Growth enhancing policy is the means to sustain the environment

by

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Abstract: We study Ramsey second-best optimal policy in a general equilibrium model of growth with renewable natural resources. Natural resources are depleted by private economic activity, but they can also be maintained by public policy. The government uses distorting taxes to finance infrastructure services and cleanup policy. Policy instruments (the tax rates and the allocation of tax revenue between infrastructure and cleanup) are chosen by solving a Ramsey-type policy problem. The more the representative citizen cares about the environment, the more growth-enhancing policies a Ramsey government should choose.

Keywords: Second-best policy, natural resources, economic growth

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1. INTRODUCTION

Is economic growth bad for the environment? Environmental groups believe that a slowdown in economic activity is needed to save the environment. Several economists, on the other hand, tend to agree that wealth and greenery can be positively correlated. Can the economy grow without damaging the environment? Should environmental concerns limit growth?

Stokey (1998) has studied whether long-term growth is feasible and, in turn, optimal when pollution occurs as a by-product of output. Stokey has studied a social planner’s problem (where the planner makes all choices including pollution), as well as the possibility of implementation of the first-best allocation resulting from the social planner’s problem. But what happens when, for some reason, the first-best allocation is unattainable? In this case, the government has to design a second-best optimal policy. What is the best policy? Should the government give priority to environmental policies over growth-enhancing policies? Should a green government choose its policies so as to put a limit to growth?

We study Ramsey second-best optimal policy in a general equilibrium model of growth augmented with renewable natural resources. The setup, although stylized, is relatively realistic. Natural resources are depleted by private economic activity, but they can also be maintained by public policy. The government uses the collected tax revenue to finance infrastructure services and cleanup policy.\footnote{Examples of infrastructure services include roads, airports, urban development, hospitals, police, etc (see Barro, 1990). Examples of cleanup policy include policies that protect, conserve and generate (via innovation) the natural resources, as well as policies that provide the right environmental incentives. All these are costly activities that require public funds.} The former (infrastructure services) provides production externalities to firms and is the engine of long-term growth. The latter (cleanup policy) improves environmental quality and produces external welfare benefits for households. Policy instruments (the tax rate on polluting activities and the allocation of tax revenue between infrastructure and cleanup policy) are chosen optimally. To the extent that there are externalities at market level and indirect policy control at government level, this is not a social planner’s problem.

We work in four steps. We first solve for a competitive decentralized equilibrium (CDE), which is for any feasible policy. Second, we endogenize policy by assuming that
the paths of policy instruments are chosen by a benevolent government that takes into account the CDE, where the latter includes the optimal behaviour of private agents. In other words, we solve for a typical Ramsey second-best allocation (RSBA). Third, to have a benchmark, we also solve for a first-best allocation (FBA). Fourth, we compare the properties of RSBA and FBA. In all cases, we study nontrivial economies where effective cleanup policy is inferior to pollution technology.

Focusing on the long run, our results are as follows. First, the Ramsey government can lead the economy to sustainable balanced growth (namely, a situation in which the economy is capable of long-term growth without damaging the environment). Thus, long-term growth is socially optimal. Second, and more interestingly, the more the representative citizen cares about the environment, the more growth-enhancing policies the Ramsey government finds it optimal to choose. Specifically, the more the citizen cares about the environment, the higher should be the share of tax revenue allocated to infrastructure vis-à-vis cleanup, the lower the income tax rate, and the higher the sustainable balanced growth rate. Third, contrary to the RSBA, in the FBA, the more the citizen cares about the environment, the more environmental friendly allocation of resources the social planner finds it optimal to choose.

The intuition behind these results is as follows. In a second-best situation where private agents ignore externalities and policymakers lack lump-sum policy instruments, when private agents care about the environment, this requires extra revenue for cleanup policy and this can only be achieved by large tax bases and high growth. Ramsey-type policymakers realize all this and choose their policy instruments accordingly, in the sense that they give priority to growth. By contrast, in a first-best situation, the social planner first hits a relatively high growth rate, and in turn allocates some of the available social resources to the environment, where the degree of allocation increases with how much we value the environment relative to consumption or other goods.

Therefore, not only there is no tradeoff between economic growth and environmental quality in the long run, but also only growing economies can afford to improve environmental quality. This is consistent with the general belief that to fund the
governments’ policy goals on health, redistribution, the environment and the rest, we need tax receipts and this can be achieved by growth-enhancing policies.\textsuperscript{2}

To check the robustness of our results, we consider two further cases. In the first, we switch from income taxes to consumption taxes. This is because the latter are, in general, less distorting than the former so that a model with consumption taxes could be closer to the first-best allocation. In the second case, we add public debt. This is because, if the asymptotic income tax rate is zero, the model may look like a lump-sum/first-best problem in the long run. Our main results remain robust. In the case of consumption taxes, although the relation between long-run growth and the consumption tax rate is different from the relation between long-run growth and the income tax rate (the relation is negative with income taxes, while it is positive with consumption taxes), we still find that the more the citizen cares about the environment, the more growth-enhancing policies the Ramsey government finds it optimal to choose. In the case of public debt, the problem is not reduced to a first best even in the long run. This is because, with imperfections at CDE level (for instance, pollution externalities), the Ramsey government needs distorting tax policy instruments all the time including the long run. Hence the Ramsey government finds it optimal to behave as before. In conclusion, to the extent that we do not solve a social planner’s problem, the main results hold.

The rest of the paper is as follows. Section 2 solves the baseline model with income taxes. Section 3 solves the social planner’s problem. Section 4 studies the cases with consumption taxes and public debt. Section 5 reviews the literature. Section 6 concludes. Technical details are in Economides and Philippopoulos (2007).

2. A GENERAL EQUILIBRIUM MODEL WITH NATURE AND POLICY

2.1 Description of the model

Consider a closed economy populated by private agents (a representative household and a representative firm) and a government. Households purchase goods, work and save in the

\textsuperscript{2}Our result is also consistent with the evidence of e.g. Grossman and Krueger (1995) that the deterioration of the environment is stopped and reversed as income rises (this is better known as “environmental Kuznets curve”). It is also consistent with cross-country reports that wealth certainly matters in the sense that per-head income is highly correlated with greenery (see e.g. \textit{The Economist}, January 27\textsuperscript{th} 2001, pp. 86-89).
form of capital. They get utility from private consumption and the stock of natural resources. Firms produce output by using private inputs (capital and labor) and public infrastructure. In doing so, they pollute the environment. Private agents take natural resources and public infrastructure as given. The government imposes taxes on the polluting firms’ output, and then uses the collected tax revenue to finance public infrastructure and cleanup policy.

The timing is as follows. First, the government chooses policy. Second, private agents make their decisions by taking as given prices, policy variables and natural resources. We assume a commitment technology on the part of the government so that it chooses policy once-and-for-all by solving a Ramsey problem. We also assume continuous time, infinite horizons and perfect foresight.

Since our aim is to solve a rather rich Ramsey policy problem, we choose a stylized setup. For instance, the model is a linear AK model at economy-wide level and we abstract from several “real-world” environmental details. Specifically, we build upon Barro’s (1990) tractable model of endogenous growth and optimal policy, which is a version of the AK model. We also focus on the long run (see the last section for a brief discussion of transition dynamics).

2.2 Household’s behavior

The representative infinitely-lived household maximizes intertemporal utility:

\[
\int_0^\infty [u(C,N)] e^{-\rho t} dt
\]

(1a)

where \( C \) is private consumption, \( N \) is the stock of economy-wide natural resources and \( \rho > 0 \) is the rate of time preference. The utility function \( u(.) \) is increasing and concave in its two arguments. For tractability, we use an additively separable function of the form:\(^4\)

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\(^3\) Our main results do not change if taxes are imposed on households’ income, or if we use pollution taxes (pollution is a by-product of output).

\(^4\) Our main results do not change if we use a more general utility function like \( \frac{[C^\gamma N^{1-\gamma}]^{1-\sigma}}{1-\sigma} \), where \( \sigma \geq 0 \), as in Stokey (1998); see Economides and Philippopoulos (2007, appendix G).
\[ u(C, N) = \nu \log C + (1 - \nu) \log N \]  

(1b)

where \( 0 < \nu < 1 \) is a parameter.

Households save in the form of capital, \( K \), receiving a rate of return, \( r \). They also supply inelastically one unit of labor services and get labor income, \( w \). Further, they receive dividends, \( \pi \). The flow budget constraint of the household is:

\[ \dot{K} + C = rK + w + \pi \]  

(2a)

where a dot over a variable denotes a time derivative. The initial capital stock \( K_0 \) is given. We assume zero capital depreciation.

The household acts competitively by taking prices, policy and natural resources as given. The control variables are the paths of \( C \) and \( K \), so that the first-order conditions include the constraint (2a) and the familiar Euler equation:

\[ \dot{C} = C(r - \rho) \]  

(2b)

2.3 Firm’s behavior

As in the model introduced by Barro (1990), public services provide production externalities to private firms and technology takes a Cobb-Douglas form at the firm’s level. Thus, the production function of the representative firm is:

\[ Y = AK^\alpha l^{1-\alpha} G^{1-\alpha} \]  

(3)

where \( Y \) is output, \( K \) and \( l \) are capital and labor inputs, \( G \) is public production services, and \( A > 0 \) and \( 0 < \alpha < 1 \) are parameters.

The representative firm maximizes profits, \( \pi \):

\[ \pi = (1 - \tau)Y - rK - wl \]  

(4)
where $0 < \tau < 1$ is a tax rate on firms’ output.

The firm acts competitively by taking prices, policy and natural resources as given. The control variables are $K$ and $I$, and the standard first-order conditions are:

\[ r = \alpha(1 - \tau) \frac{Y}{K} \]  
\[ w = (1 - \alpha)(1 - \tau) \frac{Y}{I} \]  

2.4 Motion of natural resources

The stock of renewable natural resources, $N$, evolves over time according to:

\[ \dot{N} = \delta N - P + \theta E \]  

where the parameter $\delta \geq 0$ is the rate of regeneration of natural resources, $P$ is environmental damage (see below), $E$ is public cleanup (see below) and the technology parameter $0 \leq \theta \leq 1$ measures the effectiveness of cleanup policy. The initial stock $N_0$ is given. Thus, natural resources can be renewed by regeneration and public policy.

We assume that $P$ is a by-product of final output produced. Specifically,

\[ P = sY \]  

that is, one unit of output generates $0 \leq s < 1$ units of pollution. Thus, $s$ is a technology parameter that quantifies the detrimental effect of economic activity on the environment. Note that (6) and (7) assume a linear relationship among economic activity, pollution, cleanup policy and the change in natural resources. This is as in e.g. John and Pecchenino (1994) and keeps the model linear at economy level (see (9a)-(9c) below).
2.5 Government budget constraint and the role of policy

On the revenue side, the government taxes the polluting firm’s output at a rate $0 < \tau < 1$. On the expenditure side, it spends $G$ on infrastructure and $E$ on cleanup policy. Assuming a balanced budget, we have:

$$G + E = \tau Y$$

(8a)

where, at each instant, only two out of the three policy instruments ($\tau, G, E$) can be set independently. Equivalently, we can write (8a) as:

$$G = b \tau Y$$

(8b)

$$E = (1 - b) \tau Y$$

(8c)

where $0 < b \leq 1$ is the fraction of tax revenue used to finance infrastructure and $0 \leq (1 - b) < 1$ is the fraction that finances cleanup. Thus, at each instant, policy can be summarized by $\tau$ and $b$.

2.6 Competitive decentralized equilibrium (CDE)

In a CDE: (i) households maximize utility and firms maximize profits; (ii) all constraints are satisfied; (iii) all markets clear. This holds for any feasible policy, where the latter is summarized by the paths of the independent policy instruments, $0 < \tau < 1$ and $0 < b \leq 1$.

Combining (1)-(8), it is straightforward to show that a CDE is given by:

$$\dot{C} = C \left[ \alpha (1 - \tau) A^{\alpha} (b \tau)^{\frac{1-a}{\alpha}} - \rho \right]$$

(9a)

$$\dot{K} = (1 - \tau) A^{\alpha} (b \tau)^{\frac{1-a}{\alpha}} K - C$$

(9b)

$$\dot{N} = \delta N - [s - \theta (1 - b) \tau] A^{\alpha} (b \tau)^{\frac{1-a}{\alpha}} K$$

(9c)
Equations (9a)-(9c) give the motion of consumption \((C)\), capital \((K)\) and natural resources \((N)\) as functions of policy instruments \((0 < \tau < 1 \text{ and } 0 < b \leq 1)\). Actually, since the model is \(AK\) at economy-wide level\(^5\) and thus allows for long-term growth, we can solve for ratios only, \(c \equiv \frac{C}{K}\) and \(x \equiv \frac{K}{N}\). It is straightforward to show that the dynamics of (9a)-(9c) are equivalent to the dynamics of (10a)-(10b) written below:

\[
c = c^2 - \left[(1 - \alpha)(1 - \tau)A^\alpha (b \tau)^{1 - \alpha} + \rho\right]c \tag{10a}
\]
\[
x = \left[s - \theta(1 - b)\tau A^\alpha (b \tau)^{1 - \alpha}x^2 + (1 - \tau)A^\alpha (b \tau)^{1 - \alpha} - \delta - c\right]x \tag{10b}
\]

where (10a)-(10b) constitute a two-equation system in the paths of \(c\) and \(x\).

Before we choose policy, it is useful to point out some properties of the CDE. We focus on the long run. By setting \(\frac{c}{c} = \frac{x}{x} \equiv 0\) in (10a)-(10b), one gets unique solutions for the long-run CDE values of \(c\) and \(x\). Note that \(C\), \(K\) and \(N\) grow at the same rate, denoted as \(\gamma\) (this is typical of \(AK\) models in which all quantities grow at the same rate in the long run).\(^6\) This common rate, \(\gamma\), has to be zero or positive. When \(\gamma\) is zero, the economy does not grow and the stock of natural resources remains unchanged. When \(\gamma\) is positive, a long-term growth is possible with the stock of renewable natural resources also growing at the same rate.\(^7\) Obviously, as (9a) shows, the value of \(\gamma\) depends crucially on the value of policy instruments \((\tau, b)\).

\(^5\) Output is \(Y = A^\alpha (b \tau)^{1 - \alpha}K\). If \(N = s \equiv 0\) and \(b = 1\), we get Barro’s (1990) model.

\(^6\) Inspection of (9a)-(9c) reveals that we cannot have a case in which \(C\) and \(K\) grow at the same positive rate, while \(N\) remains constant. This is not so restrictive. What is important is that the model can allow for positive and zero long-run growth without reducing the stock of natural resources. This is a sustainable equilibrium.

\(^7\) Renewable natural resources can indeed grow at a positive rate. This applies to living organisms like fish, forests, cattle and to some extent water and atmospheric systems, which have a natural capacity to assimilate and cleanse themselves. In addition to biological regeneration, renewable resources can grow in size over time thanks to environmental policy and innovation. Innovation can help even fossil fuels (oil,
The above are summarized by:

**Result 1:** Depending on the value of policy instruments \((\tau, b)\) and parameters \((\alpha, A, \rho, \delta, \theta, s)\), there is a unique long-run CDE in which \(C, K, N\) can grow at the same constant non-negative rate \((\gamma)\).

The plausibility of the above result, and in particular whether a growing CDE is possible, is confirmed below. At this stage, it is useful for what follows to point out that the balanced growth rate \((\gamma)\) increases monotonically with the share of tax revenue allocated to infrastructure vis-à-vis cleanup \((b)\), while the effect of the income tax rate \((\tau)\) on \(\gamma\) follows its well-known inverted-U pattern. Specifically, as in Barro (1990), equation (9a) implies that the effect of \(\tau\) on \(\gamma\) is negative (resp. positive) when \(\tau > 1 - \alpha\) (resp. \(\tau < 1 - \alpha\)). Actually, when we endogenize policy below, we show that along the optimal (Ramsey) policy path \(\tau > 1 - \alpha\) (the optimal tax rate will be higher than \(1 - \alpha\) because the government has also to finance public cleanup). Thus, the two policy instruments will affect \(\gamma\) in opposite directions.

### 2.7 Ramsey problem

We now endogenize policy as summarized by the paths of the income tax rate, \(0 < \tau < 1\), and the allocation of tax revenue between infrastructure and cleanup policy, \(0 < b \leq 1\). By choosing policy, the government will try to control for externalities and raise funds optimally to finance its activities. The government chooses the paths of \(0 < \tau < 1\) and \(0 < b \leq 1\) to maximize the household’s utility in (1a)-(1b) subject to the CDE in (9a)-(9c).

The current-value Hamiltonian, \(H\), is:

\[
H \equiv v \log C + (1 - v) \log N + \lambda C \frac{1}{\alpha} \left[ (1 - \tau) A^\alpha (b \tau)^{\frac{1 - \alpha}{\alpha}} - \rho \right] + 
\]

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gas, etc) and non-energy minerals (copper, bauxite, etc) not to run out in the long run: new sources are found, the efficiency of extraction goes up, existing supplies are used more economically, and new substitutes are invented (see e.g. *The Economist*, January 22\textsuperscript{nd}, 2005). For renewable and non-renewable natural resources, and their growth capacities in particular, see e.g. Perman et al. (2003).
+ \lambda_k \left[ (1 - \tau)A^{\alpha} (b \tau)^{\frac{1-\alpha}{\alpha}} K - C \right] + \lambda_n \left[ \delta N - [s - \theta (1 - b) \tau] A^{\alpha} (b \tau)^{\frac{1-\alpha}{\alpha}} K \right] \tag{11}

where \lambda_c, \lambda_k, \lambda_n are dynamic multipliers associated with (9a), (9b), (9c) respectively.

The first-order conditions of the Ramsey problem include the constraints (9a)-(9c) and the optimality conditions with respect to \tau, b, C, K, N:

\begin{align*}
(1 - \alpha - \tau) (\alpha \lambda_c C + \lambda_k K) &= -[\theta (1 - b) \tau - s (1 - \alpha)] \lambda_n K \tag{12a} \\
(1 - \alpha)(1 - \tau)(\alpha \lambda_c C + \lambda_k K) &= [(1 - \alpha)(s - \theta \tau) + \theta b \tau] \lambda_n K \tag{12b} \\
\dot{\lambda}_c &= \rho \lambda_c - \frac{\nu}{C} - \lambda_c \left[ \alpha (1 - \tau)A^{\alpha} (b \tau)^{\frac{1-\alpha}{\alpha}} - \rho \right] + \lambda_k \tag{12c} \\
\dot{\lambda}_k &= \rho \lambda_k - \lambda_k (1 - \tau)A^{\alpha} (b \tau)^{\frac{1-\alpha}{\alpha}} + \lambda_n [s - \theta (1 - b) \tau] A^{\alpha} (b \tau)^{\frac{1-\alpha}{\alpha}} \tag{12d} \\
\dot{\lambda}_n &= \rho \lambda_n - \frac{(1 - \nu)}{N} - \delta \lambda_n \tag{12e}
\end{align*}

These necessary conditions are completed with the addition of a transversality condition that guarantees utility is bounded. A sufficient condition for this to hold is:

\begin{align*}
\gamma + \delta < \rho \tag{12i}
\end{align*}

which states that in the long run the consumption growth rate, \gamma, plus the rate of regeneration of natural resources, \delta, should be less than the rate of time preference, \rho.

2.8 Stationary Ramsey second-best allocation

We transform the variables to make them stationary. After some experimentation, we define \( c \equiv \frac{C}{K}, \ x \equiv \frac{K}{N}, \ \psi \equiv \lambda_c C, \ \phi \equiv \lambda_k K \) and \( \Omega \equiv \lambda_n N \). Thus, \psi, \phi and \Omega measure respectively the social value of economy-wide consumption, capital and natural resources, and \( c \) and \( x \) are the ratios defined in the analysis of CDE above. It is
straightforward to show that the dynamics of (9a)-(9c) and (12a)-(12e) are equivalent to the dynamics of (13a)-(13g) presented below:

\[
\begin{align*}
\cdot c & = c^2 - \left[ (1 - \alpha)(1 - \tau)A^\alpha (b \tau)^{1-\alpha} + \rho \right] c \tag{13a} \\
\cdot x & = \left[ s - \theta (1-b) \tau \right] A^\alpha (b \tau)^{1-\alpha} x^2 + \left[ (1 - \tau) A^\alpha (b \tau)^{1-\alpha} - \delta - c \right] x \tag{13b} \\
\cdot \psi & = -\nu + \phi c + \rho \psi \tag{13c} \\
\cdot \phi & = \Omega x \left[ s - \theta (1-b) \tau \right] A^\alpha (b \tau)^{1-\alpha} + (\rho - c) \phi \tag{13d} \\
\cdot \Omega & = - (1 - \nu) + \rho \Omega - \left[ s - \theta (1-b) \tau \right] A^\alpha (b \tau)^{1-\alpha} \Omega x \tag{13e} \\
\theta \tau \beta & = (1 - \alpha)(\theta - s) \tag{13f} \\
(1 - \alpha - \tau)(\alpha \psi + \phi) & = - [\theta (1-b) \tau - s(1 - \alpha)] x \Omega \tag{13g}
\end{align*}
\]

where (13a)-(13g) constitute a seven-equation system in the paths of \( \tau, b, c, x, \psi, \phi, \Omega \). This is a stationary Ramsey second-best allocation (RSBA). We next study this economy in the long run.

### 2.9 Long run of Ramsey second-best allocation

In the long run, \( \dot{\tau} = \dot{b} = \dot{c} = \dot{x} = \dot{\psi} = \dot{\phi} = \dot{\Omega} = 0 \) in (13a)-(13g). Let denote the resulting long-run values of \( \tau, b, c, x, \psi, \phi, \Omega \) as \( \tilde{\tau}, \tilde{b}, \tilde{c}, \tilde{x}, \tilde{\psi}, \tilde{\phi}, \tilde{\Omega} \). Thus, variables with tildes denote long-run values in RSBA. We will present numerical solutions.\(^8\)

We use the following baseline parameter values: \( \alpha = 0.5 \) (where \( 0 < \alpha < 1 \) is the productivity of private capital in the production function), \( A = 1 \) (where \( A > 0 \) is aggregate factor productivity in the production function), \( \delta = 0.015 \) (where \( \delta > 0 \) is the

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\(^8\) It is also possible to establish analytical conditions sufficient for existence of a long-run solution (see Economides and Philippopoulos, 2007, appendix B.2).
rate of regeneration of natural resources), \( \rho = 0.04 \) (where \( \rho > 0 \) is the rate of time preference), \( \theta = 1 \) (where \( 0 < \theta \leq 1 \) is the technology parameter that quantifies the effectiveness of cleanup policy) and \( s = 0.5 \) (where \( 0 \leq s < 1 \) measures the detrimental effect of economic activity on the environment; see equations (6)-(7) above). Throughout
the paper, except otherwise stated: (i) Key results are robust to the parameter values chosen. (ii) We use the same parameter values throughout the paper. (iii) Only real solutions are presented. (iv) The transversality condition (12i) is satisfied. (v) The economy is not shrinking in the long run, namely the long-run growth rate is nonnegative. (vi) \([s - \theta(1 - b)\tau] > 0\), which is a non-trivial solution area (when \([s - \theta(1 - b)\tau] < 0\), we have a “too good to be true” economy in the sense that effective cleanup policy, \(\theta(1 - b)\tau\), is stronger than the polluting effect of production, \(s\)).

We will experiment with alternative values of \( \nu \). We focus on \( \nu \) because this is the interesting parameter in our setup; it measures how much agents value consumption vis-à-vis environmental quality (see (1b) above). Table 1 reports numerical solutions for varying values of \( \nu \) in the region \( 0.1 \leq \nu \leq 0.7 \) (for \( \nu > 0.7 \), the economy is shrinking).

Table 1 here

Inspection of numerical results implies the following: (a) The solution is well defined. For instance, \( 0 < \tilde{\tau} < 1 \), \( 0 < \tilde{b} \leq 1 \), \( \tilde{c} > 0 \), \( \tilde{x} > 0 \) and \( \tilde{y} \geq 0 \). (b) When \( \tilde{y} > 0 \), long-term growth is not only possible (as it happens at the level of CDE) but also optimal. (c) Consider now the effects of \( \nu \). In the region \( 0.1 \leq \nu \leq 0.7 \), as \( \nu \) falls (i.e. we care more about the environment relative to private consumption), it is optimal to allocate more tax revenue to infrastructure (\( \tilde{b} \) rises) and to tax less (\( \tilde{\tau} \) falls). This implies that public investment as a share of output (\( \frac{\tilde{G}}{\tilde{Y}} \)) is independent of \( \nu \), while cleanup as a share of output (\( \frac{\tilde{E}}{\tilde{Y}} \)) falls, and all this turns out to be good for the balanced growth rate (i.e. \( \tilde{y} \) rises as \( \nu \) falls). Therefore, the lower is \( 0 < \nu < 1 \), the lower the tax rate (\( \tilde{\tau} \)), the higher
the share allocated to infrastructure vis-à-vis cleanup \((\tilde{b})\), and the higher the balanced growth rate \((\tilde{\gamma})\).

Combining results, the more the representative citizen cares about the environment (i.e. the lower is \(\nu\)), the more growth-enhancing policies the Ramsey government finds it optimal to choose. Intuitively, when citizens care about the environment, this requires extra revenue for cleanup policy and can only be achieved by large tax bases and high growth. Ramsey-type policymakers realize all this and choose their policy instruments \((\tau\) and \(b\)) accordingly. As a result, the higher gets eventually the balanced growth rate, \(\tilde{\gamma}\). Also, the lower is \(\nu\), the lower are \(\tilde{x} = \frac{\tilde{K}}{N}\) and \(\tilde{C} = \frac{\tilde{C}}{N}\). That is, the environment improves relative to both physical capital and private consumption. When policy is optimally chosen, growth and greenery are not inconsistent.

The above are summarized by:

**Result 2:** Depending on the value of parameters \((\nu, \rho, \delta, \alpha, A, \theta, s)\), we have a long-run Ramsey second-best allocation (RSBA) in which the economy grows at a constant positive rate without damaging the environment. Thus, long-term growth is not only feasible (as at the level of CDE) but also socially optimal. In this RSBA, the more the citizens care about the environment, the more growth-enhancing policies the Ramsey government finds it optimal to choose.

### 3. FIRST-BEST ALLOCATION

This section solves for the reference case of a first-best allocation (FBA). Now a social planner chooses directly the paths of \(C, K, N, G\) and \(E\) (respectively, consumption, capital, natural resources, resources assigned to infrastructure and resources assigned to cleanup) to maximize (1a)-(1b) subject to:

\[
\dot{K} = AK^{\alpha - 1} - C - G - E \quad (14a)
\]

\[
\dot{N} = \delta N - sAK^{\alpha} + \theta E \quad (14b)
\]
where (14a) is the resource constraint and (14b) is the motion of natural resources.

3.1. Social planner’s problem and stationary first-best allocation

The current-value Hamiltonian, $H$, of this problem is:

$$H = \nu \log C + (1 - \nu) \log N + \lambda_k [AK^\alpha G^{1-\alpha} - C - G - E] + \lambda_n [\delta N - sAK^\alpha G^{1-\alpha} + \theta E]$$

(15)

where $\lambda_k$ and $\lambda_n$ are new dynamic multipliers associated with (14a) and (14b).

Deriving the first-order conditions with respect to $C, G, E, \lambda_k, K, \lambda_n, N$, and using the stationary auxiliary variables $c = \frac{C}{K}$, $x = \frac{K}{N}$, $g = \frac{G}{K}$ and $e = \frac{E}{K}$, we have:

$$\frac{c}{c} = \frac{\alpha A(\theta - s)}{\theta} g^{1-\alpha} - \rho - Ag^{1-\alpha} + c + g + e$$

(16a)

$$\frac{x}{x} = Ag^{1-\alpha} - c - g - e - \delta + sAg^{1-\alpha} x - \theta e x$$

(16b)

$$g = \left[ A(1 - \alpha)(\theta - s) \right]^{\frac{1}{\theta}}$$

(16c)

$$cx = \frac{\nu \left( \frac{\alpha A(\theta - s)}{\theta} g^{1-\alpha} - \delta \right)}{\theta(1 - \nu)}$$

(16d)

where (16a)-(16d) constitute a system of four equations in the paths of $c$, $x$, $g$ and $e$.

Note that in turn the consumption growth rate can be given by:

$$\frac{\dot{C}}{C} = \frac{\alpha A(\theta - s)}{\theta} g^{1-\alpha} - \rho$$

(16e)

This is a stationary first-best allocation (FBA). We next study this economy in the long run.
3.2 Long run of first-best allocation

In the long run, \( \frac{\dot{c}}{c} = \frac{\dot{x}}{x} = \frac{\dot{g}}{g} = \frac{\dot{e}}{e} \equiv 0 \) in (16a)-(16d). Let denote the resulting long-run values of \( c, x, g, e \) as \( \bar{c}, \bar{x}, \bar{g}, \bar{e} \). Thus, variables with bars denote long-run values in FBA. To make our results comparable to those in the previous section, we present numerical solutions. The parameter values used are the same as above. We also report the resulting solutions of \( \frac{\bar{C}}{N} \equiv \bar{cN}, \frac{\bar{G}}{\bar{Y}} \) and \( \frac{\bar{E}}{\bar{Y}} \), as we did in the RSBA. Table 2 reports numerical solutions for varying values of \( \nu \) in the full region, \( 0.1 \leq \nu \leq 0.9 \).

Table 2 here

Inspection of our numerical results reveals the following: (a) The solution is well defined, \( \bar{c} > 0, \bar{x} > 0, \frac{\bar{G}}{\bar{Y}} > 0, \frac{\bar{E}}{\bar{Y}} > 0 \) and \( \bar{\nu} \geq 0 \). (b) A positive balanced growth rate (\( \bar{\nu} > 0 \)) is independent of \( \nu \). This differs from the Ramsey second-best allocation (RSBA), where the balanced growth rate did depend on \( \nu \) (see Table 1 above). (c) The balanced growth rate of FBA is higher than that of RSBA (compare the long-run values of \( \gamma \) in Tables 1 and 2). (d) The positive ratio \( \frac{\bar{G}}{\bar{Y}} \) is independent of \( \nu \). This was also the case in RSBA - actually their numerical values are the same (see Tables 1 and 2). (e) The nonnegative ratio \( \frac{\bar{E}}{\bar{Y}} \) does depend on \( \nu \). Specifically, \( \frac{\bar{E}}{\bar{Y}} \) decreases with \( \nu \). Recall that, in RSBA, the effect of \( \nu \) on the long-run value of \( \frac{E}{Y} \) was the opposite; namely, in a RSBA, \( \frac{E}{Y} \) was increasing in \( \nu \) (see Table 1).

Combining results, a FBA is characterized by a higher balanced growth rate than a RSBA. Also, the FBA is characterized by better environmental quality relative to private goods than a RSBA (compare e.g. the long-run values of \( x \) in Tables 1 and 2). In

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9 See Economides and Philippopoulos (2007, appendix C) for a more detailed study of the FBA.
addition, the more the citizen cares about the environment (the lower is $\nu$), the more environmental friendly allocation of resources a social planner finds it optimal to choose (the higher is the long-run value of $\frac{E}{Y}$). By contrast, recall that the more the citizen cares about the environment, the more growth-enhancing policies a Ramsey government finds it optimal to choose. Therefore, the social planner first hits a relatively high growth rate, and in turn allocates some of the available social resources to the environment, where the degree of allocation increases with how much we value the environment relative to consumption or other goods.

The above are summarized by:

**Result 3:** Depending on the value of parameters $(\nu, \rho, \delta, \alpha, A, \theta, s)$, we have a unique long-run first-best allocation in which the economy grows at a constant positive rate without damaging the environment. The first-best allocation is characterized by a higher balanced growth rate and better environment relative to private goods than a Ramsey second-best allocation (RSBA). The more the citizen cares about the environment, the more environmental friendly allocation of resources a social planner finds it optimal to choose; this differs from a RSBA where policy control is only indirect.

### 4. ROBUSTNESS

To check the robustness of our results, we consider two further setups. In the first, we switch from the income tax used in section 2 to a consumption tax. This is because the latter is, in general, less distorting than the former so that a model with consumption taxes could be closer to the first-best allocation. In the second setup, we add public debt. This is because, if the asymptotic income tax rate is zero, the model may look like a lump-sum/first-best problem in the long run. To save on space, we will present the main results only (details for the two setups are in Economides and Philippopoulos, 2007, sections D and E respectively).
4.1 Consumption taxes

The model is as in section 2 but we replace the output tax paid by firms with a consumption tax paid by households. The household’s budget constraint is now:

\[ \dot{K} + (1 + \tau)C = rK + w + \pi \]  

(17)

where 0 < \tau < 1 denotes now a consumption tax rate.

The government budget constraint changes from (8a) to:

\[ G + E = \tau C \]  

(18a)

or, equivalently, (8b)-(8c) change to:

\[ G = b \tau C \]  

(18b)

\[ E = (1 - b)\tau C \]  

(18c)

The new competitive decentralized equilibrium (CDE) is:

\[ \dot{C} = C \left[ \alpha A \left( b \tau \frac{C}{K} \right)^{1-\alpha} - \rho - \frac{\dot{\tau}}{1 + \tau} \right] \]  

(19a)

\[ \dot{K} = A \left( b \tau \frac{C}{K} \right)^{1-\alpha} K - (1 + \tau)C \]  

(19b)

\[ \dot{N} = \delta N - sA \left( b \tau \frac{C}{K} \right)^{1-\alpha} K + \theta (1 - b)\tau C \]  

(19c)

which is for any feasible policy, where the latter is summarized by the paths of \tau and b.

Working as in subsection 2.6, it is easy to show that the two policy instruments affect the long-run growth rate (\gamma) in the same direction; either \[ \frac{\partial \gamma}{\partial \tau} > 0 \] and \[ \frac{\partial \gamma}{\partial b} > 0 \], or
\[ \frac{\partial \gamma}{\partial \tau} < 0 \text{ and } \frac{\partial \gamma}{\partial b} < 0 \] (for details, see Economides and Philippopoulos, 2007, appendix D.1). Actually, it is the former area that makes sense (and this is confirmed below when we choose policy). This is as in the literature (see Turnovsky, 1995, chapter 13.5): an increase in government expenditure on infrastructure financed by means of a consumption tax is growth-enhancing. Recall that this is different from the case with income taxes in which the two policy instruments affected the long-run growth rate in opposite directions, at least along the optimal (Ramsey) path.

The government chooses the paths of \( \tau \) and \( b \) to maximize the household’s utility subject to the CDE in (19a)-(19c). Deriving the first-order conditions for \( \tau, b, C, K, N \), and transforming the variables to make them stationary (specifically, we define \( c \equiv \frac{C}{K}, x \equiv \frac{K}{N}, \psi \equiv \lambda_c C, \phi \equiv \lambda_k K \) and \( \Omega \equiv \lambda_n N \), where \( \lambda_c, \lambda_k \) and \( \lambda_n \) are dynamic multipliers associated with (19a), (19b) and (19c) respectively), we get a seven-equation system in the paths of \( \tau, b, c, x, \psi, \phi, \Omega \). This is a new stationary RSBA.

In the long run of this economy, \[ \frac{\dot{\tau}}{\tau} = \frac{\dot{b}}{b} = \frac{\dot{c}}{c} = \frac{\dot{x}}{x} = \frac{\dot{\psi}}{\psi} = \frac{\dot{\phi}}{\phi} = \frac{\dot{\Omega}}{\Omega} = 0. \] But since the two policy instruments \( (\tau, b) \) work in the same direction, it is not possible to get a solution when both of them are chosen optimally. We therefore consider two cases. In the first, \( b \) is chosen optimally while \( \tau \) is set exogenously (we drop the optimality condition for \( \tau \)). In the second, \( \tau \) is chosen optimally while \( b \) is set exogenously (we drop the optimality condition for \( b \)). Numerical solutions for the two cases (see Tables 3 and 4 in Economides and Philippopoulos, 2007) imply that the main results do not change: in both cases, the more the citizen cares about the environment (i.e. \( \nu \) falls), the more growth-enhancing policies the Ramsey government finds it optimal to choose.

### 4.2 Adding public debt

This subsection adds public debt \( (D) \) to the model in section 2. We expect our single income tax to inherit the features of a capital income tax. To facilitate comparison with the literature, we follow Chamley (1986) as close as possible.

The flow budget constraint of the household changes from (2a) to:
\[ \frac{\dot{K}}{+} + \frac{\dot{D}}{+} + C = r(K + D) + w + \pi \] 

(20)

where for simplicity the return to government bonds equals the return to capital. The initial stocks \( K_0, D_0 \) are given.

The government budget constraint changes from (8a) to:

\[ \dot{D} = rD + G + E - \tau Y \]

(21)

The new competitive decentralized equilibrium (CDE) is:

\[ \begin{align*}
\dot{C} &= C(\bar{\tau} - \rho) \\
\dot{K} &= AK^\alpha G^{1-\alpha} - C - E - G \\
\dot{N} &= \delta N - sAK^\alpha G^{1-\alpha} + \theta E \\
\dot{D} &= \bar{\tau}D + G + E - AK^\alpha G^{1-\alpha} + \frac{\bar{\tau}K}{\alpha}
\end{align*} \]

(22a)-(22d)

where \( \bar{\tau} \equiv (1-\tau)\alpha A \left( \frac{G}{K} \right)^{1-\alpha} \) denotes the net return to assets. The CDE is for any feasible policy, where the latter is summarized by the paths of \( \tau, G, E \), or equivalently \( \bar{\tau}, G, E \).

The government chooses the paths of \( \bar{\tau}, G, E \) to maximize the household’s utility subject to the CDE in (22a)-(22d). Deriving the first-order conditions for \( \bar{\tau}, G, E, C, K, N, D \), and transforming the variables to make them stationary (specifically, we define \( c \equiv \frac{C}{K}, x \equiv \frac{K}{N}, g \equiv \frac{G}{K}, e \equiv \frac{E}{K}, d \equiv \frac{D}{K}, \psi \equiv \lambda_c C, \phi \equiv \lambda_d K, \Omega \equiv \lambda_n N \) and \( \Pi \equiv \lambda_d D \), where \( \lambda_c, \lambda_k, \lambda_n, \) and \( \lambda_d \) are dynamic multipliers associated with (22a), (22b), (22c), (22d) respectively), we get a ten-equation system in the paths of \( \bar{\tau}, g, e, c, x, d, \psi, \phi, \Omega, \Pi \). This is a new stationary RSBA.
The above described system has two qualitative features. First, working as in Chamley (1986, p. 616), it is straightforward to show that the Ramsey tax rate can be positive even in the long run. The long-run tax rate is zero only in the special case in which $s = 0$ and $\alpha = 1$. This is consistent with previous results in the literature. In particular, the first condition ($s = 0$) eliminates the external effects from private agents’ actions (see (6)-(7) above). The second condition ($\alpha = 1$) undoes the effects from exogenous labor supply. In general, it is known that the zero limiting capital tax rate result does not hold when there are “imperfections”, like externalities or exogenous labor supply.\(^{10}\) Thus, we cannot be in a lump-sum/first-best situation in the long run. Second, as Chamley (1986, p. 617) points out, the comparison of the intertemporal first-order condition of the private sector (22a), and the intertemporal first-order condition of the government for public debt, shows that they are identical\(^{11}\) so that the marginal excess burden measured in units of private consumption, $C\lambda_d$, is constant over time. This implies that, to get a full solution to the Ramsey policy problem, one has to solve simultaneously for the long run and the whole optimal path including period 0 (as is obviously also the case in the primal approach to the Ramsey problem). Here, as we have done so far, we focus on the long run. In this case, since there is a “missing equation” in the long run because assets from the viewpoint of the household and bonds from the viewpoint of the government are perfect substitutes, a relatively simple way around this problem is to assume that public debt is exogenously given in the long run. For instance, we set $\frac{D}{Y} \equiv 0.63$ in the long run, which is the average value of the public debt-to-output ratio in the US over 1980-2003. Note that this assumption is not too restrictive since there is need for distorting taxation even in the long run.

\(^{10}\) Jones et al. (1993) also get a positive long-run optimal tax rate on capital in a model with exogenous labor supply. See Guo and Lansing (1999) for an explanation of this, as well as of several other findings in the literature. Other surveys include Chari and Kehoe (1999) and Ljungqvist and Sargent (2000, chapter 12). We report that when we solve our model with labor supply endogenously chosen, we only need $s = 0$ to get zero income taxation in the long run.

\(^{11}\) Equivalently, in terms of the stationary model, the equations for $\frac{d}{d}$, $\frac{c}{c}$ and $\frac{\Pi}{\Pi}$ are linearly dependent.
Therefore, a long-run solution can be obtained by setting
\[
\begin{align*}
\frac{\dot{\phi}}{\phi} &= \frac{\dot{\Pi}}{\Pi} = \frac{\dot{\Omega}}{\Omega} \\
\dot{\phi} &= \phi \\
\dot{\Pi} &= \Pi \\
\dot{\Omega} &= \Omega \\
\dot{c} &= c \\
\dot{e} &= e \\
\dot{x} &= x \\
\dot{\psi} &= \psi
\end{align*}
\]
where \( d = 0.63 Ag^{1-\alpha} \). Numerical solutions (see Table 5 in Economides and Philippopoulos, 2007) imply that as \( \nu \) falls (i.e. as we care more about the environment relative to private consumption), it is optimal to reduce both the income tax rate (i.e. \( \bar{\tau} \) falls) and cleanup spending as a share of output (i.e. \( \bar{E} \gamma \) falls). These policy choices turn out to be good for the balanced growth rate (i.e. \( \tilde{\gamma} \) rises, as \( \nu \) falls). Therefore, again, the key message of Result 2 goes through.

The same numerical solutions confirm that when \( \alpha = 1 \) and \( s = 0 \), we get \( \tau = 0 \). This is the Chamley result of limiting zero (capital) income tax rates. The same solution implies that, in this case, the balanced growth rate is independent of \( \nu \), while \( \bar{e} \) (and hence \( \bar{E} \gamma \)) decreases with \( \nu \). Thus, the properties of this special case resemble those of the first-best allocation in section 3.

5. COMPARISON WITH THE LITERATURE

The literature on the link among growth, nature and policy is rich and still growing. A paper close to ours is Stokey (1998). Our model differs because: (i) We solved for a RSBA in the sense that the government chooses its policy subject to the CDE, while Stokey (see section 6 in her paper) has studied whether different policy instruments can implement the FBA resulting from the social planner’s problem. As we have shown, RSBA and FBA have very different properties and macroeconomic implications. (ii) We used a richer menu of distorting policy instruments including the allocation of tax revenue between infrastructure and cleanup. Thus, the tax proceeds are used to finance activities that affect the allocation of resources. In Stokey, by contrast, the proceeds are rebated to households as a lump-sum subsidy.

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12 We omit the equation for \( \frac{d}{d} \) because it is linearly dependent with the equations for \( \frac{c}{c} \) and \( \frac{\Pi}{\Pi} \).

13 Recall that variables with tildes denote long-run values.
Another related paper is Jones and Manuelli (2001), who have also emphasized that policy choices should be taken seriously in dynamic general equilibrium models of growth and environmental degradation. But, in their model, the proceeds from the tax are thrown away. They also focus on the comparison of voting over effluent charges and over direct regulation of technology.

Finally, our work is related to previous theoretical results in e.g. Philippopoulos and Economides (2003) that, concerning the provision of public goods, many policy lessons may change once one moves from static to dynamic frameworks.

6. CONCLUDING REMARKS

We studied Ramsey second-best optimal economic policy in a general equilibrium model of endogenous growth augmented with renewable natural resources. Economic policy took the form of public infrastructure and cleanup policy both being financed by distorting taxes. We focused on the implications of second-best optimal policy for long-run sustained growth. We showed that with the right policy not only there is no tradeoff between economic growth and environmental quality in the long run, but also that only growing economies can afford to improve environmental quality. The more we care about the environment, the more growth-enhancing policies the Ramsey government should choose. This differs from a first-best allocation.

We also report some results along the transition path (see Economides and Philippopoulos, 2007, appendices A.2, B.3 and C.3, for details). The CDE, irrespectively of whether the economy grows or not in the long run, is dynamically unstable. Thus, a decentralized economy, in which private agents do not internalize the effects of their actions on the environment, cannot converge to a well-defined long run. This supports environmentalists’ concerns. On the other hand, the RSBA is saddlepath stable (at least, in the baseline model in section 2). Thus, since the Ramsey government internalizes externalities, it manages to resolve the instability problem arising at the level of CDE.

Therefore the general result is that economic policy, if designed properly, can achieve a lot of good things. In our setup, it can lead to sustainable balanced growth and stabilize an otherwise dynamically unstable economy.
REFERENCES


Economides G. and A. Philippopoulos (2007): Technical Appendix to “Growth enhancing policy is the means to sustain the environment”, available on [www.aueb.gr/users/gecon/workingpapers.htm](http://www.aueb.gr/users/gecon/workingpapers.htm)


Table 1: Effect of $\nu$ on long-run RSBA with income taxes

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$\bar{\tau}$</th>
<th>$\bar{\beta}$</th>
<th>$\bar{c}$</th>
<th>$\bar{x}$</th>
<th>$\bar{\psi}$</th>
<th>$\bar{\phi}$</th>
<th>$\bar{\Omega}$</th>
<th>$\bar{c}x = \bar{C} / \bar{N}$</th>
<th>$\bar{G} / \bar{Y}$</th>
<th>$\bar{E} / \bar{Y}$</th>
<th>$\bar{\gamma}$</th>
</tr>
</thead>
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<td>0.092</td>
<td>0.046</td>
<td>0.397</td>
<td>0.904</td>
<td>23.698</td>
<td>0.004</td>
<td>0.250</td>
<td>0.326</td>
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<td>0.2</td>
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<td>0.090</td>
<td>0.101</td>
<td>1.020</td>
<td>1.749</td>
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<td>0.009</td>
<td>0.250</td>
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<td>0.011</td>
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<td>0.089</td>
<td>0.168</td>
<td>1.908</td>
<td>2.512</td>
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<td>0.251</td>
<td>3.103</td>
<td>3.169</td>
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Notes: $\alpha = 0.5, \ A = 1, \ \delta = 0.015, \ \rho = 0.04, \ \theta = 1, \ s = 0.5$. 
Table 2: Effect of $\nu$ on long-run FBA

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$\bar{c}$</th>
<th>$\bar{x}$</th>
<th>$\bar{x} = \bar{C} / \bar{N}$</th>
<th>$\bar{G} / \bar{Y}$</th>
<th>$\bar{E} / \bar{Y}$</th>
<th>$\bar{y}$</th>
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</thead>
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<td>0.427</td>
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</tbody>
</table>

Notes: $\alpha = 0.5$, $A = 1$, $\delta = 0.015$, $\rho = 0.04$, $\theta = 1$, $s = 0.5$. 