Price competition in a differentiated products duopoly under network effects

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1. Introduction

Motivation and overview. In telecommunications and other information systems, network effects play a crucial role as, by their nature, the value these systems generate depends on the number of users. At the same time, in a given market there may be competing products (or services) and hence strategic pricing becomes important. These products (or services) will typically be viewed by the market as differentiated along a horizontal dimension (that is, some users prefer one product and some the other) or a vertical (that is, all users view a product as superior) dimension. In fact, we argue that whenever network effects are product-specific, product differentiation becomes important. In this paper, we provide an analysis of price competition between horizontally and vertically differentiated products.
differentiated duopoly products when there are network effects. In addition to comparing the prices set by the firms, consumers may choose a product because it has higher quality, because they prefer its particular variety (location), or because they expect that a larger number of other users will choose the same product (and hence they expect its value to increase via the network effect).

Relation to the key literature references. The paper is related to two distinct literatures, one on network effects and the other on product differentiation. A seminal analysis of network effects in oligopoly is provided by Katz and Shapiro (1985). They study competition in quantities (not prices, which is what we do), and in their model, consumers’ expectations about these quantities are formed before the firms make their choices (whereas we also consider the case where the firms’ choices influence the consumers’ expectations). Our work is closely related to Grilo et al. (2001) who, as far as we know, are the first to study network effects when product differentiated duopolists compete in prices. Besides some specific modeling differences, our analysis can be viewed as complementing theirs and providing some important further results. First, unlike Grilo et al. (2001), we also consider the case where expectations are not influenced by prices and we contrast how the equilibria differ from the case where they are influenced by the prices. Thus, we emphasize the role that expectations play for the market outcome. Second, we allow for horizontal and vertical differentiation to both exist at the same time; we focus on circumstances in which the low-quality product can survive (or even dominate) in the market and the crucial role that the process of expectations formation plays for that to happen. Third, we provide a (continuity) refinement that leads to a unique equilibrium in the case where expectations can be influenced by prices and network effects are strong.

Model description and main results. We set up the simplest model that allows us to explore the issue at hand: a static duopoly where suppliers of differentiated products compete in prices. To capture horizontal differentiation, we employ the standard modification by d’Aspremont et al. (1979) of Hotelling’s (1929) “linear-city” with quadratic transportation costs. In addition, we introduce the possibility of quality differences; thus, in addition to horizontal differentiation, there may be an independent dimension of vertical product differentiation, such that all consumers view quality in the same way (see e.g. Vettas, 1999). Finally, we enrich the model by adding a network effect, so that given each product’s characteristics and price, each consumer would prefer the product more widely used. We take as given the differentiation between the products and we examine how price competition takes place under network effects. A number of important questions arise, not only from a theory but also from a business strategy and a policy viewpoint: When is it that equilibria have to be symmetric and when can they be asymmetric (in prices or market shares)? When will both firms be active in the market and when can we expect one of the two to be excluded? Is it possible that products of lower quality obtain a higher market share? How is the equilibrium affected if firms can influence consumers’ expectations? Does the ability of firms to commit to the prices they set contribute to an equal or an unequal sharing of the market and does such a commitment tend to favor high- or low-quality firms? Are there conditions that give rise to a multiplicity of equilibria and what is the role of the formation of expectations for distinguishing among such equilibria?

In this paper, we emphasize the role of consumers’ expectations formation. Throughout the analysis, consumers’ expectations about firms’ market shares are required to be fulfilled in equilibrium, to be “rational”. However, we distinguish between two alternatives. First, we examine the case where these expectations cannot be influenced by the prices set by the firms. In this case, expectations are formed before the prices are set (or equivalently, even if expectations are formed after some prices are set, that does not matter because firms do not commit to these prices). Second, we examine the case where consumers’ expectations can be influenced by the prices set by the firms; this should be the case when firms commit to the prices.

When expectations are not influenced by the prices, our main results are as follows. If the products have equal quality and for relatively weak network effects, the only equilibrium is that the firms share the market equally. In contrast, for stronger network effects, there can also be asymmetric equilibrium configurations, with one of the two firms capturing the entire market. The threshold, above which asymmetric equilibria with only one active firm arise, is determined by comparing the strength of the network effect to the transportation cost (or, equivalently, to the importance of product differentiation). The two possible asymmetric equilibria are extreme (in the sense that one firm captures the entire market) and no other asymmetric equilibria (with the market shared unequally) exist. However, when, in addition to horizontal
characteristics, the products differ with respect to their qualities, we find that, if the network effect is relatively weak, the high-quality firm captures a larger share of the market and the low-quality firm a smaller one, depending on the quality difference. If the network effect is relatively strong, either the low-quality firm or the high-quality firm may capture the entire market; however, for the low-quality firm to capture the entire market, the quality difference cannot be too large relative to the network effect.

When expectations are influenced by the prices, competition tends to be more intense relative to the previous case. When the two products have the same quality and the network effect is relatively weak, the firms share the market equally and both make positive profits. In contrast, if the network effect is relatively strong, there is a tendency for one firm to capture the entire market. In this case, we emphasize the crucial role for the construction of the equilibrium of the expectations’ formation following the observation by consumers of out-of-equilibrium prices. We characterize a continuum of equilibria and show how the set of equilibrium prices shrinks to a single point if expectations are required to move in a continuous manner (then, one firm captures the entire market). Under quality differentiation, with weak network effects, the high-quality firm captures a larger market share than its rival, or captures the entire demand, if the quality difference is large enough. If the network effect is strong, again there may be a continuum of equilibria, but the high-quality firm is favored.

By comparing the various scenarios examined, we see that to obtain an equilibrium where the firms share the market asymmetrically when the expectations are not influenced by prices, the qualities of the products have to be different. When prices can influence the equilibrium, however, we can have asymmetric equilibria even without quality differences. So, unequal market shares are consistent either with unequal inherent qualities or with expectations influenced by the prices set. Also, when expectations are influenced by the prices, the firms tend to compete with greater intensity than in the case when expectations are not influenced by the prices, leading to lower equilibrium prices and profits. Both when prices influence consumers’ expectations (for weak or strong network effects) and when they do not (for weak network effects), it is possible that the market is shared, with the high-quality firm obtaining a larger share than its rival. Importantly, when expectations are influenced by prices, the high-quality firm is favored more and its market share increases relative to when expectations are not influenced by prices. Further, when expectations are influenced by prices, it becomes easier to obtain a corner equilibrium (where one firm, low- or high-quality, serves all the market). Continuity of expectations is also shown to have a dramatic effect, by reducing the continuum of equilibria that one obtains under strong network effects to a unique one.

In our model, with unit demands, prices do not affect total welfare. A social planner would either choose to have all buyers served by the high-quality firm (if the quality difference is high or if network affects are strong) or by both firms when the unit transportation cost is high. The equilibria we derive tend to be efficient under strong network effects and high-quality differences. Since the threshold for strong network effects is smaller when expectations are influenced by the prices (under our continuity restriction), we find that it is easier to reach an efficient outcome under expectations that are affected by prices. Moreover, we look at the case of market-wide network effects, and find that the equilibrium prices and profits tend to be more competitive under product-specific network effects and when expectations are influenced by the announced prices.

The core policy and empirical implications of the paper are related to how expectations are influenced by the prices. In markets where the prices of products and services can be changed easily, often or without cost, it makes sense that the consumers should not base the formation of their expectations about the market shares on the observed prices. The opposite will tend to be true when prices cannot be changed easily, often and without cost. Then the formation of expectations will have to take the observed prices seriously. Whether prices can be changed often and costlessly or not depends, of course, on various features of the firms’ environment: technological (e.g. how often new price lists can be produced?), institutional (e.g. how do consumers receive price information?) or regulatory (e.g. is permission by a sectoral regulator required for a price increase or decrease and how fast can this be obtained?). In a setting like ours, any policy intervention that will enhance the ability of firms to commit to their prices (by making price changes harder to take place), will be beneficial, since they will tend to lead to lower equilibrium prices and also to a higher probability of an efficient outcome.6

The remainder of the paper is as follows. In Section 2, we set up the basic model allowing for horizontal and vertical differentiation and network effects. In Section 3, we examine equilibrium behavior when consumers’ expectations about the market shares cannot be influenced by the firms’ prices. In Section 4, we study the equilibrium when consumers’ expectations are influenced by the prices that firms charge. In Section 5 we refine the equilibria by imposing a continuity restriction. Section 6 compares the equilibrium sets across the different cases. We discuss efficiency and compare product specific network effects to market-wide network effects. Section 7 concludes. Some derivations are in the Appendix.

2. The basic model

We consider competition between two firms, A and B, in a simple Hotelling model. Demand is represented by a con-

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6 For instance, in markets with network effects but where for other reasons (e.g. fighting “durable-goods-type” problems) firms may have established commitment devices so that prices do not change easily (such as in software or websites access pricing), or where regulation makes such changes difficult (e.g. in fixed-line telephony or some energy markets) prices should be considered by buyers before forming expectations about market shares. In other markets, like in mobile telephony or financial exchanges, prices tend to change more often (and not even to have a linear or other simple structure) so it is harder for buyers to consider them as fixed when trying to predict market shares.
tinuum of consumers with total mass equal to 1. The two firms are located at the extremes of a [0,1] linear city: firm A is located at point 0 and firm B at point 1. In addition to different locations, firms also have potentially different product qualities. The product quality of firm A is denoted by \( q_a \) and that of firm B by \( q_b \). Without loss of generality we set \( q_a = 0 \) and \( q_b = q > 0 \), that is, firm B is the “high-quality” firm and A the “low-quality” firm, while \( q \geq 0 \) is simply the product quality difference. Firms use simple linear pricing and seek to maximize their own profit. For simplicity, we set costs equal to zero.

Consumers are uniformly distributed along the unit interval. Each consumer selects one unit of the product of one of the two firms. Every consumer derives utility from the network size of the firm he has chosen. Since these sizes are actually determined after consumers make their choices, consumers choose a product based on their expectations about each firm’s network. In equilibrium, a consumer located at point \( x \in [0,1] \) will be indifferent between the two products when their “generalized” costs are equal:

\[
p_A + tx^2 - bx^2 = p_B + t(1-x)^2 - b(1-x^2) - q. \tag{1}
\]

where \( p_i \) is firm \( i \)'s price, \( i = A, B, t > 0 \) is an index of the per unit transportation cost, \( b > 0 \) measures the network effect and \( x^2 \) is the number of customers expected to purchase firm A’s product. If, for a given consumer located at \( x \), the left hand side of (1) is above (respectively, below) its right hand side, that consumer will purchase product B (respectively, A). Solving Eq. (1) for \( x \), we obtain the location of the indifferent consumer as

\[
x = \frac{p_A - p_B + b(2x^2 - 1) + t - q}{2t}. \tag{2}
\]

If the solution in (2) falls within \([0,1]\), then a consumer indifferent between the two products exists; otherwise, all consumers prefer one of the two products.

We have a two stage game. First firms announce their prices, subsequently consumers, based on their expectations and the announced prices maximize their utility by selecting a unit of a product. We are looking for a subgame perfect Nash equilibrium and solve backwards. First the demand function is determined and then, given this demand, firms’ profits have to be maximized with respect to the prices. We solve the model assuming rational (in other words, fulfilled in equilibrium) expectations. Of course, the formation of these rational expectations plays a crucial role in the derivation of the demand function and therefore of the equilibrium. In particular, expectations can be influenced by the announced prices or they may be formed prior to the announcement of prices. We analyze both cases and then we further refine the equilibrium set based on a continuity requirement.

\[3.\text{Equilibrium with expectations not influenced by prices}\]

In this section, we analyze the case where consumers’ expectations about market shares (that is, about the network effect), while fulfilled in equilibrium, are not influenced by the announced prices. This corresponds to cases where consumers believe that the firms cannot commit to the prices they announce, or when the consumers’ expectations are formed before prices become observable. Then the firms do not view the prices as an instrument to manipulate these expectations.

To proceed to the analysis we first examine price competition for fixed consumers’ expectations. Effectively, these expectations serve to create an asymmetry between the two firms in addition to their otherwise different product qualities. The equilibrium derivation is presented in A. It is shown that when firms A and B compete taking as given consumers’ expectations that A’s market share is \( x^* \in [0,1] \), prices are set in equilibrium so that the implied market share for firm A is

\[
x^*(x^*) = \begin{cases} 1 & \text{if } b(2x^* - 1) - q > 3t \\ \frac{1}{3} \frac{b(2x^* - 1) - q}{b^3} & \text{if } -3t < b(2x^* - 1) - q < 3t \\ 0 & \text{if } b(2x^* - 1) - q \leq -3t. \end{cases} \tag{3}
\]

with firm B having share equal to \( 1 - x^*(x^*) \).

When consumers’ expectations are that the market shares are not extreme, both firms will indeed make positive sales.\(^8\) If consumers expect that the market share differences exceed a given threshold, then the firm expected to have a larger market share indeed captures the entire market. A high value of \( t \) tends to make both firms have positive sales in equilibrium, while high values of \( b \) or \( q \) tend to make the favored firm capture the entire market.\(^9\)

We now proceed by requiring that the consumers’ expectations are rational (fulfilled in equilibrium), while they cannot be influenced by the prices set by the firms. We impose rationality by substituting \( x^* = x^* \) into expression (3). There are three cases to consider. Case 1: For relatively weak network effects (\( b < 3t \)), we obtain

\[
x^* = \begin{cases} \frac{1}{2} \frac{q}{b^3} & \text{if } 0 < q < 3t - b \\ 0 & \text{if } q \geq 3t - b. \end{cases} \tag{4}
\]

Thus, firm B (the high-quality firm) has a larger market share than its rival if the quality difference is relatively small (\( 0 < q < 3t - b \)), still both firms make positive sales. If the quality difference is relatively strong (\( q \geq 3t - b \)),

\[8\] The same assumption is also followed in many contributions in the networks literature. For example the well-known analysis of Katz and Shapiro (1985) also follows, in its greater part, the same assumption that expectations are not influenced by the firms’ strategic choices (quantities in that analysis).

\[9\] A firm that prices at zero and makes no sales provides the required competition to discipline the behavior of the rival firm in a corner equilibrium. It could be also viewed as a firm that is ready to enter a contestable market.

\[10\] With fixed expectations, the model becomes one where horizontally differentiated products are also viewed by (all) consumers as of different qualities (see Vettas, 1999).
firm B captures the entire market. The corresponding equilibrium prices are \(^\text{11}\)

\[
p_A^* = \begin{cases} \frac{t - \frac{2q^3}{1 - \beta}}{C_0} & \text{if } 0 \leq q < 3t - \beta \\ 0 & \text{if } q \geq 3t - \beta 
\end{cases}
\]

(5)

and

\[
p_B^* = \begin{cases} \frac{t + \frac{q}{1 - \beta}}{C_0} & \text{if } 0 \leq q < 3t - \beta \\ \beta - t + q & \text{if } q \geq 3t - \beta 
\end{cases}
\]

(6)

Case 2: For relatively strong network effects (\( \beta > 3t \)), we obtain

\[
x' = \begin{cases} \{0, \frac{1}{2}, 1\} & \text{if } q = 0 \\ \{0, 1\} & \text{if } 0 < q < \beta - 3t \\ 0 & \text{if } q \geq \beta - 3t 
\end{cases}
\]

(7)

Thus, when there is no quality difference either firm may capture the entire market, or the two firms may equally share the market. When the quality difference between the two firms is relatively small (\(0 < q < \beta - 3t\)), either the high- or the low-quality firm may capture the entire market. When the quality difference is relatively strong, however, equilibrium is possible only when the high-quality firm captures the entire market. The corresponding equilibrium prices are

\[
p_A^* = \begin{cases} 0 & \text{if } x' = 0 \\ t & \text{if } x' = \frac{1}{2} \\ \beta - t - q & \text{if } x' = 1 
\end{cases}
\]

(8)

and

\[
p_B^* = \begin{cases} \beta - t + q & \text{if } x' = 0 \\ t & \text{if } x' = \frac{1}{2} \\ 0 & \text{if } x' = 1 
\end{cases}
\]

(9)

Case 3: For the borderline case \( \beta = 3t \), we have to distinguish two scenarios: when \( q = 0 \), any \( x' = x' \in [0, 1] \) satisfies expression (3), and so any expectation about market shares can be fulfilled in equilibrium. However, when \( q > 0 \) we necessarily have \( x' = 0 \), the only expectation that can be fulfilled in equilibrium is when consumers expect that the high-quality firm captures the entire market. The associated equilibrium prices are as follows. For \( q = 0 \) we have \( p_A^* = 2tx' \) and \( p_B^* = 2t(1 - x') \). For \( q > 0 \) we have \( p_A^* = 0 \) and \( p_B^* = 2t + q \). We summarize:

**Proposition 1.** When consumers’ expectations are not influenced by the prices, equilibrium is as follows. Case 1: For weak network effects (\( \beta < 3t \)), if \( 0 < q < 3t - \beta \) there is a unique equilibrium where both firms have positive market share (\( x' = \frac{1}{2} - \frac{q}{2(3t - \beta)} \)), with the high-quality firm capturing a larger market share, while if \( q \geq 3t - \beta \), there is a unique equilibrium where the high-quality firm captures the entire market. Case 2: For strong network effects (\( \beta > 3t \)), if \( q = 0 \) either firm captures the entire market or they may share the market equally; if \( q < \beta - 3t \) either the high- or the low-quality firm may capture the entire market; if \( q \geq \beta - 3t \) the high-quality firm captures the entire market. Case 3: When \( \beta = 3t \), if \( q = 0 \) any expectation can be fulfilled in equilibrium, while if \( q > 0 \) the high-quality firm captures the entire market.

Let us discuss the structure of the equilibria we have derived. To obtain some insights into the problem let us focus on the special case when \( q = 0 \), no quality difference. Then, in equilibrium firm A’s market share is equal to

\[
x' = \begin{cases} \frac{1}{2} & \text{if } \beta < 3t \\ \text{any } x' \in [0, 1] & \text{if } \beta = 3t \\ 0 \text{ or } \frac{1}{3} \text{ or } 1 & \text{if } \beta > 3t 
\end{cases}
\]

(10)

We see that, for any given network effect \( \beta \) or transportation cost \( t \), if consumers expect that the two firms will equally share the market, this expectation is fulfilled in equilibrium. For the case where the network effect is weak (\( \beta < 3t \)), this expectation of equal sharing is the only one that can be fulfilled in equilibrium. When the network effect is strong (\( \beta > 3t \)), in equilibrium, three expectations concerning market shares can be fulfilled: either one of the two firms captures the entire market, or the two firms share the market equally. No other expectation can be supported in equilibrium (e.g. one with unequal market sharing).\(^\text{12}\) Finally, when \( \beta = 3t \) any expectation can be fulfilled in equilibrium. This borderline case is the only case where both firms can make positive sales without sharing the market evenly. Thus, when consumers’ expectations are not influenced by prices and there is no quality difference, the presence of network effects does not dramatically modify price competition relative to the no network effects case. In such a case, firms would share the market equally with prices equal to \( t \); if network effects are weak this remains the only equilibrium behavior. If network effects are strong, the possibility that one of the firms captures the entire market also emerges in equilibrium (with prices extreme enough to support the equilibrium).

Now let us focus on the role of the quality difference. When is it that the high-quality firm necessarily captures the whole market in equilibrium and when is it that the low-quality firm can get some share or the whole market? Collecting the various cases from above, we see that if the quality difference is high enough (relative to the network effect), the high-quality firm always captures the entire market. The low-quality firm, may also capture the entire market, but only when the quality difference is small and the network effect is strong. Moreover, the two firms never share the market equally, unless their qualities are exactly equal. The high-quality firm gets more customers than its rival, but both firms have positive market shares if the network effect is weak and the quality difference is small.

4. Equilibrium when expectations are influenced by prices

Now, we examine how prices and market shares are formed when firms can manipulate consumers’ expectations. Firms announce their prices and consumers take into

\(^{11}\) It follows by substitution into the equilibrium prices expression with fixed expectations in \( A \).

\(^{12}\) When \( \frac{1}{2} < x' < 1 \), firms’ price competition implies \( x > x' \), while when \( 0 < x' < \frac{1}{2} \) it implies \( x < x' \).
consideration the announced prices when forming their expectations.\textsuperscript{13} In this scenario, firms announce their prices, knowing that these announcements will influence consumers’ expectations about each firm’s market share. Then, consumers form their expectations, taking as given the prices that have been announced and choose which product to purchase.\textsuperscript{14}

Although our models differ with respect to their parametric set up,\textsuperscript{15} our analysis in this section is parallel to that in Section 4 in Grilo et al. (2001) and the nature of our equilibria in this section is essentially identical to theirs. Therefore, we keep the analysis as short as possible, only to the extent required to state the precise results, as they emerge in the case of our model.\textsuperscript{16}

We start by substituting $x = x$ in expression (2) to obtain

\[ x = \frac{p_B - p_A - \beta + t - q}{2(t - \beta)}. \]  

(11)

This expression is crucial for the equilibrium construction. It corresponds to the location of the indifferent consumer (if one exists) when the prices are $p_A$ and $p_B$ and when all the consumers believe that the market share of firm A is indeed given by expression (11). The idea here is that, since firms can influence consumers’ expectations via their prices, these prices should be used when deriving the expected market shares (which, of course, will be equal in equilibrium to the actual market shares). It is important to observe that if $t > \beta$, a higher price by a firm is associated with a lower market share for that firm. In contrast, if $t < \beta$, an increase in a firm’s price is associated with an increase in its market share. With strong network effects the only way that the firms may share the market is if the one with the higher quality-adjusted price also has a larger market share; otherwise, all consumers would prefer one of the two firms.

We start with the $t > \beta$ case. We note that Eq. (11) implies that $x \in (0, 1)$ only if $\beta - t + q < p_B - p_A < t - \beta + q$. If $p_B - p_A > t - \beta + q$, then all consumers choose firm A and if $\beta - t + q > p_B - p_A$, they all choose firm B. Collecting all possibilities, when consumers form their expectations after observing firms’ prices, and when these expectations are fulfilled, we find that firm A’s market share, when $t > \beta$, is

\[ x = \begin{cases} 0 & \text{if } p_B - p_A \leq \beta - t + q \\ \frac{p_B - p_A - \beta + t - q}{2(t - \beta)} & \text{if } \beta - t + q \leq p_B - p_A \leq t - \beta + q \\ 1 & \text{if } p_B - p_A \geq t - \beta + q. \end{cases} \]  

(12)

Proceeding to the equilibrium construction when network effects are relatively weak, we obtain the following result (details are in B).

**Proposition 2.** Suppose consumers’ expectations are influenced by the prices and $t > \beta$. If $0 < q \leq 3(t - \beta)$ there is a unique equilibrium where both firms have positive market shares $\left( x = \frac{1}{2} - \frac{q}{2(t - \beta)} \right)$, the high-quality firm captures a larger market share than its rival and the equilibrium prices are $p_A^* = t - \beta + q$ and $p_B^* = t - \beta - q$. If $q > 3(t - \beta)$, there is a unique equilibrium where the high-quality firm captures the entire market and the equilibrium prices are $p_A^* = 0$ and $p_B^* = t - q > 0$.

Let us now turn to the case of strong network effects ($t < \beta$) where the analysis becomes much richer, since both firms’ demand functions can be increasing in their own prices. Our aim is to highlight the role of expectations formation. Firm A’s market share, when consumers behave rationally and $t < \beta$, is

\[ x = \begin{cases} 0 & \text{if } p_B - p_A \leq \beta - t + q \\ \frac{p_B - p_A - \beta + t - q}{2(t - \beta)} & \text{if } \beta - t + q \leq p_B - p_A \leq t - \beta + q \\ 1 & \text{if } p_B - p_A \geq t - \beta + q. \end{cases} \]  

(13)

Note that firm A’s market share is given by expression (13) when the prices are $p_A$ and $p_B$ and when consumers believe that firm A’s market share is indeed given by expression (13). We observe that when $t - \beta + q \leq p_B - p_A \leq t - q$, three different expectations concerning the market shares can be consistent with equilibrium: when consumers observe that the price difference belongs to this interval, they may believe that all consumers will choose firm A, or that all consumers will choose firm B, or that $p_B - p_A - \beta < t - q$ consumers will choose firm A and the rest will choose firm B. We also observe that, when the price difference belongs to this interval, an increase in the price of a firm is associated with an increase in its market share.

In this case, because network effects are so strong that they may dominate the other strategic considerations, the requirement that expectations are rational (fulfilled in equilibrium) is too weak to significantly restrict by itself the set of equilibria. It becomes crucially important also how we treat the formation of expectations following an observation of out-of-equilibrium prices.\textsuperscript{17} We require therefore that expectations after observing some price deviation are formed in a way that corresponds to some equilibrium behavior for the consumers. More precisely, this implies that the market share expectations following any prices should obey Eq. (13). Note that these expectations when prices satisfy $t - \beta + q \leq p_B - p_A \leq t - q$ can take three different values, a feature of the problem

\textsuperscript{13} Consumers should be influenced by the announced prices if firms commit to them. Whether this is possible or not depends on the institutional and technological details in each market. If firms could easily raise their prices, an announcement of a low early price would not be a credible signal interpreted by the consumers as corresponding to a larger market share.

\textsuperscript{14} Katz and Shapiro (1985), briefly also examine in the Appendix of their paper the case where expectations are formed after the firms’ strategic choices have been made. Our analysis is different since the firms compete in prices and the implied quantities (representing the network effect) are only determined in equilibrium and, thus, influenced by the strategic choices only indirectly, not “chosen” directly by the firms, as in Katz and Shapiro (1985).\textsuperscript{2}

\textsuperscript{15} Grilo et al. (2001) have a richer parametric network effect function (a linear and a quadratic term) and a wider set of locations on the line but we allow for quality differences independently of the horizontal differentiation.

\textsuperscript{16} The details are available by the authors upon request.

\textsuperscript{17} If, for instance, we allowed for expectations to be unconstrained in the event that some out-of-equilibrium prices are observed, then we could support a very large set of prices as an equilibrium - we would simply have to specify that consumers react to any deviation by shifting their expectations to believing that the deviating firm will have zero market share. Clearly, such an approach would be extreme.
that significantly enriches the analysis: the consumers may believe that the market is shared according to the stated rule in (13), or that one or the other firm captures the entire market.\footnote{We will refine our equilibrium in the subsequent section by becoming more restrictive with respect to the formation of the out of equilibrium expectations.}

Two inequalities play an important role, corresponding to when price undercutting is not desirable for a firm, that is, when sharing the market is more profitable than capturing the entire market:

\[ p_A \left( 1 - \frac{p_A - \beta + t - q}{2(t - \beta)} \right) \geq p_B - (\beta - t) + q. \]  

(15)

We obtain:

**Proposition 3.** With strong network effects (\( \beta > t \)) and consumers’ expectations influenced by the prices set by the firms, multiple equilibria exist. Any combination of \( p_A \leq \beta - t - q \) and \( p_B \leq \beta - t + q \) constitutes an equilibrium with associated market shares for firm A equal to \( x_A = \frac{p_B - p_A - \beta - t + q}{2(t - \beta)} \) and \( x_A = 1 \). Any combination of \( p_A \geq \beta - t - q \) and \( p_B \geq \beta - t + q \) that satisfy Eq. (14) constitutes an equilibrium with associated market shares \( x_A = 0 \) and \( x_A = \frac{p_B - p_A - \beta - t + q}{2(t - \beta)} \). Any combination of \( p_A \approx \beta - t - q \) and \( p_B \approx \beta - t + q \) that satisfy Eq. (15) constitutes an equilibrium with associated market shares \( x_A = \frac{p_B - p_A - \beta - t + q}{2(t - \beta)} \).

Let us illustrate the logic behind the above Proposition by focusing on a particular pair of candidate equilibrium prices. Take for a given \( p_B \in [q, \beta - t] \) the pair of prices formed by that \( p_B \) and by some \( p_A = p_B - q \). This pair of prices could possibly be associated with three equilibrium market shares for firm A: \( x_A = \frac{p_B - p_A - \beta - t + q}{2(t - \beta)}, x_A = 0 \), and/or \( x_A = 1 \). Let us check first whether this specific combination of prices can be supported as an equilibrium with \( x = \frac{1}{2} \). An increase in firm A’s price can lead consumers to believe that firm A will lose all its customers (\( x = 0 \)) and a decrease in its price decreases its market share and its profit. The same reasoning holds for firm B, so we conclude that there are expectations that support \( p_A = p_B - q \) with \( x = \frac{1}{2} \) as an equilibrium. Can the same prices constitute an equilibrium with \( x = 0 ? \) For firm A, the same argument holds as when \( x = \frac{1}{2} \). Concerning firm B, an increase as well as a decrease in B’s price may lead consumers to believe that B will never attract any customers. Finally, can the same prices constitute an equilibrium with \( x = 0 ? \) The same arguments apply as with \( x = 1 \), but we need to reverse the roles of the firms. Similar reasoning holds for any given pair of prices when \( p_A \leq \beta - t - q \) and \( p_B \leq \beta - t + q \). The equilibrium combinations of prices are depicted in Fig. 1.

The fact that different consumers’ market shares expectations can be consistent with different prices plays a strong role in determining the attractiveness of each product and allows us to support in equilibrium a large set of prices and different market shares. In particular, it is important to note that, as shown in Fig. 1, the equilibrium set becomes “narrower” as we move up. The higher the prices are, the smaller their difference has to be to obtain an equilibrium. If prices are high and their difference is large, then there is room for each firm to profitably deviate to a lower price. Thus, when the price difference becomes large enough, the only consistent expectation is that the lower priced firm captures the entire market. If the candidate equilibrium prices are already low, there is no room for such a profitable deviation to an even lower price. For high prices, however, such a deviation would be profitable and thus, the higher the prices are, the closer to each other they have to be for an equilibrium to exist.

Up to now, for the strong network effect case, we have studied the formation of expectations and the associated equilibria imposing only the weak requirement that expectations (following out-of-equilibrium prices) are restricted to follow some equilibrium behavior. A large set of equilibria emerges, as we have seen. In the next section, we proceed to refine the set of equilibria by imposing a stricter requirement on expectations, continuity.

5. Refinement of equilibria: continuity of expectations

Proposition 3 shows that a multiplicity of equilibria emerges under strong network effects when expectations are only restricted to not being arbitrary out of the equilibrium path. It is important to study how additional restrictions on the way expectations can be formed can further refine the set of equilibria. The more restrictive we become as to how the expectations are formed, the smaller the set of equilibria we can obtain. Grilo et al. (2001), to refine the set of equilibria, suggest an “invariance axiom” (in their Section 4, Proposition 4), but still multiple equilibria arise. They show that the firm with the higher price always captures the entire market, but without necessarily charging the maximum possible price. Therefore, as Grilo et al. (2001) also suggest (in their ft. 11), a more selective refinement may be desirable.
In order to refine the set of equilibria in our model, we impose the restriction on the formation of expectations that expectations move “continuously” when this is possible: what this means is that, small changes of the equilibrium prices lead to small changes in the expectations about the market shares. This is relevant for the candidate equilibrium prices when their difference falls within the \((t - \beta + q, \beta - t + q)\) interval. Then, as already discussed, without any restrictions, three market shares are consistent with these prices. By continuity, we require that if there is a small price deviation (out of equilibrium), consumers’ expectations do not discretely “jump” to another “branch” of \((13)\), when this is not necessary (that is, when it is possible to move continuously).\(^{19}\) This is a reasonable restriction if one believes that consumers do not alter their expectations drastically when they observe a small change in some price. Importantly, we show that under strong network effects, by adding this continuity requirement we can only have an equilibrium where one of the firms captures the entire market. Thus, our analysis highlights the crucial role of expectations formation for the characterization of the equilibrium behavior.

We turn now to the details of the analysis. When the network effects are strong \((t < \beta)\) and the quality difference is also small \((q < \beta - t)\) we cannot have an interior solution: each firm would have an incentive to increase its price and (by the continuity of expectations) increase its market share and its profit. Can we have a corner solution? If we set \(p_A = 0\) and \(p_B = \beta - t + q\), all consumers would choose firm B. We need to check whether firm B has an incentive to deviate (obviously, to a higher price, as its market share cannot be increased further). By expression \((13)\), we observe that, if firm B increases its price, it loses its clients and makes zero profit. We conclude that for \(p_A = 0\), firm B optimally charges \(p_B = \beta - t + q\). Now, let us check firm A’s behavior. Firm A has no incentive to decrease its price below zero, since that would lead to a loss. If firm A increases its price and if consumers continue to believe that firm B will capture the entire market, then firm A will not attract any customers and, therefore, will continue having zero profit. We conclude that \(p_A = 0\) and \(p_B = \beta - t + q\) are indeed equilibrium prices and firm B captures the entire market. The same argument holds, symmetrically, for \(p_A = \beta - t - q\) and \(p_B = 0\), where firm A captures the entire market. Could we also have an equilibrium when \(p_A > 0\) and \(p_B = p_A + \beta - t + q\)? Firm B has no reason to deviate, by the same logic as in the previous case. On the other hand, firm A has an incentive to decrease its price in order to attract the entire market and make positive profit. We conclude that we cannot have an equilibrium when both firms charge strictly positive prices.

For the case where the qualities of the two products are different enough, that is \(q \geq (\beta - t)\), firm A will never attract the entire market in equilibrium since \(p_A = \beta - t - q < 0\) and firm B always prefers to capture the entire market than to share it with firm A. In this case, when expectations are continuous we can only have a unique equilibrium where \(p_A = 0\) and \(p_B = \beta - t + q\) and firm B, the higher quality firm, attracts the entire market. We conclude:

**Proposition 4.** Suppose that \(\beta > t\) and consumers’ expectations respond in a continuous manner to price changes. If \(q < \beta - t\), we obtain two equilibria where either firm can capture the entire market. Firm A captures the entire market when \(p_A = \beta - t - q\) and \(p_B = 0\) and firm B captures the entire market when \(p_A = 0\) and \(p_B = \beta - t + q\). If \(q > \beta - t\), there is a unique equilibrium where the high-quality firm captures the entire market and the equilibrium prices are \(p_A = 0\) and \(p_B = \beta - t + q\).

Thus, the set of price equilibria shrinks dramatically and only corner equilibria survive when we impose some reasonable continuity assumption on the formation of expectations.\(^{20}\) Some further discussion is in order. Starting from the general case with expectations affected by prices, when firms differ in two dimensions, that is, with respect to both location and quality \((q > 0)\) and when the network effect is relatively weak \((t < \beta)\), our results depend on the magnitude of the quality difference between the two firms. The high-quality firm always attracts more customers than its rival and finds it more profitable to capture the entire market if the quality difference is high enough \((q \geq 3(t - \beta))\). When both firms have positive market shares, the high-quality firm charges a higher price compared to its rival, and its price increases as the quality difference increases. When the network effect is relatively strong \((t < \beta)\), we may obtain a continuum of equilibria, depending on the off-the-equilibrium expectations’ formation. When we focus on the case where expectations can change (responding to out-of-equilibrium prices) only in a continuous manner, we find that, if the quality difference is relatively low \((q < \beta - t)\), either firm can capture the entire market but we cannot have an interior equilibrium. Further, we observe that to have a unique equilibrium where the high-quality firm captures the entire market, we need a higher quality difference \((q > \beta - t)\). Finally, note that this quality level is lower than the quality difference needed for a unique equilibrium in the case of relatively weak network effects.

### 6. Comparison of equilibria and efficiency

It is useful to compare the various cases that emerge in our analysis and summarize the results. Table 1 shows the equilibrium market shares. The equilibrium prices are indicated here only for the case of weak networks effects and

---

\(^{19}\) Thus, starting from a situation where the market is shared (and of course this has to be reflected in the expectations), if the price of one or both firms is changed marginally, the expectations about market shares will be that the market will continue to be shared (with slightly modified shares) and will not suddenly change to that one firm will get all the market (assuming, of course the price is not changed to a level where this corner outcome could be the only equilibrium expectation).

\(^{20}\) The Grilo et al. (2001) refinement requires “invariance,” meaning that consumers cannot change their expectations when the difference between the two prices does not change (in other words, if both prices increase or decrease by the same amount the expectations should not change). This refines significantly the set of equilibria but allows equilibria where both firms price at zero and in case one firm captures the whole market by charging a positive price, this price (and the corresponding profit) is not uniquely determined. Our continuity requirement rules out all other equilibria other than corner equilibria and the prices are uniquely determined.
low-quality differences, since it is only then that the prices differ between the two cases. For the case of strong network effects when expectations are influenced by the announced prices, we report the equilibria that emerge under the continuity refinement.

We note that the low-quality firm (firm A) may capture the entire market ($x = 1$) only under strong network effects and small quality differences. We observe that this case is met for a lower threshold with respect to the network effect when expectations are influenced by the announced prices. The high-quality firm, however, may dominate the market ($x = 0$) both under weak and under strong network effects. This is achieved for a wider range of parameter values when expectations are influenced by the prices. When firms share the market, the high-quality firm captures a larger market share, compared to the case where expectations are not influenced by prices.

We also note that when expectations are influenced by prices, $t > \beta$ (that is, the network effects are weak) and $q < 3(t - \beta)$, the equilibrium prices are lower than the prices firms charge when these prices cannot influence consumers’ expectations. The intuition behind this result is that, when firms know that their prices will influence consumer’s expectations, they compete more intensely and are willing to lower their prices to indirectly gain profit through an increase in their network effect (that is, the market share that consumers expect for them). Given the price of one firm, the rival wishes to undercut its price to strategically alter consumers’ expectations in its favor. Consumers that observe a firm to have a lower price than its rival, expect that a larger number of other consumers will choose this firm, so the market share of this firm will be larger than its rival’s (and so will be its overall attractiveness, due to the network effect). With both firms acting this way, we obtain lower equilibrium prices compared to the case of expectations not influenced by prices. On the other hand, in the case where $t < \beta$, we can only have corner solutions since the network effect is so strong that dominates all other differentiation and drives consumers to believe that a single firm will capture the entire market. Crucially, we have shown that imposing the requirement that expectations do not move discontinuously when consumers face out-of-equilibrium prices is enough to deliver this drastic reduction of the equilibrium set.

**Efficiency and profits.** Let us turn now to the efficiency properties of the various equilibria that emerge. In our model, the market is covered and we have unit demand, thus the prices do not affect total welfare (higher prices do not cause a deadweight loss). Total welfare depends on the network effect and the transportation cost and can be measured by their difference. In addition, we have to take into consideration the different qualities of the two firms. If a social planner had to choose between the two corner outcomes, he would always prefer the dominance of the high-quality firm in the market, firm B. To see why total welfare is maximized, we have to compare the case when firm B dominates the market, with the case where both firms make sales. In the former case, the market benefits through the strong network effect and the higher quality, while in the latter through a reduced aggregate transportation cost.

When firm B dominates the market, the total network benefit is $\frac{1}{2}t^3 + 3t^2 \beta$ and the quality benefit is $q$. Therefore, total welfare is proportional to $W_1 = 1 - \frac{t}{2} + q$. When both firms are active, and $x$ is firm A’s market share, the total network benefit is $\frac{1}{2}t^3 + 3t^2 \beta + q(1 - x)^2 = \frac{1}{2}(1 - 2x + 2x^2)$ and the quality benefit is $q(1 - x)$, therefore, total welfare is proportional to $W_2 = (\beta - \frac{t}{2}) (1 - 2x + 2x^2) + q(1 - x)$. Comparing the two cases, we see that it is efficient for one firm to capture the market ($W_1 > W_2$) either when $\beta > \frac{t}{2}$ or when $\beta < \frac{t}{2}$ and $q > \frac{t}{2}$. When $\beta < \frac{t}{2}$ and $0 < q < t - 2\beta$, the social planner would prefer the two firms to share the market with firm A’s market share being equal to $x = \frac{1}{2} - \frac{\beta - t}{t - 2\beta}$.

We can now proceed to discuss the efficiency of equilibria we have derived, relative to the above derived benchmark. Under weak network effects, equilibria are efficient either when $q = 0$, where both firms equally share the market, or under high quality differences, where firm B captures the entire market. In this case, we see that it is easier to reach an efficient equilibrium when expectations are influenced by the announced prices, compared to when expectations are not influenced by the prices, since the threshold for high-quality is smaller. Under strong network effects, we only have corner equilibria, but we need to have a high enough quality difference in order for firm B to be the one that captures the entire market. In this case, the threshold for high-quality is lower when expectations are not influenced by the announced prices, but on the other hand, the threshold for strong network effects is lower under influenced expectations.

While, as we have discussed, the level of prices does not affect total welfare, in order to understand when the market is more competitive and consumers benefit, we can look at the price level. Also, it is useful to compare the mar-
are weak and quality differences are small.\textsuperscript{23}

The equilibrium prices and profits when network effects are influenced by the announced prices. For brevity we report only the cases of product-specific network effects where expectations are not influenced by prices and where expectations are influenced by the announced prices. For brevity we report only the equilibrium prices and profits when network effects are weak and quality differences are small.\textsuperscript{23}

We have studied a simple duopoly model where the two products are horizontally and vertically differentiated and exhibit network effects. We consider two scenarios of the (rational) formation of expectations about market shares, depending on the possible commitment of the prices that firms set. We examine both possibilities and show that the analysis and the results are qualitatively different.

When the firms cannot commit to the prices they announce, consumers’ expectations cannot be influenced by these prices. In this case, if the two firms are only horizontally differentiated and the network effect is weak, the only equilibrium is that the firms share the market equally; if the network effect is strong, either firm may capture the entire market, or they share the market equally. When the two products have the same quality we can never have an asymmetric “interior” equilibrium; if both firms make positive sales, sharing must be equal. If the two firms also differ in their product quality, we can have an asymmetric equilibrium where both firms have positive market shares, and the high-quality firm attracts more customers. However, under certain conditions (small quality difference and strong network effects), the low-quality firm can capture the entire market.

When we turn to the case where firms’ prices influence consumers expectations, we observe that, when network effects are relatively weak, firms compete more intensely compared to the previous case. This is because firms recognize that they can each manipulate the beliefs consumers have about the sharing of the market and, thus, by lowering their price they can attract consumers not only directly (because they become less expensive) but also indirectly (because of the coordination of beliefs). When the network effect is strong, we have emphasized how the equilibrium behavior depends on the way that market shares expectations are formed when out-of-equilibrium price deviations are observed. In general there is a continuum of equilibria and the higher the prices, the smaller the difference between the prices has to be. We also emphasize that, if expectations are required to respond to price deviations in a continuous manner, there can only be corner equilibria with one firm capturing the entire market and the prices are uniquely determined. In addition, when the qualities of the two firms differ enough, in the unique equilibrium it has to be that the high-quality firm captures the entire demand.

By emphasizing the crucial role of the rational formation of expectations for the eventual equilibrium (prices, profits and market shares), and in particular for when the low-quality firm may dominate in the market, we provide insights important for strategy decisions and policy design in network industries. First, the market outcomes expected crucially depend on whether the technology and institutional environment is such that it forces or allows firms to commit to the prices they set or not. In the former case, prices can be used to influence the consumers’ expectations and tend to be in equilibrium lower than in the latter case. It is also shown to be crucial if expectations tend to respond in a smooth manner to changes in the announced prices (in which case the market will tend to be dominated by a single firm, possibly the lower quality one) or if they can be drastically manipulated by small price changes (in which case multiple equilibrium exist and the eventual outcome will be harder to predict from a practical viewpoint).

Expanding our analysis in the direction of employing alternative product differentiation structures, dynamic competition, and by endogenizing the horizontal or vertical product differentiation decisions of firms, offers very promising possibilities for further research.

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\begin{table}
\centering
\caption{Comparison of prices and profits.}
\begin{tabular}{|l|l|l|}
\hline
Case & Expectations not influenced & Case II & Expectations influenced \\
\hline
Case I & Market-wide networks & Case III \\
\hline
$3t > \beta$ and $q < 3r - \beta$ & $t > \beta$ and $q < 3(t - \beta)$ & $\beta = 0$ and $q < 3t$ \\
$P_A = t - \frac{b}{C_0}$ & $P_A = t - \beta - \frac{b}{2}$ & \\
$P_B = t + \frac{b}{C_0}$ & $P_B = t + \beta + \frac{b}{2}$ & \\
$\pi_A = \frac{(q - 3r + \beta)^2}{2(3 - \beta)}$ & $\pi_A = \frac{(q - 3\beta)^2}{2(3 - \beta)}$ & \\
$\pi_B = \frac{(q - 3r - \beta)^2}{2(3 - \beta)}$ & $\pi_B = \frac{(q - 3\beta)^2}{2(3 - \beta)}$ & \\
\hline
\end{tabular}
\end{table}

\textsuperscript{22} For a more detailed analysis see Griva and Vettas (2004).

\textsuperscript{23} Under strong network effects, or under weak network effects and high-quality differences, only one firm can capture the entire market (under the restriction of continuity) and therefore prices and profits are always higher compare to the case where $\beta = 0$. 

\begin{itemize}
\item For a more detailed analysis see Griva and Vettas (2004).
\item Under strong network effects, or under weak network effects and high-quality differences, only one firm can capture the entire market (under the restriction of continuity) and therefore prices and profits are always higher compare to the case where $\beta = 0$.
\end{itemize}
Appendix A. Price equilibria under fixed expectations

We present here the construction behind Eq. (3). Let us take as given that all consumers expect firm A’s share to be $x^e \in [0,1]$. Then, firm A’s market share, as a function of the prices, is

$$
x = \begin{cases} 
0 & \text{if } p_B - p_A < q - t - \beta(2x^e - 1) \\
\frac{p_B - p_A + \beta(2x^e - 1) - t - q}{2t} & \text{if } q - t - \beta(2x^e - 1) < p_B - p_A < q + t - \beta(2x^e - 1) \\
1 & \text{if } p_B - p_A \geq q + t - \beta(2x^e - 1). 
\end{cases}
$$

As a consequence, firm A maximizes its profit,

$$
\pi_A = \begin{cases} 
0 & \text{if } p_B - p_A < q - t - \beta(2x^e - 1) \\
\frac{p_B - p_A + \beta(2x^e - 1) - t - q}{2t} & \text{if } q - t - \beta(2x^e - 1) < p_B - p_A < q + t - \beta(2x^e - 1) \\
p_B & \text{if } p_B - p_A \geq q + t - \beta(2x^e - 1), 
\end{cases}
$$

with respect to $p_A$ and firm B maximizes its profit,

$$
\pi_B = \begin{cases} 
p_B & \text{if } p_B - p_A < q - t - \beta(2x^e - 1) \\
\frac{p_B(1 - \frac{p_B - p_A + \beta(2x^e - 1) - t - q}{2t})}{2t} & \text{if } q - t - \beta(2x^e - 1) < p_B - p_A < q + t - \beta(2x^e - 1) \\
0 & \text{if } p_B - p_A \geq q + t - \beta(2x^e - 1), 
\end{cases}
$$

with respect to $p_B$.

We first examine the case where $x \in (0,1)$. The second-order conditions show that the profit functions are strictly concave, as $\frac{\partial^2 \pi_A}{\partial p_A^2} = \frac{\partial^2 \pi_B}{\partial p_B^2} = -\frac{\beta}{2} < 0$. Then, via the first-order conditions, we derive the reaction functions:

$$
p_A = R_A(p_B, x^e) = \frac{p_B + \beta(2x^e - 1) + t - q}{2} 
$$

and

$$
p_B = R_B(p_A, x^e) = \frac{p_A - \beta(2x^e - 1) + t + q}{2}.
$$

By solving the system of the reaction functions, we obtain the equilibrium prices (given the expected market shares):

$$
p_A(x^e) = t + \frac{\beta(2x^e - 1) - q}{3} \quad (A.1)
$$

and

$$
p_B(x^e) = t - \frac{\beta(2x^e - 1) - q}{3} \quad (A.2)
$$

By substituting Eqs. (A.1) and (A.2) into (2) we obtain

$$
x^e(x^e) = \frac{1}{2} + \frac{\beta(2x^e - 1) - q}{6t}.
$$

We observe that for $-3t < \beta(2x^e - 1) - q < 3t$, both firms have positive market shares. We reach a corner, where firm A captures the entire market, when $\beta(2x^e - 1) - q \geq 3t$ (note that for this inequality to hold, expectations must satisfy $x^e > \frac{1}{2}$, since $q$ can only take non negative values). And we reach a corner, where firm B captures the entire market, when $\beta(2x^e - 1) - q \leq -3t$.

Let us now explore possible corner solutions. Consider, first, the case where $\beta(2x^e - 1) - q \geq 3t$. If $p_A = \beta(2x^e - 1) - q - t$, firm B has zero demand if it charges any non negative price. In this case, firm B achieves its maximum profit, of zero, by charging $p_B = 0$. Next, we need to show that $p_A = \beta(2x^e - 1) - q - t$ is a best response for firm A. Firm A solves $\max_{p_A} \pi_A = \pi_B(x)$, where $x$ is its market share:

$$
x = \begin{cases} 
\frac{t - p_B + \beta(2x^e - 1) - q}{2t} & \text{if } p_A \geq \beta(2x^e - 1) - q - t \\
1 & \text{if } p_A \leq \beta(2x^e - 1) - q - t.
\end{cases}
$$

As long as $p_A \leq \beta(2x^e - 1) - q - t$ and $x^e \geq \frac{1}{2}$, the optimal price for firm A is

$$
p_A^* = \beta(2x^e - 1) - q - t, \quad (A.3)
$$

the highest price that gives the entire market to firm A. If firm A were to lower its price, it would not increase its market share, since firm A already captures the entire market: hence, the lower price would lead to a decreased profit. If $p_A \geq \beta(2x^e - 1) - q - t$, firm A shares the market with firm B and its profit decreases. It follows that the best response to $p_B = 0$ is given by expression (A.3), the maximum price that allows firm A to capture the entire market.

Consider now the case where $\beta(2x^e - 1) - q \leq -3t$. If $p_B = -\beta(2x^e - 1) + q + t$, firm A has zero demand if it charges a non negative price. In this case, firm A maximizes its profit by charging $p_A = 0$ and obtains zero profit. Next, we show that $p_B = -\beta(2x^e - 1) + q + t$ is a best response for firm B when $p_A = 0$. Firm B solves $\max_{p_B} \pi_B = \pi_A(1 - x)$ where $x$ is

$$
x = \begin{cases} 
\frac{t - p_B + \beta(2x^e - 1) - q}{2t} & \text{if } p_B \geq -\beta(2x^e - 1) + q - t \\
0 & \text{if } p_B \leq -\beta(2x^e - 1) + q - t.
\end{cases}
$$

As long as $p_B \leq -\beta(2x^e - 1) + q - t$, the optimal price for firm B is

$$
p_B^* = -\beta(2x^e - 1) + q - t. \quad (A.4)
$$

that is, the highest price that gives the entire market to firm B. If firm B decreased its price, it would not increase its sales, since firm B already captures the entire market. Hence, such a price decrease is not profitable. If $p_B \geq -\beta(2x^e - 1) + q - t$, firm B shares the market with firm A and its profit function is decreasing in $p_B$. It follows that the best response to $p_A^* = 0$ is given by expression (A.4), the maximum price that allows firm B to capture the entire market.

To sum up, when firms A and B compete taking as given consumers’ expectations that A’s market share is $x^e \in [0,1]$, the equilibrium prices are
p^A_A(x') = \begin{cases} 
\beta(2x' - 1) - q - t & \text{if } \beta(2x' - 1) - q \geq 3t \\
\frac{t + \frac{\beta(2x' - 1) - q}{3} - q}{4} & \text{if } -3t < \beta(2x' - 1) - q < 3t \\
0 & \text{if } \beta(2x' - 1) - q \leq -3t 
\end{cases}

p^B_B(x') = \begin{cases} 
0 & \text{if } \beta(2x' - 1) - q \geq 3t \\
\frac{t - \frac{\beta(2x' - 1) - q}{3} - q}{4} & \text{if } -3t < \beta(2x' - 1) - q < 3t \\
-\beta(2x' - 1) + q - t & \text{if } \beta(2x' - 1) - q \leq -3t 
\end{cases}

### Appendix B. Equilibrium derivation when expectations are influenced by prices and t > β

When expectations are influenced by prices and t > β, firm A maximizes its profit, 

πₐ = \begin{cases} 
0 & \text{if } p_b - p_A \leq \beta - t + q \\
P_A & \text{if } p_b - p_A \geq \beta - t + q 
\end{cases}

with respect to pₐ. Similarly, firm B maximizes its profit, 

πₜ = \begin{cases} 
P_B & \text{if } p_b - p_A \leq \beta - t + q \\
0 & \text{if } p_b - p_A \geq \beta - t + q 
\end{cases}

with respect to pₜ.

The best response correspondence of firm A is \(^{24}\)

pₐ = Rₐ(pₜ) = \begin{cases} 
p_b - t + \beta - q & \text{if } p_b \geq 3(t - β) + q \\
\frac{p_b - t + \beta - q}{2} & \text{if } \beta - t + q \leq p_b \leq 3(t - β) + q \\
\text{any price} \geq p_b + t - β - q & \text{if } p_b < \beta \\
-t + q & \text{if } p_b < \beta 
\end{cases}

and is derived as follows. If firm A is to capture the entire market, the maximum price it can charge is pₐ = pₜ + t + β - q. Now assume that each firm has positive market share. Then, its best response function, derived from the first-order condition corresponding to the relevant branch of the profit function, is

pₐ = Rₐ(pₜ) = \frac{p_b - t + β + t - q}{2}. \quad (B.1)

Firm A will charge this price as long as \(β - t + q \leq p_b - p_A \leq t - β + q\) and, after substituting expression (B.1) and solving with respect to pₜ, we obtain \(β - t + q \leq p_b \leq 3(t - β) + q\). If \(p_b \geq 3(t - β) + q\), firm A finds it more profitable to capture the entire market than to share it with firm B. On the other hand, if \(p_b \leq β - t + q\), firm A cannot compete with firm B because if it tries to share the market it ends up with losses. In this case, firm A prefers to make no sales and charges a price \(p_A \geq p_B + t - β - q\). The best response correspondence of firm B, 

pₜ = Rₜ(pₐ) = \begin{cases} 
p_a - t + β + q & \text{if } p_a \geq 3(t - β) + q \\
\frac{p_a - t + β + q}{2} & \text{if } \beta - t + q \leq p_a \leq 3(t - β) + q \\
\text{any price} \geq p_a + t - β - q & \text{if } p_a < β - t + q. 
\end{cases}

can be derived in a similar way as for A, where now firm B’s best response function, if the market is shared, is

pₜ = Rₜ(pₐ) = \frac{p_A - β + t + q}{2}.

As long as both firms make positive sales, the best response correspondences of firms A and B intersect when \(p_a = t - β - \frac{q}{2} + \frac{b}{2} \) and, in equilibrium, the indifferent consumer is located at \(x = \frac{-q - b}{2\sigma} \). To verify that the equilibrium prices are indeed \(p_a = t - β - \frac{q}{2} + \frac{b}{2} \) and \(p_b = t - β + \frac{q}{2} \), we solve for that no firm has an incentive to deviate and capture the entire market. Given that \(p_a = t - β - \frac{q}{2} + \frac{b}{2} \), firm B captures the entire market when it charges a price \(p_b\) such that \(t - β - \frac{q}{2} + \frac{b}{2} \leq p_b \leq t - β + \frac{q}{2} \). Solving this equation with respect to \(p_b\), we find that firm B captures the entire market by charging \(p_b = \frac{q}{2} \). Firm B has no incentive to deviate if its profit when it captures the entire market is smaller than its profit when it shares the market, which is when \(\frac{q}{2} \leq \frac{1}{2\sigma} \). Solving for \(q\) we find that firm B prefers to share the market when \(0 \leq q \leq 3(t - β) \). So, when \(t > β \) and \(0 < q < 3(t - β) \), the two firms share the market and the equilibrium prices are \(p_a = t - β - \frac{q}{2} + \frac{b}{2} \) and \(p_b = t - β + \frac{q}{2} \).

If \(q \geq 3(t - β) \) and \(t > β \), firm B finds it more profitable to capture the entire market than to share it with firm B. We proceed by calculating the first derivative of the profit function \(πₜ = pₜ(1 - \frac{p_b - p_a - β - t}{3}) \) with respect to \(pₜ\), after replacing \(p_a = 0\) and \(p_b = β - t + q\). We obtain \(\frac{dπₜ}{dpₜ} = \frac{q - 3(t - β)}{2}\) and observe that it becomes negative when \(q > 3(t - β) \). So, in this case, firm B’s profit function decreases when \(p_b > t + q\). We conclude that, when \(t > β \), \(q > 3(t - β) \) and \(p_a = 0\), firm B maximizes its profit when it captures the entire market by charging \(p_b = t - q + q\). A higher price and sharing the market with its rival, would decrease its profit. Similar is the logic that holds for firm A. We conclude that, for \(t > β \) and \(q > 3(t - β) \), the equilibrium prices are \(p_a = 0\) and \(p_b = β - t + q > 0\) and firm B captures the entire market.

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24 When both firms have positive market shares, the second-order conditions imply that both firms’ profit functions are strictly concave since \(\frac{d^2π}{dp_a^2} = \frac{d^2π}{dp_b^2} = -\frac{1}{2}σ < 0\) and since we assume here that \(t > β \).

25 Note that, in this case where consumers’ expectations are influenced by firms’ prices, parameters \(t\) and \(β\) are multiplied by zero, which is the location of the indifferent consumer when firm B captures the entire market.

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### References

