OPTIMAL PROTECTION OF PROPERTY RIGHTS
IN A GENERAL EQUILIBRIUM MODEL OF GROWTH

by
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Abstract: We incorporate weak property rights into an otherwise standard general equilibrium model of growth and second-best optimal economic policy. In this setup, the state plays two of its key roles: it protects property rights and provides public services. The government chooses economic policy (namely, the income tax rate and the allocation of tax revenues between public services and enforcement of law) to maximize the economy’s growth rate. The focus of our analysis is on how weak property rights open the door to multiple decentralized competitive equilibria, the different properties of those equilibria, the characteristics of second-best optimal policies and their general equilibrium implications.

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I. INTRODUCTION

It is widely recognized that the incentive to work, produce, study, invest, etc, depends on whether property rights, or rights of ownership, are certified and protected. The less secure are the property rights, the lower this incentive in the first place. This is, in turn, bad for economic growth. 

It is also recognized that the degree of security of property rights is endogenous. Although there can be several intuitive determinants of that degree, it is the government that ultimately certifies and protects property rights. Activities such as police services, courts, prisons, national defense, the design of patent rights, the stability of laws and institutions etc (i.e. “the law”) are typically the job of the state. But to provide such services, the state needs tax revenues. Thus, as it is the case with all public services, there is a tradeoff: on the one hand, weak property rights distort incentives and lead to resource misallocation; on the other hand, the funding of government policies to protect property rights requires tax revenues which are unavoidably raised by distorting taxes (see also Barro and Sala-i-Martin, 1995, pp. 159-161). In other words, the government has to design a second-best policy. Then, a natural question to ask is “what is the best economic policy?” Answering this question will also provide answers to questions, like “what should be the size of public sector?”, or “how many resources should the government allocate to the protection of property rights vis-à-vis other public services?”, or “should the government go for full protection of property rights?”.

To answer these questions, we incorporate ill-defined property rights into an otherwise standard general equilibrium model of endogenous growth and second-best

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1 For a survey of property rights, see e.g. Drazen (2000, pp. 343-345 and 439-444).
2 See e.g. Mauro (1995), Knack and Keefer (1995), Hall and Jones (1999) and Rodrik (1999). For a survey of property rights and growth, see e.g. Drazen (2000, chapter 11, sections 4-6).
3 These determinants include the technologies of production and conflict, the number of contenders and the kind of asymmetries (see e.g. Hirshleifer, 1991 and 1995, and Skaperdas, 1992); the private costs of property rights protection (see e.g. de Meza and Gould, 1992, and Grossman and Kim, 1996); the probability of maintaining ownership in one’s output as an increasing function of government expenditure on the protection of property rights (see e.g. Barro and Sala-i-Martin, 1995, pp. 159-161); the size of wealth to be appropriated and the degree of inequality (see e.g. Benhabib and Rustichini, 1996); the elasticity of intertemporal substitution (see e.g. Lane and Tornell, 1996, in a model with redistributive struggle); the efficiency wages paid to corruptible bureaucrats (see e.g. Acemoglu and Verdier, 1998, in a model where bureaucrats’ corruption arises due to asymmetric information); the effectiveness of predation technology versus the state’s ability to deter predators (see e.g. Grossman, 2002); etc.
optimal policy (see e.g. Barro, 1990, and Barro and Sala-i-Martin, 1995, chapter 4.4). Our model has two distinct features: First, individuals can expropriate each other’s output in an attempt to increase their own personal wealth. That is, in addition to standard decisions like consumption, saving, etc, individuals also choose optimally the allocation of their time between productive work and extractive activities. Second, the government uses the collected income tax revenues to finance the provision of public production services (maintenance of roads, airports, etc) and the enforcement of law (police, courts, prisons, etc). The former is the engine of long-term (endogenous) growth. The latter captures the role of the state as the key protector of property rights. Then, in line with the endogenous growth literature, we solve a normative exercise: we assume that the government chooses the income tax rate, as well as the allocation of tax revenues between infrastructure and enforcement of law, to maximize the economy’s growth rate. In doing so, it tries to correct the social inefficiencies caused by decentralized private economic behavior.

The main results are as follows. We first solve for a Decentralized Competitive Equilibrium (DCE) for any feasible economic policy. Weak property rights lead to multiple DCE. Technically, multiplicity arises from strategic complementarities in illegal activities (see Mauro, 2002, in a similar context). Intuitively, if each self-interested individual expects other individuals to be particularly aggressive, it is optimal for him to act similarly and so we end up in a relatively bad equilibrium characterized by low work effort, weak property rights and low growth. But, depending on expectations about other individuals’ behavior, it is equally possible to end up in a relatively good equilibrium characterized by higher work effort, stronger property rights and higher growth. Both equilibria are self-fulfilling.

The two DCE have different comparative static properties. Specifically, if we are in the aggressive DCE, an increase in the size of public sector further distorts individuals’

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4 See, among many others, Baumol (1990) and Murphy et al. (1991), where individuals decide how to allocate their time and effort between productive activities (work, innovation, entrepreneurship, etc) and socially unproductive activities (rent seeking, tax evasion, stealing, organizing crime, etc).

5 Mauro (2002) also provides real-world examples of strategic complementarities in anti-social activities. As he explains (p. 5), the main idea behind strategic complementarities is that “if many people steal, then the probability of any one of them being caught will be low”. Tirole’s (1996, p. 18) motivation is that “our individual incentives depend also on those of the groups we are associated with”. Cooper and John (1988) have shown that strategic complementarities are necessary for multiple equilibria. See below for details.
incentives by making them more aggressive and pushing them deeper to illegal activities. By contrast, if we are in the good DCE, the opposite occurs: a larger size of public sector improves individuals’ incentives. The effects of the allocation of tax revenues between infrastructure and enforcement of law are symmetrically opposite; namely, a higher share allocated to the enforcement of law improves incentives and growth in the bad DCE, while it deteriorates incentives and growth in the good DCE. The intuition of these comparative static results is discussed in detail below (see subsection III.6). At this stage, we just point out that these results suggest that, when an economy is trapped into a bad equilibrium with ill-behaved economic agents and low economic activity, it is better to reduce tax rates and give emphasis to the enforcement of law relative to public good provision. By contrast, when an economy enjoys well-behaved economic agents and high economic activity, it is better to increase tax rates and use the tax revenues to finance mainly the provision of public goods.

We then endogenize economic policy and study the resulting General Equilibrium (GE). When a growth-maximizing government chooses the income tax rate and the allocation of tax revenues between enforcement of law and infrastructure, there are three results. First, the government manages to resolve the expectations coordination problem arising at the level of a DCE. Thus, in the presence of two market imperfections (namely, production externalities and ill-defined property rights), and with two policy instruments at its disposal, a growth-maximizing government manages to coordinate private expectations and select a single equilibrium. Second, although it manages to resolve the expectations coordination problem, the government cannot lead the economy to a GE with fully protected property rights. Actually, the features of the GE with optimally chosen policy resemble the features of the bad DCE. This happens because, in a growth model with endogenous tax bases, taxes are distortionary rather than lump sum, so that we have a second-best policy problem. And, as is well known, second-best policy problems are problems of indirect policy control. Third, the government finds it optimal to rely more heavily on the share of tax revenues allocated between infrastructure and enforcement of law, rather than on the income tax rate. This is because the latter is “more distortionary” than the former. Specifically, while the share of tax revenues has only substitution effects, the income tax rate affects both saving and substitution decisions.
(where substitution effects are attempts to avoid taxes by substituting non-taxed for taxed activities).

What is the paper’s contribution? Although the public economics literature has always emphasized that the primary role of the state is to protect property rights (see e.g. Atkinson and Stiglitz, 1980, p. 5), there has not been so far a formal analysis of this key role of the state in a dynamic general equilibrium model of growth and second-best optimal policy. This is the gap our paper fills in. The model closest to ours is the one presented by Barro and Sala-i-Martin (1995, pp. 159-161). However, Barro and Sala-i-Martin do not study the micro-foundations of property rights; also, the government does not provide infrastructure services and does not choose the allocation of tax revenues between different public services. Finally, the other papers mentioned in footnote 3 above focus on different mechanisms for the protection of property rights or are static (see below for further details).

The rest of the paper is as follows. Section II discusses the model. Section III solves for a decentralized competitive equilibrium given policy. The optimum policy problem is in Section IV. Section V concludes.

II. INFORMAL DESCRIPTION OF THE MODEL

This section presents the main characteristics and assumptions of the model.

First, we build on Barro’s (1990) model of long-term growth and endogenously chosen fiscal policy. We introduce weak property rights into this model by assuming that individuals can extract from other individuals’ output to increase their own personal wealth. Specifically, we assume that when a firm produces an amount of output, $y$, only a fraction of it, $py$, can be actually appropriated by that firm, because the rest, $(1-p)y$,
can be taken away by households.\textsuperscript{9} Thus, $0 < p \leq 1$ measures the degree of property rights.

Second, expropriation comes at a private cost.\textsuperscript{10} Specifically, following most of the literature on rent seeking, we assume that it requires time and effort (see Tullock, 1967). The fraction of firms’ output extracted by each household increases with the time and effort that this household allocates to extractive activities, as well as the average time and effort allocated to extractive activities by all households, while it decreases with the fraction of government revenue that goes to the protection of property rights. The assumption that the fraction extracted by each household increases with the average time and effort that all households allocate to extractive activities reflects strategic complementarities in illegal activities. This is as in e.g. Mauro (2002).

Third, the government imposes income taxes and uses the collected tax revenues to finance the protection of property rights, as well as the provision of infrastructure services. Following most of the endogenous growth literature (see e.g. Barro, 1990), we assume that, when the government chooses economic policy, its objective is to maximize the economy’s growth rate. The sequence of events is as follows: policy is chosen first, and in turn private agents make their decisions acting simultaneously and competitively. Thus, the government acts as a Stackelberg leader vis-à-vis private agents.

We will now formalize this story.

\textbf{III. DECENTRALIZED COMPETITIVE EQUILIBRIUM}

We will work with backward induction. Thus, this section solves for a decentralized competitive equilibrium for any feasible economic policy. Economic policy will be endogenized in the next section.

It is convenient to start with the expropriation technology.

\textsuperscript{9} There are many ways of modeling expropriation. For instance, firms may attempt to expropriate each other’s capital or output, or households may attempt to expropriate each other’s assets, or a combination of both. The specific way of modeling “who takes away from whom” is not important because households are also firm-owners in this class of models. Here we choose a simple way of formalizing weak property rights.
III.1 The expropriation technology

At each instant, each household $i = 1, 2, ..., I$ has access to an equal share of firms’ output, denoted by $\frac{Y}{I}$. Also, at each instant, each $i$ has one unit of time available and then allocates a fraction $0 < \theta^i \leq 1$ of this unit to productive work and the rest $0 \leq (1 - \theta^i) < 1$ to extraction. We assume that the fraction of $\frac{Y}{I}$ extracted by household $i$ (denoted by $0 \leq e^i < 1$) increases with the effort that $i$ allocates to extractive activities, $(1 - \theta^i)$, as well as the average time allocated to extractive activities by all households, $\frac{\sum_i (1 - \theta^i)}{I} \equiv (1 - \bar{\theta})$, while it decreases with the fraction of total tax revenues earmarked for the enforcement of law, denoted as $b$ (where $b$ is specified in equation (8b) below). To incorporate our story in a tractable way, we use the form:

$$e^i = \frac{1}{1 + \frac{vb}{(1 - \theta^i)(1 - \bar{\theta})}}$$

(1)

where $0 < v \leq 1$ is a unit conversion parameter that transforms tax revenues into effective protection of property rights. Notice that if $\theta^i = 1$, then $e^i = 0$, as it should be.

We can now model the behavior of firms, households and the government.

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10 Breaking the law, bribing, employing lawyers, hiring bodyguards, etc, are costly activities.
11 We assume that each household has access to $y/I$, rather than to $y$, so as to avoid scale effects in equilibrium.
12 We assume that it is the average, rather than the sum, of illegal activities that provides externalities. This is to avoid scale effects in equilibrium. This is not important.
13 This extraction technology is similar to Grossman’s (2002, equation (2)). As Grossman points out, this is a black box like any production function. Note that Grossman models appropriation, not expropriation.
III.2 Firms’ behavior

We assume a single firm (this is only for simplicity). This firm maximizes profits, $\pi$:

$$\pi = py - rk - wl$$  \hspace{1cm} (2)

where $0 < p \leq 1$ is the fraction of appropriability of the firm’s own output; $y$, $k$ and $l$ are respectively the firm’s output, capital and labor; and $r$ and $w$ are respectively the market interest rate and wage rate.

At the firm’s level, the production function takes a Cobb-Douglas form:

$$y = Ak^{\alpha}l^{1-\alpha}G^{1-\alpha}$$  \hspace{1cm} (3)

where $A > 0$ and $0 < \alpha < 1$ are parameters and $G$ is per capita government production services.\(^{15}\) This is a widely used production function (see e.g. Barro and Sala-i-Martin (1995, chapter 4)).

The firm acts competitively by taking prices, policy variables and aggregate outcomes as given.\(^{16}\) The first-order conditions for $k$ and $l$ are simply:

$$r = \alpha p \frac{y}{k}$$ \hspace{1cm} (4a)

$$w = (1 - \alpha) p \frac{y}{l}$$ \hspace{1cm} (4b)

so that, with constant returns to scale at the firm’s level, profits are zero.

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\(^{14}\) A more general specification would be $e^t = \frac{1}{1 + \frac{\lambda}{\nu} - (1 - \theta)^{\mu}}$, where $0 \leq \lambda \leq 1$ and $0 \leq \mu \leq 1$. Here we set $\lambda = \mu = 1$ so that extractive actions are strategic complements. If $\lambda = 1$ and $\mu = -1$, we would have strategic substitutes. If $\lambda = 1$ and $\mu = 0$, we get Grossman’s (2002) technology.

\(^{15}\) We assume that it is the average $\bar{G} = \frac{G}{I}$, rather than the total $G$, that provides externalities, in order to avoid scale effects in equilibrium. This is not important.

\(^{16}\) Thus, the firm takes $p, r, w, \bar{G}$ as given.
III.3 Households’ behavior

Each household $i$ maximizes intertemporal utility:

$$\int_0^\infty \left( \frac{(c^i)^{1-\sigma}}{1-\sigma} \right) e^{-\rho t} \, dt$$  \hspace{1cm} (5)

where $c^i$ is $i$’s consumption, $\sigma > 0$ (with $\sigma \neq 1$) is a degree of intertemporal substitution and $\rho > 0$ is a discount factor.

Each household $i$ consumes $c^i$ and saves $a^i$ in the form of an asset. Also, as said above, it is endowed with one unit of effort time at each instant,\(^{17}\) and allocates a fraction $0 < \theta^i \leq 1$ of this unit to productive work, while the rest, $0 \leq (1 - \theta^i) < 1$, goes to extractive activities. Thus, household $i$’s budget constraint is:

$$a^i + c^i = (1 - \tau)(ra^i + w\theta^i) + e^i \frac{y}{I}$$ \hspace{1cm} (6)

where $e^i$ is as in equation (1) above, a dot over a variable denotes a time derivative, and $0 < \tau < 1$ is a proportional income tax rate common to all households. Notice that there are taxes on legal activities only.

Each household $i$ acts competitively by taking prices, policy variables and aggregate outcomes as given.\(^{18}\) If we combine the first-order conditions for consumption, saving and extraction ($c^i, a^i, \theta^i$), we have:

$$\dot{c}^i = c^i \left( \frac{(1-\tau)r - \rho}{\sigma} \right)$$  \hspace{1cm} (7a)

$$(1-\tau)w = \frac{\nu b(1-\theta)}{\nu b + (1-\theta')^2} \frac{y}{I}$$  \hspace{1cm} (7b)

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\(^{17}\) Since the total effort time is exogenous, leisure is not included in the instantaneous utility function in (5). This is for simplicity.

\(^{18}\) Thus, each household $i$ takes $r, w, \tau, b, \nu, y, (1-\theta)$ as given.
where (7a) is the familiar Euler equation for the isoelastic case, and (7b) implies that net returns from work and expropriation are equal in equilibrium.

III.4 Government budget constraint and the role of economic policy

The government uses total tax revenues, \( \tau(\sum_i a_i + w\sum_i \theta^i) \), to finance two activities: it spends \( P \) on the protection of property rights and \( G \) on the provision of infrastructure services. Assuming a balanced budget, the budget constraint is:

\[
P + G = \tau(\sum_i a_i + w\sum_i \theta^i) \quad (8a)
\]

Without loss of generality, we assume that a fraction \( 0 < b < 1 \) of total tax revenues finances the provision of \( P \), while the rest, \( 0 < (1-b) < 1 \), finances the provision of \( G \). Thus, instead of (8a), we can write:

\[
P = b \tau(\sum_i a_i + w\sum_i \theta^i) \quad (8b)
\]

\[
G = (1-b) \tau(\sum_i a_i + w\sum_i \theta^i) \quad (8c)
\]

where inspection of (8a)-(8c) reveals that the independent policy instruments at each point in time are \( \tau \) and \( b \).\(^\text{19}\)

III.5 Decentralized competitive equilibrium (given economic policy)

We now solve for a Decentralized Competitive Equilibrium (DCE). This is for any feasible economic policy. In a DCE: (i) Each individual firm and household maximize their own profit and utility respectively by taking prices, policy variables and aggregate outcomes as given. (ii) All markets clear. This means \( \sum_i \theta^i \) in the labor market and

\(^\text{19}\) We have assumed that it is the share of tax revenues, \( b \), that matters for the protection of property rights in equation (1) rather than the level of resources, \( P \). This is for algebraic simplicity. The literature also uses similar convenient functional specifications (see e.g. Barro and Sala-i-Martin, 1995, pp. 159-161).
\[ k = \sum_i a_i \] in the capital market.\(^{20}\) (iii) Individual decisions are consistent with aggregate decisions. This means \((1 - p)y = \sum_i e_i \frac{Y_i}{I_i}\). Namely, the amount stolen from the firms equals the amount grabbed by households. (iv) All constraints, including the economy’s resource constraint, are satisfied. For simplicity, we will focus on a symmetric DCE, i.e. individuals are alike ex post.\(^{21}\) Thus, from now on, the superscript \(i\) can be omitted and the variables \((c, k, \theta)\) in (9a)-(9c) below will denote per capita values.

Combining (1)-(8), it is straightforward to show that a DCE is summarized by:\(^{22}\)

\[
\begin{align*}
\dot{c} &= c \left[ 1 - \sigma \frac{\alpha B(\tau, b) - \rho}{\tau} \right] \\
\dot{k} &= \left[ 1 - \tau + \frac{(1 - \theta)^2}{vb} \right] B(\tau, b)k - c \\
(1 - \alpha)(1 - \tau)[(1 - \theta)^2 + vb] &= \theta(1 - \theta)
\end{align*}
\]

where \( B(\tau, b) \equiv A^{\frac{1}{\sigma}}[\theta\theta(1 - b)]^{\frac{1}{\sigma}} \frac{vb}{vb + (1 - \theta)^2} \] \(\geq 0\). Notice that consistency of individual and aggregate decisions implies that the equilibrium fraction stolen is \((1 - p) = e = \frac{(1 - \theta)^2}{vb + (1 - \theta)^2}\). This is well defined, \(0 \leq (1 - p) < 1\).

Equations (9a)-(9c) give the paths of \((c, k, \theta)\) for any economic policy, where the later is summarized by the tax rate \(0 < \tau < 1\) and the allocation of tax revenues between enforcement of law and infrastructure, \(0 < b < 1\).

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\(^{20}\) Recall that there is a single firm and \(i = 1, 2, \ldots, I\) households.

\(^{21}\) Solving for a symmetric equilibrium is rich enough to capture incentive problems and show how non-cooperative and cooperative equilibria differ. See e.g. Cooper and John (1988).

\(^{22}\) Equation (9a) follows from the Euler in (7a). Equation (9b) follows if we combine the budget constraints (2), (6) and (8). That is, (9b) is the economy’s resource constraint, \(\dot{k} + c + P + G = y\), where the solution for output is \(y = \left[ \frac{(1 - \theta)^2 + vb}{vb} \right] B(\tau, \theta)k\). Equation (9c) follows mainly from (4b) and (7b).
Notice that the economy-wide output is linear in capital. Thus, the model is a variant of the $AK$ model. Actually, if $\theta = \nu = 1$ and also $b = 0$ so that $B(\tau) \equiv A^\alpha \tau^{1-\alpha}$, we get Barro’s (1990) model as a benchmark. Also notice that while it is the social return to capital that matters in the resource constraint (9b), it is the private return that drives saving decisions in the Euler equation (9a). That is, with production externalities, there is a wedge between private and social returns. This is a standard result.

An advantage of the model is its simplicity. Equation (9c) is a quadratic in $\theta$ only. Once we solve (9c) for $\theta$, equation (9a) can give the so-called balanced growth path along which consumption and capital grow at the same constant rate, defined as

$$\gamma \equiv \frac{c}{c} = \frac{k}{k};$$

in turn, equation (9b) can give the consumption-to-capital ratio, $\frac{c}{k}$. Thus, the key part of the solution is the value of $\theta$ in (9c). It is straightforward to show (see Appendix A) that there can be two solutions for $0 < \theta < 1$ in (9c). Therefore, we have:

**Proposition 1:** Given economic policy (summarized by $0 < \tau < 1$ and $0 < b < 1$), weak property rights lead to multiple (two) Decentralized Competitive Equilibria.

**Proof:** See Appendix A.

Therefore, given economic policy, weak property rights lead to multiple (two) solutions for the fraction of effort that individuals allocate to work relative to extraction ($\theta$), and in turn two solutions for the associated DCE. Without loss of generality, we denote these two solutions for $\theta$ as $0 < \theta_1 < \theta_2 < 1$. Inspection of (9a) and (9b) reveals that equilibria with high work effort ($0 < \theta_2 < 1$) are associated with a higher balanced growth rate and a higher consumption-to-capital ratio than equilibria with low work effort ($0 < \theta_1 < 1$). Hence, we call the former “good” and the latter “bad” DCE. Notice that multiplicity takes the form of an expectations coordination problem.\(^{23}\)

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\(^{23}\) Thus, multiplicity is independent of initial conditions. In particular, the predetermined capital stock plays no role in (9c). Instead, it is the choice of the jump variables that puts the economy on either one of the two equilibria. See Evans et al. (1998) for an intuitive interpretation of expectational indeterminacy.
The multiplicity of DCE arises from strategic complementarities in illegal activities in equation (1). This is as in Tirole (1996) and Mauro (2002). It is straightforward to show that, without strategic complementarities, equation (9c) changes to $\theta(1-\alpha)(1-\tau)[(1-\theta)+\nu b] = \theta$, which gives a unique $\theta$. The interpretation of multiple DCE is as follows. Given the possibility of weak property rights, extraction becomes the dominant strategy (both solutions for $\theta$ are less than one). This happens because self-interested individuals realize that there is a contestable pie available. Hence, they always find it optimal to devote some of their private resources to extractive activities. In addition, there is an expectations coordination problem depending on expectations about the society’s behavior. Specifically, if individual $i$ expects other individuals to be particularly aggressive, $i$’s potential return to extractive activities increases (because of strategic complementarities, higher extractive effort by others increases the marginal return to higher extractive effort by $i$). Hence, $i$’s incentive to be aggressive gets stronger, and we end up in a relatively bad equilibrium with high extractive activity and low work effort. But it is equally possible to end up in a relatively good equilibrium with lower extractive activity and higher work effort depending on expectations about other individuals’ behavior. Thus, we have two self-fulfilling perfect foresight equilibria.

**III.6 Numerical solution and comparative statics of DCE**

To confirm the plausibility of our theoretical results, we provide numerical solutions for (9a)-(9c). This will also allow us to get comparative static results. We choose the following parameter values: $\alpha = 0.8$, where $\alpha$ is the productivity of private capital vis-à-vis public infrastructure in the firm’s production function (3); $\rho = 0.04$, where $\rho$ is the time discount factor in equation (5); $\sigma = 2$, where $\sigma$ is the degree of intertemporal substitution in equation (5); $A = 1$, where $A$ is aggregate productivity in (3); and $\nu = 1$, where $\nu$ converts tax revenues into effective protection of property rights in equation (1). Concerning policy instruments, we set $\tau = 0.2$, where $\tau$ is the income tax rate, and $b = 0.5$, where $b$ is the share of total tax revenues used for the protection of property rights vis-à-vis public infrastructure. All these are commonly used parameter values.

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24 We report that numerical results are robust to the parameter values chosen.
Then, we get the following results. Equation (9c) gives two solutions for $\theta$: $\theta_1 = 0.22$ and $\theta_2 = 0.91$. In turn, the low work effort solution (i.e. $\theta_1 = 0.22$) implies a low degree of property rights, $(1 - E_1) = 0.46$, and a low balanced growth rate, $\gamma_1 = 0.03$, relative to the high work effort solution (i.e. $\theta_2 = 0.91$) which gives $(1 - E_2) = 0.98$ and $\gamma_2 = 0.15$.

We continue with comparative static results by focusing on the effects of policy instruments, $\tau$ and $b$. We start with the effects of the tax rate, $0 < \tau < 1$. The numerical simulations above imply $\frac{\partial \theta}{\partial \tau} < 0$ around the low work effort DCE associated with $\theta_1$, and $\frac{\partial \theta}{\partial \tau} > 0$ around the high work effort DCE associated with $\theta_2$. Thus, the two DCE have opposite comparative static properties. If it so happens to be in a bad DCE, a higher tax rate (or equivalently a larger public sector) leads to lower work effort. By contrast, if it so happens to be in a good DCE, a higher tax rate (or equivalently a larger public sector) leads to higher work effort. The intuition is as follows. In general, there are two effects from an increase in the tax rate. First, a higher tax rate pushes individuals away from legal to illegal activities since the latter are untaxed. Second, an increase in the tax rate leads to higher tax revenues; this allows the government to finance higher infrastructure services and enforcement of law, both of which improve the return to productive work and so give the right incentives. The combination of these two opposite effects will determine the net, equilibrium outcome. If we are in the bad equilibrium case, in which work effort is low and hence economic activity and tax revenues are initially low, the first effect dominates so that an increase in the tax rate further distorts individuals’ incentives. By contrast, if we are in the good equilibrium case, in which work effort is high and hence economic activity and tax revenues are initially high, the second effect dominates so that an increase in the tax rate further improves individuals’ incentives.

25 This is usually the case in models with multiple equilibria. See e.g. Lockwood and Philippopoulos (1994) and the references cited therein.

26 For given $\theta$, the solution for output implies $\frac{\partial y}{\partial \tau} > 0$ (see footnote 22 above).
Then, the effect of the tax rate, $\tau$, on the balanced growth rate, $\gamma$, follows naturally. Equation (9a) implies \[ \frac{\partial \gamma}{\partial \tau} = \frac{\partial \gamma}{\partial \theta} \frac{\partial \theta}{\partial \tau} + \frac{\partial \gamma}{\partial \theta} \frac{\partial \theta}{\partial \tau}. \] That is, an increase in $\tau$ exerts two effects on $\gamma$: (i) A direct Laffer-curve type effect, $\frac{\partial \gamma}{\partial \tau}$. This is Barro’s (1990) result and arises from a tradeoff between higher productive government services and higher taxes required to finance these services. (ii) An indirect effect through the effort allocated to work, $\theta$. This can be positive if we are in the good equilibrium case where $\frac{\partial \theta}{\partial \tau} > 0$, or negative if we are in the bad equilibrium case where $\frac{\partial \theta}{\partial \tau} < 0$. Actually, we report that numerical simulations imply that the total net effect, $\frac{\partial \gamma}{\partial \tau}$, is a Laffer curve.

We continue with the effects of $0 < b < 1$. These are symmetrically opposite to those of $\tau$. Specifically, the same numerical simulations imply $\frac{\partial \theta}{\partial b} > 0$ and $\frac{\partial \gamma}{\partial b} > 0$ around the low work effort DCE associated with $\theta_1$, while $\frac{\partial \theta}{\partial b} < 0$ and $\frac{\partial \gamma}{\partial b} < 0$ around the high work effort DCE associated with $\theta_2$. In other words, an increase in the share of tax revenues earmarked for the protection of property rights improves ceteris paribus the incentive to work as well as the economy’s growth rate in the bad DCE, while the opposite happens in the good DCE. The intuition is analogous to that behind the effects of the tax rate. That is, if we are in the bad equilibrium case, in which the tax revenue effect is relatively small, a higher $b$ improves incentives and this is good for growth. By contrast, if we are in the good equilibrium case, in which the tax revenue effect is relatively big, a higher $b$ reduces the tax base\(^{27}\) and hence the government’s ability to provide infrastructure and law, which reduces the return to legal work and thus has an adverse effect on the incentive to work and economic growth.

\(^{27}\) For given $\theta$, the solution for output implies $\frac{\partial y}{\partial b} < 0$ (see footnote 22 above).
III.7 Summary of this section

We solved for a Decentralized Competitive Equilibrium (DCE). This is described by (9a)-(9c) and holds for any feasible policy. The latter is summarized by the tax rate, $0 < \tau < 1$ and the allocation of tax revenues between enforcement of law and infrastructure, $0 < b < 1$. The next section will endogenize $\tau$ and $b$. Before we move on to optimal policy, we should point out that DCE are socially inefficient. This is due to two market imperfections: (a) production externalities cause a wedge between private and social returns and lead to inefficiently low investment; (b) ill-defined property rights distort individuals’ incentives and push them to extractive activities which are socially harmful.

IV. SECOND-BEST OPTIMAL POLICY AND GENERAL EQUILIBRIUM

We now endogenize policy, as summarized by the two independent policy instruments $0 < \tau < 1$ and $0 < b < 1$. Following most of the related endogenous growth literature, we assume that the optimum policy problem for the government is to choose its policy instruments in such a way as to maximize the economy’s balanced growth rate, $\gamma$.\textsuperscript{28} In doing so, the government will try to correct the market imperfections mentioned above and also raise tax revenue in the most efficient way.

IV.1 Solution

The government chooses $\tau$ and $b$ to maximize the consumption growth rate in (9a) by taking into account the determination of $\theta$ from (9c). This is a static second-best policy problem.\textsuperscript{29} After some algebra (see Appendix B), the first-order conditions for $\tau$ and $b$ are respectively:

\textsuperscript{28} The objective of a well-meaning government could alternatively be to maximize households’ lifetime utility, as in the Ramsey optimal tax problem. We choose to solve for a growth maximizing policy because it is a simpler problem and, more importantly, because it allows us to make our results directly comparable to Barro’s well-known result; namely, that the socially optimal income tax rate equals the productivity of public services, $(1-\alpha)$. Also, recall that in Barro’s basic model, the tax rate that maximizes the growth rate is also the tax rate that maximizes the utility of the representative household. This equivalence does not hold when there are market imperfections in the form of weak property rights like in our paper.
\[(1 - \alpha - \tau)[2(1 - \alpha)(1 - \tau)(1 - \theta) + 1 - 2\theta] = \tau(1 - \alpha)(1 - \theta)[1 + 2(1 - \tau)] \] 
\[(1 - \alpha - \tau) b \nu (1 - b)(1 - \tau) = \tau(1 - \theta)[b - (1 - \theta)[1 - \tau(1 - b)]] \]

Equations (10a)-(10b), jointly with (9c), are a system in $\tau, b$ and $\theta$. If we solve for $\tau, b$ and $\theta$, then (9a) and (9b) give the balanced growth rate and the consumption-to-capital ratio. This is therefore a general equilibrium solution with endogenously chosen policy.

IV.2 Numerical solution of general equilibrium

The non-linear system (9c), (10a) and (10b) cannot be solved analytically. We therefore resort to numerical solutions by using the same parameter values as before in the case of a decentralized competitive equilibrium (see subsection III.6 above). That is, we again set $\alpha = 0.8, \rho = 0.04, \sigma = 2, A = 1$ and $\nu = 1$\textsuperscript{30}. Then, the system (9c), (10a) and (10b) gives $\tau = 0.12$ for the income tax rate, $b = 0.79$ for the share of tax revenues used to finance the protection of property rights vis-à-vis public infrastructure, and $\theta = 0.32$ for the share of effort allocated to work. In turn, this solution gives $(1 - E) = 0.63$ for the degree of property rights and $\gamma = 0.04$ for the balanced growth rate. Notice that this unique general equilibrium solution for $\theta, (1 - E)$ and $\gamma$ is between the two corresponding solutions at the level of a DCE (see subsection III.6 above). Thus, $0.22 < \theta = 0.32 < 0.91, 0.46 < (1 - E) = 0.63 < 0.98$ and $0.03 < \gamma = 0.04 < 0.15$. Actually, the unique general equilibrium solution for $\theta, (1 - E)$ and $\gamma$ is closer numerically to the bad, aggressive DCE ($\theta_1 = 0.22, (1 - E_1) = 0.46$ and $\gamma_1 = 0.03$).

Consider three key features of the solution. First, we got a unique general equilibrium. That is, the government has managed to resolve the expectations coordination problem arising at the level of a DCE. Therefore, in the presence of two market imperfections, and with two policy instruments at its disposal, a growth-maximizing government manages to coordinate private expectations and push the economy to a single general equilibrium. This is an equilibrium selection result. Observe $\tau_1 = 0.22$ and $E_1 = 0.46$.

\textsuperscript{30} Hence, we do not have time inconsistency problems. This is as in Barro (1990) and the related literature.

\textsuperscript{30} Numerical results are robust to the parameter values chosen. We will report economically admissible solutions only (for instance, tax rates between zero and one).
that here it is the government - which acts as a Stackelberg leader and is equipped with enough policy instruments - that plays the role of an expectations coordination device.31

Second, although the government internalizes everything as a Stackelberg leader, it cannot fully protect property rights (that would be the case in which $\theta = 1$ and $E = 0$ in general equilibrium). That is, although it manages to resolve the expectations coordination problem, the government finds it impossible to lead the economy to Barro’s second-best outcome with fully protected property rights.32 This is because the government does not have lump-sum policy instruments at its disposal. Notice that this result is similar to that in Acemoglu and Verdier (1998), who also find that it is not optimal to enforce all property rights. On the other hand, this is different from Grossman (2002), where the state can choose its tax policy so as to deter everybody from being a predator. Our results differ because here we have a second-best policy problem so that policy control is indirect. By contrast, in Grossman, taxes on exogenous endowments are essentially lump sum so that his policy problem can be thought as a first-best one.

Third, the chosen tax rate, $\tau = 0.12$, is lower than Barro’s, $\tau^{Barro} = 0.2$. This is because a higher tax rate further distorts individuals’ incentives (in the area of the aggressive DCE). As a result, the government finds it optimal to rely more heavily on the other policy instrument available, $b$, rather than on the tax rate, $\tau$. The latter is “more” distortionary than the former. Specifically, $b$ does not affect saving decisions (see (7a) above). By contrast, $\tau$ has both saving and substitution effects, where substitution effects are attempts to avoid taxes by substituting non-taxed for taxed activities - see (7b) above.33

31 Recall that there are at least three other ways of eliminating expectational indeterminacy of this form: First, by introducing costs of adjustment into agents’ decision-making (see e.g. Krugman, 1991). Second, by introducing learning mechanisms (see e.g. Evans et al., 1998). Third, by introducing a policy intervention, or a policy regime switch, at some future date (see e.g. Lockwood and Philippopoulos, 1994).

32 We work as follows: Barro’s optimal tax rate is $\tau^{Barro} = (1 - \alpha)$. Using the same parameter values as above, this gives $\tau^{Barro} = 0.2$ and $\gamma^{Barro} = 0.19$ in general equilibrium. Then, we try to choose the policy instruments, $\tau$ and $b$, so as to make the realized balanced growth rate in (9a) equal to 0.19. There are no economically admissible solutions for $0 < \tau < 1$ and $0 < b < 1$ that can achieve this.

33 Kneller et al. (1999) also distinguish distortionary from non-distortionary taxes on the basis of whether taxes affect saving/investment decisions and hence the rate of growth.
V. CONCLUSIONS AND EXTENSIONS

We incorporated weak property rights into a general equilibrium model of growth and second-best optimal policy. In this setup, the state played two important roles: it provided infrastructure and protected property rights. To the extent that we allowed the state to play its primary role (i.e. to act as an enforcer of property rights by allocating resources to policing, etc), we filled a gap in the public economics literature.

Since the main results have been written in the Introduction, we close with a possible extension. Here, as in most of the literature, we took the possibility of weak property rights as given. Namely, we assumed that extraction is possible, and then studied its implications for incentives, macroeconomic outcomes and optimal policy. An interesting extension would be to explain how the possibility of extraction arises in the first place. As far as we understand, this criticism applies to most common-pool models. That is, most of the literature assumes that there is a possibility of common access to a social resource, and this in turn opens the door to anti-social behavior (in the form of violating property rights, rent seeking, lobbying, stealing, etc). Different papers differ in what the common-pool resource is assumed to be (for a survey, see Drazen, 2000, chapter 10.7). However, this possibility is taken as given.
APPENDICES

Appendix A: Equation (9c)

Equation (9c) is written as:

\[ \theta^2 - \left[ \frac{1+2(1-\alpha)(1-\tau)}{1+(1-\alpha)(1-\tau)} \right] \theta + \frac{(1-\alpha)(1-\tau)(1+\nu\theta)}{1+(1-\alpha)(1-\tau)} = 0 \]  

(A.1)

Assuming that the discriminant is positive, there are two real and distinct roots, say \( \theta_1 \) and \( \theta_2 \), where

\[ \theta_1 + \theta_2 = \frac{1+2(1-\alpha)(1-\tau)}{1+(1-\alpha)(1-\tau)} > 0 \]  

(A.2a)

\[ \theta_1 \theta_2 = \frac{(1-\alpha)(1-\tau)(1+\nu\theta)}{1+(1-\alpha)(1-\tau)} > 0 \]  

(A.2b)

Hence, both roots are positive, \( \theta_1 > 0 \), \( \theta_2 > 0 \). It is also easy to show that \( \theta_1 + \theta_2 < 2 \) and \( (1-\theta_1)(1-\theta_2) > 0 \). Combining results, it follows \( 0 < \theta_1 < \theta_2 < 1 \).

Appendix B: Growth maximizing policy

The government solves:

\[
\max_{\tau,b} \frac{c}{c} = \sigma^{-1}[\alpha(1-\tau)B(\tau,b)-\rho]
\]

(B.1)

subject to the solution for \( \theta \) from equation (9c) and where \( B(\tau,b) \) is defined in the main text. The first-order conditions are simply:

\[
(1-\tau)B_\tau(\tau,b) = B(\tau,b) \]  

(B.2a)

\[ B_b(\tau,b) = 0 \]  

(B.2b)

Total differentiation of (9c) implies:

\[
\frac{\partial \theta}{\partial \tau} = -\frac{(1-\alpha)[(1-\theta)^2 + \nu\theta]}{2(1-\alpha)(1-\tau)(1-\theta) + 1-2\theta} \]

(B.3a)

\[
\frac{\partial \theta}{\partial b} = \frac{\nu(1-\alpha)(1-\tau)}{2(1-\alpha)(1-\tau)(1-\theta) + 1-2\theta} \]

(B.3b)

Using (B.3a) and (B.3b) into (B.2a) and (B.2b), we get (10a) and (10b) in the text. In turn, (9c), (10a) and (10b) constitute a three-equation system in \( \tau, b \) and \( \theta \).
REFERENCES


