Are exchange rate bands better than fixed exchange rates? The imported credibility approach

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Abstract

This paper explicitly incorporates exchange rate bands into the Barro–Gordon model of inflation. This is a non-trivial extension. Focusing on Markov-perfect equilibria, we specify the properties of the optimal policy functions, exchange rate dynamics and hence the arguments for and against target zones. The attractiveness of alternative exchange rate regimes (i.e. fixed, flexible and target zones) and the position of exchange rates inside the band depend on the inflationary performance of the center country.

Keywords: Exchange rate bands; Dynamic policy games; Inflation

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1. Introduction

Theories of inflation stress how the limited ability of governments to precommit to price stability results in high inflation without employment benefits (see Barro and Gordon, 1983a,b). One of the institutional mechanisms to improve the outcome of the Barro–Gordon (BG) game is participation in a regime of exchange-rate commitments. The idea is that exchange-rate commitments offer member-countries the opportunity to gain credibility by tying their monetary policy to that of a low-inflation center country (see Giavvazi and Giovannini, 1987 and Giavvazi and Pagano, 1988). However, one weakness of this imported

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1Exchange rate commitments are reflected in private expectations only if they are credible.
credibility model is that it treats the exchange-rate-regime as a strict fixed-rate regime, so that the comparison is between flexible and fixed rates. Obviously, this is not always true. For instance, in the case of the European ERM, member-countries have a range of manoeuvre for independent monetary policy within a pre-specified exchange rate band, while at the same time are required to intervene in the foreign exchange markets to prevent the exchange rate from moving outside the band (see Svensson, 1994).

The target zone (TZ) model is a nonlinear compromise between fixed and flexible exchange rates. Following Krugman (1991), TZ models have applied the mathematical theory of optimal bounded control to study the effect of the band on exchange rate determination. The basic TZ model assumes that exchange rate dynamics are driven by an exogenous unregulated Brownian motion, and that policymakers alter this stochastic process when the exchange rate hits the edges of the band (marginal intervention) or even when the exchange rate is in the interior of the band (intra-marginal intervention). However, one weakness of this model is the way it treats monetary policy: the latter is used only to maintain the TZ. Obviously, this is not true. Monetary policy is endogenously chosen and used both for stabilizing the economy and maintaining the TZ.

The present paper explicitly incorporates exchange rate bands into the Barro-Gordon model of inflation. In particular, we assume that the monetary authorities play a BG game with wage-setters, but strategies are constrained by a TZ for the exchange rate. This is a non-trivial extension because – contrary to the basic TZ model – monetary policy is now strategically chosen. It is also different from the basic BG game, because now optimal strategies depend on the position of the exchange rate within the TZ. There is now a genuine state variable that creates an intertemporal tradeoff in inflation. Focusing on Markov-perfect equilibria, we specify the properties of the optimal policy functions for exchange rates and hence the arguments for and against target zones.

Throughout the paper, we assume that the trend of foreign inflation is always expected to be less than the domestic inflation bias, which would arise in a BG model without a TZ. This is in the line of the ‘imported credibility’ model. However, we allow for short-run stochastic shocks which can either reinforce this trend (e.g. when the center country enjoys a disinflation shock) or reverse it (e.g. when the center country suffers from a positive inflation shock).

Our results are as follows. To expose the logic of the model, we consider first the deterministic case where the center country’s inflation is always lower than the domestic inflation bias. In this case, if the exchange rate lies initially in the interior of the TZ, the monetary authorities overinflate relative to the basic BG model so as

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to push the exchange rate toward the upper boundary of the TZ.\textsuperscript{3} Once there, they follow an accommodating policy by holding the exchange rate fixed at the upper boundary and keeping domestic inflation equal to foreign inflation.

The above benchmark case seems to suggest that the TZ is nonoptimal, i.e. it is better to use fixed exchange rates rather than a TZ. However, this is true only in the deterministic case. On the contrary, we show that in the stochastic case, a TZ may be superior to both fixed and flexible exchange rates, because it stabilizes domestic inflation when the center country is subject to random inflation shocks. If the center country enjoys a disinflation shock, the domestic country behaves as described above in the deterministic case (i.e. the shock merely reinforces the depreciation of the exchange rate towards the upper boundary). However, if the center country suffers from a positive inflation shock, the strategic behaviour of the domestic country ensures it has lower inflation and its exchange rate appreciates by moving into the interior of the band (assuming PPP holds). If the TZ is small enough, a sequence of high inflation shocks in the center country can take the exchange rate to the lower boundary of the band.

Therefore, a welfare comparison of alternative exchange rate regimes implies the following. If the center country enjoys stable low inflation, the domestic country chooses fixed exchange rates rather than a TZ, as being in the interior of the TZ sustains relatively high inflation. However, if the center country suffers from random inflation shocks, a TZ is superior to both fixed and flexible exchange rate regimes. This happens because a TZ allows the domestic country to import the anti-inflationary credibility of the center country but not its price variability. Also, the domestic country is better off the wider the TZ band. A large band gives more room for stabilizing the economy against high foreign inflation shocks.

The rest of the paper is organized as follows. Section 2 introduces the economic environment. Section 3 solves for the special case of the BG model. Section 4 solves for the general BG-TZ model. Section 5 closes the paper. Mathematical proofs are gathered in Appendix A and Appendix B.

2. The economic environment

The government plays a Barro–Gordon (BG) game with wage-setters, but strategies are constrained by a credible target-zone (TZ) for the exchange rate. In particular, the government plays a BG game when the exchange rate is inside the band, but it holds the exchange rate fixed when the latter hits the boundaries of the band.\textsuperscript{4}

\textsuperscript{3}The exchange rate is defined as units of domestic currency per foreign currency. Therefore, when the exchange rate increases (i.e. depreciates), it moves towards the upper boundary. When it decreases (i.e. appreciates), it moves towards the lower boundary.

\textsuperscript{4}This is what we call absorbing barriers. See Froot and Obstfeld (1991) for other scenarios.
Consider a small open economy, in which a private and a public sector interact over a number of discrete time-periods, \( t = 1, 2, \ldots \). The private sector consists of a constant group of trade-union members \((n^*)\), who unilaterally set one-period nominal wages \((w_t)\). The public sector is a central bank, which sets the price level \((p_t)\). However, a pre-specified and credible TZ for the exchange rate \((e_t)\) constrains the choice of \(p_t\). The exchange rate must lie between a lower and an upper boundary. Without loss of generality, we assume

\[
e < e_t < 0.
\]

Perfect substitutability between domestic and foreign goods implies that the policy instrument \((p_t)\), together with the exogenous foreign price level \((p_t^*)\), determine the exchange rate \((e_t)\). Thus,

\[
e_t = p_t - p_t^*,
\]

where, all variables will be in natural logs.

The payoff to the union in the entire game depends negatively on deviations of actual employment \((\ell_t)\) from target \((n^*)\). Thus, the union minimizes \(1/2 \sum_{t=1}^{\infty} \delta^{t-1} [\ell_t - n^*]^2\), where \(0 < \delta < 1\) is the discount factor.

The central bank cares about the whole labour force \((n)\) and dislikes inflation \((p_t - p_t^*)\). For simplicity, the inflation target is zero. Thus, the bank minimizes \(1/2 \sum_{t=1}^{\infty} \delta^{t-1} [(p_t - p_t^*)^2 + \alpha(\ell_t - n)^2]\), where \(\alpha > 0\) is the relative weight placed on the employment target. As in BG, we assume \(n > n^*\).

The foreign price level \((p_t^*)\) follows an exogenous stochastic process

\[
p_t^* = \pi_t^* + p_{t-1}^* + \eta_t,
\]

where \(\pi_t^*\) is foreign trend inflation, and \(\eta_t\) is a stationary white noise error process defined over some bounded support.

The order of events within any period \(t\) is as follows. After the realization of \(p_t^*\), wage setters set \(w_t\). Following this, the bank sets \(p_t\) subject to the TZ constraint. We assume that both wage setters and the bank know the realization of \(p_t^*\), when they set \(w_t\) and \(p_t\). Finally, firms choose employment \((\ell_t)\) given real wages \((w_t - p_t)\), so that

\[
\ell_t = \ell_t^d = p_t - w_t.
\]

We will solve for the perfect equilibrium where the current actions of the players \((w_t, p_t)\) depend on the game history only through the current state of the economy, which here is summarized by the state variables \((p_{t-1}, p_t^*)\). That is, we will solve for a Markov-perfect equilibrium.

Let \(U(p_{t-1}, p_t^*)\) be the expected loss function of the union at time \(t\). Using standard programming techniques, \(U(\cdot)\) is defined by the recursive equation
where $E_t$ denotes the mathematical expectation subject to the union’s information set at time $t$. The solution to Eq. (5) will define a wage strategy of the form $w_t = w(p_{t-1}, p_t^*)$.

After wage setters choose $w_t$, the bank plays. Let $B(p_{t-1}, p_t^*, w_t)$ be the loss function of the bank at time $t$. It is defined by the recursive equation

$$B(p_{t-1}, p_t^*, w_t) = \min_{p_t} \left[ \frac{1}{2}(p_t - p_{t-1})^2 + \frac{\alpha}{2}(p_t - w_t - n)^2 + \delta E_t B(p_t, p_{t+1}^*, w_{t+1}) \right],$$

where the solution to Eq. (6), subject to Eqs. (1) and (2), will define an optimal price strategy of the form $p_t = p(p_{t-1}, p_t^*, w_t)$.

This completes the description of the game. Section 4 will analyze the complete BG-TZ model, Eqs. (1)–(6). However, before doing this, we first analyze in Section 3 the basic BG model without a TZ. We do this for two reasons. First, it provides some intuition for the equilibrium pricing process and the solution strategy used to derive the equilibrium in the general BG-TZ case. Second, it allows clear comparisons between the alternative exchange rate regimes.

3. The Barro–Gordon model (BG)

In the BG model, the exchange rate and hence Eq. (1), Eq. (2) and Eq. (3) play no role when we solve Eq. (5) and Eq. (6). Assuming that $U(\cdot) = U(p_t)$ and $B(\cdot) = B(p_{t-1}, w_t)$ are continuously differentiable, the optimal wage $w_t$ solves

$$E_t \left[ (p_t - w_t - n) \left( \frac{\partial p_t}{\partial w_t} - 1 \right) + \delta U(p_t) \frac{\partial p_t}{\partial w_t} \right] = 0, \quad (7a)$$

where the union takes into account that by changing $w_t$, the government’s price $p_t$ changes according to $p_t = p(p_{t-1}, w_t)$. Given $(w_t, p_{t-1})$, the government’s optimal price $p_t$ satisfies

$$(p_t - p_{t-1}) + \alpha(p_t - w_t - n) + \delta E_t \left[ \frac{\partial}{\partial p_t} B(p_t, w_{t+1}) \right. + \left. \frac{\partial}{\partial w_{t+1}} B(p_t, w_{t+1}) \right] = 0. \quad (7b)$$

The problem is to find functional forms $w_t = w(p_{t-1})$ and $p_t = p(p_{t-1}, w_t)$ that satisfy Eq. (5), Eq. (6) and Eqs. (7a) and (7b). Intuition suggests identifying the

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Assuming a bounded solution exists. For any function $f(x)$, $f'(x)$ will denote the first derivative.
following solution. Since employment depends only on current prices, then with stationary strategies, the expected equilibrium losses will be independent of the previous period price level. Thus, it is anticipated that \( U'(p_t) = 0 \) and \( (d/dp_t)E_tB(p_t,w_{t+1}) = 0 \). Then, Eq. (7a) and Eq. (7b) imply the linear rules \( w_t = p_{t-1} + \alpha n - (1 + \alpha) n^u \) and \( p_t = (p_{t-1} + \alpha w_t + \alpha n)/(1 + \alpha) \). Along the equilibrium path, the reduced-form solution is

\[
\begin{align*}
p_t - p_{t-1} &= \alpha(n - n^u), \tag{8a} \\
\epsilon_t - \epsilon_{t-1} &= \alpha(n - n^u) - (\pi^* + \eta_t), \tag{8b} \\
\ell_t &= p_t - w_t = n^u, \tag{8c}
\end{align*}
\]

so that there is a positive inflation bias and the union attains its employment target. The latter implies \( U(\cdot) = 0 \) (which satisfies the assumed restriction \( U'(p_t) = 0 \)). Moreover, with these policy rules,

\[
E_tB(p_t,w_{t+1}) = \frac{\alpha(1 + \alpha)}{2(1 - \delta)}(n - n^u)^2, \tag{8d}
\]

which satisfies the assumed restriction \( (d/dp_t)E_tB(p_t,w_{t+1}) = 0 \).

This characterizes equilibrium payoffs and strategies in a perfect equilibrium. Notice that in this game the union gets its first-best outcome. The union’s optimal strategy is to set \( w_t = p(p_{t-1},w_t) - n^u \). Since the government’s optimal action is to set \( p_t = p(p_{t-1},w_t) \), actual employment equals the union’s target employment level, while the economy has positive wage and price inflation. This result will also hold in the next section, except the government’s pricing strategy is also constrained by the TZ on the exchange rate.

4. The Barro–Gordon target-zone model (BG–TZ)

We now solve the complete model Eqs. (1)–(6). We solve for a Markov-perfect equilibrium, where the government uses a pricing strategy \( p_t = p(p_{t-1},p_t^*,w_t) \). By the above argument, if an equilibrium exists, the union will always achieve its first-best by setting \( w_t = p(p_{t-1},p_t^*,w_t) - n^u \) and its loss function will satisfy \( U(\cdot) = 0 \). Assuming that a solution for \( w_t \) exists, this defines a wage pricing rule of the form \( w_t = \hat{w}(p_{t-1},p_t^*) \).

It is convenient to define the government’s expected loss function for the next period as

\[U(\cdot) = \int U(p_t,\epsilon_t,\ell_t) \phi(\epsilon_t,\ell_t) \, d\epsilon_t \, d\ell_t\]

This is true only in the unconstrained BG model. On the contrary, in the BG–TZ model the loss function is a function of the previous period price level because the initial position within the TZ matters for the game. See below.
Using Eq. (9b), Eq. (6) can be redefined as

\[ B(p_{t-1}, p^*_t, w_t) = \min_{p_t} \left[ \frac{1}{2} (p_t - p_{t-1})^2 + \frac{\alpha}{2} (p_t - w_t - n)^2 + \delta V(p_t, p^*_t) \right]. \]  

(10)

We shall show below that in equilibrium \( V(\cdot) \) there is a series of convex quadratic functions (see Appendix B). Hence, \( V(\cdot) \) is differentiable almost everywhere. However, \( V(\cdot) \) is not a convex function. It is convenient, for what follows, to define the minimand in Eq. (10) as

\[ Z = \frac{1}{2} (p_t - p_{t-1})^2 + \frac{\alpha}{2} (p_t - w_t - n)^2 + \delta V(p_t, p^*_t). \]  

(11)

It will be shown that if an optimal interior choice of \( p_t \) exists, then \( V(\cdot) \) is differentiable and convex at \( p_t \). Hence the necessary first-order condition for an interior minimum in Eq. (10) is

\[ (p_t - p_{t-1}) + \alpha (p_t - w_t - n) + \delta \frac{\partial}{\partial p_t} V(p_t, p^*_t) = 0, \]  

(12a)

while necessary conditions for the corner solutions to be optimal are

\[ p_t - p^*_t \quad \text{only if} \quad p_t^* - p_{t-1} + \alpha (p_t^* - w_t - n) + \delta \frac{\partial}{\partial p_t} V(p_t^*, p^*_t) \leq 0, \]  

(12b)

\[ p_t = p_t^* + \epsilon \quad \text{only if} \quad p_t^* + \epsilon - p_{t-1} + \alpha (p_t^* + \epsilon - w_t - n) \]  

\[ + \delta \frac{\partial}{\partial p_t} V(p_t^* + \epsilon, p^*_t) \geq 0. \]  

(12c)

Along the equilibrium path, where \( w_t = \hat{w}(p_{t-1}, p_t^*) = p_t - n^* \) Eq. (12a) and Eq. (9b) imply

\[ p_t - p_{t-1} = \alpha (n - n^*) - \delta \frac{\partial}{\partial p_t} V(p_t, p^*_t), \]  

(13a)

\[ V(p_{t-1}, p^*_t) = E_{t-1} \left[ \frac{1}{2} (p_t - p_{t-1})^2 + \frac{\alpha}{2} (n - n^*)^2 + \delta V(p_t, p^*_t) \right], \]  

(13b)

(and similarly for the corner solutions). Using Eq. (2) in Eq. (13a) and Eq. (13b), we get

\[ e_t - e_{t-1} = \alpha (n - n^*) - (\pi^* + \eta_t) - \delta \frac{\partial}{\partial e_t} V(e_t, p^*_t). \]  

(14a)
\[ V(e_{t-1}, p_{t-1}^*) = E_{t-1} \left[ \frac{1}{2} (e_t - e_{t-1} + \pi^* + \eta_t)^2 + \frac{\alpha}{2} (n - n^*)^2 + \delta V(e_t, p_t^*) \right], \]

(14b)

where \( p_t \) follows Eq. (3).

Now, because \((\pi^* + \eta_t)\) has been assumed to be independent of \( p_{t-1}^* \), we consider a stationary equilibrium, where the loss function \( V(e_{t-1}, p_{t-1}^*) \) is independent of \( p_{t-1}^* \) given the exchange rate \( e_{t-1} \) (see Appendix A). In other words, in a stationary equilibrium, the foreign price level only affects domestic prices through the exchange rate.

Using this restriction, Eqs. (14a) and (14b) reduce to

\[ e_t - e_{t-1} = \alpha (n - n^*) - (\pi^* + \eta_t) - \delta V'(e_t), \]

(15a)

\[ V(e_{t-1}) = E_{t-1} \left[ \frac{1}{2} (e_t - e_{t-1} + \pi^* + \eta_t)^2 + \frac{\alpha}{2} (n - n^*)^2 + \delta V(e_t) \right], \]

(15b)

while the corner solutions are

\[ e_t = 0 \text{ if } -e_{t-1} - \alpha (n - n^*) + (\pi^* + \eta_t) + \delta V'(0) \leq 0, \]

(15c)

\[ e_t = e \text{ if } e - e_{t-1} - \alpha (n - n^*) + (\pi^* + \eta_t) + \delta V'(e) \geq 0. \]

(15d)

Eqs. (15a)–(15d) define the necessary conditions for the equilibrium strategies. Notice that the BG solution for \( e_t \), Eq. (8b), and the value function, Eq. (8d), satisfy Eqs. (15a) and (15b), but do not satisfy the boundary conditions (15c)–(15d) for all \( t \). In general, \( V(e) \) will not be a constant in the BG–TZ framework. As a result, the difference in policy rules between the two regimes depends on the sign of \( V'(e_t) \). We will refer to this as the strategic effect.

The evolution of prices and exchange rates (according to Eqs. (15a)–(15d) and Eq. (2)) depends crucially on the properties of the value function \( V(e) \) in general, and the sign of \( V'(e_t) \) in particular. Our main task is to evaluate the function \( V(e) \) subject to the boundary conditions (15c)–(15d). Solving for \( V(e) \) is not possible in general. However, as we shall show, we can get an analytical solution for the deterministic case. \( \eta_t = 0 \). Then, for particular parameter values, we analyze the stochastic case.

In what follows, we will assume \( \pi^* < \alpha (n - n^*) \). That is, the trend of foreign inflation, \( \pi^* \), is always expected to be less than the domestic inflation bias which would arise without a TZ, \( \alpha (n - n^*) \). This is a standard assumption in the imported credibility literature in which high-inflation countries borrow the higher credibility of the center country.
4.1. Deterministic case ($\eta_t = 0$)

To expose the logic of the model, we consider first the deterministic case in which the center country's inflation is always lower than the domestic inflation bias which would arise without a TZ. In this case, we have:

**Proposition 1.** Suppose $\eta_t = 0$ for all $t$ with probability 1. Then, in a Markov-perfect equilibrium:

1. When $e_{i-1} = 0$ (i.e. we start from the upper boundary of the TZ), domestic inflation equals foreign inflation ($p_i - p_{i-1} = \pi^*$) and there are no exchange rate changes ($e_i - e_{i-1} = 0$). When $e_{i-1} < 0$ (i.e. we start from the interior of the TZ), domestic inflation is strictly higher than foreign inflation ($p_i - p_{i-1} > \pi^*$) and the exchange rate strictly depreciates ($e_i - e_{i-1} > 0$).
2. As $e_i e_{i-1} \to -\infty$, the value function and the policy rule tend to the BG solution.
3. $V(e)$ is a minimum at the upper boundary, $e = 0$.

**Proof.** See Appendix B.

In what follows, we discuss the implications of Proposition 1 for the motion of the economy. When $e_{i-1}$ is sufficiently close to its upper boundary 0, the TZ is a binding constraint on the domestic government's monetary policy (and will continue to bind in the future). However, when $e_{i-1}$ is sufficiently away from its upper boundary so that $e_i < 0$ is optimal, then $V'(e_i) < 0$. Then, comparing Eq. (8b) with Eq. (15a) reveals that, if the TZ is non-binding, the strategic effect causes exchange rate depreciation (and hence price inflation) to be strictly greater in the BG-TZ model than in the BG model. That is, the government effectively pushes the exchange rate toward the upper boundary.

The intuition is clear. Recall that the underlying assumption is $\pi^* < \alpha(n - n^*)$. The aim of the government is to reduce domestic inflation to foreign inflation $\pi^*$. However, this is credible only at the upper boundary. Therefore, if we start in the interior of the band, the forward-looking government marginally overinflates relative to the BG outcome so as to push the exchange rate toward the upper boundary. Once there, it follows an accommodating policy holding the exchange rate fixed and keeping domestic inflation at foreign inflation $\pi^*$.

In the proof to Proposition 1 we have also shown that as $e_{i-1}$ decreases (i.e. if we start far away from the upper boundary), the number of time-periods before the TZ is a binding constraint, increases. In the limit as $e_{i-1} \to -\infty$, the strategic effect becomes very small and we get the BG solution.

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7The lower boundary is irrelevant in the deterministic case where there is always a positive inflation bias due to BG interactions. However, see below in the stochastic case.
To summarize the deterministic case, where $\pi^* < \alpha(n - n^*)$, the government has an incentive to overinflate relative to the BG case so as the exchange rate depreciates to reach its upper boundary. Since being in the interior of the TZ sustains relatively high inflation, the government should choose a fixed exchange rate regime rather than a TZ, and sets $e = 0$.8

We now consider the optimal exchange rate regime when the center country is subject to random inflation shocks.

4.2. Stochastic case ($\eta \neq 0$)

The previous section suggests that the TZ is not optimal. Inside the TZ, the strategic effect is inflationary which seems costly. However, this interpretation is misleading. This section introduces stochastic inflation shocks and shows that the TZ stabilizes domestic inflation when the foreign country is subject to random inflation shocks. This reduces the domestic government’s expected loss through inflation. That is, in the long run, it has the same average inflation as the foreign country, but lower inflation variance.

We again assume $\pi^* < \alpha(n - n^*)$, but now $\eta \neq 0$ in Eq. (3) above. For simplicity, we assume that there are two states for $\eta$. Each realization is iid. With probability $0 \leq q \leq 1$, low (denoted by the subscript $l$) foreign inflation is realized, where $\pi^* + \eta_l = \pi_l^*$. With probability $1 - q$, high (denoted by the subscript $h$) foreign inflation is realized, where $\pi^* + \eta_h = \pi_h^*$. Then, $\pi^* = q\pi_l^* + (1 - q)\pi_h^* < \alpha(n - n^*)$.

Intuition suggests the following mechanism. If $\pi_l^* = \pi_l^*$, Eq. (15a) suggests a large increase (i.e. depreciation) in $e_l$. Not only is the foreign inflation trend less than the domestic inflation bias, $\pi^* < \alpha(n - n^*)$, but in addition the foreign country enjoys a disinflation shock which reinforces the motion of the exchange rate towards the upper boundary. On the other hand, if $\pi_h^* = \pi_h^*$, the exchange rate is more likely to fall (i.e. appreciate). In a stationary equilibrium with inflation shocks, exchange rates may now lie in the interior of the band.

As in the deterministic case, we have to solve for the loss function $V(e)$ and the policy rule in Eqs. (15a)-(15d). Unfortunately, this problem is not analytically tractable for the general case. However, an analytic solution exists for appropriately chosen parameter values. One example which depicts the basic qualitative results is the following. Suppose that the domestic inflation bias that would arise without a TZ is $\alpha(n - n^*) = 0.06$ and the discount rate is $\delta = 0.95$. Also suppose $\pi_l^* = 0.04$ with probability $q = 0.875$, and $\pi_h^* = 0.12$ with probability $1 - q = 0.125$, so that expected foreign inflation is $\pi^* = 0.05 \leq 0.06$.

We will consider two cases: a large TZ with a 5% band-width, and a small TZ.

8In Coles and Philippopoulos (1995), we provide a numerical example which shows all the above results.
The essential difference is that in the large band case, the lower boundary $e$ never binds on government’s monetary policy. This is not the case with a small TZ.

**Case 1. A large TZ.** Suppose that the TZ allows 5% currency fluctuation so that $e \in [-0.05, 0]$. Let $e_{t,i}$ (respectively $e_{t,h}$) denote the period $t$ exchange rate if the center country has low (respectively high) inflation. Given $e_{t-1} \in [-0.05, 0]$, the solution to Eqs. (15a)–(15d) is

$$e_{t,i} = \begin{cases} 0 & \text{if } \pi_t^* = \pi_{t,i}^* , \\ e_{t,h} = -0.0092 + 0.53e_{t-1}, & \text{if } \pi_t^* = \pi_{t,h}^* , \end{cases}$$

where the government’s loss function is

$$V(e_{t-1}) = 0.0304 - 0.0449e_{t-1} + 0.467e_{t-1}^2 + \frac{\alpha(n - n)^2}{2(1 - \delta)}$$

and the last term is the standard BG loss due to positive unemployment.

If the foreign country does well ($\pi_t^* = \pi_{t,i}^* $), the exchange rate jumps (i.e. depreciates) immediately to the upper boundary 0. On the other hand, if the foreign country does badly ($\pi_t^* = \pi_{t,h}^* $), the exchange rate may appreciate or depreciate depending on its initial position. In particular, if $e_{t-1} \geq -0.0196$, then $e_{t,h} \leq e_{t-1}$, i.e. the exchange rate appreciates. However, if $e_{t-1} < -0.0196$, then both $e_{t,i}$ and $e_{t,h} > e_{t-1}$, i.e. the exchange rate depreciates for both high and low foreign inflation (this happens because the strategic effect dominates). Therefore, in this large TZ case, the lower boundary never binds and observed exchange rates always lie in the interval $[-0.0196, 0]$. In other words, the exchange rates of high inflation economies would lie in the upper half of this band.

The government’s loss function, given by Eq. (16c), decreases with $e$ and is at its lowest when $e = 0$. In particular, when $e = 0$,

$$V(0) = 0.0304 + \frac{\alpha(n - n)^2}{2(1 - \delta)} .$$

**Case 2. A small TZ.** In a small TZ, the lower boundary $e$ may bind when the center country has high inflation. If $e \geq -0.0196$, and for the same parameter values chosen above, it can be shown that the government’s loss function is a

\[\text{Note that Eq. (16c) implies } V'(e) = -0.0449 + 0.934e \text{ which is larger (in absolute value), the more negative } e \text{ is. Recall that } V'(e) \text{ is the strategic effect.}\]

\[\text{Here, it is the strategic interaction that pushes the exchange rate towards the upper boundary of the TZ rather than in the middle of it (however, see the stochastic case). Thus, we get the same predictions for the exchange rate with the basic TZ model, but for different reasons. In the basic TZ model, exchange rate dynamics are driven by an exogenous Brownian motion. Then, the smooth-pasting properties imply low exchange-rate variability near the edges of the band, which keeps the exchange rate in that neighborhood for a relatively long time once it gets there.}\]
series of quadratic functions similar to those described in the proof of Proposition 1. A simple illustrative example arises if we assume \( e \in [-0.009, -0.007] \). Let \( e^* = 0.01 + 1.83e \). Then, it can be shown that the TZ splits into two regions, \([e, e^*]\) and \((e^*, 0]\), with the following properties:

1. For \( e_{t-1} \in (e^*, 0] \), \( e_{t, l} = 0 \) and \( e_{t, h} \in (e, e^*) \). That is, low foreign inflation causes \( e = 0 \) to bind, while high foreign inflation results in an appreciation of the currency within the TZ.

2. For \( e_{t-1} \in [e, e^*] \), \( e_{t, l} = 0 \) and \( e_{t, h} = e \). That is, low foreign inflation causes \( e = 0 \) to bind as before, but now high foreign inflation causes the lower boundary \( e \) to bind. In other words, a sequence of high inflation shocks in the center country takes the exchange rate to the lower boundary of the TZ.

As before, it is possible to show that the government’s loss function is decreasing with \( e \). For purposes of comparison with Eq. (16d), the expected loss, when \( e = 0 \), is

\[
V(0 \mid e) = 0.0311 + 0.0195e + 0.2462e^2 + \frac{\alpha(n - n^u)^2}{2(1 - \delta)}.
\]  

(17)

Since in a small TZ the lower boundary \( e \) may bind, the government’s expected loss now depends on the width of the TZ (compare Eq. (17) with Eq. (16d)). For \( e \in [-0.009, -0.007] \), which is the range of values for which this equilibrium exists, this loss decreases as the band-width increases.\(^1\) This is discussed further in the next section.

4.3. Welfare comparisons of alternative exchange rate regimes

We now compare the loss under a TZ with the loss under fixed and flexible exchange rates. Under fixed and flexible exchange rates, the losses (for the same parameter values chosen above) are\(^2\)

\[
V(\text{fix}) = 0.0320 + \frac{\alpha(n - n^u)^2}{2(1 - \delta)} \quad \text{fixed exchange rates},
\]

(18a)

\[
V(\text{flex}) = 0.0360 + \frac{\alpha(n - n^u)^2}{2(1 - \delta)} \quad \text{flexible exchange rates}.
\]

(18b)

\(^1\)For example, ignoring the last term on the RHS of Eq. (17), the loss when \( e = -0.009 \) is 0.030904 which is less than the loss when \( e = -0.007 \), which is 0.030936.

\(^2\)0.0320 in Eq. (18a) is also the loss of the foreign country. Eq. (18b) gives the loss in the unconstrained BG solution.
Comparing these losses with the loss under the large TZ described by Eq. (16d) above, implies that the large TZ is superior to both fixed exchange rates and flexible exchange rates (i.e. 0.0304<0.0320<0.0360). This happens because a TZ allows the domestic country to import the anti-inflationary credibility of the center country but not the price variability of the latter.

For example, consider the large TZ described above with \( e_{-1} = 0 \). If the foreign country has high inflation, \( \pi^*_h = 0.12 \), then domestic inflation rises to 0.111 which is less than foreign inflation (i.e. 0.111<0.12), and the exchange rate appreciates to \( e_t = 0.009 \). If in the next period the foreign country does well, a strong anti-inflation stance becomes credible again. Foreign inflation is now \( \pi^*_f = 0.04 \), and the exchange rate returns to \( e_t = 0 \), which constrains domestic inflation to 0.049. Most importantly, the variance of inflation in the domestic country is lower than the variance of inflation in the center country. In this stochastic case, the strategic effect reduces inflation variance and hence reduces the government’s expected loss.

For the same stabilizing argument, a small TZ also dominates the fixed and flexible exchange rate regimes. However, a large TZ is better for the government than a small TZ.\(^{13}\) The difference is that if the center country keeps having high inflation shocks, an appreciating exchange rate sustains lower domestic inflation. Of course, if the lower boundary of the TZ binds, this effect disappears and domestic inflation equals the high foreign inflation – the government is no longer able to smooth out these foreign inflation shocks, and so is worse off. For the example given, the government’s expected losses strictly decrease the wider the TZ. This can only correspond to a lowering of the variance of domestic inflation.

This completes the examination of the stochastic case.\(^{14}\)

5. Conclusions, limitations and extensions

This paper has explicitly incorporated exchange rate bands into the BG model of inflation. Our policy game differs from the TZ literature, where monetary policy is used only to maintain the TZ. It also differs from the BG literature, where the position of the exchange rate in the interior of the TZ does not matter. We saw that, even in the simplest possible context, this is not a trivial extension. However,

\(^{13}\)See the discussion below Eq. (17). Also, compare the loss under the large TZ given by Eq. (16d) with the loss under the small TZ given by Eq. (17); ceteris paribus, 0.0304<0.0311.

\(^{14}\)In Coles and Philippopoulos (1995), we also compute some comparative statics which show that: (1) an increase in the difference between the employment targets of the government and wage-setters leads to an increase in inflation variance, and (2) an increase in the variance of foreign inflation increases the desirability of a TZ relative to other exchange rate regimes.
we managed to specify exchange rate dynamics and hence the arguments for and against target zones.

We saw that the position of the exchange rate and the relative desirability of alternative exchange rate regimes depends on the inflationary performance of the center country. Therefore, the role of the center country is crucial. Events (like the Vietnam war and the Great Society program in the case of Bretton Woods, or the German unification in the case of the European Monetary System) leading to a higher inflation equilibrium relative to the peripheral economies may jeopardize the leadership position of the center country and change the rules of the game.

There are several limitations on our results. First, in our model, a fixed exchange rate is optimal in the deterministic case, while, in the stochastic case, the domestic country is better off with a large TZ. This begs the question: what determines the optimal width of the band? Answering this would require the analysis of a two-country game. In that case, the lower boundary for one country is the upper boundary for the other. Determining the optimal TZ would depend on the distribution of inflation shocks across countries, the policy targets in each country and the nature of the game being played by the respective governments.

Second, we have assumed fully credible bands and so we have not specified how foreign exchange intervention takes place. In reality, central banks defend exchange rate bands by using foreign reserves and interest rates, where such controls are costly and hence the TZ may not necessarily be fully credible.

Third, we have abstracted from realignment risks (see Svensson, 1991 and Bertola and Caballero, 1992). However, if there is a positive probability of realignment when the exchange rate hits the upper boundary of the TZ, we do not expect our qualitative results to change, but the strategic effect would be weaker and we might have real effects on employment. Fourth, there is clearly an element of strategic interaction between central banks and speculators associated with foreign exchange intervention (see Bhattacharya and Weller, 1992). Here, we have abstracted from such interactions.

Throughout the paper, we consider only foreign inflation shocks. However, these shocks can be thought as relative ones, i.e. how important foreign shocks are relative to domestic ones.

In Coles and Philippopoulos (1995), we extend the above BG-TZ model by introducing costly foreign exchange intervention. We show that the threat of speculative attacks may cause the government to follow a tight monetary policy to keep the exchange rate in the interior of the band rather than allow it to move close to the upper boundary. The need to reduce costly intervention may be sufficient to make it credible that the government will take an anti-inflationary stance, even though the upper boundary is non-binding. However, such a regime will still stabilize domestic inflation and may be preferred to fixed exchange rates.

For instance, if nominal wage contracts are a weighted average of detenting the band and realigning it, but the government defends the TZ, then real wages turn out to have been too high and employment falls.
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Appendix A

Suppose we consider a finite horizon model which terminates at period T. Since $V_T = 0$, Eq. (14b) implies

$$V_{T-1}(\cdot) = E_{T-1}[1/2(e_T - e_{T-1} + \pi^* + \eta_T)^2 + \alpha/2(n - n^u)^2 + 0],$$  \hspace{1cm} (A.1)

where, $e_T$ is given by Eq. (14a) or by the corner solution. Conditioned on $e_{T-1}$, it follows that $\pi^*$, $\eta_T$ and $e_T$ are all independent of $p^*_{T-1}$. Hence, given $e_{T-1}$, $V_{T-1}(\cdot)$ is also independent of $p^*_{T-1}$, i.e. $V_{T-1}(e_{T-1}, p^*_{T-1}) = V_{T-1}(e_{T-1})$.

Next consider period $t = T - 2$. In this case,

$$V_{T-2}(\cdot) = E_{T-2}[1/2(e_{T-2}^* - e_{T-2} + \pi^* + \eta_{T-2})^2 + \alpha/2(n - n^u)^2 + \delta V_{T-1}(e_{T-2})].$$  \hspace{1cm} (A.2)

Again, given $e_{T-2}$, it follows that $\pi^*$, $\eta_{T-1}$ and $e_{T-1}$ (defined by Eq. (14b)) are all independent of $p^*_{T-2}$. Hence, $V_{T-2}(\cdot)$ is also independent of $p^*_{T-2}$, given $e_{T-2}$. Hence $V_{T-2}(e_{T-2}, p^*_{T-2}) = V_{T-2}(e_{T-2})$. By induction $V_{T-s}(\cdot) = V_{T-s}(e_{T-s})$ for all $s > 0$. As $s \to \infty$, $V_{T-s}(\cdot)$ tends to the stationary solution (15).

Appendix B

We will show that the value function $V(e)$ is a sequence of quadratic functions, $V_s(e) = a + b e + c e^2 + [\alpha(n - n^u)^2]/[2(1 - \delta)]$. Each section is convex and decreasing. However, the value function $V(e)$ is not convex and is not decreasing. The subscript 's' will refer to the number of periods before the TZ becomes a binding constraint on the government’s optimal pricing strategy. Let $D_s$ denote the set of exchange rates where, using the optimal pricing policy, the TZ becomes a binding constraint in $s$ periods time.

For convenience, we redefine the notation in Eqs. (15a)–(15d) by setting $x = e_r$ and $e = e_{r+1}$. Hence our objective is to solve

$$x + \delta V'(x) = e + \alpha(n - n^u) - \pi^* \quad \text{(policy rule)},$$  \hspace{1cm} (B.1)

$$V(e) = 1/2(x - e + \pi^*)^2 + \alpha/2(n - n^u)^2 + \delta V(x) \quad \text{(value function)},$$  \hspace{1cm} (B.2)
over the domain \([e_0, 0]\), where Eq. (B.1) holds for an interior solution for \(x\) (otherwise, the corner solution holds as given by Eqs. (15c) and (15d)).

Because \(\pi^* < \alpha(n-n^u)\), we guess that the exchange rate will always increase. Hence we construct a solution by assuming that \(x(e) > e\) for all \(e < 0\) and \(x(0) = 0\).

Suppose, given \(e < 0\), that in the optimal strategy it will take \(s + 1\) further periods to reach the upper boundary. If we denote \(V(\cdot) = V_{s+1}(\cdot)\) for this case, then Eq. (B.1), Eq. (B.2) imply the recursive equations

\[
x_s + \delta V'_s(x_s) = e + \alpha(n-n^u) - \pi^* \quad \text{(policy rule),} \tag{B.3}
\]

\[
V_{s+1}(e) = 1/2(x_s - e + \pi^*)^2 + \alpha/2(n-n^u)^2 + \delta V_s(x_s) \quad \text{(value function).} \tag{B.4}
\]

Assuming \(V_s(\cdot)\) is quadratic, as described above, Eq. (B.3), Eq. (B.4) define the following recursive equations for \(c_s, b_s\) and \(a_s\):

\[
c_{s+1} = \frac{\delta c_s}{1 + 2\delta c_s}, \tag{B.5}
\]

\[
b_{s+1} = \frac{\delta [b_s - 2c_s \pi^*]}{1 + 2\delta c_s}, \tag{B.6}
\]

\[
a_{s+1} = \delta a_s + \frac{(\pi^*)^2}{2} + \frac{[\alpha^2(n-n^u)^2 - (\pi^* + \delta b_s)^2]}{2(1 + 2\delta c_s)} \tag{B.7}
\]

We are now in a position to solve for \(V(e)\). If \(e = 0\) and \(x(0) = 0\), then Eq. (B.2) implies

\[
V(0) = \frac{(\pi^*)^2 + \alpha(n-n^u)^2}{2(1 - \delta)} > 0. \tag{B.8}
\]

Now, suppose for some \(e < 0\), the optimal strategy is to choose \(x(e) = 0\). In that case, \(V(e) = V_s(e)\) where, by Eq. (B.2),

\[
V_s(e) = 1/2(-e + \pi^*)^2 + \alpha/2(n-n^u)^2 + \delta V(0) \tag{B.9}
\]

and it follows that \(V_1(0) = V(0)\). Now, if \(x(e) = 0\) is optimal, then the corner condition (15c) must hold. Let \(e_1 = [-\alpha(n-n^u) - (1 - \delta)\pi^*]\) and define \(D_1 = [e_1, 0]\) which is nonempty. By construction of \(e_1\), if \(V(\cdot) = V_1(\cdot)\) for \(e \in D_1\), then Eq. (15c) holds for \(e \geq e_1\) and does not hold for \(e < e_1\). Moreover, for \(e \geq e_1\), there does not exist an interior solution for \(x(e)\) in Eq. (B.1) (assuming \(x(e) > e\)). Hence we have a candidate solution: \(V(\cdot) = V_1(\cdot)\) for \(e \in D_1\), where the optimal policy rule is \(x(e) = 0\). For \(e < e_1\), Eq. (14b) does not hold and the optimal strategy is to set \(x(e) < 0\).
Anticipating an induction proof, notice that \( V_1(\cdot) \) is quadratic where \( c_1 = 0.5, \)
\( b_1 = -\pi^* \) and \( a_1 = [(\pi^*)^2]/[2(1-\delta)] \).

Now suppose \( e < e_1 \) and the optimal strategy is to set \( x(e) \in D_1 \). In this case we
are two steps away from \( e = 0 \), and \( V(\cdot) = V_2(\cdot) \) is defined by Eq. (B.3) and Eq.
(B.4) with \( s = 1 \). Since \( V_1 \) is quadratic, then \( V_2 \) must be quadratic, and its
coefficients are defined by Eq. (B.5), Eq. (B.6) and Eq. (B.7) with \( s = 1 \). It is
straightforward to show that \( V_2(e_1) = V_1(e_1) \) but \( V'_2(e_1) > V'_1(e_1) \) as drawn in Fig. 1
(where 0 is always the upper boundary).

This complicates the analysis somewhat. \( V(\cdot) \) is not convex and hence Eq. (13a)
does not completely characterize the government’s optimal pricing strategy. The
definition for \( Z \) in Eq. (11) is rewritten following the notation of the appendix as

\[
Z = 1/2(x - e + \pi^*)^2 + \alpha/2(x + p_t^* - w_t - n)^2 + \delta V(x),
\]

where \( Z \) is the government’s loss by choosing \( x = p_t - p_t^* \), given the announced \( w_t \).

To understand the problem caused by the nonconvexity above, suppose that
\( V(\cdot) = V_2(x) \) for \( e \in [e_2, e_1) \), where \( e_2 \) is to be defined, and \( V = V_1(x) \) for \( e \in [e_1, 0] \).
Given \( e, p_t^* \) and \( w_t \), \( Z(x, \cdot) \) has a structure as shown in Fig. 2.

There may be two local minima. Hence Eq. (15a) in the text admits too many
potential solutions and we have to refine the argument as follows. Suppose, given
\( e \), the union anticipates that the government will set \( x = x_2(e) \) and so sets
\( w_t = x_2(e) + p_t^* - n'' \) (so that expected employment equals the union’s target
employment). Then, Eq. (B.10) becomes

\[
\hat{Z}(x,e) = 1/2(x - e + \pi^*)^2 + \alpha/2(x - x_2(e) + n'' - n)^2 + \delta V(x).
\]

Fig. 1.
By construction of \( x_2(\cdot) \) and the definition of \( B(\cdot) \) in Eq. (10), the minimum of \( \hat{Z} \) for \( x < e_1 \) must occur at \( x = x_2 \). Let \( x' \) denote the value of \( x \) which minimizes \( \hat{Z} \) for \( x > e_1 \). It can be shown that, as \( e \) increases, the minimum at \( x' \) decreases more quickly than the minimum at \( x_2 \). For \( e \) large enough we can reach a situation as in Fig. 3.

In this case, \( x_2 \) cannot be an equilibrium (even though it satisfies the first-order
conditions (B.3)-(B.4)). If the union sets \( w_i = x_2(e) + p_i^* - n'' \), the government deviates by setting \( p_i - p_i^* = x' > x_2 \). The union should increase \( w_i \). It can be shown by using Eq. (B.10), that as \( w_i \) increases, the minimum to the right of \( e_1 \) falls below the minimum to the left of \( e_1 \). Hence if in the above case the union sets \( w_i = x_1(e) + p_i^* - n'' \), the government’s best response is to set \( x = x_1(e) \) which forms an equilibrium.

Being loose with notation, define \( e_2 \) where \( x_2(e), e) = \hat{Z}(e', e) \). Then we have the following candidate solution: \( V = V_2(\cdot) \) for \( e \in [e_2, e_1) = D_2 \). By construction, if \( e \in D_2 \) (which implies \( e \geq e_2 \)), the union anticipates \( x = x_2(e) \) and sets wages accordingly. The government’s best response is \( x = x_2(e) > e_1 \). For \( e < e_2 \), the union anticipates \( x = x_2(e) \in D_1 \), sets wages accordingly and the government’s best response is to set \( x = x_2 \). Thus, the union’s wage rule is not continuous across \( e = e_2 \) and it can be shown that \( V(\cdot) \) is not continuous at \( e_2 \). In fact, \( V(\cdot) \) has a structure as shown in Fig. 4.

Now consider \( s = 2 \) in Eq. (B.3) and Eq. (B.4) which defines \( V_3(\cdot) \) and \( x_3(\cdot) \). Suppose \( V(\cdot) = V_3(\cdot) \) for \( e \in [e_3, e_2) \) where \( e_3 \) is to be defined. Notice by construction of \( e_2 \), that if \( e < e_2 \), there exists a wage strategy \( w_i = x_3(e) + p_i^* - n'' \) which dominates any wage higher than that. Suppose, however, the union considers setting \( w_i = x_3(e) + p_i^* - n'' \). Then, for \( s = 2 \), Eq. (B.11) is rewritten as

\[
\hat{Z}_{s+1}(x, e) = \frac{1}{2}(x - e + \pi^*)^2 + \alpha/2(x - x,_{s+1}(e) + n'' - n)^2 + \delta V(x).
\]

(B.12)

For \( e < e_2 \), \( \hat{Z}_3(\cdot) \) has a structure as shown in Fig. 5, where \( x' \) is the argument
which minimizes $Z_3$ for $x > e_2$, which must occur in the interval $[e_2, e_1)$ (by the construction of $e_2$, and that $e < e_2$ and $x_3(e) < x_2(e)$). Define $e_3$ where $\hat{Z}_3(x_3(e), e) = \hat{Z}(x', e)$. It follows that if $e \in [e_2, e_3)$, the union anticipates $x = x_2(e) \in D_2$, sets wages accordingly, and the government’s best response is to set $x = x_2$. For $e < e_3$, the union anticipates $x = x_3(e) \in D_2$, sets wages accordingly and the government’s best response is to set $x = x_3 \in D_2$. Hence we have the equilibrium $V(\cdot) = V_3(\cdot)$ for $e \in [e_3, e_2)$.

The induction follows straightforwardly from this. If $V_{s-1}(\cdot)$ is quadratic, then $V_s(\cdot)$ is also quadratic with coefficients given by Eq. (B.5), Eq. (B.6), Eq. (B.7), with starting values $a_1$, $b_1$ and $c_1$ given above. It follows that $\{c_s\}$ is a positive, decreasing sequence which declines exponentially to zero as $s \to \infty$. It can also be shown that $b_s = -2n \pi^* c_s$, so that $b_s$ is negative for all $s$ and also tends to zero as $s \to \infty$. Given the solution for $b_s$ and $c_s$, it also follows that $V_s$ is a convex function for all $s$, whose minimum is at $e = n \pi^* > 0$. Hence, $V'_s(\cdot) < 0$ for all $e < 0$.

A solution to Eq. (B.1) and Eq. (B.2) is given by $V(\cdot) = V_s(\cdot)$ for $e \in [e_s, e_{s-1})$, where $e_s$ is defined by $\hat{Z}_s(x_s(e), e) = \hat{Z}_s(x', e)$. Since $V'_s(\cdot) < 0$ for all $s$, Eq. (B.3) implies $x_s(e) > e$ for all $e \in D_{s+1}$, for all $s > 0$ (as assumed). Notice that the nonconvexity of $V(\cdot)$ at $e_{s-1}$ implies that the union’s wage policy is not continuous at $e = e_s$, where $x_s(e_s) < e_{s-1} < x_{s-1}(e_s)$. This implies that the value function is discontinuous at $e_s$, where $V(e_s^+) > V(e_s^-)$ for all $s > 1$. (Given the union raises wages by $x_{s-1}(e_s) - x_s(e_s)$ across $e_s$, notice that by Eq. (B.10), $Z(\cdot)$ must increase with $w$ for all $x \geq x_s(e_s)$. Since the government chooses $x = x_{s-1}(e_s) > x_s(e_s)$ (while $x_s(e_s)$ was optimal before), then $V(\cdot)$ must increase by a discrete amount at $e_s$.

Since $V'_s(\cdot) < 0$, then Eq. (B.3) implies $x_s - e > \alpha(n - n^*) - \pi^*$. Hence, in
general, \( e_{t-1} - e_t > \alpha(n-n^*) - \pi^* \). Moreover, as \( s \to \infty \), \( e_s \to -\infty \) and \( a_s \to [\alpha^2(n-n^*)^2]/[2(1-\delta)] \) and \( V_s(\cdot) \) tends to the BG solution in Eq. (8d). Finally, since inflation is greater than \( \pi^* \) for \( e < 0 \), \( V(\cdot) \) must be at its global minimum at \( e = 0 \).

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