MACROECONOMIC DYNAMICS AND OPTIMAL EXTERNAL BORROWING IN AN IMPERFECT WORLD CAPITAL MARKET*

by

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I INTRODUCTION

Since the mid-1970s, the discussion on external debt has been at the centre of macroeconomic policy. The burden of the debt has dramatically increased because of the large increase in real interest rates in world capital markets. It seems that an increasing demand for borrowing and macroeconomic policies in DCs are behind the high real interest rates. This raises the question of a policy towards external debt, since the decision to service the debt reduces growth and consumption.

On a theoretical level, the problem of external debt has been analysed in an intertemporal optimization context. However, the results of the benchmark small open economy model are not attractive. The reason is that this model cannot produce endogenous dynamics. There is not a steady state unless it so happens that the time preference rate equals the world interest rate factor but, then, consumption is constant along the optimal path and the steady state is not unique. To remedy these unattractive results, different studies vary in the way in which they deviate from the benchmark model. For instance, in a small open economy facing given world interest rates, Obstfeld (1981) assumed endogenous time preference rates and Blanchard (1985) imposed finite horizons.

This paper focuses on the role of real interest rates to analyse an alternative deviation from the benchmark small open economy model that leads to well-behaved dynamics. The world capital market is assumed to be imperfect, so that as an indebted country expands its borrowing it must pay higher interest rates. This is in the spirit of Bardhan (1967) and Hamada (1969) and consistent with what actually happens. To the extent that interest rates are endogenous, they affect the endogenous dynamics of debt, current

* Manuscript received 15.6.89; final version received 5.3.90.
† This paper is part of my Ph.D. thesis. I would like to thank George Alogoskoufis, David Begg, John Drifflin, an anonymous referee and the Editor for their valuable comments. All errors are mine.
1 In particular, the increase in world interest rates is mainly due to a reduction in world savings (as a result of unduly optimistic policies in LDCs and the U.S. deficit) and to macroeconomic policies in DCs (tight money, easy fiscal policy and increased marginal efficiency of investment). See van Wijnbergen (1985) and Cohen (1985).
account, output and consumption. In addition, they transmit the effects of world-wide changes in macroeconomic policies to the domestic economy.

Adopting a debtor's perspective, the main results are as follows. A debtor economy has a tendency to converge to a well-defined steady state where the debt service burden is paid when, first, total and marginal debt repayments are both increasing functions of the amount of borrowing and, second, either it cares about the future or the dominant diagonal condition holds. This saddlepath condition is consistent with the optimal growth theory as surveyed in McKenzie (1981). Therefore, the endogeneity of world interest rates produces well-behaved dynamics and offers an alternative to endogenous time preference rates, finite horizons, or enriched utility functions.

Consider the motion of the economy along this saddlepath. An initial period characterized by high consumption financed by external borrowing must be followed by periods of decreasing borrowing, current account surpluses and high output relative to consumption for the debt service to be repaid. However, once having placed the economy on its convergent path, along which the decreasing debt leads to decreasing marginal debt repayments, the surplus of output over consumption can start decreasing over time.

This framework can be used to analyse the dynamic response of an indebted economy to macroeconomic policies in creditor countries. Policies that put an upward permanent pressure on world interest rates require an initial sharp decline in consumption to pay the increased debt service and place the economy on the new saddlepath. However, in the long run this exogenous shift, by increasing the cost of borrowing, lowers the optimal level of debt and allows the economy to enjoy a higher level of consumption. If the economy becomes more patient, i.e., it cares more about the future, the dynamic responses are the same.

Section II models the economy. Section III solves for the saddlepath. Section IV discusses the motion of the economy over time. The dynamic effects of exogenous changes are analysed in Section V. Section VI closes the paper. Mathematical proofs are gathered in an Appendix.

II The Economy

In order to focus on the essentials the model is as simple as possible. Consider a deterministic market-clearing real open economy that is centrally planned. International trade in both goods and assets is allowed. For simplicity there is no capital and the only assets are world bonds which allow lending or borrowing. There are no restrictions on this world market except that all debts must be repaid. The social planner is endowed with perfect foresight and an infinite discrete time horizon.

\footnote{In this context, there is complete equivalence between a command economy and a decentralized one (see Abel and Blanchard, 1983).}
Consider the world asset market. One-period bonds, \( b \), pay a rate of return, \( r > 0 \). Thus, if the country borrows \( b_{t-1} \) at time \( t-1 \), it has to repay \((1+r_{b_{t-1}})b_{t-1}\) the next period \( t \). As in Bardhan (1967) and Hamada (1969), it is assumed that the debtor economy faces an inverse supply function of borrowing, \( r_t = r(b_t) \), where \( r_b(.) > 0, r_{bb}(.) > 0 \). In the symmetric case of a creditor country, the economy would face a demand for borrowing, \( r_t = r(b_t) \), where now \( r_b(.) < 0, r_{bb}(.) < 0 \). In what follows, we shall deal with a debtor economy.

To save on notation, denote the total debt repayment by \( f(b) \). Thus, \( (1+r(b_{t-1}))b_{t-1} = f(b_{t-1}) \). The properties of \( f(.) \) follow directly from the properties of \( r(.) \), so that \( f_b(.) > 0, f_{bb}(.) > 0 \). The sign of these derivatives has a simple economic interpretation: total and marginal debt repayments are both increasing functions of the amount of borrowing.

Since there is no capital, domestic output of the single traded good is produced only by labour. Assuming a linear production function, one unit of output is produced by one unit of labour, \( l \).

Denoting consumption by \( c \), the budget constraint at time \( t \) is simply

\[
b_t - b_{t-1} = c_t + r(b_{t-1})b_{t-1} - l_t
\]

which is the balance of payments. The current account deficit is financed by the capital account surplus (a debtor economy), or the current account surplus finances the capital account deficit (a creditor economy).

To close the planner's problem, his instantaneous utility function is assumed to be separable in consumption and labour effort, \( u(c_t) - v(l_t) \).

Assume \( u_c(.) > 0, u_{cc}(.) < 0, u_{ccc}(.) > 0 \), and \( c_t > 0 \) since \( \lim_{c \to 0} u_c(.) = \infty \).

Also, assume \( v_l(.) > 0, v_{ll}(.) > 0, v_{lll}(.) < 0 \), and \( \lim_{l \to -\infty} v_l(.) = \infty \).

Maximizing \( \sum_{t=0}^{\infty} \delta^t[u(c_t) - v(l_t)] \) with respect to \( \{c_t, l_t, b_t\}_{t=0}^{\infty} \) subject to (1) and an initial debt, \( b_{-1} \), the optimality conditions are

\[
\begin{align*}
\text{(2a)} & \quad u_c(c_t) = v_l(l_t) \\
\text{(2b)} & \quad u_c(c_t) = \delta f_b(b_t)u_c(c_{t+1}) \\
\text{(2c)} & \quad b_t = f(b_{t-1}) + c_t - l_t \\
\text{(2d)} & \quad b_{-1} \equiv b_{-1} \\
\text{(2e)} & \quad \lim_{t \to \infty} d_t b_t = 0
\end{align*}
\]

\( ^4 \text{Thus, it is assumed that the only endogenous variable that affects the interest rate is borrowing.} \)

\( ^5 \text{For exogenous effects, see Section V.} \)
where \( f(b_t) \equiv \{1 + r(b_t)\}b_t; d_t \equiv \prod_{j=1}^{t} (1 + r_{j-1})^{-1}, \) with \( r_t = r(b_t) \) and \( d_0 \equiv 1; \) and \( 0 < \delta < 1 \) is the exogenous time preference rate.

(2a) states that the marginal rate of substitution between leisure and consumption equals their relative price (here, unity). The intertemporal condition (2b) says that the marginal debt repayment, \( f(b_t) \), equals the discounted marginal revenue, \([u_c(c_t)]/[\delta u_c(c_{t+1})]\). (2c) and (2d) are the budget constraint and the initial condition. Finally, solvency requires the satisfaction of the terminal condition (2e) which rules out the possibility of unbounded utility by following an ever-rising borrowing policy and meeting all debt repayments through further borrowing. As in Obstfeld (1981), (2e) holds if the paths of \( \{c_t, l_t, b_t\} \) \( \infty \) are such that in each period the present value of labour income net of debt (i.e., the present value of wealth), \( \sum_{s=t}^{\infty} d_l s - d_l s b_{t-1}, \) equals the present value of consumption, \( \sum_{s=t}^{\infty} d_s c_s > 0. \)

Having derived the equilibrium dynamics, we now turn to their behaviour.

III ENDOGENOUS DYNAMICS

Examination of the equilibrium dynamics (2) will be constructed in three steps. In a preliminary step, some substitutions will reduce (2) to a mathematically-manageable form familiar to the optimal growth theory. The second step discusses the steady state. The third step analyses the behaviour of the economy around the steady state.

Consider the first step. Solving (2a) for output/employment, we get

\[
l_t = u_c^{-1}\{u_c(c_t)\} \equiv L(c_t) \quad (3)
\]

where the properties of \( u(.) \) and \( v(.) \) imply that \( L_c(.) = 0, L_{cc}(.) < 0. \) Then, (2c) implies that

\[
c_t - l_t \equiv c_t - L(c_t) = b_t - f(b_{t-1}) \quad (4)
\]

or

\[
c_t = \kappa^{-1}\{b_t - f(b_{t-1})\} \quad (5)
\]

where the properties of \( \kappa(c_t) \equiv c_t - L(c_t) \) follow from those of \( L(.) \).

Substituting (5) for consumption into (2b), the equilibrium dynamics can be reduced to a second-order difference equation in borrowing, \( b. \)

\[
u_t[\kappa^{-1}\{b_t - f(b_{t-1})\}] = \delta f_t(b_t) u_t[\kappa^{-1}\{b_{t+1} - f(b_t)\}] \quad (6)
\]

Consider next the steady state, denoted by \( b^\infty, c^\infty \) and \( l^\infty. \) With \( b_t = b^\infty \) for all \( t, (6) \) results in a unique solution \( b^\infty = f_b^{-1}(1/\delta) > 0. \) Then, it follows from (4) that \( l^\infty - c^\infty = r(b^\infty)b^\infty, \) where \( l^\infty \) and \( c^\infty \) are given by (3) and (5). In
the steady state, the surplus of output over consumption must equal interest payments for debt service.

The final step analyses the endogenous dynamics around the steady state. Before starting, some notation will be very convenient. The reader familiar with optimal growth theory can observe that (6) can be written as

$$V_1 \{b_t - f(b_{t-1})\} + \delta V_2 \{b_{t+1} - f(b_t)\} = 0$$

(7)

where the properties of the function $V(.t) \equiv V\{b_t - f(b_{t-1})\}$ along the optimal path are given in Appendix (1).

When evaluated at the steady state, the eigenvalues of the Jacobian equation, which describes the motion of the model, are the solutions of

$$V_{12} + (V_{11} + \delta V_{22}) \lambda + \delta V_{21} \lambda^2 = 0$$

(8)

Evaluating these partial derivatives (given in Appendix (1)) at the steady state, it directly follows that:

**Proposition 1.** At least in the neighbourhood of the steady state, the overall maximand $V(.)$ is a twice differentiable and strictly concave function. Also, symmetry of positive off-diagonal Jacobian elements holds, as introduced by the optimal growth literature and surveyed in McKenzie (1981). Thus, $V_{11}, V_{22} < 0, V_{12} = \lambda_2 > 0$, and $V_{11} V_{22} - V_{12}^2 > 0$.

Going back to (8), Appendix (2) proves that:

**Proposition 2.** Proposition 1 guarantees a positive discriminant and hence real eigenvalues. Thus, complex dynamics are ruled out.6

We can now examine the asymptotic behaviour of the linearized model. Since it can be checked that both roots, $\lambda_1$ and $\lambda_2$, are positive, we prove in Appendix (3) following Benhabib and Nishimura (1985):

**Proposition 3.** Propositions 1 and 2 imply that

(i) $0 < \lambda_1 < 1$ and $\lambda_2 > 1$, when $V_{12} (1 + \delta) + V_{22} + \delta V_{11} < 0$

(ii) $\lambda_1, \lambda_2 > 1$, when $V_{12} (1 + \delta) + V_{22} + \delta V_{11} > 0$.

If condition (i) holds, the model exhibits saddlepath stability. Observe that saddlepath requires this supplementary condition (i), which is independent of concavity, symmetry of off-diagonal elements and sign restrictions guaranteed by the structure of the model itself. These results are quite consistent with McKenzie (1981). It is easily proved that condition (i) is satisfied if we care about the future, that is, if the time preference rate, $\delta$, is sufficiently close to 1.7 This result has been proved by Levhari and Liviatan (1972) and Scheinkman (1976). Alternatively, condition (i) is directly satisfied

6For complex dynamics, see Grandmont (1986).

7By substituting the derivatives of $V(.)$—given in Appendix (1)—evaluated at the steady state into condition (i).
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by the dominant diagonal condition.\textsuperscript{8} This result, independent of the time preference rate, has been proved by Araujo and Scheinkman (1977).

Therefore, if condition (i) of Proposition 3 holds, external borrowing evolves over time according to the rule \((t \geq 0)\)

\[ b_t = b^\infty + (b_{-1} - b^\infty) \lambda_1^{t+1} \]  

(9a)

or

\[ b_t = \lambda_1 b_{t-1} \]  

(9b)

where \(0 < \lambda_1 < 1\) is the stable root, \(b_{-1}\) is the initial condition (2d), and \({}\) denotes deviations from the steady state. In turn, (5) and (3) can be solved for consumption and output, respectively.

This completes the saddlepath solution. To summarize, our solution guarantees that a debtor economy has a tendency to converge to a well-defined steady state where the debt service burden is paid. This is ensured when, first, total and marginal debt repayments are both increasing functions of the amount of borrowing and, second, either we care about the future or the dominant diagonal condition holds. This is a logical result.

Before we move on, it is worth comparing our results with those of the relevant literature. Consider first the steady state. In the benchmark small open economy model (where there is a given world interest rate, \(\bar{r}\), condition (2b) would rule out the possibility of a steady state, unless it so happened that the time preference rate, \(\delta\), were equal to \(1/(1 + \bar{r})\). However, in this case, the economy has no endogenous dynamics, since the paths of consumption, output and welfare are flat; also there are only equations (2a) and (2c) to determine \(b^\infty\), \(c^\infty\) and \(I^\infty\), so that the steady state is not unique.\textsuperscript{9} The endogeneity of world interest rates produces endogenous dynamics and a unique steady state and, therefore, plays the same role with endogenous time preference rates (see Obstfeld, 1981) and finite horizons (see Blanchard, 1985).\textsuperscript{10}

Consider next the condition that guarantees saddlepath stability. Obstfeld (1981) assumes that the time preference rate depends on contemporaneous utility. Stability is ensured when domestic residents care less about the future as utility and consumption increase today.\textsuperscript{11} Blanchard (1985) ensures stability by assuming that for each individual the world interest rate is less than the discount factor plus the probability of death. Otherwise,

\textsuperscript{8}See McKenzie (1981, p. 1337).

\textsuperscript{9}See fn. 2. These results also hold in more general models with capital accumulation (see Blanchard, 1983, and Giovannini, 1988).

\textsuperscript{10}For other mechanisms that produce endogenous dynamics, see Liviatan (1981) and Dornbusch (1983). The former introduced debt in the utility function and the latter changes in the prices of non-traded goods.

\textsuperscript{11}See Obstfeld and Stockman (1985, p. 963). Remember that if \(\delta\) and \(\theta\) are the time preference rates in discrete and continuous time respectively, then \(\theta = (1 - \delta)/\delta\). Then, the condition is \(\delta_\theta(u) < 0\).
individual consumption would increase at a rate larger than the probability of death and hence aggregate consumption would increase forever.\textsuperscript{12}

IV THE MOTION OF THE ECONOMY OVER TIME

This section discusses the implications of our solution for the dynamics of external borrowing, current account, consumption and output. Inspecting (9) reveals that the optimal path of debt is monotone. Assume that the economy starts with an initial debt higher than the steady state one, \( \bar{b}_{-1} \equiv (b_{-1} - b^\infty) > 0 \). Then, since \( 0 < \lambda_1 < 1 \), all the time the external debt is decreasing, \( b_{t-1} > b_t > b_{t+1} \) and so on.\textsuperscript{13} Monotonic saddlepath stability and absence of cycles is a standard mathematical result in this class of models.\textsuperscript{14} The intuition becomes clear from the solution. We proved that \( V(t) = V\{b_t - f(b_{t-1})\} \) implies \( V_{12}(t) > 0 \). The higher the amount of past borrowing, \( b_{t-1} \), the higher the marginal utility of today borrowing, \( V_1(t) \), and hence from the concavity of the utility function the less the amount of today borrowing, \( b_t \). Then, \( b_{t-1} > b_t \) implies \( b_t > b_{t+1} \) and so on (for a general proof, see Benhabib and Nishimura, 1985).

Since the capital account does not change sign along the whole saddlepath, (4) reveals that the current account does not change either. Assume that the economy starts with a relatively high debt, \( (b_{-1} - b^\infty) > 0 \), which has been used to finance a high level of consumption. This initial period must be followed by periods of decreasing debt and current account surpluses, \( l_t - c_t - r(b_{t-1})b_{t-1} = (1 - \lambda_1)b_{t-1} > 0 \), for the debt service to be repaid. In fact, Cohen (1985) has calculated the required current account surpluses that would make solvency possible in a number of LDCs.

It is interesting to examine the saddlepath relation between the surplus of output over consumption, \( (l_t - c_t) \), and past borrowing, \( b_{t-1} \). Conditions (9) and (4) imply that along the saddlepath \( (l_t - c_t) \) is an increasing and convex function of \( b_{t-1} \).\textsuperscript{15} The meaning of the first derivative is clear. An initial period of relatively high debt must be followed by periods of high output relative to consumption and, as we said before, decreasing borrowing and current account surpluses. Consider the implications of the second derivative. The decreasing debt leads to decreasing marginal debt repayments and this allows a decreasing surplus of output over consumption, \( (l_t - c_t) > (l_{t+1} - c_{t+1}) > 0 \),

\textsuperscript{12}See Blanchard (1985, p. 230).

\textsuperscript{13}The results are symmetric if we start from an initial position where the debt is less than the steady-state one, \( \bar{b} \equiv (b_{-1} - b^\infty) < 0 \).

\textsuperscript{14}See Kamien and Schwartz (1981, Part II, Section 9).

\textsuperscript{15}Thus, \( \frac{\partial(l_t - c_t)}{\partial b_{t-1}} = [1 - \lambda_1 + r_s(b_{t-1})b_{t-1} + r(b_{t-1})] > 0 \) and

\[ \frac{\partial^2(l_t - c_t)}{\partial b_{t-1}^2} = [r_s(b_{t-1})b_{t-1} + 2r_s(b_{t-1})] > 0. \]
and so on. Once having placed the economy on its convergent path, the surplus can start decreasing, although \((l_t - c_t) > 0\) all the time due to monotonicity. Finally, along the saddlepath \(l_t\) is increasing and convex in \(b_{t-1}\), while \(c_t\) is decreasing and concave in \(b_{t-1}\). This completes the algebraic proof.\(^{16}\)

To reinforce this proof and following the standard practice, we show the relevant phase diagram. Recall that the motion of the economy is given by (2b) and (2c). Start from (2b). Consumption is stationary, \(c_t = c_{t+1}\), whenever \(b^\omega = f_b^{-1}(1/\delta) > 0\). In the \((c,b)\) plane, the locus \(c_t = c_{t+1}\) is then a vertical line. The properties of \(f(.)\) imply for \(b_t > b^\omega\), \(\delta f_b(b_t) > \delta f_b(b^\omega) = 1\). This implies that \(u_c(c_t)\) is decreasing or \(c_t\) is increasing. On the other hand, for \(b_t < b^\omega\), \(c_t\) is decreasing. These movements of \(c\) are represented by small vertical arrows.

Consider now (2c). The slope and curvature of the locus \(b_t = b_{t-1}\) follow from the properties of \(l^\omega - c^\omega = r(b^\omega)b^\omega\). Above this locus, \(b_t\) increases as \(c_t\) increases. Below this locus, \(b_t\) decreases. These movements of \(b\) are shown by small horizontal arrows.

The steady state, \(E\), is the point of intersection of the two loci, and from the arrows it is obvious that this steady state is a saddlepoint. The planner chooses the stable root \(0 < \lambda_1 < 1\) to place the economy on the unique convergent path to the steady state. The negative and concave association

![Fig. 1 The Saddlepath](image)

\(^{16}\)What is the value of the world interest rate? Since \(r_t = r(b_t)\), the solution for \(b_t\) given by (9) results in a solution for \(r_t\). The properties of \(r(.)\) imply that the marginal debt repayment, \(f_b(b_t) = [1 + r(b_t) + r_b(b_t)b_t]\), is higher than the average debt repayment, \([f(b_t)]/b_t = [1 + r(b_t)]\).

Since the domestic country faces a positively-sloped supply of capital, \(r_t = r(b_t)\), it exploits its market power and pays a price \([1 + r(b_t)]\) when it borrows \(b_t\), which is less than its marginal debt repayment.
between past borrowing and consumption along the saddlepath follows from the algebraic analysis. If we start at point $O$, where a relatively high debt, $(b_{-1} - b^\infty) > 0$ has financed a high level of consumption, consumption has to fall vertically below $O$ to point $P$ to place the economy on the saddlepath. Then, the economy travels along the saddlepath to $E$.

Finally, compare the dynamics of adjustment in this model with those in the models of Obstfeld (1981) and Blanchard (1985). Not surprisingly, the results are symmetric (these models discuss the case of a creditor country). However, as explained in the previous section, the source of the endogenous dynamics and the stability conditions are quite different from those in this paper.

To complete this comparison, consider the steady-state position of the domestic country relative to the rest of the world. Fig. 1 seems to imply that the economy is always a net debtor in the steady state, i.e., its net foreign assets are negative. However, the real answer is given by the properties of the interest rate function, $r_t = r(b_t) > 0$. We proved that at the steady state $\delta \cdot f_b(b^\infty) = 1$ or $b^\infty = f_b^{-1}(1/\delta) > 0$, where $f_b(b^\infty) = 1 + r(b^\infty) + r_t(b^\infty)b^\infty$. It follows that $r(b^\infty) + r_t(b^\infty)b^\infty = (1 - \delta)/\delta \equiv \theta$. This condition can describe two steady-state asset positions. If $0 < r(b^\infty) < \theta$, $r_t(b^\infty)b^\infty > 0$. In this case, $b^\infty$ represents debt (the country as a debtor faces a positively-sloped supply of world capital, $r_t(b^\infty) > 0$). On the other hand, if $r(b^\infty) > \theta > 0$, $r_t(b^\infty)b^\infty < 0$. In this case, $b^\infty$ represents assets (the country as a creditor faces a negatively-sloped demand for world capital, $r_t(b^\infty) < 0$). These results are consistent with the standard conditions of the relevant literature (see Blanchard, 1985, p. 230). Therefore, even if we have adopted a debtor's perspective in the analysis of solvency, the model is generally capturing the case of a creditor economy as well.

V Dynamic Responses to the World Capital Market and Time Preference

The framework developed so far can be used to analyse the effects of exogenous changes in world interest rates or time preferences on the dynamics of an indebted economy.

We start from exogenous changes in world interest rates. The large increase in debt since the mid-1970s in many LDCs has been caused—not only by unduly optimistic levels of spending financed by external borrowing—but also by large increases in the world interest rates due to macroeconomic policies in the lending DCs. The most obvious candidates seem to be tight money, easy fiscal policies and increased marginal efficiency in these countries (see van Wijnbergen, 1985).

\[17\text{Remember that if } \delta \text{ and } \theta \text{ are the time preference rates in discrete and continuous time respectively, then } \theta = (1 - \delta)/\delta.\]
Assume that the inverse supply function of world capital, \( r_t = r(b_t) \), has the form \( r_t = Ab_t^\alpha \), where \( A > 0 \) and \( \alpha > 1 \). The term \( A \) is the channel for transmitting the effects of the world capital market to the domestic country.

Start from the steady state. Along the \( c_t = c_{t+1} \) locus, \( b^\infty = \int b^{-1}(1/\delta) \). Our particular example implies that \( b^\infty = \left\{ \frac{1-\delta}{\delta(1+\alpha)A} \right\}^{1/\alpha} \) and then it is clear that \( \partial b^\infty / \partial A < 0 \). Thus, in the \((c, b)\) plane, a permanent increase in \( A \) shifts the \( c_t = c_{t+1} \) locus to the left (see Fig. 2). Along the \( b_t = b_{t-1} \) locus, \( l^\infty - c^\infty = r(b^\infty)b^\infty \). Our example implies that \( l^\infty - c^\infty = A(b^\infty)^{\alpha+1} \) and then it can be proved by using the value of \( b^\infty \) that \( \partial(l^\infty - c^\infty) / \partial A < 0 \). Thus in the \((c, b)\) plane, a permanent increase in \( A \) shifts the \( b_t = b_{t-1} \) locus to the right.

Exogenous changes that put an upward permanent pressure on world interest rates (\( A \) increases from \( A_1 \) to \( A_2 \)) move the economy's long-run equilibrium, from \( E_1 \) to \( E_2 \). This exogenous shift, by increasing the cost of borrowing, lowers the optimal level of long-run debt, \( b^\infty(A_1) > b^\infty(A_2) \). Then, a lower debt service burden allows the economy to enjoy a higher level of consumption, \( c^\infty(A_1) < c^\infty(A_2) \).

Consider the movement towards the new steady state. Assume that the economy's initial position is the steady state \( E_1 \), and \( A \) increases from \( A_1 \) to \( A_2 \). Since the debt is a backward-looking variable and cannot jump, consumption has to fall sharply from \( E_1 \) to \( M \) to pay the increased debt service and place the economy on the new saddlepath. The motion along this saddlepath has been analysed in the previous section.

Finally, consider the effects of a permanent increase in the time preference rate, \( \delta \), which is equivalent to assuming that the economy cares more about the future. It is clear from the solution, \( b^\infty = \left\{ \frac{1-\delta}{\delta(1+\alpha)A} \right\}^{1/\alpha} \), that the
qualitative dynamic response is the same. As before, an increase in $\delta$ shifts the $c_t = c_{t+1}$ locus to the left and the $b_t = b_{t-1}$ locus to the right. There is an initial sharp decline in consumption but, in the long run, the debt is lower and consumption is higher.

VI CONCLUSIONS

This paper studied the dynamics of an extension of the benchmark small open economy which incorporated a richer specification of the world asset market. In particular, it was assumed that total and marginal debt repayments are both increasing functions of the amount of borrowing. The inherent logic of the model placed the economy on its saddlepath toward a unique steady state and produced interesting dynamic paths of output, consumption, current account and external borrowing.

The model can be extended in many ways. Perhaps an important extension would be to study the above problem and, specifically, the endogenous debt repayment, not only in terms of solvency but also in terms of strategic interaction between the debtor country and the creditor institutions.

APPENDIX

(1) Properties of $V(t) = V\{b_t - f(b_{t-1})\}$ along the Whole Optimal Path

$$V_1(t) = u_c[K^{-1}\{b_t - f(b_{t-1})\}] > 0$$

$$V_2(t+1) = -f_0(b_t)u_c[K^{-1}\{b_{t+1} - f(b_t)\}] < 0$$

$$V_{12}(t) = -u_{cc}(c_t)f_0(b_{t-1})/\kappa_c(c_t) > 0$$

$$V_{21}(t+1) = -u_{cc}(c_{t+1})f_0(b_{t+1})/\kappa_c(c_{t+1}) > 0$$

$$V_{11}(t) = u_{cc}(c_t)/\kappa_c(c_t) < 0$$

$$V_{22}(t+1) = -u_{cc}(c_{t+1})f_0(b_{t+1}) + u_{cc}(c_{t+1})[f_0(b_{t+1})]^2/\kappa_c(c_{t+1}) < 0$$

(2) Proof of Proposition 2

The sign of the discriminant depends on the sign of

$\{V_{22} + (\delta V_{11})^2 - 2\delta V_{12}\} + \{2\delta(V_{11}V_{22} - V_{12}^2)\}.$

Concavity of $V(.)$ guarantees that both the first term (indirectly) and the second term (directly) are positive.

(3) Proof of Proposition 3

Starting from the fact that both roots, $\lambda_1$ and $\lambda_2$, are positive, the sign of $\lambda_1 + \lambda_2 - 2$ is checked. It can be shown that Proposition 1 results in $\lambda_1 + \lambda_2 - 2 > 0$, so that at least one of the two roots is greater than unity. Then, the sign of $(\lambda_1 - 1)(\lambda_2 - 1)$ is examined. It directly follows that condition (i) implies a negative sign, so that one root, say $\lambda_1$, has to be less than unity. On the other hand, condition (ii) implies a positive sign, so that both roots have to be greater than unity. Keep in mind that, as proved by Levhari and Liviatan (1972), the strict concavity of $V(.)$ at the steady state guarantees that the Jacobian possesses no unit roots.
REFERENCES


