Ramsey tax-spending policy
in a model of endogenous growth

by

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\textbf{Abstract:} We augment Barro’s (1990) popular model of optimal fiscal policy and endogenous growth by adding public debt. The main result (that it is optimal to keep the income tax rate flat over time) ceases to hold. In particular, we restore the celebrated property of Ramsey policy. Also, by getting a full analytical and numerical solution to the Ramsey problem, we quantify policies and allocations across different tax policy regimes.

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1. **Introduction**

The design of taxation and expenditures is a key policy problem. Barro (1990) introduced a simple model to study this problem. Thanks to its analytical tractability, this model has become a workhorse in the theoretical literature on optimal fiscal policy and endogenous growth. In Barro’s model, there is a trade-off between productivity-enhancing public services and distorting income taxes required to finance these services. A basic result is that a benevolent government finds it optimal to keep the income tax rate flat over time. This differs from the celebrated result of the optimal public finance literature, referred to as Ramsey, that the optimal tax structure is one where the tax rate on (capital) income is high in the initial periods and roughly zero in the latter periods (see e.g. Chamley, 1986, and Judd, 1985).

This paper closes the gap between Barro’s model and the Ramsey tradition. We do so by simply adding public debt and utility-enhancing public services into Barro’s model. We then study the properties of the second-best optimal tax-spending-debt policy mix and its implications for the macroeconomy over time.

We do not only study qualitative properties as is usually the case in the Ramsey literature, but we also provide an analytical and numerical solution to the full Ramsey problem.1 By getting not only the celebrated qualitative result mentioned above, but also a quantitative solution of the whole optimal path of Ramsey policies and allocations, is of interest in its own right.2

The main results are as follows. (i) The introduction of public debt restores the celebrated Ramsey result: the optimal tax rate is not flat over time.3 (ii) Except the tax rate, and despite the latter’s big changes across the three tax policy regimes (exogenous tax rates in the initial period; this is followed by a period of heavy

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1 A qualitative analysis provides important insights but cannot yield definite results. What is the value of the non-zero tax rate(s) in the initial period(s) of heavy taxation? How do public spending, private consumption, etc, change across different tax regimes?

2 In the Ramsey policy problem, one cannot solve first for the long run and in turn study the transition period, as is typically the case in growth models. Instead, one has to solve simultaneously for the long run and the movement to the long run. There are no many papers that get such a full solution. Chari et al. (1994) are a notable exception; but they work with a neoclassical growth model rather than with an endogenous growth model like Barro’s. See below for details.

3 Benhabib et al. (2001) have derived the same qualitative result by generalizing Barro’s (1990) model in a different way. They have shown that once one deviates from Barro’s model by changing the production function from Cobb-Douglas to CES, the flat tax rate property ceases to hold.
taxation; and zero taxation thereafter), macroeconomic allocations jump at their long-run values in the period of heavy taxation, and stay there thereafter. (iii) Comparing macroeconomic allocations between the initial period and the period of heavy taxation, there is a big drop in private consumption, and a rise in public investment, as shares of capital or output, in the period of heavy taxation (and this continues thereafter). Thus, it is optimal to let private consumption bear most of the cost of high taxation and give priority to growth-promoting policies. On the other hand, it is optimal to keep the share of public consumption almost flat over time.

Sections 2 and 3 solve for a competitive equilibrium and Ramsey policy.

2. The economy and competitive equilibrium

We combine Barro (1990) and Chamley (1986). To solve the Ramsey policy problem, we will use the so-called primal approach (see e.g. Lucas, 1990, Chari et al., 1994, and Ljungqvist and Sargent, 2000, chapter 12). This means that the competitive equilibrium will be summarized by the resource constraint holding at every date and an intertemporal implementability (budget) constraint at time 0.

2.1 Household, firm and the government

Using for simplicity an additively separable utility function, the household maximizes:

$$\sum_{t=0}^{\infty} \beta^t (\nu \ln C_t + (1-\nu)\ln H_t)$$

(1)

where $C_t$ and $H_t$ are private and public consumption, $0 < \beta < 1$ and $0 < \nu < 1$.

Household’s budget constraint is:

$$K_{t+1} - K_t + B_{t+1} - B_t + C_t = (1 - \tau_t)(w_t + r_t K_t + r_t B_t + \pi_t)$$

(2)

where $K_{t+1}$ and $B_{t+1}$ are the end-of-period stocks of capital and bonds, $0 < \tau_t < 1$ is the income tax rate, $r_t$ is the asset return, $w_t$ is the wage rate (the household supplies inelastically one unit of labor services in each period) and $\pi_t$ is firm’s profit. The
initial stocks, $K_0$ and $B_0$, are given. For simplicity, capital and bonds pay the same return ex ante, and we assume zero capital depreciation. The household chooses \( \{C_t, K_{t+1}, B_{t+1}\}_{t=0}^{\infty} \) to maximize (1) subject to (2).

Firm’s profits are:

\[
\pi_t = Y_t - r_t K_t - w_t l_t
\]

where $Y_t$ is output and $l_t$ is labor input. As in Barro (1990),

\[
Y_t = AK_t^{\alpha} l_t^{1-\alpha} G_t^{1-\alpha}
\]

where $G_t$ are productivity-enhancing public services, $A > 0$ and $0 < \alpha < 1$. The firm chooses $K_t$ and $l_t$ to maximize (3) subject to (4) in each $t$.

Government’s budget constraint is:

\[
G_t + H_t + (1 + r_t)B_t = \tau_t (w_t + r_t K_t + r_t B_t + \pi_t) + B_{t+1}
\]

2.2 **Competitive equilibrium (CE)**

Given the paths of the independent policy instruments \( \{\tau_t, H_t, G_t\}_{t=0}^{\infty} \), a CE is an allocation \( \{C_t, K_{t+1}, B_{t+1}\}_{t=0}^{\infty} \) and prices \( \{r_t, w_t\}_{t=0}^{\infty} \), such that the household’s and the firm’s problems are solved, markets clear and the government budget is satisfied.

The CE can be summarized by the resource constraint holding in each period and a single implementability (budget) constraint in period 0:

\[
C_t + G_t + H_t + K_{t+1} - K_t = AK_t^{\alpha} G_t^{1-\alpha}
\]

\[
\frac{1}{1-\beta} = \frac{(K_0 + B_0)[1 + \alpha(1-\tau_0)AK_0^{\alpha-1}G_0^{1-\alpha}]}{C_0}
\]

where we have eliminated prices and taxes (apart from $\tau_0$, which is exogenously given to make the problem nontrivial; see Ljungqvist and Sargent, 2000, p. 323).
If we drop public consumption and public debt, our model is Barro’s. Regarding the Ramsey literature, the difference is that here we use a single income tax and (at social level) a linear $AK$ technology. The former is not important because an income tax inherits the features of a capital income tax. An $AK$ technology has the advantage that it implies no transition dynamics within each tax policy regime (see below); this reduces the complexity of the problem without affecting the key points. On the other hand, here we also choose the two types of public spending.

3. Ramsey policy problem

The government chooses $\{C_t, K_{t+1}, H_t, G_t\}_{t=0}^{\infty}$ to maximize (1) subject to (6a)-(6b). The Lagrangean is:

$$\sum_{t=0}^{\infty} \beta^t \left[ \ln C_t + (1-\nu) \ln H_t + \lambda \left[ AK_t^\alpha G_t^{1-\alpha} - C_t - G_t - H_t - K_{t+1} + K_t \right] \right] + \phi \left[ \frac{1}{1-\beta} - (K_0 + B_0) \left[ 1 + (1-\tau_0) \alpha K_0^{\alpha-1} G_0^{\alpha-1} \right] \right]$$

(7)

where $\phi$ is an atemporal multiplier associated with (6b) and $\lambda_t$ is a dynamic multiplier associated with (6a).

At $t \geq 1$, the first-order conditions imply:

$$\frac{(1-\nu)}{K_t} \frac{C_t}{K_t} = \nu \frac{H_t}{K_t}$$

(8a)

$$\frac{G_t}{K_t} = \left[ (1-\alpha) A \right]^{1/\alpha}$$

(8b)

$$\frac{C_{t+1}}{C_t} = \beta \left[ 1 + \alpha A \left( \frac{G_{t+1}}{K_{t+1}} \right)^{1-\alpha} \right]$$

(8c)

while, the same first-order conditions at time 0 imply:

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4 See Chari et al. (1994) for a model with transitional dynamics within tax policy regimes.
\[(1 - \nu) \frac{K_0}{H_0} = \Gamma_0 \quad (9a)\]

\[\Gamma_0 \left[ (1 - \alpha)A \left( \frac{G_0}{K_0} \right)^{-\alpha} - 1 \right] = \phi \left( \frac{K_0}{C_0} \right) (1 - \alpha) (1 - \tau_t) \alpha A \left( \frac{K_0}{G_0} \right)^{-\alpha} \left( 1 + \frac{B_0}{K_0} \right) \quad (9b)\]

\[\Gamma_0 = \beta \nu \frac{K_1}{K_0} \frac{K_0}{C_1} [1 + \alpha A \left( \frac{G_1}{K_1} \right)^{1-\alpha}] \quad (9c)\]

where \(\Gamma_0 \equiv \nu \frac{K_0}{C_0} + \phi \left( \frac{K_0}{C_0} \right)^{2} \left[ 1 + \frac{B_0}{K_0} \right] \left[ 1 + \alpha (1 - \tau_t) A \left( \frac{G_0}{K_0} \right)^{1-\alpha} \right].\)

In addition, (6a-b) hold all the time, \(t \geq 0\).

3.1. **Qualitative implications for the tax rate and policy regimes**

Without exogenous upper bounds on the tax rate, there can be only one period with nonzero taxation, and this is at \(t = 1\). Actually, it is straightforward to show (see e.g. Chari et al., 1994, pp. 629-630) that, in this class of utility functions, the optimal tax rate is zero at \(t = 2\) onward. Thus, there can be three tax policy regimes that correspond to \(t = 0\), \(t = 1\) and \(t \geq 2\), where \(\tau_t\) is exogenously given at \(t = 0\) and \(\tau_t = 0\) at \(t \geq 2\).

3.2. **The full Ramsey system**

We work in two steps. First, we combine the first-order conditions - equations (8a-c), (9a-c) and (6a-b) - so as to satisfy continuity across policy regimes. As said, there can be three distinct policy regimes, which correspond to periods \(t = 0\), \(t = 1\) and \(t \geq 2\). Second, since the \(AK\) model allows for long-term growth, we need to transform variables so as to make them stationary. In particular, we define \(\frac{C_t}{K_t} \equiv c_t\), \(\frac{H_t}{K_t} \equiv h_t\), and \(\frac{G_t}{K_t} \equiv g_t\) at any \(t \geq 0\). Note that, as in the basic \(AK\) model, after period 2 there are no transitional dynamics. Therefore, we have the system:

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5 If there are exogenous upper bounds on the tax rate, it is optimal to set it at its upper bound for as many periods as necessary and zero thereafter. See Chari et al. (1994, p. 630).
\[(1 - \nu) = \Gamma_0 h_0 \] 

\[\Gamma_0 \left[ (1 - \alpha) A g_{0^{-\alpha}} - 1 \right] c_0 = \phi \left( 1 + \frac{B_0}{K_0} \right) \alpha (1 - \alpha) A g_{0^{-\alpha}} (1 - \tau_0) \] 

\[\Gamma_0 c_1 \frac{k_1}{k_0} = \beta v [\alpha A g_1^{1-\alpha} + 1] \] 

\[(1 - \nu) c_1 = \nu h_1 \] 

\[g_1 = \left[ (1 - \alpha) A \right]^{\frac{1}{\alpha}} \] 

\[\frac{c}{c_1} = \frac{\beta (1 + \alpha A g_{1^{-\alpha}})}{1 + A g_{1^{-\alpha}} - c_1 - g_1 - h_1} \] 

\[(1 - \nu) c = \nu h \] 

\[g = \left[ (1 - \alpha) A \right]^{\frac{1}{\alpha}} \] 

\[1 = \frac{\beta (1 + \alpha A g_{1^{-\alpha}})}{1 + A g_{1^{-\alpha}} - c - g - h} \] 

\[\frac{1}{1 - \beta} = \frac{\left( 1 + \frac{B_0}{K_0} \right) \left[ 1 + (1 - \tau_0) \alpha A g_{0^{-\alpha}} \right]}{c_0} \] 

where subscripts denote time periods, while variables without time subscripts denote long-run values (here the long run is reached at \( t = 2 \)). We have ten equations in ten unknowns, \( c_0, h_0, g_0, c_1, h_1, g_1, c, h, g, \phi \). This is given the initial stocks \( (B_0, K_0) \) and the initial tax rate \( \tau_0 \).

3.3. **Numerical solution of (10a-j)**

As a baseline case, we set the following parameter values and initial conditions: \( \alpha = 0.75, \nu = 0.85, A = 1, \beta = 0.9, \frac{B_0}{K_0} = 0.25 \) and \( \tau_0 = 0 \). The solution is in Table 1. We also report the implied tax rates, \( \tau_i \) (they follow from household’s Euler equations).
Table 1: Ramsey policies and allocations

<table>
<thead>
<tr>
<th></th>
<th>$c_i$</th>
<th>$h_i$</th>
<th>$g_i$</th>
<th>$\tau_i$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>0.172</td>
<td>0.020</td>
<td>0.065</td>
<td>0 (set)</td>
<td></td>
</tr>
<tr>
<td>$t = 1$</td>
<td>0.125</td>
<td>0.022</td>
<td>0.157</td>
<td>0.988</td>
<td>0.636</td>
</tr>
<tr>
<td>$t \geq 2$</td>
<td>0.125</td>
<td>0.022</td>
<td>0.157</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Matlab 7.0

All solution values make sense. For instance, the tax rate in the first period is positive, $\tau_1 = 0.988$, while it is zero in the long run ($t \geq 2$). The value of the atemporal multiplier associated with the implementability constraint, $\phi$, is positive.

The main results have been written in the Introduction. Here, we only note some important special cases. First, Barro’s model (in which there is no public consumption nor bonds so that the government budget constraint is $G_t = \tau_t Y_t$, where $Y_t = A^{\sigma} \tau_t^{1-\alpha} K_t$) implies that the optimally chosen income tax rate is flat over time and equals the productivity of public services, $1 - \alpha$. In our setup, this follows from (10e) and (10h). Thus, it is optimal to follow for ever the tax regime chosen at $t = 1$. Second, the inclusion of public consumption is not important to our results. Third, the above results combined imply that it is the presence of government bonds that restores the Ramsey property.7

Finally, we report that results are robust to changes in the values of parameters and initial conditions.

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6 See footnote 5 above.
7 Lansing (1999) shows that the long-run optimal tax rate on capital income is not zero when utility is logarithmic (as in our model) and the government budget is balanced. But when he adds public debt, the zero long-run tax rate is recovered. Although our setups differ (he uses a Judd-type neoclassical growth model), our qualitative result is similar: once we add public debt, the long-run (capital) income tax rate gets zero.
REFERENCES


