The Basic RBC model with Dynare

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Abstract

This paper briefly describes the basic RBC model without steady-state growth.It uses dynare software to simulate the model under standard RBC parameter values.Finally,it briefly describes the basic commands of dynare for stochastic and deterministic simulation for the basic RBC model.

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1 The Model

1.1 Households

$$\max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right) \right\} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \gamma \ln C_t + (1-\gamma) \ln\left(1 - N_t\right) \right\}$$

subject to:

$$w_t N_t + r_t K_{t-1} + \mathcal{R}_t = C_t + I_t + T_t \tag{1}$$

$$K_t = (1-\delta) K_{t-1} + X_t \tag{2}$$

The behaviour of the representative household is described by the following three equations:

[Marginal Utility of Consumption] :

$$\frac{\gamma}{C_t} = \lambda_t \tag{3}$$

[Labour Supply] :

$$\frac{1-\gamma}{\gamma} \frac{C_t}{1-N_t} = w_t \tag{4}$$

[Euler for Capital] :

$$\lambda_t = \beta \lambda_{t+1} \left(1 - \delta + r_{t+1}\right) \tag{5}$$

1.2 Firms

Profit maximization:

$$\max_{\{K_{t-1}, N_t\}} \mathcal{R}_t = e^{A_t} \left(K_{t-1}\right)^a \left(N_t\right)^{1-a} - w_t N_t - r_t K_{t-1} \tag{6}$$

[Labour Demand] :

$$w_t = (1-a) \frac{Y_t}{N_t} \tag{7}$$

[Demand for Capital] :

$$r_t = a\frac{Y_t}{K_{t-1}} \tag{8}$$

1.3 Government

Government budget constraint is balanced each period:

$$T_t = G_t \tag{9}$$

2 Equilibrium:

We have the following vector with 8 endogenous variables $\begin{bmatrix} Y_t & C_t & N_t & X_t & K_t & r_t & w_t & \lambda_t \end{bmatrix}'$ and a vector of two exogenous shocks $\begin{bmatrix} A_t & G_t \end{bmatrix}'$ which satisfy the following 8 equations:

[Marginal Utility of Consumption] :

$$\frac{\gamma}{C_t} = \lambda_t \tag{10}$$

[Labour Supply] :

$$\frac{1-\gamma}{\gamma}\frac{C_t}{1-N_t} = w_t \tag{11}$$

[Euler for Capital] :

$$\lambda_t = \beta\lambda_{t+1}\left(1-\delta+r_{t+1}\right) \tag{12}$$

[Law motion of Capital] :

$$K_t = \left(1-\delta\right)K_{t-1} + X_t \tag{13}$$

[Production Function] :

$$Y_t = e^{A_t}K_{t-1}^{a}N_t^{1-a} \tag{14}$$

[Labour Demand] :

$$w_t = \left(1-a\right)\frac{Y_t}{N_t} \tag{15}$$

[Demand for Capital] :

$$r_t = a\frac{Y_t}{K_{t-1}} \tag{16}$$

[Resource Constraint] :

$$Y_t = C_t + X_t + G_t \tag{17}$$

3 Steady-state :

[Marginal Utility of Consumption] :

$$\frac{\gamma}{C} = \lambda \tag{18}$$

[Labour Supply] :

$$\frac{1-\gamma}{\gamma}\frac{C}{1-N} = w \tag{19}$$

[Euler for Capital] :

$$1 = \beta\left(1 - \delta + r\right) \tag{20}$$

[Law motion of Capital] :

$$X = \delta K \tag{21}$$

[Production Function] :

$$Y = e^{A} K^{a} N^{1-a} \tag{22}$$

[Labour Demand] :

$$w = \left(1 - a\right)\frac{Y}{N} \tag{23}$$

[Demand for Capital] :

$$r = a\frac{Y}{K} \tag{24}$$

[Resource Constraint] :

$$Y = C + X + G \tag{25}$$

4 Some Hints

i)You always finish a command with a semicolon.

ii)State endogenous and exogenous variables must be in $t-1$ period.While forward looking variables must be in $t+1$ period.

iii)In the first-order approximation your policy functions must be the same with the solvek function Klein(1998) method.

iv)You can enter a comment after //.

5 Introduce the model in Dynare

5.1 Endogenous Variables & Structural Parameters

In order to introduce the equations of your model in Dynare,firstly you should denote some global variables that will play the role of the endogenous variables,the exogenous shocks,the structural parameters and the steady-state values of your model.In the standard RBC model we have the following vector of endogenous variables $\left[\begin{array}{cccccccc} Y_t & C_t & N_t & X_t & K_{t-1} & r_t & w_t & \lambda_t \end{array}\right]'$.Moreover,we assume that economy is hitted by two exogenous states $\left[\begin{array}{cc} A_t & s_t^g \end{array}\right]'$ which are governed by two AR(1) processes,finally we have two exogenous iid processes $\left[\begin{array}{cc} \varepsilon_t^a & \varepsilon_t^g \end{array}\right]'$.In Dynare you must denote two blocks of variables the endogenous variables(endogenous control and states and exogenous states) and the exogenous shocks (the iid processes) with the following commands:

Example 1

```
var y c n x k r w lam a sg;     //Endogenous variables & AR processes
varexo ea eg;                   //Exogenous shocks
```

Finally,you must denote which letters will be treated as structural parameters or as steady-state values by Dynare.In the RBC model will have $\left[\begin{array}{cccccc} \beta & \alpha & \gamma & \delta & \rho_a & \rho_g \end{array}\right]'$ and the steady state value $[\overline{s}^g]'$.

Example 2

```
parameters gam beta delta alpha rhoa rhog sgs;   // Parameters
```

If you introduce your model in a first order approximation you will also need to denote the steady state values of your endogenous variables.In this case you will use as many more parameters as the steady-state values of your model.Notice that you should use two different letters one for the endogenous variable (y) and one for its steady-state value (ys).In this case the commands are:

Example 3

```
var y c n x k r w lam a sg;                                                //Endogenous variables
varexo ea eg;                                                              //Exogenous shocks
parameters gam beta delta alpha rhoa rhog sgs ys cs ns xs ks rs ws lams;   //Parameters
```

Before you enter the equations of the model you must assign a value to each of the parameters that you denoted above,i.e.:

Example 4

```
gam=0.3;
beta=0.99;
delta=0.025;
.....
```

5.2 Equations

You can write the equations of the model in Dynare with three different forms:i)in levels,ii)in exp-log form and iii)in a first order approximation form or linear form.The choice of the form depends on your needs.Dynare uses Schmitt-Grohe & Uribe 2004 in order to find a first and a second order approximation of your model.The second moments that Dynare calculates will generally be different in the two forms (i),(iii).Notice that all endogenous/exogenous states variables must be written in period $t-1$ so as to be considered as states by Dynare.

5.2.1 In Levels

In this case,dynare will calculate steady-state values of the endogenous variables and then will take a first (and second) Taylor order approximation around this steady-state.The commands are:

Example 5

```
model; //with this command i denote that i will introduce the model in levels
gam/c=lam;// (1) Marginal Utility/Lagrange Multiplier
((1-gam)/gam)*(c/(1-n))=w;  //(2) Labour Supply
lam=beta*lam(+1)*(1-delta+r(+1));  //(3) Euler for Capital
k=(1-delta)*k(-1)+x;  //(4) Law motion of Capital
```

An important feature of Dynare is that if an endogenous variable is a state variable you must denote by introducing in period (-1),or before.Thus,capital is written k(-1).

Example 6

```
y=exp(a)*(k(-1))^alpha*n^(1-alpha);  //(5) Production Function
w=(1-alpha)*(y/n); //(6) Labour Demand
r=alpha*(y/k(-1)); //(7) Demand for Capital
y=c+x+sg*y; //(8) Resource Constraint
a=rhoa*a(-1)+ea; // (9) AR(1) Productivity shock
log(sg)=(1-rhog)*log(sgs)+rhog*log(sg(-1))+eg; //(10) AR(1) Government Expenditures
end;
```

5.2.2 In Approximated/Linear form

You introduce the first order approximation of your equations,i.e. the resource constraint is:

Example 7

model (linear);

...

*ys*y=cs*c+xs*x+sgs*ys*(y+sg);*

....

end;

5.3 Steady-state

If you have introduced your equations in levels form.Dynare will calculate the steady-state of your model using......However,you must give initial values for all your endogenous variables which are quite accurate.We follow the following procedure in order to find the initial values for the steady state.We use these values for the basic calibration of the model:

β	α	γ	δ	ρ_a	ρ_g
0.99	0.3	0.3	0.025	0.95	0.95

We solve for the steady state:

$$r = \frac{1}{\beta} - 1 + \delta$$

$$K = \frac{a}{r} Y$$

We set initially $Y = 1, e^A = 1, A = 0$.

$$X = \delta K$$

$$N = \left[\frac{Y}{AK^a} \right]^{\frac{1}{1-a}}$$

$$w = (1 - a) \frac{Y}{N}$$

$$C = \frac{\gamma}{1-\gamma} \left(\frac{1-N}{N} \right) Y (1 - a)$$

$$G = Y - C - X$$

$$\lambda = \frac{\gamma}{C}$$

We use this values to give the initial values for steady state.The commands are:

Example 8

```
initval;
...
c=0.6824;
sg=0.2;
a=0;ea=0;eg=0;
...
end;
steady;
```

6 Simulations

6.1 Deterministic

Dynare uses a relaxation algorithm (see Juillard 1996) in order to solve numerically the non-linear equations.In this case you must introduce your equations in levels.Dynare produce series for all the endogenous variables of the model when the economy is hitted by a temporary or permanent exogenous deterministic shock.The model is deterministic so that agents have perfect foresight for the exogenous shock.

6.1.1 Temporary Shock

In this case the economy is hitted by a temporary shock perfectly predictable for one or many periods and then returns back to the initial steady-state of the economy.The commands are:

Example 9

```
shocks;
var sg;  // A shock to the ratio of government spending to output
periods 1:3 3:4; //For periods 1:3 government spending is reduced to 0.3 and for periods
3:4 is reduced to 0.25 then goes back to initial steady-state
values 0.3 0.25;
end;
```

6.1.2 Permanent

If the shock is permanent the economy will converge to the new steady-state.You must specify the new steady-state:

Example 10
endval;
....
sg=0.2;
...
end;

If you want to study a permanent decrease in period 1 to the ratio of government spending with respect to output you can assign the same values to all the endogenous variables of the model and the new value for the exogenous permanent shock (s^g).

6.2 Stochastic

In the case you want Dynare to first-order approximate your model around a steady-state and perform simulations,compute moments proceed to variance decomposition and graph irfs you can call Dynare to do it by following the illustrated steps.Firstly,you must specify the standard deviations of the exogenous iid shocks that will hit your economy.The commands are:

Example 11
shocks;
var ea; //the productivity(TFP) shock
stderr 0.007; // the standard deviation of TFP shock
end;

Then you call Dynare to first (or second) order approximate your model.Dynare uses Schmitt-Grohe & Uribe 2004 algorithm in order to approximate the equations of the model.Notice that when you first order approximate your model their solution is identical with Klein 1998 and thus policy functions must be the same.The command for stochastic simulation is:

Example 12 *stoch_simul(hp_filter=1600,order=1,IRF=0,nomoments);*

This command has many options the most important of them are:

First or Second order Approximation	order=1 or 2
The λ coefficient on HP-filter	hp_filter=1600(Quarterly),14400(yearly)
The No of simulated series	periods=1000
No IRFs	IRF=0
Does not print second moments	nomoments

7 Results-Output of Dynare

7.1 Deterministic Case

In the workspace you can find:

oo_.	contains all the variables
y,c,x,...	the endogenous simulated series
ys0	initial steady-state

7.2 Stochastic Case

i) The irfs of each variable, say "y", with respect to exogenous shock "ea" is saved in a vector with the name "y_ea".

ii) All the important outputs of Dynare are saved in a global variable which is called "oo_",so you can either call:

The steady state values	"oo_.steady_state"
The variance-covariance matrix	"oo_.var"
The mean of the endogenous variables	"oo_.mean"
The variance Decomposition	"oo_.gamma_y{7}"
The IRFs	"oo_.irfs"

7.2.1 Compute Relative Volatilities

i) Log-Linear form (Command : model (linear);)

The steady-state and the mean values of all the endogenous variables are zero. So we just divide the standard deviation (square root of the diagonal of variance-covariance matrix) with the standard deviation of output.

Command : statistic=100*sqrt(diag(oo_.var(1:8,1:8)))/oo_.var(1,1) ;

ii) In Levels form

The mean of the endogenous variables have a non-zero value. So we use the following formula:

$$relvol(x_t) = \frac{\frac{[var(x_t)]^{\frac{1}{2}}}{mean(x_t)}}{\frac{[var(y_t)]^{\frac{1}{2}}}{mean(y_t)}}$$

where x_t :is each endogenous variable of the model, and y_t : output.

Commands:

Example 13

```
statistic=100*sqrt(diag(oo_.var(1:8,1:8)))/oo_.var(1:8) ;
rel=statistic(1:8)./statistic(1,1)
```

Useful References

Tommaso Mancini Griffoli 'User Guide'.

Francisco Barillas,Ricardo Colacito et al.'Practicing Dynare'.

Marco Ratto.'Analysing DSGE models with global sensitivity analysis'.Computational Economics 2008

Stephanie Schmitt-Grohe and Martin Uribe.'Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function'.Journal of Economic Dynamics and Control, vol. 28, January 2004, pp. 755-775.

Links

http://www.dynare.org

http://www.dynare.org/phpBB3/(Dynare forums,you can find very useful examples and ask your questions,for sure you will receive answers.)