On informative advertising and product differentiation

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Abstract

We study informative advertising within a random utility, non-localized competition model of product differentiation. Quasi-concavity of profits may fail, as each firm may prefer to deviate to a high price, targeting consumers who only become informed about its own product. When a symmetric equilibrium exists, it is unique. Increasing the number of firms may increase or decrease the equilibrium price. Advertising tends to be suboptimal when product differentiation is low and excessive otherwise. We also revisit Grossman and Shapiro [Grossman, G., Shapiro, S., 1984. Informative advertising with differentiated products. Review of Economic Studies 51, 63–81], focusing on similarities in constructing a symmetric equilibrium in the two models, but also illustrating differences in their welfare properties.

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1. Introduction

In differentiated products markets, consumers typically do not know all the competing product varieties or their prices. Since each consumer’s purchasing choices are made from a
restricted set, an important role exists for advertising, as firms may wish to inform consumers about their products and prices. In this paper we study informative advertising within a product differentiation oligopoly model with non-localized competition. We build on Perloff and Salop’s (1985) “random utility” model, one that has also provided the basis for important analyses in the field.\(^1\) In this setting we introduce informative advertising, as first formalized by Butters (1977). Naturally, in such a model, firms’ pricing and advertising decisions are jointly endogenous.\(^2\)

The main features of our model are as follows. A number of firms compete with differentiated products and are able to inform the consumers via costly advertising about the existence of their products and their prices. Firms choose prices and advertising intensities simultaneously. Consumers choose to purchase a unit of the product that leaves them with the highest net surplus. We examine the existence of a symmetric equilibrium and explore its comparative statics and other properties. Specifically, our main focus is on symmetric equilibria where all consumers would purchase one unit when they receive an ad from at least one firm.

Understanding how informative advertising interacts with a random utility non-localized model of product differentiation is important, as such a model represents one of the most straightforward ways to examine advertising in oligopoly. The analysis presents some technical challenges: while constructing the symmetric equilibrium, we find that the profit function may not be quasi-concave in each firm’s own price over the entire range of relevant prices. Equilibrium existence is, thus, not guaranteed and the solution to the first-order conditions represents an equilibrium only for a certain range of the parameters, although local concavity holds. The economic intuition for the possible lack of quasi-concavity is important. Under standard assumptions about informative advertising, consumers that have not received an advertisement from a given firm cannot purchase its product. Thus, with positive probability, some consumers have only received advertisement from one and only one firm — this firm then enjoys monopoly power relative to these “captive” consumers and may have an incentive to raise its price to a high level. Of course, whether such a high price strategy is profitable or not depends (among other factors) on how many consumers become informed from advertising, which is endogenously determined. It follows that, when constructing a symmetric equilibrium, we have to examine possible deviations not only locally but also to much higher levels.

While profit functions are not, in general, quasi-concave in price, a symmetric equilibrium does exist for a large space of parameters and we prove that, if it exists, it is unique. When we examine the comparative statics of the equilibrium, we find among other results that an increase in the number of firms does not necessarily reduce the equilibrium price. Still, increasing the number of firms decreases the per-firm profit, implying that, if entry were endogenized, there would be a

\(^1\) See e.g. Wolinsky (1986) and Anderson et al. (1992) for a review.

\(^2\) Advertising in our model informs consumers about both a product’s existence and its price. As we discuss in detail below, the closest work to ours is by Grossman and Shapiro (1984) who have studied advertising in the context of Salop’s (1979), localized competition, “circle” model. Alternative formulations exist. In Meurer and Stahl (1994) consumers observe prices while firms decide whether to inform them about product characteristics; in Bester and Petrakis (1995) consumers know that two firms exist and the price of the product in their region but only learn the price of the other firm once they receive an ad; in Baye and Kovenock (1994) advertising may inform consumers about a commitment to a lower price than that of the rival. Anderson and Renault (2006) study advertising content (i.e. product vs. price information, or both). Of course, besides direct information, advertising also plays other roles, including signaling (e.g. Kihlstrom and Riordan, 1984), coordination (Bagwell and Ramey, 1994) or persuasion (Dixit and Norman, 1978; Bloch and Manceau, 1999), to mention just a few. See Bagwell (2005) for a review of research on advertising, as well as a collection of papers on “Advertising and Differentiated Products” by Baye and Nelson (2001) that includes evidence from a number of markets.
unique zero profit equilibrium number of firms. We also explore the role of the advertising cost: an increase in this cost reduces the equilibrium advertising levels and increases prices; importantly, it may either decrease or increase the equilibrium profit.

Informative advertising can be studied within a variety of underlying oligopoly models. One may wonder whether the “complication” described above about the shape of the profit functions in our setting is not also present in other models of informative advertising, in particular in the well-known Grossman and Shapiro (1984) model. We show that, in fact, it is present but, by focusing on local deviations around the symmetric equilibrium, previous studies have implicitly assumed that a deviation to a high price is not profitable. We illustrate this potential complication in the Grossman and Shapiro (1984) model. We characterize the profit functions over the entire range of relevant prices and discuss the second-order conditions. Since consumers do not know about a product if they have not received information, which firms may be the effective closest neighbors of each firm for each consumer becomes random. Due to the failure, in general, of quasi-concavity, results presented for the characterization of a symmetric equilibrium hold only for a restricted parameter space, while for other cases the intuition from these is not valid. We also observe that the underlying models — the Salop (1979) model on a circle and the Perloff and Salop (1985) random utility model with uniform value distributions — may lead to similar equilibrium behavior when a symmetric equilibrium with informative advertising exists.

The main welfare issue here is whether equilibrium advertising is above or below the socially optimal level. Welfare depends both on the advertising intensity of the firms and on prices. Consumers who do not receive an ad cannot trade (so no surplus is realized). Since products are differentiated, it is also important how close is the match between each consumer and the product that advertising allows to be chosen. We show that equilibrium advertising may be above or below the optimum. In particular, the market tends to overprovide advertising when product differentiation is high and to underprovide advertising when it is low. The intuition is that the lower product differentiation is, the higher the intensity of price competition and the lower the profit that each firm may expect, even when a large number of consumers receive its ads. Thus, the incentive to provide (costly) advertising is reduced for each firm. Welfare maximization, on the other hand, requires that a significant number of ads are being sent, as it is efficient for trade to take place even when product differentiation is low. We also discuss similarities and differences in the welfare properties of our model and Grossman and Shapiro (1984). Advertising can be above or below the optimal in that model too, depending on the extent of product differentiation. However, for parameters set so that the symmetric equilibrium conditions are parallel, the welfare evaluation of the equilibrium may be different in the two models.

The remainder of the paper is organized as follows. Section 2 sets up the basic model. Section 3 is the core of the analysis; it examines the structure of the profit functions, constructs the symmetric equilibrium and examines its comparative static properties. Section 4 endogenizes the number of products/firms by deriving the equilibrium where firms enter upon paying an entry cost. In Section 5 we revisit the equilibrium construction in the Grossman and Shapiro (1984) model. We also compare the equilibrium conditions between the two models and discuss how informative advertising works under different product differentiation assumptions. In Section 6

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3 The lack of quasi-concavity is noted in some different but related models, such as Ireland’s (1993) model of firms choosing advertising and then prices with homogeneous products and Banerjee and Kovenock’s (1999) study of localized and non-localized competition with consumer lock-in. That charging a high price directed to only some consumers may be profitable for the firms is already central in Varian’s (1980) “sales” model where some consumers are informed and some uniformed and firms play mixed strategies.
we compare the equilibrium to the social optimum. Section 7 concludes. Some derivations and additional details are relegated to an Appendix that is available at the Journal’s website (and references throughout the text are made to this Appendix).

2. The model

There are \( n \geq 2 \) firms, each selling a single product. The population of consumers is normalized to have measure one and each consumer has unit demand. A given consumer \( \kappa \) that consumes one unit of the product of firm \( i \) has value \( v^\kappa_i \). The values \( v^\kappa_i \), for each product \( i \) and each consumer \( \kappa \) are random draws from a distribution \( F(v) \), with corresponding density \( f(v) \). To simplify the analysis, we further assume that the values \( v^\kappa_i \) are i.i.d. uniform random variables in \([a, b]\). Each firm has constant per unit production cost \( c \), with \( 0 \leq c < a \), and sets a per unit price for its product.

Consumers are not aware of the existence of the products and their prices and, as a result, there is a role for informative advertising. Firm \( i \) chooses its advertising intensity \( \phi_i \in [0, 1] \), where \( \phi_i \) is the proportion of consumers that receive the advertisement ("ad", for short) of firm \( i \). All consumers are equally likely to receive a particular ad. A consumer \( \kappa \) that receives an ad from firm \( i \) becomes perfectly informed about his valuation, \( v^\kappa_i \), and the price \( p_i \). For each firm, the cost of advertising is given by an advertising expenditure function \( A(\phi_i) \). Following Grossman and Shapiro (1984), we assume that the advertising technology is such that \( A: [0, 1] \rightarrow \mathbb{R}^+ \) has continuous second derivative, with \( A' > 0 \) and \( A'' > 0 \) and, thus, it is increasingly more expensive to reach more consumers.\(^4\) We also assume that \( A'(0)+c<a \) and, thus, the total (production plus advertising) cost of supplying one unit to at least one consumer is certainly lower than his value for the unit. Advertising is the only way a consumer may learn about a product and its price. Thus, if a consumer has not received an ad from a given firm, this consumer has zero demand for this firm’s product.

Our basic model corresponds to the following game. First, all the firms choose simultaneously their prices, \( p_i \), and advertising intensities, \( \phi_i \). Then, choosing from among all the firms from which he has received an ad, each consumer \( \kappa \) purchases one unit from the firm that offers the highest net surplus \( v^\kappa_i - p_i \) (assuming this purchase leaves the consumer with a non-negative net surplus; otherwise he buys nothing). We look for a symmetric (subgame-perfect) Nash equilibrium of the game where each firm maximizes its profit, and each consumer maximizes his surplus.

3. Equilibrium

We proceed to the construction of a symmetric equilibrium of the game outlined above. Whereas the existence of asymmetric equilibria may not, in general, be ruled out, focusing on symmetric equilibria allows us to make our analysis directly comparable to earlier results while, at the same time, allows us to highlight the main strategic features of the problem.

Assuming all other \((n-1)\) firms choose the same price \( p \) and advertising intensity \( \phi \), the profit function of firm \( i \) when it sets price \( p_i \) and chooses advertising intensity \( \phi_i \) is

\[
\pi_i(p_i, p, \phi_i, \phi) = (p_i-c)D_i(p_i; p, \phi_i, \phi) - A(\phi_i),
\]

\(^4\) A formulation along these lines was first proposed by Butters (1977).
where

\[ D_i(p_i, p, \phi_i, \phi) = \phi \sum_{k=0}^{n-1} \frac{(n-1)!}{k!(n-1-k)!} (1-\phi)^{n-1-k} \phi^k \Pr(p_i, p, k) \]  

(2)

is firm \( i \)'s demand. \( \Pr(p_i, p, k) \) is defined as the probability that a consumer that receives the ads of firm \( i \) and of \( k \) other firms chooses to buy from firm \( i \).

The demand function of firm \( i \) can be thought of as constructed in two steps: first we ask, given the firms’ advertising strategies, what is the probability that any given consumer sees the ads of a certain number of firms; second, we ask, given the prices and that consumer’s (realization of) values for the products, what is the probability the consumer will choose to purchase a particular product. There are \((n-1)!/(k!(n-1-k)!))\) different combinations with which a consumer may receive the ads of \( k \) firms other than firm \( i \), and each such combination occurs with probability \((1-\phi)^{n-1-k} \phi^k \).

Let us now proceed by assuming that \( p<a \). Thus, we are after constructing an equilibrium where all consumers purchase one unit of the product if they have received at least one ad (of course, that \( p<a \) holds will have to be verified in equilibrium for each parameters’ configuration). As for the probabilities \( \Pr(p_i, p, k) \), given that the valuations are distributed according to the c.d.f. \( F \) on \([a, b]\), when the typical consumer compares the price-value combinations of \( k+1 \geq 2 \) products (firm \( i \) with price \( p_i \), and the other \( k \) products with price \( p \)) he purchases from firm \( i \) with probability equal to

\[ \Pr(p_i, p, k) = \begin{cases} 1 & \text{if } p_i \leq p-b+a \\ \int_a^{b-p_i-p} f^k(v-p_i+p) f(v) \, dv + \int_{b-p_i-p}^b f(v) \, dv & \text{if } p_i \in [p-b+a, p] \\ \int_a^{b-p_i-p} f^k(v-p_i+p) f(v) \, dv & \text{if } p_i \in [p, p+b-a] \\ 0 & \text{if } p_i > p+b-a. \end{cases} \]  

(3)

Recall that the probability each consumer sees the ad of a given firm is independent from that of seeing the ad of another firm. Note that the first probability in the definition of the demand functions reflects the stochastic nature of the advertising technology (it is not known whether a given consumer will receive an ad that has been sent or not). The second probability captures the consumer heterogeneity: each consumer’s decision is deterministic, but the values are distributed in the population.

Our construction here is consistent with other studies of product differentiation where consumers purchase the best of the available products. However, equilibria with \( p \in [a, b] \) may also exist. In the case of perfect information about products and prices — which would correspond to \( \phi=1 \) in our model — it can be shown that a single symmetric equilibrium exists. Though we have not formally proved an analogous result in our model (due to the complexity of the relevant functions), our numerical results indicate that a single symmetric equilibrium exists for each set of the parameters involving either \( p<a, p>a \) or \( p=a \). Our paper focuses on equilibria with \( p<a \). The equilibrium construction, however, and the main results are qualitatively similar if we extend the analysis to the case when not all informed consumers purchase — see Christou and Vettas (2003) for the details. Note, that whether the price exceeds \( a \) or not may matter for an empirical approach to the relevant issues. In particular, the option of purchasing an “outside” good in empirical discrete choice models of demand may be quite important — see e.g. Berry (1994) and Berry et al. (1995). See also Goeree (2004) for a study of competition when advertising influences the set of products that consumers can choose from and an application to the personal computer industry.

The analysis of the derivation is relatively standard and is therefore omitted. For a detailed analysis see e.g. Anderson et al. (1992).
If the consumer has received ads from no other firms \((k=0)\), he purchases from firm \(i\) if and only if his value of the product exceeds its price. Thus, we have

\[
\Pr(p_i, p, 0) = \begin{cases} 
1 & \text{if } p_i \leq a \\
1 - F(p_i) & \text{if } p_i \in [a, b] \\
0 & \text{if } p_i \geq b.
\end{cases}
\] (4)

For \( F \) uniform on \([a, b]\), expressions (3) and (4) can be written as follows. When \(k \geq 1\), the relevant probability is

\[
\Pr(p_i, p, k) = \begin{cases} 
\frac{1}{(b-a)(k+1)} & \text{if } p_i \leq p - b + a \\
\frac{(p - p_i)^k (k+1) (p-p_i)}{(b-a)(k+1)} & \text{if } p_i \in [p - b + a, p] \\
\frac{1}{k+1} \left( \frac{b-a-p_i+p}{b-a} \right)^{k+1} & \text{if } p_i \in [p, p + b - a] \\
0 & \text{if } p_i > p + b - a,
\end{cases}
\] (5)

and when \(k = 0\) it is

\[
\Pr(p_i, p, 0) = \begin{cases} 
1 & \text{if } p_i \leq a \\
\frac{b-p_i}{b-a} & \text{if } p_i \in [a, b] \\
0 & \text{if } p_i \geq b.
\end{cases}
\] (6)

3.1. Necessary conditions for a symmetric equilibrium

As we discuss in more detail below, the profit function for each firm \(i\) has continuous derivatives with respect to \(p_i\) and \(\phi_i\). Thus, a symmetric equilibrium, with \(\phi \in (0, 1)\), will be determined by the first-order conditions \(\partial \pi_i(p, p, \phi, \phi) / \partial p_i = 0\) and \(\partial \pi_i(p, p, \phi, \phi) / \partial \phi_i = 0\). We now turn to the analysis of these necessary first-order conditions.

Regarding the first-order conditions with respect to price, we proceed by setting all prices other than \(p_i\) equal to \(p\) and all advertising levels equal to \(\phi\). From expression (1) we obtain

\[
\frac{\partial \pi_i(p_i, p, \phi, \phi)}{\partial p_i} = D_i(p_i, p, \phi, \phi) + (p_i - c) \frac{\partial D_i(p_i, p, \phi, \phi)}{\partial p_i},
\] (7)

where, from Eq. (2),

\[
\frac{\partial D_i(p_i, p, \phi, \phi)}{\partial p_i} = \phi \sum_{k=0}^{n-1} \frac{(n-1)!}{k!(n-1-k)!} (1-\phi)^{n-1-k} \phi^k \frac{\partial \Pr(p_i, p, k)}{\partial p_i}.
\] (8)

From expressions (5) and (6) we can derive \(\partial \Pr(p_i, p, k) / \partial p_i\) and establish that it is a continuous function of \(p_i\) for \(p_i < a\). This property implies that \(\partial \pi_i(p_i, p, \phi, \phi) / \partial p_i\) is also a continuous function of \(p_i\) in this interval. As a result, if the profit function \(\pi_i(p_i, p, \phi, \phi)\) is maximized with respect to \(p_i\) at some level \(p_i < a\), it must be that \(\partial \pi_i(p_i, p, \phi, \phi) / \partial p_i\) equals zero.
At a symmetric equilibrium this maximum is achieved when $p_i = p$, which when substituted to expressions (8) and (2) yields

$$\frac{\partial D_i(p, p, \phi, \phi)}{\partial p_i} = \phi \left( -\frac{1}{b-a} \right) (1-(1-\phi)^{n-1}),$$

and

$$D_i(p, p, \phi, \phi) = \frac{1-(1-\phi)^n}{n}. \tag{10}$$

Since $p < a$, a consumer buys if and only if he has received an ad from at least one firm. The probability that a given consumer sees no ad is $(1-\phi^n)$ and thus the numerator in expression (10) measures all the consumers that purchase. Due to symmetry in equilibrium, the firms share the market equally.

Substituting expressions (9) and (10) in Eq. (7), and by setting $\frac{\partial \pi_i(p, p, \phi, \phi)}{\partial p_i}$ equal to zero we obtain the candidate equilibrium price as a function of $\phi$:

$$p = \frac{(b-a)}{n\phi} \frac{1-(1-\phi)^n}{1-(1-\phi)^{n-1}} + c. \tag{11}$$

In Appendix 1 (Result 1) we show that the right-hand side of link to Eq. (11) is strictly decreasing in $\phi$. This implies that at an equilibrium with $\phi < 1$, the equilibrium price is greater than $(b-a)/n + c$, which is the equilibrium price in the corresponding perfect information case, in the Perloff and Salop (1985) model.

We now turn to the first-order conditions with respect to the advertisements. We proceed in an analogous fashion as above. From Eq. (2), we have $D_i(p, p, \phi_i, \phi) = \phi_i (1-(1-\phi)^n)/(n\phi)$ and, by substituting in Eq. (1), we obtain:

$$\frac{\partial \pi_i(p, p, \phi_i, \phi)}{\partial \phi_i} = (p-c) \frac{1-(1-\phi)^n}{n\phi} - A'(\phi_i). \tag{12}$$

Since this derivative of the profit function is a continuous function of $\phi_i$, at a symmetric equilibrium with $\phi \in (0, 1)$, it must be equal to zero when evaluated at $\phi_i = \phi$ or, equivalently, we have:

$$\frac{1-(1-\phi)^n}{n\phi} (p-c) = A'(\phi). \tag{13}$$

We conclude that, if an interior equilibrium exists, conditions (11) and (13) must hold simultaneously and the system of these two equations can be solved to determine the equilibrium values $(p, \phi)$.\textsuperscript{8}

\textsuperscript{8} If our maintained assumption $(a-c) > A'(0)$ does not hold, expression (13) cannot be satisfied for any positive advertising level. Since $[1-(1-\phi)^n]/n\phi < 1$ and $\phi \in (0, 1)$, the left-hand side of Eq. (13) would then always be less than the right-hand side (given that $A'' > 0$).
The next step in the analysis is to establish that, for each set of parameter values, there is a unique pair \((p, \phi)\) that may constitute a symmetric equilibrium. To show this we first solve Eq. (11) for \((p-c)\) and substitute it into Eq. (13) to obtain:

\[
\frac{(b-a)(1-(1-\phi))}{n^2\phi^2} = A'(\phi).
\]  

(14)

For convenience of expression, let us define \(L(\phi)\) as the left-hand side of Eq. (14). For \(\phi \in (0, 1)\), this function satisfies the following properties: (i) \(L(\phi)\) is continuous. (ii) \(L(\phi)\) is strictly decreasing in \(\phi\) (see Appendix 1). (iii) \(\lim_{\phi \to 0} L(\phi) = +\infty\) (apply l’Hôpital’s rule), and \(\lim_{\phi \to 1} L(\phi) = (b-a)/n^2>0\). It follows that, since \(A''(\phi)>0\), if there is a value of \(\phi\) that solves Eq. (14), this value is unique. Since the right-hand side of Eq. (11) is strictly decreasing in \(\phi\), uniqueness of \(\phi\) implies also uniqueness of a \(p\) that solves the first-order conditions.

The left-hand side of Eq. (14) tends to \((b-a)/n^2\) as \(\phi\) tends to 1. It follows that, if \(A'(1) \leq (b-a)/n^2\), there is no pair \((p, \phi)\) that satisfies both Eqs. (11) and (13) at the same time. The condition \(A'(1) \leq (b-a)/n^2\) implies that the marginal cost of advertising is less than its marginal benefit for every symmetric price schedule — recall that the right-hand side of Eq. (11) is decreasing in \(\phi\). This would further imply that, in any symmetric equilibrium, \(\phi=1\). Combining the above, we obtain the following result.

**Proposition 1.** If a symmetric equilibrium exists with \(p<a\) and \(\phi \in (0, 1)\), it is unique and characterized by Eqs. (11) and (13). If \(A'(1)>(b-a)/n^2\), there is a unique pair \((p, \phi)\) that satisfies the first-order conditions (11) and (13). If \(A'(1) \leq (b-a)/n^2\), then there is no pair \((p, \phi)\) that satisfies these conditions. In such a case, if a symmetric equilibrium exists with \(p<a\), it involves \(\phi=1\) and \(p=(b-a)/n+c\).

We next examine the second-order conditions of the problem.

### 3.2. Sufficient conditions

What may prevent the unique pair \((p, \phi)\) that solves Eqs. (11) and (13) from constituting an equilibrium is that some firm \(i\) may have a profitable deviation when all other firms choose \((p, \phi)\). As we will now show, this possibility comes about because the profit function of firm \(i\) is not, in general, quasi-concave in \(p_i\). So, even when the necessary conditions are satisfied and sufficiency conditions may hold locally, a symmetric equilibrium may fail to exist. The following result refers to the second-order conditions of the firms’ maximization problem.

**Lemma 1.** \(\pi(p_i, p, \phi_i, \phi)\) is strictly concave in \(\phi_i\) but is not always quasi-concave in \(p_i\).

**Proof.** The first part of the Lemma 1 follows from the fact that the demand function is linear in \(\phi_i\) and that \(A''(\phi_i)>0\). The second part can be proved via numerical counter-examples (see below). □

Let us now examine in more detail the crucial feature of the lack, in general, of quasi-concavity of the profit function. The profit function given in Eq. (1) is the weighted sum of \((p_i-c)\Pr(p_i, p, k), k=0, 1, \ldots, n-1\). Each function \((p_i-c)\Pr(p_i, p, k), k=1, \ldots, n-1\), is quasi-concave in \(p_i\) (see e.g. Caplin and Nalebuff, 1991) but is maximized at a different price, so that \(\pi(p_i, p, \phi, \phi)\) does not have to be quasi-concave. The product \((p_i-c)\Pr(p_i, p, 0)\), in particular, plays a key role. Assume that \(p+b-a<a\) and
consider \( p_i \in [p + b - a, a] \), that is, firm \( i \) sells to the consumers that receive only its ad.\(^9\) It follows that, in this region, the profit is given by \( \pi_i(p_i, p, \phi, \phi) = (p_i - c)\phi(1 - \phi)n - 1 - A(\phi) \), which is strictly increasing in \( p_i \). Thus, in such a case, there is an interval of prices with values above \( p \) for which each function \( \pi_i(p_i, p, \phi, \phi) \) is strictly increasing in \( p_i \), which means that \( \pi_i(p_i, p, \phi, \phi) \) is not quasi-concave in \( p_i \).

Two examples help illustrate the point (see Fig. 1). We employ the parameter values \( n = 7, b = 10, c = 0 \), and the advertising cost function \( A(\phi_i) = \gamma \ln(1 - \phi_i)/\ln(1 - r) \) with \( \gamma = 0.5 \) and \( r = 0.1 \).\(^10\) In both cases, the profit function \( \pi_i(p_i, p, \phi, \phi) \) is not quasi-concave. In Case I, we assume \( a = 6 \), and, then, the corresponding candidate equilibrium pair \( (p, \phi) \approx (1.767, 0.334) \), as derived by Eqs. (11) and (13), does appear to be consistent with a symmetric equilibrium.\(^11\) In the second case, we assume \( a = 7 \), and, then, the candidate pair \( (p, \phi) \approx (1.517, 0.294) \) is clearly not a symmetric equilibrium strategy profile. In this second case, there is clearly a profitable deviation to a higher price, even though the first-order conditions are satisfied and the sufficiency conditions hold locally.\(^12\)

Similar quasi-concavity complications exist in other related and well-known models of informative advertising (see Section 5 below). Thus, for each application of the model, a numerical analysis has to be carried out to confirm that the first-order conditions indeed describe an equilibrium. Fig. 2 illustrates, numerically calculated, \( \pi_i(p_i, p, \phi_i, \phi) \) as a function of \( p_i \) and \( \phi_i \) for the same set of parameters used above for Fig. 1 other than the value of \( a \) (for ease of presentation, the figures have been truncated to report only positive profit levels). In Case I \( (a = 6.4) \) the first order conditions indeed give us the unique symmetric equilibrium \( (p, \phi) \approx (1.67, 0.32) \), whereas in Case II \( (a = 6.8) \) they do not, as there is a profitable deviation from the candidate point \( (p, \phi) \approx (1.57, 0.30) \) to a higher price.

Note that the problem of the possible non-existence becomes more severe as product differentiation vanishes and it becomes certain that an equilibrium does not exist when \( a = b \). Formally, we show first that for \( a = b \) there is no symmetric pure strategy equilibrium.\(^13\) If the

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\(^9\) Even the consumer with the highest valuation for the product of firm \( i \) would prefer to buy any other product (provided he had received an ad) even if he had the lowest possible valuation for that product: \( b - p_i \leq a - p \), that is, \( p_i \geq p + b - a \).

\(^10\) This advertising cost function is borrowed from Grossman and Shapiro (1984, p. 66). In this specification, \( \gamma \) can be viewed as the cost of an ad per person.

\(^11\) Still, we cannot conclude from the diagram that the given \( (p, \phi) \) is the equilibrium strategy profile because a firm may change both its price and advertising. See the numerical examples corresponding to Fig. 2 where profit is calculated as a function of both \( p \) and \( \phi \).

\(^12\) We do not claim here that the profit function is not quasi-concave for all parameter values. The important feature is that the function is generally not quasi-concave and that the candidate maximum for the symmetric equilibrium is not always the global best response.

\(^13\) We expect that a mixed strategy equilibrium exists in this case. For such equilibria in related models see e.g. Ireland (1993) and Stahl (1994).
candidate equilibrium involved $\phi > 0$ and $p > c$, a firm could profitably deviate to a slightly lower price than $p$ so as to capture all consumers informed by that firm. If $\phi > 0$ and $p = c$, then all firms would make losses, and any one of them could profitably deviate to $\phi = 0$. Finally, if $\phi = 0$, there are no informed consumers and firms make zero profit. Then, each firm, unilaterally, has a profitable deviation to a positive price and advertising level (by our maintained assumption $A' (0) + c < a$). Now, when the difference $(b - a)$ is positive but small, we can use the fact that the relevant conditions (11) and (13) are continuous in $\phi$ when $\phi > 0$. By the continuity of the first order conditions it follows that equilibrium fails to exist for relatively low but strictly positive values of $b - a$ (see Appendix 2). We can summarize

**Proposition 2.** There exists no symmetric equilibrium when product differentiation is small enough.

For the following steps in our analysis, we proceed by implicitly restricting the parameters so that our conditions indeed characterize an equilibrium.$^{14,15}$

3.3. Comparative statics

We examine here the comparative static properties of the equilibrium we have characterized above.

(i) $\frac{d\phi}{dn} < 0$.

In more concentrated markets, each firm advertises more. Eq. (14) determines implicitly the equilibrium level of advertisements as a function of $n$. Since the derivative of the left-hand side of Eq. (14) is decreasing in $\phi$ and in $n$ (see Appendix 1), and $A'' > 0$, it follows by implicit differentiation of Eq. (14) that $d\phi / dn < 0$. The intuition behind this result is that informative

---

$^{14}$ Ideally, one would like to partition the parameter space, specifying exactly the regions where a symmetric equilibrium exists. However, the problem is too involved to allow us to do this. For instance, one may think that starting from the candidate equilibrium, a high price deviation would be more desirable for a firm when parameters imply an increased probability that consumers only receive an ad from that firm, which in turn depends on $\phi$. However, the possible profitability of a deviation also depends on the level of the candidate equilibrium price, which is jointly endogenous with $\phi$.

$^{15}$ We have solved numerically some examples for cost asymmetries with $n = 2$. Depending on the parameters, there are cases where increasing the unit production cost difference allows us to obtain an equilibrium (which may not exist for a smaller difference) and also cases where the higher unit cost firm may set a lower price (and advertise less) than its lower cost rival.
advertising increases price competition: for a given advertising intensity, an increase in $n$ increases the expected number of ads reaching each consumer, which intensifies price competition, causing a negative impact on the price. This, in turn, reduces the marginal profitability of advertising, leading to a lower level of advertising by each firm. To see this, fix the value of $\phi$ in expression (12). Given that the price, $p$ (see expression (11)) and the ratio $(1−(1−\phi)^n)/(n\phi)$ are decreasing in $n$, an increase in $n$ makes the derivative of expression (12) negative. Thus, each firm has a direct incentive to reduce its advertising expenditure. A secondary effect, caused by the advertising decrease by all firms, gives each firm a lower incentive to further decrease its ads, or even an incentive to increase them. Given that $p$ is a decreasing function of $\phi$, after the decrease in the ads, “in the first round”, $p$ goes up increasing the derivative of expression (12). The series of the effects goes on indefinitely, with a negative total effect.

\[ \text{(ii)} \frac{d\phi}{d(b-a)}>0. \]

The result is directly obtained by implicit differentiation of Eq. (14). Underlying this result is that a larger $(b-a)$ implies weaker price competition, which increases the marginal profitability of advertising for each firm. Thus, firms tend to advertise more in markets with more heterogeneous products.

\[ \text{(iii)} \frac{dp}{dn} \geq 0. \]

The relation between the number of firms and the equilibrium price can be either positive or negative. The equilibrium price is given by Eq. (11). There are two effects on $p$ when $n$ increases, moving in opposite directions. On the one hand, an increase in $n$ has a “direct” negative impact on $p$ (by Eq. (11) and Appendix 1, Result 2). On the other hand, an increase in $n$ causes a decrease in $\phi$ (see comparative statics result (i) above), that in turn tends to increase $p$. Thus, this “indirect” effect is positive. The final impact can be either positive or negative depending on the parameter values and the form of the advertising cost function.\footnote{Numerical examples that illustrate both cases can be easily constructed.}

\[ \text{(iv)} \frac{dp}{d(b-a)}>0. \]

By expression (11) we observe that the “direct” effect of an increase in $(b-a)$ on the price is positive. Still, the total effect is not obvious as it depends jointly on the sensitivity of the price and the advertising on each other and on the other parameters of the model. For example, an increase in $(b-a)$ increases $\phi$ but this tends to reduce the price. In Appendix 3 we establish that the net effect of an increase of $(b-a)$ on $p$ is positive.

Next, we extend our comparative static analysis with a study of the relation of the basic economic variables of our model with the unit cost of ads. To do this, we assume that the cost of informing a percentage $\phi$ of the consumers requires paying for $A(\phi)$ ads, or $\gamma A(\phi)$ money units, where $\gamma$ is the price of each ad. Then, we see that a more costly advertising technology implies a lower advertising intensity by each firm and a higher price, while the effect on profits is ambiguous. In particular, we have:

\[ \text{(v)} \frac{d\phi}{d\gamma}<0. \]
By total differentiation of Eq. (14), we obtain
\[
\frac{\partial}{\partial \phi} \left( \frac{(b-a)(1-(1-\phi)^n)^2}{n^2 \phi^2 \left(1-(1-\phi)^{n+1}\right)} \right) d\phi = \gamma A \phi d\phi + A d\gamma.
\]

Since the left-hand side of Eq. (14) is decreasing in \( \phi \) (see Appendix 1), and \( A \phi, A > 0 \), it follows that \( d\phi/d\gamma < 0 \).

(vi) \( dp/d\gamma > 0 \).

By Eq. (11) we observe that the equilibrium price depends on \( \gamma \) only indirectly through \( \phi \), and thus \( dp/d\gamma = (dp/d\phi)(d\phi/d\gamma) \). Since \( d\phi/d\gamma < 0 \) and \( dp/d\phi > 0 \), the result follows immediately.

(vii) \( d\pi/d\gamma \geq 0 \).

The equilibrium profit can either increase or decrease in \( \gamma \): The direct effect on profit is negative: an increase in \( \gamma \) increases the cost of reaching any given measure of \( \phi \) consumers. But there are also two indirect effects, through \( \phi \) and through \( p \). Specifically, the fact that \( d\phi/d\gamma < 0 \) tends to reduce the advertising cost but also total demand. On the other hand, the positive relationship between the equilibrium price and the per ad cost tends to increase profit. The net impact depends on the parameters and the form of the advertising cost function (and examples indeed show that both cases are possible).

4. Endogenizing the number of firms

Consider a “free entry” equilibrium where any firm can enter the market by paying an entry cost \( K > 0 \). Formally, we add to the initial game a stage in which firms decide whether to enter the market or not. We examine the subgame-perfect Nash equilibrium of this enlarged game. Given the number of firms, and provided that a symmetric equilibrium exists, our analysis above implies that
\[
\pi^*(n) = \frac{1-(1-\phi)^n}{n} (p-c) - A(\phi) - K = \frac{1-(1-\phi)^n}{n\phi} (p-c) - A(\phi) - K,
\]
where \( \pi^*(n) \) denotes the equilibrium profit of each firm, and \( \phi \equiv \phi(n) \) and \( p \equiv p(n) \) are the equilibrium advertising level and equilibrium price, respectively, as functions of \( n \).

We can establish that the following property holds.

**Lemma 2.** In the symmetric equilibrium, the profit of each firm is strictly decreasing in the number of firms in the market.

**Proof.** It suffices to show that \( d\pi^*(n)/dn < 0 \). Specifically, we have
\[
\frac{d\pi^*(n)}{dn} = \frac{d}{dn} \left( \frac{1-(1-\phi)^n}{n\phi} (p-c) - A(\phi) \right) \frac{d\phi}{dn} = \phi \frac{d}{dn} \left( \frac{1-(1-\phi)^n}{n\phi} (p-c) \right) + \frac{1-(1-\phi)^n}{n\phi} (p-c) \frac{d\phi}{dn} - A(\phi) \frac{d\phi}{dn} = \phi A'(\phi) \frac{d\phi}{dn} + A'(\phi) \frac{d\phi}{dn} - A(\phi) \frac{d\phi}{dn}, \text{ by Eq. (13)}
\]
\[
= \phi A'(\phi) \frac{d\phi}{dn} < 0. \quad \square
\]
The above result directly implies that the equilibrium number of firms determined by the condition \( \pi(n) = 0 \) is unique:

**Proposition 3.** The “free entry” equilibrium, if it exists, is unique.

5. Informative advertising based on localized competition

Thus far, we have examined informative advertising in a model of product differentiation based on non-localized competition using a symmetric random utility model. In this Section, we revisit earlier work on informative advertising in oligopoly, where the underlying model is one of localized competition.\(^{17}\) In particular, we analyze some aspects of the well-known model of Grossman and Shapiro (1984) that have not received the attention that they may deserve and we study analogies between the equilibria of the two types of models. A special case of this model is discussed in Tirole (1988) and we refer to this analysis as well. The possible non-existence of a symmetric equilibrium in our model is a common feature to the Grossman and Shapiro (1984) model, too. The intuition above still holds: for certain parameter values, a firm would prefer to charge a higher price and sell to the consumers that have been informed only about a small set of products (including its own) rather than charging the candidate equilibrium price.

The Grossman and Shapiro (1984) model involves \( n \) firms located at equal distances around a circle, as in the Salop (1979) model of horizontal product differentiation.\(^ {18}\) Consumers are uniformly located around the circle. The other aspects of the model, including the informative advertising and pricing decisions of the firms, are as described in the main part of the present paper. We start with the derivation of the demand function of firm \( i \) at a candidate symmetric equilibrium, \( D_i(p_i, p, \phi, \phi) \), defined over the entire range of prices, \( p_i \).

As the transportation cost is assumed to be linear in distance, the demand functions already exhibit discontinuities at certain prices like under certainty.\(^ {19}\) Following Grossman and Shapiro (1984), we partition the consumer population into \( n \) groups, where the \( k \)th group is the set of consumers to whom the representative firm offers the \( k \)th highest surplus among the \( n \) firms, assuming all consumers are fully informed. The demand function is determined by the sizes of these \( n \) groups. Suppose, without loss of generality, that firm \( i \) is located at point 0 and firm \( i+k \) is located at point \( k/n \). Suppose that all firms but \( i \) set the same price, \( p \). We denote by \( N_k \) the size of the \( k \)th group of consumers and by \( z_k \) the location of the consumer that is indifferent between buying from firm \( i \) and from firm \( i+k \); where \( z_k = k/(2n) + (p - p_i)/(2t) \). For \( p_i \) close enough to \( p \) we have \( N_1 = 2z_1, N_2 = 2(z_2 - z_1) = 1/n, N_3 = 2(z_3 - z_2) = 1/n, \ldots, N_n = 2(z_n - z_{n-1}) = 1/n, \ldots, N_n = 1 - 2z_{n-1} \). In order for the consumers of group \( N_k \) to purchase from firm \( i \), it must be that the consumers of that group receive an ad from firm \( i \), but they do not receive an ad from any of the firms that give them surplus greater than the \( k \)th highest surplus, that firm \( i \) gives to them. This occurs with probability \( \phi(1 - \phi)^{k-1} \). It follows that

\[
D(p_i, p, \phi, \phi) = \sum_{k=1}^{n} \phi(1 - \phi)^{k-1}N_k. \tag{15}
\]

As noted above, expression (15) represents the demand of firm \( i \) for \( p_i \) close enough to \( p \). This is the case explicitly considered in Grossman and Shapiro (1984). However, the demand function should be modified for prices not close to \( p \). We provide here this analysis. The underlying logic is the same as in the perfect information case with linear transportation costs: if the consumer

\(^{17}\) See Deneckere and Rothschild (1992) for a study of the relation between models of localized and non-localized competition.

\(^{18}\) This model was first introduced by Vickrey (see e.g. Vickrey, 1999, as pointed out in a foreword by Anderson and Braid).

\(^{19}\) Similar to those one has in a linear city model; see D’Aspremont et al. (1979).
located at the same point as firm \( i \) marginally prefers to buy from firm \( j \), then all consumers prefer to do so. In particular, consider \( p_i \) greater than \( p \). The price \( p_i \) that makes the consumer located at point 0 indifferent between buying from firm \( i \) or from firm \( i+1 \), is \( p_i = p + t/n \).\(^{20}\) It follows that, for \( p_i > p + t/n \), there are fewer than \( n \) groups of consumers that may demand the product of firm \( i \). Specifically, for \( p_i \in [p + t/n, p + 2t/n] \), there are \( n-2 \) such groups.\(^{21}\) This is because prices are such that the product of firm \( i \) is dominated by the products of firm \( i+1 \) and of firm \( i-1 \), that is, all consumers that become informed by either firm \( i+1 \) or firm \( i-1 \) prefer to buy either product \( i+1 \) to product \( i \) or \( i-1 \) to \( i \). It follows that the ranking for the product of firm \( i \) starts from position 3. The relevant groups are now \( N_1 = 2z_2, N_2 = 2(z_3-z_2) = 1/n, \ldots, N_k = 2(z_{k+1} - z_k) = 1/n, N_{n-2} = 1 - 2z_{n-2} \), and the demand is

\[
D(p_i, p, \phi, \phi) = \sum_{k=1}^{n-2} \phi(1-\phi)^{k+1}N_k.
\]

More generally, for \( p_i \in [p + (r-1)t/n, p + rt/n] \) the respective groups are \( N_1 = 2z_r, N_2 = 2(z_{r+1} - z_r) = 1/n, \ldots, N_k = 2(z_{k+(r-1)} - z_k) = 1/n, \ldots, N_{n-2(r-1)} = 1 - 2z_{n-r} \), and the demand is

\[
D(p_i, p, \phi, \phi) = \sum_{k=1}^{n-2(r-1)} \phi(1-\phi)^{k+2(r-1)-1}N_k.
\]

As \( p_i \) increases, we finally reach a point where product \( i \) is dominated by all other products. The critical price is \( p + t(n-1)/(2n) \) if the number of firms, \( n \), is odd, and \( p + t/2 \), if \( n \) is even.\(^{22}\) Thus, if \( n \) is odd, we have \( D(p_i, p, \phi, \phi) = \phi(1-\phi)^{n-1} \) for \( p_i \in [p - t(n-1)/(2n), R - t/2] \) and, if the number of firms is even, we have \( D(p_i, p, \phi, \phi) = \phi(1-\phi)^{n-1} \) for \( p_i \in [p - t/2, R - t/2] \). For \( p_i \in [R - t/2, R] \) demand is given by \( \phi(1-\phi)^{n-1}2(R-p_i)/t \) and, for \( p_i > R \), demand is zero.

It remains to consider prices \( p_i \) lower than \( p \). Once again, a continuous decrease in \( p_i \) gradually reduces the number of groups of consumers to which firm \( i \) may sell. However, in this case, this is because, by reducing its price, firm \( i \)’s product dominates its closer rivals’ product, implying a discontinuously increasing demand function (in the same vein as in the perfect information case).

Incorporating imperfect information (and informative advertising) tends to make competition with firms located at a greater distance more intense, hence, the candidate equilibrium price is lower than that under certainty.\(^{23}\) However, under uncertainty, the problem of non-existence of an equilibrium is exacerbated. Not only may prices lower than the candidate equilibrium represent a profitable deviation in a discontinuous way (as they can be even in the case under certainty) but the fact that a portion of consumers becomes informed only about a limited number of firms including firm \( i \), provides a possible basis for firm \( i \) to profitably deviate to a higher price. Eventually, the existence of a symmetric equilibrium depends jointly on all parameters

\(^{20}\) This price is the solution to \( u - p_i = u - t/n - p \).

\(^{21}\) The price \( p_i = p + 2t/n \) makes the consumer located at point 0 indifferent between buying from firm \( i \) or firm \( i+2 \), that is, it solves \( u - p_i = u - 2t/n - p \).

\(^{22}\) This is because, the maximum transportation cost a consumer can save is \( t/2 \), in the case where \( n \) is even, that is, when there is a firm located at a diametrically different location from firm \( i \) — at distance \( 1/2 \). When \( n \) is odd, the maximum distance a firm may have from firm \( i \) is \((n-1)/(2n)\), and, accordingly, the maximum transportation cost advantage of that firm equals \((n-1)/(2n)\).

\(^{23}\) Under full information, the price equilibrium conditions can be determined relative to the closest two rivals and a deviation to a higher price never pays. Along these lines, Grossman and Shapiro (1984, p. 67) point out that under imperfect information “competition is no longer localized”.

Fig. 3 depicts two representative cases. In Case I, we use the parameter values $R=2.5$, $n=6$, $t=1$, $c=0$ and the advertising cost function $A(\phi_i) = \gamma r \ln(1-\phi_i)/\ln(1-r)$, with $\gamma = 0.5$ and $r=0.1$. We observe that the (unique) candidate equilibrium pair $(p_i, \phi_i) \approx (0.9445, 0.194)$ is not a symmetric equilibrium profile: with $\phi_i = \phi$ there is a profitable price deviation to $p_i=2=R-t/2$. On the other hand, in Case II the candidate equilibrium pair $(p_i, \phi_i) \approx (96.55, 0.67)$ is indeed a symmetric equilibrium profile. The set of parameters we use in Case II is $R=250$, $n=8$, $t=250$, $c=50$, $\gamma=3$, $r=0.1$; with the same advertising cost function as above (a set of parameters also used in Grossman and Shapiro, 1984).

A special case of the Grossman and Shapiro (1984) model is discussed in Tirole (1988, ch. 7). For illustration purposes, the Hotelling (1929) linear city model with two firms at the endpoints is used there as a basis. The fact that firms are located at opposite endpoints of the linear city eliminates one of the reasons for possible equilibrium non-existence, namely, possible discontinuities associated with the incentive of firms to undercut one another and capture the total number of (partly) informed consumers. However, since in equilibrium firms choose to inform a strict subset of the consumers, the second reason discussed above for possible non-existence remains: if the reservation price is high enough, each firm has a unilateral incentive to set a much higher price (compared to the candidate equilibrium one) and sell only to the consumers that are only aware of the existence of its product. So, like in the more general case of the circle model, the characterization of a symmetric equilibrium in this case only holds for a limited range of the parameters (see Appendix 4).

There is an analogy between the conditions that determine the price and advertising intensity at the candidate symmetric equilibrium in each of the two models, Grossman and Shapiro (1984) and the one we have presented in Section 3 with random utility. Under full information, the equilibrium price in the Salop (1979) “circle” model is $p=c+t/n$, where $t$ is the per-unit of distance transportation cost while, in the random utility model of Perloff and Salop (1985) with valuations uniformly distributed on $[a, b]$, the equilibrium price is $p=c+(b-a)/n$. Thus, the distance $(b-a)$ plays a role comparable to that of $t$, essentially representing the intensity of

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24 As in Section 3, examining the local properties of the profit functions is not enough to guarantee existence, as deviations to much higher or lower prices may be profitable. So, while Grossman and Shapiro (1984) observe that profits are concave functions, locally, around the equilibrium profit (see their footnote 11), this property does not imply existence of an equilibrium even when the first-order conditions are satisfied.

25 For $n=2$, the circle model is equivalent to the linear city model with the two firms located at the endpoints.
product differentiation.

So, despite the fact that the demand and profit functions are overall quite different in the two models, the candidate symmetric equilibrium prices are identical. Since (when we turn to uncertainty) advertising enters in the same way in the two models, it comes as no surprise that this similarity carries over, and the candidate symmetric equilibrium pair \((p, \phi)\) is characterized by the two first-order conditions:

\[
p = \frac{x}{n\phi} \frac{1-(1-\phi)^n}{1-(1-\phi)^{n-1}} + c \tag{16}
\]

and

\[
\frac{1-(1-\phi)^n}{n\phi} (p-c) = A'(\phi), \tag{17}
\]

where \(x = t\) in the Grossman and Shapiro (1984) model and \(x = b - a\) in our model. Thus, the unit transportation cost in the circle model (twice the maximum transportation cost that a consumer may incur in a circle of unit length) plays exactly the same role as the maximum difference (in consumers’ valuations for any two products) in the random utility model. Given that the equilibrium conditions can be viewed as parallel in this way, the comparative static properties of the two models for the symmetric equilibria derived will also go in the same direction. In the next Section we examine whether the similarity of the symmetric equilibrium conditions also implies analogous welfare properties.

6. Welfare

We now turn to the comparison between the market equilibrium and the socially optimal level of advertising. We do this primarily in the context of our model, although we also provide some comments on the related properties of the Grossman and Shapiro (1984) model. So we first return to our model, presented in Section 2. Suppose a social planner chooses the level of advertising \(\phi\), taking as given the number of firms in the market. Total surplus can be then expressed as

\[
W(\phi; n) = \sum_{k=1}^{n} \frac{n!}{k!(n-k)!} \left(1-\phi\right)^{n-k} \phi^k M(k) - \left\{ \frac{(1-(1-\phi)^n)}{n} c + A(\phi) \right\} n, \tag{18}
\]

where

\[
M(k) = \int_{a}^{b} v \left\{ k(F(v))^{k-1} f(v) \right\} dv = b - \frac{b-a}{k+1}. \tag{19}
\]

Implicit in the above welfare function is the assumption that the socially optimal price is below \(a\), that is, each consumer that receives some ad buys a unit of some product. The function \(M(k)\) represents the expected value of the maximum valuation of a consumer that receives \(k\) ads. The sum in expression (18) represents the expected (gross) value for each consumer and is constructed

\[\text{26 Importantly, however, } (b-a) \text{ represents, for the non-localized competition model, the maximum gross utility difference possible when purchasing different products, whereas, in the localized competition context, } t/2 \text{ represents the maximum possible transportation cost. When moving from a setting where all consumers know all products to one of uncertainty and informative advertising we observe that the two models are affected in different ways. In the random utility model the maximum difference in values for a given consumer is always } (b-a) \text{ regardless of how many products the consumer has been informed about. On the circle this difference depends on how many products the consumer has been informed about. Under perfect information and symmetric pricing this difference is } t/n.\]

\[\text{27 See the Appendix of the Grossman and Shapiro (1984) paper for the equilibrium conditions of that model.}\]
in a similar way as Eq. (2). Finally, the total cost incurred by each firm (corresponding to the terms inside the curly brackets in Eq. (18)) consists of the production and the advertising costs.\footnote{Welfare depends both on advertising and prices. As long as prices are below $a$, they represent transfers between consumers and firms, but they also affect the quality of the matches between consumers and products. With unequal prices some consumers would end up purchasing products that involve a worse match than what could be attained with equal prices. So we proceed here assuming that the planner imposes the same price for all products and below $a$. Also, we look for a symmetric solution to the welfare maximization problem — for the examples we have used, we have confirmed that the welfare maximizing solution is indeed symmetric.}

In our model, advertising contributes to welfare in two ways: First, consumers are not aware of the products unless they have received an ad. When no ad is received no trade surplus is realized. The more people have received (at least) one ad, the larger the number of trades that will take place (and the increase in welfare). Second, since products are heterogeneous, the larger the number of ads a consumer receives, the wider the set of products among which he can choose the one he will purchase and, thus, the higher the value that will be realized. So, advertising both allows trades to take place and also improves the quality of the matching between consumers and products. Importantly, note that the first role of advertising (allowing trade to take place) does not depend on the difference $b - a$, but only on the average value from a trade, in contrast to its second role (improve the quality of matches) which vanishes in the limit as $a$ tends to $b$.

Turning now to a more formal treatment, the basic problem that one faces in the analysis is due to the complexity of the underlying equations (both for the welfare and the equilibrium problems), which does not enable us to derive closed-form solutions. Thus, we have to rely on numerical techniques (Appendix 5 reports some calculations for the relatively simpler case where $n=2$). Such numerical approaches here are relatively robust and straightforward because the welfare function is concave in $\phi$ (see Appendix 6). Having tested a wide variety of parameter configurations, while we cannot state a formal Proposition, we report:

**Summary of numerical results:** For a given number of firms, when the difference $(b - a)$ is high (small), the market tends to overprovide (underprovide) informative advertising as compared to the socially optimal level.

In other words, the market tends to over or underprovide informative advertising in a systematic way, depending on the degree of product differentiation. Fig. 4 presents results that illustrate the above finding. The advertising cost function is the same as in the examples in Section 3.2 of the paper, and the parameter values are $b=200$, $c=0$, $r=0.1$ and $\gamma=2$. In the Figure we can distinguish four regions. When $a$ takes values outside the shaded regions $R_1$ and $R_2$, the first-order conditions fail to imply a symmetric equilibrium. Specifically, when $a$ is low enough, there is no symmetric equilibrium with $p<a$, that is, where all informed consumers purchase. On the other hand, for $a$ high enough, the candidate equilibrium price is relatively low and each firm wants to deviate to a higher price, hence there is no symmetric price equilibrium. If the pair $(a, n)$ lies within region $R_1$, that is, if for given $n$ product differentiation is relatively high, the market equilibrium level of ads, $\phi^*$, exceeds the socially optimal level, $\phi^o$. The opposite happens when the pair $(a, n)$ lies within region $R_2$. The intuition is as follows. As the products become less differentiated, the market equilibrium price falls (as per comparative statics result (iv) in Section 3.3). This implies a decrease in the firms’ profit margins, which in turn reduces the firms’ incentives to provide information to possible buyers. Put differently, the “business-stealing” effect among firms becomes weaker. On the other hand, whereas the welfare gain through improved matching is also reduced (since the products tend to be not very differentiated), the welfare gain associated with an increase in the number of trades remains unchanged (as long as $a>c$, that is, trade is always efficient). Thus, there is a positive role for advertising even when products are relatively homogeneous, whereas the profit incentive for each
firm is very low. This explains why the market tends to underprovide ads as the difference \( b - a \) becomes small. The opposite forces are at work when the market is characterized by relatively high product differentiation and, thus, high profit margins. In that case, each firm has a strong unilateral incentive to advertise and expand its own market share, and this implies an equilibrium advertising level that is too high from a social welfare point of view.

We discuss now how the equilibrium advertising compares to the welfare optimal in the context of the Grossman and Shapiro (1984) model. First, as in our model (with random utility), the market may under or overprovide advertising. This property does not follow from the analysis in the original Grossman and Shapiro (1984) paper. Second, when comparing the two models, based on random utility and the circle model, we find both similarities and differences in the welfare properties of the equilibrium advertising. While the intensity of product differentiation matters in both models for the welfare evaluation of the equilibrium and while, as discussed above, the equilibrium conditions of the two models appear quite similar, the two welfare maximization problems are not parallel. To illustrate more clearly the main forces at work, let us examine more closely the welfare functions for the simpler case with \( n = 2 \) products. The welfare function of our model (from Eq. (18)) can be written as

\[
W(\phi; n = 2) = \phi^2 \left( b - \frac{b-a}{3} - c \right) + 2\phi(1-\phi) \left( b - \frac{b-a}{2} - c \right) - 2A(\phi),
\]

whereas for the Grossman and Shapiro (1984) model it is given by

\[
W^{GS}(\phi; n = 2) = \phi^2 \left( s - \frac{t/2}{4} - c \right) + 2\phi(1-\phi) \left( s - \frac{t/2}{2} - c \right) - 2A(\phi).
\]

In contrast to our results, the Grossman and Shapiro (1984) paper suggests that the market overprovides advertising irrespective of the degree of product differentiation. Their formal analysis is conducted under the assumption of large \( n \), but the paper also reports (p. 76) that the conclusion appears to hold true even for smaller values of \( n \). This result is not generally correct, however, as can be checked by numerical examples. Tirole (1988) already has pointed out that we may have underprovision of advertising, as well. Whether advertising is excessive or suboptimal is one of the core themes in this general area. In the classic Butters (1977) paper on informative advertising, equilibrium advertising by firms producing a homogeneous product is socially optimal. In important extensions, Stegeman (1991) shows that advertising is inadequate when valuations are heterogeneous and Stahl (1994) finds a similar result with downward sloping individual consumer demands. See also Anderson et al. (1992) and Bagwell (2005) for a discussion of this issue.

This function can be found in Tirole (1988), with only a minor expositional difference. In order to retain the analogy between parameters \( t \) and \( b - a \) and the price equilibrium equations given by Eq. (16), we assume here that the distance between the two firms in the linear city, as studied by Tirole, is normalized to 1/2 instead of 1.
In the above Eq. (21), $s$ represents the consumers’ reservation price for each product. The terms multiplied by $\phi^2$ and $2\phi(1-\phi)$ represent the average surplus created when the consumers receive ads from two firms and from one firm, respectively. In order to compare the two models, we may pair $s$ (that is, the maximum gross consumer surplus in the localized competition model) with $b$ (that is, the maximum gross consumer surplus in the non-localized competition model). Note that, with $b-a$ set equal to $t$, the average distance of informed consumers from their optimal purchases are smaller in the localized competition model: these distances are $t/4$ and $t/8$ compared to $(b-a)/2$ and $(b-a)/3$, respectively, for the consumers that receive one and two ads. Also note that, the marginal welfare gains of (more) information for the average consumer, as measured by the distance from the optimal product, are different in the two models. Specifically, in the localized competition model, more information implies a greater decrease in the average transportation, since the latter is reduced from $t/4$ to $t/8$, which amounts to a 50% decrease and $t/8$ in absolute value. In contrast, in the non-localized competition model, the corresponding numbers are 33.3% and $(b-a)/6$. Thus, although the matching is improved in absolute terms more in the non-localized competition model, in percentage terms the opposite is true.

Advertising affects welfare differently in the two models because of differences in how values are determined. In the localized competition model, consumer valuations for any two products are negatively correlated since, as the consumer gets closer to one firm, he gets further away from the other. This is not the case in the random utility model, where the consumer valuations for any two products are drawn from independent distributions. Overall, whether the market overprovides advertisements in the one or the other context depends on all the parameters. The conclusion that can be drawn from our analysis here corresponds to the two points we have mentioned above. First, that the market may underprovide advertising in both models, especially when product differentiation is low; in Grossman and Shapiro (1984) as $t$ becomes smaller, the firms’ incentives to advertise are reduced, whereas the social incentive to inform consumers, and create demand for the products, remains positive. Second, that the welfare properties of the two models are not parallel. In particular, even with $n=2$, we see that, if we set $t=b-a$ to couple the first-order conditions of the two models, a given level of $(b-a)$ or $t$ does not necessarily imply over or underprovision of advertising in both models.\footnote{For example, for $b=s=200$, $c=0$, $r=0.1$, $\gamma=2$, and for the advertising cost function of Section 3.2, when $b-a=t=75$, both models have a symmetric equilibrium with $p \approx 44$ and $\phi = 0.92$. It can be calculated that this equilibrium level of ads is below the optimal (welfare maximizing) advertising level in our model but it is above the optimal advertising level in the Grossman and Shapiro (1984) model.}

\section{Conclusion}

The model examined in this paper represents one of the most straightforward ways to study informative advertising in oligopoly. Firms compete strategically both in prices and in informative advertising. In equilibrium, a higher level of advertising tends to lower prices, as consumers gain information about more products. Higher cost of advertising leads to higher prices but may either

\footnote{Another welfare analysis may be about the comparison of the socially optimal number of firms with the number of firms in the “free-entry” equilibrium (based on Proposition 3). Since the advertising decisions affect social welfare, we have to distinguish between the case where firms compete in advertising, which leads to a “second-best” outcome, and the case where the social planner selects both the number of firms and the advertisement levels, which leads to the “first-best”. Numerical results we have derived (see Christou and Vettas, 2003, for details) suggest that the market overprovides product variety (which also happens under perfect information — see Anderson et al., 1992, p. 206) and that in the presence of imperfect information this gap may further increase as compared to the perfect information case. The differences between the first-best and the second-best numbers that we found were relatively small.}
increase or decrease the firms’ profits. More firms may lead to either a higher or a lower price. Compared to the social optimum the market tends to overprovide advertising when the degree of possible product differentiation is large, and to underprovide it otherwise.

That a firm may have an incentive to deviate to a higher price than the one prescribed in the candidate symmetric equilibrium, rather than being an anomalous case, appears a robust characteristic of informative advertising models of product differentiation. The underlying force is that, for some parameter values, the (candidate) equilibrium advertising level is such that it makes it likely that a large enough share of consumers will be informed about only one product. As a result, a firm may wish to exploit its monopoly power over these captive consumers and increase its profit by charging a high price. It follows that, when establishing a symmetric price and advertising equilibrium, attention should be given to discrete jumps in prices and not only to local deviations. This feature is also present in models based on product differentiation with localized competition and we provide a further analysis of the Grossman and Shapiro (1984) model.

Some additional aspects of advertising in oligopoly may fit in the general framework proposed here, even though they have to be left out of the present analysis. Such topics, for future work, may include cases when consumers are informed about the existence of products while advertising informs them only about prices, cases of vertical (quality) product differentiation, and cases where firms could choose their advertising content.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.ijindorg.2006.08.004.

References