Sharing Resources and Costs in Grids

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Joint work with Richard Weber

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Two main problems:

- The way the system resolves resource conflicts depends on declarations by agents.
- Since facility cost must be shared, agents like to free-ride.

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Can we compute the optimal policies?

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How to share resources and recover costs?

- Easy when we know utilities of participants.
- But in practice participants have private information! If the policy of the system depends on this information, they will disclose it to the best of their advantage.

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- Lots of free ridding at the NE! How can we improve the system?
- Relate the scheduling of the server to the contribution of the agents! E.g., processor sharing with coefficients proportional to the y_is.
- Which is the optimal scheduling policy?

A simple example of full information

- Discrete time t,
- Facility of size Q (or to be determined), fixed cost c per time slot,
- In slot t agent i has utility $\theta_{i,t}\sqrt{x_i}$, $\theta_{i,t}$ iid, $U[0,1]$.

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Resource sharing problem: At each time t allocate the resource to maximize value to agents, obtain payments to cover the cost.

$$\underset{\{x_i\}}{\text{maximize}} \sum_{i=1}^{n} \theta_{i,t}\sqrt{x_i} \text{ such that } \sum_{i=1}^{n} x_i \leq Q.$$

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Solution:

$$x_i = \frac{\theta_{i,t}^2}{\sum_{k=1}^{n}\theta_{k,t}^2} Q,\ V_{i,t} = \frac{\theta_{i,t}^2}{\sqrt{\sum_{k=1}^{n}\theta_{k,t}^2}}\sqrt{Q},\ V_t = \sqrt{\sum_{k=1}^{n}\theta_{k,t}^2}\sqrt{Q}\,.$$

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If $E[V_t] > c$, fix payments $p_{i,t} = c/n$. This may not work if $\theta_{i,t} = \theta_i$ for all t (it may not be ex-post rational).

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- In a variation of the model, agents are on/off with a certain probability. How do we obtain this information?

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An agent should profit more by participating in this system than by building his own facility.

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4. The resource sharing and the payment policies take into account the information provided in (*),(**).

Example: allocating a single item

We wish to share a single machine between 2 agents. Each day agent i has utility $\theta_{i,t}$, where $F_1 = U[0,1]$ and $F_2 = U[0,2]$, considered known to system operator. How de we allocate the machine and take payments to cover the cost c?

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- Choose p so that expected payment per time slot is c.

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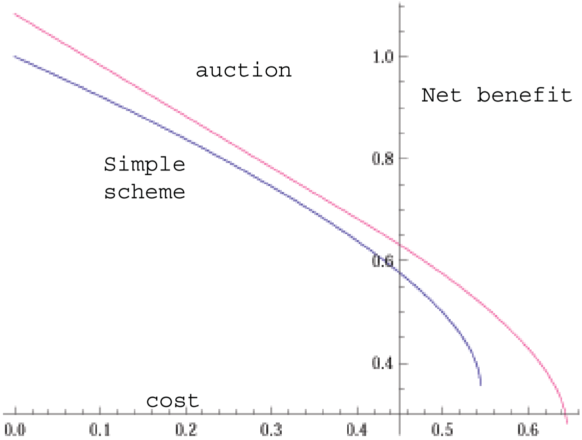
Solution maximizes sum of expected agent utilities conditional on recovering c on the average, is incentive compatible. Note that agent 2 wins if $\theta_{2,t} > \theta_{1,t} + b$.

Comparing the policies



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- Open problem: optimal scheme if we do not know the F_is.
- Lets look at some other fun problems!

Building a facility from scratch

A different model: a facility of size Q and cost $c(Q) = Q$ (per slot) is formed by initial contributions of agents. These are incentivized to contribute because their contribution will affect the amount of resources they will get at run time. Probably a good model for virtual Grid infrastructures.

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- Agents declare their θ_is and system runs according to posted policy.

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It turns out that the solution of the Mechanism Design problem implies a simple 'effective bandwith' tariff for type i agents:

- System guarantees resource y for a contribution of $\alpha_i y$.
- Agent i indirectly declares its θ_i by selecting y to maximize $\max_y\{\theta_i u(y) - \alpha_i y\}$.
- No information on F_i required!

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Results for $\alpha = .8$, $u(x) = 10 - 1/x$

scheme	social welfare	values of q_1, q_2
stand alone	$r\alpha - 2\sqrt{\alpha}$	$\sqrt{\alpha}$
	6.21115	0.894427
central planner	$r\alpha - \sqrt{2\alpha(1+\alpha)}$	$\sqrt{\alpha(1+\alpha)/2}$
$s = \frac{1}{2}(1 + 1/\alpha)$	6.30294	0.848528
proportional division	$r\alpha - \frac{\sqrt{\alpha}\,(3+5\,\alpha)}{2\,\sqrt{1+3\,\alpha}}$	$\frac{1}{2}\sqrt{\alpha(1+3\,\alpha)}$
$s = 1$	6.30225	0.824621
equal division	$r\alpha - \frac{3}{2}\sqrt{\alpha(1+\alpha)}$	$\frac{1}{2}\sqrt{\alpha(1+\alpha)}$
$s = 0$	6.2	0.6

Results for $\alpha = .8$, $u(x) = 10 - 1/x$

scheme	social welfare	values of q_1, q_2
stand alone	$r\alpha - 2\sqrt{\alpha}$	$\sqrt{\alpha}$
	6.21115	0.894427
central planner	$r\alpha - \sqrt{2\alpha(1+\alpha)}$	$\sqrt{\alpha(1+\alpha)/2}$
$s = \frac{1}{2}(1 + 1/\alpha)$	6.30294	0.848528
proportional division	$r\alpha - \frac{\sqrt{\alpha}\,(3+5\,\alpha)}{2\,\sqrt{1+3\,\alpha}}$	$\frac{1}{2}\sqrt{\alpha(1+3\,\alpha)}$
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$s = 0$	6.2	0.6

Open problem: how do these results generalize? How much information is needed to determine the optimal s?

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The problems that are to be solved by agents 1 and 2, respectively, are to maximize with respect to q_1 and q_2

$$nb_1(q) = \alpha_1 \left[\alpha_2 Eu\left(\frac{\sum_1^n q_i}{M+2}\right) + (1-\alpha_2) Eu\left(\frac{\sum_1^n q_i}{M+1}\right)\right] - q_1\,,$$

$$nb_2(q) = \alpha_2 \left[\alpha_1 Eu\left(\frac{\sum_1^n q_i}{M+2}\right) + (1-\alpha_1) Eu\left(\frac{\sum_1^n q_i}{M+1}\right)\right] - q_2$$

where $M=$ number of agents $3, \ldots, n$ that are present. Since $\alpha_1(1-\alpha_2) > \alpha_2(1-\alpha_1)$ it follows that $\frac{\partial}{\partial q_1} nb_1(q) = 0 \implies \frac{\partial}{\partial q_2} nb_2(q) < 0.$

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Given $f(t)$, one can recover the corresponding $x(t)$ and $y(t)$, by taking $q(s) = -f(s) + sf'(s)$ and $u(x(s)) = f'(s)$. These tariffs are incentive compatible.

An optimal control formulation for $u(y) = y^{1/k}$

When n is large we wish to design $f(\cdot)$ so as to maximize

$$\int_0^1 f(s)\,ds$$

subject to a constraint that says that incoming contributions and outgoing allocations are equal, i.e.,

$$\int_0^1 [sx(s) - q(s)]\,ds = \int_0^1 [sf'(s)^k + f(s) - sf'(s)]\,ds \leq 0\,.$$

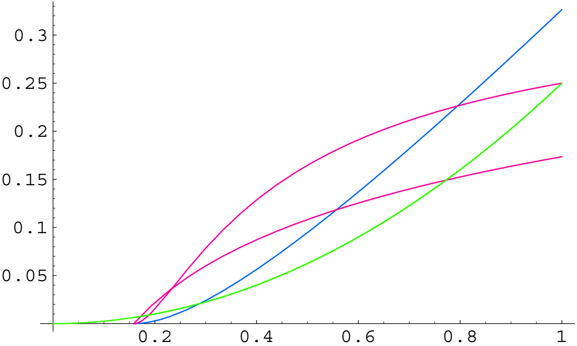
An optimal control formulation for $u(y) = y^{1/k}$

Let $z_1(t) = f(t)$, $z_2(t) = f'(t)$. Our problem becomes

$$\text{maximize} \int_0^1 z_1(t)\,dt \text{ subject to}$$

$$\begin{aligned} 0 &= w + \int_0^1 [s z_2(s)^k + z_1(s) - s z_2(s)]\,ds \\ z_1'(t) &= z_2(t) \\ z_2'(t) &= v(t) \\ z_1(t) &\geq 0 \\ z_2(t) &\geq 0 \\ w &\geq 0\,. \end{aligned}$$

An optimal control formulation for $u(y) = y^{1/k}$



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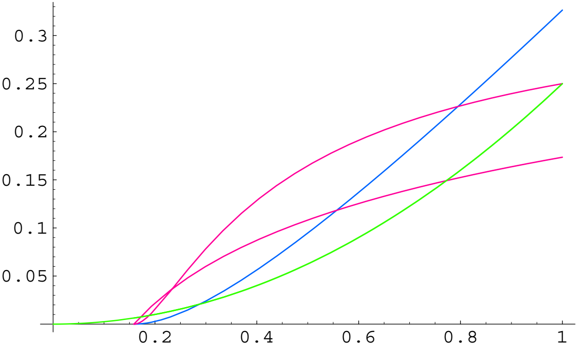

Agents with small α (less than $\alpha^* = 0.1585$) are prevented from participating.

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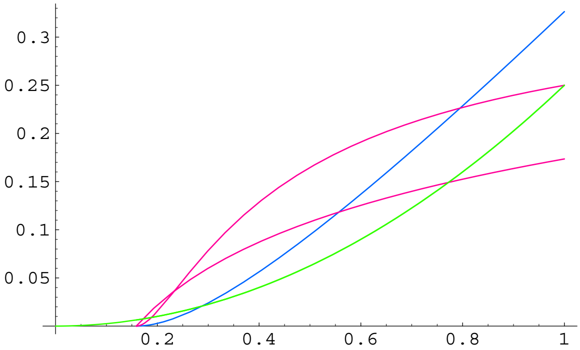

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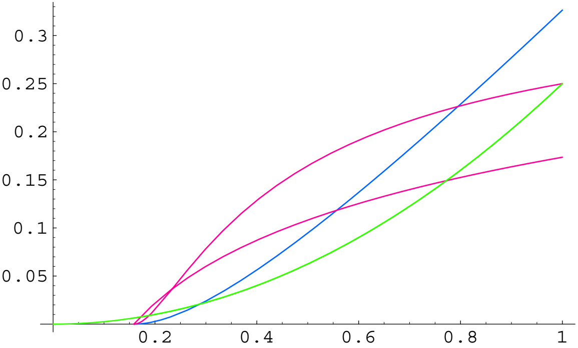

Agents with small α (less than $\alpha^* = 0.1585$) are prevented from participating. Some agents may prefer self-provisioning. Agents with large αs benefit more.

Conclusions

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- THANK YOU!!