

Asymptotics for Provisioning Problems of Peering Wireless LANs with a Large Number of Participants

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- how to get the right incentives...
 - ... without too much work
- the basic economic problem
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- some simpler economic solutions for large n
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Motivation

- **WLAN roaming**: is a **public good**
 - to be provisioned amongst a number of participants who are able to communicate information about their private preferences for the good
 - This provisioning is to be done in a manner that is incentive compatible, rational and feasible (**Mechanism Design**)
- We show that as the number of participants becomes large
 - the solution of the provisioning problem, **when exclusions are possible**, can be approximated by solving a simpler problem with a policy based on fixed entrance fees
 - The solution of the simpler problem is within $o(n)$ of the solution of the original problem

Basic insight

- p2p WLAN roaming is a *public good* problem
 - all peers benefit from the contribution of any single peer
 - but contribution is costly
 - obtaining roaming by one peer does not prevent another peer from obtaining roaming (no congestion effects)
 - positive externality creates an incentive to *free-ride* on efforts of others
 - a peer's incentive is to offer little coverage in the common pool and requests lots of roaming access from others

Implications

- Implication: “free market” solution is inefficient
 - each peer maximises own net benefit
 - actions affect others
 - hence private optimum differs from social optimum
- Classical solution: apply prices or rules to modify behaviour
 - each peer pays/is paid according to the effect it has on others
 - generally requires a different price/rule for each peer
- Problem: requires lots of information
 - e.g., Lindahl prices require global information about all users’ costs and benefits

What to do?

- How can the system/planner/network manager get this information?
 - if lucky, can gather data about users
 - otherwise, users **must be given incentives** to reveal relevant information to planner
- **Mechanism Design**: set prices/rules to encourage users to tell truth

Use Mechanism Design?

- Well-developed economic theory; but solutions typically
 - don't achieve full efficiency (users get something for their info)
 - **very complex, dependent on fine details**
 - **require large amounts of info to be passed to centre**
- Does it have to be this hard? approximations?
 - 2 key characteristics of p2p networks
 - **large**: Gnutella and Kazaa: millions of users, Napster: 40–80m subscribers; up to 5m simultaneous users
 - **heterogeneous**: bandwidth, latency, availability and degree of sharing vary across peers by 3–5 orders of magnitude

Mechanism Design

- Planner: maximize welfare/efficiency
- Agents: maximize net benefit
 - agents have information that planner does not
- 3 constraints:
 - ICC: incentive compatibility
 - PC: participation
 - FC: feasibility
- General results:
 - loss of efficiency due to private information
 - requires lots of info passed
 - complex, depends on fine details

Example

Amount of coverage: Q Cost : $c(Q)$ Agent i : $\theta_i u(Q), F(\theta_i)$

1. System planner chooses and posts

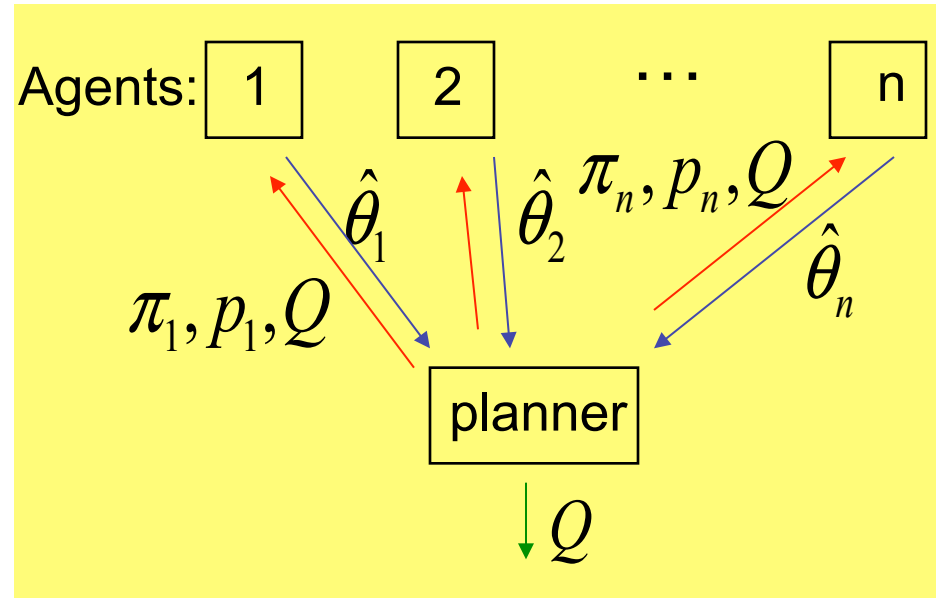
$$Q(\theta), \{p_i(\theta)\}, \{\pi_i(\theta)\}$$

so that

$$\text{FC: } \sum_i \pi_i(\theta) p_i(\theta) = c(Q(\theta))$$

$$\text{PC: } E_{-i}[\theta_i u(Q(\theta)) - p_i(\theta)] \geq 0$$

$$\text{ICC: } NB_i(\theta_i) \geq NB_i(\hat{\theta}_i)$$



2. Agents declare their valuations $\theta_1, \theta_2, \dots, \theta_n$

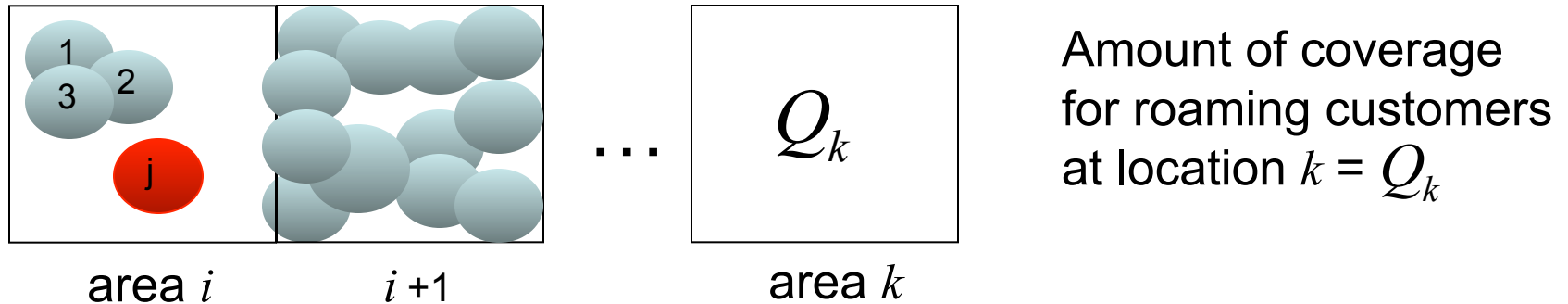
3. Planner chooses $Q(\theta)$, collects payments $\{p_i(\theta)\}$, enforces $\{\pi_i(\theta)\}$

Instead of monetary payments, use payments made “in kind”

Large systems are simpler

- Size helps!
 - simplifies mechanism, limits per capita efficiency loss
- Theorem: A very simple mechanism
“contribute F if join, 0 otherwise”
is nearly optimal when the network is large
- Why?
 - in a large network it is hard to get people pay more than a minimum
- Other major benefits:
 - Low informational benefit, easy to apply in a large class of examples

Peering of WLANs



The j th WLAN owner in area i has utility $\theta_{ij} \sum_{l=1}^L u_l(Q_l)$, where $\theta_{ij} \text{ iid } (F_i)$

Only WLANs in area i can contribute for the cost of maintaining Q_i

Cost of providing coverage Q_i in a area = $c(Q_i)$

Payment = monetary or “in kind”: amount of coverage contributed by a WLAN owner to roaming customers of other WLANs

The model

The optimization problem is to maximize

$$\int_{\Theta} \sum_{i=1}^L \left[\sum_{j=1}^{n_i} \pi_{ij}(\boldsymbol{\theta}) \theta_{ij} \sum_{\ell=1}^L u_{i\ell}(Q_{\ell}(\boldsymbol{\theta})) - c_i(Q_i(\boldsymbol{\theta})) \right] dF(\boldsymbol{\theta})$$

subject to conditions of

1) feasibility $E_{\boldsymbol{\theta}} \left(\sum_{j=1}^{n_i} \pi_{ij}(\boldsymbol{\theta}) p_{ij}(\boldsymbol{\theta}) - c_i(Q_i(\boldsymbol{\theta})) \right) \geq 0, \forall i$

2) individual rationality $\theta_{ij} V_{ij}(\theta_{ij}) - P_{ij}(\theta_{ij}) \geq 0$

3) incentive compatibility $\theta_{ij} V_{ij}(\theta_{ij}) - P_{ij}(\theta_{ij}) \geq \theta_{ij} V_{ij}(\hat{\theta}_{ij}) - P_{ij}(\hat{\theta}_{ij})$

where

$$V_{ij}(\theta_{ij}) = \int_{\Theta_{-ij}} \pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) \sum_{\ell} u_i(Q_{\ell}(\theta_{ij}, \boldsymbol{\theta}_{-ij})) dF(\boldsymbol{\theta}_{-ij})$$
$$P_{ij}(\theta_{ij}) = \int_{\Theta_{-ij}} \pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) p_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) dF(\boldsymbol{\theta}_{-ij}).$$

the model (cont.)

which is equivalent to problem $P(n)$: maximize

$$\int_{\Theta} \sum_{i=1}^L \left[\sum_{j=1}^{n_i} \pi_{ij}(\boldsymbol{\theta}) \theta_{ij} \sum_{\ell=1}^L u_{i\ell}(Q_{\ell}(\boldsymbol{\theta})) - c_i(Q_i(\boldsymbol{\theta})) \right] dF(\boldsymbol{\theta})$$

$$\text{s.t. } \int_{\Theta} \sum_{j=1}^{n_i} \pi_{ij}(\boldsymbol{\theta}) g_i(\theta_{ij}) \sum_{\ell} u_{i\ell}(Q_{\ell}(\boldsymbol{\theta})) - c_i(Q_i(\boldsymbol{\theta})) dF(\boldsymbol{\theta}) \geq 0 \quad \forall i$$

$$\text{where } g_i(\theta_{ij}) = \theta_{ij} - \frac{1 - F_i(\theta_{ij})}{f_i(\theta_{ij})}$$

Lemma: Lagrangian methods work: maximize the Lagrangian

$$\int_{\Theta} \sum_{i=1}^L \left[\sum_{j=1}^{n_i} \pi_{ij}(\boldsymbol{\theta}) (\theta_{ij} + \lambda_i g_i(\theta_{ij})) \sum_{\ell=1}^L u_{i\ell}(Q_{\ell}(\boldsymbol{\theta})) - (1 + \lambda_i) c_i(Q_i(\boldsymbol{\theta})) \right] dF(\boldsymbol{\theta})$$

The asymptotic result

- Define problem $\hat{P}(n)$:

$$\text{maximize } \sum_{i=1}^L \left[n_i \sum_{\ell=1}^L u_{i\ell}(Q_\ell) \int_0^1 \pi_i(\theta_i) \theta_i dF_i(\theta_i) - c_i(Q_i) \right]$$

subject to the L constraints

$$n_i \sum_{\ell=1}^L u_{i\ell}(Q_\ell) \int_0^1 \pi_i(\theta_i) g_i(\theta_i) dF_i(\theta_i) - c_i(Q_i) \geq 0$$

over the L scalars $\{Q_i\}$ and the L functions $\{\pi_i\}$

Theorem: $\hat{\Phi}_n \leq \Phi_n \leq \hat{\Phi}_n + o(n)$

and the optimizing values of $\hat{P}(n)$ define the fixed fee policy for the original problem

The limiting problem

- Finally we need to solve

$$\text{maximize}_{Q_1, \dots, Q_L, \theta_1^*, \dots, \theta_L^*} \sum_{i=1}^L \left[n_i \sum_{\ell} u_i(Q_{\ell}) \int_{\theta_i^*}^1 (1 - F_i(\theta_i)) d\theta_i - c_i(Q_i) \right]$$

$$\text{subject to } n_i(1 - F_i(\theta_i^*)) \theta_i^* \sum_{\ell} u_i(Q_{\ell}) - c_i(Q_i) \geq 0, \forall i$$

- The optimal policy is for a peer of location i to contribute a fixed fee (possibly not monetary)

$$\theta_i^* \sum_l u_{il}(Q_l^*)$$

Further work

- Multiple rounds
- unknown distributions
- more accurate modelling of utility and cost
 - relate to size of footprint, max number of roaming customers, bandwidth usage
 - sensitivity issues
- how to solve the limiting problem in practice
- enforce exclusions, check contributions