar manner for the remaining deviations we can confirm that no unilateral or multilateral deviation from $G$ is profitable. The payoffs of the grand coalition consist the $\delta$–core of the game $(N, u)$. To verify this we present the payoffs of the various coalition structures in the following table. We only present the payoffs of the coalition structures that involve deviations compatible with the coalition structure core:

The payoffs of the grand coalition consist the $\delta$–core of $(N, u)$.

<table>
<thead>
<tr>
<th></th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${1, 2, 3}, {4}$</td>
<td>0.2033</td>
<td>0.1133</td>
<td>0.2033</td>
<td>0</td>
</tr>
<tr>
<td>${1, 2, 4}, {3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${1, 3, 4}, {2}$</td>
<td>0.1133</td>
<td>0.19</td>
<td>0.3133</td>
<td>0.2133</td>
</tr>
<tr>
<td>${2, 3, 4}, {1}$</td>
<td>0.1</td>
<td>0.0767</td>
<td>0.2667</td>
<td>0.0667</td>
</tr>
<tr>
<td>${1, 2}, {3, 4}$</td>
<td>0.005</td>
<td>0.015</td>
<td>0.36</td>
<td>0.05</td>
</tr>
<tr>
<td>${1, 3}, {2, 4}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${1, 4}, {2, 3}$</td>
<td>0.21</td>
<td>0.115</td>
<td>0.205</td>
<td>0</td>
</tr>
<tr>
<td><strong>$G$</strong></td>
<td><strong>0.1875</strong></td>
<td><strong>0.2775</strong></td>
<td><strong>0.2875</strong></td>
<td><strong>0.0875</strong></td>
</tr>
</tbody>
</table>

According to (12), the tax rates are

$$(t_1^0, t_2^0, t_3^0, t_4^0) = (0.662, 0.294, 0.394, -0.35).$$

Notice that the rest of the countries subsidize the participation of the country 4 in the grand coalition for this sharing scheme as well. Finally, as the conditions of the Proposition 2 also hold in this
economy, the payoffs of the grand coalition are in the $\gamma$–core of the game $(N,u)$:

The payoffs of the grand coalition consist the $\gamma$–core of $(N,u)$.

<table>
<thead>
<tr>
<th></th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${1,2}, {3}, {4}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${1,3}, {2}, {4}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${1,4}, {2}, {3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${2,3}, {1}, {4}$</td>
<td>0.2</td>
<td>0.115</td>
<td>0.205</td>
<td>0</td>
</tr>
<tr>
<td>${2,4}, {1}, {3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${3,4}, {1}, {2}$</td>
<td>0</td>
<td>0.01</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>${1,2,3}, {4}$</td>
<td>0.2033</td>
<td>0.1133</td>
<td>0.2033</td>
<td>0</td>
</tr>
<tr>
<td>${1,2,4}, {3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${1,3,4}, {2}$</td>
<td>0.1133</td>
<td>0.19</td>
<td>0.3133</td>
<td>0.2133</td>
</tr>
<tr>
<td>${2,3,4}, {1}$</td>
<td>0.1</td>
<td>0.0767</td>
<td>0.2667</td>
<td>0.0667</td>
</tr>
</tbody>
</table>

$G = \begin{bmatrix} 0.1875 & 0.2775 & 0.2875 & 0.0875 \end{bmatrix}$

**Example 3** In this example, we consider an economy where the projects have the following local benefits, externalities and costs,

<table>
<thead>
<tr>
<th>$b_i$</th>
<th>$c_i$</th>
<th>$e_{1i}$</th>
<th>$e_{2i}$</th>
<th>$e_{3i}$</th>
<th>$e_{4i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.9</td>
<td>0</td>
<td>0.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>$Country$</td>
<td>2</td>
<td>1</td>
<td>1.1</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.1</td>
<td>-0.1</td>
<td>-0.1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
<td>1.1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

When we consider the Nash bargaining solution sharing scheme and the $\gamma$–core notion of coalition stability, the countries 3 and 4 are not able to improve their payoffs by defecting from the grand
coalition to form \{3, 4\}:

The payoffs of the grand coalition consist the \(\gamma\)–core of \((N, u)\).

<table>
<thead>
<tr>
<th>(\pi)</th>
<th>(u_1)</th>
<th>(u_2)</th>
<th>(u_3)</th>
<th>(u_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1, 2}, {3}, {4}</td>
<td>0.15</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>{1, 3}, {2}, {4}</td>
<td>0.1</td>
<td>0</td>
<td>-0.1</td>
<td>0</td>
</tr>
<tr>
<td>{1, 4}, {2}, {3}</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>-0.1</td>
</tr>
<tr>
<td>{2, 3}, {1}, {4}</td>
<td>0.1</td>
<td>0.2</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>{2, 4}, {1}, {3}</td>
<td>0.1</td>
<td>0.2</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>{3, 4}, {1}, {2}</td>
<td>0</td>
<td>0.1</td>
<td>0.255</td>
<td>0.255</td>
</tr>
<tr>
<td>{1, 2, 3}, {4}</td>
<td>0.1667</td>
<td>0.2667</td>
<td>-0.033</td>
<td>0</td>
</tr>
<tr>
<td>{1, 2, 4}, {3}</td>
<td>0.1667</td>
<td>0.2667</td>
<td>0</td>
<td>-0.033</td>
</tr>
<tr>
<td>{1, 3, 4}, {2}</td>
<td>0.3367</td>
<td>-0.1</td>
<td>0.1367</td>
<td>0.1367</td>
</tr>
<tr>
<td>{2, 3, 4}, {1}</td>
<td>0.2</td>
<td>0.4367</td>
<td>0.1367</td>
<td>0.1367</td>
</tr>
</tbody>
</table>

\[ G = \begin{pmatrix} 0.5275 & 0.6275 & 0.3275 & 0.3275 \end{pmatrix} \]

However, this is not true when we consider the \(\delta\)–core notion of coalition stability. In such case, the countries 1, 2 continue cooperating when 3, 4 defect from the grand coalition and fund the project 2 that generates positive externalities for the deviating countries. The payoffs of the various coalition structures are:

The payoffs of the fragmented coalition structure \{\{1, 2\}, \{3, 4\}\} consist the \(\delta\)–core of \((N, u, \{\{1\}, \{2\}, \{3\}, \{4\}\})\).

<table>
<thead>
<tr>
<th>(u_1)</th>
<th>(u_2)</th>
<th>(u_3)</th>
<th>(u_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1, 2}, {3}, {4}</td>
<td>0.1667</td>
<td>0.2667</td>
<td>-0.033</td>
</tr>
<tr>
<td>{1, 2, 3}, {4}</td>
<td>0.1667</td>
<td>0.2667</td>
<td>0</td>
</tr>
<tr>
<td>{1, 3}, {2}, {4}</td>
<td>0.3367</td>
<td>-0.1</td>
<td>0.1367</td>
</tr>
<tr>
<td>{2, 3}, {1}, {4}</td>
<td>0.2</td>
<td>0.4367</td>
<td>0.1367</td>
</tr>
<tr>
<td>{1, 2}, {3, 4}</td>
<td>\textbf{0.05}</td>
<td>\textbf{0.15}</td>
<td>\textbf{0.355}</td>
</tr>
<tr>
<td>{1, 3}, {2}, {4}</td>
<td>0.1</td>
<td>0.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>{1, 4}, {2}, {3}</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ G = \begin{pmatrix} 0.5275 & 0.6275 & 0.3275 & 0.3275 \end{pmatrix} \]

Despite the fact that the payoffs that generates the Nash bargaining solution sharing scheme are in the \(\gamma\)–core but not in the \(\delta\)–core of the game \((N, u)\), the payoffs of the optimal sharing scheme
have both the $\gamma$–core and the $\delta$–core properties:

The payoffs of the grand coalition consist the $\gamma$–core and the $\delta$–core of $(N, u)$.

$$\hat{\pi} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \\ \{1,2\}, \{3\}, \{4\} & 0.15 & 0.25 & 0 & 0 \\ \{1,3\}, \{2\}, \{4\} & 0.1 & 0 & -0.1 & 0 \\ \{1,4\}, \{2\}, \{3\} & 0.1 & 0 & 0 & -0.1 \\ \{2,3\}, \{1\}, \{4\} & 0.1 & 0.2 & -0.1 & -0.1 \\ \{2,4\}, \{1\}, \{3\} & 0.1 & 0.2 & -0.1 & -0.1 \\ \{3,4\}, \{1\}, \{2\} & 0 & 0.1 & 0.255 & 0.255 \\ \{1,2,3\}, \{4\} & 0.2 & 0.1 & 0.1 & 0 \\ \{1,2,4\}, \{3\} & 0.2 & 0.1 & 0 & 0.1 \\ \{1,3,4\}, \{2\} & 0.2033 & -0.1 & 0.2033 & 0.2033 \\ \{2,3,4\}, \{1\} & 0.2 & 0.37 & 0.17 & 0.17 \\ \{1,2\}, \{3,4\} & 0.05 & 0.15 & 0.355 & 0.355 \\ \{1,3\}, \{2,4\} & 0.1 & 0.2 & -0.1 & 0 \\ \{1,4\}, \{2,3\} & 0.1 & 0.2 & 0 & -0.1 \\ \end{array}$$

$G \begin{array}{cccc} 0.6275 & 0.3275 & 0.4257 & 0.4275 \end{array}$

**Example 4** We consider the economy of the example 3, but here we make some changes to the externalities of the projects 1 and 2. Specifically, we increase $e_{31}, e_{41}$ from $-0.1$ to $0$ and decrease $e_{32}, e_{42}$ from $0.1$ to $-0.075$. The countries 3,4 are better off when they defect from the grand coalition.
under the $\gamma$–core notion of coalition stability:

The payoffs of the fragmented coalition structure $\{\{3,4\},\{1\},\{2\}\}$ consist the $\gamma$–core of $(N,u,\{\{3,4\},\{1\},\{2\}\})$

\[\begin{array}{cccc}
\pi & 0.1 & 0.2 & 0 & 0 \\
\{1,2\}, \{3\}, \{4\} & 0.15 & 0.25 & -0.075 & -0.075 \\
\{1,3\}, \{2\}, \{4\} & 0.1 & 0.2 & 0 & 0 \\
\{1,4\}, \{2\}, \{3\} & 0.1 & 0.2 & 0 & 0 \\
\{2,3\}, \{1\}, \{4\} & 0.1 & 0.2 & 0 & 0 \\
\{2,4\}, \{1\}, \{3\} & 0.1 & 0.2 & 0 & 0 \\
\{3,4\}, \{1\}, \{2\} & 0 & 0.1 & 0.355 & 0.355 \\
\{1,2,3\}, \{4\} & 0.1083 & 0.2083 & 0.0083 & -0.075 \\
\{1,2,4\}, \{3\} & 0.1083 & 0.2083 & -0.075 & 0.0083 \\
\{1,3,4\}, \{2\} & 0.3033 & 0.1 & 0.2033 & 0.2033 \\
\{2,3,4\}, \{1\} & 0 & 0.4033 & 0.2033 & 0.2033 \\
\end{array}\]

\[G = \begin{pmatrix} 0.4525 & 0.5525 & 0.3525 & 0.3525 \end{pmatrix}\]

However, this is not true when we consider the $\delta$–core notion of coalition stability. In such case, the countries $1,2$ continue cooperating when $3,4$ defect from the grand coalition and fund the project 2 that generates positive externalities for the deviating countries. The payoffs of the various coalition structures are:

The payoffs of the grand coalition consist the $\delta$–core of $(N,u)$.

\[\begin{array}{cccc}
\pi & u_1 & u_2 & u_3 & u_4 \\
\{1,2,3\}, \{4\} & 0.1083 & 0.2083 & 0.0083 & -0.075 \\
\{1,2,4\}, \{3\} & 0.1083 & 0.2083 & -0.075 & 0.0083 \\
\{1,3,4\}, \{2\} & 0.3033 & 0.1 & 0.2033 & 0.2033 \\
\{2,3,4\}, \{1\} & 0 & 0.4033 & 0.2033 & 0.2033 \\
\{1,2\}, \{3,4\} & 0.05 & 0.15 & 0.28 & 0.28 \\
\{1,3\}, \{2,4\} & 0.1 & 0.2 & 0 & 0 \\
\{1,4\}, \{2,3\} & 0.1 & 0.2 & 0 & 0 \\
\end{array}\]

\[G = \begin{pmatrix} 0.4525 & 0.5525 & 0.3525 & 0.3525 \end{pmatrix}\]

Hence, in this economy the payoffs of the grand coalition are in the $\delta$–core but not in the $\gamma$–core of the game $(N,u)$. Despite this, the payoffs of the optimal sharing scheme have both the $\gamma$–core
and the $\delta$–core properties:

The payoffs of the grand coalition consist the $\gamma$–core and the $\delta$–core of $(N,u)$.

<table>
<thead>
<tr>
<th>Coalition</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${1,2}$, ${3}$, ${4}$</td>
<td>0.15</td>
<td>0.25</td>
<td>-0.075</td>
<td>-0.075</td>
</tr>
<tr>
<td>${1,3}$, ${2}$, ${4}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${1,4}$, ${2}$, ${3}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${2,3}$, ${1}$, ${4}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${2,4}$, ${1}$, ${3}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${3,4}$, ${1}$, ${2}$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.355</td>
<td>0.355</td>
</tr>
<tr>
<td>${1,2,3}$, ${4}$</td>
<td>0.1333</td>
<td>0.2333</td>
<td>-0.0417</td>
<td>-0.075</td>
</tr>
<tr>
<td>${1,2,4}$, ${3}$</td>
<td>0.1333</td>
<td>0.2333</td>
<td>-0.075</td>
<td>-0.0417</td>
</tr>
<tr>
<td>${1,3,4}$, ${2}$</td>
<td>0.2367</td>
<td>0.1</td>
<td>0.2367</td>
<td>0.2367</td>
</tr>
<tr>
<td>${2,3,4}$, ${1}$</td>
<td>0</td>
<td>0.3367</td>
<td>0.2367</td>
<td>0.2367</td>
</tr>
<tr>
<td>${1,2}$, ${3,4}$</td>
<td>0.05</td>
<td>0.15</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>${1,3}$, ${2,4}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${1,4}$, ${2,3}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$G$</td>
<td>0.44</td>
<td>0.54</td>
<td>0.365</td>
<td>0.365</td>
</tr>
</tbody>
</table>

**Example 5** We consider the economy of the example 4, but here we increase $e_{12}$ from 0.2 to 0.25, $e_{32}, e_{42}$ from $-0.075$ to 0.1, $e_{13}, e_{23}$ from $-0.1$ to 0 and $e_{43}$ from 0.81 to 0.94. We examine the $\gamma$–core notion of coalition stability when the countries adopt the optimal sharing scheme. We find that the countries 3 and 4 are unable to improve their payoffs if they leave the grand coalition in
order to form \{3,4\}:

The payoffs of the grand coalition consist the \(\gamma\)-core of \((N,u)\).

\[
\begin{array}{cccc}
\hat{\pi} & u_1 & u_2 & u_3 \\{1,2\},\{3\},\{4\} & 0.175 & 0.275 & 0.1 & 0.1 \\
\{1,3\},\{2\},\{4\} & 0.1 & 0.2 & 0 & 0 \\
\{1,4\},\{2\},\{3\} & 0.1 & 0.2 & 0 & 0 \\
\{2,3\},\{1\},\{4\} & 0.1 & 0.2 & 0 & 0 \\
\{2,4\},\{1\},\{3\} & 0.1 & 0.2 & 0 & 0 \\
\{3,4\},\{1\},\{2\} & 0.1 & 0.2 & 0.42 & 0.42 \\
\{1,2,3\},\{4\} & 0.15 & 0.25 & 0.15 & 0.1 \\
\{1,2,4\},\{3\} & 0.15 & 0.25 & 0.1 & 0.15 \\
\{1,3,4\},\{2\} & 0.38 & 0.2 & 0.28 & 0.28 \\
\{2,3,4\},\{1\} & 0.35 & 0.5133 & 0.3133 & 0.3133 \\
\end{array}
\]

\[
\begin{array}{cccc}
G & 0.76 & 0.61 & 0.51 & 0.51 \\
\end{array}
\]

Nevertheless, this is not true when we consider the \(\delta\)–core notion of coalition stability. In such case, the countries 1,2 continue cooperating when 3,4 defect from the grand coalition and fund the project 2 that generates positive externalities for the deviating countries. The payoffs of the various coalition structures are:

The payoffs of the fragmented coalition structure \{\{1,2\},\{3,4\}\} consist the \(\delta\)-core of \((N,u,\{\{1,2\},\{3,4\}\})\).

\[
\begin{array}{cccc}
\{1,2,3\},\{4\} & 0.15 & 0.25 & 0.15 & 0.1 \\
\{1,2,4\},\{3\} & 0.15 & 0.25 & 0.1 & 0.15 \\
\{1,3,4\},\{2\} & 0.38 & 0.2 & 0.28 & 0.28 \\
\{2,3,4\},\{1\} & 0.35 & 0.5133 & 0.3133 & 0.3133 \\
\{1,2\},\{3,4\} & 0.175 & 0.275 & 0.52 & 0.52 \\
\{1,3\},\{2,4\} & 0.1 & 0.2 & 0 & 0 \\
\{1,4\},\{2,3\} & 0.1 & 0.2 & 0 & 0 \\
G & 0.76 & 0.61 & 0.51 & 0.51 \\
\end{array}
\]

Despite the fact that the payoffs that generates the optimal sharing scheme are in the \(\gamma\)–core but not in the \(\delta\)–core of the game \((N,u)\), the payoffs of the Nash bargaining solution sharing scheme
have both the γ—core and the δ—core properties:

The payoffs of the grand coalition consist the γ—core and the δ—core of \((N, u)\).

\[
\begin{array}{cccc}
\hat{\pi} & u_1 & u_2 & u_3 & u_4 \\
\{1,2\},\{3\},\{4\} & 0.1 & 0.2 & 0 & 0 \\
\{1,3\},\{2\},\{4\} & 0.1 & 0.2 & 0 & 0 \\
\{1,4\},\{2\},\{3\} & 0.1 & 0.2 & 0 & 0 \\
\{2,3\},\{1\},\{4\} & 0.1 & 0.2 & 0 & 0 \\
\{2,4\},\{1\},\{3\} & 0.1 & 0.2 & 0 & 0 \\
\{3,4\},\{1\},\{2\} & 0.1 & 0.2 & 0.42 & 0.42 \\
\{1,2,3\},\{4\} & 0.1833 & 0.2833 & 0.0833 & 0.1 \\
\{1,2,4\},\{3\} & 0.1833 & 0.2833 & 0.1 & 0.0833 \\
\{1,3,4\},\{2\} & 0.38 & 0.2 & 0.28 & 0.28 \\
\{2,3,4\},\{1\} & 0.35 & 0.5133 & 0.3133 & 0.3133 \\
\{1,2\},\{3,4\} & 0.175 & 0.275 & 0.52 & 0.52 \\
\{1,3\},\{2,4\} & 0.1 & 0.2 & 0 & 0 \\
\{1,4\},\{2,3\} & 0.1 & 0.2 & 0 & 0 \\
\end{array}
\]

\[
G = 0.6225 \quad 0.7225 \quad 0.5225 \quad 0.5225
\]

**Example 6** We consider the economy of the example 5 and decrease \(e_{32}, e_{42}\) from 0.1 to −0.075.

For the optimal sharing scheme, the countries 3,4 are better off when they defect from the grand
coalition under the $\gamma$–core notion of coalition stability.

The payoffs of the fragmented coalition structure \{\{3,4\},\{1\},\{2\}\} consist the $\gamma$–core of $(N,u,\{\{3,4\},\{1\},\{2\}\})$.

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1,2}, {3}, {4}</td>
<td>0.15</td>
<td>0.25</td>
<td>–0.075</td>
<td>–0.075</td>
</tr>
<tr>
<td>{1,3}, {2}, {4}</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>{1,4}, {2}, {3}</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>{2,3}, {1}, {4}</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>{2,4}, {1}, {3}</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>{3,4}, {1}, {2}</td>
<td>0.1</td>
<td>0.2</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>{1,2,3}, {4}</td>
<td>0.15</td>
<td>0.25</td>
<td>–0.025</td>
<td>–0.075</td>
</tr>
<tr>
<td>{1,2,4}, {3}</td>
<td>0.15</td>
<td>0.25</td>
<td>–0.075</td>
<td>–0.025</td>
</tr>
<tr>
<td>{1,3,4}, {2}</td>
<td>0.38</td>
<td>0.2</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>{2,3,4}, {1}</td>
<td>0.1</td>
<td>0.48</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>$G$</td>
<td>0.5725</td>
<td>0.6725</td>
<td>0.3975</td>
<td>0.3975</td>
</tr>
</tbody>
</table>

but not under the $\delta$–core notion of coalition stability.

The payoffs of the grand coalition consist the $\delta$–core of $(N,u)$.

<table>
<thead>
<tr>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1,2,3}, {4}</td>
<td>0.15</td>
<td>0.25</td>
<td>–0.025</td>
</tr>
<tr>
<td>{1,2,4}, {3}</td>
<td>0.15</td>
<td>0.25</td>
<td>–0.075</td>
</tr>
<tr>
<td>{1,3,4}, {2}</td>
<td>0.38</td>
<td>0.2</td>
<td>0.28</td>
</tr>
<tr>
<td>{2,3,4}, {1}</td>
<td>0.1</td>
<td>0.48</td>
<td>0.28</td>
</tr>
<tr>
<td>{1,2}, {3,4}</td>
<td>0.175</td>
<td>0.275</td>
<td>0.345</td>
</tr>
<tr>
<td>{1,3}, {2,4}</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>{1,4}, {2,3}</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>$G$</td>
<td>0.5725</td>
<td>0.6725</td>
<td>0.3975</td>
</tr>
</tbody>
</table>

Thus, in this economy the payoffs of the grand coalition are in the $\delta$–core but not in the $\gamma$–core of the game $(N,u)$. Despite this, the payoffs of the Nash bargaining solution sharing scheme have
both the $\gamma$–core and the $\delta$–core properties:

The payoffs of the grand coalition consist the $\gamma$–core and the $\delta$–core of $(N,u)$.

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${1,2},{3},{4}$</td>
<td>0.15</td>
<td>0.25</td>
<td>-0.075</td>
<td>-0.075</td>
</tr>
<tr>
<td>${1,3},{2},{4}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${1,4},{2},{3}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${2,3},{1},{4}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${2,4},{1},{3}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${3,4},{1},{2}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>${1,2,3},{4}$</td>
<td>0.125</td>
<td>0.225</td>
<td>0.025</td>
<td>-0.075</td>
</tr>
<tr>
<td>${1,2,4},{3}$</td>
<td>0.125</td>
<td>0.225</td>
<td>-0.075</td>
<td>0.025</td>
</tr>
<tr>
<td>${1,3,4},{2}$</td>
<td>0.38</td>
<td>0.2</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>${2,3,4},{1}$</td>
<td>0.1</td>
<td>0.48</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>${1,2},{3,4}$</td>
<td>0.175</td>
<td>0.275</td>
<td>0.345</td>
<td>0.345</td>
</tr>
<tr>
<td>${1,3},{2,4}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${1,4},{2,3}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$G = \begin{pmatrix} 0.535 & 0.635 & 0.435 & 0.435 \end{pmatrix}$

**Example 7** We consider the economy of the example 1, but in this case change the externalities of the projects 2, 3 and 4 in the following manner:

<table>
<thead>
<tr>
<th>$b_i$</th>
<th>$c_i$</th>
<th>$e_{1i}$</th>
<th>$e_{2i}$</th>
<th>$e_{3i}$</th>
<th>$e_{4i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.9</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Country</td>
<td>2</td>
<td>1</td>
<td>1.1</td>
<td>0.21</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.1</td>
<td>0.2</td>
<td>-0.1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
<td>1.1</td>
<td>0</td>
<td>0</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Regardless of the sharing scheme that the countries adopt, 3 and 4 are better off if they form the coalition $\{3,4\}$ rather than cooperate with 1, 2. No country has incentives to leave its current coalition in the fragmented coalition structure $\{(1,2),\{3,4\}\}$ and there does not exist neither a two-member nor three member coalition that can generate a surplus high enough to improve upon the payoffs of all of its members. The payoffs of the fragmented coalition structure $\{(1,2),\{3,4\}\}$
consist the $\delta$–core of the game $\langle N, u, \{\{1, 2\}, \{3, 4\}\} \rangle$. The payoffs of the various coalition structures for Nash bargaining solution sharing scheme this case are,

The payoffs of the fragmented coalition structure $\{\{1, 2\}, \{3, 4\}\}$ consist the $\delta$–core of $\langle N, u, \{\{12\}, \{3, 4\}\} \rangle$.

<table>
<thead>
<tr>
<th>Coalition Structure</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${1, 2}, {3}, {4}$</td>
<td>0.155</td>
<td>0.165</td>
<td>0.33</td>
<td>0.0</td>
</tr>
<tr>
<td>${1, 3}, {2}, {4}$</td>
<td>0.15</td>
<td>0.01</td>
<td>0.25</td>
<td>0.1</td>
</tr>
<tr>
<td>${1, 4}, {2}, {3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${2, 3}, {1}, {4}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${2, 4}, {1}, {3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${3, 4}, {1}, {2}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>${1, 2, 3}, {4}$</td>
<td>0.17</td>
<td>0.18</td>
<td>0.27</td>
<td>0.2</td>
</tr>
<tr>
<td>${1, 2, 4}, {3}$</td>
<td>0.17</td>
<td>0.18</td>
<td>0.3</td>
<td>0.17</td>
</tr>
<tr>
<td>${1, 3, 4}, {2}$</td>
<td>0.133</td>
<td>0.11</td>
<td>0.233</td>
<td>0.133</td>
</tr>
<tr>
<td>${2, 3, 4}, {1}$</td>
<td>0.31</td>
<td>0.176</td>
<td>0.266</td>
<td>0.166</td>
</tr>
<tr>
<td>${1, 2}, {3, 4}$</td>
<td>0.155</td>
<td>0.165</td>
<td>0.35</td>
<td>0.225</td>
</tr>
</tbody>
</table>

$G$ | 0.2025 | 0.2125 | 0.3025 | 0.2025 |

while the corresponding payoffs for the optimal sharing scheme are,

The payoffs of the fragmented coalition structure $\{\{1, 2\}, \{3, 4\}\}$ consist the $\delta$–core of $\langle N, u, \{\{12\}, \{3, 4\}\} \rangle$.

<table>
<thead>
<tr>
<th>Coalition Structure</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${1, 2}, {3}, {4}$</td>
<td>0.155</td>
<td>0.165</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>${1, 3}, {2}, {4}$</td>
<td>0.15</td>
<td>0.0</td>
<td>0.25</td>
<td>0.1</td>
</tr>
<tr>
<td>${1, 4}, {2}, {3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${2, 3}, {1}, {4}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${2, 4}, {1}, {3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${3, 4}, {1}, {2}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>${1, 2, 3}, {4}$</td>
<td>0.17</td>
<td>0.08</td>
<td>0.37</td>
<td>0.2</td>
</tr>
<tr>
<td>${1, 2, 4}, {3}$</td>
<td>0.1367</td>
<td>0.1467</td>
<td>0.3</td>
<td>0.2367</td>
</tr>
<tr>
<td>${1, 3, 4}, {2}$</td>
<td>0.1667</td>
<td>0.11</td>
<td>0.2667</td>
<td>0.1667</td>
</tr>
<tr>
<td>${2, 3, 4}, {1}$</td>
<td>0.31</td>
<td>0.1767</td>
<td>0.2667</td>
<td>0.1667</td>
</tr>
<tr>
<td>${1, 2}, {3, 4}$</td>
<td>0.155</td>
<td>0.165</td>
<td>0.35</td>
<td>0.25</td>
</tr>
</tbody>
</table>

$G$ | 0.31 | 0.11 | 0.3 | 0.2 |
We derive the appropriate tax rates for the Nash bargaining solution sharing scheme from (11),

\[ (t^*_1, t^*_2, t^*_3, t^*_4) = (0.53, 0.47, 1.95, -0.95) \]

and for the optimal sharing scheme from (12),

\[ (t^*_1, t^*_2, t^*_3, t^*_4) = (0.5275, 0.4725, 1.9545, -0.9545) \].

Due to the very high externality of the project 4 for the country 3, the latter has incentives to subsidize the participation of the country 4 in the coalition \( \{3, 4\} \).

**Example 8** Here, the projects have the following local benefits, externalities and costs,

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|
|   |   |   |   |   |   |
| 1 |   |   |   |   |   |
| 2 |   |   |   |   |   |
| 3 |   |   |   |   |   |
| 4 |   |   |   |   |   |

The project 4 may have a very high externality for the country 3, but the surplus of the coalition \( \{3, 4\} \) is not high enough. 3 is better off when it cooperates with 1 and 2. As the conditions of the Corollaries 5 and 6 hold, the payoffs of the fragmented coalition structure \( \{\{1, 2, 3\}, \{4\}\} \) consist the \( \delta \) – core of the game \( (N, u, \{\{1, 2, 3\}, \{4\}\}) \) for both of the sharing schemes. Specifically, under
the Nash bargaining solution sharing scheme, the payoffs of the various coalition structures are

<table>
<thead>
<tr>
<th>Coalition Structure</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${1, 2}, {3}, {4}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>${1, 3}, {2}, {4}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>${1, 4}, {2}, {3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>${2, 3}, {1}, {4}$</td>
<td>0.165</td>
<td>0.1325</td>
<td>0.0225</td>
<td>0</td>
</tr>
<tr>
<td>${2, 4}, {1}, {3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>${3, 4}, {1}, {2}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>${1, 2, 3}, {4}$</td>
<td><strong>0.17</strong></td>
<td><strong>0.18</strong></td>
<td><strong>0.07</strong></td>
<td>0</td>
</tr>
<tr>
<td>${1, 2, 4}, {3}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>${1, 3, 4}, {2}$</td>
<td>0.1133</td>
<td>0.11</td>
<td>0.0133</td>
<td>0.1133</td>
</tr>
<tr>
<td>${2, 3, 4}, {1}$</td>
<td>0.1</td>
<td>0.1233</td>
<td>0.0133</td>
<td>0.1133</td>
</tr>
<tr>
<td>${1, 2}, {3, 4}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>${1, 3}, {2, 4}$</td>
<td>0.1</td>
<td>0.11</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>${1, 4}, {2, 3}$</td>
<td>0.165</td>
<td>0.1325</td>
<td>0.0225</td>
<td>0</td>
</tr>
<tr>
<td>$G$</td>
<td>0.1375</td>
<td>0.1475</td>
<td>0.0375</td>
<td>0.1375</td>
</tr>
</tbody>
</table>
As far as the optimal sharing scheme is concerned, we have

The payoffs of the fragmented coalition structure \( \{(1,2,3),(4)\} \) consist the \( \delta \)-core of \((N,u,\{(1,2,3),(4)\})\).

<table>
<thead>
<tr>
<th>Coalition</th>
<th>( u_1 )</th>
<th>( u_2 )</th>
<th>( u_3 )</th>
<th>( u_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1,2}, {3}, {4}</td>
<td>0.1</td>
<td>0.11</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>{1,3}, {2}, {4}</td>
<td>0.1</td>
<td>0.11</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>{1,4}, {2}, {3}</td>
<td>0.1</td>
<td>0.11</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>{2,3}, {1}, {4}</td>
<td>0.165</td>
<td>0.1325</td>
<td>0.0225</td>
<td>0</td>
</tr>
<tr>
<td>{2,4}, {1}, {3}</td>
<td>0.1</td>
<td>0.11</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>{3,4}, {1}, {2}</td>
<td>0.1</td>
<td>0.11</td>
<td>0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>{1,2,3}, {4}</td>
<td>0.2133</td>
<td>0.1583</td>
<td>0.0483</td>
<td>0</td>
</tr>
<tr>
<td>{1,2,4}, {3}</td>
<td>0.1</td>
<td>0.11</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>{1,3,4}, {2}</td>
<td>0.113</td>
<td>0.11</td>
<td>0.013</td>
<td>0.1133</td>
</tr>
<tr>
<td>{2,3,4}, {1}</td>
<td>0.1</td>
<td>0.1567</td>
<td>0.0467</td>
<td>0.0467</td>
</tr>
<tr>
<td>{1,2}, {3,4}</td>
<td>0.1</td>
<td>0.11</td>
<td>0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>{1,3}, {2,4}</td>
<td>0.1</td>
<td>0.11</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>{1,4}, {2,3}</td>
<td>0.165</td>
<td>0.1325</td>
<td>0.0225</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ G = \begin{pmatrix} 0.1625 & 0.1725 & 0.0625 & 0.0625 \end{pmatrix} \]

Also, according to (11) and (12), the appropriate tax rates are

\[
(t_1^*, t_2^*, t_3^*, t_4^*) = (0.32, 0.35, 0.33, 0)
\]

and

\[
(t_1^0, t_2^0, t_3^0, t_4^0) = (0.3069, 0.3537, 0.3394, 0).
\]

Appendix B

Proof of Lemma 1. We first prove that the aggregate of the tax rates that assigns the Nash Bargaining solution sharing scheme is equal to 1. We sum the tax rates of all the members of the coalition \( C \), as given by (11). Recall that \( C \) may not fund the projects of all of its participants and for this reason, in (11) we distinguish between the members that have their projects funded in \( C \) and the ones that they do not. Below, the term \( \sum_{j\in F(x,C)} b_j \) is the sum of the local benefits of the projects that are funded by \( C \),
\[
\sum_{i \in C} t_i^* = \frac{1}{A} \left[ \sum_{j \in F(\pi, C)} b_j + \sum_{i \in C} \left( \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, \mathcal{N} \setminus C)} e_{ij} \right) - \sum_{i \in C} (d_i(\pi_C, C) + \frac{s^*(C)}{|C|}) \right]
\]

\[
= \frac{1}{A} \left[ \sum_{j \in F(\pi, C)} b_j + \sum_{i \in C} \left( \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, \mathcal{N} \setminus C)} e_{ij} \right) - \sum_{i \in C} d_i(\pi_C, C) - \frac{1}{|C|} \sum_{i \in C} s^*(C) \right]
\]

\[
= \frac{1}{A} \left[ \sum_{j \in F(\pi, C)} b_j + \sum_{i \in C} \left( \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, \mathcal{N} \setminus C)} e_{ij} \right) - \sum_{i \in C} d_i(\pi_C, C) - \frac{1}{|C|} \times |C| \times s^*(C) \right]
\]

If we replace \(s^*(C)\) from (4) we get:

\[
\sum_{i \in C} t_i^* = \frac{1}{A} \left[ \sum_{j \in F(\pi, C)} b_j + \sum_{i \in C} \left( \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, \mathcal{N} \setminus C)} e_{ij} \right) - \sum_{i \in C} d_i(\pi_C, C) - s^*(C) \right].
\]

We further replace \(w(\pi, C)\) from (2) and have:

\[
\sum_{i \in C} t_i^* = \frac{1}{A} \left[ \sum_{j \in F(\pi, C)} b_j + \sum_{i \in C} \left( \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, \mathcal{N} \setminus C)} e_{ij} \right) - \sum_{i \in C} (b_j + \sum_{i} e_{ij} - c_j) - \sum_{i \in C} \sum_{j} e_{ij} \right].
\]

Since \(\sum_{i} \sum_{j} e_{ij} = \sum_{j} \sum_{i} e_{ij}\) is a property of the summation,

\[
\sum_{i \in C} = \frac{\sum_{j \in F(\pi, C)} c_j}{A} = \frac{A}{A} = 1,
\]
as \( A = \sum_{j \in F(\pi, C)} c_j \).

We work in a similar manner in order to prove that the tax rates of the optimal sharing scheme exhibit the same property. We sum the tax rates of all the members of \( C \), as given by (12). Recall again that \( C \) may not fund the projects of all of its participants and for this reason, in (??) we distinguish between the members that have their projects funded in \( C \) and the ones that they do not,

\[
\sum_{i \in C} t_i^o = \frac{1}{A} \left[ \sum_{j \in F(\pi, C)} b_j + \sum_{i \in C} \left( \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, N \setminus C)} e_{ij} \right) - \sum_{i \in C} (u_i(\pi_{C \setminus \{i\}}, \{i\}) + s^o(C)) \right]
\]

\[
= \frac{1}{A} \left[ \sum_{j \in F(\pi, C)} b_j + \sum_{i \in C} \left( \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, N \setminus C)} e_{ij} \right) - \sum_{i \in C} u_i(\pi_{C \setminus \{i\}}, \{i\}) - \frac{1}{|C|} \sum_{i \in C} s^o(C) \right]
\]

\[
= \frac{1}{A} \left[ \sum_{j \in F(\pi, C)} b_j + \sum_{i \in C} \left( \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, N \setminus C)} e_{ij} \right) - \sum_{i \in C} u_i(\pi_{C \setminus \{i\}}, \{i\}) - s^o(C) \right].
\]

When we replace \( s^o(C) \) from (7) we have:

\[
\sum_{i \in C} t_i^o = \frac{1}{A} \left[ \sum_{j \in F(\pi, C)} b_j + \sum_{i \in C} \left( \sum_{j \in F(\pi, C)} e_{ij} + \sum_{j \in F(\pi, N \setminus C)} e_{ij} \right) - w(\pi, C) \right].
\]

Finally, we follow the same steps we did in the previous part of the proof and get that \( \sum_{i \in C} t_i^o = 1. \)
Proof of Proposition 1. In order to prove (15), we consider that a group of countries leaves the grand coalition and forms the finer coalition $C$. As the countries that deviate expect the non-deviating countries to break up into singletons, such a deviation leads to the coalition structure

$$\pi = \{ C, \ldots, \underbrace{\{i_1\}, \ldots, \{i_t\}}_{\text{non-deviating countries}} \}. \]$$

A random member of $C$, for instance $i$, has incentives to deviate from $G$ as long as

$$u_i(G, G, t_i^*) < u_i(\pi, C, t_i^*).$$

We replace $u_i(G, G, t_i^*)$ and $u_i(\pi, C, t_i^*)$ from (6)

$$d_i(\tilde{\pi}, \{i\}) + \frac{s^*(G)}{n} < d_i(\tilde{\pi}_C, \{i\}) \Leftrightarrow \frac{s^*(C)}{|C|} > \frac{s^*(\tilde{\pi})}{|C|}. \]$$

Since in this case $\tilde{\pi}_C \equiv \tilde{\pi}$, we get

$$d_i(\tilde{\pi}, \{i\}) + \frac{s^*(G)}{n} < d_i(\tilde{\pi}, \{i\}) \Leftrightarrow \frac{s^*(C)}{|C|} > \frac{s^*(\tilde{\pi})}{|C|}. \]$$

Proof of Proposition 2. In order to prove (16), we consider that a group of countries leaves the grand coalition and forms the finer coalition $C$. As the countries that deviate expect the non-deviating countries to break up into singletons, such a defection leads to the coalition structure

$$\pi = \{ C, \ldots, \underbrace{\{i_1\}, \ldots, \{i_t\}}_{\text{non-deviating countries}} \}. \]$$

A random member of $C$, for instance $i$, has incentives to deviate from $G$ as long as

$$u_i(G, G, t_i^o) < u_i(\pi, C, t_i^o).$$

We replace $u_i(G, G, t_i^o)$ and $u_i(\pi, C, t_i^o)$ from (10) and get that

$$u_i(\tilde{\pi}, \{i\}) + \frac{s^o(G)}{n} < u_i(\tilde{\pi}, \{i\}) \Leftrightarrow \frac{s^o(C)}{|C|} > \frac{s^o(\tilde{\pi})}{|C|}. \]$$
Proof of Proposition 3. In order to prove (17), we consider that a group of countries leaves the grand coalition and forms the finer coalition $C$. As the countries that deviate expect the non-deviating countries to stay together, such a deviation leads to the coalition structure $\pi = \{C, N \setminus C\}$. A random member of $C$, for instance $i$, has incentives to deviate from $G$ as long as

$$u_i(G, G, t^*_i) < u_i(\pi, C, t^*_i).$$

We replace $u_i(G, G, t^*_i)$ and $u_i(\pi, C, t^*_i)$ from (6)

$$d_i(\hat{\pi}, \{i\}) + \frac{s^*(G)}{n} < d_i(\hat{\pi}_C, \{i\}) + \frac{s^*(C)}{|C|}.$$  

Any profitable project of $C$ in $\hat{\pi}_C$ is also profitable in $\hat{\pi}$ and vice versa. We replace $d_i(\hat{\pi}, \{i\})$ and $d_i(\hat{\pi}_C, \{i\})$ from (3). If $i \notin F(\hat{\pi}, C), F(\hat{\pi}_C, C)$ we have

$$b_i + \sum_{j \in F(\hat{\pi}, C)} e_{ij} + \sum_{j \in F(\hat{\pi}, N \setminus C)} e_{ij} - c_i + \frac{s^*(G)}{n} < b_i + \sum_{j \in F(\hat{\pi}_C, C)} e_{ij} + \sum_{j \in F(\hat{\pi}_C, N \setminus C)} e_{ij} - c_i + \frac{s^*(C)}{|C|}.$$

But, if $i \notin F(\hat{\pi}, C), F(\hat{\pi}_C, C)$ we have

$$\sum_{j \in F(\hat{\pi}, C)} e_{ij} + \sum_{j \in F(\hat{\pi}, N \setminus C)} e_{ij} + \frac{s^*(G)}{n} < \sum_{j \in F(\hat{\pi}_C, C)} e_{ij} + \sum_{j \in F(\hat{\pi}_C, N \setminus C)} e_{ij} + \frac{s^*(C)}{|C|}.$$  

They both reduce to

$$\sum_{j \in F(\hat{\pi}, C)} e_{ij} + \sum_{j \in F(\hat{\pi}, N \setminus C)} e_{ij} + \frac{s(G)}{n} < \sum_{j \in F(\hat{\pi}_C, C)} e_{ij} + \sum_{j \in F(\hat{\pi}_C, N \setminus C)} e_{ij} + \frac{s(C)}{|C|}.$$  

The members of $C$ are singletons in the coalition structures $\hat{\pi}$ and $\hat{\pi}_C$. They fund the same projects and generate the same externalities for the country $i$.  

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Thus,
\[
\frac{s^*(G)}{n} + \sum_{j \in F(\hat{\pi}, C)} e_{ij} < \frac{s^*(C)}{|C|} + \sum_{j \in F(\hat{\pi}, N \setminus C)} e_{ij} - \sum_{j \in F(\hat{\pi}, N \setminus C)} e_{ij}.
\]

**Proof of Proposition 4.** In order to prove (18), we consider that a group of countries leaves the grand coalition and forms the finer coalition \(C\). As the countries that deviate expect the non-deviating countries to stay together, such a deviation leads to the coalition structure \(\pi = \{C, N \setminus C\}\). A random member of \(C\), for instance \(i\), has incentives to deviate from \(G\) as long as
\[
u_i(G, G, t_i^0) < \nu_i(\pi, C, t_i^0).
\]
We replace \(\nu_i(G, G, t_i^0)\) and \(\nu_i(\pi, C, t_i^0)\) from (10)
\[
u_i(\pi_{G \setminus \{i\}}, \{i\}) + \frac{s^0(G)}{n} < \nu_i(\pi_{C \setminus \{i\}}, \{i\}) + \frac{s^0(C)}{|C|}.
\]
We replace \(\nu_i(\pi_{G \setminus \{i\}}, \{i\})\) and \(\nu_i(\pi_{C \setminus \{i\}}, \{i\})\) from (Utility). If \(i \in F(\pi_{G \setminus \{i\}}), F(\pi_{C \setminus \{i\}})\) we have
\[
\left(\sum_{j \in F(\pi_{G \setminus \{i\}})} e_{ij} - c_i + \frac{s^0(G)}{n} < b_i + \sum_{j \in F(\pi_{C \setminus \{i\}})} e_{ij} - c_i + \frac{s^0(C)}{|C|}\right)
\]
\[
\Leftrightarrow \sum_{j \in F(\pi_{G \setminus \{i\}})} e_{ij} + \frac{s^0(G)}{n} < \sum_{j \in F(\pi_{C \setminus \{i\}})} e_{ij} + \frac{s^0(C)}{|C|}.
\]
This inequality also holds when \(i \notin F(\pi_{G \setminus \{i\}}), F(\pi_{C \setminus \{i\}})\) and for this reason we get that
\[
\frac{s^0(G)}{n} < \frac{s^0(C)}{|C|} - \left(\sum_{j \in F(\pi_{G \setminus \{i\}})} e_{ij} - \sum_{j \in F(\pi_{C \setminus \{i\}})} e_{ij}\right).
\]

**References**


Mitigation of Climate Change, Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, Cambridge University Press.


