Asymptotics for Provisioning Problems of Peering Wireless LANs with a Large Number of Participants

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- why incentives matter
- how to get the right incentives...
  ... without too much work
- the basic economic problem
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- some simpler economic solutions for large n
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Motivation

- **WLAN roaming**: is a **public good**
  - to be provisioned amongst a number of participants who are able to communicate information about their private preferences for the good
  - This provisioning is to be done in a manner that is incentive compatible, rational and feasible (**Mechanism Design**)
- We show that as the number of participants becomes large
  - the solution of the provisioning problem, when exclusions are possible, can be approximated by solving a simpler problem with a policy based on fixed entrance fees
  - The solution of the simpler problem is within $o(n)$ of the solution of the original problem

Basic insight

- p2p WLAN roaming is a *public good* problem
  - all peers benefit from the contribution of any single peer
  - but contribution is costly
  - obtaining roaming by one peer does not prevent another peer from obtaining roaming (no congestion effects)
  - positive externality creates an incentive to *free-ride* on efforts of others
  - a peer's incentive is to offer little coverage in the common pool and requests lots of roaming access from others

Implications

- Implication: “free market” solution is inefficient
  - each peer maximises own net benefit
  - actions affect others
  - hence private optimum differs from social optimum
- Classical solution: apply prices or rules to modify behaviour
  - each peer pays/is paid according to the effect it has on others
  - generally requires a different price/rule for each peer
- Problem: requires lots of information
  - e.g., Lindahl prices require global information about all users’ costs and benefits

What to do?

- How can the system/planner/network manager get this information?
  - if lucky, can gather data about users
  - otherwise, users **must be given incentives** to reveal relevant information to planner
- Mechanism Design: set prices/rules to encourage users to tell truth

Use Mechanism Design?

- Well-developed economic theory; but solutions typically
  - don't achieve full efficiency (users get something for their info)
  - **very complex, dependent on fine details**
  - **require large amounts of info to be passed to centre**
- Does it have to be this hard? approximations?
  - 2 key characteristics of p2p networks
    - **large**: Gnutella and Kazaa: millions of users, Napster: 40–80m subscribers; up to 5m simultaneous users
    - **heterogeneous**: bandwidth, latency, availability and degree of sharing vary across peers by 3–5 orders of magnitude

Mechanism Design

- Planner: maximize welfare/efficiency
- Agents: maximize net benefit
  - agents have information that planner does not
- 3 constraints:
  - ICC: incentive compatibility
  - PC: participation
  - FC: feasibility
- General results:
  - loss of efficiency due to private information
  - requires lots of info passed
  - complex, depends on fine details

Example

Amount of coverage: Q Cost : $c(Q)$ Agent i: $\theta_i u(Q), F(\theta_i)$

1. System planner chooses and posts

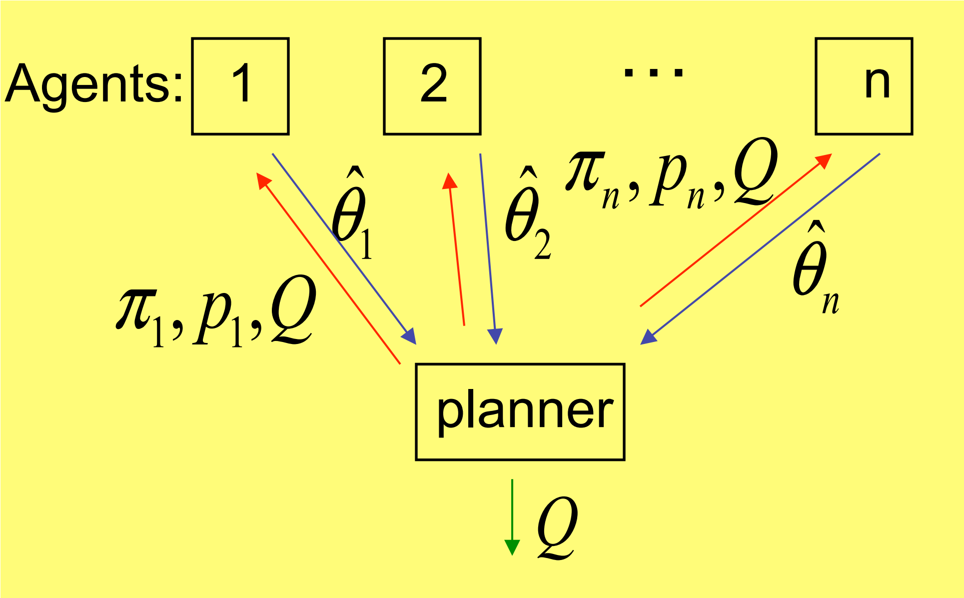
$$Q(\theta), \{p_i(\theta)\}, \{\pi_i(\theta)\}$$

so that

FC: $\sum_i \pi_i(\theta) p_i(\theta) = c(Q(\theta))$

PC: $E_{-i}[\theta_i u(Q(\theta)) - p_i(\theta)] \geq 0$

ICC: $NB_i(\theta_i) \geq NB_i(\hat{\theta}_i)$



2. Agents declare their valuations $\theta_1, \theta_2, \ldots, \theta_n$

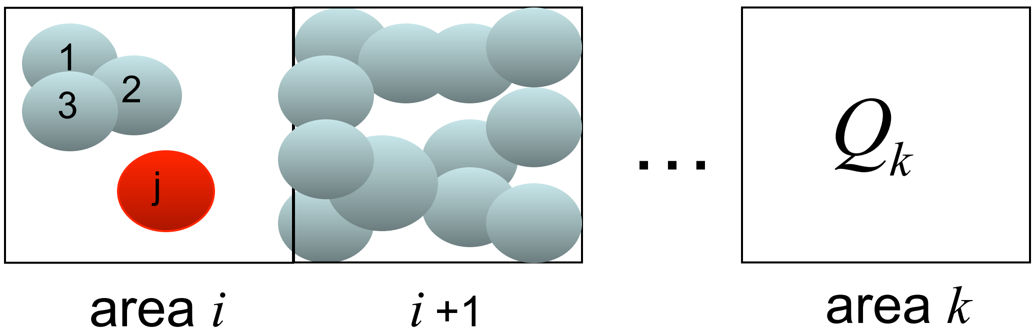
3. Planner chooses $Q(\theta)$**, collects payments** $\{p_i(\theta)\}$**, enforces** $\{\pi_i(\theta)\}$

Instead of monetary payments, use payments made "in kind"

Large systems are simpler

- Size helps!
  - simplifies mechanism, limits per capita efficiency loss
- <u>Theorem</u>: A very simple mechanism
  "contribute F if join, 0 otherwise"
  is nearly optimal when the network is large
- Why?
  - in a large network it is hard to get people pay more than a minimum
- Other major benefits:
  - Low informational benefit, easy to apply in a large class of examples

Peering of WLANs



Amount of coverage for roaming customers at location k = Q_k

The jth WLAN owner in area i has utility $\theta_{ij} \sum_{l=1}^{L} u_l(Q_l)$, where θ_{ij} $iid\ (F_i)$

Only WLANs in area i can contribute for the cost of maintaining Q_i

Cost of providing coverage Q_i in a area = $c(Q_i)$

Payment = monetary or "in kind": amount of coverage contributed by a WLAN owner to roaming customers of other WLANs

The model

The optimization problem is to maximize

$$\int_{\Theta} \sum_{i=1}^{L} \left[\sum_{j=1}^{n_i} \pi_{ij}(\boldsymbol{\theta})\theta_{ij} \sum_{\ell=1}^{L} u_{i\ell}(Q_\ell(\boldsymbol{\theta})) - c_i(Q_i(\boldsymbol{\theta})) \right] dF(\boldsymbol{\theta})$$

subject to conditions of

1) feasibility $E_{\boldsymbol{\theta}} \left(\sum_{j=1}^{n_i} \pi_{ij}(\boldsymbol{\theta}) p_{ij}(\boldsymbol{\theta}) - c_i \left(Q_i(\boldsymbol{\theta}) \right) \right) \geq 0, \forall i$

2) individual rationality $\theta_{ij} V_{ij}(\theta_{ij}) - P_{ij}(\theta_{ij}) \geq 0$

3) incentive compatibility $\theta_{ij} V_{ij}(\theta_{ij}) - P_{ij}(\theta_{ij}) \geq \theta_{ij} V_{ij}(\hat{\theta}_{ij}) - P_{ij}(\hat{\theta}_{ij})$

where

$$V_{ij}(\theta_{ij}) = \int_{\Theta_{-ij}} \pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) \sum_{\ell} u_i(Q_\ell(\theta_{ij}, \boldsymbol{\theta}_{-ij}))\, dF(\boldsymbol{\theta}_{-ij})$$

$$P_{ij}(\theta_{ij}) = \int_{\Theta_{-ij}} \pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) p_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij})\, dF(\boldsymbol{\theta}_{-ij})\,.$$

the model (cont.)

which is equivalent to problem $P(n)$: maximize

$$\int_{\Theta} \sum_{i=1}^{L} \left[\sum_{j=1}^{n_i} \pi_{ij}(\boldsymbol{\theta})\theta_{ij} \sum_{\ell=1}^{L} u_{i\ell}(Q_\ell(\boldsymbol{\theta})) - c_i(Q_i(\boldsymbol{\theta})) \right] dF(\boldsymbol{\theta})$$

s.t. $$\int_{\Theta} \sum_{j=1}^{n_i} \pi_{ij}(\boldsymbol{\theta}) g_i(\theta_{ij}) \sum_{\ell} u_{i\ell}(Q_\ell(\boldsymbol{\theta})) - c_i\left(Q_i(\boldsymbol{\theta})\right) dF(\boldsymbol{\theta}) \geq 0 \quad \forall i$$

where $g_i(\theta_{ij}) = \theta_{ij} - \dfrac{1 - F_i(\theta_{ij})}{f_i(\theta_{ij})}$

Lemma: Lagrangian methods work: maximize the Lagrangian

$$\int_{\Theta} \sum_{i=1}^{L} \left[\sum_{j=1}^{n_i} \pi_{ij}(\boldsymbol{\theta})(\theta_{ij} + \lambda_i g_i(\theta_{ij})) \sum_{\ell=1}^{L} u_{i\ell}(Q_\ell(\boldsymbol{\theta})) - (1+\lambda_i)c_i\left(Q_i(\boldsymbol{\theta})\right) \right] dF(\boldsymbol{\theta})$$

The asymptotic result

- Define problem $\hat{P}(n)$:

  maximize $$\sum_{i=1}^{L}\left[n_i \sum_{\ell=1}^{L} u_{i\ell}(Q_\ell)\int_0^1 \pi_i(\theta_i)\theta_i\, dF_i(\theta_i) - c_i\left(Q_i\right)\right]$$

  subject to the L constraints

  $$n_i \sum_{\ell=1}^{L} u_{i\ell}(Q_\ell)\int_0^1 \pi_i(\theta_i) g_i(\theta_i)\, dF_i(\theta_i) - c_i\left(Q_i\right) \geq 0$$

  over the L scalars $\{Q_i\}$ and the L functions $\{\pi_i\}$

  Theorem: $\hat{\Phi}_n \leq \Phi_n \leq \hat{\Phi}_n + o(n)$

  and the optimizing values of $\hat{P}(n)$ define the fixed fee policy for the original problem

The limiting problem

- Finally we need to solve

$$\underset{Q_1,\ldots,Q_L,\theta_1^*,\ldots,\theta_L^*}{\text{maximize}} \sum_{i=1}^{L} \left[n_i \sum_\ell u_i(Q_\ell) \int_{\theta_i^*}^{1} (1 - F_i(\theta_i))\, d\theta_i - c_i(Q_i) \right]$$

subject to $n_i(1 - F_i(\theta_i^*))\, \theta_i^* \sum_\ell u_i(Q_\ell) - c_i(Q_i) \geq 0, \forall i$

- The optimal policy is for a peer of location i to contribute a fixed fee (possibly not monetary)

$$\theta_i^* \sum_l u_{il}(Q_l^*)$$

Further work

- Multiple rounds
- unknown distributions
- more accurate modelling of utility and cost
  - relate to size of footprint, max number of roaming customers, bandwidth usage
  - sensitivity issues
- how to solve the limiting problem in practice
- enforce exclusions, check contributions