The Office of the Deputy Prime Minister, Department for Transport, and Department of the Environment, Food and Rural Affairs

A Social Time Preference Rate for Use in Long-Term Discounting

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OXERA

Executive Summary

In the calculation of present values of costs and benefits of public sector projects and policies, future values are multiplied by a *discount factor* that is calculated from the social discount rate (social time preference rate). This practice is described in the Treasury's Green Book. In the past it has always been the practice to use the same discount rate over short timescales as for long timescales. Yet, discounting at even a modest rate, such as 3.5%, reduces the value of costs (or benefits) several hundred years hence to almost zero. This disenfranchises future generations from consideration in today's decisions.

Recent work on discounting over the long term has now made it clear that constant rate discounting has only a limited justification, and that it is possible to make recommendations for better practice. The recent literature argues that discount rates vary with time and that, in general, they decline as the time horizon increases. There are several strands to the arguments.

The first set of arguments derives from empirical observations of how people actually discount the future. There is some evidence that individuals' time preference rates are not constant over time, but decrease with time. Individuals are observed to discount values in the near future at a higher rate than values in the distant future. While some evidence still supports time-constant discount rates, the balance of the empirical literature suggests that discount rates decline in a hyperbolic fashion with time.

The second set of arguments in favour of time-varying discount rates derives from uncertainty about economic magnitudes. Two parameters have been selected for the main focus of this approach. The first is the discount rate itself. The argument is that uncertainty about the social weight to be attached to future costs and benefits—ie, the discount *factor*—produces a certainty-equivalent discount *rate* which will generally be declining with time. The second uncertain parameter is the future state of the economy as embodied in uncertainty about future consumption levels. Under certain assumptions, this form of uncertainty also produces a time-declining discount rate.

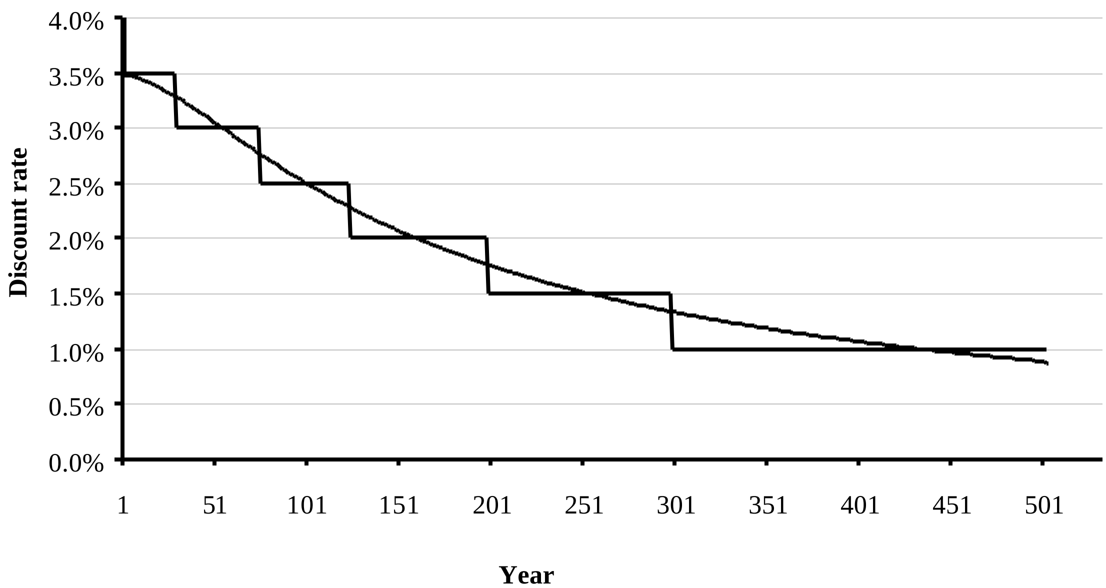
The third set of arguments for time-declining discount rates does not derive from empirical observation or from uncertainty. Instead, this approach—the 'social choice' approach—directly addresses the concerns of many that constant-rate discounting shifts unfair burdens of social cost on to future generations. It adopts specific assumptions (axioms) about what a reasonable and fair balance of interests would be between current and future generations, and then shows that this balance can be brought about by a time-declining discount rate.

Any one, or all of these arguments support the hypothesis that the social time preference rate declines with time. There have been a number of attempts to construct models to quantify the shape of this decline, and, in some cases, to test them empirically. These include those by Weitzman (uncertainty in the discount rate), Newell and Pizer (uncertainty in the discount rate proxied by uncertainty in the interest rate), Gollier (uncertainty in the rate of growth of consumption), and Li and Löfgren and Chichilnisky (fairness between generations as a matter of social choice).

This report concludes that the results from Newell and Pizer's modelling of interest rates could be used as the basis for policy guidance on a social time preference rate for the UK. Although the results were based on US interest rates rather than UK rates, and although

the report indicates that some future refinements to the method are justified, Newell and Pizer's data offers an empirically based path for the discount rate. The results are shown in the figure below—a discount rate that declines to 1.0% in the long term. The discount rate has been set to start at 3.5%, in line with the draft Treasury Green Book guidance.

Suggested change in discount rate changes over time, reflecting uncertainty



For ease of application in policy, this declining path in the discount rate is translated into an approximate step schedule of rates, also shown in the figure. The step schedule is listed in the table below.

Suggested step schedule of discount rates

Period of years	Discount rate (%)
0–30	3.5
31–75	3.0
76–125	2.5
125–200	2.0
201–300	1.5
301+	1.0

Contents

1. Introduction

1.1 Purpose of this paper

The ODPM, DfT and Defra commissioned this study from OXERA to explore recent developments in the social discount rate literature, and make them accessible to an informed audience. Discounting is the process by which the value attached by people to goods and services that arise at different times can be compared. The requirements of the project were to provide:

- an explanation of the context and main issues relating to long-term discounting;
- a review of recent contributions in this area, explained in reasonably accessible language;
- a commentary on the significance of this new literature for the choice of long-term social discount rates in the UK, taking the expected HM Treasury discount rate of 3.5% as the starting point;
- identification of the main alternative regimes for long-term discount rates which are consistent with the literature and with 3.5% as the short-term rate;
- the selection of a long-term discount rate; and
- illustration of the effects of the alternative time-varying discount rate regimes on three areas of policy concern: transport infrastructure; global warming; and nuclear energy.

Part I of this paper covers the first five requirements, Part II covers the last.

Within Part I of this paper, the commentary begins with a brief introduction to the philosophical debate about the justice of discounting long-term values. The reader is encouraged to reach a view by the end of this commentary as to whether a positive discount rate is just and necessary. If the reader decides it is not, then the social time preference discount rate chosen is zero, and the report shows that this has particular implications for long-run welfare-maximisation. If the reader accepts the arguments for positive discounting, then the following chapters discuss the arguments for a positive discount rate which declines with time. This is contrasted with the more traditional view that there is a single social time preference rate which is invariant with time.

The Treasury's draft Green Book introduces the social time preference rate as follows:

> 'people prefer to receive goods and services sooner rather than later, and to bear costs later rather than sooner. This is known as 'social time preference' and the social time preference rate is the rate which reflects the value society places on consumption of goods and services now, compared with consumption in the future.'[1]

[1] HM Treasury (2002), page 51.

Individuals reveal their time preference rate through market decisions, such as lending and borrowing. For individuals as a whole**,** their time preference rate is equal to the real interest rate on lending or borrowing money. However, for society, it is society's willingness to trade off consumption now with consumption later, potentially over periods of time, of a lifetime or longer. The social time preference rate may differ from individuals' time preference rates, but there is no equivalent observable market rate for the social time preference rate. Fortunately, the social time preference rate can be broken down into separate components, which may allow further insights to be made.

This paper briefly recaps how the social time preference rate can be built up from these elements, a topic which is also covered in the Treasury's draft Green Book, Annex 7. When this is done, the Treasury concludes, the social time preference rate is around 3.5% per annum,[2] but it notes that 'where...impacts occur over the long term…a lower discount rate may be appropriate'.

> Over the very long term, discounting will reduce the present values of costs and benefits occurring far in the future. These values are also likely to be affected by inherent uncertainty. A reduction in the discount rate over the very long term might be the most suitable way of tackling these uncertainties. Decisions should be made on a case-by-case basis.

This paper examines the theoretical analysis supporting this statement, and shows the effect of uncertainty on the discount rate in the long term. The recommendations for policy-makers are summarised in Figure 6.1 and Table 6.1 in the conclusions section.

1.2 Conventional discounting

Benefits (and costs) occurring in time *t* are conventionally discounted as follows:

$$PV(B_t) = \frac{B_t}{(1+s)^t} \qquad (1.1)$$

where PV refers to the present value, B is benefit, t is time, and s is the social time preference rate. For the purposes of this report, s is the social time preference rate advanced in HM Treasury (2002a), and its derivation is not of immediate concern. The expression:

$$d_t = \frac{1}{(1+s)^t} \qquad (1.2)$$

[2] This compares with 3% in Germany, and 8% in France—see Evans and Sezer (2002) and the rate used until recently in the UK, 6%—see HM Treasury (1997).

is the *discount factor*. Another way of showing the link between the social time preference (discount)[3] rate and the discount factor is to write[4]:

$$s = \frac{d_{t-1}}{d_t} - 1 \qquad (1.3)$$

In this formulation, *s* is a constant and does not vary with time.

It will prove important to maintain a clear distinction between the discount rate, *s*, and the discount factor. It is the *discount factor* that constitutes the social weight being applied to benefits and costs at different points in time. A crucial implication of this basic point is that any uncertainty about the weight to be given to future interests will be uncertainty about the discount factor *d*. Counter-intuitively, this uncertainty about the discount *factor* can imply a discount *rate* that declines with time.

1.3 Impacts of conventional discounting

The bias in favour of the present in discounting is easily illustrated. Imagine a project with long-run benefits or costs. At a social discount rate of 3.5%, £1 of benefit (cost) in years 50, 100, and 200, has a present value respectively of:

at *time* = 50, present value = 0.18

at *time* = 100, present value = 0.03

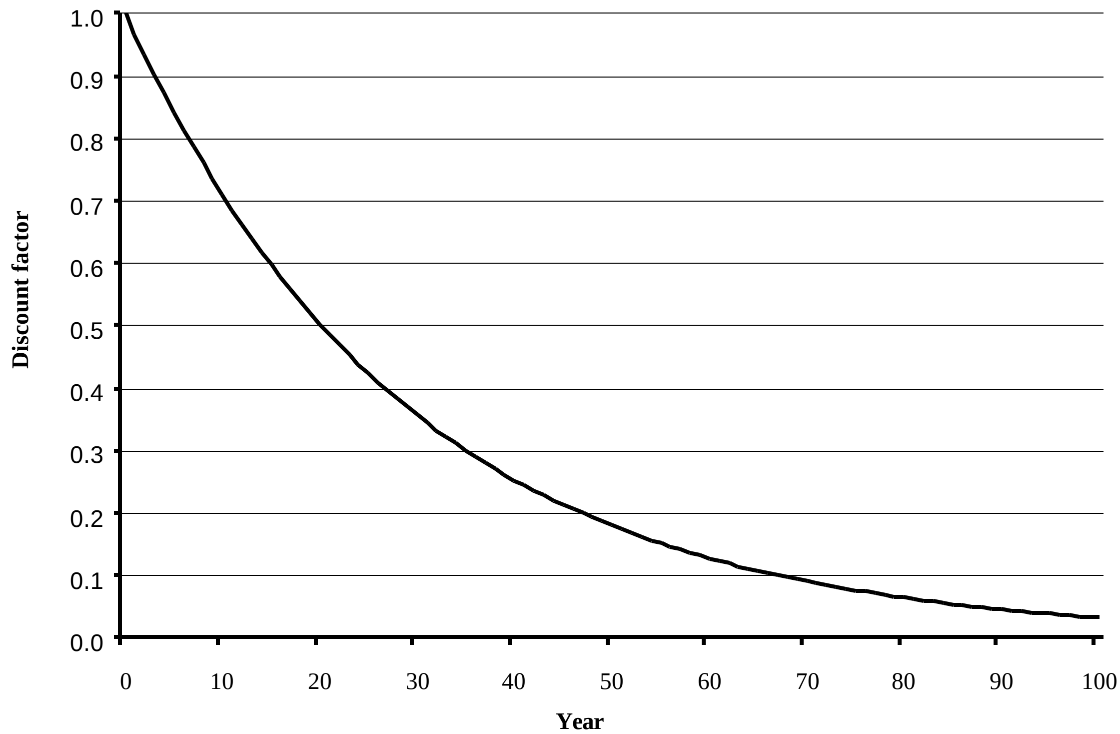
at *time* = 200, present value = ~0

The £1 of benefit (cost) is reduced to 18 pence for *time* = 50, to 3 pence for *time* = 100, and effectively to nothing for *time* = 200. Figure 1.1 shows the discount factors obtained using a constant discount rate of 3.5%.

[3] This paper frequently uses the term ‘discount rate’ in place of the term ‘social time preference rate’.

[4] By virtue of the fact that $d_2 = \frac{d_1}{(1+s)}$ and so on.

Figure 1.1: Discount factors obtained with a constant discount rate of 3.5%



The conventional, or ‘discounted utility’ formulation of the intertemporal choice problem was introduced by Samuelson (1937). The discount rate is a single number which is intended to capture all the forces giving rise to time discounting. Frederick et al. (2002) note that Samuelson himself never intended that the discounted utility model be taken as the norm, either as a description of actual behaviour or in terms of social decision-making[5]. They suggest that its mathematical convenience was instrumental in its adoption, as with the further formalisations by Koopmans (1960). A defence of the model has, however, been advanced by a number of authors—see, for example, Mas-Colell et al. (1995).

There are two potential interpretations of the discounted utility approach. The ‘positivist’ interpretation is that constant discount rates *describe* actual behaviour—that is, intertemporal choices can be predicted on the assumption that people possess constant discount rates. This interpretation tends to fall foul of the empirical evidence, which is not generally consistent with the assumption of constant rates (see Chapter 3). The second interpretation is normative, and suggests that, however individuals behave, social

[5] This raises the issue of what the purpose of Samuelson’s paper was. Frederick et al (2002) suggest it was to show how earlier work, which had faced difficulties in being extended to more than two periods, could be developed on a multi-period basis, and to show that the assumption of cardinal utility was required to illustrative intertemporal choices.

decisions ought to be made on the basis of the discounted utility model. Several reasons might be advanced for the normative approach. Constant discount rates possess certain desirable properties (such as time-consistency) which it might be argued should characterise rational social decision-making. Similarly, even if the empirical evidence suggested that individual discount rates are very high (as is often the case), society might wish to use a lower discount rate for social decisions. For example, society might not wish to deplete natural resources at the rates that would be implied by high individual discount rates.

The special assumptions underlying the discounted utility model are discussed at length in Frederick et al. (2002). The most important are:

- *utility independence*: if two wholly different time paths of utility have the same present value, the individual is assumed to be indifferent between the two time paths. It is easy to imagine why this may not hold in practice, since individuals are unlikely to be indifferent between a time path in which all the utility is 'bunched' into one period, compared with a time path where it is spread out evenly over time. Yet both could have the same present value.

- *consumption independence*: marginal rates of substitution between consumption in any two periods are assumed to be independent of consumption in any other period. Consumption goods cannot be complementary across more than one time period. Again, it is easy to see how this assumption might be violated in practice: consumption yesterday might well affect the value of consumption in some other period.

- *stationarity*: any identical activity generates the same utility regardless of when it occurs. Another way of saying this is that preferences are assumed not to change over time. Again, fairly self-evidently, we know that preferences do change over time, so this assumption is also likely to be violated in practice.

- *independence from forms of consumption*: the discount function is not affected by the form of consumption, that is, what it is that is being discounted. Frederick et al. (2002) review the literature that suggests individuals discount different things at different rates, thus refuting the assumption.

- *non-variation with time*: the discount rate is assumed not to vary with time. As noted above, some argue that this assumption is more for analytical convenience than logic, but issues of time-consistency[6] are argued by others to be an important

[6] Time consistency can be summarised as follows. Let there be two consumption paths $(c_t,\ldots c_T)$ and $(c'_t\ldots c'_T)$. Let $c_t = c'_t$. Time consistency requires that if $U_t((c_t,c_{t+1}\ldots c_T) = U_t(c'_t,c'_{t+1}\ldots c'_T)$ then $U_{t+1}(c_{t+1}\ldots c_T) = U_{t+1}(c'_{t+1}\ldots c'_T)$. The demonstration that constant discount rates are time consistent was first made by Strotz (1956).

feature of constant discount rates. Also as noted, if time-consistency is regarded as a feature of rationality, we may wish society to behave according to this presumption of rationality, even if individuals do not behave in that fashion.

Overall, then, the (constant) discounted utility model has advantages and disadvantages. The main disadvantages are that it appears neither comprehensively to describe real-world behaviour, nor to be consistent with some aspects of what many would see as desirable social decision-making (because it militates against the welfare of future generations). The main advantages are (a) analytical convenience and (b) ‘rationality’ of preferences (time-consistency).

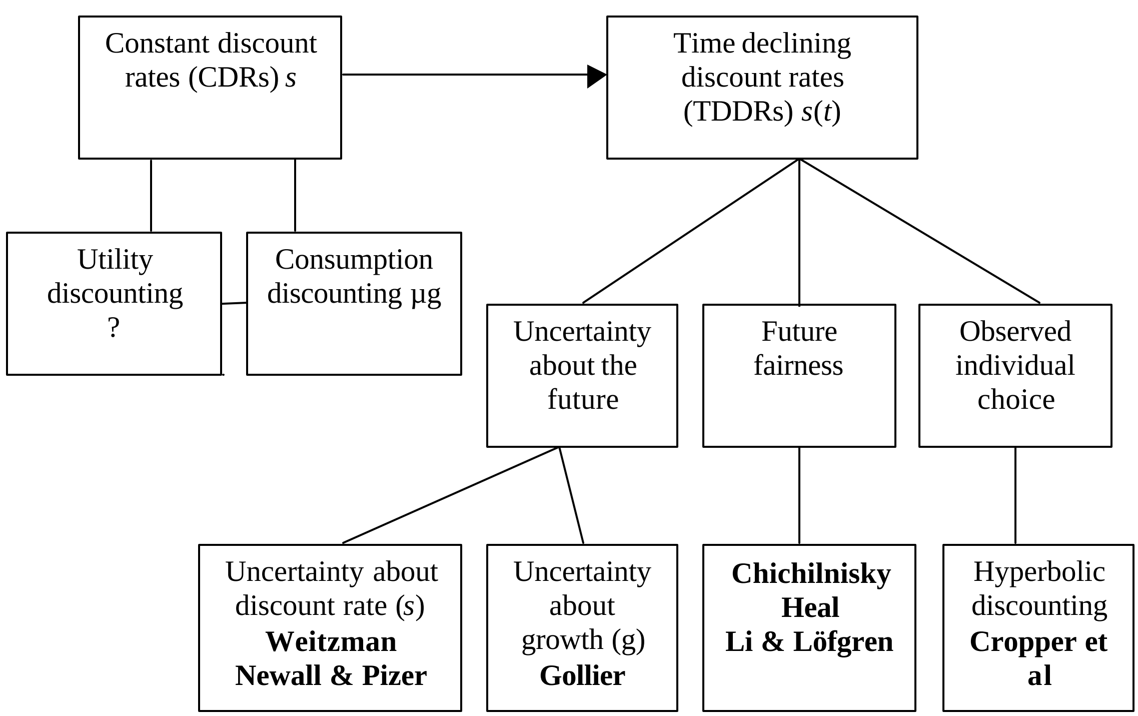
1.4 Guide to the rest of the report

The principal focus of this report is a set of recent literature on the social time preference rate for long time periods. It suggests that the effective discount rate is not constant over time but declines with time. This paper examines three issues:

- the theoretical basis for supposing that discount rates are decreasing functions of time;
- the extent to which the theoretical literature gives practical guidance on the time-path of effective discount rates;
- the practical implications for government policy of adopting time-varying discount rates.

Section 2 discusses the ethical issues associated with discounting at a positive rate, and at a zero rate. Section 3 introduces a decomposition of the social time preference rate into components—the left-hand side of Figure 1.2.

Figure 1.2: A survey of the discounting literature



Section 4 of the report explains the theoretical advances in understanding the time-path of discount rates—the right-hand side of Figure 1.2. These advances reveal three powerful reasons why the social time preference rate might decline as the time horizon extends. First, uncertainty about the future, whether in terms of future economic growth, or future social time preference rates themselves, results in a declining rate. Second, considerations of intergenerational equity and future fairness argue against a discount rate that gives the present generation dictatorship over future ones. Third, experimental work by both psychologists and economists on individual choice has recently revealed that, in many contexts, individuals discount the future at a declining rate, and that the discount rate follows a hyperbolic path.

While all three arguments give rise to declining social time preference rates, they do not imply the same functional form for the decline. Also, while some are amenable to empirical testing, others contain parameters which drive the results but which cannot be observed. Careful scrutiny of the evidence leads to recommendations for policy appraisal in the conclusions section, section 6.

Section 7 looks at the potential implications of time-varying discount rates through illustrations in various policy sectors: nuclear power new build; investment in air and road transport infrastructure; flood defence; and climate change mitigation.

2. Should the Social Discount Rate be Zero?

2.1 The ethics of discounting

The practice of discounting the future has long been controversial both within the economics profession, and in the philosophical critiques of welfare economics and its counterpart, cost–benefit analysis. Discounting involves lowering the weight given to a unit of cost or benefit in the future compared with the present. The further into the future the costs and benefits occur, the lower the weight tends to be. The higher the discount rate, and hence the lower the weight, the less likely it is that investments or policies incurring short-run costs and long-run benefits will be sanctioned by cost–benefit analysis. Conversely, projects with short-run benefits and long-run costs are more likely to be sanctioned by cost–benefit analysis. The resulting 'tyranny of the present' is well known. In both cases, the interests of current generations appear to override those of future generations. How far this bias towards the interests of the current generation matters depends on the ethical stance taken with respect to the interests of future generations. If those interests are depicted as 'rights', then discounting has a strong ethical bias. If, at the very least, current generations should have some regard to the interests of the long-run future, due account needs to be taken of the intergenerational effects of positive discounting. This report is not concerned with the choice of an ethical stance towards the interests of future generations, except to say that achieving the goal of sustainable development, widely espoused in UK government policy, will require some attention to be paid to the intergenerational effects of current decisions.

Most of the debate about discounting has centred on how to derive a rate of discount that reflects society's preferences and the fact of capital productivity, given various distortions such as taxes.[7] As Norgaard and Howarth (1991) put it: '…the bulk of the literature struggled primarily with the correctness of the discount rate rather than the correctness of discounting'.[8] Arguments to the effect that there should be 'no discounting' are not arguments against discounting as such, but reduce to arguments in favour of using a specific discount rate; namely zero. Few economists suggest that the discount rate is zero. Nonetheless, there has been a questioning of positive discounting in some quarters. For example, a number of authors have argued for zero discounting across generations, but positive discounting within a generation.[9] Environmentalist critiques of positive discounting tend to focus on its alarming consequences for notions of inter-generational fairness. Typically, such critiques argue for low positive rates of discount, or zero discounting. Philosophical critiques tend to share the same concern about inter-generational fairness, and argue that the reasons for positive discounting are either empirically invalid, or not applicable in contexts where consequences cross generations, rather than occurring within a generation (eg, Barry, 1983). Other critiques suggest that

[7] The major surveys are Lind et al. (1982); and Portney and Weyant, (1999).

[8] Norgaard and Howarth (1991), page 90.

[9] See Norgaard and Howarth (1991); Howarth and Norgaard (1990; 1993); Page (1977, 1978), and Sandler and Smith (1976).

what is being discounted matters: for example, while *commodity* benefits and costs may be discounted, future *wellbeing* should not be discounted.[10]

2.2 Zero discounting as a solution to present bias

One widely-countenanced solution to the present bias of positive discounting is to discount at zero per cent, often described as ‘no discounting’. The usual method of justifying this view is to offer the reasons advanced by economists for positive discounting and then refute them, so implying that the only rational procedure is to discount at zero per cent.

A typical example of such a criticism from a philosopher would be that of O’Neill (1993), who focuses on the Ramsey formula for the social time preference rate, $s = ? + \mu.g$.[11]

First, pure time preference, *?*, is rejected by O’Neill because (a) it confuses discounting by an individual within his or her own lifetime with discounting across different generations, and (b) it is irrational anyway. The first argument suggests that any one individual may discount a future benefit because of the sheer passage of time, but that this is not the same thing as saying that a future generation will similarly value the future benefit at the discounted value perceived by the first generation. It is legitimate to discount one’s own utility, but not legitimate to discount utility across generations, since the latter involves discounting *someone else’s* wellbeing. An analogous argument is advanced by Broome (1991). He argues that ‘impartiality’ is a requirement of utilitarianism, and that impartiality means that both ‘no time can count differently from any other’, and ‘…good at one time cannot count differently from good at another. Hence…the [pure time] discount rate must be nought’.[12]

It can be argued that cost–benefit analysis espouses not utilitarianism, but inter-temporal efficiency, and there are many paths of development that meet that requirement. Choosing between them involves making some judgement about equity (ie, fairness), rather than utilitarianism as such. More importantly, the O’Neill–Broome arguments overlook one of the roles played by utility discounting. Suppose there are two individuals, A and B, both with identical marginal utility of income (consumption) schedules: for a given amount of aggregate income, the rule for maximising joint utilities would be that each be given half the available income. At that point, the two marginal utilities of income for A and B would be identical, and aggregate utility would be maximised. Now suppose that, for some reason, B secures less utility from their share of available income than A does (for example, B may face higher prices, or have some impairment). Giving each the same income level now seems unfair, since B is worse-off than A in utility terms. An equity judgement, based on utilitarian considerations, would dictate the transfer of some income from A to B. In fact, however, utilitarianism implies the exact opposite: namely, that

[10] See Broome (1992), chapter 3.
[11] Described in more detail in the next chapter.
[12] Broome (1991), p.921.

income be transferred from B to A. This is because A secures higher marginal utility than B for increments to A's income (Sen, 1973). Now if A is the future generation, and B the current one, A will tend to be richer, because of technological progress, if for no other reason. Yet, using the utilitarian argument, shifting resources from the current generation, B, to the potentially richer future generation, A, would be justifiable. Some form of correction for inequality aversion is required, and arguably pure time preference provides this. Pure time preference should not be zero.[13]

Taking the expression $\mu.g$ in the social time preference rate equation, O'Neill objects to consumption discounting because of its assumption that future generations will be wealthier than the present one; '...there are good grounds for believing that the average wealth of future generations might be a great deal less than that of current generations'.[14] This would be relevant if, for example, the more catastrophic scenarios associated with global warming came about. But the assumption underlying O'Neill's judgement that future generations will be worse off is that wealth has a single determinant: the availability of natural resources. This might be tenable if it is true that natural resources are an absolutely constraining input on the economic system, as some people believe. However, for future generations to be worse off, they would have both to face a natural capital constraint, and experience the failure of technological progress. The likelihood of a combination of these factors seems distant. Nonetheless, it is quite correct to say that if g is negative, then $\mu.g$ will be negative, and the social time preference rate could then be negative.

O'Neill's third objection to social time preference rate being positive rests on the belief that uncertainty about the future should not enter the discount rate. While every individual faces mortality, society at large does not, or so it is argued. What actually happens is that 'life chances' enter the value of *?*, the pure time preference rate. Even if life chances are omitted from *?*, we are left with the other arguments for making *?* > 0. But, if risks of societal extinction are not zero; it seems odd to argue that environmental damage in the future might reduce wealth, without also recognising that the human species may face extinction. While there are telling points in O'Neill's criticism of positive discounting, it does not support the rejection of a positive social discount rate, nor the rejection of non-zero pure time preference.

Some economists (eg, Broome, 1992) and philosophers (eg, Parfit, 1984) have taken a different approach to the justification for zero discounting. They hold that it is possible to discount real commodities, but not wellbeing. Consider a fine, rocky landscape that cannot readily be converted into a different quantity through time. It could be argued that the value of this landscape does not change with time and it therefore has a zero 'own interest rate'. Compare this to a natural resource, such as a fishery, which can change its value through time by increasing in size. The argument is that some things should be

[13] An objection to the role being played here by the pure time preference rate is that it is an ad hoc adjustment of utilitarianism. Rawls (1972) was of the view that it is better to jettison the underlying utilitarian model.
[14] O'Neill (1993), p.52.

discounted and some should not. As a general rule, anything that generates 'constant wellbeing' should not be discounted. But a basic reason why wellbeing and commodities are not readily distinguishable from each other is that resources are needed to generate wellbeing, and they could be used to produce other wellbeing generated by commodities. Again, it is unclear that differentiating between wellbeing and commodities justifies zero discounting.

2.3 The implications of zero discounting

The philosophical critiques of positive discounting refute the arguments in favour of positive discounting, without asking what would happen if discount rates were zero. This issue is explored in Olson and Bailey (1981). Their analysis is confined to the utility discount rate. Even if utility discount rates are zero, arguments for consumption discounting remain. The Olson Bailey context is an economy with a positive interest rate, say *r*, and a utility discount rate *?*. If society is concerned to maximise utility (wellbeing) over time, discounted at the pure time preference rate, and it faces a budget constraint set by income, but allowing for positive interest rates, then setting *?* = 0 has surprising results. As Appendix 1 demonstrates, as long as interest rates are positive, zero discounting implies that there are situations in which current generations should reduce their incomes to subsistence level in order to benefit future generations.

The Olson–Bailey argument reflects a more general concern about zero discounting, and, for that matter, low discounting generally. The effect of lowering the discount rate is to increase the amount of saving that the current generation should undertake. The lower the discount rate, the more future consumption matters, and hence more savings and investment should take place in the current generation's time period. Thus, while lowering the discount rate appears to take account of the wellbeing of future generations, it implies bigger and bigger sacrifices of current wellbeing. Indeed, Koopmans (1965) showed that, however low the current level of consumption is, further reductions in consumption would be justified in the name of increasing future generations' consumption. The implication here is that there will be a very large number of future generations, so that whatever the increment in savings now, and whatever its cost to the current generation, future gains would substantially outweigh current losses. The logical implication of zero discounting is the impoverishment of the current generation. This finding would of course relate to every generation, so that, in effect, each successive generation would find itself being impoverished in order to further the wellbeing of the next. The Rawls criterion (Rawls, 1972), that the aim should be to maximise the wellbeing of the poorest group in society, would reject such a policy of current sacrifice, since the sacrifice would be made by the poorest generation. The opposite tension remains: discounting at any positive rate opens up the possibility that future generations will, at some point, be worse off than the current one.

2.4 Property rights across generations

Page (1977, 1988), Howarth and Norgaard (1990; 1993), and Norgaard and Howarth (1991), offer a critique of discounting that explicitly addresses the issue of discounting within and across generations. The implication of their argument is fairly easily stated. As is well known, the notion of an efficient (or Pareto efficient) allocation of resources occurs when it is impossible to reallocate resources so that those who gain can compensate those who lose, and still be better off. But there are very many such efficient points, and it is impossible to choose between them without making some explicit judgement about the 'deservingness' of different individuals or groups in society. Such a judgement is embodied in a *social welfare function*, and, while certain social welfare functions are commonly used, there are no obvious rules determining which social welfare function should be used. Social welfare functions basically 'weight' the utilities of individuals, and there are many ways this can be done. Some weighting rules might invoke notions of 'rights' to certain resource, such as health, education, a minimum standard of living, etc.

Whichever social welfare function is chosen, and given all the possibilities in the economy for generating wellbeing, there will be a set of prices that would produce the best social welfare. Included in those prices will be the rate of interest. Howarth and Norgaard's argument is that what is efficient through time will depend on how rights to resources are distributed through time. Just as the choice of any one (Pareto) efficient point now depends on how the utilities of individuals are weighted, so the choice of an efficient path of resource through time depends on how we weight the utilities of different generations. The interest rate (discount rate) is not an exogenous variable in the determination of such allocations, but is determined by the allocation of resources through time. A different configuration of rights will produce a different discount rate. Put another way, the discount rate is a function of the allocation of property rights between generations. The optimal use of resources is dependent not just on market conditions (perfect competition), but *also* on the choice of the correct distribution of resource rights. This result challenges the basic presumption that all that is needed for optimal resource use through time is the efficient functioning of markets. Competitive markets are needed for *efficiency*, but what is *optimal* also depends on how property rights are distributed. It follows that the discount rate is indeterminate; it too depends on how resource rights are allocated. Inverting the argument, choosing any social discount rate amounts to choosing the intergenerational distribution of resources.

The practical implication of the Howarth-Norgaard approach is that decisions about the wealth to be transferred between generations (how much we leave for the next generation), is an issue of *intergenerational equity,* which should not be evaluated using *efficiency* criteria alone. Therefore, the discount rate should not be chosen so as to determine such intergenerational transfers. Indeed, it is inefficient to do so. Assuming some 'intergenerational wealth plan' exists, all prices, including the discount rate, will be determined. As these authors say, 'If we are concerned about the distribution of welfare across generations, then we should transfer wealth, not engage in inefficient investments'.[15]

While Howarth and Norgaard do not spell out their own preferred rules for the transfer of wealth across generations, the economic literature on sustainable development has done so. The earliest statement of such rules appears to be the work of Page (1977). In modern parlance, what Page called 'livability criteria' have become known as 'constant capital

[15] Norgaard and Howarth (1991), p.98.

rules'.[16] Adopting some objective that requires per capita wellbeing to rise through time, or, rather, that the conditions necessary to achieve this should be ensured by each generation, leads to the rule that total capital stocks equal to, or greater than, those inherited by each generation, should be passed on to the next. Such a rule holds for each generation to ensure sustainability. Insofar as capital stocks can be improved, in terms of their welfare-creation potential, technological change might permit a declining inherited stock of natural resources. Population change typically acts as an offset to the benefits of technological change. Two variants of the rule have emerged, the first being as stated here, and known as 'weak sustainability'. The second, so-called 'strong sustainability', not only requires that the stock of all capital assets should rise through time, but that stocks of specific assets, usually environmental capital, should not decline. Strong sustainability is designed to reflect the view that there are limits to substitution between forms of capital.

What happens to the discount rate in such models is not always clear. Page (1977, 1988) argues that once the capital stock conditions are met, the net present-value criterion can be adopted for resource allocation decisions within any generation. Effectively, then, welfare maximisation operates within a broad intergenerational constraint, relating to a notion of sustainable development. In turn, this reflects a powerful ethical position being taken on the rights or interests of future generations. Presumably, the discount rate is then given either by the single generation's own social time preference, or the rate of capital productivity.

2.5 Implications of the critique of positive discounting

Probably the most important message to emerge from the philosophical and economic critique of positive discounting is the separation of rules about the allocation of resources (wealth) across generations, from rules about allocating resources over time, within a generation. The latter issue is addressed by adopting a social discount rate with the usual adjustments for market distortions. The former, it is argued, cannot be addressed efficiently by choosing a discount rate. Rather, some rule for intergenerational wealth transfers is required, independent of the choice of discount rate. Such a critique is consistent with the sustainable development literature. This debate concerns the value of d, rather than the value of s, provided g is positive (an expectation of increasing future consumption).

The sustainability literature argues that loading the task of achieving both efficiency and equity goals onto one parameter, the discount rate, is itself inefficient. However, formulating wealth transfer rules is not straightforward. The alternative would appear to be to adopt the same approach as in the past, (ie, use the discount rate to reflect both goals) but *not* demand the discount rate to be constant through time. This is the approach addressed in the remainder of the report.

[16] See Markandya and Pearce (1991); Atkinson et al. (1997).

3. The Ramsey Formula

3.1 Introduction

In empirical studies of discounting, what is observed is a discount factor—an inter-temporal weight—and hence a discount rate can be inferred from these weights. Most of the empirical literature is surveyed in Frederick et al. (2002).[17] The authors note that there is enormous variability in empirical estimates, and that many of the rates are extremely high. They attribute these outcomes to the many factors involved in determining inter-temporal choice. As such, substantial variability in estimated discount rates should not be a surprise, since rates vary with context, and with the type of benefit or cost being discounted, etc. Any attempt to 'decompose' or 'deconstruct' discount rates is therefore hazardous. With these caveats in mind, in this section, the decomposition of the standard social time preference rate, *s*, is introduced.

The standard formula for determining the social discount rate based on social time preference is given by the Ramsey equation $s = r + m.g$,[18] where *?* is the rate at which individuals discount future utility (well-being, welfare); μ is the elasticity of the marginal utility of income (consumption) schedule; and g is the projected rate of growth of per-capita real consumption. The origin of this equation is in models of individual savings behaviour, in which individuals maximise utility (welfare) over time. *s* will be equated to the interest rate *r* in general when agents optimally allocate their incomes between savings and consumption.

Pearce and Ulph (1999) provide an overview of the empirical estimates of the individual components. They further decompose the equation into:

$$s = d - L + m.g$$

where *d* is now the 'true' utility discount rate—that is, the rate at which utility is discounted independently of any risks to life, and *L* is the rate of change in life chances. If life chances become less, the overall utility discount rate, *?*, will become larger.

Each element is examined in turn below.

3.2 The 'pure' time preference rate, d

As noted in the text, some economists (including Ramsey himself) and philosophers regard d as having the value zero. While superficially attractive in many respects, zero rates produce many paradoxes, including the prospect of justifying the condemnation to misery of successive generations in the name of benefiting later generations (Olson & Bailey, 1981). Other paradoxes are addressed in Heal (1998, chapter 5). Work by Scott

[17] Frederick et al. (2002) provides an excellent overview of the multi-motive models of discount rates.
[18] Ramsey (1928).

(1977, 1989) suggests that long-run savings behaviour in the UK is consistent with a value of d of 0.5. This component of the social time preference rate is the least amenable to empirical analysis, but the literature suggests that the range is 0.0–0.5.

There is no easy way in which to trace how d will vary over time. Any attempt to do so would probably involve the inspection of savings models constructed at different points in time.

3.3 Changing life chances, *L*

Pearce and Ulph (1999) point out that the correct interpretation of changing life chances is that relating to whole generations, rather than the higher discount rates that may ensue from individuals simply getting older.[19] The latter has been the subject of a fairly extensive literature (eg, Kula, 1985) but appears not to be relevant to the derivation of the social time preference rate. Newbery (1992) estimates a value of life risks at 1%, which is close to the current ratio of total deaths in the UK to total population (1.1%), see Table 3.1, and is also close to Kula's value of 1.2%, in Kula (1987).

Table 3.1: Projected and recent death rates in the UK

Year	Deaths per capita	Percentage of population dying each year
1991	0.0111	1.11%
2000	0.0102	1.02%
2011	0.00984	0.98%
2021	0.00999	1.00%

Source: Office of National Statistics.

This suggests that a current value for life chances would be 1.1%, with a projected change in the near future to 1%.

3.4 The elasticity of marginal utility of consumption, m

The classic source of estimates of μ is Stern (1977). Recent reviews are Pearce and Ulph (1999) and Cowell and Gardiner (1999).[20] Some authors regard μ as unobservable—for example, McKenzie (1983). Estimates of μ can only be derived if either some restrictions are placed on the underlying utility functions (essentially, additive separability), or μ is seen as an inequality aversion parameter implicit in some set of social decisions. Not surprisingly, then, the value of μ has proven to be controversial, with the literature also being unnecessarily confusing, due to a failure to distinguish the different bases for

[19] For the interpretation of the discount rate as a probability of survival to the next period, see Mas-Colell et al. (1995, chapter 20).

[20] For a very recent attempt to estimate m, see Evans and Sezer (2002)

estimates, to problems in Stern's original survey and to the disregard of Stern's own caveats.

Stern's survey suggested a range of value from 0 to 10, with a 'concentration of estimates' around a value of 2. However, close inspection of Stern's article shows that the higher values for μ come from a study of South Korea in 1970. In fact, Stern's estimates contain only two estimates relating to the UK, both from work by Brown and Deaton, and producing values of 2–2.8, the basis for these being 'judgemental'. Other estimates based on systems of demand equations—for example, Kula 1985, 1987—use an incorrect formula (the ratio of income elasticity of demand to price elasticity of demand) to derive μ.[21] Stern's analysis of savings behaviour is also problematic, because of a misspecified equation, as noted by Scott (1989). Rather than a value of $\mu = 5$, as suggested by Stern, Scott suggests that the correct value based on savings data 1951–1973 is 1.5. Stern concludes that 'enough has been said to prevent any reader taking such numbers away for direct use in cost benefit analysis… We are merely illustrating ways of going about it' (1977, p. 244). While Stern's own warning suggests that no reliance should be placed on the estimates in his survey, it is clear that selected values of μ have been taken from that survey and used to justify a value range of 1.5–2.0.

Pearce and Ulph (1999) show that savings models, such as those developed by Blundell et al. (1994), point to a value of $\mu = 0.8$. Cowell and Gardiner (1999) similarly suggest that work on savings behaviour implies a value of μ 'just below or just above one' (p. 31). They look at UK tax schedules to see what social decisions might imply about μ as a social inequality aversion parameter, and suggest that this work implies a range of 1.2–1.4; and that experimental work produces values of around 4. They conclude that 'a reasonable range seems to be from 0.5… to 4' (p. 33). Values such as 4, however, imply a quite dramatic degree of inequality aversion. To see this, consider two individuals, rich (R) and poor (P), with utility functions of the form:

$$U_i = \frac{Y_i^{1-m}}{1-m} \qquad \text{i = R,P}$$

The ratio of the two *marginal* utilities is given by:

$$\left[\frac{Y_P}{Y_R}\right]^m$$

Suppose, just for illustration, that the income of the rich individual is ten times that of the poor one, $Y_R = 10Y_P$. The range of social values is shown in Table 3.2, corresponding to various values of μ.

Table 3.2: The ratio of loss in utility by the rich individual to the gain by the poor when wealth is transferred at the margin

μ =	**0.5**	**0.8**	**1.0**	**1.2**	**1.5**	**2.0**	**4.0**
Loss to R as a fraction of gain to P	0.31	0.16	0.10	0.06	0.03	0.01	~0

This shows that, at $\mu = 4$, the social value of extra income to R is zero. At $\mu = 1$, a marginal unit of income to the poor, P, is valued ten times the marginal gain to the rich, R. At $\mu = 2$, the relative valuation is 100 times. In this illustration, then, values even of $\mu = 2$ do not seem reasonable. A value of $\mu = 1$ does seem feasible. Overall, looking at the implied values of μ in savings behaviour and at the illustration above, values of μ in the range 0.5–1.2 seem reasonable.

Projections of μ are clearly impossible to make without projecting either savings behaviour or social aversion to inequality.

3.5 The rate of growth of per-capita consumption, *g*

Pearce and Ulph (1999) suggest that observations of very long-term runs of per-capita consumption data will overcome some of the problems of using shorter runs of past data. For example, g will be understated if there is a switch to leisure from consumption, and will be overstated if there are social costs of consumption.

Since growth rates tend to be projected only for short-term periods, projecting long run values of g is not straightforward. Past growth rates represent the best information. These are shown in Table 3.3 for the UK. The last 100–180 years' data suggests a value for g 1.3–1.6.

Table 3.3: Percentage per-annum growth in GDP per capita in the UK

Period	Average per-annum growth in GDP per capita, %
1500–1820	0.27
1820–1870	1.26
1870–1913	1.01
1913–1950	0.92
1950–1973	2.44
1973–1998	1.79
1500–1998	0.65
1820–1998	1.35
1870–1998	1.39
1913–1998	1.58
1950–1998	2.10

Source: Calculated from data in Maddison (2001).

3.6 The social discount rate, *s*

Bringing the estimates together suggests the following, shown in Table 3.4.

Table 3.4: Estimates of the value of the elements of the social time preference rate

	This study	HM Treasury (2002a)	Blundell (1993, 1994)
d	0.0–0.5	1.3–1.5 based on Scott (1	
L	1.0–1.1		
μ	0.8–1.1	1.0	0.8–1.4
g	1.3–2.1	2.0	
s	**2.0–3.9**	**3.5**	

This gives the range s = 2.4–3.4%. Use of the 1950–98 economic growth rate, 2.1%, would raise the upper bound of this range to 3.9%.

Up to this point, this paper has rehearsed arguments about the magnitude of the social time preference rate, and has not considered whether that rate is constant over time. The discussion which now follows is the main subject of this paper—the social time preference rate in the long term—and considers whether the estimate of the social time preference rate derived above is suitable for use for the appraisal of long-term effects.

4. Evidence for a Declining Social Time Preference Rate

This chapter begins the investigation of arguments for time-varying discount rates. The suggestion that discount rates vary inversely with time is not new. They were, for example, analysed in early work by Elster (1979). Hyperbolic rates were identified in experimental work by Thaler (1981).[22] Other contributions are noted in the survey by Frederick et al. (2002). Nonetheless, it only in recent years that the rationale for time-varying rates has been discussed extensively.

4.1 Uncertainty about the future social time preference rate

4.1.1 Introduction

To obtain the present value of cost or benefit at time *t*, the future value is multiplied by the discount factor, where the discount factor is $1/(1+r)^t$. If the value of *r* is uncertain, and could take any value within a range, then it is necessary to estimate the range of discount factors associated with these values of *r*, before deriving the range of possible present values of the effect. When *r* has a low value, the discount factor will be large (closer to unity), and the present value of the cost or benefit will be a large proportion of its future value. When a high discount rate is used, the discount factor will be small (closer to zero), and the present value of the effect will be smaller, closer to zero. To obtain the expected or certainty-equivalent present value, it is necessary to take an average of the present values of the effect, with each scenario weighted by its probability of being realised. When this is done, the values from the scenarios with low discount rates contribute more to the weighted average. The simple consequence of compounding over time is to reduce to zero the importance of those scenarios with high discount rates. In effect, the power of exponential discounting reduces the importance of future scenarios with high discount rates.

While it may be possible to estimate present values for every scenario, suppose it were desirable to be able to estimate a present value without examining every scenario. In this case, a certainty-equivalent discount factor would be used to convert the future value into a present value. The certainty-equivalent discount rate for constructing discount factors is the harmonic mean of the discount factors and declines over time, falling at the limit to the lowest conceivable rate.[23] While the mathematics described above is not new, the insight to discounting is ascribed to Weitzman, whose paper states that '...In the limit [as time goes to infinity], the properly averaged *certainty-equivalent* discount factor corresponds to the minimum discount rate'.[24] Put succinctly, Weitzman's observation is that when there is an uncertain discount rate, the correct discount rate for a particular time period—the certainty-equivalent discount rate—can be found by taking the average of the

[22] Note that some of the literature, (eg, Frederick et. al. (2002)), refers to *any* time-declining discount rate as 'hyperbolic', whereas we reserve the term here for time-declining discount rates consistent with a discount factor that has a specific hyperbolic form, as in Figure 4.3.
[23] The mathematical proof is shown in Appendix A2.
[24] Weitzman (1998), p.206.

discount *factor*, rather than the discount *rate* itself. A numerical example illustrates the result. In Table 4.1, there are ten potential scenarios $(j=10)$, and each scenario is manifested with equal probability: $p_1 = p_2 = ... = p_{10} = 0.1$. It is easy to see that the certainty-equivalent discount rate approaches the lowest discount rate of the ten scenarios considered, 1%. In year 200 the rate has fallen to 1.16%, and by year 500 this rate has fallen 1.01%. Weitzman's key result is that the limit of the certainty-equivalent discount rate, as *t* goes to infinity, is 1% in this example.

Table 4.1: Numerical example of Weitzman's declining certainty-equivalent discount rate

Interest rate	Discount factors in period *t*				
Scenarios	10	50	100	200	500
1%	0.91	0.61	0.37	0.14	0.01
2%	0.82	0.37	0.14	0.02	0.00
3%	0.74	0.23	0.05	0.00	0.00
4%	0.68	0.14	0.02	0.00	0.00
5%	0.61	0.09	0.01	0.00	0.00
6%	0.56	0.05	0.00	0.00	0.00
7%	0.51	0.03	0.00	0.00	0.00
8%	0.46	0.02	0.00	0.00	0.00
9%	0.42	0.01	0.00	0.00	0.00
10%	0.39	0.01	0.00	0.00	0.00
Certainty-equivalent discount factor, $\tilde{A}_t$	0.61	0.16	0.06	0.02	0.00
Certainty-equivalent discount rate, $\tilde{r}_t$	4.73%	2.54%	1.61%	1.16%	1.01%

Note: for the definitions of $\tilde{A}_t$ and $\tilde{r}_t$, see Appendix A2.

The next question is why the future social time preference rate is uncertain.

4.1.2 The causes of uncertainty in the social time preference rate

The Ramsey formula for *s* represents the consumers decisions concerning saving and consumption. *s* will be equated with *r*—that is, the productivity of capital—in equilibrium. This equality represents the meeting of two sides of the economy, in some sense, the consumption side and the production side. It is the productivity of capital that Weitzman focuses on as being the source of uncertainty.

Weitzman (1998) argues that, in the distant future, there is uncertainty in the productivity of capital which is driven by the factors determining the marginal productivity of capital—capital accumulation, the degree of diminishing returns, the state of the environment, the state of international relations, the level and pace of technological progress, and the degree of substitutability of accumulable for non-accumulable factors. These uncertainties in the factors of future productivity, in Weitzman's formulation, result in discount rates that decline with the time horizon. Weitzman assumes risk neutrality, and focuses on the effect of random variation in the discount rate on investment decisions.

Although uncertainty in the productivity of capital will be linked to growth at some level, it is probably not helpful to think about these together. Hence, uncertainty in growth is dealt with separately, in sub-section 4.2.

4.1.3 The measurement of uncertainty in the social time preference rate

The uncertainty in the social time preference rate could be measured from a proxy for s, such as the highest available risk-free rate on borrowing or the lowest risk-free rate on lending.[25] This approach has attracted the attention of several authors, who have discussed the question of which interest rate to use for the analysis of past behaviour (see for example, Portney and Weyant, 1999; Arrow et al., 1996; Lind 1982). Plausible candidates include rates of return from bonds and other debt instruments, equities, or direct investment. A recent study by Newell and Pizer used the US market interest rate for long-term, high-quality, government bonds (primarily US treasury bonds). Based on these nominal rates, they created a series of real interest rates by subtracting a measure of expected inflation. Then they converted them to their continuous compound equivalents.

Newell and Pizer (2000, 2001) used these data to create a simulation of future interest rates in the US, with the same uncertainty observed in historical interest rates. They simulate the rate of decline of the certainty-equivalent discount rate with time for the USA.

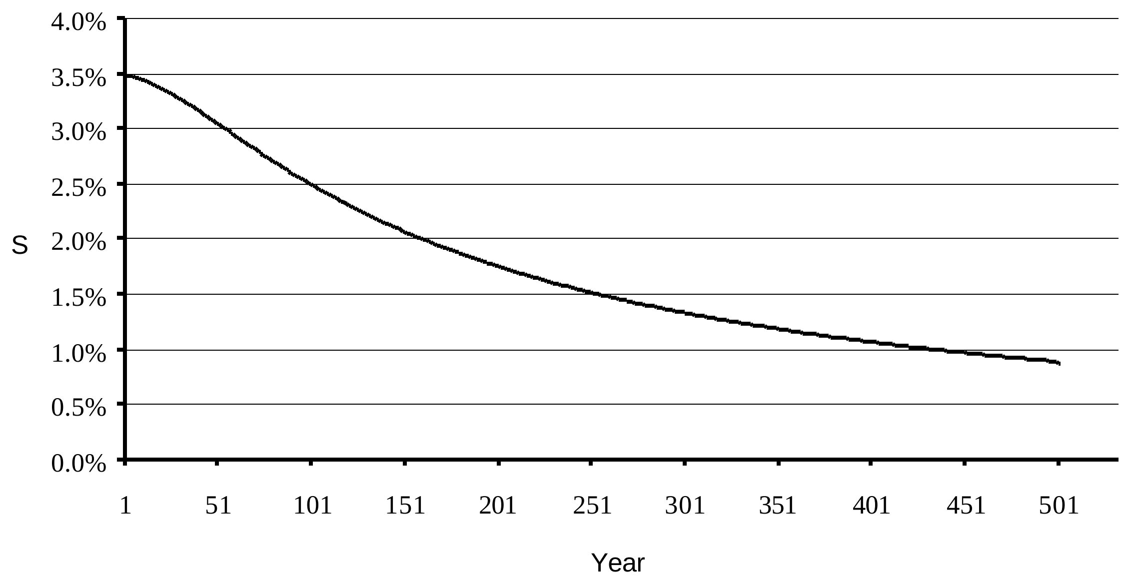
Two of the crucial conditions underlying Weitzman's result are that the discount rate is *uncertain* and that it is *highly persistent*: that is, the expectation must be that periods of low (high) rates will tend to be followed by additional periods of low (high) rates. Newell and Pizer found significant empirical evidence that this had been the case historically. The simulation is described in detail in Appendix A3. In simulating the future certainty-equivalent rates, they found that from a starting value of 4%, the certainty-equivalent rate falls below 1% 400 years hence.

However, Newell and Pizer's statistical approach contained some weaknesses—for example, it could not test robustly for crucial aspects of interest rate behaviour, such as whether the path of interest rates followed a random walk or were mean reverting. The statistical tests applied to their data set were inconclusive, because they were biased towards accepting the hypothesis of a random walk model. If this work were to be repeated, it would be important to try to test the underlying interest rate behaviour statistically, and therefore to approach the statistical analysis in a way that would lead to unbiased and powerful tests. This would include tests for instability (structural breaks) in interest rate behaviour over a long time periods of time.

[25] HM Treasury (2002).

Nevertheless, the Newell and Pizer study, provides a means of relating variation in historic interest data to the certainty-equivalent value of *s* over time. Figure 4.1 shows an adaptation of their results, starting at 3.5%.

Figure 4.1: The path of the social time preference rate, *s*



Source: Adapted from Newell and Pizer.

4.2 Uncertainty about economic growth

When uncertainty about future growth is introduced, argues Gollier (2002a, 2002b), the argument for high discount rates on the basis of the wealth effect, $m.g$, is weakened. In the presence of uncertainty, individuals' degree of acceptance of risk (or, conversely, their degree of aversion to risk) becomes an important determinant of the social time preference rate, and the significance of the term m increases. It no longer represents only the desire for consumption smoothing, but also individuals' acceptance of risk.

$$m = R(c)$$

$R(c)$ has a specific interpretation in the consumer behaviour literature. It represents the index of relative risk aversion, which measures individuals' aversion to risk relative to their wealth, in this case measured by consumption. This term can be increasing, decreasing, or constant with respect to changes in wealth, or in this case, growth in per capita consumption. In the presence of uncertainty about future growth, individuals make decisions based on their attitude towards risk, and these decisions determine s.

The effect of uncertain growth on the level of the discount rate depends on the change in risk preferences, $R(c)$, with wealth. Gollier (2002b) shows that when individuals display *prudence*, the socially efficient discount rate is reduced with increasing wealth. An individual is prudent if his or her propensity to save increases with future income risk (Kimball 1990). This effect can be described as the 'prudence effect', and can be understood as individuals valuing income in future periods more, as a response to anxiety about future income risks.

Thus, when individuals are prudent, there are two opposing effects on the level of the social time preference rate over time: the wealth effect (desire for income smoothing), and attitude to risk. Gollier goes on to show that, over time, there is also an interaction of these aspects of individuals' preferences. He is able to define the set of preferences that leads to a social time preference discount rate which declines over time. His explanation is as follows, and is set out in more detail in Appendix A5.

First, note that the social time preference rate is equal to the equilibrium interest rate in a competitive economy. The variation of interest rates in the future is called the yield curve, and in equilibrium represents the optimal discount rate for future costs and benefits. A declining yield curve means that discount rates should be declining over time.

When individuals face uncertainty about the future, short-term interest rates are determined by the interaction of the wealth effect and risk preferences as described. As such, the shape of the yield curve is dependent upon the risk surrounding future growth, the way in which these risks interact over time, and the attitude of individuals within the economy to these growth risks.

A declining yield curve will result if individuals expect future consumption, or growth, to be significantly lower than at present. In a situation in which growth rates are similar across time periods, the rationale for declining social optimal discount rates is driven by the preferences of the individuals in the economy, rather than by expectations of growth. Gollier derives the conditions under which the yield curve is declining under different assumptions concerning the likelihood of recession.

When the assumption is made that there is no risk of recession,[26] the discount rate will decline where individuals exhibit decreasing aversion to risk as wealth increases. Many studies have found empirical evidence to show that people have such preferences.[27] For example, there is evidence showing that the share of wealth invested in risky assets increases with income in most developed countries.

However, these observations are insufficient for the result to hold when the risk of recession is introduced. Indeed, the conditions on individual preferences required for the economy to exhibit discount rates which decline with time become increasingly complex, unintuitive, and empirically difficult to test.

In a recent paper, Gollier develops the analysis further, and states in his recent draft paper:[28]

[26] Gollier's definition of recession is zero or negative *per capita* growth. Specifically, his no-recession result states that if the random growth is within a positive interval that does not include 0, then declining relative risk aversion has to be assumed.

[27] See, for example, Ogaki and Zhang (2000).

[28] Gollier and Zechkhauser (2002).

> ... when agents have different rates of impatience, using a *constant rate* [emphasis added] to discount cash-flow occurring at different dates—as done in most cost–benefit analyses—is an inefficient investment decision criteria. Moreover, under the widely accepted assumption of decreasing absolute risk aversion, a *constant* [emphasis added] collective discount rate would favour too much short-term investment projects with respect to those yielding benefits over a longer time horizon.

He goes on to recommend using the risk-free rate for medium-term horizons (5% in the case of France), converging to a low value in the long run.

4.3 The social choice literature

4.3.1 Introduction

The results in sub-sections 4.1 and 4.2, that the social time preference rate, s, is a declining function of the time horizon, were driven by the existence of uncertainty in the social time preference rate and economic growth, combined with certain risk preferences. The following contributions from Chichilnisky and Heal (1997), and Li and Löfgren (2000), derive from the social choice literature, and introduce the notion of intergenerational equity and sustainability. They show that a declining discount rate, r, is consistent with a rule whereby current (future) generations must always take into account the wellbeing of future (current) generations; the 'non-dictatorship' of one generation over another.

4.3.2 Chichilnisky's approach

Chichilnisky (1997), argues that for any positive discount rate there will always be a dictatorship of the present over the future, in the conventional constant discounting model, since the discount *factor* tends towards zero in the long run, making future generations irrelevant to the decision maker. She introduces two axioms for sustainable development, that, in combination, require that neither the present nor the future should play a dictatorial role in society's choices over time. She also characterises the preferences that satisfy these axioms. These axioms require that the ranking of alternative consumption paths is sensitive both to what happens in the present and immediate future, and to what happens in the very long run. Sensitivity to the present means that there is no date before which changes have no effect on the ranking (double negative). Sensitivity to the long-run future means that there is no date after which changes do not matter, in the sense of affecting the ranking. The mathematical details of her model are presented in Appendix A6.

Dasgupta (2001, Chapter 6), among others, points out that there are technical problems with Chichilnisky's formula. One difficulty is that the formula amounts to an announcement that the future will be discounted in a conventional manner (although it might be discounted with a declining discount rate), but that, after a point—the so-called 'switching date'—remaining effects will not be discounted (ie, it will be discounted at a zero rate). Dasgupta (2001) shows that it is always possible to improve aggregate wellbeing by postponing the switching date. Consequently, the objective function fails to prescribe any optimal path for consumption and savings. Note also that the Chichilnisky formula says little about how to choose the relevant weight for the 'near' future, and hence the weight for the far distant future.

However, while the Chichilnisky formula may not be immediately policy-relevant, it is an ingenious attempt to square the underlying tensions in discounting: the failure of high discount rates to exhibit what many would see as sufficient concern about the future, and the failure of low discount rates to be fair to the present.

4.3.3 Li and Löfgren's approach

In contrast to Chichilnisky (1997), and Heal (1995), who treat present and future generations as separate entities in the objective function of the decision maker, Li and Löfgren (2000) treat the future in a different and novel way.

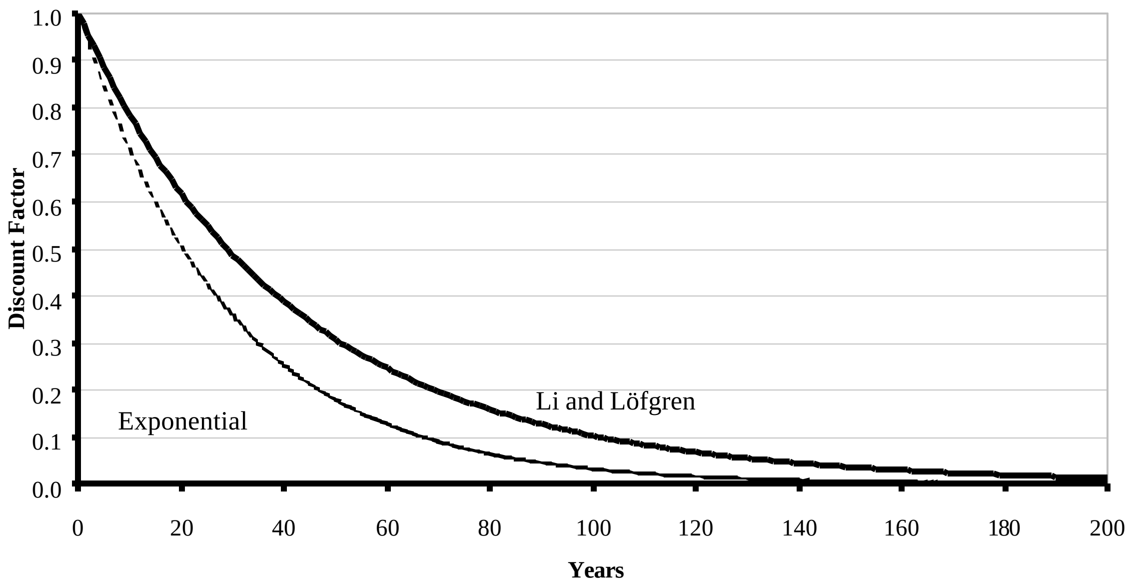
Li and Löfgren's society consists of two individuals, a utilitarian and a conservationist, each of which make decisions over the inter-temporal allocation of resources. The utility functions of these two individuals are identical, and again have consumption (c), and the resource stock (this time, s), as their arguments. The important difference between these two decision makers is that they are assumed to discount future utilities at different rates. The utilitarian, who wants to maximise the present value of his or her utility (U_1), has a rate of time preference equal to $r > 0$. The conservationist, who derives utility from conserving the stock of the natural resource, has a rate of time preference equal to $\delta = 0$, and maximises his or her utility (U_2). The overall societal objective is to maximise a weighted sum of wellbeing for both members of the society, given their different respective weights upon future generations (see Appendix A6).

Unlike the utilitarian discount function, which tends to zero as time reaches towards infinity, the weighted discount function tends to the weight for the far distant future. Hence Li and Löfgren's model results in a positive welfare weight for the conservationist. Consequently, this model does not involve any dictatorship of present over future generations. Moreover, as the utilitarian's welfare level is explicitly considered, there will not be any dictatorship of the future over the present. Thus, the model explicitly considers intergenerational equity.

Within this framework, the individual with the lowest discount rate will in general dominate the far-distant future, meaning that, over time, the discount rate will be a declining function of the time horizon.

Li and Löfgren, and Chichilnisky are concerned with sustainability and intergenerational equity, and consider future paths of welfare or *utility*. Thus, their models deal with the utility discount rate, r, which makes them difficult to use for discounting future cash flows. However, they both generate declining discount rates (see Appendix A6). Figure 4.2 shows this decline as the long-run discount factor converges to the weight attached to the conservationist, which has been set equal to 1/3. An exponentially-declining discount rate is shown for comparison.

Figure 4.2: Discounting utility using the Li and Löfgren model



Source: OXERA calculations.

To put the Li and Löfgren model to use requires a choice of a discount rate for the conservationist and the utilitarian, and a choice of weights attached to each of these preferences for the future. Unfortunately, there is no guidance in the literature about the choice of such weights.

4.4 Hyperbolic discounting

4.4.1 Empirical evidence

There is strong experimental evidence that individuals discount the future in their daily choices about consumption; that they apply a declining discount rate, and that the decline of the discount rate follows a hyperbolic path. The economic literature has used hyperbolic discounting with considerable success to explain otherwise difficult and irrational phenomena, such as drug addiction, procrastination, and under-saving. For these reasons, it has become an important body of literature.

Frederick et al. (2002) list the available studies on empirical estimates of discount rates. Studies are either revealed preference (studies of actual behaviour) or stated preference (questionnaire or experimental studies). Many of the studies reviewed show discount rates that decline as the time horizon is extended. They also show that, for this same set of studies with time-varying discount rates, this pattern holds across the studies. However, out of a wider sample of 42 studies, 25 report rates that are hyperbolic, the remaining 17 reporting conventional constant rates. Moreover, once studies with time horizons of less than one year are removed from the sample, the regression line of discount rates with time horizon shows a zero slope—the discount rate does not vary with time horizon.

While not directly relevant to the issue of whether rates are constant or time-varying, Frederick et al. (2002) also note many ‘anomalies’ with the empirical work. For example, some studies find that benefits may be discounted more heavily than costs, and small amounts may be discounted more heavily than large amounts. They conclude that:

> '..procedures used to measure time preference consistently fail to isolate time preference, and instead reflect, to varying degrees, a blend of both pure time preference and other theoretically distinct considerations…' (p.389).

Unlike conventional discounting, which is exponential, in hyperbolic discounting the weight assigned to each period, d_t, declines as a hyperbolic function of time, as noted by Loewenstein and Prelec (1992).[29]

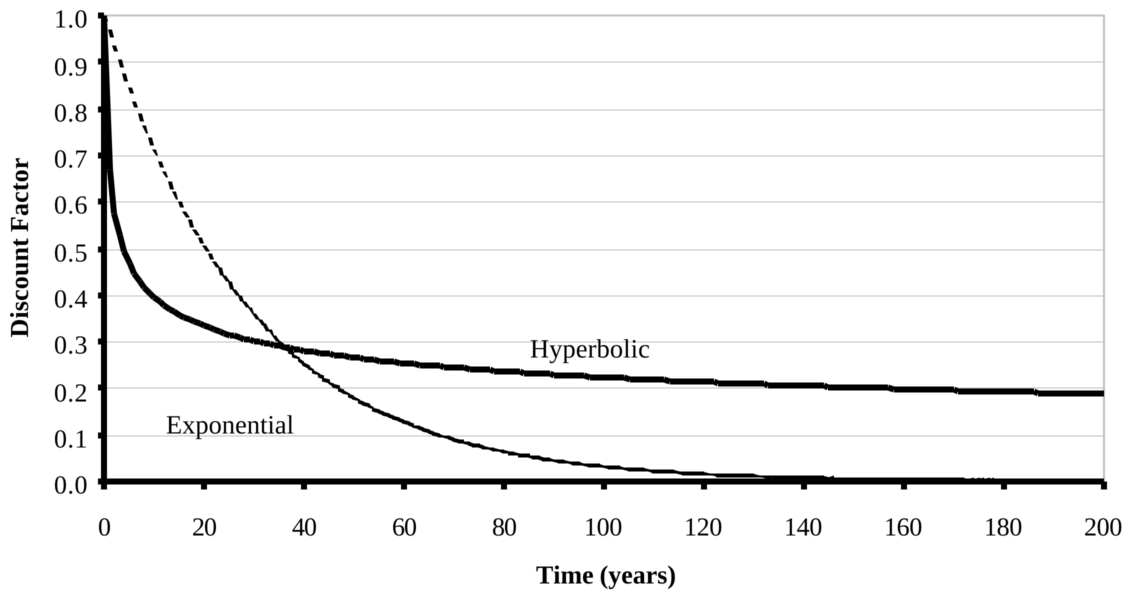
$$d_t = \frac{1}{(1+kt)^{h/k}}$$

The parameter h reflects 'time perception'. If $h = 0$, individual time periods are perceived as passing extremely fast: infinitely so, in fact. As h tends to ∞, time is not perceived to pass at all. The parameter k measures the deviation of the hyperbolic discounting function from the standard exponential model. As k approaches zero, d_t approaches the exponential function.

The effect, compared with exponential discounting, is to lower the discount *factor* for near-term gains and losses, and to raise it for distant gains and losses. Put another way, it reduces the value of near-term effects, and increases the value of distant effects, compared with an exponential discount rate. The result is shown in Figure 4.3, which compares hyperbolic rates with exponential rates.

[29] See Appendix A7 for further details.

Figure 4.3: Hyperbolic and exponential discount factors



Source: OXERA calculations.

4.4.2 Hyperbolic discounting

The main difficulty with hyperbolic discounting lies, once again, in the specification of appropriate parameters. These parameters can be measured empirically. The results are that in the equation below, $k = 4$, and $h = 1$.[30]

$$d_t = \frac{1}{(1+kt)^{h/k}}$$

These parameters imply rapidly declining discount rates, starting from a very high rate, and do not seem to be reasonable for policy purposes. Hence, hyperbolic discounting, while providing a representation of the discount rates implicit in individual behaviour, is perhaps less well suited to policy applications of social choice. In particular, the initial discount rates, being extraordinarily high, are well beyond a reasonable upper bound for the social time preference rate.

[30] See references cited in Harris and Laibson (2001).

5. Some Issues with Time-varying Discount Rates

This section covers a series of unrelated practical issues:

- time inconsistency;
- when in time the discount rate begins to decline, or the definition of the long term; and
- state preferences.

5.1 Time inconsistency

Dynamic (time) inconsistency refers to a situation where plans made at one point in time are contradicted by later behaviour. The identification of this possibility is often credited to Strotz (1956), and has been the subject of papers by a number of writers, including Kydland and Prescott (1977), and Dasgupta (2001). They have shown that declining discount rates can be intertemporally inefficient, or 'intergenerationally incongruent'. Here are two illustrations.

- To be congruent, generation '*A*' chooses a policy, and generation '*B*' acts in accordance with it. Generation *A* does not revise what generation *B* planned. If, generation *A*'s plans are revised by generation *B*, then generation *A* will not have optimised its behaviour: what it intended for generation *B* will turn out to have been wrong. Thus, time-dependent discount rates turn out to be incongruent, with the result that, over long time periods, wellbeing is not maximised.

  Other writers, such as Henderson and Bateman (1995), see the process of changing the discount rate as time moves on as legitimate. People, they say, do not see themselves living in absolute, but in relative time. Revising and re-evaluating plans as time moves on is consistent with behavioural studies, and with the value judgement that what ought to be done by way of discounting should reflect what people actually do.

- Intuitively, individual A could prefer one apple today to two apples tomorrow, but, at the same time, prefer two apples in 31 days to one apple in thirty days. Why this leads to inconsistency is obvious. If A makes a consumption plan according to these preferences, A will plan to receive two apples in 31 days, but then, as time passes and that day approaches, A will change his or her mind and choose to get the one apple one day earlier. A's initial plans are inconsistent with his or her subsequent actions.

Unless government can make a once-and-for-all self-binding commitment to the policy rule it chooses initially, private-sector agents will presumably expect government to re-optimise at later dates. In this case, the private-sector agents may realise that, when future time periods become the present, the government could have an incentive to deviate from the policy rule in period 1, even in the absence of external shocks to the economy, and even if government is benevolently attempting to maximise social welfare. One reason that time-inconsistency in policy can arise is that a policy rule announced in period 1 may encourage private-sector agents to commit to certain actions over the near term. Once private-sector agents have committed to these actions, however, government may then find it desirable to shift to a new policy rule.

Time-inconsistency has a number of implications. If private agents realise that government may deviate from its current announced policy rule in future time periods, then government might lose its credibility. That is, private-sector agents might not condition their choices on government's announced policy rule choice for period 1, but might rather consider the likelihood of possible deviations from this policy rule. Moreover, private-sector agents would also presumably realise that any policy rule chosen by a re-optimising government in some future time period, (t), will depend to some extent on their choices in periods before (t).

There is no easy resolution of this issue. Incongruence, or dynamic inconsistency, results in consumption and savings plans that are sub-optimal for all generations. Heal (1998) proves that almost all types of declining discount rates result in time inconsistency.[31]

5.2 When in time the discount rate's decline begins

Weitzman (1998) suggests that the discount rate should not decline immediately, because interest rates are certain in the immediate future while there are financial instruments offering guaranteed returns. The longest time horizon for financial instruments is generally for government bonds, which yield up to 30 years in the future. Beyond this there are few assets which can be used reliably to measure the discount rate. From this perspective it is reasonable to assume that uncertainty concerning future interest rates begins after the market ceases to offer a valuation, perhaps beyond 30 years. The period beyond 30 years, after *T*, would then be treated as the 'distant future'. However, against this position, it can be argued that the interest rates offered on long-term bonds include compensation for the long-term illiquidity of the principal, and that once this is stripped out, the interest rate would be seen to decline from year zero, as a result of uncertainty.

5.3 State preference: the covariance of discount rates and economic factors

It is possible to anticipate some of the effects of growth of consumption on prices. One such effect is the rising relative valuation of environmental damage or benefit.

Basic cost–benefit decision rules take the form of maximising the net present benefits:

$$\sum_{0}^{T}\left(B_t - C_t - E_t\right)\left(1+s\right)^{-t}$$

[31] See Hepburn (2002).

where B is benefits, C is non-environmental costs, and E is environmental costs (externalities).

The innovation suggested by time-varying discount rates would be to change this formula to:

$$\sum_{0}^{T} \left(B_t - C_t - E_t\right).\left(1 + s(t)\right)^{-t}$$

that is, s becomes a (declining) function of t.

This adjustment should be distinguished from a further modification relating to the *valuation* of benefits and costs. In all cost–benefit equations, the relative value of a benefit and/or cost may well change over time. One obvious reason for this would be that relative values will change as incomes change. Illustrating with environmental costs of pollution, the value of the externalities in any period t is given by:

$$E_t = \sum_{i,t} p_{i,t}.Q_{i,t}$$

where p is the unit value of the externality (pounds per tonne pollutant etc.) and Q_i is the quantity of the ith pollutant (tonnes CO_2 etc).

The unit value (shadow price) of pollution may rise with time. The relevant parameter is the *income elasticity of willingness to pay* (e_{WTP}).[32] While this is often assumed, implicitly or explicitly, to be equal to unity, the available evidence suggests that values like 0.3–0.4 are more likely.[33] Hence:

$$p_t = p_0.(1 + e_{WTP}.y)$$

where y is the rate of growth of per capita income. Hence the externality value in period t is:

$$E_t = \sum_{i,t} p_0 (1 + e_{WTP}.y)^t.Q_{i,t}$$

Note that *both* adjustments—time-varying discount rates, and rising relative valuations of E—are potentially legitimate modifications to the way cost–benefit analysis is normally carried out, and it is best to keep the two adjustments separate. For example, if time-

[32] Note this is not the same as the income elasticity of *demand*, see Flores and Carson (1997).

[33] For a comprehensive review see Pearce (2002b). Rising relative shadow prices of environment and *declining* shadow prices for ‘development’ benefits are integral to the modified cost-benefit procedures set out by Krutilla and Fisher (1975). The income elasticity of willingness to pay may vary between goods and services. For health services, for example, the figure may be different from the figure of 0.3–0.4 for environment services.

varying discount rates are not judged to be justified, then the relative argument remains and should be evaluated on its own merits.[34]

The practical consequence of these adjustments are that a test should be made to see whether the cost or benefit prices have per capita income as their arguments. If they do, and are strongly affected by income levels, then it may be necessary to adopt the ‘state preference’ approach mentioned above, adjusting prices in each discount rate scenario to reflect the level of consumption growth in that scenario.

[34] Confusion is easily generated. For example, some analysts deduct the rising value of the damage from the discount rate to obtain a ‘net’ discount rate, sometimes called an ‘environmental’ discount rate. This is justified computationally but could quickly obscure the separate arguments in the cost-benefit formula.

6. Conclusions

6.1 Summary

This section draws together the theory presented in sections 2–5, emphasising the main advantages and disadvantages of each approach, and highlighting their applicability for policy making. Finally, it presents guidelines for implementation.

6.1.2 Uncertainty in the discount rate

Weitzman (1998) examined the impact of uncertainty in the social time preference rate directly. His approach made no particular assumption about the source of the uncertainty in the discount rate, but to be made useful for policy application, the uncertainty had to be characterised. One attempt at characterisation has been made so far. Newell and Pizer derived a distribution of discount rates by running thousands of simulations of future interest rates, using summary statistics based on past rates. Although it might be possible to improve the statistical approach used by Newell and Pizer, and it might be applied to UK data, it provides a basis for a time-variant profile of s.

6.1.2 Uncertainty in economic growth

Gollier explores the consequences of the level of consumption in the future being uncertain. The theory of his approach is elegant. From a practical perspective, however, the difficulties in implementing the Gollier approach are numerous, as many of the parameters necessary to specify the variation of the discount rate over time are unknown.

Moreover, once a risk of recession is incorporated into Gollier's basic model, the conditions attached to results of the model become complex, and may be impossible to test. So, although Gollier's work contains elegant economic theory, with interesting implications, at the moment its direct usefulness for policy-making is limited.

6.1.3 Social choice

The arguments for a declining discount rate that are founded upon intergenerational equity come closest to dealing directly with concerns that constant discounting simply represents the current generation's selfish refusal to consider the welfare of future generations.

Although Chichilnisky (1997) and Heal (1998) contribute significant advances in economic theory by creating models of intergenerational fairness, they offer little assistance to the policy-maker. In contrast, Li and Löfgren (2000) offer a model that could be applied to policy issues, once several parameters have been specified, and initial simplifying assumptions have been made. Their model combines the time preferences of individuals to give a declining rate for discounting utility.

It might be possible for the parameters in Li and Löfgren's model, from which the time-variant path of r could be derived, to be chosen by a political process. Political representatives could be asked to make ethical judgements about the relative importance of 'development' and 'conservation'.

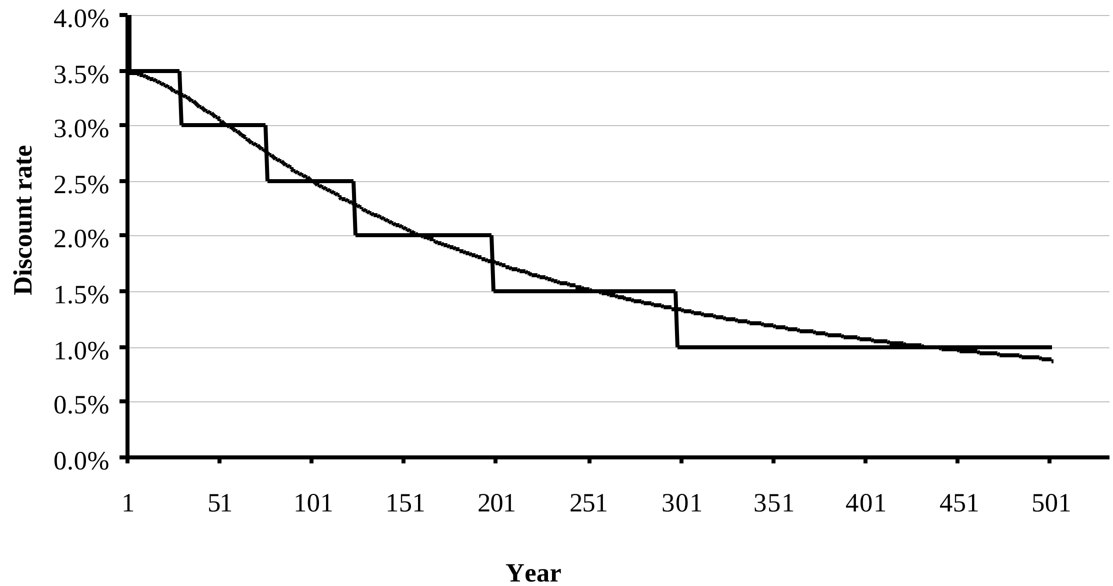
6.2 Conclusions

There are powerful reasons for choosing a declining social time preference rate. This conclusion is supported by robust recent theoretical work, which has taken several different approaches to the subject.

Although there is a paucity of empirical evidence on the pattern of the rate's decline, it may be better to use those data which are available rather than to continue practising discounting with a non-declining rate in the long term. The data best suited the policy-makers' need were produced by Newell and Pizer. When set to commence at 3.5%, these data give a time-variant discount rate as shown in Figure 6.1.

A stepped schedule of rates can be created to approximate to the curve above. Figure 6.1 includes a schedule. Figure 6.2 shows the equivalent discount factors for the continuous function and the schedule.

Figure 6.1: Continuous declining rate and step schedule of rates



Note: based on data supplied by Newell and Pizer.

Figure 6.2: The discount factors associated with the continuous and step schedules

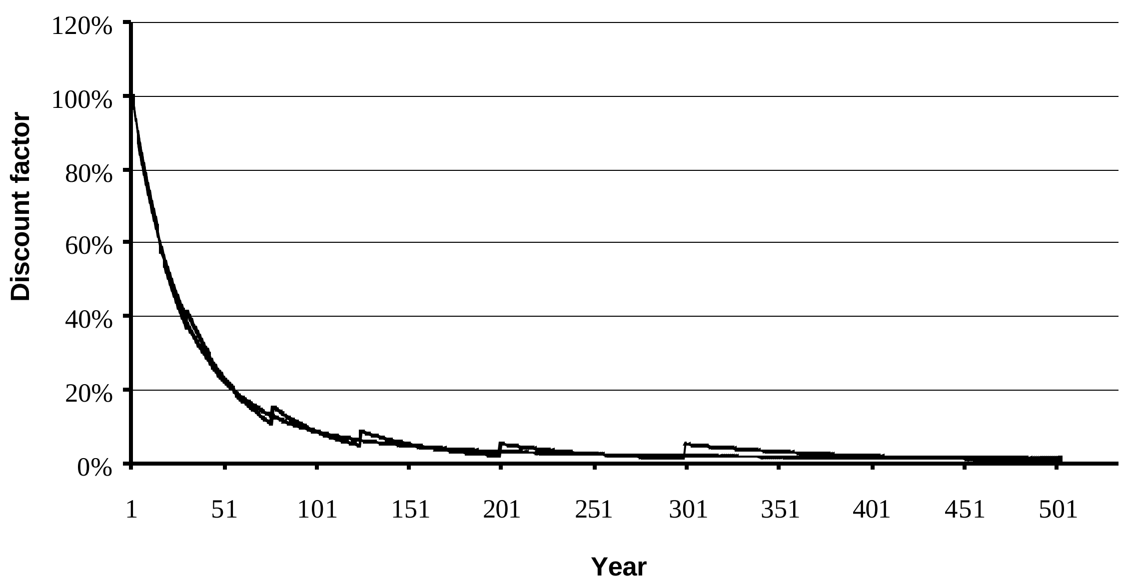


Figure 6.2 reveals that the step schedule shown in Figure 6.1 and Table 6.1 offers a good approximation to the continuous function.

Table 6.1: Suggested step schedule of discount rates

Period of years	Discount rate
0–30	3.5%
31–75	3.0%
76–125	2.5%
125–200	2.0%
201–300	1.5%
301+	1.0%

7. Part II Illustrations

7.1 Introduction

In this section, the discounting theories presented earlier are applied to topical policy questions with the aim of showing how declining discount rates will affect the appraisal of relatively long-term government policies, programmes, and projects.

The case studies considered have been chosen to be topical and illustrative, in that they show the effect of declining discount rates:

- is small, over a short (eg, 30-year) period, but large over a very long period;
- can influence the choice of a policy or project;
- does not always support the option perceived as best for the environment;
- can affect financial planning, in both public and private sectors;
- may result in a re-evaluation of hurdle cost–benefit ratios or budgets in the public sector.

For each case study, three discounting regimes are compared:

- a flat discount rate of 6%;
- a flat discount rate of 3.5%;
- a declining discount rate starting at 3.5%.

The examples are illustrative of changes in discount rates only, and ignore other elements of the changes in the Treasury's draft Green Book that go in tandem with these, such as optimism bias, and greater emphasis on benefits valuation. These are beyond the scope of this study.

Each of the following sections presents a case study. In subsection 7.2, declining discount rates are applied to cost estimates for nuclear new build. In subsection 7.3, two case studies from the transport sector are examined, one road and one air infrastructure. Subsection 7.4 contains an application of very long-term discounting with respect to damage from climate change. In subsection 7.5, an approach to discounting in flood-defence investment is proposed, and in the final subsection, health is discussed.

7.2 Nuclear industry

The change from a flat to a declining discount rate regime will have three effects on the nuclear industry. First, it will affect the economics of new build. Second, it will alter the evaluation of the options for nuclear-waste storage and disposal. Finally, it will affect the funding requirements for the management of future nuclear liabilities. This report only considers the first of these impacts. However, the impacts of time-varying discount rates should be evaluated for the other nuclear industry impacts in any follow-up work, as the effects are likely to be important.

7.2.1 Build and decommissioning

New nuclear build in the UK is being considered as a possible contribution to the UK's energy supply and climate change strategies. In 1998, the House of Commons Trade and Industry Committee recommended that: 'A formal presumption be made now, for purposes of long-term planning, that new nuclear plant may be required in the course of the next two decades.' This recommendation has been supported by a joint working group of the Royal Society, and the Royal Academy of Engineering. More recently, the Performance and Innovation Units report, *The Energy Review*, recommended that the nuclear option should be kept open.[35]

The PIU noted that decisions about new nuclear build are 'relatively insensitive to back end costs even if back end costs are highly uncertain.'[36] The basis for this conclusion is clear:

> Back end costs are incurred a relatively long time after the plant is constructed, even if decommissioning is prompt and wastes are treated quickly and comprehensively…the final cost of back end operations can be discounted at whatever rate of accumulation of funds can be reasonably assumed. At present, BE uses an average 3.5% (real) funds accumulation rate, and the arithmetic of even such low rates such as 3.5% is powerful.

The result of this powerful arithmetic is that the present value costs of decommissioning are relatively insignificant in the cost–benefit analysis. Once declining discount rates are employed, however, the present value costs of decommissioning increase. For the purposes of illustration, the following assumptions were made:[37]

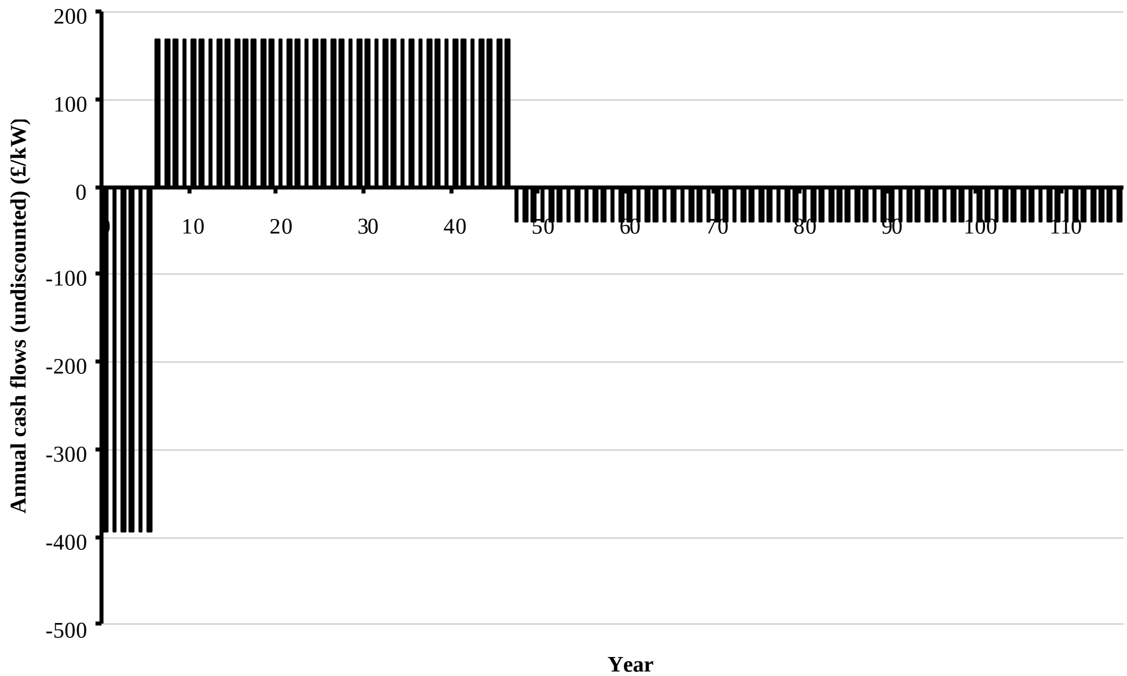
- construction cost of £2,250/kW in 2000 money, and a load factor of 0.85, based on PIU (2001) Working Paper on *The Economics of Nuclear Power*;
- variable operating and maintenance cost of 0.6p/kWh, and fuel cost of 0.4p/kWh, in 1993 money, based on the submission of the NUCG to the 1995 White Paper on *The Prospects for Nuclear Power in the UK*;
- fixed operating costs of 1.5% of construction cost (OXERA rough estimate);
- construction period of six years, reactor lifetime of 40 years, and a decommissioning programme over the following 70 years;
- decommissioning costs of £40/kW per year over the 70-year period, implying a total of £2,800/kW (undiscounted);[38]
- electricity revenues of 4.2 p/kWh so that the base-case scenario (flat discounting at 6%), just breaks even.

These assumptions generate the cash-flows presented in Figure 7.1. The first six years show the front-end costs of construction and commissioning, followed by the 40-year

[35] Performance and Innovation Unit (2002a).
[36] Performance and Innovation Unit (2002b).
[37] It is noted that new build costs might be substantially below the costs quoted here if the estimates made by some manufacturers turn out to be correct. Here we have chosen to use estimates based on actual historical costs.
[38] This estimate is uncertain. Note that at a constant 6% discount rate, this is equivalent to assuming decommissioning costs of 0.05 p/kWh. The PIU note that 'Currently waste policy is possibly even more uncertain than it was in 1995 (given the abandonment of the NIREX Rock Characterisation Facility in 1997). It is therefore impossible to estimate waste management costs in any useful way at present': Performance and Innovation Unit (2002b).

operating life, when the plant is producing positive cash flows, and finally a 70-year period of expenditure on decommissioning.

Figure 7.1: Cash flows for nuclear build and decommissioning

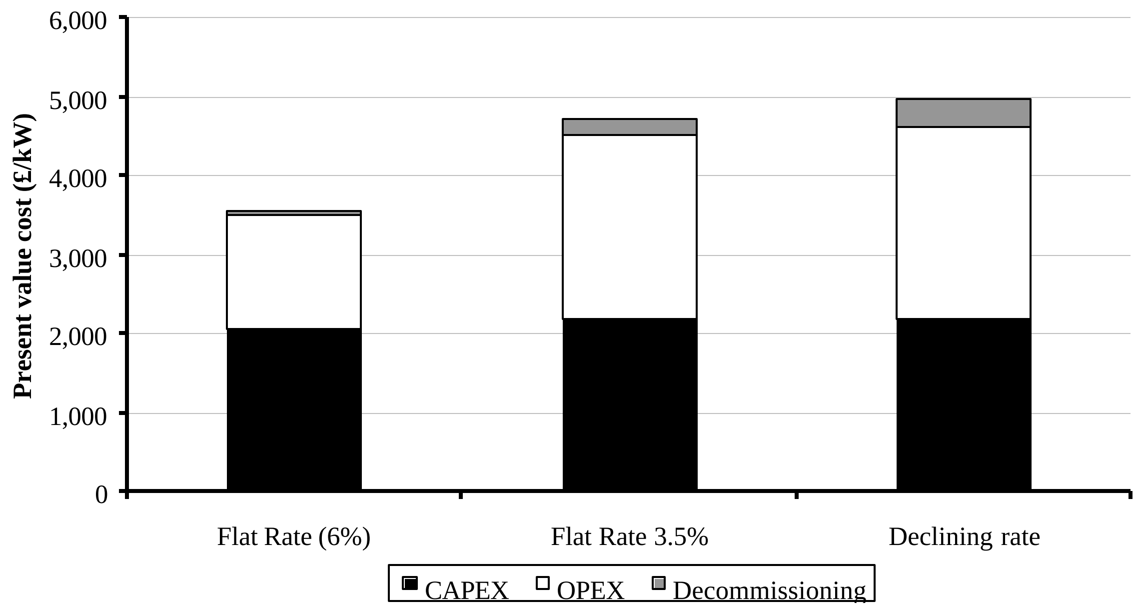


Source: OXERA calculations.

The impact of declining discount rates in the nuclear case is twofold. On the one hand, lower discount rates place more weight on far-distant accruals of costs and benefits. The present-value costs of decommissioning more than double once declining discount rates are employed. As Figure 7.2 illustrates, decommissioning costs change little from approximately 45£/kW and 210£/kW, with flat discount rates of 6% and 3.5% respectively, under the declining discount rate schedule.

As Figure 7.3 reveals, the declining discount rates hardly change the electricity price required for the plant to break even, relative to the 3.5% flat discount rate scenario.

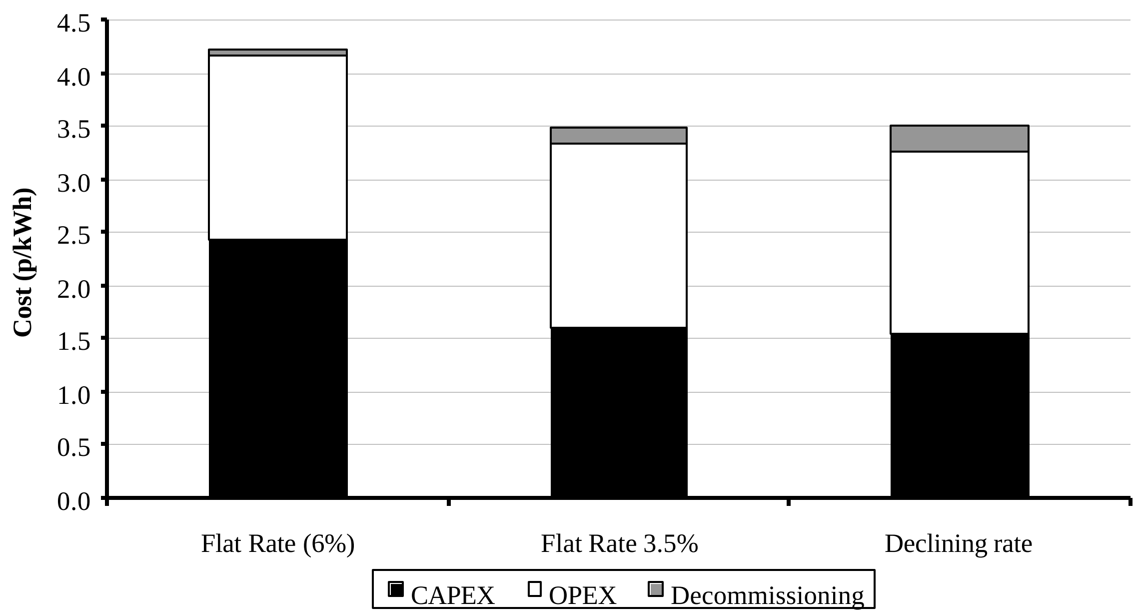
Figure 7.2: Lifetime cost (present value) of nuclear plant



Source: OXERA calculations.

There is another interesting feature of this analysis. An apparent paradox appears when comparing Figures 7.2 and 7.3. The 3.5% rate figures show higher lifetime costs, yet require a lower electricity price to break even. This is because the 3.5% rate discounting places higher weights on the period of revenue generation than discounting at 6%, and hence requires less revenue to cover the investment costs.

Figure 7.3: Break-even electricity prices (NPV = 0)



Source: OXERA calculations.

These calculations illustrate the minimal effect of applying a declining discount rate to new nuclear build. The assumptions employed here are illustrative. More accurate, up-to-date data on the capital, operating, and decommissioning costs of new-build nuclear plants would be required for policy-making purposes.

7.3 Transport

7.3.1 Road Infrastructure

Major highway schemes (those with a gross cost of over £5m) are appraised according to a standardised costs–benefit approach, which compares the discounted costs and benefits from new infrastructure investment.[39] Transport investment is characterised by significant front-end costs, and a stream of future benefits (particularly in travel-time saved). Therefore, switching from a flat to a declining discount rate would be expected to make road infrastructure investment more attractive.

OXERA took data from three road case studies, covering a range of different types of investment, namely:

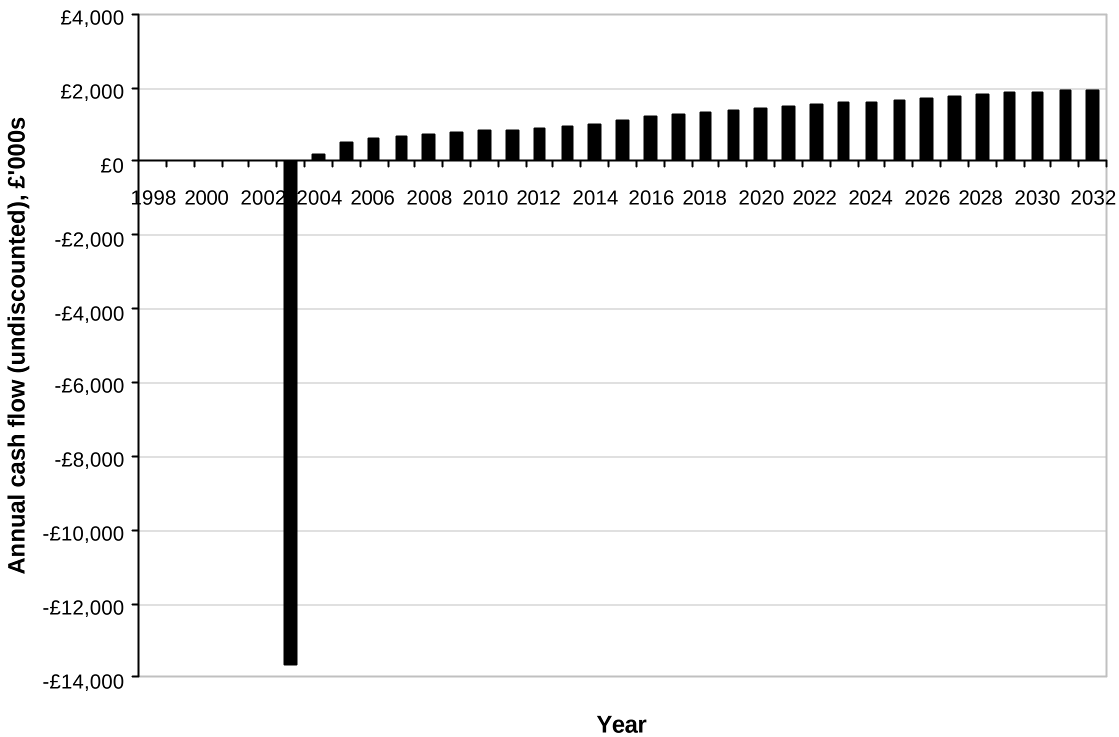
- A2 widening from Bean to Cobham (widening a dual three-lane road to dual four);
- A3 Thursley grade separation (a safety scheme); and
- A34 Chieveley (a major junction upgrade at M4/A34 junction).

These three case studies, while different in nature, all have standard cash-flow profiles characterised by up-front costs followed by a stream of benefits. The net cash flows for the investment in A2 widening are shown in Figure 7.4.

The result is that the application of declining discount rates increases the present value of the investment. Figure 7.5 shows the benefit cost ratio (BCR) of A2 widening for the five discounting frameworks. Employing the declining discount rate schedule increases the BCR of the investment from 1.45, under a constant 3.5% discount rate, to 1.54, (corresponding to a net present value increase from approximately £5.5m to £6.1m) for the ‘low benefit’ scenario.

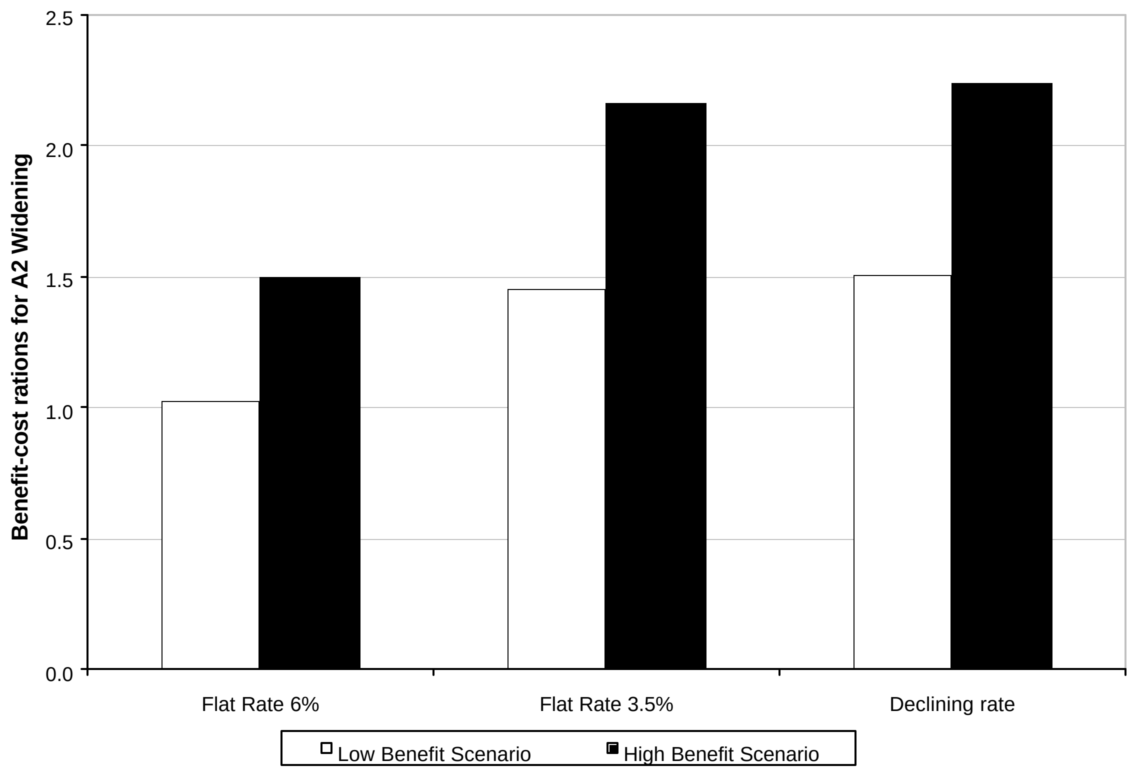
[39] Department for Transport (2002).

Figure 7.4: Net cash flows for A2 widening, £'000s



Source: Highways Agency

Figure 7.5: Benefit–cost ratios of A2 widening



Source: Highways Agency and OXERA calculations

Table 7.1: Benefit–cost ratios of three road schemes

BCR (based on £1998) Low benefit scenario	**A2 Widening**	**A3 Thursley safety upgrade**	**A34 Chieveley junction upgrade**
Flat rate 6%	1.02	2.58	4.85
Flat rate 3.5%	1.45	3.45	7.30
Declining rate schedule	1.50	3.54	7.81

Source: Highways Agency and OXERA calculations

These three examples provide evidence that declining discount rates will make road investment more likely to pass a cost–benefit test. Indeed, declining discount rates will make investment in all transport infrastructure that provides long-term benefits more attractive. Air infrastructure is a good example.

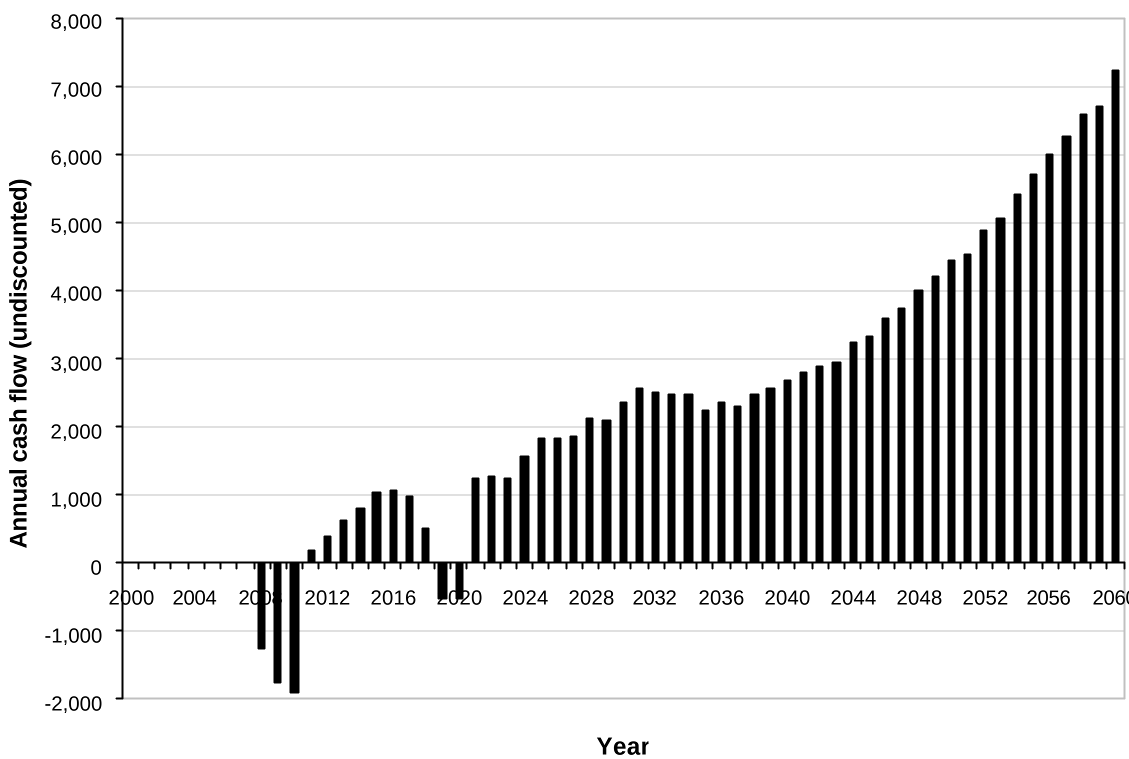
7.3.2 Air infrastructure

The South East and East of England Regional Air Services Study (SERAS) aims to provide a better understanding of the demand for, and constraints on, airports, and air service development, in the South East and East of England.[40]

As part of the SERAS Stage Two study, an economic assessment measured the relative impacts of each SERAS package. OXERA compared the net benefits to users (generated and existing), and producers, against the capital costs of the packages, under constant and declining discount rates.

The undiscounted net cash flows from a typical SERAS package are presented in Figure 7.6. The two periods of negative cash flow reflect capital expenditure on a new runway at Heathrow from 2008, and on a new runway at Stansted from 2018.

Figure 7.6: Annual net cash flows of SERAS Packages, £m



Source: SERAS economic appraisal

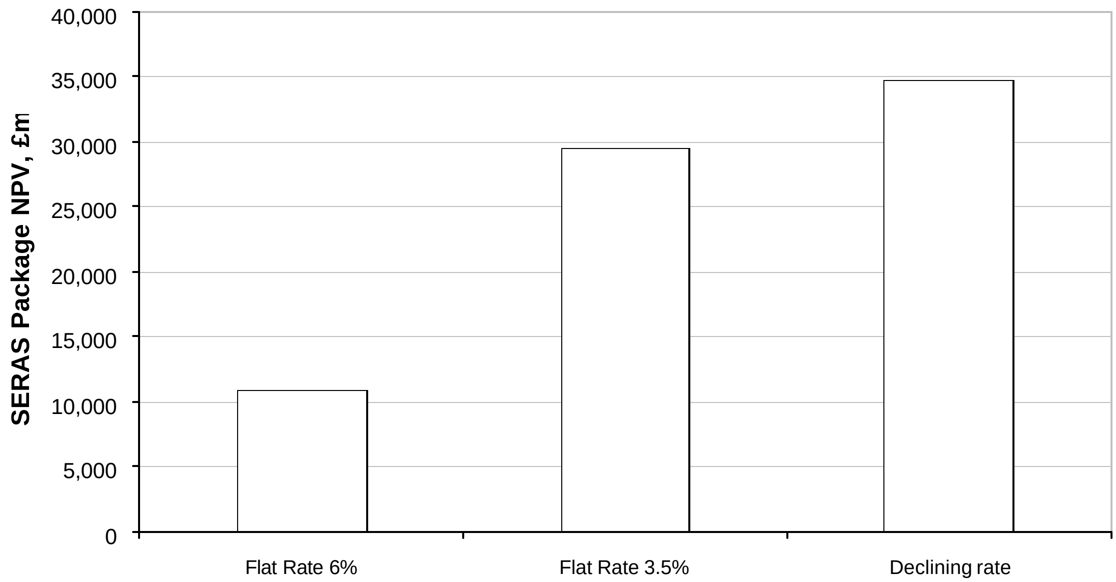
The analysis of the impact of declining discount rates is relatively straightforward. As the packages involve up-front capital investment, followed by a long stream of benefits, lower discount rates make the packages look more attractive, as, indeed, do declining

[40] Department of Transport, Local Government and the Regions (2002).

discount rates. Indeed, when compared with a flat discount rate of 6%, a discount rate of 3.5% increases package NPV from approximately £11bn to approximately £29bn, as shown in Figure 7.7. Employing the declining discount rate schedule increases the NPV further, to £35bn.

Overall, it is clear from the analysis of road and air case studies that declining discount rates will make transport infrastructure investment more attractive.

Figure 7.7: Net present value of a typical SERAS package, £m



Source: SCAM model for SERAS economic appraisal.

7.4 Climate change mitigation

Whether investment in climate change mitigation is supported by cost–benefit analysis depends, critically, upon the estimate of the social cost of carbon. The social cost of carbon is an estimate of the present monetary value of damage done by anthropogenic carbon-dioxide emissions. Therefore, the discounting framework employed can be expected to have a significant impact upon the social cost of carbon, and hence upon the attractiveness of investment in climate change mitigation.

As Clarkson and Deyes (2002) note, when a lower discount rate is employed, the present value of the marginal damage from emissions invariably increases. For instance, the marginal damage values determined using the Fund 1.6 model of Tol (1999), increase from \$20/tC to \$42/tC to \$109/tC, as the discount rate declines from rates of 5% to 3% to 1% respectively.[41]

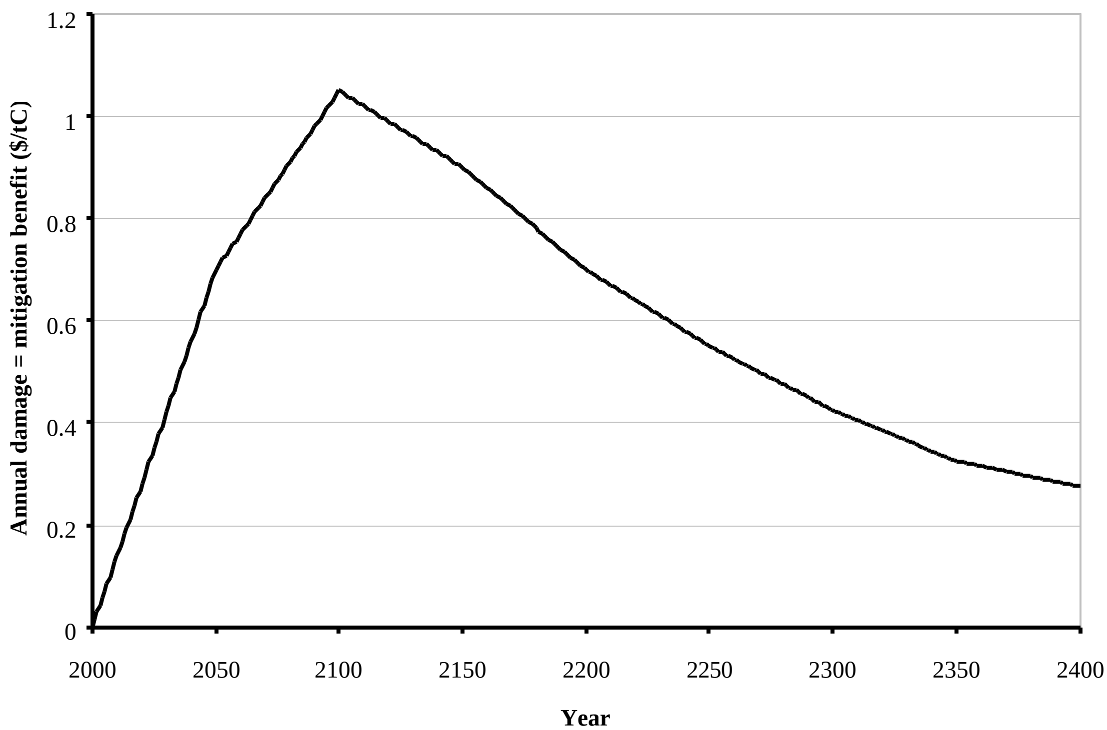
There are two reasons for the increase in the social cost of carbon as the discount rate declines. First, the costs of climate change will be incurred far into the future, so the use of a lower discount rate increases the weight placed on these costs; second, the economic damage from CO_2 emissions is dependent upon the stock of carbon in the atmosphere (and also on the rate of economic growth). The stock of carbon in the atmosphere will increase as time passes (as will the level of economic output), implying that the marginal damage from carbon is increasing with time. Lower discount rates therefore place higher emphasis on the future, where the marginal damage of emissions is higher.

Newell and Pizer present a time profile of the benefit of reducing one tonne of carbon emissions in 2000, based on the DICE model of Nordhaus and Boyer.[42] The illustration in this section is based on a linear approximation of the Nordhaus and Boyer profile, shown in Figure 7.7. The damages increase steeply in the next century, but then slowly decline from 2100 onwards.

[41] Pearce (2002a) notes that the FUND 2.0 model, as employed by Tol and Downing (2000) reflects the more recent literature on adaptation. However, the present report is not concerned accurately to determine the social cost of carbon. Its purpose is to illustrate the impact upon such social cost estimates of employing declining discount rates. The general trend in the FUND 1.6 model—that employing lower discount rates increases the social cost of carbon—remains valid, irrespective of the particular model employed.

[42] Newell and Pizer (2001).

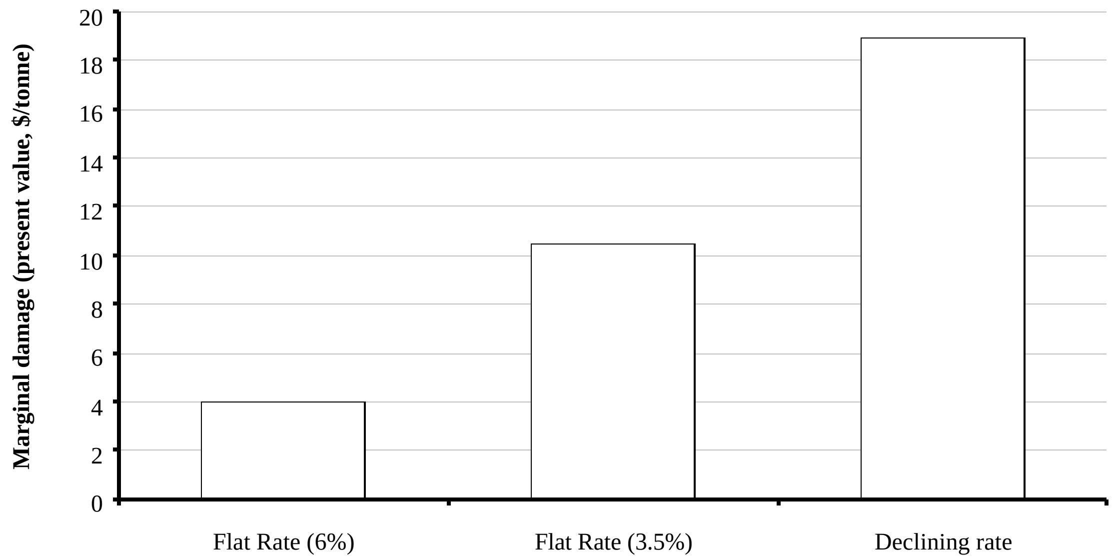
Figure 7.7: Time profile of damage from the emission of a tonne of carbon in the form of carbon dioxide



Source: Newell and Pizer (2001)

The stream of damage over the next 400 years can be discounted back to obtain the marginal damage of a tonne of carbon emitted in 2000. The choice of discount rate regime will therefore affect the present value of the social cost of carbon, and results are presented in Figure 7.8. The estimates vary from approximately $4/tC at a 6% flat discount rate, to $10/tC under the 3.5% flat rate, to $19/tC under the declining discount rate. The use of a declining discount rate almost doubles the valuation of damage.

Figure 7.8: Discounted marginal damage of carbon emissions



Source: OXERA calculations based on Newell and Pizer (2001).

The conclusion from this analysis is that the use of declining discount rates will increase the estimate of the social cost of carbon. An equivalent analysis could be performed on a variety of different underlying climate change models, including the FUND 1.6, FUND 2.0, DICE and the OPEN models.[43] The central conclusion is that, irrespective of the underlying model, estimates of the social cost of carbon are likely to increase by about one third when a declining discount rate is used.

Such an impact on estimates of the social cost of carbon would have formidable implications for policy in several areas. A few examples will suffice. First, whereas we noted that the economics of nuclear build could be adversely affected by time-varying discount rates, a higher social cost of carbon would make nuclear build more attractive in social value terms. Hence, a full reappraisal of nuclear build requires a number of interacting effects to be evaluated, and this should be the subject of future work. Second, higher social costs of carbon would make it more likely that commitments to Kyoto targets and beyond would pass a cost–benefit test (Pearce, 2002a). Third, higher carbon damage costs would affect evaluations of different fuel uses, (eg, diesel versus gasoline). Fourth, measures of 'green' national product and environmentally modified measures of labour and factor productivity would be affected.

[43] See Tol (1999), Nordhaus and Boyer (2000).

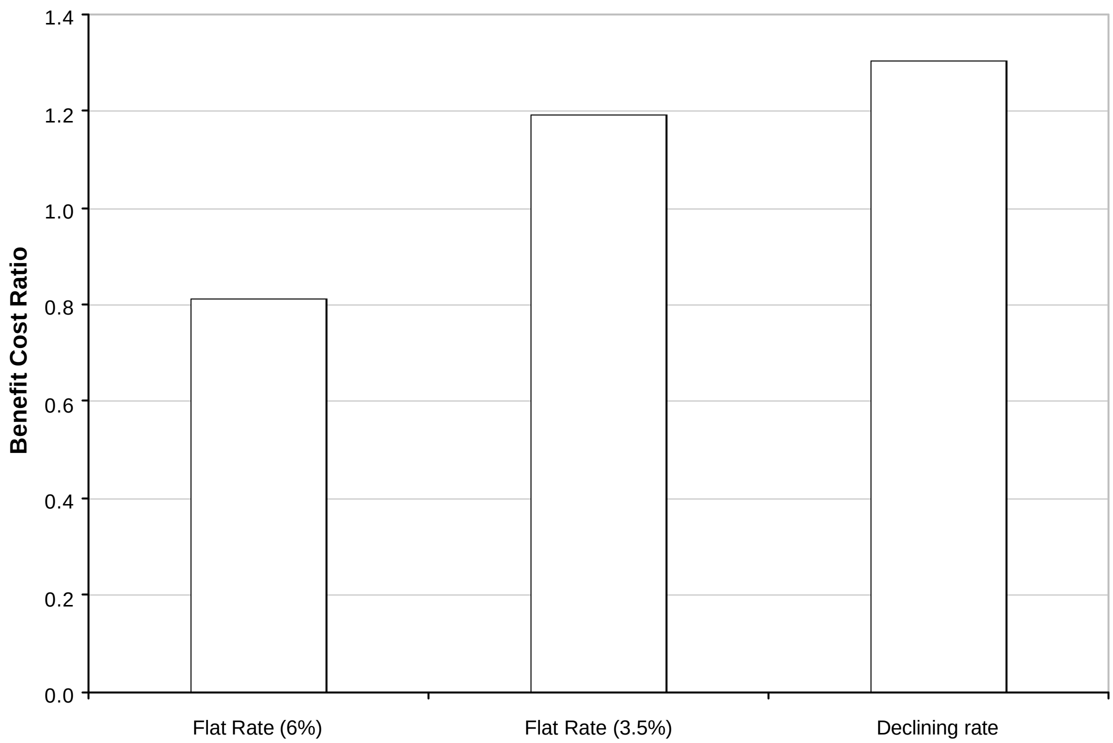
7.5 Flood defences

Declining discount rates may also have an effect on the economics of flood protection. Over the last ten years, flood-defence investment has been characterised by an annual expenditure that has been assumed to offset significant damage—a cost–benefit ratio much greater than unity.

OXERA was supplied with a stochastic model by Binne, Black & Veatch and the Environment Agency designed to assess the costs and benefits of investment in a particular cell (protected area) of flood defences for Shrewsbury. The model determines the net benefit of investment by comparing the damage suffered in a 'do nothing' scenario with damage where flood defences have been constructed. The benefits can then be compared with the costs of constructing and maintaining the defences.

Employing a 6% discount rate gives the result that flood defence investment does not pass the cost–benefit analysis. However, a benefit cost ratio (BCR) of approximately 1.2 is obtained with a 3.5% discount rate, and 1.3 with a declining rate, as shown in Figure 7.9.

Figure 7.9: Benefit–cost ratio for a particular cell of flood defences in Shrewsbury



Source: Shrewsbury FAS project estimates and OXERA calculations.

7.6 Health

Conventional procedures in cost–benefit analysis require that the money value of health benefits, and the money value of the costs needed to achieve those benefits, both be discounted at the same common rate. There has been some debate in the discounting literature about the validity of discounting future 'lives' at all, on the grounds that lives are 'equal' regardless of who lives them. This debate is outside the scope of this report. In the health economics literature there have been further debates about:

- dealing with 'latency', that is, how to value a health impact X years hence, including an extension to otherwise expected life years;
- dealing with rising relative valuations of good health states.

7.6.1 Latency

The correct procedure for dealing with latency is to discount any future health benefit at the time it occurs, back to the time when the expenditure is to be made to secure the benefit. Thus, if someone is aged 40 now, and is asked for his/her willingness to pay to secure a beneficial impact at age 65, the relevant calculation is:

$$WTP_{40,65} = p.\ WTP_{65,65}/(1+s)^{25}$$

where the first subscript denotes the age when WTP is sought, and the second indicates the time when the benefit accrues. The value p is the probability that the individual will survive from the age of 40 to the age of 65. Thus WTP at age 40 for a benefit at age 65 is equal to the WTP for someone aged 65 for an immediate benefit, the latter being discounted at the social time preference rate, and multiplied by the probability of survival in the intervening period.[44]

The impact of time-varying discount rates on the present value of latent health benefits would depend on the latency period. For the example shown, near-term discount rates would not vary significantly over, say, 30 years, because of the greater certainty of future discount rates and future economic activity in that period. Where latent effects extend beyond that time-horizon, however, present values would increase relative to the constant discounting case. This would have some policy implications for decisions about health R&D where effects could be greatest in the long-term. The same may be true for environmental improvements affecting human health, changes in health behaviour, etc. Overall, we would expect time-varying discount rates to justify some reallocation of expenditure from measures with immediate benefits to measures with long-term benefits.

7.6.2 Rising valuations of high health states over time

The correct procedure for dealing with health benefits and costs has been set out by Gravelle and Smith (2001), see also Jones-Lee and Loomes (1995). The central issues can be elicited by considering the basic cost-benefit equation:

$$NPV = \sum_t [B_t - C\ ]/(1+s)^t$$

This can be rewritten as:

$$NPV = \sum_t \frac{B_0(1+g.e)^t}{(1+s)^t} - \sum_t \frac{C_t}{(1+s)^t}$$

where B_0 is the value of benefits in the initial period, g is the growth rate of income (per capita) and e is the *income elasticity of willingness to pay for health.*[45] An approximation of the left hand side of this equation is:

$$PV(B) \approx \sum_t \frac{B_0}{(1+s-g.e)^t} = \sum_t \frac{B_0}{(1+s-k)^t}$$

where k is the growth rate of the value of a unit of health benefits and $k = g.e$.

The term $(1 + s - k)$ is often referred to as a 'net' discount rate, or sometimes in the environmental literature as an 'environmental discount rate'. However, it is advisable to keep s and g separate for presentational purposes.

[44] Cropper and Sussman, (1990), Hammitt (2000).

[45] Note that B is in money terms and hence is measured by WTP. The appropriate notion of elasticity is therefore the elasticity of WTP (price times quantity) not the conventional elasticity of demand (quantity only).

The observation made by Gravelle and Smith (2001) is that benefits in time t should already allow for a rising relative valuation so that $B_t = B_0.(1+g.e)^t$. No further adjustment is required and s is the proper and common discount rate for costs and benefits. The previous equations show that this procedure is the same as taking benefits in an initial year and compounding them forward at a growth rate $k = g.e$, which in turn is equivalent (approximately) to discounting constant annual benefits equal to the initial year benefits at a 'net' discount rate of $s - k$.

The argument in the health economics literature for using a *lower* discount rate for health 'benefits' arises when benefits are not monetised—ie, when the context is one of cost-effectiveness rather than cost-benefit. If, instead of *B*, a measure of physical benefit, say life-years saved *L*, is used, then there is no apparent way in which the *value* of *L* can be increased to allow for a rising relative valuation. This is because the notion of a rising relative value only makes sense if the benefit is measured in money units, *B*, rather than 'physical' units, *L*. In practice, however, *L* can have a rising relative value so that, say, a life year now might be worth $(1 + k)$ life years in one year's time, and so on. The idea of leaving life years in their original (initial year) units and lowering the discount rate, *s*, is then formally equivalent to allowing for the rising relative value of *L*.

The implications of a time-varying discount rate for health appraisal is then fairly straightforward. If the context is one of cost-benefit analysis, the procedure is to adopt a value of *s* that is declining. Care has to be taken to ensure that the *value* of health benefits is properly accounted for—ie, that B_t reflects rising valuations if appropriate. These rising relative valuations do not affect the underlying discount rate, $s(t)$. As with the latency issue, the longer the time horizon over which health benefits accrue, the greater will be the present value of those benefits if $s(t)$ is a declining function of time than if *s* were held constant.

Appendix 1: Implications of zero discounting

In this appendix we show that provided interest rates are positive, zero utility discounting implies that current generations should reduce their incomes to subsistence level to benefit future generations.

Formally, the individual is assumed to maximise utility, *U*, where

$$\max U = \sum_{t=0}^{T} \frac{U(C_t)}{(1+ r)^t}$$

where *C* is consumption, *t* is time and *?* is the pure time preference rate (utility discount rate). *U* is maximised subject to a constraint

$$\sum_{t=0}^{T} \frac{Y^* - C_t}{(1+r)^t}$$

*Y** is an expected income stream. Over all periods, income minus consumption (savings) must be zero[46].

The resulting condition[47] is

$$\frac{U'(C_t)}{U'(C_0)} = \frac{(1+ r)^t}{(1+r)^t}$$

The marginal utilities of consumption in each period must equal the ratio shown, and the ratio incorporates the utility discount rate and the interest rate.

If *?* = 0, the equation becomes

$$\frac{U'(C_t)}{U'(C_0)} = \frac{1}{(1+r)^t}$$

Now let *t* get very large. The right-hand side of the equation above is simply a discount factor, so as *t* gets large, the discount factor gets very small. In the limit (with *t* tending to infinity) the equation will be zero. This means that $U'(C_0)$ itself tends to infinity, (ie, the marginal utility of consumption in the current period goes to infinity). In turn, this means that consumption in the current period tends to zero, or, at the very least, to some minimal subsistence level. The conclusion is that as long as interest rates are positive, zero utility

[46] The formal basis for this constraint is that it is the equation of an intertemporal budget line. For the background see Varian (1996), Chapter 10.

[47] To prove this, work with just two periods, 0 and 1. Then U = U(C_0)+ U(C_1)/(1+?). Given the budget equation for two periods as well, maximise U subject to the constraint.

discounting would imply that current generations should reduce their incomes to subsistence level, in order to benefit future generations.

Appendix 2: Weitzman's formulation

A2.1 An illustrative example

Recall that the discount factor for a time period *t*, A_t is given by equation (A2.1)

$$A_t = \frac{1}{1+r_1} * \frac{1}{1+r_2} * \frac{1}{1+r_3} * ...... * \frac{1}{1+r_t} \tag{A2.1}$$

where, in the conventional approach, *r* is constant for all t. When *r* is uncertain, however, we may face several potential states of the world, each with an associated discount rate and probability of occurrence. For example, imagine two potential future states of the world, state a and state b, each with associated interest rates, r_1 and r_2, and probabilities of being realised, p_1 and p_2, where $p_1 + p_2 = 1$. Assuming that r_1 and r_2 are constant across time in each scenario, the associated discount factors for each scenario are:

$$A_{1t} = \frac{1}{(1+r_1)^t}$$

and

$$A_{2t} = \frac{1}{(1+r_2)^t}$$

Weitzman (1998) states that the certainty-equivalent discount *factor* should be used in such cases, rather than the simple average of the interest *rates* (r_1 and r_2), weighted by their probabilities. In this example, the certainty-equivalent discount factor is

$$\tilde{A}_t = \frac{1}{2}\left(p_1 A_{1t} + p_2 A_{2t}\right) \tag{A2.2}$$

By extension, the general expression of the certainty-equivalent discount factor is

$$\tilde{A}_t = \frac{1}{n}\sum_{i=1}^{n} p_i \frac{1}{(1+r_{it})^t} \tag{A2.3}$$

Where, as before $\sum p_j = 1$.[48] This term describes the expected present value of an extra dollar paid out or taken in at time *t*, for an individual who desires a fixed benefit in period

[48] The more general case for a continuous distribution of states of the world is shown in Appendix A2.2.

t and currently faces an uncertain, random, discount rate. The certainty-equivalent marginal discount *rate* associated with (A2.3) is given by

$$\tilde{r}_t = -\frac{\tilde{A}_t - \tilde{A}_{t-1}}{\tilde{A}_t} \qquad \text{(A2.4)}$$

Weitzman (1998) shows that under certain plausible circumstances, as t becomes very large, $\tilde{r}_t$ tends to the lowest rate observed in the scenarios considered. Formally, this result can be expressed as:

$$\lim_{t\to\infty} \tilde{r}(t) = r_{\min} \qquad \text{(A2.5)}$$

The certainty-equivalent *marginal* discount rate $\tilde{r}_t$ should not be confused with the certainty-equivalent *average* discount rate. The certainty-equivalent *marginal* discount rate is the rate of decline of the discount factor. It is the discount rate that is applied to the next time-interval.[49] In contrast, the average discount rate is a measure of the mean discount rate, applied over the entire time horizon. For instance, if the *marginal* discount rate starts at 3.5%, and declines to 0% by year 50, then the *average* discount rate over the period will be somewhere between the two extremes. Indeed, it turns out that the average discount is the 'harmonic' mean of $(1+r_1)^t$ and $(1+r_2)^t$. This implies that $\tilde{r}_t$ is the solution to

$$\frac{1}{(1+\tilde{r}_t)^t} = \frac{1}{n}\sum_{i=1}^{n} p_i \frac{1}{(1+r_{it})^t}$$

A2.2 The general formulation

In Weitzman's formulation there are n possible future scenarios, and n associated values for the interest rate, indexed by $j = 1,2,...,n$. The analysis is undertaken in continuous time, as opposed to discrete analysis. Scenario j in equation A2.6 has a probability of occurrence equal to p_j, where $p_j > 0$ and $\sum p_j = 1$. The discount factor for scenario j is given by

$$a_j(t) \equiv \exp\left(-\int_0^t r_j(t)\,dt\right), \quad j = 1,..., n \qquad \text{(A2.6)}$$

[49] In this report, and elsewhere in the literature, 'discount rate' refers to the marginal discount rate unless otherwise stated.

where $r_j(t)$ is the discount or interest rate at time t in scenario j.[50] The discount rate is close to today's interest or discount rate for small t, but approaching some other limit as time goes to infinity; (ie, $r_j^* \equiv \lim r_j(t)$). That is, from today's perspective the far-distant-future interest rate is a (non mean-reverting) random variable. The certainly-equivalent discount factor (ie, how much the extra dollar of the investment paid out or taken in at time t is worth today) is

$$A(t) \equiv \sum p_j a_j(t) \tag{A2.7}$$

The certainly-equivalent instantaneous discount rate, and the certainty-equivalent far-distant-future discount rate, are given by equations A2.8 and A2.9, respectively:

$$R(t) \equiv \frac{\dot{A}(t)}{A(t)} \tag{A2.8}$$

$$R^* = \lim_{t \to \infty} R(t) \tag{A2.9}$$

Given the above mathematical formulation, Weitzman's proposition is:

$$R^* = r^*_{\min} \tag{A2.10}$$

The proposition tells us that the interest rate for discounting events in the distant future should be its lowest possible limiting value, which corresponds to the properly-averaged certainty-equivalent discount factor. The result is partially dependent upon the discount rate being a non-mean reverting random variable, (ie, it exhibits persistence over time, and does not tend towards its long-run mean).

A2.3 An entertaining experiment: Weitzman's survey of economists

Weitzman provides an entertaining application of his central result in a follow-up article.[51] Weitzman asked over 2000 economists to give their 'professionally considered gut feeling' on the appropriate discount rates for climate change mitigation.

The results approximately followed a gamma distribution, with mean discount rate of 4%, and standard deviation of 3%. Taking this distribution as a reflection of the uncertainty about future states of the world, Weitzman then calculated the relevant average discount factor, and hence the implicit discount rate. He found that the discount rate declined approximately as shown in Table A2.1.

[50] Note that this need not be constant across time as assumed for scenarios in the main text.
[51] Weitzman (2001).

Table A2.1: Weitzman's rates obtained from fitting survey data to a gamma distribution

Time from present	Label	Marginal discount rate (%)
1–5 years	Immediate future	4
6–25 years	Near future	3
26–75 years	Medium future	2
76–300 years	Distant future	1
More than 300 years	Far-distant future	0

Appendix 3: Newell and Pizer's Model

A3.1 The model

The general form of the random walk model used by Newell and Pizer is shown in equations (A3.1) and (A3.2). This model is used because it captures uncertainty in the interest rate and allows for persistence. As shown by equations (A3.1) and (A3.2), the interest rate, r_t is modelled as being dependent upon the rate in the previous period, r_{t-1}, and a random component, e_t, which represents random changes to the interest rate. These random changes are related to changes in the previous period, e_{t-1}, as shown in equation (A3.2).

$$r_t = r_{t-1} + e_t \tag{A3.1}$$

$$e_t = r e_{t-1} + x_t \tag{A3.2}$$

As above, the interest rate reflects the long-run risk-free rate, which will approximate to the social time preference rate, s. Equation (A3.2) shows that the current changes in the interest rate are driven by the current random change, x_t , and changes that occurred in the past. r measures the extent to which past changes in the interest rate persist in future periods. A value of r near one means that the changes in the interest rate can persist over time so that we will observe the interest rate staying consistently low or high for many periods. In contrast, a value of r near zero means that a period of abnormally high interest rates are equally likely to be followed by high or low rates. It is the former condition that is required for the Weitzman result to hold in practice, and that Newell and Pizer test.

A3.2 Forecasting the certainty-equivalent discount rate

Newell and Pizer estimate the parameters of the random walk model above, and use their results to simulate the time path of future interest rates. The random walk model is driven by the random changes x_t, and thus different manifestations of x_t will generate different predicted paths for future interest rates. Similarly, different values of r will generate different predictions.

In order to construct the uncertainty of the interest rate in the future, Newell and Pizer simulate 100,000 of such paths using US data. Each of these simulations can be thought of as representing potential future scenarios. Based on these simulations they numerically calculate the certainty-equivalent interest rate, $\tilde{r}_t$, in a manner similar to Weitzman (1998). It should be noted that this approach assumes that the uncertainty regarding the discount rate begins immediately, but that its effects are gradual. In this environment there is no need to choose a value for T. Box A3.1 details the precise steps used by Newell and Pizer for constructing the numerical approximation to the certainty-equivalent discount rate.

Box A3.1: The procedure for simulating certainty-equivalent interest rates

To simulate 100,000 possible future discount rate paths for each model, starting in 2002 and extending 400 years into the future:

1. Begin with point estimates of initial values for the interest rate, and estimate the joint covariance matrix from the random walk model (the estimates of the variances and covariances of the parameters of the model (eg, in the case of equation (4.2) this would simply be the variance of r)).
2. Assume, for each simulated interest rate path, that the parameters of the joint covariance matrix are jointly normally distributed, and draw values for each parameter. This allows the random walk to vary over time, and not be strictly governed in the long-run by an initial r.
3. Draw values for stochastic shocks *x*.
4. Create the shocks e_t by defining e_t in each time period, using equation (4.2), or more complicated structures including more lagged values of r.
5. Use the simulated values of e to simulate discount rates for the random walk model based on equation (4.1), and (4.2), or a more complicated error structure.
6. Compute the expected certainty-equivalent discount factor $E[P_t]$ as described in appendix 4.
7. Compute the certainty-equivalent discount rate as the discrete approximation to equation (V) in appendix 4 given by $\tilde{r}_t = -\dfrac{dE[P_t]}{E[P_{t+1}]-1}$

Newell and Pizer provide the following explanation of their model. Noting that the certainty-equivalent discount factor, $E[P_t]$, and rate of discount, $\tilde{r}_t$, between adjacent periods at time *t* are given by equations A3.3 and A3.4, respectively

$$E[P_t] = E[\exp(-\sum_{s=1}^{t} r_s)] \tag{A3.3}$$

$$\tilde{r}_t = -\frac{dE[P_t]/dt}{E[P_t]} \tag{A3.4}$$

Following Newell and Pizer (2001, page 8), equation A3.3 is evaluated using equation A3.8 to separate it into two parts, depending on the two components of discount rate uncertainty, the mean discount rate and e_t

$$E[P_t] = E[\exp(-ht)] \cdot E\{\exp(-\sum_{s=1}^{t} e_s)\} \tag{A3.5}$$

Assuming that η is distributed normally mean $\bar{h}$ and variance s_h^2, one can show that the first part of A3.3 is equal to

$$E[\exp(-ht)] = \exp\left(-\bar{h}t + \frac{t^2 s_h^2}{2}\right) \tag{A3.6}$$

The second term is given by

$$E[P_t] = \exp(-\bar{h}t + \frac{t^2 s_h^2}{2})\exp\left(\frac{s_x^2}{2(1-r)^2}\left(t - \frac{2(r - r^{t+1})}{1-r} + \frac{r^2 - r^{2t+2})}{1-r^2}\right)\right) \tag{A3.7}$$

Combining equations A3.6 and A3.7, gives us A3.8; that is

$$\tilde{r}_t = -\bar{h} - ts_h^2 - s_x^2\Omega(r,t) \tag{A3.8}$$

This shows how the certainty-equivalent discount rate is a function of ρ, time, and the variance of the error terms e_t and ξ_t. It is these terms that are estimated by the random walk model for the numerical estimation of $\tilde{r}_t$. Lastly, the specific form of $W(r,t)$ is

$$\Omega(r,t) = \frac{1 - r^2 + 2\log(r)\, r^{t+1}(1 + r - r^{t+1})}{2(1-r)^3(1+r)} \tag{A3.9}$$

Appendix 4: Gollier's Model

A4.1 The basic framework

Gollier uses a simple growth model, in which in each period consumers are endowed with consumption levels c_t, where consumption in the future is considered uncertain due to uncertain growth rates. Growth follows a random walk pattern. In an uncertain world, the decision-maker must construct expectations about the future in order to determine the optimal discount rate. The decision maker's objective is as follows

$$\max U(z_t) = \sum_{t=0}^{\infty} b^t E[u_t(\tilde{z}_t)] \qquad \text{(A4.1)}$$

$\tilde{z}_t$ represents the fact that future consumption levels are uncertain, since they contain the random growth variable $\tilde{g}_t$, and $E[.]$ represents the expectation operator, and shows that the individuals are making a judgement about what they think will happen in the uncertain future. In this environment, the optimal discount rate in period zero for costs and benefits accruing in period one becomes

$$1+s_1 = \frac{u'(c_0)}{bE[u'(c_0\tilde{g}_1)]} \qquad \text{(A4.2)}$$

Analogously, the optimal discount rate, s_2, in period zero for costs and benefits that accrue in period two becomes

$$(1+s_2)^2 = \frac{u'(c_0)}{b^2 E[u'(c_0\tilde{g}_1\tilde{g}_2)]} \qquad \text{(A4.3)}$$

where the expected level of consumption in period two is represented by $E[c_0\tilde{g}_1\tilde{g}_2]$, (ie, consumption is increased by random growth in periods one and two). Thus, the optimal discount rate for costs and benefits in period t, upon investments made today, can be represented in general by

$$(1+s_t)^t = \frac{u'(c_0)}{b^t E\left[u'\left(c_0\prod_{i=1}^{t}\tilde{g}_i\right)\right]} \qquad \text{(A4.4)}$$

where the term $\prod_{i=1}^{t}\tilde{g}_i$ represents the product of all random growth from now until period t, and thus $E\left[c_0\prod_{i=1}^{t}\tilde{g}_i\right]$ represents expected consumption in period t. It is within this framework that Gollier analyses the conditions under which the optimal discount rate s_t is declining over time.

A4.2 The yield curve

As in the text, Gollier decomposes the socially optimal discount rate, s_2, from (A5.3) as follows. First define the short-term gross interest rate that will prevail in period $t=1$, as a function of current consumption, $r(c)$ as follows (where $c = c_0\tilde{g}_1$)

$$r(c) = \left(1+s_1^1\right) = \frac{u'\left(c_0\tilde{g}_1\right)}{bE\left[u'\left(c_0\tilde{g}_1\tilde{g}_2\right)\right]} \tag{A4.5}$$

where s_1^1 is the net short term interest rate. Note that this is simply the analogous socially optimal discount rate for returns in period $t=2$, but taken from the perspective of period $t=1$ rather than $t=0$. From this we can see that the future interest rate s_1^t determines the current (ie, at $t=0$) long-term interest rate s_2 [52]

$$\left(1+s_2\right)^2 = \left(1+s_1\right)E\left[\left(1+s_1^1\right)\right] \tag{A4.6}$$

The shape of the yield curve comes from a comparison of s_1 and s_2. This is analogous to equation (A4.5).

It should be evident from the above that the (expected) short-term interest rates are increasing in (expected) growth, (ie, $r(c)$ is decreasing in g_t). Thus a declining yield curve could arise from a situation in which the expected growth in period $t=2$, $\tilde{g}_2$, is much lower than expected growth in period $t=1$, $\tilde{g}_1$ [53]. This suggests that there should be a declining discount rate over time in the presence of expected future slowdowns in economic growth.

A4.3 No recession result

When no recession is assumed, and growth is similar across periods Gollier shows that in order for the yield curve to be declining in the level of consumption *c*, the agents of the

[52] In fact the true representation is: $\left(1+s_2\right)^2 = \left(1+s_1\right)E\left[\tilde{W}.\left(1+s_1^1\right)\right]$, where $\tilde{W} = \dfrac{E\left[u'\left(c_0\tilde{g}_1\right)\right]}{u'\left(c_0\tilde{g}_1\right)}$, and reflects the ratio of expected and actual (after the realisation of random growth) marginal utilities in time period $t=1$. In expectation, this term is equal to 1 and the term can be expressed as in (9).

[53] Formally, Gollier suggests that this could be the case when the distribution of $\tilde{g}_2$ is first order stochastically dominated by that of $\tilde{g}_1$. That is, the expected value of $\tilde{g}_2$ is always less than or equal to that of $\tilde{g}_1$.

economy must have preferences which are characterised by decreasing relative risk aversion. This result comes from the first derivative of (A4.5) with respect to c, and determining when this derivative is negative. The first derivative of (A4.5) can be stated as:

$$zr'(c) = r(c)\left[R(c\tilde{g}_2) - R(c)\right] \tag{A4.7}$$

where $R(c)$ represents the index of relative risk aversion of the agents in period one, and $R(c\tilde{g}_2)$ the equivalent term in period two. Clearly (A4.7) is strictly negative when $R(c\tilde{g}_2) < R(c)$. Since we have assumed that there is no risk of recession in the economy, it is clear that $c\tilde{g}_2 > c$, thus wealth increases as time passes. It follows that the index of relative risk aversion must be decreasing over time and with wealth, in order for the yield curve, and the socially optimal discount rate, to be declining.

A4.4 Recession result

Relative prudence is measured by

$$P(c) = -c\frac{u'''(c)}{u''(c)} \tag{A4.8}$$

A4.5 Simulation method

The discount rate at time t is shown in (A4.9)

$$(1+d_t)^t = \frac{u'(c)}{b^t Eu'\left(c\prod_{i=1}^{t}\tilde{g}_i\right)} \tag{A4.9}$$

Gollier’s simulation employs a power utility function for which the first derivative is shown in (A4.10).

$$u'(c) = (c-k)^{-g} \tag{A4.10}$$

Where k represents some minimum or subsistence level of consumption. Assuming a simple case in which $g = 1$, $b = .99$, and growth is 75% and 0% with probability 0.63 and 0.37 respectively leads to an explicit expression for (A4.9):

$$(1+d_t)^t = \frac{(c_0-k)^{-1}}{0.99^t * \left[0.63 * 1.75^{t*}(c_0 * 1.75^t - k)^{-1} + 0.37 * (c_0-k)^{-1}\right]} \tag{A4.11}$$

Notice that the utility function displays DRRA and the distribution of future growth is non-negative with certainty, (ie, there is no risk of recession). These are the sufficient conditions for a declining yield curve.

Appendix 5: Chichilnisky's Approach

Chichilnisky's criterion can be represented in the following objective function[54]

$$\max_{c,s} a\int_0^\infty u(c_t, q_t)e^{-rt}dt + (1-a)\lim_{t\to\infty} u(c_t, q_t) \tag{A5.1}$$

Utility (u) is a function of consumption (c) and the resource stock (q) at each time period (t) and e^{-rt} is the conventional exponential discount factor. $u(c_t, q_t)$ is assumed the same for all dates t, so that generations are assumed to be the same in the way in which they rank alternatives.

Intuitively, the *lim* term reflects the sustainable utility level attained by a particular policy decision regarding c_t and s_t. This can be interpreted as the well-being of generations in the far distant future. Chichilnisky's approach is a mixture of the two approaches seen so far: a generalisation of the discounted utilitarian approach, mixed with an approach that ranks paths of consumption and natural resource use according to their long-run characteristics or sustainable utility levels. This criterion can be applied under the two main axioms regarding the ranking of alternative utility paths[55]. Notice that $\alpha \in (0,1)$, can be interpreted as the weights that the decision-maker applies to each component of the criterion, with α providing the weight upon the present generation, and $(1-\alpha)$ representing the weight placed upon the future generation.

Another way of understanding this approach is to observe that it is not possible to attach equal weights to consumption in all periods, present and future, because if one did, the weights would sum to infinity. A natural response is therefore to concentrate some weights on the present $\int_0^\infty u(c_t, s_t)e^{-rt}dt$ and some on the future $\lim_{t\to\infty} u(c_t, s_t)$. This is precisely what the Chichilnisky criterion achieves.

[54] This section draws from Heal (1998), Chapter 5.

[55] Strictly, the model only has a solution when these axioms hold.

Appendix 6: The Li and Löfgren approach

This objective function employed by Li and Löfgren is represented in equation (A6.1).

$$U = aU_1 + (a - 1)U_2 = \int_0^\infty u(c_t, s_t)p(t)dt \tag{A6.1}$$

where

$$U_1 = \int_0^\infty u(c_t, s_t)\exp(-rt)dt \tag{A6.2}$$

$$U_2 = \lim \int_0^\infty u(c_t, s_t)\exp(-dt)dt \tag{A6.3}$$

As before, α and $(1 - \alpha)$ are the weights, which weigh the objectives of each decision maker. $r(t)$ is the discount factor, which is a function of time. More explicitly, the discount factor is derived from a weighted sum of the discount rates of the utilitarian and the conservationist; (ie, $p(t) = \exp(\ln\{ b\exp(-dt) + a\exp(-rt)\})$.

A6.2 Li and Löfgren utility discount rate

The effective utility discount rate in Li and Löfgren is given by

$$a(t) = \frac{-\ln\{(1 - a)\exp(-dt) + a\exp(-rt)\}}{t} \tag{A6.4}$$

A time profile of discount rates can therefore be found by merely selecting the discount rates for the conservationist and the utilitarian, d and r respectively. For example, if the conservationist discounts the future at a rate of zero: $d = 0$, the discount factor becomes:

$$p(t) = (1 - a) + a\exp(-rt) \tag{A6.5}$$

In the distant future when t is large, this term has a minimum value of $(1 - a)$, the weight attached to the conservationist, or future generations. It is in this way that the effective discount *rate* can be thought of as declining over time to zero.

Appendix 7: Hyperbolic Discounting

Following Loewenstein and Prelec (1992), define the discount factor to be:

$$d_t = \frac{1}{(1+kt)^{h/k}} \tag{A7.1}$$

The parameter h reflects 'time perception'. If h = 0, individual time periods are perceived as passing extremely fast: infinitely so, in fact. As h tends to ∞, time is not perceived to pass at all. The parameter k measures the deviation of the hyperbolic discounting function from the standard exponential model. As k approaches zero, d_t approaches the exponential function.

Several forms of hyperbolic discount function can be derived from the previous equation. If k = 1

$$w_t = \frac{1}{(1+t)^h} \tag{A7.2}$$

This is the form used by Harvey (1986). A popular form of this equation is

$$w_t = \frac{1}{t^h} \tag{A7.3}$$

which is used, for instance, in the work of Cropper et al. (1992, 1994).

If h/k = 1

$$w_t = \frac{1}{(1+k.t)} \tag{A7.4}$$

This is the form used by Mazur (1987).

These functional forms are *hyperbolic*, as opposed to exponential. To gauge the effects of hyperbolic discounting, fix an arbitrary date, say t = 10, and find the values of h and k in the generalised Loewenstein-Prelec equation above that, which would give the same result as conventional discounting at some rate, say s = 0.05. In other words, find the values of h and k that solve

$$\frac{1}{(1+s)^{10}} = \frac{1}{(1+10)^h} = \frac{1}{(1+10k)} \tag{A7.5}$$

If s = 0.05 and t = 10, then h = 0.202 and k = 0.0568. Setting t = 1 and then t = 50 the equation now results in

$$w_{t=1} = \frac{1}{2^{0.202}} = 0.869$$

and

$$w_{=50t} = \frac{1}{51^{0.202}} = 0.454 \tag{A7.6}$$

Similar calculations can be carried out for the other equation.

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