Optimal Protection of Property Rights in a General Equilibrium Model of Growth

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Abstract
We incorporate weak property rights into an otherwise standard general equilibrium model of growth and second-best optimal policy. In this setup, the state plays two of its key roles: it protects property rights and provides public services. The government chooses policy (the income tax rate, as well as the allocation of collected tax revenues between law enforcement and public services) to maximize the growth rate of the economy. The focus of our analysis is on how weak property rights generate multiple decentralized competitive equilibria, the different properties of these equilibria, and the implications of second-best optimal policies.

Keywords: Property rights; growth; second-best policy

JEL classification: D7; D9; H3

I. Introduction
It is widely recognized that the incentives to work, produce, study, invest, etc. depend on whether property rights are certified and protected. The lower the security of property rights, the smaller these incentives. This, in turn, is bad for economic growth.¹

It is also recognized that the degree of security of property rights is endogenous. Although there can be several intuitive determinants of this

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See e.g. Knack and Keefer (1995), Mauro (1995), Hall and Jones (1999) and Rodrik (1999). For a survey of property rights and growth, see e.g. Drazen (2000, Ch. 11, Secs. 4–6).
degree, it is the government that ultimately certifies and protects property rights. Activities such as police services, courts, prisons, the design of patent rights, the stability of laws and institutions, etc. (i.e., “the law”) are typically the responsibility of the state. But to provide such services, the state needs tax revenues. Thus, as is the case with all public services, there is a tradeoff: weak property rights distort incentives and lead to resource misallocation, while the funding of government policies to protect property rights requires tax revenues which are unavoidably raised by distorting tax instruments; see Barro and Sala-i-Martin (1995, pp. 159–161). Then, it is natural to ask questions like “what is the best economic policy?” and “what are the macroeconomic implications of such a policy?”.

We incorporate ill-defined property rights into an otherwise standard general equilibrium model of growth and second-best optimal policy; see Barro (1990). Our model has three distinct features. First, individuals can expropriate each other’s output in an attempt to increase their own personal wealth. That is, in addition to standard decisions on consumption, saving, etc., individuals also choose the optimal allocation of their time between productive work and illegal extractive activities. Second, the government uses collected income tax revenues to finance the provision of public production services (such as maintenance of roads and airports) and law enforcement (such as police, courts and prisons). The former provides infrastructure externalities to firms and is the engine of long-term growth. The latter captures the role of the state as the key protector of property rights. Then, in line with the endogenous growth literature, we solve a normative exercise: the government chooses the income tax rate, as well as the allocation of tax revenues between infrastructure and law enforcement, to maximize the growth rate of the economy. In so doing, it tries to correct the inefficiencies caused by decentralized private behavior. A third feature of the model is that there are external effects from social interactions, in the sense that the average level of illegal activities in the economy affects the behavior of each individual agent. The way this social externality is

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2 These determinants include the technologies of production and conflict, the number of contenders and the kind of asymmetries, see e.g. Hirshleifer (1991, 1995) and Skaperdas (1992); the private costs of property rights protection, see e.g. de Meza and Gould (1992) and Grossman and Kim (1996); the probability of maintaining ownership in one’s output as an increasing function of public expenditure on the protection of property rights, see e.g. Barro and Sala-i-Martin (1995, pp. 159–161); the size of wealth to be appropriated and the degree of inequality, see e.g. Benhabib and Rustichini (1996); the elasticity of intertemporal substitution, see e.g. Lane and Tornell (1996) in a model with redistributive struggle; the efficiency wages paid to corruptible bureaucrats, see e.g. Acemoglu and Verdier (1998) in a model where bureaucrats’ corruption arises due to asymmetric information; the effectiveness of predation technology versus the state’s ability to deter predators, see e.g. Grossman (2002); etc.

3 The assumption that aggregate behavior exerts an external effect on individual behavior is as in the expanding literature on social non-market interactions; see e.g. Cooper (1999),
modeled can lead to strategic complementarities, or strategic substitutabilities, in violating property rights. The former has been particularly widespread in capturing the idea that when the others break the law, it pays to do the same.

The main results are as follows. We first solve for a decentralized competitive equilibrium (DCE), which holds for any feasible policy. Strategic complementarities in violating property rights can generate multiple, Pareto-ranked DCE. Specifically, our results show that when strategic complementarities generate increasing returns to scale in the expropriation technology at the economy level, there are two self-fulfilling perfect-foresight equilibria associated with different welfare implications and policy recipes. This is a situation with coordination failure; cf. Tirole (1996) in a model of corruption and Murphy, Shleifer and Vishny (1993), Mauro (2004) and Park, Philippopoulos and Vassilatos (2005) in models with rent seeking. Multiplicity suggests that economies with the same fundamentals can end up with very different outcomes. They can end up in a relatively bad equilibrium characterized by low work effort, ill-defined property rights and low growth; but it is equally possible to end up in a relatively good equilibrium characterized by higher work effort, stronger property rights and higher growth.

The two DCE also have different comparative-static properties. Our results suggest that, when an economy is trapped in a bad equilibrium with ill-behaved economic agents and low economic activity, it is better for incentives and growth to reduce tax rates and give priority to law enforcement over public-good provision. By contrast, when an economy enjoys relatively well-behaved economic agents and high economic activity, it is better to increase tax rates and use the tax revenues to finance public-good provision.

We then endogenize policy. When a growth-maximizing government chooses the income tax rate, as well as the allocation of tax revenue between law enforcement and infrastructure, there are three results. First, the government manages to resolve the expectations coordination problem which arises at the level of DCE. Thus, in our setup, the government can serve as a selection equilibrium device. Second, although it manages to resolve the expectations coordination problem, the government’s selection options are limited; for example, it cannot lead the economy to an equilibrium

Glaeser and Scheinkman (2003) and Eaton (2004). Tirole’s (1996, p. 18) motivation is that “our individual incentives depend also on those of the groups we are associated with”.

4 This is as in the growth literature where external increasing returns to scale are important devices for multiplicity. Related macro papers that study the link between complementarities, returns to scale and multiplicity are discussed in Section III below. At this early stage, we simply point out that our results are consistent with the game-theoretic results in Cooper and John (1988) where strategic complementarities are a necessary condition for multiple decentralized equilibria.
with fully protected property rights. Third, as the returns to scale in illegal activities increase, the government finds it optimal to tax less and use the collected tax revenue to finance law enforcement rather than infrastructure. Among other things, this implies that, with weak property rights, the optimal tax rate is lower than Barro’s (1990) reference rate.

What is the contribution of our paper? The public economics literature has always emphasized that the primary role of the state is to protect property rights; see e.g. Atkinson and Stiglitz (1980, p. 5). So far, however, this key role of the state has not been analyzed formally in a general equilibrium model with second-best optimal policy. Our paper fills this gap and the framework used here may facilitate the study of various issues related to weak property rights. A model close to ours may be found in Barro and Sala-i-Martin (1995, pp. 159–161). However, they do not study the microfoundations of property rights and, in their model, the government does not provide infrastructure services or choose the allocation of tax revenues between different functions. The other papers mentioned earlier in footnote 2 focus on different mechanisms for the protection of property rights or use a static approach. Moreover, we show the importance of social interactions in illegal activities and their role in macroeconomic multiplicity.

The rest of the paper is organized as follows. The model is described in Section II. We then solve for a decentralized competitive equilibrium for given policy in Section III. The optimum policy problem is considered in Section IV. Section V concludes.

II. Informal Description of the Model

We now introduce the main features of the model.

First, we build on Barro’s (1990) model of long-term growth and optimal policy. We incorporate weak property rights in this model by assuming that individuals can extract from other individuals’ output to increase their own personal wealth. Here we assume that households can extract from firms’ output.\(^5\) Specifically, if for simplicity there is a single firm and \(i = 1, 2, \ldots, I\) households, the firm can appropriate only a fraction \(0 < p \leq 1\) of its output produced, \(y\), because the rest, \((1 - p)y\), can be taken away by households. Thus, \(0 < p \leq 1\) is the degree of property rights.

Second, expropriation comes at a private cost. If household \(i\) has one unit of time available at each point in time, he/she can allocate a fraction \(0 < \theta^i \leq 1\) of this unit to productive work and the rest \(0 \leq (1 - \theta^i) < 1\) to

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\(^5\) There are many ways of modeling expropriation. For instance, firms may attempt to expropriate each other’s capital or output, or households may attempt to expropriate each other’s assets, or a combination of both. The specific way of modeling “who takes away from whom” is not important because households are also firm owners in this class of general equilibrium models.

extraction. This is as in e.g. Baumol (1990), Murphy, Shleifer and Vishny (1991), Hirshleifer (1995) and Grossman and Kim (1996), in the sense that individuals decide how to allocate their private resources between productive and unproductive activities. The value of $\theta^i$ is chosen optimally, jointly with consumption and saving.

Third, the fraction of firms’ output extracted by each household increases with the time that this household allocates to extractive activities, while it decreases with the fraction of government revenue that goes to the protection of property rights. We also allow the fraction of firms’ output extracted by each household to be affected by the economy-wide average time allocated to extractive activities by all households. The way this social interaction is modeled leads to strategic complementarities, or strategic substitutabilities, in illegal activities. Strategic complementarity (substitutability) means that the optimal level of activity chosen by an individual increases (decreases) with the level of activity chosen by society, thus inducing more (less) illegal action on his part as well.

Fourth, the government imposes income taxes and uses the collected tax revenue to finance the protection of property rights and the provision of infrastructure services. Following most of the endogenous growth literature, as in e.g. Barro (1990), we assume that the government’s objective is to maximize the growth rate of the economy. The sequence of events is as follows: policy is chosen first, after which private agents make their decisions acting simultaneously and competitively.

We now formalize this story.

### III. Decentralized Competitive Equilibrium

Here, we solve for a decentralized competitive equilibrium for any policy. Policy is endogenized in the next section. It is convenient to start with expropriation technology.

**Expropriation Technology**

To obtain a tractable model of expropriation, we follow Grossman (2002). Individual $i$’s effective illegal activity, denoted as $s^i$, is:

$$s^i = \frac{(1 - \theta^i)^\lambda(1 - \theta)^\mu}{b^\phi},$$

where $0 \leq (1 - \theta^i) < 1$ is the time spent by $i$ in violating property rights,

$$\sum_{i=1}^I (1 - \theta^i) I \equiv (1 - \theta)$$

is the average time spent in violating property rights in the economy, $b$ is the fraction of tax revenue earmarked by the government for law enforcement.
(where \(b\) is specified in equation (8b) below), and \(\lambda, \mu\) and \(\phi\) are parameters. Specifically, \(0 < \lambda < 1\) measures the effectiveness of \(i\)'s own extractive efforts, \(\mu\) is the effect of social interactions and \(0 < \phi < 1\) is a measure of policy effectiveness in protecting property rights (see below for details and range of parameters).

The illegal activity, \(s^i\), is translated into extraction, \(e^i\), by:

\[
e^i = \frac{s^i}{1 + s^i}.
\]

(1b)

Thus, if \(i\) decides to allocate \(0 \leq (1 - \theta^i) < 1\) to illegal activities, a fraction \(0 \leq e^i < 1\) can be taken away, and only the rest, \(0 < (1 - e^i) \leq 1\), is appropriated by the producer. Functions like (1b) are widely used in the literature. 7

When \(\theta^i = 1\), \(e^i = 0\) and \(p = 1\).

**Firms’ Behavior**

For notational simplicity, there is a single firm. This firm maximizes profits, \(\pi\), given by:

\[
\pi = pY - rK - wL,
\]

(2)

where \(0 < p \leq 1\) is the degree of property rights; \(Y\), \(K\) and \(L\) denote the firm’s output, capital input and labor input; and \(r\) and \(w\) are the interest rate and wage rate. Uppercase letters denote aggregate quantities and lowercase per capita quantities.

At the firm level, the production function takes a Cobb–Douglas form:

\[
Y = AK^\alpha L^{1-\alpha} g^{1-\alpha},
\]

(3)

where \(A > 0\) and \(0 < \alpha < 1\) are parameters and \(g\) is per capita public productive services; see e.g. Barro and Sala-i-Martin (1995, Ch. 4).

The firm acts competitively. The first-order conditions for \(K\) and \(L\) are:

\[
r = \alpha p \frac{Y}{K}
\]

(4a)

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6 We assume that it is the fraction, rather than the level, of public expenditures that matters. This is because the model is of the AK type at the economy-wide level (see equations (9a)–(9c) below) so that only ratios and rates can be specified in equilibrium. As argued above, we use this setup for tractability. Barro and Sala-i-Martin (1995, p. 160) and Grossman (2002) also use “fractions” in similar functions.

7 As Grossman (2002, p. 36) argues, “it is a generic black box that conceals the process of predation, just as the standard generic production function conceals the process of production”. Actually, our equations (1a)–(1b) are a generalization of equation (2) in Grossman (2002); specifically, we get Grossman’s extraction technology if \(\mu = 0\) (note, however, that Grossman models appropriation, not expropriation). Hirshleifer (1995) also uses ad hoc functional relations to model the “technology of conflict”.

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\[ w = (1 - \alpha)p \frac{Y}{L}, \]  

(4b)

so that, with constant returns to scale at the firm level, profits are zero.

**Households’ Behavior**

Each household \( i \) maximizes intertemporal utility:

\[ \int_0^\infty \left( \frac{(c^i)^{1-\sigma}}{1-\sigma} \right) e^{-\rho t} dt, \]

(5)

where \( c^i \) is \( i \)'s consumption, \( \sigma > 0 \) (with \( \sigma \neq 1 \)) is a degree of intertemporal substitution and \( \rho > 0 \) is a discount factor.

Each household \( i \) consumes \( c^i \) and saves \( a^i \) in the form of an asset. It is also endowed with one unit of effort time at each point in time, and allocates a fraction \( 0 < \theta^i \leq 1 \) of this unit to productive work, while the rest, \( 0 \leq (1 - \theta^i) < 1 \), goes to extractive activities. Thus, \( i \)'s budget constraint is:

\[ \dot{a}^i + c^i = (1 - \tau)(r a^i + w \theta^i) + e^i \frac{Y}{I}, \]

(6)

where \( e^i \) follows from (1a) and (1b) above,\(^8\) a dot over a variable denotes a time derivative, and \( 0 < \tau < 1 \) is an income tax rate common to all households. Note that there are taxes on legal activities only and that we assume full capital depreciation.

Each household \( i \) acts competitively. The first-order conditions for consumption, saving and extraction \( (c^i, a^i, \theta^i) \) give:

\[ \dot{c}^i = c^i \left( \frac{(1 - \tau)r - \rho}{\sigma} \right) \]

(7a)

\[ (1 - \tau)w = \frac{b^\phi \lambda (1 - \theta^i) \lambda^{-1} (1 - \theta^i) \mu}{[b^\phi + (1 - \theta^i) \lambda (1 - \theta^i) \mu]^2} \frac{Y}{I}, \]

(7b)

where (7a) is the familiar Euler equation for the isoelastic case, and (7b) implies that net returns from work and expropriation are equal in equilibrium.

Two details should be noted. First, it is straightforward to show that the second-order conditions of the above maximization problem also require \( 0 < \lambda \leq 1 \) in equation (1a).\(^9\) Second, total differentiation of (7b) implies

\[ \frac{Y}{I}. \]

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\(^8\) We assume that each \( i = 1, 2, \ldots, I \) has access to an equal share of the single firm’s output, \( Y/I \).  

\(^9\) The condition \( 0 < \lambda \leq 1 \) is required for concavity of the (atemporal part in the) transition equation (6), where \( e^i \) is as in (1a) and (1b), with respect to \( \theta^i \). Recall that if the objective function (5) and the transition equation in (6) are both jointly concave in the controls, and
that the sign of the partial $\partial(1-\theta^i)/\partial(1-\theta)$, i.e., the effect of the average illegal action taken by all agents on the optimal illegal action of an individual agent, depends on the signs of the parameter $\mu$ and the expression $[b^\phi - (1-\theta^i)^\lambda(1-\theta)^\mu ]$. If this partial is positive (negative), $(1-\theta^i)$ and $(1-\theta)$ are strategic complements (substitutes), in the sense that an increase (decrease) in the illegal action taken by all agents increases (decreases) the productivity of an agent’s own illegal action, thus inducing more (less) action on his part as well. In what follows, we work in the region where $[b^\phi - (1-\theta^i)^\lambda(1-\theta)^\mu ] > 0,$ so that the sign of the partial is the sign of $\mu$. In other words, if $\mu > 0$, $(1-\theta^i)$ and $(1-\theta)$ are strategic complements; if $\mu < 0$, $(1-\theta^i)$ and $(1-\theta)$ are strategic substitutes; if $\mu = 0$, there are no social interactions (this is the case in Grossman, 2002).

**Government Budget Constraint and the Role of Economic Policy**

The government uses its tax revenue, $\tau(r \sum_i a^i + w \sum_i \theta^i)$, to finance two activities: it spends $D$ on the protection of property rights and $G$ on the provision of infrastructure. The budget constraint is:

$$D + G = \tau \left( r \sum_i a^i + w \sum_i \theta^i \right).$$  \hspace{1cm} (8a)

Without loss of generality, we assume that a fraction $0 < b < 1$ of total tax revenues finances the provision of $D$ and the rest, $0 < (1 - b) < 1$, finances the provision of $G$. Thus, we decompose (8a) into:

$$D = b\tau \left( r \sum_i a^i + w \sum_i \theta^i \right)$$  \hspace{1cm} (8b)

$$G = (1 - b)\tau \left( r \sum_i a^i + w \sum_i \theta^i \right),$$  \hspace{1cm} (8c)

where inspection of (8a)–(8c) reveals that the independent policy instruments at each point in time are $\tau$ and $b$. Recall that it is the share of tax with a non-negative multiplier associated with (6), the necessary conditions are also sufficient for optimality.

\[10\] We confirm that this condition (that contains endogenous variables) is satisfied in equilibrium. It requires the enforcement of law to be effectively stronger than illegal activities. Similarly, in Hirshleifer (1995), the economy breaks down when conflict is sufficiently strong. If this condition is not satisfied, we get counterintuitive properties. This is not surprising since if it is violated, $s^i$ in (1a) is higher than unity (equation (1a) just translates $0 \leq (1-\theta^i) < 1$ into $s^i$, so that the latter should also be between zero and unity). Grossman (2002) also uses functional forms which guarantee that similar variables are between zero and unity.
revenue, $b$, that matters for the protection of property rights in equation (1a) rather than the associated level, $D$.

**Decentralized Competitive Equilibrium**

We now solve for a decentralized competitive equilibrium (DCE). This holds for any feasible economic policy. In a DCE: (i) Each firm and household maximize their own profit and utility respectively by taking prices, policy variables and aggregate outcomes as given. (ii) All markets clear; this means $L = \sum_i \theta^i$ in the labor market and $K = \sum_i a^i$ in the capital market. (iii) Individual decisions are consistent with aggregate decisions. This implies $(1 - p)Y = \sum_i e^i(Y/I)$. Namely, the amount seized from the firm(s) equals the amount seized by households. (iv) All constraints, including the resource constraint of the economy, are satisfied. For simplicity, we focus on a symmetric DCE, i.e., individuals are alike *ex post*. Thus, from now on, the superscript $i$ can be omitted and the variables $(c, k, \theta)$ in (9a)–(9c) below will denote per household values.

Combining (1)–(8), it is straightforward to show that a DCE is given by:

\[
\dot{c} = c \left( \frac{(1 - \tau)\alpha B(\tau, b) - \rho}{\sigma} \right) \quad (9a)
\]

\[
\dot{k} = \left( 1 - \tau + \frac{(1 - \theta)^{\lambda+\mu}}{b^\phi} \right) B(\tau, b)k - c \quad (9b)
\]

\[
(1 - \alpha)(1 - \tau)[(1 - \theta)^{\lambda+\mu} + b^\phi] = \theta \lambda (1 - \theta)^{\lambda+\mu-1}, \quad (9c)
\]

where

\[
B(\tau, b) \equiv A^{1/\alpha} [\tau \theta (1 - b)]^{(1-\alpha)/\alpha} \left( \frac{b^\phi}{b^\phi + (1 - \theta)^{\lambda+\mu}} \right)^{1/\alpha} > 0.
\]

Equation (9a) follows from the Euler equation in (7a) and the solutions for $y$ and $p$. Equation (9b) follows from combining (2), (6) and (8). That is, (9b) is the economy’s resource constraint, $\dot{k} + c + d + g = y$, where

\[
y = \left( \frac{b^\phi + (1 - \theta)^{\lambda+\mu}}{b^\phi} \right) B(\tau, \theta)k.
\]

Equation (9c) follows from (4b) and (7b). Note that consistency of individual and aggregate decisions implies that the equilibrium fraction stolen is

\[
(1 - p) = e = \frac{(1 - \theta)^{\lambda+\mu}}{b^\phi + (1 - \theta)^{\lambda+\mu}},
\]

which is well defined with $0 \leq (1 - p) < 1$. Note also that the economy-wide output is linear in capital. Thus, the model is a variant of the AK

model. Actually, if $\theta = 1$ and also $b = 0$, we obtain Barro's (1990) model as a benchmark.

Equations (9a)–(9c) give the paths of $(c, k, \theta)$ for any policy, where policy is summarized by the tax rate, $0 < \tau < 1$, and the allocation of tax revenues between law enforcement and infrastructure, $0 < b < 1$. An advantage of the model is its simplicity. Equation (9c) is an atemporal non-linear equation in $\theta$ only. Once we solve (9c) for $\theta$, (9a) can give the so-called balanced growth path along which consumption and capital grow at the same constant rate, defined as $\dot{c}/c = \dot{k}/k \equiv \gamma$; in turn, (9b) can give the consumption-to-capital ratio, $c/k$. Thus, at the DCE level, the key part of the solution is the value of $\theta$ in (9c). In the Appendix we show that when a solution exists:

**Proposition 1.** Equations (9a)–(9c) imply: (i) if $\lambda + \mu > 1$, weak property rights lead to multiple (two) Pareto-ranked DCE; (ii) if $0 \leq \lambda + \mu \leq 1$, there is a unique DCE.

**Proof:** See the Appendix.

The assumption that $\lambda + \mu > 1$ implies that social returns to scale in the expropriation technology (1a) are increasing.\(^{11}\) Note that, with $0 < \lambda \leq 1$ from the second-order conditions of the households’ problem, the condition $\lambda + \mu > 1$ requires $\mu > 0$; recall that $\mu > 0$ is a condition for strategic complementarities in the expropriation technology. Note also that even if $\mu > 0$, it is possible to have $\lambda + \mu \leq 1$ when the value of $\lambda$ is small enough. Of course, if $\mu < 0$ (which is a condition for strategic substitutabilities), we always have $\lambda + \mu \leq 1$. Thus, strategic substitutability rules out multiplicity. After combining results, we have:

**Corollary 1.** When strategic complementarities ($\mu > 0$) generate social increasing returns to scale in expropriation ($\lambda + \mu > 1$), there are multiple (two) Pareto-ranked DCE. Thus, strategic complementarities are a necessary condition for multiplicity.

\(^{11}\) Let us rewrite (1a) in a symmetric equilibrium as

$$s((1 - \theta), b) = \frac{(1 - \theta)^{\lambda+\mu}}{b^\phi}.$$  

Then, for any scalar $\varepsilon$, we have $s(\varepsilon(1 - \theta), \varepsilon b) = \varepsilon^{\lambda+\mu-\phi}s$. Thus, with $0 \leq \phi \leq 1$, $\lambda + \mu > 1$ is a necessary condition for increasing returns to scale at the economy level. If we treat the policy instrument, $b$, as a parameter (which is the case at the level of a DCE), $s(\varepsilon(1 - \theta), b) = \varepsilon^{\lambda+\mu}s$, so that $\lambda + \mu > 1$ is also sufficient for increasing returns to scale. Note that—as in the growth literature—the economy can exhibit social increasing returns even if returns at the level of the individual are not increasing ($0 < \lambda \leq 1$).
Large enough external effects from social interactions—in the form of strategic complementarities—can thus generate social increasing returns in breaking the law, and this leads to multiple equilibria. It should be kept in mind that a technology of increasing returns in (1a) does not necessarily mean that self-interested agents eventually seize more and are better off. The contestable pie, \( Y \), is endogenous and is damaged by extractive behavior.

Our result is consistent with the game-theoretic results in Cooper and John (1988) and Cooper (1999) that strategic complementarities are a necessary condition for multiple symmetric Nash equilibria. It is also consistent with the macroeconomic literature, where the key to indeterminacy (at least in one-sector growth models) is increasing returns to scale, usually arising from externalities in production or from monopolistic competition; see Benhabib and Farmer (1999). Furthermore, it is consistent with the results of d’Aspremont, Dos Santos Ferreira and Gerard-Varet (1995) and Cooper (1999, Ch. 3) who also obtain multiplicity given enough strategic complementarity to generate increasing returns to scale.

What then are the implications of multiplicity? When \( \lambda + \mu > 1 \), weak property rights can lead to multiple (two) solutions for the fraction of effort that individuals allocate to work relative to extraction (\( \theta \)) and, in turn, two solutions for the associated DCE. Without loss of generality, let us denote the two solutions for \( \theta \) as \( 0 < \theta_1 < \theta_2 \leq 1 \). Inspection of (9a) and (9b) reveals that equilibria with high work effort (\( 0 < \theta_2 \leq 1 \)) are associated with a higher balanced growth rate and a higher consumption-to-capital ratio than equilibria with low work effort (\( 0 < \theta_1 < 1 \)). Hence, we call the former “good” and the latter “bad” DCE.

What is the intuition behind multiplicity? Note first that, given the possibility of weak property rights, self-interested individuals may find it optimal to devote some of their private resources to extractive activities. But, in addition, there might be an expectations coordination problem depending on the aggregate behavior of society. If the social externality from strategic complementarities is large enough to generate increasing returns to scale in extractive efforts at the social level, there is expectational indeterminacy. We can end up in a relatively bad equilibrium with high extractive activity.

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12 What is the idea behind social increasing returns in (1a)? As the intensity of illegal actions increases (agents allocate their energies to illegal practices and not to productive activities), extraction can become easier because illegal acts that might have appeared shocking earlier will begin to look less shocking and may even begin to be tolerated. Extraction can also reduce or distort the ability of the government to play its fundamental role in enforcing contracts and protecting property rights. It also reduces tax bases and the ability of the government to carry out necessary public expenditures on education. See e.g. Tanzi (2002) for an intuitive discussion in the similar case of corruption and Murphy et al. (1993) in the case of rent seeking.
and low work effort; but it is equally possible to end up in a relatively good equilibrium with lower extractive activity and higher work effort. Multiplicity can arise when the rate of return to an action (or an asset) and its quantity co-move. This is possible under individual rationality only when there are external forces that act to reinforce the decisions of individuals. In our setup, there might be an expectations coordination problem because if all agents were to simultaneously increase their illegal actions, the related rate of return would tend to increase, thereby justifying the higher rate of return on these illegal actions; see e.g. Benhabib and Farmer (1999) in growth models.

It is also interesting to study the effects of policy instruments, i.e., the tax rate, $0 < \tau < 1$, and the share of tax revenues earmarked for the protection of property rights, $0 < b < 1$ (recall that policy instruments are given at the level of DCE). We focus on case (i) of Proposition 1, which is the relatively rich case with two DCE. In the Appendix it is shown that $\frac{d\theta}{d\tau} < 0$ and $\frac{d\theta}{db} > 0$ around the low work effort DCE (associated with $\theta_1$), while (under one relatively mild condition that is satisfied in the numerical solutions reported below) $\frac{d\theta}{d\tau} > 0$ and $\frac{d\theta}{db} < 0$ around the high work effort DCE (associated with $\theta_2$).

Thus, the two DCE appear to have opposite comparative-static properties. If it happens to be in the low-$\theta$ DCE, a higher $\tau$, or a lower $b$, lead to lower work effort. By contrast, if it happens to be in the high-$\theta$ DCE, a higher $\tau$, or a lower $b$, lead to higher work effort. The intuition is as follows. In general, there are two effects from an increase in the tax rate. First, a higher tax rate pushes individuals away from legal to illegal activities since the latter are untaxed. Second, an increase in the tax rate leads to higher tax revenues. This allows the government to finance higher infrastructure services and law enforcement, both of which improve the return to productive work and give the right incentives. The combination of these two opposite effects will determine the net effect. If we are in the bad equilibrium, in which work effort is low and hence economic activity and tax revenues are initially low, the first effect dominates so that an increase in the tax rate further distorts individuals’ incentives. By contrast, if we are in the good equilibrium, in which work effort is high and hence economic activity and tax revenues are initially high, the second effect dominates so that an increase in the tax rate further improves individuals’ incentives. The effects of $b$ are symmetrically opposite. If we are in the bad equilibrium, in which the tax revenue effect is relatively small, a higher $b$ improves incentives. By contrast, if we are in the good equilibrium, in which the tax revenue effect is relatively large, a higher $b$ reduces the tax base and

---

13 In case (ii) of Proposition 1, i.e., when $0 \leq \lambda + \mu \leq 1$, the unique solution has the same properties as the low work effort equilibrium of case (i). See also below.
hence the government’s ability to provide infrastructure and law enforcement, which reduces the return to legal work and thus has an adverse effect on incentives. Therefore, the policy lessons pointed out in the introduction follow.

**Numerical Solution and Comparative Statics of DCE**

We also solve (9a)–(9c) numerically. We choose the following parameter values: $\alpha = 0.8$, where $\alpha$ is the productivity of private capital vis-à-vis infrastructure in the firm’s production function (3); $\rho = 0.04$, where $\rho$ is the time discount factor in (5); $\sigma = 2$, where $\sigma$ is the degree of intertemporal substitution in (5); $A = 1$, where $A$ is aggregate productivity in (3); and $\phi = 0.2$, where $\phi$ is a measure of policy effectiveness in law enforcement in (1a). Concerning the policy instruments, we set $\tau = 0.2$ for the income tax rate (which would be its optimal value in a Barro-type model) and $b = 0.5$ which is a rather neutral value for the share of tax revenues allocated to the protection of property rights vis-à-vis infrastructure. Most of these parameter values are commonly used (results are robust to the parameter values chosen). Concerning the key parameters, $\lambda$ and $\mu$, and following the theoretical results in the Appendix, we consider various combinations of $\lambda$ and $\mu$. This enables us to study the cases described in Proposition 1 and Corollary 1 as well as shown analytically in the Appendix. The cases are when (a) strategic complementarities ($\mu > 0$) generate increasing returns to scale in illegal activities ($\lambda + \mu > 1$); (b) there are strategic complementarities but no increasing returns to scale ($\lambda + \mu \leq 1$); (c) individual and social actions are strategic substitutes ($\mu < 0$).

Solutions are reported in Table 1. Numerical results confirm the analytical results of Proposition 1 and Corollary 1. When $\lambda + \mu > 1$, two solutions for $\theta$ emerge. We also report the solutions for the degree of property rights, $p$, and the balanced growth rate, $\gamma$. As long as there is a unique $\theta$, higher social externalities, $\mu$, lead monotonically to higher $p$ and higher $\gamma$. In the region with two DCE, the DCE associated with the low $\theta$ behaves similarly, while the DCE associated with the high $\theta$ behaves oppositely.

Tables 2 and 3 report the effects of the two policy instruments $0 < \tau < 1$ and $0 < b < 1$. We focus on the relatively rich case with two DCE (specifically, we work with $\lambda = \mu = 1$). The results confirm the arguments in the

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14 The parameter $\alpha$ is the weight of private capital relative to public infrastructure services in private production. Following most of the literature, we choose $\alpha = 0.8$. As Barro and Sala-i-Martin (1995, p. 79) point out, “high values of $\alpha$ are reasonable if we take a broad view of capital to include the human components”. Our qualitative results do not change if we set lower values as in the real business cycle (RBC) literature. The properties of DCE have been shown analytically, so they hold for any $0 < \alpha < 1$.

15 In the last row of Table 1, we also report the popular quadratic case, $\lambda = \mu = 1$. © The editors of the Scandinavian Journal of Economics 2007.
### Table 1. Decentralized competitive equilibrium (DCE)

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\mu$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
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<th>$p_2$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
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<tbody>
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<td>0.966</td>
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</table>

Notes: $A=1$, $\alpha=0.8$, $\rho=0.04$, $\sigma=2$ and $\phi=0.2$. Also $\tau=0.2$ and $b=0.5$.

### Table 2. Effects of $\tau$ on DCE

<table>
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<tr>
<th>$\tau$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$\gamma_1$</th>
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<td>0.144</td>
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<td>0.546</td>
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Notes: $A=1$, $\alpha=0.8$, $\rho=0.04$, $\sigma=2$, $\phi=0.2$, $\lambda=1$ and $\mu=1$. Also $b=0.5$.

### Table 3. Effects of $b$ on DCE

<table>
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<tr>
<th>$b$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$p_1$</th>
<th>$p_2$</th>
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<td>0.047</td>
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<td>0.958</td>
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Notes: $A=1$, $\alpha=0.8$, $\rho=0.04$, $\sigma=2$, $\phi=0.2$, $\lambda=1$ and $\mu=1$. Also $\tau=0.2$.
preceding subsection that \(d\theta/d\tau < 0\) and \(d\theta/db > 0\) in the low-\(\theta\) DCE, and \(d\theta/d\tau > 0\) and \(d\theta/db < 0\) in the high-\(\theta\) DCE. Note also that the effect of \(\tau\) on \(\gamma\) is monotonically decreasing in the low-\(\theta\) DCE, while it follows a Laffer-curve pattern in the high-\(\theta\) DCE. The effects of \(b\) are symmetrically opposite to those of \(\tau\).

IV. Second-best Optimal Policy

We now endogenize policy, as summarized by the time paths of the two policy instruments, \(0 < \tau < 1\) and \(0 < b < 1\). We assume that the optimum problem for the government is to choose its policy instruments to maximize the balanced growth rate of the economy, \(\gamma\).\(^{16}\) In so doing, the government will try to correct the market imperfections (arising from externalities and weak property rights) and raise tax revenue efficiently.

**Optimal Policy**

The government chooses \(\tau\) and \(b\) to maximize the growth rate in (9a) by taking into account (9c) for \(\theta\). The first-order conditions for \(\tau\) and \(b\) are (see the Appendix):

\[
(1 - \alpha - \tau)[(\lambda + \mu)(1 - \alpha)(1 - \tau)(1 - \theta)^{\lambda + \mu - 1} + \lambda(1 - \theta)^{\lambda + \mu - 1} - (\lambda + \mu - 1)\lambda \theta(1 - \theta)^{\lambda + \mu - 2}] = \tau(1 - \alpha)(1 - \theta)^{\lambda + \mu - 1}[\lambda + (\lambda + \mu)(1 - \tau)]
\]

(10a)

\[
(1 - \alpha - \tau)\phi b^\phi(1 - b)(1 - \tau) = \tau(1 - \theta)^{\lambda + \mu - 1}[\theta \lambda b - \phi(1 - \tau)(1 - \theta)(1 - b)].
\]

Equations (10a) and (10b), jointly with (9c), are a system in \(\tau\), \(b\) and \(\theta\). If we solve for \(\tau\), \(b\) and \(\theta\), then (9a) and (9b) can give the balanced growth rate and the consumption-to-capital ratio. This is a growth-maximizing allocation (GMA).

\(^{16}\)The objective of a well-meaning government could alternatively be to maximize households’ lifetime utility, as in the Ramsey problem. In this paper, we choose to solve for a growth-maximizing policy mainly because it allows us to make our results directly comparable to Barro’s well-known result that the income tax rate should be equal to the productivity of public services, \((1 - \alpha)\). The key features reported below do not change qualitatively when we solve for a Ramsey allocation, where the government chooses the paths of \(\tau\) and \(b\) to maximize (5) subject to (9a)–(9c)—results are available on request. It is worth pointing out that, in Barro’s basic model, the tax rate that maximizes the growth rate is also the tax rate that maximizes household’s utility. This equivalence may not hold when there are imperfections in the form of weak property rights, as in our paper.
Numerical Solution and Comparative Statics of Growth-maximizing Allocation

The non-linear system (9c), (10a) and (10b) cannot be solved analytically. We therefore resort to numerical solutions by using the same parameter values as in the case of a DCE. That is, we set $\alpha = 0.8$, $\rho = 0.04$, $\sigma = 2$, $A = 1$, $\phi = 0.2$. We study various combinations of $\lambda$ and $\mu$ so as to capture the cases described in Proposition 1 and Corollary 1. Numerical solutions are reported in Table 4. We focus again on the case $\lambda + \mu > 1$, i.e., the relatively rich case with two solutions at the level of DCE.

The first result is that we obtain a unique GMA. In other words, by choosing its tax-spending policy instruments ($\tau$ and $b$) optimally, a growth-maximizing government manages to resolve the expectations coordination problem arising at the level of DCE. This happens because, when the government steps in, it acts as a Stackelberg leader that knows there are two DCE and is equipped with enough policy instruments. It can thus play the role of an equilibrium selection device. Note that the number of policy instruments available can be important here. For instance, we report that if the government chooses optimally only the value of $\tau$ (while the value of $b$ is set exogenously, say, at its DCE numerical value in Table 1), we obtain two GMAs. In other words, if the government has only a relatively distorting policy instrument (like the tax rate, $\tau$)\(^{17}\) at its disposal, it cannot cope with the coordination problem. Therefore, as Atkinson and Stiglitz (1980, p. 14) point out, in second-best policy problems, the nature of the solution may be

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\mu$</th>
<th>$\tau$</th>
<th>$b$</th>
<th>$\theta$</th>
<th>$p$</th>
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Notes: $A = 1$, $\alpha = 0.8$, $\rho = 0.04$, $\sigma = 2$ and $\phi = 0.2$.

\(^{17}\) The tax rate, $\tau$, is “more” distorting than $b$. Specifically, $b$ does not directly affect saving decisions (see (7a) above). By contrast, $\tau$ has both direct saving and substitution effects, where substitution effects are attempts to avoid taxes by substituting non-taxed for taxed activities (see (7a) and (7b) above).

critically dependent on the policy instruments available to the government. The same authors also indicate that another equally important aspect could be the objective of the government. We report, however, that our key results do not change when, for instance, we solve for a Ramsey allocation.\(^{18}\)

The second result has to do with the value of \(\theta\). Our numerical solutions imply that, in the region \(\lambda + \mu > 1\), in which there are two Pareto-ranked DCE, the government can implement the low-\(\theta\) equilibrium only. In other words, although the government manages to select a single outcome by acting as a Stackelberg leader, it cannot push the economy to the high-\(\theta\) equilibrium. An explanation is that the government does not have non-distorting policy instruments at its disposal, so that its selection options are limited. Actually, in all cases, with or without multiple DCE, the government finds it impossible to reproduce Barro’s reference outcome with fully protected property rights (\(\theta = 1, e = 0\) and so \(\tau = 1 - \alpha = 0.2\) and \(\gamma = 0.19\)). Generally speaking, this is a second-best policy problem and hence a problem of indirect control only.

The third result relates to the comparative-static properties of GMA. For all combinations of \(\lambda\) and \(\mu\) in Table 4 (with or without multiple DCE), as the overall degree of extraction rises (i.e., as \(\lambda + \mu\) increases), the government finds it optimal to tax less and allocate more resources to law enforcement (i.e., \(\tau\) falls and \(b\) rises monotonically). Among other things, this implies that the GMA properties resemble those of the bad DCE and that the chosen tax rate is lower than Barro’s reference optimal rate. Intuitively, in the area of the bad DCE, a higher \(\tau\) further distorts incentives, while a higher \(b\) improves them. So an optimizing government acts accordingly.

V. Concluding Remarks and Extensions

We have incorporated weak property rights into a general equilibrium model of growth and second-best optimal policy. To the extent that we allowed the state to play its primary role (i.e., to act as an enforcer of property rights by allocating resources to police, courts, etc.), we have filled a gap in the public economics literature. In so doing, we also emphasized the importance of social interactions in (il)legal activities.

Since the main results were described in the introduction, we close with a possible extension. In our paper, the government performed useful functions only. However, low-level government officials (e.g. bureaucrats) as well as high-level officials (e.g. ministers) can misuse their public power to enforce

\(^{18}\) In different models, it is the optimizing government that causes multiplicity; see e.g. Cooper (1999, p. 131) and Park and Philippopoulos (2004). Thus, one should not be tempted to draw any general conclusions regarding the role of the government as a coordination device.

property rights, in the sense that they can use it for private benefit and rent extraction. It would thus be interesting to add some symmetry between the behavior of private agents and that of government agents. To our knowledge, this has not yet been done in a general equilibrium dynamic setup with optimizing (private and government) agents and weak property rights.

### Appendix

**Study of Equation (9c)**

Let us define the LHS and RHS of (9c) as $L(\theta; \tau, b)$ and $R(\theta; \tau, b)$ respectively. We have the first-order derivatives:

$$L_\theta(\cdot) = -(1 - \tau)(1 - \alpha)(\lambda + \mu)(1 - \theta)^{\lambda+\mu-1}$$

(A1a)

$$R_\theta(\cdot) = \lambda(1 - \theta)^{\lambda+\mu-2}[1 - (\lambda + \mu)\theta]$$

(A1b)

and the second-order derivatives:

$$L_{\theta\theta}(\cdot) = (1 - \tau)(1 - \alpha)(\lambda + \mu)(\lambda + \mu - 1)(1 - \theta)^{\lambda+\mu-2}$$

(A2a)

$$R_{\theta\theta}(\cdot) = -\lambda(\lambda + \mu - 2)(1 - \theta)^{\lambda+\mu-3}[1 - (\lambda + \mu)\theta]$$

$$-\lambda(\lambda + \mu)(1 - \theta)^{\lambda+\mu-2}.$$  

(A2b)

Inspection of the above equations reveals that the sign and magnitude of $(\lambda + \mu)$ are crucial to existence and uniqueness of a solution. To cover all interesting possibilities, one has to distinguish between the following four cases:

**Case 1.** $\lambda + \mu > 1$, where $0 < \lambda \leq 1$ and $\mu > 0$. Evaluating $L(\cdot)$ and $R(\cdot)$ at $\theta = 0$ and $\theta = 1$, we have (given $\tau$ and $b$):

$$L(0) = (1 - \tau)(1 - \alpha)(1 + b^\phi) > R(0) = 0$$

(A3a)

$$L(1) = (1 - \tau)(1 - \alpha)b^\phi > R(1) = 0$$

(A3b)

$$L(0) > L(1) > 0.$$  

(A3c)

In turn, with $\lambda + \mu > 1$, (A1a) implies $L_\theta(\cdot) < 0$, while (A1b) implies $R_\theta(\cdot) > 0$ when $\theta < 1/(\lambda + \mu)$, and $R_\theta(\cdot) < 0$ when $\theta > 1/(\lambda + \mu)$. Also, (A2a) implies $L_{\theta\theta}(\cdot) > 0$. Combining results, if

$$R\left(1\over\lambda + \mu\right) > L\left(1\over\lambda + \mu\right),$$

that is if:

$$\left(1\over\lambda + \mu\right)\lambda\left(\lambda + \mu - 1\over\lambda + \mu\right)^{\lambda+\mu-1} > (1 - \tau)(1 - \alpha)\left(b^\phi + \left(\lambda + \mu - 1\over\lambda + \mu\right)^{\lambda+\mu}\right)$$

(A4)
there exist two solutions for \( \theta \),

\[
0 < \theta_1 < \frac{1}{\lambda + \mu} < \theta_2 \leq 1.
\]

Thus, (A4) is a sufficient condition for the existence of two DCE.

In the rest of this case, we study the comparative-static properties of \( \theta_1 \) and \( \theta_2 \) with respect to the exogenous (at DCE level) \( \tau \) and \( b \). Equation (9c) implies the partials:

\[
\theta_\tau = -\frac{(1-\alpha)((1-\theta)^{\lambda+\mu} + b^\phi)}{(1-\theta)^{\lambda+\mu-1}[(\lambda + \mu)(1-\alpha)(1-\tau) + \lambda - (\lambda + \mu - 1)\lambda\theta(1-\theta)^{-1}]}
\]

(A5a)

\[
\theta_b = \frac{\phi b^{\phi-1}(1-\alpha)(1-\tau)}{(1-\theta)^{\lambda+\mu-1}[(\lambda + \mu)(1-\alpha)(1-\tau) + \lambda - (\lambda + \mu - 1)\lambda\theta(1-\theta)^{-1}]},
\]

(A5b)

Hence, the signs of \( \theta_\tau \) and \( \theta_b \) depend on the sign of the expression:

\[
(\lambda + \mu)(1-\alpha)(1-\tau) + \lambda - (\lambda + \mu - 1)\lambda\theta(1-\theta)^{-1}.
\]

(A6)

In particular, if:

\[
\theta < \frac{\lambda + (\lambda + \mu)(1-\alpha)(1-\tau)}{\lambda(\lambda + \mu) + (\lambda + \mu)(1-\alpha)(1-\tau)} \Rightarrow \text{ (A6) is positive } \Rightarrow \theta_\tau < 0, \theta_b > 0,
\]

(A7a)

while if:

\[
\theta > \frac{\lambda + (\lambda + \mu)(1-\alpha)(1-\tau)}{\lambda(\lambda + \mu) + (\lambda + \mu)(1-\alpha)(1-\tau)} \Rightarrow \text{ (A6) is negative } \Rightarrow \theta_\tau > 0, \theta_b < 0,
\]

(A7b)

It turns out that the difference

\[
\frac{\lambda + (\lambda + \mu)(1-\alpha)(1-\tau)}{\lambda(\lambda + \mu) + (\lambda + \mu)(1-\alpha)(1-\tau)} - \frac{1}{\lambda + \mu}
\]

is always positive (where, as seen above, \( 1/(\lambda + \mu) \) is the value of \( \theta \) that maximizes \( R(\theta) \)). Therefore, either

\[
0 < \theta_1 < \frac{1}{\lambda + \mu} < \frac{\lambda + (\lambda + \mu)(1-\alpha)(1-\tau)}{\lambda(\lambda + \mu) + (\lambda + \mu)(1-\alpha)(1-\tau)} < \theta_2 \leq 1 \quad \text{(A8a)}
\]

or

\[
0 < \theta_1 < \frac{1}{\lambda + \mu} < \frac{\lambda + (\lambda + \mu)(1-\alpha)(1-\tau)}{\lambda(\lambda + \mu) + (\lambda + \mu)(1-\alpha)(1-\tau)} < \theta_2 < 1. \quad \text{(A8b)}
\]

If (A8a) holds, \( \theta_1 \) displays the comparative-static properties in (A7a) and \( \theta_2 \) the properties in (A7b). If, on the other hand, (A8b) holds, both \( \theta_1 \) and \( \theta_2 \) behave as in (A7a). Combining these two possibilities, it follows that \( \theta_1 \) always behaves as in (A7a), while \( \theta_2 \) has ambiguous comparative-static properties. Whether (A7a) or (A7b) is the case

for $\theta_2$ depends on the sign of the following expression:

$$R \left( \frac{\lambda + (\lambda + \mu)(1 - \alpha)(1 - \tau)}{\lambda(\lambda + \mu) + (\lambda + \mu)(1 - \alpha)(1 - \tau)} \right)$$

$$- L \left( \frac{\lambda + (\lambda + \mu)(1 - \alpha)(1 - \tau)}{\lambda(\lambda + \mu) + (\lambda + \mu)(1 - \alpha)(1 - \tau)} \right).$$

In particular, if this expression is positive, which is equivalent to

$$\lambda \lambda + \mu (\lambda + \mu - 1)^{\lambda+\mu-1} [\lambda + (\lambda + \mu)(1 - \alpha)(1 - \tau)]$$

$$- (1 - \alpha)(1 - \tau) \left( b^\phi + \frac{\lambda^{\lambda+\mu}(\lambda + \mu - 1)^{\lambda+\mu}}{(\lambda + \mu)^{\lambda+\mu}[\lambda + (1 - \alpha)(1 - \tau)]^{\lambda+\mu}} \right) > 0,$$

(A9)

$\theta_2$ behaves as in (A7b). If the opposite holds, $\theta_2$ behaves as in (A7a). Although we cannot show analytically that (A9) holds, we can report that all numerical simulations we have experimented with indicate it does.

**Case 2.** $\lambda + \mu = 1$, where $0 < \lambda \leq 1$ and $\mu \geq 0$. In this case, we have:

$$L(0) = (1 - \tau)(1 -\alpha)(1 + b^\phi) > R(0) = 0 \quad (A10a)$$

$$L(1) = (1 - \tau)(1 -\alpha)b^\phi > 0 \quad (A10b)$$

$$R(1) = \lambda > 0 \quad (A10c)$$

In turn, (A1a) and (A1b) imply $L_\theta(\cdot) < 0$ and $R_\theta(\cdot) > 0$, respectively, while (A2a) and (A2b) imply $L_{\theta\theta}(\cdot) = R_{\theta\theta}(\cdot) = 0$. Therefore, if $R(1) > L(1)$, that is, if:

$$\lambda > (1 - \tau)(1 -\alpha)b^\phi,$$

(A11)

there exists a unique solution, $0 < \theta < 1$. Condition (A11) is a sufficient condition for existence and uniqueness of a DCE. Regarding the comparative-static properties of $\theta$ with respect to $\tau$ and $b$, it can be shown that if $0 < \lambda + \mu \leq 1$, (A6) above is always positive; hence, from (A5a) and (A5b), $\theta_\tau < 0$ and $\theta_b > 0$.

**Case 3.** $0 < \lambda + \mu < 1$, where $0 < \lambda \leq 1$. In this case, we have:

$$L(0) = (1 - \tau)(1 -\alpha)(1 + b^\phi) > R(0) = 0 \quad (A12a)$$

$$L(1) = (1 - \tau)(1 -\alpha)b^\phi > 0 \quad (A12b)$$

$$R(1) \to +\infty, \quad \text{as } \theta \to 1. \quad (A12c)$$

In turn, (A1a) and (A1b) imply $L_\theta(\cdot) < 0$ and $R_\theta(\cdot) > 0$ respectively. Also, (A2a) implies $L_{\theta\theta}(\cdot) < 0$. Therefore, there exists a unique well-defined solution, $0 < \theta < 1$. The comparative-static properties of $\theta$ with respect to $\tau$ and $b$ are similar to those in Case 2.
Case 4. \(\lambda + \mu = 0\), where \(0 < \lambda \leq 1\) and \(\mu < 0\). In this case, we have:

\[
L(0) = (1 - \tau)(1 - \alpha)(1 + b^\phi) > R(0) = 0 \quad (A13a)
\]

\[
L(1) = (1 - \tau)(1 - \alpha)(1 + b^\phi) > 0 \quad (A13b)
\]

\[
R(1) \to +\infty, \quad \text{as } \theta \to 1. \quad (A13c)
\]

In turn, (A1a) and (A1b) imply \(L_{\theta}(\cdot) = 0\) and \(R_{\theta}(\cdot) > 0\) respectively. Also, (A2b) implies \(R_{\theta\theta}(\cdot) > 0\). Therefore, there exists a unique solution, \(0 < \theta < 1\). The comparative-static properties of \(\theta\) with respect to \(\tau\) and \(b\) are again similar to those in Case 2 above.

**Second-best Optimal Policy**

The growth-maximizing government solves:

\[
\max_{\tau, b} \frac{\dot{c}}{c} = \sigma^{-1}[\alpha(1 - \tau)B(\tau, b) - \rho], \quad \text{(A14)}
\]

subject to the solution for \(\theta\) from equation (9c) denoted as \(\theta = \theta(\tau, b)\), and subject to:

\[
B(\tau, b) \equiv A^{1/\alpha}[\tau\theta(1 - b)]^{(1-\alpha)/\alpha} \left(\frac{b^\phi}{b^\phi + (1 - \theta)^{\lambda + \mu}}\right)^{1/\alpha} > 0. \quad \text{(A15)}
\]

The first-order conditions with respect to \(\tau\) and \(b\) are, respectively:

\[
(1 - \tau)B_\tau(\tau, b) = B(\tau, b) \quad \text{(A16a)}
\]

\[
B_\tau(\tau, b) = 0, \quad \text{(A16b)}
\]

where

\[
B_\tau(\tau, b) = B(\tau, b) \left(\frac{1 - \alpha}{\alpha \tau} + \frac{(1 - \alpha)\theta}{\alpha \theta} + \frac{(\lambda + \mu)(1 - \theta)^{\lambda + \mu - 1}\theta}{\alpha[b^\phi + (1 - \theta)^{\lambda + \mu}]}\right) \quad \text{(A17a)}
\]

\[
B_b(\tau, b) = B(\tau, b) \left(-\frac{(1 - \alpha)}{1 - b} + \frac{(1 - \alpha)\theta b}{\theta} + \frac{\phi b^{-1}(1 - \theta)^{\lambda + \mu} + (\lambda + \mu)(1 - \theta)^{\lambda + \mu - 1}\theta b}{b^\phi + (1 - \theta)^{\lambda + \mu}}\right), \quad \text{(A17b)}
\]

and where total differentiation of (9c) implies (A5a) and (A5b). Using (A17a), (A17b), (A5a) and (A5b) in (A16a) and (A16b), we get (10a) and (10b) in the text. In turn, (9c), (10a) and (10b) constitute a three-equation non-linear system in \(\tau\), \(b\) and \(\theta\). This is solved numerically as explained in the text (see Table 4).
References


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