Monetary and fiscal activism in general equilibrium *

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July 4, 2018

Abstract

This paper studies the possible merits of macroeconomic policy activism. By policy activism, we refer both to how much to adjust policy instruments in light of recent changes in real activity (the output gap) and to the array of policy instruments employed for this task. We study a wide array of fiscal and monetary policy actions (standard and non-standard) within a unified DSGE setting, which includes all the main building blocks of an economy (households, firms, private banks, a Central Bank and the Treasury). Activism is contrasted to the case of passive policy with no reaction to the business cycle. Our main policy result is that a combination of monetary and fiscal policy activism can help the real economy. This happens when tax-sending policy instruments, open market operations by the central bank and remittances to Treasury react to the output gap at a moderate degree and, at the same time, the policy nominal interest rate is kept constant at a low level.

Keywords: Monetary policy, fiscal policy, stabilization.

JEL classification: E44, E5, E63

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*I am indebted to my thesis supervisor, Apostolis Philippopoulos, for his help and guidance throughout this project. I have particularly benefited from comments by Vangelis Vassilatos and seminar participants at Athens University of Economics and Business. I would also like to thank Petros Varthalitis for many discussions and help with Dynare. All errors are mine.

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1 Introduction

The approach to macroeconomic policy has changed radically and several times since the 1960s. After the belief in Keynesian type intervention in the 1960s and 1970s, most economists believed that the emphasis should shift to the role of policy in supporting price stability (regarding monetary policy) and economy’s growth potential (regarding fiscal policy). This changed radically during the 2007-8 world crisis. Since 2008, reacting to the severe economic downturn, most governments and central banks around the world have adopted an unprecedented peacetime countercyclical policy expansion.\(^1\)

The efficacy of this policy is still a debated issue. To put the same thing differently, does monetary and fiscal activism help? By activism, we typically mean expansionary policies during periods of recession. Specifically, by activism, as stated in an early contribution by Okun (1972), we refer both to how much to adjust policy instruments in light of recent changes in economic activity and to the array of policy instruments employed for this task. The former has to do with the magnitude of the reaction to macroeconomic indicators like the output gap. The latter has to do with the number of policy instruments used, and, in particular, with the inclusion of new, or unconventional, instruments like quantitative easing (QE) and various asset purchase programs.\(^2\)

This paper studies a wide array of fiscal and monetary policy actions (standard and non-standard), as well as, their interactions, in a DSGE model which includes all the main building blocks of an economy (households, firms, private banks, a Central Bank and the Treasury).\(^3\)

Activism, as defined above, will then be contrasted to the case of passive policy with no reaction of policy instruments to the output gap. Our work differs from most of the related literature because, while there is a rich literature on the role of monetary policy and a rich literature on the role of fiscal policy, there has not been a systematic analysis of the stabilization efficacy of the main monetary and fiscal policy instruments within a unified micro-founded general equilibrium framework. That is, the focus here is on the instrument problem: which instrument to employ and how to use it in recessionary periods.

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\(^1\)See Feldstein (2009) for a brief review of changes in the role of policy since the World War II.

\(^2\)See Cúrdia and Woodford (2010a, 2010b) for conventional and unconventional monetary policies in a DSGE framework, as well as the debate on this. This issue has become even more complex in the Eurozone, where there is a single central bank for different countries. For instance, in the debate on the European debt crisis, there are those who argue that a necessary feature of governance in the Eurozone is that the ECB can act as lender of last resort in the government bond markets (see e.g. De Grauwe, 2013). On the other hand, there are those who argue that the ECB’s bond-buying program is harmful for the Eurozone (see e.g. Sinn, 2010, and Sinn and Wollmerschauer, 2011).

\(^3\)For the classification between standard and non-standard monetary policy measures we follow Lenza et al. (2010).
Following most of the related literature (see e.g. Schmitt-Grohé and Uribe, 2005 and 2007), we will follow a Taylor rule-like approach to policy. In particular, we will allow the policy instruments to react endogenously to a small number of easily observable macroeconomic indicators and, mainly, since the focus is on countercyclical actions taken to reduce the 2007-8 economic downturn, to the output gap. We adopt such a feedback approach to policy because this is closer in spirit to activism; as the word “activism” itself declares, action means shifts in policy triggered by changes in the state of the economy.\(^4\) Besides, there is empirical support that the endogenous reaction of policy instruments to deviations of indicators from their targets, namely the magnitude of feedback policy coefficients, changes over the business cycle.\(^5\)

Our medium-scale DSGE model consists of a private sector and a policy sector. In the private sector, there are three different groups of households, while firms are distinguished into firms producing final goods, intermediate goods and capital goods. Private banks intermediate between savers and borrowers and, since giving loans is relatively costly, introduces a financial friction as in e.g. Cúrdia and Woodford (2010a, 2010b). In the policy sector, there are a Central Bank, which has a wide range of conventional and unconventional policy instruments in its hands, and the Treasury, which sets the standard tax-spending policy instruments. The model is described in detail in subsection 2.1.

We will work in two steps. In the first step, we feed the above described DSGE model with a variety of shocks widely used by the business cycle literature.\(^6\) We find that a single adverse temporary shock to total factor productivity, propagated by the internal dynamics of our model, can, on its own, account relatively well for the behavior of the main macroeconomic variables in the recent recession period. However, we need to clarify that our aim is not to reproduce the sub-prime crisis or the disruptions of any specific financial market; our aim is just to get a depression and then study what fiscal-monetary policy can (or cannot) do about this.

In the second step, assuming that transition dynamics are driven by the above TFP shock, we, first, allow for monetary policy activism. This means that standard (policy interest rate) and non-standard monetary policy instruments (open market operations (OMOs) and divi-

\(^4\)A feedback rule-like approach differs methodologically from the recent literature on policy multipliers, that usually studies the macroeconomic effects of exogenous policy shocks, and, in particular, temporary shocks to policy rules (see e.g. Coenen et al., 2012, and Coenen et al., 2013, for rich reviews of fiscal policy multipliers in the euro area and the US). See below for more details on the effects of policy shocks. A rule-like approach also differs from the early Keynesian approach to policy, which also supported discretionary actions (see e.g. Hofmann and Bogdanova (2012), Taylor (2014)).

\(^5\)See e.g. the discussion in Coenen et al. (2012).

\(^6\)See e.g. Chari et al. (2007).
dends from the Central Bank to the Treasury), as well as extra unconventional ones used by central banks nowadays (extra loans to private banks and direct loans to private firms), are allowed to react to the output gap. We experiment with one instrument at a time, as well as, with combinations. We find that both standard and non-standard monetary activism, although they have demonstrable effects on financial conditions, its effects on the real macro economy are not straightforward. Furthermore, unconventional QE-type monetary programs appear to be effective complements to conventional policies only when specific credit frictions and re-balancing portfolio effects are present (this is similar to the results in Araújo et al. (2015), where the effects of monetary policies depend critically on the type and degree of collateral constraints). All this is relative to passive policy. Also, all this is without fiscal activism.

Then, as a second line of defense against an economic downturn, we also allow for fiscal activism. This means that, in the above described monetary framework, fiscal policy instruments (public spending and tax rates on consumption, labor and profits) are also allowed to react to the output gap. We now find that there can be real benefits from a fiscal-monetary policy mix. In particular, policy activism helps the real economy when fiscal (tax-sending) policy instruments and OMOs react to the output gap at a moderate degree and, at the same time, the nominal interest rate is reacts aggressively to the output gap. This mix can perhaps rationalize the policies having been followed by most central banks and treasuries in several countries since 2008.

The rest of the paper is organized as follows. Section 2 sets up the model. Parameterization and the steady state solution are in section 3. This solution will serve as the departure for our policy experiments. Section 4 deals with the first step as discussed above. Section 5 studies monetary activism. Section 6 presents the effects of fiscal-monetary policy mixes. Section 7 closes the paper.

2 Model

2.1 Brief literature review on monetary transmission mechanism

We consider a medium-scale closed economy DSGE model with a private and a policy sector. The policy sector includes both monetary and fiscal policy authorities. Before we present the model, first descriptively and then formally, we provide a short review of the literature on the modelling of monetary policy and explain how our model relates to this literature (the modelling of fiscal policy is standard and less controversial – it is the efficiency of various fiscal policy instruments that is more controversial).
Monetary policy is transmitted to the real economy through two core channels; the interest-rate and the credit channel. The traditional interest-rate channel of monetary policy describes the neoclassical links between policy interest rates and other asset prices, which, in turn, affect agents’ consumption and investment decisions. The transmission mechanism relies on frictions, like portfolio adjustment problems or wage and price rigidities (see e.g. Walsh, 2010, chapters 5 and 6). Here, we will adapt the latter and, specifically, we will assume Rotemberg-type nominal rigidities.

However, as is widely recognized (see e.g. Walsh, 2010, chapter 11 and in particular p. 511), to the extent that one wishes to study the real effects of the “actual implementation” of monetary policy, one cannot treat the nominal money supply or the interest rate on government bonds (as we typically do in macro models) as variables controlled by the monetary authorities. Instead, one has to use variables like the policy interest rate or OMOs or QE-type policies, as actual policy instruments. This requires the inclusion of financial markets. The latter means the introduction of agent heterogeneity (e.g. agents are distinguished between borrowers and lenders, or there are several types of borrowers and various types of lenders), which allows for financial intermediation.\(^7\)

In this framework, the credit channel arises through imperfections in financial markets; these include problems of adverse selection, moral hazard, asset heterogeneity (e.g. there are bank and non-bank sources of lending), transaction (monitoring or agency) costs, etc.\(^8\) Financial frictions result in interest rate differentials, which, in turn, determine credit. Here, to keep the model tractable, we will model financial heterogeneity and frictions in a simple way by assuming that households differ in their time discount factor (see also e.g. Moore and Kiyotaki, 1997, Iacoviello, 2005, Cúrdia and Woodford, 2010a and 2010b and Gavin, 2015,) and that private banks, which can play a central role in the transmission of monetary policy (see e.g. Walsh, 2010, chapter 10.6.1 for a review of the bank channel), face transaction costs that are increasing in loans and decreasing in reserves and deposits (see also e.g. Uribe and Yue, 2006 and Cúrdia and Woodford, 2010a and 2010b); however, we also study the case in which financial frictions take the form of agency problem between borrowers and lenders (see e.g. Gertle and Kiyotaki, 2010, and Gertler and Karadi, 2011).

\(^7\)Borrowing and lending cannot occur in a representative-agent world so that agents must differ in some way that gives rise to borrowers and lenders. Also, not all borrowers and not all lenders are alike. Popular ways of producing agent heterogeneity include differences in time discount rates (see e.g. Moore and Kiyotaki, 1997, and Iacoviello, 2005), technologies (see e.g. Ajello, 2016, and Brunnermeier and Sannikov, 2014) etc.

\(^8\)For reviews of financial frictions in macro models, see e.g. Walsh (2010, chapter 10.5) and Duncan and Nolan (2017).
2.2 Informal description of the model

The private sector consists of three different groups of households. The first group of households is assumed to consume, work, hold currency and save in form of bank deposits and government bonds; for convenience, we will refer to them as “savers”. The second group of households is assumed to consume, work, hold currency and, being the owners of private firms, receive the profits made by them; for convenience, we will refer to these households as “capitalists”. The third group is assumed to consume, work, hold currency and, being the owners of private banks, receive the profits made by them; for convenience, we will refer to these households as “bankers”.\(^9\) On the production side, private firms are distinguished into firms producing final goods, intermediate goods and capital goods. As in most of the related literature, we model final goods and capital goods firms in a stylized way, while intermediate goods firms act monopolistically and are also assumed to hold currency and borrow from private banks so as to finance their investment and labour costs. Since private firms are owned by capitalists, firms’ time preference rate equals capitalists’ inter-temporal marginal rate of substitution in consumption. Similarly, since private banks are owned by bankers, banks’ time preference rate equals bankers’ inter-temporal marginal rate of substitution in consumption. To get reasonable differences across different interest rates, as well as, positive steady state values of financial variables, we assume that bankers are more patient than savers who are in turn more patient than capitalists.\(^10\) Private banks intermediate between lenders and borrowers, where this financial intermediation introduces a friction a la Cúrdia and Woodford (2010a, 2010b, 2011) according to which giving loans implies an extra cost. We shall start by assuming that private banks make loans to private firms and hold reserves at the central bank as assets, while they receive deposits from households on the liability side. Loans made to firms, deposits received by households and reserves held at the central bank pay different interest rates. At a later stage, with respect to private banks, we shall enrich their liability side by allowing them to receive excess liquidity or loans provided by the central bank (see also below). For simplicity, we assume away interbank lending.

The policy or government sector consists of a Central Bank and the Treasury. We shall start by assuming that the Central Bank holds government bonds on the asset side, while, on

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\(^9\)For similar assumptions of agents heterogeneity, see e.g. Kiyotaki and Moore (1997), Cúrdia and Woodford (2010a, 2010b, 2011), Dib (2010), Brunnermeier and Sannikov (2014), Güntner (2015), Clerc et al. (2015) and Philippopoulos et al. (2017), while e.g. Gertler and Kiyotaki (2011) work with a consolidated family or household consisting of different types of family members.

\(^10\)The main reason for having different types of agents with different degrees of impatience to consume is to allow for financial intermediation, so that borrowers cannot borrow directly from savers at a single interest rate. This facilitates an equilibrium spread between the lending and deposit rate. See Woodford (2010) for a criticism on pre-crisis models with a single interest rate.
the liability side, there is the currency held by the non-bank public (households and private firms) and reserves held by private banks at the central bank. Within this baseline set up, and following e.g. Hall and Reis (2015), Del Negro and Sims (2015) and Benigno and Nisticò (2017), the policy instruments of the central bank include the nominal interest rate paid on reserves (the so-called policy rate), the amount of government bonds purchased by the central bank (the so-called open-market operations) and the remittances or dividends from the central bank to the government. At a later stage, to mimic the unconventional extra measures taken by most central banks since the financial crisis started in 2007-8, we shall enrich the asset side of the central bank by incorporating excess liquidity or loans to the banking sector, as well as, direct loans to the non-bank private sector. In other words, we include both standard and non-standard monetary policy instruments. Finally, the Treasury, or the government, issues government bonds and levies distorting taxes to finance public spending. As said, government bonds can be held by private agents and by the central bank. We assume away the possibility of default on public debt.

Note that, following the literature on DSGE models with financial intermediation and financial frictions, we explicitly study the role of the central bank’s balance sheet. Thus, monetary policy, in the form of interest rate and reserve supply policy, is transmitted indirectly via private banks but it is also transmitted directly to the macro-economy through the purchase of government bonds, the provision of extra liquidity and direct lending to the private sector. Besides, money will not be neutral in the transition because of Rotemberg-type nominal fixities faced by intermediate goods firms, as in New Keynesian DSGE literature.

2.3 Households

In this subsection, we model the three types of households.

**Households as savers**

There are \( N^h \) identical households-savers indexed by superscript \( h = 1, 2, 3, ..., N^h \). Each one of them maximizes lifetime utility given by:

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11 Purchase of public sector securities “on a large scale”, provision of reserves to the banking sector “in excess of regular liquidity needs” and “direct lending operations for the non-bank private sector or purchase of private sector assets” are among the instruments used by most central banks since the financial crisis started in 2007-2008. These policy interventions are also known as quantitative easing (QE). See e.g. Caglar et al. (2011) and McMahon et al. (2015) for different QE policies in different countries. For monetary policy in the euro area, see e.g. European Central Bank (2011 and 2015). The critical role of interest bearing reserves during the recent financial crisis is highlighted in the Bank of England CCBS Handbook (2015) and the paper of Reis (2016). Pattipeilohy (2016) classifies the central banks of major advanced economies as “treasuries holders” in terms of asset composition and “bankers’ bank” in terms of liability composition (i.e. higher liabilities vis-à-vis the domestic private than the public sector), at the end of 2015. Caglar et al. (2011) reports that reserves in the Bank of England have tripled since the introduction of QE policies in 2010.
\[
\sum_{t=0}^{\infty} (\beta^h)^t u(c_t^h, u_t^h, g_t)
\]

where \(c_t^h\) is \(h\)'s consumption, \(u_t^h\) is \(h\)'s hours of work, \(g_t\) is real public spending and \(0 < \beta^h < 1\) is a time preference rate. In our numerical solutions below, for simplicity, we will use the additively separable function \(u(c_t^h, u_t^h, g_t) = \ln c_t^h + \nu^h \ln(1 - u_t^h) + \chi_g \ln (g_t)\), where \(\nu^h, \chi_g\) are preference parameters.

Each \(h\)'s maximization is subject to the budget constraint (written in real terms):

\[
(1 + \tau_t^c) c_t^h + d_{t+1}^h + b_{t+1}^h + m_{t+1}^h = (1 - \tau_t^y) w_t u_t^h + (1 + i_t^d) \frac{p_{t-1}}{p_t} d_t^h + (1 + i_t^b) \frac{p_{t-1}}{p_t} b_t^h + \frac{p_{t-1}}{p_t} m_t^h
\]

where \(d_{t+1}^h\) and \(b_{t+1}^h\) are \(h\)'s end-of-period real bank deposits and real government bond holdings respectively, \(i_t^d\) denotes the nominal interest rate on bank deposits and \(i_t^b\) the nominal interest rate on government bonds between \(t - 1\) and \(t\), \(p_t\) is the current price level, \(m_{t+1}^h\) is \(h\)'s end-of-period real money holdings, \(u_t^h\) is \(h\)'s hours of work, \(w_t\) is the real wage rate, and \(0 < \tau_t^y, \tau_t^c < 1\) are tax rates on labor and consumption respectively.

To give money a role, we use simple cash-in-advance constraints. In this case, the cash-in-advance constraint is defined as:

\[
m_{t+1}^h \geq a^h (1 + \tau_t^c) c_t^h
\]

where \(a^h \geq 0\) is a parameter.

Each household-saver chooses \(\{c_t^h, u_t^h, d_{t+1}^h, b_{t+1}^h, m_{t+1}^h\}_{t=0}^{\infty}\) subject to the above two constraints. The first-order conditions include:

\[
\frac{1}{c_t^h} = \lambda_t^h (1 + \tau_t^c) + a^h \psi_t^h (1 + \tau_t^c)
\]

\[
\lambda_t^h = \beta_t^h \lambda_{t+1}^h (1 + i_t^d) \frac{p_t}{p_{t+1}}
\]

\[
\lambda_t^h = \beta_t^h \lambda_{t+1}^h (1 + i_t^b) \frac{p_t}{p_{t+1}}
\]

\[\text{\textsuperscript{12}}\text{For a similar modeling of money demand, see e.g. Niemann et al (2013), Auerbach and Obstfeld (2005) and Schmitt-Grohé and Uribe (2007).}\]
\[ \lambda^h_t - \psi^h_t = \beta^h \lambda^h_{t+1} \frac{p_t}{p_{t+1}} \]  \hspace{1cm} (4d)

\[ \nu^h_t \frac{1}{1 - u^h_t} = \lambda^h_t (1 - \tau^y_t) w_t \]  \hspace{1cm} (4e)

\[ \psi^h_t \left[ m^h_{t+1} - a^h (1 + \tau^c_t) c^h_t \right] = 0 \]  \hspace{1cm} (4f)

where \( \lambda^h_t \) is the multiplier associated with the budget constraint (2) and \( \psi^h_t \) is the multiplier associated with the cash-in-advance constraint (3). Thus, including the budget constraint in (2) we have 7 equations in \( \{c^h_t, u^h_t, a^h_{t+1}, b^h_{t+1}, m^h_{t+1}, \lambda^h_t, \psi^h_t\}_0^\infty \). In what follows, we will assume that the cash-in-advance constraint is binding all the time, so that \( m^h_{t+1} = a^h (1 + \tau^c_t) c^h_t \) and \( \psi^h_t \neq 0 \) at all \( t \).

**Households as capitalists**

There are \( N^e \) identical households-capitalists indexed by superscript \( e = 1, 2, 3, ..., N^e \). These households own the private firms and hence receive their profits. Each one of them maximizes lifetime utility given by:

\[ \sum_{t=0}^\infty (\beta^e)^t u(c^e_t, u^e_t, g_t) \]  \hspace{1cm} (5)

where notation is as above except that now we use the superscript \( e \) instead of the superscript \( h \). We assume \( 0 < \beta^e < \beta^h < 1 \), namely, households as capitalists are less patient than households as savers; this is, as mentioned above, in order to allow for differences between lending and deposit interest rates in equilibrium. in order to allow for differences between lending and deposit interest rates in equilibrium.\(^{13}\) We will again use \( u(c^e_t, u^e_t, g_t) = \ln c^e_t + \nu^e \ln (1 - u^e_t) + \chi_g \ln (g_t) \) for the period utility function.

The maximization is subject to the budget constraint and the cash-in-advance constraint which are:

\[ (1 + \tau^c_t) c^e_t + m^e_{t+1} = \pi^e_t + (1 - \tau^y_t) w_t u^e_t + \frac{p_{t-1}}{p_t} m^e_t \]  \hspace{1cm} (6)

\[ m^e_{t+1} \geq a^e (1 + \tau^c_t) c^e_t \]  \hspace{1cm} (7)

\(^{13}\)It is usual to assume that agents are heterogeneous in terms of their time preference rates in order to have borrowers and lenders in equilibrium. See e.g. Kiyotaki and Moore (1997), Cúrdia and Woodford (2010a, 2010b, 2011), Dib (2010), Brunnermeier and Sannikov (2014), Garin (2015), Philippopoulos et al. (2017) and many others.
where \( m_{t+1}^e \) is \( e \)'s end-of-period real money holdings, \( u_t^e \) is \( e \)'s hours of work and \( \pi_t^f \) is net profits or dividends distributed by private firms to their owners.

Each household-capitalist chooses \( \{c_t^e, u_t^e, m_{t+1}^e\} \) subject to the above two constraints. The first-order conditions include:

\[
\frac{1}{c_t^e} = \lambda^e_t (1 + \tau_t^e) + a^e \psi_t^e (1 + \tau_t^e) \tag{8a}
\]

\[
\lambda^e_t - \psi_t^e = \beta^e \lambda_{t+1}^e pt \over pt+1 \tag{8b}
\]

\[
\frac{\nu^e}{1 - u_t^e} = \lambda^e_t (1 - \tau_t^y) w_t \tag{8c}
\]

\[
\psi_t^e \left[ m_{t+1}^e - a^e (1 + \tau_t^e) c_t^e \right] = 0 \tag{8d}
\]

Thus, including the budget constraint in (6), we have 5 equations in \( \{c_t^e, u_t^e, m_{t+1}^e, \lambda_t^e, \psi_t^e\} \). In what follows, we will assume that the cash-in-advance constraint is binding all the time, so that \( m_{t+1}^e = a^e (1 + \tau_t^e) c_t^e \) and \( \psi_t^e \neq 0 \) at all \( t \).

**Households as bankers**

There are \( N^b \) identical households-bankers indexed by superscript \( b = 1, 2, 3, ..., N^b \). These households own the private banks and hence receive their profits. Each one of them maximizes lifetime utility given by:

\[
\sum_{t=0}^{\infty} \left( \beta^b \right)^t u \left( c_t^b, u_t^b, g_t \right) \tag{9}
\]

where notation is as above except that now we use the superscript \( b \) instead of the superscript \( h \) or \( e \). We assume \( 0 < \beta^e < \beta^h < \beta^b < 1 \); in other words, households as bankers are more patient than the other two types of households; this is in order to get positive amounts of loans and deposits at the steady state. We will again use \( u(c_t^b, u_t^b, g_t) = \ln c_t^b + \nu^b \ln(1 - u_t^b) + \chi_g \ln \left( g_t \right) \) for the period utility function.

The maximization is subject to the budget constraint and the cash-in-advance constraint which are:

\[
(1 + \tau_t^e) c_t^b + m_{t+1}^b = \pi_t^b + (1 - \tau_t^y) w_t u_t^b + \frac{pt-1}{pt} m_t^b \tag{10}
\]

\[
m_{t+1}^b \geq a^b (1 + \tau_t^e) c_t^b \tag{11}
\]

where \( m_{t+1}^b \) is \( b \)'s end-of-period real money holdings, \( u_t^b \) is \( b \)'s hours of work and \( \pi_t^b \) is net
profits or dividends distributed by private banks to their owners.

Each household-banker chooses \( \{c^b_t, u^b_t, m^b_{t+1}\}_0^\infty \) subject to the above two constraints. The first-order conditions include:

\[
\frac{1}{c^b_t} = \lambda^b_t (1 + \tau^c_t) + a^b \psi^b_t (1 + \tau^c_t) \quad (12a)
\]

\[
\lambda^b_t - \psi^b_t = \beta^b \lambda^b_{t+1} \frac{p_t}{p_{t+1}} \quad (12b)
\]

\[
\frac{\nu^b}{1 - w^b_t} = \lambda^b_t (1 - \tau^y_t) w_t \quad (12c)
\]

\[
\psi^b_t \left[ m^b_{t+1} - a^b (1 + \tau^c_t) c^b_t \right] = 0 \quad (12d)
\]

where \( \lambda^b_t \) is the multiplier associated with the budget constraint (10) and \( \psi^b_t \) is the multiplier associated with the cash-in-advance constraint (11).

Thus, including the budget constraint in (10), we have 5 equations in \( \{c^b_t, u^b_t, m^b_{t+1}, \lambda^b_t, \psi^b_t\}_0^\infty \). In what follows, we will assume that the cash-in-advance constraint is binding all the time, so that \( m^b_{t+1} = a^b (1 + \tau^c_t) c^b_t \) and \( \psi^b_t \neq 0 \) at all \( t \).

### 2.4 Production sector

There are three stages of production and correspondingly three types of firms. Final goods firms produce a single final good by combining differentiated intermediate goods through a Dixit-Stiglitz technology. Intermediate goods firms produce differentiated intermediate goods acting as monopolists in their own product market and by using capital goods and labour services; these firms also face Rotemberg-type nominal fixities in New Keynesian tradition. Capital goods firms use existing capital to produce new capital which is sold to intermediate goods firms. For simplicity, we use simple devices to model these production stages and, as said above, we assume that all these devices/firms are owned by capitalists who receive all profits in a lump-sum fashion.\(^{14}\)

**Production of final goods**

There are \( N^s \) identical final goods firms indexed by superscript \( s = 1, 2, 3, ..., N^s \). Each one of them produces an amount of final output, \( y^s_f \), using intermediate goods, \( y^s_i \), according

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\(^{14}\)For a similar modeling of the production sector, see e.g. Gertler and Kiyotaki (2011), Carillo and Poilly (2013), Brzoza-Brzezina et al (2013), Garín (2015) and Güntner (2015). Alternatively, as mentioned before, we could simply assume, as in Gertler and Kiyotaki (2011), that there is a single household/family and there are different types of household/family members called workers, bankers, etc. All these alternatives are not expected to change the main results (see also the discussion in Gertler and Kiyotaki (2011)).
to the Dixit-Stiglitz CES production function:

\[ y_t^s = \left[ \sum_{f=1}^{N^f} \lambda \left[ y_t^f \right]^\phi \right]^{\frac{1}{\phi}} \]  

(13)

where \( 0 < \phi \leq 1 \), \( N^f \) is the number of intermediate goods firms and \( \lambda = 1/N^f \) to avoid scale effects in equilibrium. Thus, in a symmetric equilibrium below, we will simply have \( y_t^s = y_t^f \equiv y_t \).

Under perfect competition, in each period, the firm chooses \( y_t^f \) to maximize profits:

\[ p_t y_t^s - \sum_{f=1}^{N^f} \lambda p_t^f y_t^f \]  

(14)

where \( p_t \) is the price of the final good and \( p_t^f \) is the price of each intermediate good \( f \).

Taking prices as given, the first-order condition for \( y_t^f \) is:

\[ p_t^f = \left( \frac{y_t^f}{y_t^s} \right)^{\phi-1} p_t \]  

(15)

or equivalently \( y_t^f = \left( \frac{p_t^f}{p_t} \right)^{\frac{1}{\phi-1}} y_t^s \).

Then, if profits are zero in the competitive final-goods sector, we have the price index:

\[ p_t = \left[ \sum_{f=1}^{N^f} \lambda \left[ p_t^f \right]^{\frac{1}{\phi-1}} \right]^{\frac{\phi-1}{\phi}} \]  

(16)

Thus, in a symmetric equilibrium below, we will simply have \( p_t = p_t^f \).

**Production of capital goods**

There are \( N^c \) identical capital producing firms indexed by superscript \( c = 1, 2, 3, \ldots, N^c \). Capital goods producers generate new capital, which they sell to intermediate goods firms at a price, \( q_t \). Following e.g. Güntner (2015) and Gertler and Kiyotaki (2011), we simply assume that their objective is to maximize profits defined as:

\[ \sum_{t=0}^{\infty} [q_t k_{t+1}^c - \delta k_t^c - q_t (1 - \delta) k_t^c] \]  

(16)

where \( i_t^c \) is investment, \( 0 \leq \delta \leq 1 \) is the capital depreciation rate and \( 0 < \beta^c < 1 \) is the

\[ For \text{ simplicity, we do not include capital adjustment costs.} \]
time discount rate. In equilibrium, since firms are owned by capitalists, we will set $(\beta^c)^t \equiv (\beta^e)^t \frac{C^e}{C^e_0}$ (see also e.g. Altug and Labadie, 1994, chapter 4).

The law of motion of the firm’s capital is given by:

$$k_{t+1}^c = (1 - \delta)k_t^c + \xi_t i_t^c$$

(17)

where $\xi_t$ is an investment shock similar to that in e.g. Chari et al. (2007) and Fisher (2006).

The first-order condition for $i_t^c$ implies:

$$q_t = 1$$

(18)

and in turn the capital goods firm’s profits are zero in equilibrium.

**Production of intermediate goods**

There are $N^f$ differentiated intermediate-goods firms indexed by superscript $f = 1, 2, 3, ..., N^f$. Each one of them maximizes the present discounted value of its stream of profits subject to the demand function for its own product, cash-in-advance constraint and Rotemberg-type nominal fixities.\(^\text{16}\) In particular, each firm maximizes:

$$\sum_{t=0}^{\infty} \left( \beta^f \right)^t \pi_t^f$$

(19)

where in equilibrium we will set $(\beta^f)^t \equiv (\beta^e)^t \frac{C^e}{C^e_0}$, since firms are owned by capitalists. In each period, the real profit is defined as:

$$\pi_t^f = (1 - \tau_t^\pi) \left\{ \frac{p_t^f y_t^f}{p_t} - w_t u_t^f \right\} - q_t i_t^f + i_t^{f+1} - (1 + i_t^f) \frac{p_{t-1}^f}{p_t} -$$

(20)

$$-m_t^{f+1} + \frac{p_t-1}{p_t} m_t^f - \frac{\chi}{2} \left( \frac{p_t^f}{p_t^{f-1}} - 1 \right)^2$$

where the first term in brackets on the RHS is the gross profit made by each firm and $0 < \tau_t^\pi < 1$ is the associated tax rate on gross profits (see also e.g. Turnovsky, 1995), $i_t^f$ is investment made by each intermediated goods firm and $q_t$ is the associated price of capital, $i_{t+1}^f$ is the end-of-period loans taken by the firm from private banks and $i_t^f$ is the associated nominal interest rate on these loans between $t-1$ and $t$, $m_t^{f+1}$ is the end-of-period real money

\(^{16}\text{For similar modeling of the firm and its relation to households as capitalists, see e.g. Uribe and Yue (2006) and Garin (2015).}
balances held by each firm and \( \frac{x}{2} \left( \frac{p_t}{p_{t-1}} - 1 \right)^2 \) is a standard Rotemberg-type cost function from price changes.\(^{17}\) At a later stage, we will follow e.g. Gertler and Karadi (2011, 2013), Gertler and Kiyotaki (2011) and Güntner (2015) by assuming that intermediate-goods firms are financially constrained in the sense that they have to finance their capital acquisition in advance of production. In other words, we will assume that the real amount of new loans equals the quantity of new capital at the end of each period, \( l_{t+1}^f = k_{t+1}^f \). This friction introduces feedback effects between the financial and real sectors and, through them, a direct channel of monetary policy transmission.

The motion of capital for each firm is:

\[
k_{t+1}^f = (1 - \delta)k_t^f + \xi_t i_t^f
\]

(21)

The maximization is also subject to the demand function (11), as well as a production function and a cash-in-advance constraint which are respectively:\(^{18}\)

\[
y_t^f = f \left( k_t^f, u_t^f \right) = A_t \left( k_t^f \right)^\alpha \left( u_t^f \right)^{1-\alpha}
\]

(22)

\[
m_t+1^f \geq a^f w_t u_t^f
\]

(23)

Each firm chooses \( \{u_t^f, i_t^f, k_t^f+1, l_{t+1}^f, m_{t+1}^f\}_{t=0}^\infty \) subject to the above constraints. The first-order conditions include:

\[
\left( \frac{1}{\xi_t^f} \right) \lambda_t^e = \beta^e \lambda_{t+1}^e \left( \frac{1}{\xi_{t+1}^f} \right) q_{t+1} \left( 1 - \delta \right) + \beta^e \lambda_{t+1}^e \left( 1 - \tau_{t+1}^p \right) \phi \frac{\partial y_{t+1}^f}{\partial k_{t+1}^f} - \beta^e \lambda_{t+1}^e \chi \left( \frac{p_{t+1}}{p_t} - 1 \right) \frac{p_{t+1}}{p_t} \left( \phi - 1 \right) \frac{\partial y_{t+1}^f}{\partial k_{t+1}^f} - \left( \beta^e \right)^2 \lambda_{t+1}^e \chi \left( \frac{p_{t+2}}{p_{t+1}} - 1 \right) \frac{p_{t+2}}{p_{t+1}} \left( \frac{1 - \phi}{p_{t+1}} \lambda_{t+1}^e \right) \frac{\partial y_{t+1}^f}{\partial k_{t+1}^f}
\]

(24a)

\(^{17}\)Alternatively, we could assume that firms maximize the present value of their net cash flows by making production and financial decisions where the latter include the issuance of equities and corporate bonds (bought by capitalists) and bank loans (supplied by banks). See e.g. Turnovsky (1995, chapter 11) and Altug and Labadie (1994, chapter 4) although in simpler models.

\(^{18}\)The firm’s cash-in-advance constraint is as in e.g. Uribe and Yue (2006).
Thus, including the definition for profits in (20), the law-of-motion of capital (21) and the production function in (22), we have 8 equations in 

\( n, i, k, l, m, f, y, \) at all \( t \).

In what follows, we will assume that the cash-in-advance constraint is binding all the time, so that \( m_{t+1} = a^f w_{t} u_{t}^f \) and \( \psi_{t}^f \neq 0 \) at all \( t \).

As already said, later on we will add, as a QE type policy, direct loans to these firms from the central bank.

### 2.5 Private Banks

There are \( N^b \) identical private banks indexed by superscript \( b = 1, 2, 3, ..., N^b \). In our baseline setup presented so far, private banks hold loans to firms and reserves at the central bank as assets, while their liabilities consist of the funds deposited by savers. For simplicity, as said above, we assume away an interbank market.  

As in the literature on banking, banks intermediate between lenders and borrowers and this introduces a financial friction that adds to the overall cost of credit faced by borrowers. Here, following e.g. Cúrdia and Woodford (2010a, 2010b, 2011), we model this friction in a simple way by assuming that private banks need to pay a cost to manage their assets and liabilities. In particular, this cost increases with the amount of loans made to private firms,

\[ \psi_{t}^f [m_{t+1}^f - a^f w_{t} u_{t}^f] = 0 \]

Thus, in our model, bank intermediation incurs at a simple cost as in Freixas and Rochet (1997, chapter 3), Aghion and Howitt (2009, chapter 6), Cúrdia and Woodford (2010a, 2010b, 2011), Corsetti et el. (2103).
whereas it decreases with the amount of reserves held at the central bank and the amount of
deposits received by savers.\footnote{We can add an extra financial friction like “bad loans” as in e.g. Corsetti et al. (2013), who assume that bad loans are a fraction of total loans, where this fraction increases with the government bond spread. We report that, under such loans, our results remain qualitatively the same.}

It is worth adding here that we have also experimented with alternative financial frictions like those in Gertler and Kiyotaki (2011) and Gertler and Karadi (2011, 2013), who assume an agency problem between lenders and borrowers, which results in an endogenous constraints on banks leverage ratios, and our main results do not change (see below for further details).

The bank’ cost function is assumed to be of the form:

$$\Psi\left(l^b_t, d^b_t, m^r_t\right) = \frac{\eta^l}{2} \left(l^b_t\right)^2 + \frac{\eta^d}{2} \left(d^b_t\right)^2 + \frac{\eta^m}{2} \left(m^r_t\right)^2$$  \hspace{1cm} (25)

where $l^b_t, d^b_t, m^r_t$ are respectively loans made to private firms, deposits received by house-
holds and reserves held at the central bank and where $\eta^l, \eta^d, \eta^m \geq 0$ are the associated cost
parameters. That is, we assume $\Psi^l(.) > 0, \Psi^d(.) < 0$ and $\Psi^r(.) < 0$.

Each private bank maximizes:

$$\sum_{t=0}^{\infty} \left(\beta^b\right)^t \pi^b_t$$  \hspace{1cm} (26)

where, in equilibrium we will set $\left(\beta^b\right)^t = (\beta^b)^t \frac{\lambda^b}{\lambda^b}$, since banks are owned by households as bankers.

In each period, the real profits net of costs is defined as:\footnote{Alternatively, as in the case of firms above, we could assume that banks maximize the present value of their net cash flows where their financial decisions include the issuance of equities and corporate bonds (bought by capitalists). See e.g. Gertler and Kiyotaki (2011), Gertler and Karadi (2011, 2013).}

$$\pi^b_t \equiv \frac{p_{t-1}}{p_t} \left\{ (1 + i^b_t) l^b_t - (1 + i^d_t) d^b_t + (1 + i^r_t) m^r_t \right\} +
+ d^b_{t+1} - l^b_{t+1} - m^r_{t+1} - \Psi\left(l^b_{t+1}, d^b_{t+1}, m^r_{t+1}\right)$$  \hspace{1cm} (27)

The first-order conditions imply:

\begin{align*}
\end{align*}
\[ p^b_{t+1} = \frac{\beta^b (1 + i^b_{t+1}) \frac{p_t}{p_{t+1}} - 1}{\eta^b} \]  
(29a)

\[ \left( \frac{d^b_{t+1}}{d^b_{t+1}} \right)^{-3} = \frac{\beta^b (1 + i^b_{t+1}) \frac{p_t}{p_{t+1}} - 1}{\eta^b} \]  
(29b)

\[ \left( m^r_{t+1} \right)^{-3} = \frac{1 - \beta^b (1 + i^b_{t+1}) \frac{p_t}{p_{t+1}}}{\eta^m} \]  
(29c)

According to these first-order conditions, the supply of loans given to firms and reserves held at the central bank are increasing functions of the lending and the reserve rate respectively, while the demand for deposits is a decreasing function of the deposit rate.\footnote{As mentioned above, we assume that $\beta^e < \beta^h < \beta^b$, in order to get positive amounts of loans and deposits in the steady state solution.}

Later on, we will also allow private banks to hold government bonds (on their asset side) and to receive excess liquidity or loans (the so-called discount loans) provided by the central bank (on their liability side).

### 2.6 The budget constraint of the consolidated private sector

Before we move on to policy, it is worth noting that, if we combine all the above accounting constraints and assume that $N^h = N^e = N^s = N^f = N^b = 1$, in order to avoid scale effects (so that population fractions do not matter in equilibrium), the within-period budget constraint of the consolidated private sector is:

\[(1 + \tau_t^e) \left( c^h_t + c^e_t + c^b_t \right) + q_t i^f_t + b^h_{t+1} + \frac{\chi}{2} \left( \frac{p^f_t}{p^f_{t-1}} - 1 \right)^2 + \frac{p_{t-1}}{p_t} \Psi (b^b_t, d^b_t, m^r_t) = \]

\[= (1 - \tau_t^e) \frac{p^f_t}{p_t} f \left( k^f_t, u^f_t \right) - (\tau_t^b - \tau_t^e) w_t u^f_t + \frac{p_{t-1}}{p_t} \left( 1 + i^b_t \right) b^b_t - \]

\[- \left[ m^h_{t+1} + m^e_{t+1} + m^f_{t+1} + m^b_{t+1} + m^r_{t+1} - \frac{p_{t-1}}{p_t} \left( m^h_t + m^e_t + m^f_t + m^b_t + (1 + i^b_t) m^r_t \right) \right]

\footnote{If reserves are negative ($m^r_t < 0$), i.e. private banks borrow from the central bank, they become a decreasing function of the policy rate.}
2.7 Economic policy

There are two policy authorities, the Treasury and the Central Bank.\textsuperscript{24}

The Treasury (i.e. fiscal authorities)

The within-period budget constraint of the fiscal branch of the government (the so-called Treasury) is in aggregate and real terms:

\[ g_t + \frac{p_t-1}{p_t} (1 + \pi_t^b) b_t^T = \tau^c_t (c_t^h + c_t^e + c_t^b) + (\tau^y_t - \tau^\pi_t) w_t u_t^f + \tau^\pi_t \frac{p_t}{p_t} f (k_t^f, u_t^f) + b_{t+1}^T + rcb_t \quad (31) \]

where \( b_{t+1}^T \) is the end-of-period total real public debt, \( rcb_t \) is real transfers from the central bank to the government,\textsuperscript{25} \( 0 \leq \tau^c_t, \tau^y_t, \tau^\pi_t < 1 \) are tax rates on consumption, labor income and profits respectively, \( g_t \) is total real public spending and \( \pi_t^b \) is the nominal interest rate on government bonds. Note that, in equilibrium, \( b_{t+1}^T = b_{t+1}^{cb} + b_{t+1}^h \), which means that government bonds will be purchased by private agents, \( b_{t+1}^h \), and the Central Bank, \( b_{t+1}^{cb} \).

Following usual practice, we will view the tax rates, \( \tau^c_t, \tau^y_t, \tau^\pi_t \), and government spending as share of output, defined as \( s^g_t \), as the independently set of fiscal policy instruments. Then, \( b_{t+1}^T \) will follow residually to satisfy the flow budget constraint in (31). Strictly speaking, since \( b_{t+1}^T = b_{t+1}^{cb} + b_{t+1}^h \) in equilibrium, where \( b_{t+1}^{cb} \) is treated as a monetary policy instrument (see below), it will be \( b_{t+1}^h \) that follows residually.

The Central Bank (i.e. monetary authorities)

The Central Bank’s liabilities consist of the currency held by households and firms and reserves held by banks, on which it pays the nominal interest rate \( i_t^r \). These liabilities fund in turn the Central Bank’s holdings of government debt, \( b_{t+1}^{cb} \), and the remittances or dividends given to the fiscal authority, \( rcb_t \). However, as already said, to complete the menu of monetary policy instruments used after 2007-8, we will also allow, in section 5, for loans to private banks and to private firms on the asset side of the Central Bank.

The within-period budget constraint of the Central Bank that links changes in assets and liabilities is in aggregate and real terms:

\textsuperscript{24}The analysis of the policy sector follows Walsh (2010, chapter 4) and is also consistent with the analysis in Reis (2017).

\textsuperscript{25}Central banks turn over to the government almost all of the interest earnings on their portfolio of government debt.
\begin{align*}
  b_{t+1}^c + rcb_t + \frac{p_t-1}{p_t} (m_t^h + m_t^e + m_t^f + m_t^b + (1 + \delta_t) m_t^r) & \equiv (32a) \\
  \equiv (1 + \delta_t) \frac{p_t-1}{p_t} b_t^c + m_{t+1}^h + m_{t+1}^e + m_{t+1}^f + m_{t+1}^b + m_{t+1}^r
\end{align*}

which can be rewritten in a more explanatory way as:

\begin{align*}
  \left( b_{t+1}^c - \frac{p_t-1}{p_t} b_t^c \right) + rcb_t & \equiv \delta_t \frac{p_t-1}{p_t} b_t^c + \\
  + [m_{t+1}^h + m_{t+1}^e + m_{t+1}^f + m_{t+1}^r] - \frac{p_t-1}{p_t} [m_t^h + m_t^e + m_t^f + m_t^b + (1 + \delta_t) m_t^r]
\end{align*}

(32b)

Thus, as in Walsh (2010, equation 4.2), the last two terms on the RHS denote the change in the central bank’s own net liabilities, where these liabilities are called high-powered money or the monetary base, because they are the stock of currency held by the non-bank private sector plus the reserves holdings of the banking sector.

Following Hall and Reis (2015), Del Negro and Sims (2015) and Benigno and Nisticò (2017), we will treat the nominal interest rate on reserves, \( i_t \), the real value of the central bank’s government bond purchases or equivalently its open market operations (OMOs), \( b_{t+1}^c \), and the real value of remittances from the central bank to the government, \( rcb_t \), as the independently set monetary policy instruments (as said, later on, we will also add loans made to private banks and firms as monetary policy instruments). Then, to the extent that currency held by the non-bank public as well as reserves held by private banks are demand determined, the central bank’s budget constraint in (32a) or (32b) can provide an extra equation to determine the inflation rate or the price level. The processes of the independently set monetary policy instruments are specified in subsection 2.9 below.

It is important to clarify two issues here. First, there is a rich menu of choices regarding the classification between exogenously set monetary policy instruments and the residually determined one.\textsuperscript{26} Here, we choose this specific classification (namely, the nominal interest rate on reserves, the real value of OMOs and remittances to the fiscal authorities to be the independently set monetary policy instruments) because we find it to be more intuitive as well as more consistent with the conduct of monetary policy followed in practice (see e.g. Mishkin and Eakins (1998)). We report however that we have experimented with several other

\textsuperscript{26}For example, Reis (2017, p. 7) assumes that the monetary policy consists of choices of the interest rate paid on reserves and the balance sheet policies of central bank’s government bond holdings and reserves.
classifications and the main results, especially for the real variables, do not change; these results are available upon request. Second, as is well recognized (see e.g. Walsh, 2010 chapter 4), there are several ways of determining the inflation rate or the price level including the use of money market equilibrium conditions or the intertemporal budget constraint of the consolidated public sector (the latter is the fiscal theory of the price level). Here, other things equal, the inflation rate will be determined by the central bank’s period budget constraint. Again, we report that we have experimented with several other specifications and the main results do not change; these results are available upon request.

2.8 The budget constraint of the consolidated public sector

Before we move on, it is worth noting that, if we combine the above two accounting constraints, (31) and (32a), the budget constraint of the consolidated public sector is:

\[
(1 + i_t^h) \frac{pt-1}{pt} b_t^h \left(c_t^h + c_t^e + c_t^b\right) + (1 + i_t^p) \frac{pt-1}{pt} m_t^p = \tau_t^c c_t^c + (\tau_t^w - \tau_t^f) w_t u_t^f + \tau_t^p \frac{pt}{pt} f\left(k_t^f, u_t^f\right) - g_t +
\]

\[
+ \left[ m_{t+1}^h + m_{t+1}^e + m_{t+1}^f + m_{t+1}^b \right] - \frac{pt-1}{pt} \left( m_t^h + m_t^e + m_t^f + m_t^b \right) + b_{t+1}^h + m_{t+1}^f
\]

where \(b_t^h + m_t^f\) is total liabilities of the two branches of the government (fiscal and monetary), the first term/line on the RHS is tax revenues minus government spending, the second term/line on the RHS is the so-called seigniorage revenue (see Reis (2017), section 4) and the last term/line on the RHS is the end-of-period total liabilities.\(^{27}\)

2.9 Macroeconomic equilibrium

We solve for a symmetric equilibrium in which all firms and all households belonging to a specific type are alike ex-post.

Market-clearing conditions

\(^{27}\)The above accounting relation makes clear that central bank liabilities, \(m_t^b\), should be counted as part of public debt. In other words, as Reis (2017) points out, bank reserves, being a liability of the central bank, they are also a government liability.
The market-clearing conditions in the markets for the final good, labor, investment, capital, loans, deposits and government debt are respectively (for notational simplicity and in order to avoid scale effects, we set as said before $N^h = N^e = N^s = N^c = N^f = N^b \equiv 1$ so that population fractions do not matter in equilibrium):

\[ c^b_t + c^f_t + c^h_t + i_t + g_t + \frac{\chi}{2} \left[ \frac{p_t}{p_{t-1}} - 1 \right]^2 + \frac{p_t}{p_{t-1}} (\Psi(l_t, d_t, m_t^r)) = y_t \]  \hspace{1cm} (34a)

\[ u^f_t = u^h_t + u^c_t + u_b^t \equiv u_t \]  \hspace{1cm} (34b)

\[ i^f_t = i^c_t \equiv i_t \]  \hspace{1cm} (34c)

\[ k^f_{t+1} = k^c_{t+1} \equiv k_{t+1} \]  \hspace{1cm} (34d)

\[ l^f_{t+1} = l^h_{t+1} \equiv l_{t+1} \]  \hspace{1cm} (34e)

\[ d^h_{t+1} = d^b_{t+1} \equiv d_{t+1} \]  \hspace{1cm} (34f)

\[ b^f_{t+1} = b^h_{t+1} + b^b_{t+1} \]  \hspace{1cm} (34g)

**Decentralized equilibrium system**

We can now combine all the above to solve for a decentralized equilibrium for any feasible policy. In this equilibrium, (i) all households maximize utility (ii) all firms maximize profits (iii) all constraints are satisfied and (iv) all markets clear via price flexibility.

The equilibrium system can be summarized by 31 equations in 31 endogenous variables, \{c^b_t, c^f_t, c^h_t, u^b_t, u^c_t, u^f_t, d_{t+1}, l_{t+1}, k_{t+1}, i_t, b^b_{t+1}, m^h_{t+1}, m^f_{t+1}, m^b_{t+1}, m^c_{t+1}, m^r_{t+1}, \lambda^b_t, \lambda^c_t, \lambda^f_t, \psi^b_t, \psi^c_t, \psi^f_t, \psi^h_t, \psi^r_t, \psi^e_t, \psi^s_t, \psi^m_t, l_t, d_t, b^h_t, y_t, w_t, \Pi_t \equiv p_t/p_{t-1} \}_{t=0}^{T}. The system is presented in detail in Appendix 1. This is for given the independently set policy instruments, the exogenous stochastic variables and initial values for the state variables. The next two subsections model the independently set policy instruments and the exogenous stochastic variables.
2.10 Modelling of policy instruments

Following most of the related literature, we adopt a rule-like approach to policy (see e.g. Philippopoulos et al. (2017) for a review). In particular, we assume that the policy instruments can follow feedback, or state-contingent, simple policy rules according to which, in addition to a conventional exogenous AR(1) component, they can also react to a number of endogenously determined macroeconomic indicators. These rules are:

\[ s_t^g = (1 - \rho^g) s_{t-1}^g + \rho^g s_{t-1}^g - \gamma^{g:y} \left( \frac{b_t^g}{y_{t-1}} - \frac{b_t^g}{y} \right) - \gamma^{g:y} (y_t - y) + \varepsilon_t^g \]  

(35a)

\[ \tau_t^c = (1 - \rho^c) \tau_{t-1}^c + \gamma^{c:y} \left( \frac{b_t^c}{y_{t-1}} - \frac{b_t^c}{y} \right) + \gamma^{c:y} (y_t - y) + \varepsilon_t^c \]  

(35b)

\[ \tau_t^y = (1 - \rho^y) \tau_{t-1}^y + \gamma^{y:y} \left( \frac{b_t^y}{y_{t-1}} - \frac{b_t^y}{y} \right) + \gamma^{y:y} (y_t - y) + \varepsilon_t^y \]  

(35c)

\[ \tau_t^\pi = (1 - \rho^\pi) \tau_{t-1}^\pi + \gamma^{\pi:y} \left( \frac{b_t^\pi}{y_{t-1}} - \frac{b_t^\pi}{y} \right) + \gamma^{\pi:y} (y_t - y) + \varepsilon_t^\pi \]  

(35d)

\[ b_{t+1}^{cb} = \left( 1 - \rho^{cb} \right) b_{t}^{cb} + \rho^{cb} b_{t}^{cb} - \gamma^{cb:y} (y_t - y) + \varepsilon_{t}^{b28} \]  

(35e)

\[ rcb_{t+1} = \left( 1 - \rho^{rcb} \right) rcb_t + \rho^{rcb} rcb_t - \gamma^{rcb:y} (y_t - y) + \varepsilon_{t}^{rcb29} \]  

(35f)

\[ \log (1 + i_{t+1}) = (1 - \rho^{i}) \log (1 + i_t) + \rho^{i} \log (1 + i_t) + \gamma^{i:x} \log (\Pi_t/\Pi) + \gamma^{i:y} \log (y_t/y) + \varepsilon_{t}^{i} \]  

(35g)

where \( \gamma \)'s are feedback policy coefficients, variables without time subscripts denote steady state values, the \( 0 \leq \rho \)'s \( \leq 1 \) are persistence parameters, and policy shocks, \( \varepsilon_{t}^{p} \equiv (\varepsilon_{t}^{g}, \varepsilon_{t}^{c}, \varepsilon_{t}^{y}, \varepsilon_{t}^{\pi}, \varepsilon_{t}^{b}, \varepsilon_{t}^{rcb}, \varepsilon_{t}^{i}) \), are assumed to be iid, \( \varepsilon_{t}^{p} \sim NIID (0, \sigma_p^2) \).

Two things are important in the above rules. The first is the macroeconomic indicators selected. The second is the magnitude of the feedback policy coefficients. We discuss them in turn.

---

28 Following Gertler and Karadi (2011, 2013), we could also allow central banks' purchases of government debt to react positively to the deviation of government bond spread from its steady state value. In our set up, we report that this is not important.

29 Benigno and Nistico (2017) discuss alternative specifications of the dividend rules.
Regarding macroeconomic indicators, our selection, although ad hoc, is as in most of the related literature. In other words, we allow most of the policy instruments, to react to the output gap defined as the deviation of current output from its deterministic steady state value. Notice that policy instruments react to this gap in a counter-cyclical manner; this is what we label as policy activism. In addition, we assume that the policy interest rate in (35g) can also react to inflation, as is the case in the standard Taylor rule. With respect to fiscal policy instruments, we also allow them to react to public debt imbalances defined as the deviation of the inherited public debt-to GDP ratio from its steady state value or from a target value below the data average. In general, this is required for dynamic stability.

Regarding the magnitude of the feedback policy coefficients (the \( \gamma \)'s), it is widely recognized that their values can be an important factor in the variation of dynamic determinacy and/or multipliers across different models. Since their values matter along the transition path only, we will discuss this issue after we solve for the steady state. At this stage, we just report that here, following most of the literature (see e.g. Walsh, 2010, p. 341), when we add feedback policy rules like the above to the model, we just ensure that the values of the associated feedback policy coefficients do not render the system unstable or introduce multiple equilibria.

### 2.11 Modelling of exogenous stochastic variables

Except from the policy shocks defined above, all other shocks (namely, TFP and investment shocks) are assumed to follow AR(1) stochastic processes of the form:

\[
\log A_{t+1} = (1 - \rho^A) \log A + \rho^A \log A_t + \varepsilon^A_t
\]

\[
\log \xi_{t+1} = (1 - \rho^\xi) \log \xi + \rho^\xi \log \xi_t + \varepsilon^\xi_t
\]

where variables without time subscripts denote steady state values, \( 0 \leq \rho^A, \rho^\xi \leq 1 \) are persistence parameters, while \( \varepsilon^A_t \sim NIID(0, \sigma^2_A) \) and \( \varepsilon^\xi_t \sim NIID(0, \sigma^2_\xi) \) are random variables.

---

30 Following several papers in the literature on monetary policy (see e.g. Christiano and Eichenbaum (2005), Smets and Wouters (2007), Corsetti et al. (2013) and Carrillo and Poilly (2013)), in equation (29g), we treat the gross interest rate as an instrument. Using the net rate is not important to our results.


2.12 What is next and solution methodology

The above model will be solved numerically. Section 3 presents parameter values and the steady state solution. This steady state solution will serve as the baseline for all experiments. In particular, in section 4, we will assume that the economy is initially at this steady state and experiences unanticipated shocks with known properties. We will experiment with various shocks but, as we report below in more detail, a single adverse temporary TFP shock can account relatively well for a recession scenario.\(^{33}\) Then, in the next three sections, 4, 5 and 6, transition dynamics will be driven by this TFP shock only. In section 4, we study passive policy. In section 5, we study monetary activism using standard and non-standard instruments. In section 6, we add fiscal activism. Throughout the paper, we compute linear transition dynamics using the Dynare toolbox.

3 Parameterization and steady state solution

This section presents parameter values and the resulting steady state solution.\(^{34}\)

3.1 Parameter values and policy instruments

Regarding preference and technology parameters, we will use typical values borrowed from the literature. For instance, from the Euler equations for deposits and loans, the value of time preference rates for savers, \(\beta^h\), and capitalists, \(\beta^e\), follow so as to be consistent with typical steady state values of deposit and lending rates which are 1% and 5% respectively (see also Garín, 2015). The time preference rates for bankers, \(\beta^b\), is set to a value higher than \(\beta^h\), in order ensure a positive quantity of loans and deposits at the steady state. Also, the coefficients in the cash-in-advance constraints for firms and households, \(\alpha^h, \alpha^e, \alpha^f\), are set equal to 1 as in Niemann et al. (2013) and Auerbach and Obstfeld (2005). Finally, in the private banks’ cost function, the coefficients of loans and reserves are chosen so as to get a loan to reserve ratio approximately equal to 3.

Regarding the exogenous stochastic variables of TFP and the investment shock, we set \(\rho^j = 0.95\) and \(\sigma^j = 0.1\), \(j = \alpha, \xi\), for the persistence parameters and standard deviation respectively. For symmetry, we choose the same standard deviation for all policy instruments, whereas their persistence parameters are set at 0.

\(^{33}\)We do not try to reproduce a scenario identical to the situation during the 2007-8 crisis. Our aim is to study policy activism in a recession episode.

\(^{34}\)The steady state system is presented in Appendix 2.
Concerning the steady state values of monetary variables, we set $b^c / y = 0.1$, which is approximately the QE1 in the USA, while we set the reserve interest rate at $i^r = 0.001$, which is close to the ZLB. For the steady state value of fiscal variables, as well as for preference and technology parameters not discussed so far, we will use common values in the literature.

All preference-technology parameters, parameters and policy values related to fiscal policy and parameters and policy values related to monetary policy are listed respectively in Tables 1a, 1b and 1c.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Home</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^h$</td>
<td>0.99</td>
<td>savers’ discount factor</td>
</tr>
<tr>
<td>$\beta^e$</td>
<td>0.9524</td>
<td>capitalists’ discount factor</td>
</tr>
<tr>
<td>$\beta^b$</td>
<td>0.995</td>
<td>bankers’ discount factor</td>
</tr>
<tr>
<td>$\nu^h$</td>
<td>2</td>
<td>disutility of labor for savers</td>
</tr>
<tr>
<td>$\nu^e$</td>
<td>1</td>
<td>disutility of labor for capitalists</td>
</tr>
<tr>
<td>$\nu^b$</td>
<td>1</td>
<td>disutility of labor for bankers</td>
</tr>
<tr>
<td>$\chi_g$</td>
<td>0.1</td>
<td>preference parameter for public good</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.05</td>
<td>depreciation rate of capital</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>share of capital</td>
</tr>
<tr>
<td>$\alpha^h$</td>
<td>1</td>
<td>coefficient in workers’ cash-in-advance</td>
</tr>
<tr>
<td>$\alpha^e$</td>
<td>1</td>
<td>coefficient in capitalists’ cash-in-advance</td>
</tr>
<tr>
<td>$\alpha^b$</td>
<td>1</td>
<td>coefficient in bankers’ cash-in-advance</td>
</tr>
<tr>
<td>$\alpha^f$</td>
<td>1</td>
<td>coefficient in firms’ cash-in-advance</td>
</tr>
<tr>
<td>$\chi$</td>
<td>1.2</td>
<td>coefficient in Rotemberg-type costs</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.9</td>
<td>product mark up</td>
</tr>
<tr>
<td>$\eta^l$</td>
<td>0.064</td>
<td>loans’ coefficient in cost function</td>
</tr>
<tr>
<td>$\eta^d$</td>
<td>0.005</td>
<td>deposits’ coefficient in cost function</td>
</tr>
<tr>
<td>$\eta^m$</td>
<td>0.00005</td>
<td>reserves’ coefficient in cost function</td>
</tr>
<tr>
<td>$A$</td>
<td>1</td>
<td>steady state TFP</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1</td>
<td>steady state capital quality shock</td>
</tr>
<tr>
<td>$\rho^\alpha$</td>
<td>0.95</td>
<td>persistence of TFP</td>
</tr>
<tr>
<td>$\rho^\xi$</td>
<td>0.95</td>
<td>persistence of capital quality shock</td>
</tr>
<tr>
<td>$\sigma^\alpha$</td>
<td>0.05</td>
<td>st.dev. of TFP</td>
</tr>
<tr>
<td>$\sigma^\xi$</td>
<td>0.05</td>
<td>st.dev. of capital quality shock</td>
</tr>
<tr>
<td>Parameter</td>
<td>Home</td>
<td>Description</td>
</tr>
<tr>
<td>-----------</td>
<td>------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>$s^g$</td>
<td>0.426</td>
<td>government spending as share of GDP</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.19</td>
<td>consumption tax rate</td>
</tr>
<tr>
<td>$\tau^y$</td>
<td>0.38</td>
<td>labor tax rate</td>
</tr>
<tr>
<td>$\tau^\pi$</td>
<td>0.30</td>
<td>capital tax rate</td>
</tr>
<tr>
<td>$\rho^g$</td>
<td>0</td>
<td>persistence of government spending</td>
</tr>
<tr>
<td>$\rho^c$</td>
<td>0</td>
<td>persistence of consumption tax rate</td>
</tr>
<tr>
<td>$\rho^y$</td>
<td>0</td>
<td>persistence of labor tax rate</td>
</tr>
<tr>
<td>$\rho^\pi$</td>
<td>0</td>
<td>persistence of profit tax rate</td>
</tr>
<tr>
<td>$\gamma^g,b$</td>
<td>0.05</td>
<td>response of government spending to public debt</td>
</tr>
<tr>
<td>$\gamma^g,y$</td>
<td>0</td>
<td>response of government spending to output gap</td>
</tr>
<tr>
<td>$\gamma^c,b$</td>
<td>0</td>
<td>response of consumption tax rate to public debt</td>
</tr>
<tr>
<td>$\gamma^c,y$</td>
<td>0</td>
<td>response of consumption tax rate to output gap</td>
</tr>
<tr>
<td>$\gamma^y,b$</td>
<td>0</td>
<td>response of labor tax rate to public debt</td>
</tr>
<tr>
<td>$\gamma^y,y$</td>
<td>0</td>
<td>response of labor tax rate to output gap</td>
</tr>
<tr>
<td>$\gamma^\pi,b$</td>
<td>0</td>
<td>response of profit tax rate to public debt</td>
</tr>
<tr>
<td>$\gamma^\pi,y$</td>
<td>0</td>
<td>response of profit tax rate to output gap</td>
</tr>
<tr>
<td>$\sigma^g$</td>
<td>0.05</td>
<td>st.dev. of government spending</td>
</tr>
<tr>
<td>$\sigma^c$</td>
<td>0.05</td>
<td>st.dev. of consumption tax rate</td>
</tr>
<tr>
<td>$\sigma^y$</td>
<td>0.05</td>
<td>st.dev. of labor tax rate</td>
</tr>
<tr>
<td>$\sigma^\pi$</td>
<td>0.05</td>
<td>st.dev. of profit tax rate</td>
</tr>
</tbody>
</table>
Table 1c: Monetary policy instruments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Home</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i^r$</td>
<td>0.001</td>
<td>interest rate on reserves</td>
</tr>
<tr>
<td>$b^{cb}/y$</td>
<td>0.1</td>
<td>CB government debt as % of GDP</td>
</tr>
<tr>
<td>$\rho^i$</td>
<td>0</td>
<td>persistence of policy interest rate</td>
</tr>
<tr>
<td>$\rho^b$</td>
<td>0</td>
<td>persistence of CB government debt</td>
</tr>
<tr>
<td>$\rho^{rcb}$</td>
<td>0</td>
<td>persistence of remittances</td>
</tr>
<tr>
<td>$\gamma^{i,\pi}$</td>
<td>1.5</td>
<td>response of policy interest rate to inflation</td>
</tr>
<tr>
<td>$\gamma^{i,y}$</td>
<td>0</td>
<td>response of policy interest rate to output gap</td>
</tr>
<tr>
<td>$\gamma^{cb,y}$</td>
<td>0</td>
<td>response of CB government debt to output gap</td>
</tr>
<tr>
<td>$\gamma^{rcb,y}$</td>
<td>0</td>
<td>remittances response to output gap</td>
</tr>
<tr>
<td>$\sigma^i$</td>
<td>0.05</td>
<td>st.dev. of policy interest rate</td>
</tr>
<tr>
<td>$\sigma^b$</td>
<td>0.05</td>
<td>st.dev. of CB government debt</td>
</tr>
<tr>
<td>$\sigma^{rcb}$</td>
<td>0.05</td>
<td>st.dev. of CB remittances</td>
</tr>
</tbody>
</table>

3.2 Steady state solution

Using the above parameterization, we now solve numerically for the steady state of the equilibrium system as defined above. In this steady state, we exogenously set the gross inflation rate at one, $\Pi = 1$, and we allow remittances, $rcb$, to be endogenously determined.

Also, since the policy instruments respond to deviations of macroeconomic indicators from their long-run values, the feedback policy coefficients in the policy rules play no role in the steady state.

The solution is reported in Table 2. The main macroeconomic variables and the great ratios take reasonable values which are not far away from their data average values. This solution can therefore serve as a departure point in what follows.
Table 2: Steady state solution

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^h$</td>
<td>savers’ consumption</td>
<td>0.1450</td>
</tr>
<tr>
<td>$c^e$</td>
<td>workers’ consumption</td>
<td>0.3125</td>
</tr>
<tr>
<td>$c^b$</td>
<td>bankers’ consumption</td>
<td>0.2131</td>
</tr>
<tr>
<td>$u^h$</td>
<td>savers’ hours worked</td>
<td>0.3079</td>
</tr>
<tr>
<td>$c/y$</td>
<td>consumption-to-GDP</td>
<td>0.4575</td>
</tr>
<tr>
<td>$u^e$</td>
<td>capitalists’ hours worked</td>
<td>0.3307</td>
</tr>
<tr>
<td>$u^b$</td>
<td>bankers’ hours worked</td>
<td>0.4923</td>
</tr>
<tr>
<td>$y$</td>
<td>output</td>
<td>1.4659</td>
</tr>
<tr>
<td>$m^h$</td>
<td>savers’ money holdings</td>
<td>0.1725</td>
</tr>
<tr>
<td>$m^e$</td>
<td>capitalists’ money holdings</td>
<td>0.3719</td>
</tr>
<tr>
<td>$m^b$</td>
<td>bankers’ money holdings</td>
<td>0.2536</td>
</tr>
<tr>
<td>$m^f$</td>
<td>firms’ money holdings</td>
<td>0.8277</td>
</tr>
<tr>
<td>$m^r$</td>
<td>reserves</td>
<td>0.2333</td>
</tr>
<tr>
<td>$m/y$</td>
<td>money-to-GDP</td>
<td>1.27</td>
</tr>
<tr>
<td>$k$</td>
<td>capital</td>
<td>3.0483</td>
</tr>
<tr>
<td>$k/y$</td>
<td>capital-to-GDP</td>
<td>2</td>
</tr>
<tr>
<td>$i$</td>
<td>investment</td>
<td>0.095</td>
</tr>
<tr>
<td>$i/y$</td>
<td>investment-to-GDP</td>
<td>0.1</td>
</tr>
<tr>
<td>$l$</td>
<td>loans</td>
<td>0.7</td>
</tr>
<tr>
<td>$d$</td>
<td>deposits</td>
<td>1.178</td>
</tr>
<tr>
<td>$b^h$</td>
<td>private government debt</td>
<td>0.888</td>
</tr>
<tr>
<td>$b^r/y$</td>
<td>total debt-to-GDP</td>
<td>0.7</td>
</tr>
<tr>
<td>$rcb$</td>
<td>Central Bank’s real transfers</td>
<td>0.0012</td>
</tr>
<tr>
<td>$\pi^f$</td>
<td>firms’ profits</td>
<td>0.2594</td>
</tr>
<tr>
<td>$\pi^b$</td>
<td>banks’ profits</td>
<td>0.0065</td>
</tr>
<tr>
<td>$w$</td>
<td>real wage rate</td>
<td>0.81</td>
</tr>
<tr>
<td>$i^b$</td>
<td>nominal government debt rate</td>
<td>0.01</td>
</tr>
<tr>
<td>$i^l$</td>
<td>nominal lending rate</td>
<td>0.05</td>
</tr>
<tr>
<td>$i^d$</td>
<td>nominal deposit rate</td>
<td>0.01</td>
</tr>
<tr>
<td>$i^r$</td>
<td>nominal reserves rate</td>
<td>0.001</td>
</tr>
</tbody>
</table>
4 Shocks and transition dynamics under passive policy

In this section, we start by studying the dynamic behavior of the model under passive policy. This means that none of the monetary and fiscal policy instruments is allowed to react to the output gap or to any other indicator of real economic activity. We will only allow the policy interest rate to respond to price inflation (as in the standard Taylor rule) and the government spending to-GDP-ratio to respond to the public debt-to-GDP ratio; these responses are required for dynamic stability. To put the same thing differently, if monetary policy reacts to inflation and fiscal policy does not react to the public debt gap, the equilibrium is dynamically indeterminate.

Transition dynamics will be driven by a temporary 5% negative TFP shock only. We report however that, in addition to TFP shocks, we have experimented with shocks to investment, tax rates and government spending, which are also the four types of shocks employed by Chari et al. (2007) in their business cycle study of the US economy over the postwar period. We have experimented with one shock at a time as well as with mixes of shocks at the same time. Since the addition of other shocks does not appear to add anything substantial, we will report results with TFP shocks only (transition dynamics driven by shocks to investment, tax rates and government spending shocks are presented in Appendix 3).

Simulated impulse response functions (IRFs) for the main macroeconomic variables as well as for various spreads are shown in Figure 1. Specifically, \( y \) denotes real output, \( ivv \) is for investment, \( k \) is for the capital stock, \( c \) is total private consumption, \( u \) is hours of work, \( infl \) is for price inflation, \( il \) is the nominal lending rate, \( id \) is the nominal deposit rate, \( ib \) is the nominal government bond rate, \( imb \) is the nominal reserves rate, \( l \) is for loans, \( d \) is for deposits, \( bh \) is private government debt holdings, \( bcb \) is CB's government debt holdings, \( by \) is total public debt-to-GDP ratio, \( mh \) is savers’ money holdings, \( me \) is capitalists’ money holdings, \( mf \) is firms’ money holdings, \( mb \) is bankers’ money holdings and \( mr \) is banks’ reserves.
The results in Figure 1 show that an adverse temporary TFP shock, propagated by the internal dynamics of our theoretical model, can on its own produce time paths that mimic relatively well the key features of a (“great”) recession episode. For instance, output falls by 10%, investment, consumption and work hours also fall, while the public debt-to-GDP ratio rises. Deposits increase, but banks increase their reserves at the Central Bank instead of loans. Regarding prices, the inflation rate increases, although this increase is small in magnitude and lasts for some periods after the shock, and so does the policy interest rate on reserves (since it has been assumed to react to inflation via a Taylor type rule). The rest of nominal interest rates behave similarly to the policy rate.

\[\text{We report that the behavior of the inflation rate depends on the specification of the dividends rule. For example, if we assume that the Central Bank has in its charter to pay out a dividend equal to its net income (see e.g. Hall and Reis (2015) and Benigno and Nisticò (2017)), the inflation rate exhibits a decrease of almost the same magnitude.}\]
5 Monetary activism

As a first policy experiment, we consider monetary policy only and study whether it can mitigate the recessionary effects of a negative TFP shock upon the core macroeconomic variables. In particular, we allow monetary policy instruments to react to the output gap, while fiscal policy is kept passive until the next section.

We start our policy experiments with a standard monetary policy reaction to a slowing economy; the policy rate follows a Taylor-type rule according to which the central bank reacts to inflation and the output gap. This is in subsection 5.1. Then, we continue with non-standard monetary policies. Specifically, in subsection 5.2, monetary policy is conducted via the policy interest rate on reserves and the real value of the central bank’s government bond holdings, which are the two policy instruments typically employed by central banks. Both instruments are allowed to respond relatively mildly to the output gap and, in addition, the policy interest rate reacts to inflation as in the standard Taylor rule. In subsection 5.3, we keep the policy interest rate constant at a low value and allow the OMOs only to react to the state of the economy. Subsection 5.4 is like subsection 5.2, but we also allow for countercyclical response of the third monetary policy instrument available, namely, the remittances from the central bank to the Treasury. In subsection 5.5, we assume a stronger degree of monetary activism by increasing the feedback coefficients of all three policy instruments studied so far on the output gap. To better understand the implications of each policy, and possibly identify the policy that helps more the real economy, we experiment with one instrument at a time and contrast the results in a common graph. Finally, in the last subsection 5.6, we enrich the menu of monetary policy instruments by incorporating unconventional, quantitative easing (QE) type programs. Specifically, in subsection 5.6.1, we study a policy of direct lending to private firms, while, in subsection 5.6.2, we study a discount window policy. Such QE type-policies resemble those followed by most central banks in the aftermath of the 2008 world crisis, as discussed in subsection 2.1 above.

To be able to understand the logic of our results, we will consider one policy change at a time. Also, in each new policy experiment, the new results will be compared to the passive policy studied in section 4. That is, in each figure that follows below, the dotted lines will repeat the passive case of section 4.

5.1 Standard monetary policy

As a first experiment of countercyclical monetary policy, we assume that the central bank aims to stabilize the economy and allows the policy rate to react to the deviations of output from
its steady state value. Specifically, we switch on the feedback reaction of the policy rate to the output gap and set the respective feedback policy coefficients, $\gamma^{i,y}$, set at a relatively mild degree, say 0.2, which is a value within commonly used parameter ranges.\footnote{For instance, see Leeper (1991), Schmitt-Grohe and Uribe (2007), Forni et al. (2009), Bi et al. (2013), Coenen et al. (2013), Kliem and Kriwoluzky (2014), Philippopoulos et al. (2015, 2016, 2017) and Coenen et al. (2012).}

IRFs for this scenario are reported in Figure 2, while, in the companion Figure 2a, we report the IRFs for the real interest rates, $ril, ri, rib, rid, rimb$, which are respectively the rates of loans, government bonds, deposits and reserves. As said above, transition dynamics are driven by a temporary adverse shock to TFP. Also, we include the IRFs under the passive policy studied in section 4 above.

Inspection of the IRFs in Figure 2 implies that the real effects of standard monetary activism are quantitatively small.\footnote{See also subsection 5.4 for larger scale government bond purchases by the Central Bank.} Specifically, this policy hurts the real economy on impact, while it helps it marginally in the transition (see e.g. the time paths of output, capital, work hours and consumption), mainly through the lower inflation and nominal interest rates.\footnote{Starting with Wallace (1981), the literature has shown that when OMOs trade reserves for short-term government debt, just two forms of government liabilities, they do not affect much the equilibrium outcomes. See Reis (2017).} On the positive side of effects, the public debt-to-GDP ratio falls below, whereas loans rise above, their respective steady states values in the medium run.

Summing up, the above results raise doubts about the efficacy of standard monetary activism implemented via an operational Taylor rule. Furthermore, as mentioned in subsection 2.4 above, we report that our main results remain unchanged when we introduce an agency problem between banks and their respective depositors (modeling and numerical details are available upon request).
Figure 2: Reaction to the output gap using the policy rate

Figure 2a: IRFs of real interest rates using the policy rate
5.2 Open market operations

In this subsection, we experiment with the policy interest rate and OMOs, while remittances from the central bank to the Treasury are kept constant at their steady state value. Specifically, the policy rate and bond purchases react to the output gap with the associated feedback policy coefficients, $\gamma^h$ and $\gamma^{cb}$, set at the same degree of 0.2.\footnote{Considering other indicators of recession, we could allow open market operations to react to spreads, as used in the recent literature on QE (e.g. in Gertler and Kiyotaki (2011), Gertler and Karadi (2011, 2013) and Corsetti et al (2013)). However, in our framework, spreads may not be a very informative indicator of recession (under passive policy they increase slightly and only for a few initial periods) unless additional financial frictions are added.}

Notice that, since our goal is to study the potential merits of policy activism rather than to compute the best policy mix, we apply uniformly the same value of feedback policy reaction to the output gap across all monetary policy instruments (we will do the same with fiscal policy instruments below). As said, in this section, we assume away any C.B. remittances and fiscal reaction to the output gap. In other words, the central bank purchases government bonds by extending liquidity to the banking sector, while it also allows its policy rate and to respond to the output gap. IRFs for this scenario are reported in Figure 3 and the companion Figure 3a. For comparison, we include the IRFs under the passive policy, studied in section 4, and under the simple Taylor rule, studied in section 5.1 above.

The new IRFs seem to verify the neutrality of OMOs.\footnote{The irrelevance of standard OMOs for equilibrium prices and quantities was first discussed by Wallace (1981). See, also, Benigno and Nisticò (2017) for a characterization of the theoretical conditions supporting that property.} Compared to the standard form of monetary activism of previous subsection, the portfolio effects of OMOs induce only a slightly higher decrease of private debt holdings and a subsequently lower decrease of deposits.

Summing up, our results so far indicate the need for additional countercyclical monetary policies on the top of a mild reaction of the policy interest rate and OMOs to the output gap.
Figure 3: Reaction to the output gap by using Taylor rule and OMOs

Figure 3a: IRFs of real interest rates using Taylor rule and OMOs
5.3 Fixing the policy interest rate at a low level

To mimic the constantly low interest rate policy employed during the recent financial crisis, we now assume that the central bank abandons the state-contingent Taylor rule in (35g) and, instead, keeps the nominal interest rate on reserves constant at a low, almost zero, level over time. The government spending to-GDP-ratio still responds to the public debt-to-GDP ratio (this response is required for dynamic stability). Specifically, the policy rate, $i^r$, is now kept constant at its steady state value (which is $i^r = 0.001$) all the time, i.e. we switch off the feedback response to inflation and the output gap in (35g). All other things are equal.

Figure 4: Reaction to the output gap by using OMOs while keeping the policy interest rate constant
The IRFs in Figure 4 show that this policy mix is very similar to passive policy.\textsuperscript{41} Compared to the latter, the real economy marginally benefits and this happens mainly via a higher decrease of the public debt-to-GDP ratio and an increase in work hours. On the other hand, when the central bank does not allow its interest rate instrument to co-move with inflation, it loses its ability to hit a possible inflation target and inflation falls below its steady state value after period 1. This result seems to be close to reality, where, at least in the eurozone, after ten years of extensive OMO programs there is no upsurge in inflation and still a rather sluggish recovery.

Summing up, when the policy interest rate is kept constant, OMOs have small effects on the time paths of the main macroeconomic variables. In the next section, however, we will show that the potency of such a monetary policy mix is enhanced if it is combined with fiscal policy activism.

5.4 Adding remittances to the Treasury as a weapon of counter-cyclical policy

Here, we switch on the feedback response of remittances to the output gap. In other words, we now allow all three conventional monetary instruments to react counter-cyclically to the recession. In doing so, we set $\gamma^{i,y} = \gamma^{rb,y} = \gamma^{rcb,y} = 0.2$, so that this reaction is relatively mild.

\textsuperscript{41}See Bernanke (2017).
We define this policy as *active monetary policy*.

Results are reported in Figures 5 and 5a. In these figures, for comparison, we include the passive case studied in section 4 (dotted grey line), the case of countercyclical OMOs as studied in the subsection 5.2 (black line) and the new case of active monetary policy (blue line).

The IRFs in Figure 5 imply that a countercyclical response of remittances can add small benefits to the case of OMOs (see e.g. the time paths of output, capital and work hours). This policy mix strengthens the portfolio effects of OMOs, which effects result in a sharp reduction of both private debt and total public debt-to-GDP ratio. Private agents, namely savers, being indifferent between government bonds and deposits, respond to the higher supply of governments bonds by rebalancing their portfolios towards deposits. On the other hand, consumption would be better under OMOs alone at all times, because of the adverse inter-temporal substitution effect coming from lower real interest rates that discourage savings in the short term and hence consumption over time. Furthermore, lower interest rates do not seem to trigger any movement into cash; all private money holdings suffer a higher decrease. Regarding prices, we observe a more persistent increase of the inflation rate as well as of nominal interest rates.

Summing up, when the central bank adds remittances to the Treasury as a weapon of counter-cyclical policy, it can significantly affect the total public debt. However, overall real effects of this policy appear to be negligible.
Figure 5: Active monetary policy reaction to the output gap

Figure 5a: IRFs of real interest rates using active monetary policy
Does it matter who holds government debt?

In this paragraph, we discuss how the above results of monetary activism depend on which private agent holds government debt. In our baseline model, it is households as savers (see subsection 2.2). We now modify the model by assuming that government bonds are purchased by households as capitalists, instead of households as savers, so that, perhaps, the portfolio effects can help the real economy. The idea is that the policy mix of countercyclical OMOs and remittances delivers stronger portfolio effects, which could help investment, and finally output, as capitalists switch to capital. 42

The new IRFs under monetary activism are presented in figures 6 and 6a, whereas the new equilibrium system is presented in Appendix 4.

A comparison of Figures 5 and 6 verifies the importance of the portfolio effects of OMOs; see the time paths of capital, investment and output which are now clearly above their counterparts in the passive case at least after the first negative impact effect. As a final note, we observe that the real government bond rate (now equalized to the lending rate) increases. 43

Summing up, the benefits of government debt purchases by the Central Bank depend on who holds public debt, since this shapes the portfolio effects. If there is a strong portfolio effect (meaning a switch from government bonds to capital made possible by OMOs by the central bank), this can benefit the real economy relative to the passive case.

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42 Portfolio rebalancing effects were the main explanation of ECB’s policy. For a theoretical background on these effects, see, for example Zampolli (2012) for an overview of the theory on the portfolio rebalancing effects, starting from the early works of Tobin and Friedman, or Greenwood and Vayanos (2008) and Vayanos and Vila (2009).

43 The increase in the government bond rate under OMOs has been verified empirically by Gambacorta, Hoffman, and Peersman (2011).
Figure 6: Active monetary policy reaction to the output gap when capitalists hold government debt

Figure 5: IRFs of real interest rates using active monetary policy when capitalists hold government debt
5.5 Stronger monetary activism

In this subsection, we allow for a stronger degree of monetary activism by increasing the feedback coefficients on the output gap in all monetary policy instruments employed. Specifically, we set the feedback coefficients at 0.8 instead of 0.2, used in previous experiments. We start by presenting results for one instrument at a time other things equal.

Results are reported in Figures 6 and 6a; the black line represents the case of a higher response of OMOs to the output gap ($\gamma_{rb,y} = \gamma_{i,y} = 0.2, \gamma_{cb,y} = 0.8$), the blue line the case of a higher response of remittances to the output gap ($\gamma_{cb,y} = \gamma_{i,y} = 0.2, \gamma_{rb,y} = 0.8$), and the green the case of a higher response of policy rate to the output gap ($\gamma_{cb,y} = \gamma_{rb,y} = 0.2, \gamma_{i,y} = 0.8$).

For comparison, we include the passive case studied in section 4 (dotted grey line).

Figure 6: Aggressive monetary policy reaction to the output gap

[Diagram showing various economic indicators with different colored lines for different scenarios of monetary policy reactions to the output gap.]
The IRFs of aggressive OMOs (black line) imply that there are no demonstrable real benefits of extra government bond purchases by the central bank.\textsuperscript{45} As in the case of the mild response of OMOs to the output gap (as studied in subsection 5.2), that policy hurts the real economic activity on impact, due to the increase in inflation, while, it slightly helps it in the medium-run, mainly through the big public debt-to-GDP ratio reduction.

Same results, but in a much higher magnitude, are also delivered by the higher countercyclical response of remittances to the output gap (blue line). Real variables exhibit a higher decrease in the short-run, through the persistent increase of inflation, but a faster convergence to the steady state, through the vast public debt-to-GDP ratio decline.\textsuperscript{46}

On the other hand, when the central bank puts extra weight on the response of the policy rate to the output gap (green line), this policy can benefit the real economy, at least in the medium run. For that time period, lower inflation and nominal interest rates, but higher real savings rates on savings, help private consumption, investment, hours of work and eventually output. Furthermore, the sharp decrease of the real interest rate paid on reserves, allows bank to borrow from the central bank at a very low cost, and, thus, to increase their loans provision.\textsuperscript{47}

Summing up, when we also allow for an aggressive reaction to the output gap, the most effective mix appears to be the one in which the central bank cuts strongly its nominal policy interest rate, which falls below the zero lower bound, while the use of OMOs and remittances

\textsuperscript{45}See also Reis (2016) and Blinder et al. (2017).
\textsuperscript{46}Private debt holdings become negative between periods 7 and 26.
\textsuperscript{47}Nominal policy rate becomes negative from period 5, while real reserves become negative between periods 4 and 25.
to the Treasury remains mild.

5.6 Unconventional QE-type policies

In this subsection, we continue with extra non-standard monetary policies employed by central banks since the beginning of the financial crisis in 2007. In particular, we study direct lending to private firms (subsection 5.4.1) and a discount window policy (subsection 5.4.2).

5.6.1 Direct lending to private firms

To mimic an unconventional QE type policy employed by several central banks during the recent crisis, namely, direct lending to non-financial institutions, we add central bank’s loans to private firms, denoted as $l_{cb}$, as an extra monetary policy instrument. Following Cúrdia and Woodford (2010a, 2010b), we assume that these loans pay the same lending rate as bank loans and follow a rule similar to (35e). Specifically,

$$l_{cb,t+1} = \left(1 - \rho^{lcb}\right) l_{cb,t} + \rho^{lcb} l_{cb,t} - \gamma^{lcb,y} (y_t - y) + \varepsilon_t^{lcb} \tag{37}$$

In order to be able to compare the joint policy of OMOs and direct loans to firms against the policy of OMOs alone, we set $\gamma^{lcb,y} = 0.2$ (an equal response of direct loans to the output gap) and $\rho^{lcb} = 0$. Also, we assume that at the steady the Central Bank provides just the 1% of total loans in the economy ($l_{cb} = 0.1 \times l$).

Results are reported in Figure 7, where the blue lines describe the case with direct lending to firms, other things equal relative to Figure 4.

As can be seen, central bank’s asset purchases of direct loans and government bonds, do not affect real allocations, relative to the case of government bond purchases only (the only exception is the slight increase in total loans). The reason is that these two assets, namely government bonds and loans, share close characteristics. Government bonds are chosen by savers while loans by firms, where both agents are financially unconstrained and bear no transaction costs. Thus, asset purchases cannot trigger strong portfolio effects.

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48 E.g. the Corporate Sector Purchase Program (CSPP) by ECB and Long Scale Asset Purchase programme by FED.

49 In Gertler and Kiyotaki (2011) and in Gertler and Karadi (2011, 2013), Central Bank’s private loans are a fraction, which is an increasing function of lending spread, of total loans.

50 We have not assumed any type of cost, i.e. monitoring costs, associated with CB’s direct lending to private sector, in order not to favor any particular QE policy. On the contrary, Gertler and Karadi (2013) assume smaller intermediation cost in government debt compared to private loans market.
What happens if firms are financially constrained

The implications of a policy of direct lending would change though if we follow e.g. Gertler and Karadi (2011, 2013), Gertler and Kiyotaki (2011), Güntner (2015), by assuming that intermediate-goods firms should finance their physical capital acquisition in advance of production. In other words, we will assume that the real quantity of new loans equals the quantity of new capital at the end of each period ($l_{t+1} = k_{t+1}$).

New results are presented in Figure 8, where the grey line is for the case of monetary activism and direct lending when firms are financially unconstrained (as in Figure 7) and the black line is for the same policy mix when firms are financially constrained.

The IRFs in Figure 8 show that the extra liquidity provided to private firms is now beneficial since it is directly translated into higher capital and investment. In sum, the real effects of direct lending to private firms by the central bank depend crucially upon the presence of
financial frictions and borrowing constraints in particular.\textsuperscript{51}

Figure 8: Reaction to the output gap using monetary activism plus direct lending to firms when $l_{t+1}^{f} = k_{t+1}^{f}$

The IRFs in Figure 8 show that the direct lending policy results in higher investment and inflation when firms are financially constrained.\textsuperscript{52} The extra liquidity provided to private firms is now directly translated into higher investment and capital and eventually output.

In sum, the effectiveness of direct lending to private firms by the central bank depends crucially on the presence of financial frictions and borrowing constraints in particular. This is as in e.g. Araújo et al. (2015).

\textsuperscript{51}This is as in e.g. Araújo et al. (2015).

\textsuperscript{52}Iacoviello (2005) has shown how higher inflation can be beneficial to financially constrained borrowers’ net worth when obligations are held in nominal terms.
5.6.2 Extra loans to private banks

Upon the eruption of the financial crisis of 2007-2008, most central banks extended the discount window in order to provide extra liquidity to financial institutions and thereby restore bank lending to the real economy. To mimic this type of QE policy, we add a new monetary policy instrument, central bank’s loans to private banks, denoted as $d^{cb}$. These loans are assumed to pay the same interest rate as deposits. The new monetary policy instrument follows a rule similar to (35e), which is:

$$d_{t+1}^{cb} = (1 - \rho^{dcb}) d^{cb} + \rho^{dcb} d^{cb}_t - \gamma^{dcb,y} (y_t - \gamma) + \varepsilon^d_t$$  

(38)

As in the previous experiment, for reasons of comparability, we set $\gamma^{dcb,y} = 0.2$ (an equal response of bank loans to the output gap) and $\rho^{dcb} = 0$. Also, we assume that at the steady the Central Bank provides only the 1% of total bank deposits ($d^{cb} = 0.1 * d$).

The same results and rationale, as in the case of central bank’s loans to private firms, apply also here. Central bank’s purchases of assets other than government bonds, which are held by the same agent (savers), deliver no real differences. Savers just exchange deposits for government debt.

Summing up, a policy of direct lending to private banks does not appear to benefit the real economy.

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53 The theoretical and empirical literature on banking (see e.g. Rajan, 2005, Borio and Zhu, 2012) has highlighted the importance of the “bank lending” channel for the transmission of monetary policy.
Summarizing the results of monetary policy activism studied in this section, the benefits of active monetary policies depend crucially on certain premises. Results for each main category of monetary policy have already been reported at the end of each subsection above. Here we close with some more general statements. The efficacy of the various asset purchasing programs (namely OMOs, direct loans to firms and banks) depends on their interactions with the other monetary policy instruments, on the characteristics of the public debt holders, as well as, on the existence of specific credit frictions that need to be addressed. For example, without a Taylor rule for the policy interest rate, OMOs do not seem to have any significant real effects on the economy relative to passive policy. Also, countercyclical remittances from central bank to Treasury strengthen the portfolio effects of OMOs. In addition, the extra unconventional QE type programs are potentially effective complements to conventional policies only when financial markets are stressed. Thus, for monetary activism to be effective there should be a careful consideration of market imperfections along with the intrinsic characteristics of market participants.
6 Adding fiscal activism

Based on the monetary framework of the previous section, we now also allow for active fiscal policy and study the potential real benefits of fiscal-monetary coordination.\textsuperscript{54} In particular, given the activist monetary policies studied in the previous section, we now also allow the tax rates and the government spending-to-GDP ratio to react to the output gap.

Specifically, in subsection 6.1.1, we add fiscal activism to the case of monetary activism (as studied in subsection 5.4). We name this policy as active. We find positive real effects of fiscal-monetary policy coordination. Then, in subsection 6.1.2 and 6.1.3 we study whether two alternative monetary policies, namely the policy of constant nominal policy rate (subsection 6.1.2) and the policy of stronger monetary activism, via the higher feedback response of the policy rate to the output (subsection 6.1.3), can further enhance the real benefits of active policy of subsection 6.1.1. We skip the cases of QE policies (as in subsection 5.4), which can only conditionally be beneficial to the economy (however results for all cases are available upon request).

6.1 Fiscal and monetary policy activism together

Can fiscal activism mitigate the negative impact effects, and/or reinforce the positive medium-run effects, of monetary activism?

To consider the potential merits of fiscal-monetary activism, we switch on the feedback reaction to the output gap in both fiscal and monetary policy rules and set all the associated feedback coefficients at 0.2 ($\gamma^{c;h,y} = \gamma^{i;y} = \gamma^{c;h,y} = \gamma^{g;y} = \gamma^{c;y} = \gamma^{y;y} = \gamma^{\pi;y} = 0.2$). We name this policy as \textit{active}.

The new results of active policy, in Figure 10 and 10a, are compared to passive policy as studied in section 4 and to the active monetary policy as studied in subsection 5.4. For the joint fiscal-monetary policy, we add a new IRFs shown in blue color. That is, in the figure below, the dotted line repeats the passive case of section 4, the black line reproduces the active monetary policy case of subsection 5.4 and the blue lines will illustrate the new joint monetary-fiscal action, other things equal.

Inspection of the IRFs in Figure 10 shows that fiscal activism can offset the impact costs of monetary activism. Relative to the latter, a joint policy reaction to the output gap helps private investment and consumption (this is driven by the rise in real interest rates that stimulate savings in the short term and consumption in the medium term according to the

\textsuperscript{54} Auerbach and Gorodnichenko (2017) showed that fiscal policy contributed to the US recovery after the severe slowdown of 2008-2009.
standard inter-temporal substitution effect) and work hours in the short run (this is driven by the cut in the labor tax rate according to the feedback policy rule (35c)).

On the other hand, the increasing public debt-to-GDP ratio lessens the positive medium-run benefits of monetary expansion, as well as, the persistence of any short-term benefits from fiscal expansion. Also, the higher private debt holdings decrease deposits, through the portfolio effects, and subsequently loans.

Summing up, with an active fiscal support monetary activism can help the real economy, at least in the short-run.

Figure 10: Reaction to the output gap by using both policies
6.2 Fiscal and monetary activism with constant policy rates

We repeat the policy experiment of joint fiscal-monetary activism, but now we switch off the policy rate reaction to inflation and output gap. Specifically, the policy rate, $i^r$, is now kept constant at its steady state value (which is $i^r = 0.001$) all the time, all other things are equal.

Results are shown in Figure 11 and 11a. The new results are compared again to passive policy as studied in section 4 (grey line) and, to detect additional benefits, to the active policy of the previous subsection (black line). For the new policy, we add a new IRFs shown in blue color.

Results are shown in Figure 11 and 11a. The new results are compared again to passive policy as studied in section 4 (grey line) and to the active policy of the previous subsection (black line). For the new policy, we add a new IRFs shown in blue color.

The IRFs in Figure 11 imply that the low inflation and low interest rate environment can enhance the efficacy of active policy. In particular, output decrease is almost half compared to the case of passive policy. These results are consistent with the literature. For instance, in a similar experiment, Gertler and Karadi (2013) have shown that the rise in the short-term interest rate, generated by an endogenous Taylor rule, offsets more than 80 percent of the effect of OMOs on output. Araújo et al. (2015) have found that central bank’s operations, in the absence of any change in the nominal interest rate, can affect prices and real allocations. Finally, in their review paper on fiscal stimulus, Coenen et al. (2010) have also provided evidence that fiscal multipliers increase in a variety of DSGE models when the nominal interest rate remains unchanged.
Figure 11: Reaction to the output gap using both policies
while keeping the policy interest rate constant

Figure 11a: Real interest rates using both policies
while keeping the policy interest rate constant
6.3 Fiscal and strong monetary activism

As a last policy experiment, we add fiscal activism to the case of stronger monetary activism, via a higher feedback reaction of the policy rate to the output gap (as studied in subsection 5.4). Results are reported in Figure 12 and 12a, where, as before, the new IRFs (blue line) are compared to passive policy as studied in section 4 (grey line) and to the active policy of the subsection 6.1.1 (black line).

The IRFs in Figure 12 show that the policy mix of fiscal and stronger monetary activism is relatively costly in the short-run, while it is only slightly beneficial in the medium run.

Figure 12: Fiscal and aggressive monetary policy reaction to the output gap
Figure 12a: IRFs of real interest rates using fiscal and aggressive monetary policy reaction to the output gap

Summing up, there appear to be non-negligible benefits to the real economy from fiscal-monetary activism. This is the case when tax-spending instruments on the side of fiscal policy, and OMOs and central bank’s remittances on the side of monetary policy, all react to the output gap at a moderate degree, while, at the same time, the nominal policy interest rate is kept constant at a low value rather than follow a state contingent Taylor type rule.

7 Conclusions

In this paper, using a medium scale DSGE model that included all main building blocks of a market economy, we studied the potential merits of macroeconomic (monetary and fiscal) policy activism. Since the results have been already listed in the Introduction, here we close with two potential extensions. First, it would be interesting to see what changes in an open economy context and, in particular, in a model of a currency union where the single currency plays extra roles. Second, it would be interesting to compute the optimal values of the feedback policy coefficients, in other words, to compute optimized policy rules.
References


Appendix 1: Decentralized equilibrium (for any feasible policy)

The decentralized equilibrium can be summarized by the following equilibrium conditions:

\[(1 + \tau^c_t) c^h_t + b^b_{t+1} + m^b_{t+1} = (1 - \tau^y_t) w_t u^h_t + (1 + i_t^d) \frac{p_{t-1}}{p_t} d_t + (1 + i_t^b) \frac{p_{t-1}}{p_t} p^h_t + \frac{p_{t-1}}{p_t} m^h_t \]

\[\frac{1}{c^h_t} = \lambda^h_t (1 + \tau^c_t) + a^h \psi^h_t (1 + \tau^c_t) \]

\[\lambda^h_t = \beta^h \lambda^h_{t+1} (1 + i_{t+1}) \frac{p_t}{p_{t+1}} \]

\[\lambda^l_t = \beta^h \lambda^l_{t+1} (1 + i_{t+1}) \frac{p_t}{p_{t+1}} \]

\[\lambda^l_t - \psi^l_t = \beta^h \lambda^l_{t+1} \frac{p_t}{p_{t+1}} \]

\[\frac{\nu^h}{1 - u^h_t} = \lambda^h_t (1 - \tau^y_t) w_t \]

\[m^h_{t+1} = a^h (1 + \tau^c_t) c^h_t \]

\[\frac{1}{c^e_t} = \lambda^e_t (1 + \tau^c_t) + a^e \psi^e_t (1 + \tau^c_t) \]

\[\lambda^e_t - \psi^e_t = \beta^e \lambda^e_{t+1} \frac{p_t}{p_{t+1}} \]

\[\frac{\nu^e}{1 - u^e_t} = \lambda^e_t (1 - \tau^y_t) w_t \]

\[m^e_{t+1} = a^e (1 + \tau^c_t) c^e_t \]

\[\frac{1}{c^b_t} = \lambda^b_t (1 + \tau^c_t) + a^b \psi^b_t (1 + \tau^c_t) \]

\[\lambda^b_t - \psi^b_t = \beta^b \lambda^b_{t+1} \frac{p_t}{p_{t+1}} \]

\[\frac{\nu^b}{1 - u^b_t} = \lambda^b_t (1 - \tau^y_t) w_t \]

\[m^b_{t+1} = a^b (1 + \tau^c_t) c^b_t \]
\[
\frac{1}{c_t^b} = \lambda_t^b (1 + \tau_t^c) + a^b \psi_t^b (1 + \tau_t^c) 
\]

(52)

\[
\lambda_t^b - \psi_t^b = \beta^b \lambda_{t+1}^b \frac{p_t}{p_{t+1}} 
\]

(53)

\[
\frac{\nu_t^b}{1 - u_t^b} = \lambda_t^b (1 - \tau_t^b) w_t 
\]

(54)

\[
m_{t+1}^b = a^b (1 + \tau_t^c) c_t^b 
\]

(55)

\[
\pi_t^f = (1 - \tau_t^c) \left\{ y_t - w_t \left( u_t^h + u_t^c \right) \right\} - i_t + l_{t+1} - (1 + i_t^l) \frac{p_{t+1}}{p_t} t - \]

(56)

\[
-m_{t+1}^f + \frac{p_{t-1}}{p_t} m_t^f - \frac{\chi}{2} \left( \frac{p_t}{p_{t-1}} - 1 \right)^2 
\]

\[
y_t = f \left( k_t, u_t^b, u_t^c \right) = A_t \left( k_t \right) \alpha \left( u_t^h + u_t^c + u_t^b \right)^{1-\alpha} 
\]

(57)

\[
m_{t+1}^f = a^f w_t \left( u_t^b + u_t^c \right) 
\]

(58)

\[
\left( \frac{1}{c_t^l} \right) \lambda_t^c = \beta^c \lambda_{t+1}^c \left( \frac{1}{c_{t+1}^l} \right) \left( 1 - \delta \right) + \beta^c \lambda_{t+1}^c \left( 1 - \tau_t^c \right) \phi \frac{\partial y_{t+1}}{\partial k_{t+1}} - \]

(59)

\[
- \beta^c \lambda_{t+1}^c \chi \left( \frac{p_{t+1}}{p_t} - 1 \right) \frac{p_{t+1}}{p_t} \left( \phi - 1 \right) \frac{\partial y_{t+1}}{\partial k_{t+1}} - \left( \beta^c \right)^2 \lambda_{t+2}^c \chi \left( \frac{p_{t+2}}{p_{t+1}} - 1 \right) \frac{p_{t+2}}{p_{t+1}} \frac{1 - \phi}{y_{t+1}} \frac{\partial y_{t+1}}{\partial k_{t+1}} 
\]

\[
\lambda_t^c (1 - \tau_t^c) w_t + \psi_t^c \alpha^f \lambda_t^c w_t = \lambda_t^c (1 - \tau_t^c) \phi \frac{\partial y_t}{\partial \left( u_t^h + u_t^c \right)} + \]

(60)

\[
+ \lambda_t^c \chi \left( \frac{p_t}{p_{t-1}} - 1 \right) \frac{p_t}{p_{t-1}} \left( 1 - \phi \right) \frac{\partial y_t}{\partial \left( u_t^h + u_t^c \right)} - \beta^c \lambda_{t+1}^c \chi \left( \frac{p_{t+1}}{p_t} - 1 \right) \frac{p_{t+1}}{p_t} \frac{1 - \phi}{y_t} \frac{\partial y_t}{\partial \left( u_t^h + u_t^c \right)} 
\]

\[
\lambda_t^c = \beta^c \lambda_{t+1}^c \left( 1 + i_{t+1}^l \right) \frac{p_t}{p_{t+1}} 
\]

(61)

\[
\lambda_t^c (1 - \psi_t^c) = \beta^c \lambda_{t+1}^c \frac{p_t}{p_{t+1}} 
\]

(62)
\[ k_{t+1} = (1 - \delta)k_t + \xi_t \]  
\[ \pi_t^b = \frac{p_{t-1}}{p_t} \left\{ (1 + i^b_t)l_t - (1 + i^d_t)d_t + (1 + i^r_t) m_t^b \right\} + d_{t+1} - l_{t+1} - m_{t+1}^r - \Psi(l_{t+1}, d_{t+1}, m_{t+1}^r) \]  
\[ l_{t+1} = \frac{-\beta (1 + i_{t+1}^b)}{\eta^l} \frac{p_t}{p_{t+1}} - 1 \]  
\[ (d_{t+1})^{-3} = \frac{-\beta (1 + i_{t+1}^d)}{\eta^d} \frac{p_t}{p_{t+1}} - 1 \]  
\[ (m_{t+1}^r)^{-3} = \frac{1 - \beta (1 + i_{t+1}^l)}{\eta^m} \frac{p_t}{p_{t+1}} \]  
\[ b_{t+1}^{ch} + rcb_t + \frac{p_{t-1}}{p_t} (m_t^h + m_t^i + m_t^f + m_t^b + (1 + i_t^f) m_t^r) = (1 + i_t^b) \frac{p_{t-1}}{p_t} b_t^{cb} + m_{t+1}^h + m_{t+1}^i + m_{t+1}^f + m_{t+1}^b + m_{t+1}^r + \]  
\[ g_t + \frac{p_{t-1}}{p_t} (1 + i_t^b) \left( b_t^h + b_t^{cb} \right) = \tau_t^c \left( c_t^h + c_t^e + c_t^b \right) + (\tau_t^f - \tau_t^r) w_t u_t^f + \tau_t^r y_t + b_{t+1}^h + b_{t+1}^{cb} + rcb_t \]  

The equilibrium system can be summarized by 31 equations in 31 endogenous variables, \( \{c_t^h, c_t^e, c_t^b, u_t^h, u_t^e, u_t^b, d_{t+1}, l_{t+1}, k_t, i_t, b_t^{ch}, m_{t+1}^h, m_{t+1}^i, m_{t+1}^f, m_{t+1}^b, m_{t+1}^r, \lambda_t^h, \lambda_t^e, \lambda_t^b, \psi_t^h, \psi_t^e, \psi_t^f, \psi_t^b, y_t, w_t, i_t^{cb}, i_t^d, i_t^i, \pi_t^f, \pi_t^b, \Pi_t \equiv p_t/p_{t-1} \}_t=0 \). This is for any feasible policy, as defined in subsection 2.6 above.

9 Appendix 2. Decentralized equilibrium in the steady state

In this appendix we solve for the steady state starting by writing the above system without time subscript.

We solve for the steady state starting by writing the above system without time subscript.

In the steady state, we set inflation at one, \( \Pi = 1 \), and we allow \( rcb \) to be endogenously determined.
\[(1 + \tau^c) c^h = (1 - \tau^y) w u^h + \psi^b b^h + \psi^d d \quad (70)\]

\[
\frac{1}{c^h} = \lambda^h (1 + \tau^c) + a^h \psi^h (1 + \tau^c) \quad (71)
\]

\[
1 = \beta^h (1 + \tau^d) \quad (72)
\]

\[
1 = \beta^h (1 + \tau^b) \quad (73)
\]

\[
1 - \frac{\psi^h}{\lambda^h} = \beta^h \quad (74)
\]

\[
\frac{\nu^h}{1 - u^h} = \lambda^h (1 - \tau^y) w \quad (75)
\]

\[
m^h = a^h (1 + \tau^c) c^h \quad (76)
\]

\[(1 + \tau^c) c^e = \pi^f + (1 - \tau^y) w u^e \quad (77)\]

\[
\frac{1}{c^e} = \lambda^e (1 + \tau^c) + a^e \psi^e (1 + \tau^c) \quad (78)
\]

\[
1 - \frac{\psi^e}{\lambda^e} = \beta^e \quad (79)
\]

\[
\frac{\nu^e}{1 - u^e} = \lambda^e (1 - \tau^y) w \quad (80)
\]

\[
m^e = a^e (1 + \tau^c) c^e \quad (81)
\]

\[(1 + \tau^c) c^b = \pi^b + (1 - \tau^y) w u^b \quad (82)\]

\[
\frac{1}{c^b} = \lambda^b (1 + \tau^c) + a^b \psi^b (1 + \tau^c) \quad (83)
\]

\[
1 - \frac{\psi^b}{\lambda^b} = \beta^b \quad (84)
\]

\[
\frac{\nu^b}{1 - u^b} = \lambda^b (1 - \tau^y) w \quad (85)
\]

\[
m^b = a^b (1 + \tau^c) c^b \quad (86)
\]
\[ \pi^f = (1 - \tau^\pi) \left\{ y - w (u^h + u^e) \right\} - i + il \]  

(87)

\[ y = f \left( k, u^h, u^e \right) = Ak^a \left( u^h + u^e \right)^{1-a} \]  

(88)

\[ m^f = a^f w (u^h + u^e) \]  

(89)

\[ 1 = \beta^e \left( (1 - \delta) + (1 - \tau^\pi) \frac{\partial y}{\partial k} \right) \]  

(90)

\[ \left( 1 - \tau^\pi + \psi^f a^f \right) w = (1 - \tau^\pi) \phi \frac{\partial y}{\partial (u^h + u^e)} \]  

(91)

\[ 1 = \beta^e (1 + il) \]  

(92)

\[ 1 - \beta^e = \psi^f \]  

(93)

\[ i = \delta k \]  

(94)

\[ \pi^b = il - id + ir m^r - \Psi(l, d, m^r) \]  

(95)

\[ l = \frac{\tilde{\beta}^b (1 + il) - 1}{\eta^l} \]  

(96)

\[ (d)^{-3} = \frac{\tilde{\beta}^b (1 + id) - 1}{\eta^d} \]  

(97)

\[ (m^r)^{-3} = \frac{1 - \tilde{\beta}^b (1 + ir)}{\eta^m} \]  

(98)

\[ rcb = i^b c^b - ir m^r \]  

(99)

\[ rcb = g + i^b \left( b^c + b^r \right) - \tau^c \left( c^h + c^e + c^b \right) - (\tau^y - \tau^\pi) w^f - \tau^\pi y \]  

(100)
10 Appendix 3. Transition dynamics in investment, labor tax and government spending shock under passive policy

Figure 13: Effects of a negative 5% investment shock under passive policy
Figure 14: Effects of a positive 5% government spending shock under passive policy

Figure 15: Effects of a positive 5% labor tax shock under passive policy
11 Appendix 4: Capitalists hold government debt

In this Appendix we adopt the economic framework as described in Section 2, with the only difference that government bonds are held by capitalists, \( b^c \), instead of savers.

The decentralized equilibrium can be summarized by the following equilibrium conditions:

\[
(1 + \tau_i^c) c_t^h + d_{t+1} + m_{t+1}^h = (1 - \tau_i^y) w_t u_t^h + (1 + i_t^d) \frac{p_{t-1}}{p_t} d_t + \frac{p_{t-1}}{p_t} m_t^h 
\]

\[
\frac{1}{c_t^h} = \lambda_t^h (1 + \tau_i^c) + a^h \psi_t^h (1 + \tau_i^c) 
\]

\[
\lambda_t^h = \beta^h \lambda_{t+1}^h (1 + i_{t+1}^d) \frac{p_t}{p_{t+1}} 
\]

\[
\lambda_t^h - \psi_t^h = \beta^h \lambda_{t+1}^h \frac{p_t}{p_{t+1}} 
\]

\[
\frac{\nu^h}{1 - u_t^h} = \lambda_t^h (1 - \tau_i^y) w_t 
\]

\[
m_{t+1}^h = a^h (1 + \tau_i^c) c_t^h 
\]

\[
(1 + \tau_i^c) c_t^e + b_{t+1}^e + m_{t+1}^e = \pi_t^e + \pi_t^h + (1 - \tau_i^y) w_t u_t^e + (1 + i_t^e) \frac{p_{t-1}}{p_t} b_t^e + \frac{p_{t-1}}{p_t} m_t^e 
\]

\[
\frac{1}{c_t^e} = \lambda_t^e (1 + \tau_i^c) + a^e \psi_t^e (1 + \tau_i^c) 
\]

\[
\lambda_t^e - \psi_t^e = \beta^e \lambda_{t+1}^e \frac{p_t}{p_{t+1}} 
\]

\[
\frac{\nu^e}{1 - u_t^e} = \lambda_t^e (1 - \tau_i^y) w_t 
\]

\[
\lambda_t^e = \beta^e \lambda_{t+1}^e (1 + i_{t+1}^b) \frac{p_t}{p_{t+1}} 
\]

\[
m_{t+1}^e = a^e (1 + \tau_i^c) c_t^e 
\]

70
\((1 + \tau_{t}^c) c_t^b + m_{t+1}^b = \pi^b_t + (1 - \tau_{t}^w) w_t u_t^b + \frac{p_{t-1} m_t}{p_t} \)  

(113)

\(\frac{1}{c_t^b} = \lambda_t^b (1 + \tau_{t}^c) + a^b \psi_t^b (1 + \tau_{t}^c) \)

(114)

\(\lambda_t^b - \psi_t^b = \beta^b \lambda_{t+1} \frac{p_t}{p_{t+1}} \)

(115)

\(\frac{\nu^b}{1 - \nu_t^b} = \lambda_t^b (1 - \tau_{t}^w) w_t \)

(116)

\(m_{t+1}^b = a^b (1 + \tau_{t}^c) c_t^b \)

(117)

\[
\pi_t^f = (1 - \tau_{t}^p) \left\{ y_t - w_t \left( u_t^h + u_t^e \right) \right\} - i_t + l_{t+1} - (1 + i_t) \frac{p_{t-1} l_t}{p_t} 
- m_{t+1}^f + \frac{p_{t-1} m_t^f}{p_t} - \frac{\chi}{2} \left( \frac{p_t}{p_{t-1}} - 1 \right)^2 
\]

(118)

\[y_t = f \left( k_t, u_t^h, u_t^e \right) = A_t \left( k_t \right)^\alpha \left( u_t^h + u_t^e \right)^{1-\alpha} \]

(119)

\[m_{t+1}^f = a^f w_t \left( u_t^h + u_t^e \right) \]

(120)

\[
\left( \frac{1}{\xi_{t}} \right) \lambda_t^c = \beta^c \lambda_{t+1}^c \left( \frac{1}{\xi_{t+1}} \right) (1 - \delta) + \beta^c \lambda_{t+1}^c (1 - \tau_{t+1}^w) \phi \frac{\partial y_{t+1}}{\partial k_{t+1}} - \beta^c \lambda_{t+1}^c \chi \left( \frac{p_{t+1}}{p_t} \phi \right) \frac{\partial y_{t+1}}{\partial k_{t+1}} 
\]

(121)

\[
\lambda_t^c (1 - \tau_{t}^w) w_t + \psi_t^f a^f \lambda_t^c w_t = \lambda_t^c (1 - \tau_{t}^w) \phi \frac{\partial y_t}{\partial \left( u_t^h + u_t^e \right)} + \lambda_t^c \left( \frac{p_t}{p_{t-1}} - 1 \right) \frac{p_{t-1}}{p_t} \phi \frac{\partial y_t}{\partial \left( u_t^h + u_t^e \right)} \]

(122)

\[\lambda_t^c = \beta^c \lambda_{t+1}^c (1 + i_{t+1}) \frac{p_t}{p_{t+1}} \]

(123)
\[ \lambda_t^e \left( 1 - \psi_t^f \right) = \beta^e \lambda_{t+1}^e \frac{p_t}{p_{t+1}} \]  

(124)

\[ k_{t+1} = (1 - \delta)k_t + \xi_t \]  

(125)

\[ \pi_t^b = \frac{p_{t-1}}{p_t} \left\{ (1 + \beta^b t)l_t - (1 + \beta^b l_t)dt + (1 + \beta^b e_t)m_t \right\} + dt_{t+1} - mt_{t+1} - \Psi(l_{t+1}, dt_{t+1}, m_{t+1}) \]  

(126)

\[ l_{t+1} = \frac{\beta^b (1 + \beta^b l_{t+1}) \frac{p_t}{p_{t+1}} - 1}{\eta_l} \]  

(127)

\[ (dt_{t+1})^{-3} = \frac{\beta^b (1 + \beta^b l_{t+1}) \frac{p_t}{p_{t+1}} - 1}{\eta_l} \]  

(128)

\[ (mt_{t+1})^{-3} = \frac{1 - \beta^b (1 + \beta^b l_{t+1}) \frac{p_t}{p_{t+1}}}{\eta_l} \]  

(129)

\[ b_{t+1}^c + rcb_t + \frac{p_{t-1}}{p_t} (m_t^h + m_t^f + m_t^e + m_t^b + (1 + \beta^b l_t) m_t) = \]  

(130)

\[ = (1 + \beta^b l_t) \frac{p_{t-1}}{p_t} b_t^c + m_{t+1}^h + m_{t+1}^e + m_{t+1}^f + m_{t+1}^b + m_t \]

\[ g_t + \frac{p_{t-1}}{p_t} (1 + \beta^b l_t) \left( b_t^c + b_t^b \right) = \tau_t^c \left( c_t^h + c_t^e + c_t^b \right) + (\tau_t^y - \tau_t^w) \psi_t + \psi_t \]  

(131)

The equilibrium system can be summarized by 31 equations in 31 endogenous variables, \{c_t^h, c_t^e, c_t^b, u_t^h, u_t^e, u_t^b, dt_{t+1}, l_{t+1}, k_{t+1}, \xi_t, b_{t+1}^b, m_t^h, m_t^e, m_t^f, m_t^b, m_{t+1}^h, m_{t+1}^e, m_{t+1}^f, m_{t+1}^b, \lambda_t^h, \lambda_t^e, \lambda_t^b, \psi_t^h, \psi_t^e, \psi_t^f, \psi_t^y, \psi_t^\beta, \psi_t, \psi_t^\beta, \Pi_t \equiv p_t/p_{t-1}\}_{t=0}^\infty. This is for any feasible policy, as defined in subsection 2.6 above.