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PRICING, INVESTMENTS AND MERGERS WITH INTERTEMPORAL CAPACITY CONSTRAINTS

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ABSTRACT

Pricing, Investments and Mergers with Intertemporal Capacity Constraints*

We set up a duopoly model with dynamic capacity constraints under demand uncertainty. We endogenize the investment decisions of the firms, examine their intertemporal pricing behavior, their incentives to merge, as well as the welfare implications of a merger. Whereas under known and constant demand the high capacity firm lets its low capacity rival sell out, under demand uncertainty we obtain a rich set of sales patterns. Each unit of available capacity has an option value (or opportunity cost), which depends on both firms' capacities, the current demand and the remaining horizon. This option value may be higher when the firms act non-cooperatively compared to the case when they merge to form a monopoly. Trade surplus may be higher when a merger takes place, as capacity is more efficiently managed over time. The prospect of a merger also leads to higher investment levels, as each firm wishes to appropriate a higher share of the total surplus. For some levels of the capacity instalment cost, a merger that turns the duopoly into a monopoly is welfare improving.

JEL Classification: D43, L13 and L22
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1 Introduction

Motivation

In most, if not all, markets the ability of firms to sell depends, to a larger or smaller extent, on the available capacity that they have, given their past investments and their past sales. Still, the literature has essentially ignored the relation between the dynamics of price competition and the evolution of capacities in oligopoly. In this paper, we explore the dynamics of competition when strategic sellers face capacity constraints that operate across a number of possible selling dates. We focus on the role of information (in particular that of demand uncertainty) and on the comparison between situations when all capacity is concentrated in a single seller relative to oligopoly.

If a firm’s ability to make a sale is constrained by past technology or investment choices, market competition will reflect these constraints: a firm that currently holds a small inventory (has low capacity) is more restricted in supplying the market in the present and in the following periods relative to one with a large inventory (or high capacity). Further, a firm that competes more aggressively by setting a lower current period price will most likely find itself with a lower inventory in the subsequent periods: succeeding in making a sale increases the current-period profit but decreases the firms’ ability to make a sale in the future. In this paper we study situations where past investments create constraints on how much a firm can produce and/or sell within a given number of periods.\(^1\) This setting includes situations where there are physical inventories and firms are constrained to selling a quantity that does not exceed their inventory; it also includes situations where firms’ prior investments in capacity building set a limit on how much a firm can produce within a certain time interval.\(^2\)

A number of specific examples may fit in our framework. For consulting companies that compete for projects, undertaking a project today that will take a long time and significant resources to complete implies a diminished ability to compete for other projects in subsequent periods. Thus, own and rival capacity, along with expected demand will jointly affect bids for projects. Likewise, airlines that serve a particular route are expected to be adjusting their ticket prices in the period before the flight departure, depending on residual capacities (number of seats that have not been sold) and demand conditions. Hotels also determine their rates according to the number of rooms that have been reserved, residual capacities and expected future demand. Large sellers of non perishable

\(^1\)Our focus is on the strategic aspects of the problem. Pricing under various types of intertemporal capacity constraints for a single (non strategic) seller has been a central theme in the operations research literature, usually under the name of "yield" (or "revenue") "management". For comprehensive reviews of this literature see e.g. Elmaghraby and Keskinocak (2003), McGill and Van Ryzin (1999), and Bitran and Caldentey (2003).

\(^2\)Cases where a firm can simply produce to order and without delay at each point and where current sales do not affect the firms’ ability to sell subsequently do not fall into the problem that we analyze here.
agricultural products (say sugar or wheat) also face a trade-off between selling earlier or keeping (part of) their capacity for later when demand may be higher. Hydroelectric plants should consider the usage of their water reservoirs over summer months and choose their corresponding prices according to current and expected demand. And, in fact, any seller constrained by inventories that can be sold in more than one periods (say, of natural resources or works of art) also faces similar considerations. Our analysis is relevant to such cases.³

**Brief model description and results**

We set up a simple dynamic duopoly game where each firm can make a sale in each of three periods. The specification of demand and costs is kept as simple as possible. Each firm has a maximum number of units that can be sold over the entire time horizon and seeks to maximize its aggregate profit. The two firms observe the current-period demand before they set prices but, importantly, are uncertain about future demand. The situation is repeated in the subsequent period, with the inventory of one of the firms possibly reduced as a result of making a sale in the previous period. In this setting, we first examine equilibrium pricing given the capacity constrains. Then, we endogenize the firms’ initial capacity choices.

A number of issues arise as central in specifying a firm’s path of sales and prices. First, demand considerations: if demand today appears to be low compared to expected demand tomorrow, a firm may choose to act conservatively and save its capacity for sales at a later period of possibly higher demand. Second, strategic considerations: a firm’s expected profit depends on its rival’s pricing. A firm will tend to postpone making a sale today if it expects its rival will be less aggressive tomorrow, possibly because of future strategic reasons. Likewise, anticipating low rival prices or relatively high rival capacity tomorrow implies that the firm would have a stronger incentive to make a sale today. In particular, a firm with high capacity may have an incentive to become a monopolist in later periods and thus, be less aggressive in the earlier periods. Our set-up, thus, allows the study of the composite effects of strategic interactions that emerge in such a pricing setting. We then examine each firm’s capacity choice, made at the beginning of the game. Again, in addition to the direct cost of building capacity, there are also strategic effects: a firm’s choice of capacity affects not only its own pricing strategy but also that of its rival. Finally, we examine the incentives of the two firms to merge, thus combining their capacities, and how this possibility influences the optimal investment and pricing decisions and, in turn, welfare.

Our results are related, first, to the intertemporal pricing decisions of the firms, second, to their investment decisions and, third, to the impact of mergers. Regarding the first issue, we show that the

³The same considerations emerge when the capacity-constrained agents are buyers rather than sellers: for instance, consider companies that wish to fill a fixed number of high-level positions, evaluating the trade-off between hiring a current candidate (a decision that would imply that the position would be no longer available, at least for some time) and leaving the position unfilled, hoping to find (and attract) a possibly higher-quality candidate in the future.
introduction of uncertainty over a stochastic demand creates an option value (or opportunity cost) for capacity: a firm may wish to forego a sale in the current period in expectation of higher demand in the future. This option value tends to make firms unwilling to sell in the initial periods of a pricing subgame if there is low demand, thus leading to relatively high prices. The effect of the option value of capacity may be strong enough to lead the competing firms to set equilibrium prices that, on average, exceed the prices that would be set by a monopolist. We characterize how the duopoly outcome may be inefficient relative to the socially optimal (or monopoly) allocation: capacity may remain unused and demand may remain not satisfied. As a distinct important point, we also find that the pricing and sales patterns may be quite rich, depending on the demand realizations within each period. If firms have different initial capacities and the low capacity firm cannot satisfy residual demand alone, this firm does not always sell out its capacity in equilibrium, as it would do in the constant and known demand case. There are two effects shaping equilibrium here. A high capacity firm may wish to let its rival sell out first, so that it then becomes a monopolist in the remaining periods. But, if current demand is low, the option value of capacity may be greater for the low capacity firm compared to the high capacity firm, so that the latter is the one to sell first. We characterize when, depending on the current demand, we expect the high capacity firm, or the low capacity firm, or neither of the two to make a sale.

The analysis of the investment (capacity setting) game shows that capacities are not always "strategic substitutes" but may be "strategic complements". The capacity that is the best response of a given firm is not always monotonically decreasing or increasing in the capacity of the rival firm. The net effect depends on how aggregate capacity constraints operate within the dynamic pricing problem of the firms. Regarding mergers, we consider two possibilities. When a merger can take place at the beginning of the pricing subgame only and when it can possibly take place at the beginning of each period within the pricing subgame. In either setting, the firms optimally merge at the beginning of the pricing game. However, the impact of mergers on the investment levels differs between the two settings: when a merger may take place in every period, the two firms install greater capacity. Also, for a relatively low capacity installment cost, we observe that each firm invests in capacity at a level high enough to satisfy total demand on its own. In general, when we allow for mergers we obtain investment in excess capacity. This is inefficient from the viewpoint of the merged firm and is due to the unilateral incentive that each firm has to capture a greater share of the post-merger profits.

Finally, we examine the welfare implications of a possible merger between the duopolists. For a range of the capacity cost levels, a merger may be welfare improving despite the fact that it leads to monopoly and that direct cost efficiencies are absent. The net effect of a merger on welfare depends on the relative magnitude of two opposite effects: higher trade surplus and higher cost of capacity. The positive effect on total surplus from sales is due to the price dynamics under capacity constraints
and future demand uncertainty. These factors create incentives for the firms to refrain from selling in a period with low demand. The unilateral incentives of each firm to forego a sale in order to realize higher expected profit in the future sometimes lead to no trade (i.e. lost surplus) in the duopoly case, while a monopolist in a similar situation would sell and, so, the respective surplus would be realized. This is because the opportunity cost of a unit of capacity may be lower when total capacity is owned by a single firm compared to when it is owned by two competing firms. However, when the prospect of a merger leads firms to invest in more capacity, the positive effect on total surplus in the pricing game is outweighed by the higher cost of this additional capacity. Thus, a merger is welfare improving only if it does not induce firms to invest in more capacity compared to when the possibility of a merger is absent.

Related literature

Our paper is broadly related to the literature on price competition with capacity constraints, which dates back to Edgeworth (1897); his classic work highlights the problem of a possible non-existence of a pure-strategy equilibrium in a pricing game with fixed capacities. Subsequent work on pricing under capacity constraints includes Beckman (1965), Levitan and Shubik (1972), Osborne and Pitchik (1986) and Dasgupta and Maskin (1986). An important strand of the literature endogenizes the choice of capacity prior to firms’ engaging in oligopolistic competition. In their seminal paper, Kreps and Scheinkman (1983) show that capacity precommitment followed by price competition leads to Cournot outcomes. Subsequent work has studied several variations of this setting (e.g. Davidson and Deneckere, 1986, Allen, Deneckere, Faith, and Kovenock, 2000, to mention just a couple of the related papers). One such important variation is about demand uncertainty: firms set capacities in the first stage without knowing market demand and then complete in a second stage (second period competition is in quantities in Gabzewicz and Poddar, 1997, and in prices in Reynolds and Wilson, 2000, and in de Frutos and Fabra, 2007).

The above mentioned work refers to capacity constraints that are static or operate period-by-period, that is, there is a limit on how much can be produced or sold in each period that does not depend on past decisions. In contrast, in our model, it is the (endogenous) process of capacity (inventory) reduction that provides the intertemporal link between the firms’ actions and directly shapes price dynamics. Such dynamic capacity constraints have received much less attention in the literature. Griesmer and Shubik (1963), with prices in a discrete set and Dudey (1992), allowing prices to vary continuously, have studied games where capacity-constrained duopolists face a finite

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4Strictly speaking, such a monopoly situation would be created not only by a formal merger but also if the sellers could achieve perfect collusion. For simplicity, we focus on the case of a merger.

5Other work has studied the role of capacity constraints for collusion in repeated games; see e.g. Brock and Scheinkman (1985) and Benoit and Krishna (1987). Capacity constraints may affect firms’ pricing strategies indirectly, by restricting their deviations from collusive price schemes.
sequence of identical buyers with unit demands. Dudey (1992) studies pricing under dynamic capacity constraints, with constant and known demand. Under certain conditions, the equilibrium has sellers maximizing their joint profits. A high capacity firm offers some buffering, allowing the smaller capacity firm to sell its capacity first. This work, in part, we generalize in our basic model, by considering a setting where demand is not constant across periods and future demand is unknown (we then further consider capacity choices and mergers). We show that demand uncertainty significantly modifies and enriches the equilibrium behavior: with stochastic demand it is no longer true that the smaller seller sells before the larger seller. We also highlight the associated welfare issues since, under uncertainty, the equilibrium does not have to be efficient. Biglaiser and Vettas (2004) study price competition with intertemporal capacity constraints in a two-period model without uncertainty but where buyers act strategically, choosing which seller to buy from, in their effort to avoid subsequent periods of high prices. Here, we do not model buyers as strategic. Instead we focus on the role of demand uncertainty in duopoly competition, the endogenous choice of capacities and the impact of mergers.  

The literature on (horizontal) mergers has focused on their welfare implications, a topic of high importance for both regulators and economic theorists. Williamson (1968) points out the trade-off between increasing market power and cost efficiencies that may arise from a merger. The net effect may be either positive or negative if the gain from the lower costs is higher or lower than the loss from higher prices. In a quantity competition setting, Farell and Shapiro (1990) and Levin (1990) provide sufficient conditions for a merger to be welfare improving, in terms of a weighted average of firms’ pre-merger market shares. Polasky and Mason (1998) show that positive long-run effects from mergers may be realized by allowing firms outside the merger to change their capacities in the long-run. Mergers have also been studied in price-setting frameworks. Deneckere and Davidson (1985) examine firms’ incentives to merge with price competition and differentiated products. They show that post merger prices are higher and all firms in the industry earn higher profits. Assuming firms sell homogeneous products, Baik (1995) allows for capacity adjustment after a merger. Although the focus is not on welfare issues, the paper shows that post merger prices may be lower than pre-merger, potentially implying higher consumer welfare. In general, the prevalent view in the literature is that welfare may increase if a merger results in significant cost efficiencies or if there are cost asymmetries.

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6 Ghemawat and McGaham (1998) characterize mixed strategy equilibria in a two-period duopoly with capacity constraints. Bhaskar (2001) shows that, by restricting his own demand initially, a single buyer can increase his net surplus when sellers are capacity constrained. Dudey (2006) presents conditions so that a Bertrand outcome is consistent with capacities initially chosen by the sellers. Garcia, Reitzes and Stacchetti (2001) set up an infinite horizon model to study competition between hydroelectric plants when electricity is storable. They characterize how pricing depends on the levels of water reservoirs in a Markov perfect equilibrium. In their model, the capacity level (full or empty reservoir) depends on exogenous factors (“rainfall”) and demand is constant across time and known.
between the firms. Our work adds to the mergers literature by characterizing potential inefficiencies associated with the intertemporal usage of capacity by oligopolists. Under demand uncertainty, a monopolist can potentially manage a given level of capacity more efficiently. Our results may be also interpreted in terms of cost efficiencies. While a merger is not assumed to lead to technological efficiencies, it modifies the opportunity cost of capacity utilization. When capacity is owned by a single firm, its opportunity cost may be lower compared to when the same capacity is divided between two rival firms. In this generalized sense, a merger may be viewed as leading to lower “costs”.

Structure of the paper

The paper is organized as follows. The basic model is presented in Section 2. In Section 3 we analyze equilibrium behavior in the basic model, where firms do not have the possibility to merge. First, we characterize price competition for given initial capacities and then we endogenize these capacities. Section 4 examines equilibrium (in prices and capacity choices) when mergers are allowed. In Section 5 we conduct a welfare analysis. Section 6 concludes. Some (standard but lengthy) calculations and related proofs are contained in a Technical Appendix, available by the authors upon request.

2 The basic model

We consider two firms, $A$ and $B$ that sell homogeneous products. They choose capacities once, at the beginning of the game, and then compete in prices for three periods. In each period, a buyer arrives, demanding a unit of the product. We denote the valuation of the buyer who arrives in period $t$ by $v_t$, $t = 1, 2, 3$. We further assume that the buyers’ valuations are iid draws from a uniform distribution with support $[0, 1]$. Within each period, firms set prices simultaneously, after they have observed the valuation of the current-period buyer, but without knowledge of the valuations of the future buyers. We assume that the per unit cost of capacity installment is constant and denote it by $c$ (this cost becomes sunk). Production costs within each period are assumed to be zero, for simplicity. The goal of each firm is to maximize its profit over the three periods (to simplify the notation and without loss of generality we assume no discounting of profits across periods).

Throughout the paper, a subscript $t$ refers to the period, $t = 1, 2, 3$, of the pricing subgame and superscripts $i, j$, to the firm $A$ or $B$. We denote by $(k^A_t, k^B_t)$, the capacity pair at the beginning of period $t$, by $k^i_t$ the capacity of firm $i$ in period $t$, and by $p^i_t$, the price of firm $i$ in period $t$. We denote by $\pi^i_t \equiv \pi^i_t(p^A_t, p^B_t; v_t, k^A_t, k^B_t)$ the total profit of firm $i$ from period $t$ onwards, as a function of the current-period prices of the firms, the (realized) value of $v_t$ and current capacities and also given

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7In Huck, Konrad and Mueller (2004) increased welfare does not stem from cost reductions but from better information flows regarding production levels.
that both firms follow equilibrium strategies from period \( t + 1 \) onwards. The function \( \pi^i_t \) evaluated at the equilibrium current-period prices is denoted by \( \Pi^i_t \equiv \Pi^i_t (k^A_t, k^B_t, v_t) \). The expectation of \( \Pi^i_t \) with respect to \( v_t \), \( E \left( \Pi^i_t (k^A_t, k^B_t) \right) \), represents the expected value of firm \( i \)'s equilibrium profit at the beginning of period \( t \), prior to the realization of \( v_t \). We solve the model by backwards induction, looking for subgame perfect Nash equilibria of the pricing and capacity-choice game.

3 Equilibrium pricing

In this Section we analyze the equilibrium pricing behavior of the two firms taking their initial capacity levels as given. We identify the key element that drives the price dynamics, which is the opportunity cost of the available capacity, and discuss its role in the context of any finite horizon pricing game (thus, providing insights more general than our three-period model). Then we focus on the pricing subgame of our “basic model”. For the sake of expositional clarity, we first present the case where only one firm holds positive capacity and, therefore no strategic factors affect the pricing decisions; then we also consider the strategic effects.

3.1 The monopoly case

The monopoly case may arise either because only one firm has invested in positive capacity at the beginning of the game, or because one of the firms has sold out its capacity in the previous periods. In such cases, there are no strategic considerations at play and the seller sets prices taking into account only its own capacity dynamics and the evolution of demand. As mentioned in the Introduction, such demand considerations have been studied in the “revenue management” literature where the focus has been on the pricing behavior of a single firm. In the present model, monopoly situations arise endogenously depending on the firms’ investment and pricing decisions. The study of these cases provides a clear illustration of how, in a dynamic setting with capacity constraints, the available capacity obtains an option value, which reflects the opportunity cost of selling a unit in the current period. We, thus, establish an important benchmark. Subsequently, in the analysis of duopoly, we build on the intuition developed here and show how this option value is modified by strategic considerations.

To provide some intuition for how the opportunity cost shapes the monopolist’s equilibrium behavior let us start with a two-period example. Consider a firm that holds two units of capacity, that is, it has enough capacity to satisfy demand in both periods. In this case clearly it would set the monopoly price (a price equal to the realized value) and sell a unit in each period. Now, think of a situation where the firm holds only one unit of capacity, which it can sell in any of the two periods. It is clear that, if it does not sell in the first period, it would do so in the second (last) one irrespective of the valuation of the second buyer, as there are no more opportunities to sell. Hence,
if the unit is not sold in the first period, the second period price will be equal to \( p_2 = v_2 \) and since valuations are uniformly distributed on \([0, 1]\), the expected profit is \( E (\Pi_2(1)) = Ev_1 = 1/2 \). Of course, the possibility of selling in the last period has to be taken into account when the first-period pricing decision is made. The monopolist would sell his one unit in the first period only if the profit from doing so is higher than the expected profit from selling it in the future. In other words, the firm would sell in first period if and only if \( v_1 \) exceeds \( E (\Pi_2(1)) \), which is the opportunity cost of selling this unit today. The expected equilibrium profit over both periods, before the realization of \( v_1 \), is then

\[
E (\Pi_1(1)) = \int_0^{E(\Pi_2(1))} E (\Pi_2(1)) dv_1 + \int_{E(\Pi_2(1))}^1 v_1 dv_1 = \frac{5}{8}.
\]

In general, we may distinguish between two different subcases. The first is when the firm holds enough capacity to satisfy residual demand. Consider the general case of \( T \) selling periods and think of being in period \( t \) (with \( T - t \) periods remaining until the end of the game) and a monopolist holding at least \( T - t + 1 \) units of capacity, with which he can satisfy the current as well as any future demand. In this case, the monopolist does not have any incentive to forego a sale in the current period with the view to selling in a high demand period. In other words, the opportunity cost of capacity is zero and the monopolist sells a unit of product in every period at a price equal to the current period’s valuation. We summarize the immediate result for this case:

**Lemma 1** A monopolist holding capacity \( k_t > T - t + 1 \) enough to satisfy (residual) demand sells in every period, enjoying expected profit \( E(\Pi_t(k_t)) = (T - t + 1)Ev_t \).

The other subcase arises when the monopolist holds capacity smaller than (residual) demand. In this case, the seller may have an incentive not to sell a unit of the product in the current period to a customer with low valuation, if he expects that customers with higher valuations will possibly arrive in the subsequent periods; his total profit from period \( t \) onwards is

\[
\pi_t = \max \{ v_t + E(\Pi_{t+1}(k_t - 1)), E(\Pi_{t+1}(k_t)) \},
\]

where \( v_t \) is the current period price if he sells in period \( t \). The monopolist prefers to sell in period \( t \) if and only if \( v_t \geq E(\Pi_{t+1}(k_t)) - E(\Pi_{t+1}(k_t - 1)) \). The right-hand side of this inequality represents the opportunity cost of a unit of his capacity in period \( t \), which we can denote by \( OC(k_t) \equiv E(\Pi_{t+1}(k_t)) - E(\Pi_{t+1}(k_t - 1)) \).\(^9\) We have:

\[^8\]Since we consider monopoly in this subsection, we simplify the notation by suppressing the superscript that refers to the firm.

\[^9\]Note that the initial separation of the monopoly problem in two subcases is made only to facilitate the understanding of the problem. Formally, the second subcase encompasses the first one, in which the option value of capacity is
Lemma 2 The optimal monopoly price in period $t$ with capacity smaller than residual demand, is

$$p_t = \begin{cases} 
    v_t & \text{if } v_t \geq OC(k_t) \\
    \text{some } p > v_t & \text{otherwise.}
\end{cases}$$

The monopolist’s expected equilibrium profit evaluated before the realization of $v_t$ is

$$E(\Pi_t^i(k_t)) = E\left[ \max\{v_t + E(\Pi_{t+1}^i(k_t^i - 1)), E(\Pi_{t+1}^i(k_t))\} \right] =$$

$$= \int_0^1 E(\Pi_{t+1}^i(k_t))dv_t + \int_{OC(k_t)}^1 (v_t + E(\Pi_{t+1}^i(k_t) - 1))dv_t.$$

Having described the optimal pricing behavior of the monopolist, we turn to the analysis of the situations where both firms hold positive capacity. Therefore, we now add a strategic dimension to the problem and examine how it affects the opportunity cost of the firms’ capacity which, in turn, shapes the equilibrium pricing incentives.

3.2 The duopoly case

Let us again employ our two-period example to illustrate the basic insights for the duopoly case. Consider first the case where each firm holds at least two units of capacity, that is, enough capacity for each firm to satisfy by itself the residual demand. If both firms hold positive capacity in the last period, we obtain the Bertrand outcome: zero equilibrium prices and expected profits. This means that the opportunity cost of capacity in the first period is zero, hence, the firms do not have an incentive to forego a sale; again we obtain the Bertrand outcome. In general, this result would hold for any $T$-period game, where each firm’s capacity in a given period $t$ is higher or equal to the remaining periods. We summarize equilibrium behavior in these cases:

Lemma 3 If $\min\{k_A^t, k_B^t\} \geq T - t + 1$, equilibrium prices in period $t$, are $p_t^A = p_t^B = 0$ and expected profits are $E(\Pi_t^A(k_t^A, k_t^B)) = E(\Pi_t^B(k_t^A, k_t^B)) = 0$.

We now turn to the cases where at least one of the firms cannot satisfy residual demand. Then a positive opportunity cost becomes part of the calculations, affecting the pricing incentives of both firms.

Consider, for example, the case where (again with two periods) initial capacities are $(1, 1)_1$, that is, none of the firms can satisfy total demand alone. In the second period, we distinguish two possibilities. If none of the firms has sold in the previous period we obtain the Bertrand outcome. If one of the firms has sold its unit in period 1, then the other firm becomes a monopolist in the

$$E(\Pi_{t+1}^i(k_t^i)) - E(\Pi_{t+1}^i(k_t^i - 1)) = 0,$$

since whether the monopoly sells or not in period $t$, in the next period his capacity will also suffice to satisfy residual demand.
last period (with a monopoly optimal price and expected profit). Given equilibrium behavior in the second period, the opportunity cost of firm $A$’s capacity is $E (\Pi_t^A(1,0)) - E (\Pi_t^A(0,1)) = 1/2$, provided that firm $B$ is expected to set $p^B_t \leq v_1$ (and sell). It then follows that, if $v_1 \geq 1/2$ both firms wish to sell in period 1 at a price at least equal to 1/2, with (Bertrand) competition pressing equilibrium prices down to 1/2. Regarding the case where $v_1 < 1/2$, both firms prefer their rival to sell in period $t = 1$, anticipating expected profit equal to 1/2 in the next period. Note that each firm prefers a sale to take place in period one to no sale (even if it is the one that sells), since in the latter case, its expected profit would be zero ($E (\Pi_t^A(1,1)) = 0$). Formally, each firm has to consider the case where its rival sets a price greater than $v_1$ with the view to avoiding a sale in the current period. In such a case, firm $A$’s opportunity cost is $E (\Pi_t^A(1,1)) - E (\Pi_t^A(0,1)) = 0$, that is, firm $A$ prefers to be the one that sells if the alternative is no sale at all. By the symmetry of the problem, the same argument applies for firm $B$. As a result, in the present case, there are two asymmetric equilibria in pure strategies and one symmetric equilibrium in mixed strategies. In the pure strategy equilibria one firm sets a price equal to $v_1$ and the other a price strictly greater than $v_1$. In the mixed-strategy equilibrium the firms randomize between $p = v_1$ and a price $p > v_1$.

We next discuss the role of the opportunity cost of capacity in the general $T$-period setting and characterize equilibrium prices. In the monopoly case the opportunity cost of capacity, when the pricing game has reached its $t$th period and the monopolist’s capacity is $k_t$, is readily calculated as $E(\Pi_{t+1}(k_t)) - E (\Pi_{t+1}(k_t - 1))$. In the duopoly case, if one of the firms does not sell (by setting a price greater than $v_t$) it does not immediately follow that its rival will have an incentive to sell. Therefore, we have to distinguish between two types of opportunity costs; one associated with the expectation that the rival of firm $i$ will sell, if firm $i$ does not sell in the current period (by setting, for example, a price greater than $v_t$), and one when the rival is not expected to sell (being expected to set a price greater than $v_t$). These are the following:

$$OC^A_{1} = E(\Pi^A_{t+1}(k^A_t, k^B_t - 1)) - E(\Pi^A_{t+1}(k^A_t - 1, k^B_t)),$$
$$OC^A_{0} = E(\Pi^A_{t+1}(k^A_t, k^B_t)) - E(\Pi^A_{t+1}(k^A_t - 1, k^B_t)),$$

for firm $A$, and likewise

$$OC^B_{1} = E(\Pi^B_{t+1}(k^A_t - 1, k^B_t)) - E(\Pi^B_{t+1}(k^A_t, k^B_t - 1)),$$
$$OC^B_{0} = E(\Pi^B_{t+1}(k^A_t, k^B_t)) - E(\Pi^B_{t+1}(k^A_t, k^B_t - 1)),$$

for firm $B$.\(^{10}\)

The opportunity cost $OC^i_{1}$ captures the case where, firm $i$’s rival is expected to sell in period $t$. Then, firm $i$ prefers also to sell if and only if $v_t$ exceeds $OC^i_{1}$ (the subscript “$i$” stands for “rival

\(^{10}\)These opportunity costs $OC^i_{j}$ are clearly functions of $t$, $k^A_t$ and $k^B_t$. For simplicity we suppress the arguments in the functions.
is expected to sell”). The opportunity cost \( OC^i_0 \) captures the case where, firm \( i \)'s rival is expected to not sell in period \( t \). Then, firm \( i \) prefers to sell if and only if \( v_t \) exceeds \( OC^i_0 \) (the subscript “0” stands for “rival is expected to not sell”).\(^{11}\) Since a firm cannot gain within a period more than the maximum demand valuation, 1, the opportunity costs do not exceed that valuation. However, as shown below it may be that some opportunity costs are negative.

An important feature of the problem is that each firm’s incentives to sell in period \( t \), and therefore the equilibrium prices, are determined by the current-period valuation, \( v_t \), and the relation between the two types of opportunity costs defined above. In general, the pricing decisions and the functions that relate profits to capacities can be calculated recursively once capacities, the number of time periods until the end of the game, and the probability functions of the \( v_t \)’s are specified. We next present a complete characterization of the equilibrium for the cases where \( OC^i_0 \leq OC^i_1 \), \( i \in \{A, B\} \).

As shown later in the analysis, this is always the case in our three period model.\(^{12}\)

**Proposition 1** If \( OC^i_0 \leq OC^i_1 \), \( i \in \{A, B\} \), equilibrium prices can be calculated as follows:

1. If \( v_t < \min_i \{OC^i_0\} \), then \( p^A_i, p^B_i > v_t \).
2. If \( v_t > \max_i \{OC^i_1\} \), then \( p^A_i = p^B_i = \max_i \{OC^i_1\} \), and the firm with the lowest \( OC_1 \) sells.
3. If \( v_t \in \left[ \min_i \{OC^i_0\}, \max_i \{OC^i_1\} \right] \) we discern the following subcases:
   3a. \( \min_i \{OC^i_1\} \leq \max_i \{OC^i_0\} \).
   In this case, \( OC^i_0 \leq OC^i_1 \leq OC^i_k \), where \( l, k \in \{A, B\}, k \neq l \).\(^{13}\) Equilibrium prices are \( p^k_i > v_t \) and \( p^l_i = v_t \).
   3b. \( \min_i \{OC^i_1\} \geq \max_i \{OC^i_0\} \).
   In this case, either \( OC^i_0 \leq OC^k_i \leq OC^i_1 \leq OC^k_1 \), or \( OC^i_0 \leq OC^i_1 \leq OC^k_1 \), where \( l, k \in \{A, B\}, k \neq l \).
   3b.1. If \( v_t \in \left[ OC^i_1, OC^k_1 \right] \), \( p^l_i = v_t \) and \( p^k_i > v_t \).
   3b.2. If \( v_t \in \left( \max_i \{OC^i_0\}, OC^i_1 \right) \), there are equilibria in pure strategies, where \( p^i_t = v_t \) and \( p^j_t > v_t \), as well as in mixed strategies, where firms randomize between \( p^i_t = v_t \) and \( p^j_t > v_t \). The probability of \( p^i_t = v_t \) is \( Pr^i (v_t) = \frac{2(v_t - OC^i_0)}{(v_t - OC^i_0) + (OC^j_1 - OC^i_0)} \).
   3b.3. If \( v_t \in \left[ \min_i \{OC^i_0\}, \max_i \{OC^i_1\} \right] \), the firm with the smallest \( OC_0 \) sells at a price equal to \( v_t \) and its rival sets a price greater than \( v_t \).

\(^{11}\)Note that the existence of the \( OC_0 \) type of opportunity cost allows for none of the firms’ selling in a certain period. This contrasts settings where if a firm does not sell its rival sells with certainty (see for example, Garcia et al., 2001).

\(^{12}\)In general, with an arbitrary number of periods, other cases may also arise, following same general principles. These are analyzed in a Technical Appendix available by the authors.

\(^{13}\)Throughout we fix \( k \equiv \arg \max_i \{OC^i_1\} \).
We now provide some discussion of the above results. In case (1) both firms have an incentive not to sell in period $t$ since $v_t$ is lower than both firms’ relevant opportunity costs. Hence, each firm finds it optimal to set a price greater than $v_t$. In the other extreme, case (2), $OC_1$ becomes each firm’s relevant opportunity cost: given that firm $i$’s rival is expected to sell (by setting a price lower or equal to $v_t$), firm $i$ has an incentive to set a slightly lower price so as to sell. Intuitively, successive undercutting would eliminate all prices but max${\{OC^1_i\}}$, which is the price set in equilibrium by both firms, and the firm who has the lowest opportunity cost sells.\footnote{Strictly speaking, the firm with the lowest opportunity cost can set a price lower by $\varepsilon$.} Case (3) covers all in-between cases. Figure 1 illustrates representative subcases (the firm that sells in case of a tie is indicated there by an asterisk). In case (3a), since $OC_0^l \leq OC_1^d \leq OC_0^k \leq OC_1^k$, and $v_t \in [OC_0^d, OC_1^k]$, firm $l$ would always have an incentive to sell – regardless of its expectation about the rival’s price. At the same time, given that firm $l$ is expected to sell (by setting a price smaller or equal to $v_t$), firm $k$ would not have an incentive to undercut (since $v_t < OC_1^k$). Thus, there is a continuum of equilibria where $p_l^i \in [OC_0^v, v_t]$ and $p_l^k > p_l^i$. However, all prices $p_l^k < v_t$ are weakly dominated for firm $k$. As is usual in such cases, we focus on equilibria that do not involve weakly dominated prices. Following standard arguments, the equilibrium that satisfies this condition is $p_l^i = v_t$ and $p_l^k > v_t$ and it is unique. Figure 1(a) illustrates case (3a) when $OC_1^l < OC_1^k$.

Regarding case (3b), there are two possible configurations: $OC_0^l \leq OC_0^k \leq OC_1^d \leq OC_1^k$ and $OC_0^l \leq OC_0^k \leq OC_1^d \leq OC_1^k$, representative cases of which are illustrated in Figure 1(b) and Figure 1(c), respectively. In subcase (3b.1), since $v_t \in [OC_1^i, OC_1^k]$, firm $l$ has an incentive to sell at any price greater than $OC_1^l$ irrespective of whether its rival is expected to set a price lower or greater than $v_t$. On the other hand, firm $k$ expecting $p_l^i \leq v_t$ does not wish to sell, since $v_t < OC_1^k$, and hence a dominant strategy is to charge $p_l^k > v_t$. In subcase (3b.2), since $v_t \in (\max{OC_0^i, OC_1^i})$, there are three equilibria, two in pure strategies and one in mixed strategies. Since $v_t < OC_1^i \leq OC_1^k$, given that a firm sets a price at most equal to $v_t$, its rival (weakly) prefers to set a price greater than $v_t$. Also, given that a firm sets a price greater than $v_t$, its rival strictly prefers to set a price equal to $v_t$ and sell (since $v_t > \max_{i}{\{OC_{i,0}\}}$). This argument establishes the existence of the two pure strategy equilibria where $p^i = v_t$ and $p^j > v_t$ and a mixed strategy equilibrium in which firms mix between price $v_t$ and prices greater than $v_t$. We next prove that this mixed strategy equilibrium is unique.

Recall that when $v_t \in (\max_{i}{\{OC_{0}^i, OC_{1}^i\}})$, no firm wishes to be the one that sells instead of its rival, but it prefers to make a sale if its rival is expected to not sell. Thus, if one of the firms, say firm A, randomizes between prices that do not exceed $v_t$, its rival will set only above $v_t$ prices; the best response of firm A is then to set always a price equal to $v_t$. Hence, in a mixed strategy equilibrium, both firms must randomize between two sets of prices: those equal or below $v_t$, and those greater than $v_t$. Obviously, all prices lower than $v_t$ are strictly dominated by $v_t$ – this price maximizes the

13
current period profit in case of a sale. Note also, that each firm will be indifferent between any price exceeding \( v_t \) (since the reason for such a price to be selected is to induce its rival to choose price \( v_t \) with some positive probability).

Firm A’s expected profit by setting \( p_t^A = v_t \) is

\[
\frac{1}{2} \left( [E(\Pi_{t+1}^A(k_t^A - 1, k_t^B)) + v_t] + [E(\Pi_{t+1}^A(k_t^A, k_t^B - 1))] \right) \Pr^B(v_t) + [E(\Pi_{t+1}^A(k_t^A - 1, k_t^B)) + v_t](1 - \Pr^B(v_t)),
\]

(1)

where \( \Pr^B(v_t) \) denotes the probability with which firm B sets a price equal to \( v_t \). In case of a tie we assume that each firm sells in period \( t \) with probability 1/2.

Firm A’s expected profit when setting \( p_t^A > v_t \) equals

\[
E(\Pi_{t+1}^A(k_t^A, k_t^B - 1)) \Pr^B(v_t) + E(\Pi_{t+1}^A(k_t^A, k_t^B))(1 - \Pr^B(v_t)).
\]

(2)

Since in equilibrium firm A is indifferent among all its pure strategies we obtain the value of \( \Pr^B(v_t) \) by equating (1) and (2).

For firm A, the unique value that equates (1) and (2) is

\[
\Pr^B(v_t) = \frac{2[v_t + E(\Pi_{t+1}^A(k_t^A - 1, k_t^B)) - E(\Pi_{t+1}^A(k_t^A, k_t^B))] \quad \text{v_t} + E(\Pi_{t+1}^A(k_t^A - 1, k_t^B)) + E(\Pi_{t+1}^A(k_t^A, k_t^B - 1)) - 2E(\Pi_{t+1}^A(k_t^A, k_t^B))}{2(v_t - OC_0^A)}
\]

\[
= \frac{2(v_t - OC_0^A)}{(v_t - OC_0^A) + (OC_1^A - OC_0^A)}.
\]

The same steps can be followed for the other firms and thus, we obtain for each firm \( i \)

\[
\Pr^i(v_t) = \frac{2(v_t - OC_0^i)}{(v_t - OC_0^i) + (OC_1^i - OC_0^i)}.
\]

Note that the probabilities above are valid only for the cases where both \( \Pr^A(v_t) \) and \( \Pr^B(v_t) \) belong to the open interval \((0, 1)\), that is, when \( v_t \in (\min_i\{OC_0^i\}, \max_i\{OC_1^i\}) \).

In subcase (3b.3), since \( v_t \in [\min_i\{OC_0^i\}, \max_i\{OC_1^i\}] \), the firm with the lowest \( OC_0 \) sells at price \( v_t \). Note that its rival does not have an incentive to undercut, since his \( OC_0 \) exceeds \( v_t \).

Our analysis makes it clear that the pricing problem faced by the firms is a composite one, due to the nature of the strategic interactions between the rivals. Note that, in contrast to the monopoly situation, in the presence of such strategic interactions, the pricing incentives of each firm are affected not only by its own available capacity and the level of demand, but also by the capacity of the rival firm.

A general result drawn from our analysis up to this point is that, when the current period valuation is low enough, a sale may be foregone with probability one. Other instances where there is a positive

\[\text{Pr}^i(v_t) \text{ depends on the parameters, } k_t^A, k_t^B \text{ and } v_t. \text{ We here simplify the notation, but when necessary, we will use the full notation } \text{Pr}^i((k_t^A, k_t^B), v_t).\]
probability of no sale are those in which the firms play mixed strategies. Clearly, in all these cases trade surplus is lost in the duopoly setting due to the firms’ unilateral incentives to obtain higher expected profit from selling their capacity in the future. We will elaborate further on the importance of this finding in the following sections in the context of our basic three-period model.

### 3.3 Price equilibria in the three-period model

We now turn to our basic (three-period) model and provide a detailed characterization of the price equilibrium. We proceed by analyzing the subgames initiated by all possible capacity levels. In each period we determine the equilibrium prices and compute the expected profits of the two firms before the realization of the buyer’s valuation. We present below the case where capacities in the first period are \((1, 2)_1\). This case represents a scenario where, a sale is foregone for low current-period valuation, though total capacity does not exceed total demand.

When capacities in the first period are \(k^A_1 = 1\) and \(k^B_1 = 2\), none of the firms can satisfy the total...
demand alone. The (two types of) opportunity cost can be calculated as

\begin{align*}
OC^A_1 &= E(\Pi_2^A(1, 1)) - E(\Pi_2^A(0, 2)) \approx 0.4 \\
OC^A_0 &= E(\Pi_2^A(1, 2)) - E(\Pi_2^A(0, 2)) \approx 0.375 \\
OC^B_1 &= E(\Pi_2^B(0, 2)) - E(\Pi_2^B(1, 1)) \approx 0.6 \\
OC^B_0 &= E(\Pi_2^B(1, 2)) - E(\Pi_2^B(1, 1)) \approx 0.1.
\end{align*}

Note that this case corresponds to Figure 1(c). Depending on \( v_1 \), we determine the equilibrium prices as follows (see also Proposition 1)

- If \( v_1 \geq OC^B_1 \), \( p_1^A = p_1^B = OC^B_1 \) and firm A sells.
- If \( v_1 \in [OC^A_1, OC^B_1) \), \( p_1^A = v_1 \) and \( p_1^B > v_1 \).
- If \( v_1 \in (OC^B_0, OC^A_1) \), firms randomize between \( p_1^A = v_1 \) and \( p_1^B > v_1 \) with probabilities
  \[ \Pr^A(v_1) = \frac{2+8v_1-4\ln 2}{3+4v_1-2\ln 2} \text{ and } \Pr^B(v_1) = \frac{3-8v_1}{2\ln 2-4v_1}. \]
- If \( v_1 \in [OC^B_0, OC^A_0) \), \( p_1^A > v \) and \( p_1^B = v_1 \).
- If \( v_1 < OC^B_0 \), \( p_1^A > v_1 \) and \( p_1^B > v_1 \).

It is important to discuss how equilibrium pricing depends on the current period demand. We see that, for relatively high valuations, it is firm A (the low capacity firm) that sells, for intermediate valuations any of the two firms may sell since they randomize, for relatively low valuations it is firm B (the high capacity firm) that sells, and for even lower valuations no firm sells. The underlying logic for this equilibrium behavior is as follows: for relatively high valuations, the low capacity firm would like to sell in the current period rendering its rival a monopolist for the two subsequent periods, and this outcome is exactly what firm B also wants (since it then can enjoy a monopoly profit). For intermediate valuations, both firms would like the other firm to be the one that sells: they prefer some sale to be made (to relax future competition) but the current demand is not attractive enough to make them happy to sell a unit of their capacity. For relatively low valuations, firm A does not wish to go for a sale given that it will have one more chance of getting a better price in the second period. Firm B understands this and chooses to sell a unit. For even lower valuations, firm B also does not want to sell since that would impair its strategic position in the next period \((E(\Pi_2^B(1, 1)) < E(\Pi_2^B(1, 2)))\). Thus, there is no sale. As an important benchmark, note that, if total capacity was in the hands of a single firm, it would sell a unit in each of the periods, no matter how low the customer’s valuation is.

All other subcases that emerge (the various subgames given the capacity combinations) can be analyzed in a similar way. The behavior patterns that emerge follow what has been described

16 Calculation details for all cases are in a Technical Appendix available by the authors.
in Proposition 1 and illustrated for the \((1, 2)_1\) case just above. Instead of essentially replicating details for each case, expected profits can be calculated and collected as in Table 1, to prepare the subsequent analysis.\(^{17}\) It is important, however, to discuss at this point the main qualitative features of our results.

There are two general insights that one obtains from our analysis about duopoly price competition with dynamic capacity constraints and uncertain demand. The first insight is that either the high or the low current capacity firm may be the one that in equilibrium makes a sale in a given period, depending on the current level of demand. Dudey (1992) has established an important benchmark for competition when firms have intertemporal capacity constraints and demand is constant in each period. He shows that, under some conditions, the low capacity firm sells out its capacity and does this before any sale by its high-capacity rival takes place. This argument, a type of “buffering” that the high capacity firm prefers to wait till its rival has exhausted its capacity and then cover the remaining periods, is a property that has to be modified when, in our analysis, demand becomes uncertain and stochastic. In our model, equilibrium pricing patterns become much richer: while the incentive of a high capacity firm to wait till its rival has moved out of the way still exists, additional important considerations emerge. For a high capacity firm, waiting may appear too costly if the current-period demand realization is high enough. Likewise, for a low capacity firm, selling a unit today may also be too costly (in terms of the opportunity cost of its capacity) if the current-period demand is too low. The interaction of all these effects leads to the different pricing and sales patterns that we characterize, with none, both, or one of the two firms making a sale in equilibrium.

The second insight generated by our analysis is that duopoly pricing may lead to significant inefficiencies, associated with the sub-optimal intertemporal use of capacity in the market. Importantly, in our setting (with inelastic demand and for a given initial capacity level) a monopolist would achieve the socially optimal outcome, of course, also appropriating all the surplus. In contrast, duopoly often leads to a sub-optimal use of the aggregate capacity and to lower trade surplus. This happens in cases when a monopolist would sell a unit in a given period, but such a sale would not take place under duopoly. Depending on the capacity levels and the remaining number of periods, a sale may be foregone under duopoly in one of two cases: either with probability one when the current period valuation is low enough or with some positive probability under mixed-strategy pricing (where firms randomize between selling and not selling) for intermediate levels of current-period demand. In such cases, each firm prefers not to use a unit of its capacity today, hoping that demand will be higher in the future and that the competitive conditions then will allow it to sell its unit and to enjoy higher profit.

The unilateral incentives of each duopolist to postpone a sale may exist even in cases when

\(^{17}\)Details are in a Technical Appendix available by the authors.
the aggregate capacity in the duopoly cannot cover the total demand \((k_1^A + k_1^B \leq 3)\). It follows that, under such conditions, a unit of capacity may remain unsold in equilibrium.\(^{18,19}\) Clearly, in such cases it would have been socially optimal to have all the units sold and a monopolist would choose to implement exactly this solution. Similarly, even when total capacity exceeds total demand \((k_1^A + k_1^B > 3)\) duopoly equilibrium may also imply that capacity will remain unsold and at the same time that not all the demand is satisfied (this occurs when demand realizations in the initial periods are relatively low). Again this (socially inefficient) allocation has to be contrasted with the (optimal) monopoly solution. A monopolist always sells out his capacity (up to the level of aggregate demand) and never leaves demand unsatisfied when there is still residual capacity. More precisely, a monopolist sells all of its capacity if \(k_1 \leq 3\), and serves all buyers, if \(k_1 \geq 3.\)\(^{20}\) Overall, our analysis sheds some light to a dynamic mechanism through which different market structures (monopoly vs. duopoly) influence the intertemporal pricing incentives of sellers and the allocation of capacity.\(^{21}\)

### 3.4 Capacity choices

Thus far, we have been concerned with the study of equilibrium pricing behavior. Since equilibrium profits depend on the initial capacity levels, it is important to now turn to the choices of the initial capacity levels in our model. We, therefore, study the equilibrium investment behavior where each firm chooses its own capacity anticipating equilibrium behavior in the continuation of the game.\(^{22}\)

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\(^{18}\)In particular, in cases \((1,1)_1\) and \((2,1)_1\), total demand may not be satisfied and at the same time at least one of the firms may end up with unsold capacity.

\(^{19}\)Note that in our intertemporal setting, when the sum of the firms’ capacities does not exceed total demand, equilibrium prices may be either lower than the monopoly price levels or higher and the firms may not sell out. This contrasts models with certain demand (at the price setting stage), either static (see de Frutos and Fabra, 2007) or dynamic (see Dudey, 1992) where both firms sell out at the monopoly price.

\(^{20}\)For example, in case \((1,1)_1\), if \(v_1\) and \(v_2\) turn out to be low enough, both firms may not sell neither in period 1 nor in period 2, beginning the last period with \((1,1)_3\) and, in total, only one unit of product is sold.

\(^{21}\)This contrast will be a key ingredient in our analysis of the implications of mergers and of the welfare analysis subsequently in the paper.

\(^{22}\)In the cases where in the price subgame three price equilibria emerge, we consider the one in mixed strategies described above. The main insights of our analysis are not sensitive to this assumption, as it is still true that under low current demand a firm would like its rival to make a sale. In particular, if we were to choose such an asymmetric equilibrium in subgames where one exists (with a firm "forcing" the other to make a sale), instead of a mixed strategy one, then a unit would be sold in the relevant period. However, the expectation of profits when viewing the whole game from the previous period (that is, the initial period) would reflect a willingness to postpone a sale, as this would be incorporated in the opportunity cost of capacity. Thus, with this asymmetric behavior, even though there would be no postponement of a sale (and no inefficiency) in the second period, we would have this present in the first period. Hence, the spirit of our results, based on the increased efficiency with which a single seller would allocate capacity over time relative to duopoly, would be valid in general, as well as our evaluation of the implications of a merger.
Table 1: Equilibrium expected profits

<table>
<thead>
<tr>
<th>Capacity of firm A</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(0.0)</td>
<td>(0.07)</td>
<td>(0.12)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>1</td>
<td>(0.7;0)</td>
<td>(0.52;0.52)</td>
<td>(0.5;0.85)</td>
<td>(0.45;0.88)</td>
</tr>
<tr>
<td>2</td>
<td>(1.2;0)</td>
<td>(0.85;0.5)</td>
<td>(0.5;0.5)</td>
<td>(0.75;0.5)</td>
</tr>
<tr>
<td>3</td>
<td>(1.5;0)</td>
<td>(0.88;0.45)</td>
<td>(0.5;0.75)</td>
<td>(0.0;0)</td>
</tr>
</tbody>
</table>

Table 1 presents the payoffs (equilibrium expected profits) of the two firms gross of capacity cost. By inspecting the expected profits, we observe that the high-capacity firm does not always have higher expected profit that the low-capacity firm. In particular, this is the case when considering capacities $(3, 2)_1$ and $(2, 3)_1$.\textsuperscript{23}

We examine first the investment incentives of the firms assuming zero capacity installment cost. Using Table 1 we see that there are three equilibrium capacity pairs $(2, 2)$, $(3, 2)$ and $(2, 3)$. Note that even when capacity is costless, we do not obtain an equilibrium where both firms invest in three units, since they take into account that available capacity would determine the intensity of price competition. Observe also that, when capacities are $(2, 1)_1$ and firm $A$ increases its capacity by one unit (from 2 to 3), the best response of firm $B$ is to increase its capacity by one unit as well (from 1 to 2). For other capacity pairs the opposite happens. Thus, capacities can be either strategic complements or strategic substitutes. This point has also been made by Reynolds and Wilson (2000) and de Frutos and Fabra (2007), who notice that the profit functions may not always be submodular.

The above analysis captures the strategic considerations of firms when choosing capacities. In reality, there may also be cost considerations. So, we now allow for a positive capacity installment cost, denoted by $c$, and in the following Proposition we present the equilibrium capacity configurations.\textsuperscript{24}

**Proposition 2** Depending on the capacity installment cost, equilibrium capacities are: $(2, 2)$, $(3, 2)$ and $(2, 3)$, if $c = 0$, $(2, 1)$ and $(1, 2)$, if $c \in (0.03, 0.33]$, $(1, 1)$, if $c \in (0.33, 0.52]$, $(1, 0)$ and $(0, 1)$, if $c \in (0.52, 0.7]$ and $(0, 0)$, if $c > 0.7$. If $c \in (0, 0.03]$, the firms have to randomize among positive capacity levels.

The above example indicates that when the per unit of capacity cost is relatively low (that is, not smaller than 0.03 under constant returns to scale), there is no investment in excess capacity, in the

\textsuperscript{23}This finding is in line with Dudey (1992), where it is shown that the low-capacity firm can make higher profits than the high-capacity one, as long as the low capacity is higher than one half of the total demand and the high capacity is higher or equal to total demand.

\textsuperscript{24}For a proof see the Technical Appendix.
sense that total capacity never exceeds the maximum total demand.

4 Mergers

Thus far, our analysis indicates that a duopoly may possibly perform worse than a monopoly. The driving force for this results is the firms’ incentives to restrict their sales in periods with low demand (saving their capacity for possibly higher demand realizations). Within a context of welfare assessment of a monopoly situation as compared to a duopoly, this represents one aspect of the problem. The other aspect relates to the firms’ incentives to invest. In Proposition 2 we saw that, when unit capacity costs are not too low, firms do not invest in excess capacity. What remains to be examined is whether a monopolist over or under-invests. In the context of our basic model, it is obvious that, if we let a monopoly firm decide on its capacity, given the demand specification, it will choose a socially optimal capacity. More interesting, however, is the same question when viewed within the dynamics of our model, in which the monopoly situation may emerge endogenously, as the result of a merger.

In this Section we examine the investment decisions of the two firms when they expect to possibly merge in the future. Merging allows the sellers to combine their capacities. Provided that the decision to merge has to be made after the choice of capacity, we expect this prospect to alter the firms’ incentives for investment. A critical issue relates to the time when firms may possibly decide to merge. In general, it will matter whether firms may merge only at one point in time, before all pricing activity starts or at any period after the choice of capacities. We study each of these cases one at a time, in each of the following subsections to highlight the strategic incentives. Another issue relates to the portion of the total (monopoly) profit each firm will take in case of a merger. In principle, there is a wide range of sharing rules one may employ (we can think for example of firms’ capacities as involving bargaining power). We choose to adopt a simple rule: the firms split in half the (incremental) surplus created by their merger. This appears a reasonable and relatively standard benchmark, at least in cases where each firm has a determining opinion about the merger, thus, firms have roughly equal bargaining powers.

4.1 Possibility of a merger at the beginning of the pricing subgame

In order to analyze the possibility of a merger we have to formally extend our basic model by adding a stage after the capacity choices and before the pricing subgame, in which firms decide whether or not to merge. Thus, the prospect of a subsequent merger is open at the stage where firms choose their capacities and this is what influences firms’ investment decisions.

\[25\] This is a plausible assumption since, normally, mergers are formed among established firms, that is, firms that already have a positive capacity.
As a start, observe that, since monopoly profit is always higher than total duopoly profit, a merger between the two firms is always profitable and, in equilibrium, will be realized. Thus, the firms’ investment choices will be conditional on an upcoming merger. Supposing that the merger did not occur, firms’ expected payoffs would be given by Table 1. In the present set-up, their expected profits in case of a merger are increased by an amount that is half the surplus created by the merger. The following Proposition summarizes the equilibrium capacity levels under the assumption used in the previous subsection, namely, for a constant per unit cost of capacity, $c$.\footnote{For example, if firms merge having capacities $(2, 1)_1$, firm A’s expected profit will be $E(\Pi^A_1) + \{E(\Pi^m) - [E(\Pi^A_1) + E(\Pi^B_1)]\}/2 \approx 0.925$ (where $E(\Pi^m) = 1.5$, is the profit of a monopoly firm with three units of capacity. The values of $E(\Pi^A_1)$ and $E(\Pi^B_1)$ can be seen in Table 1).}

Given this reformulation of our problem, we can calculate again the equilibrium capacities when firms rationally expect a merger to take place.

**Proposition 3** At the prospect of a potential merger, equilibrium capacities are: $(2, 2)$, if $c \leq 0.18$, $(2, 1)$ and $(1, 2)$, if $c \in (0.18, 0.33]$, $(1, 1)$, if $c \in (0.33, 0.6]$, $(1, 0)$ and $(0, 1)$, if $c \in (0.6, 0.7]$ and $(0, 0)$, if $c > 0.7$.

Compared to Proposition 2, the above Proposition shows that, when merging is possible after capacities have been chosen, there will be higher investment levels for some ranges of the capacity installment cost. This is so because the higher a firm’s capacity is, the greater its expected profit in case no merger occurs, that is, the greater its total expected profit in case of a merger.

4.2 Possibility of a merger at any stage of the pricing subgame

In this Subsection, we analyze the case in which firms can merge at any time after capacities have been chosen. Formally, we extend our basic model by adding a stage where firms may decide to merge at the beginning of each period of the pricing subgame. In such a set-up not only the investment decisions will be affected but also the pricing behavior of the firms. To see this, let us consider the case $(2, 1)_2$. The difference with the basic model stems from the fact that, if the two firms end up with positive capacities in the last period, instead of competing aggressively gaining zero expected profits, they will merge, splitting in half the total monopoly expected profit. This would alter the opportunity cost of capacity in the second period and, therefore, modify the equilibrium pricing incentives.

In Table 2 we present the expected profits of the two firms for all possible initial capacities given the merger that will take place at the beginning of the first period of the pricing subgame. A comparison between the first-period expected profits in the two versions of the model that allow for mergers, leads to the following.

\footnote{For a proof see Technical Appendix.}
Remark 1 The high-capacity firm is better off, while the low-capacity firm is worse off, if the possibility of a merger is present at the beginning of each period relatively to the possibility of a merger after the initial investment only.

<table>
<thead>
<tr>
<th>Capacity of firm A</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity of firm B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>(0.0)</td>
<td>(0.7)</td>
<td>(1.2)</td>
<td>(1.5)</td>
</tr>
<tr>
<td>1</td>
<td>(1.0)</td>
<td>(0.6)</td>
<td>(0.55)</td>
<td>(0.48)</td>
</tr>
<tr>
<td>2</td>
<td>(1.2)</td>
<td>(0.95)</td>
<td>(0.75)</td>
<td>(0.7)</td>
</tr>
<tr>
<td>3</td>
<td>(1.5)</td>
<td>(1.02)</td>
<td>(0.8)</td>
<td>(0.75)</td>
</tr>
</tbody>
</table>

Table 2: Equilibrium expected profits after a merger in the first period

On the basis of the expected profits presented in Table 2 we obtain the optimal investment levels.28

Proposition 4 When firms can merge at the beginning of each period, they do so in the first period. Then, equilibrium capacities are: (3,3), if \( c \leq 0.05 \); (2,2), if \( c \in (0.05,0.2] \); (2,1) and (1,2) if \( c \in (0.2,0.35] \); (1,1), if \( c \in (0.35,0.6] \); (1,0) and (0,1), if \( c \in (0.6,0.7] \), and (0,0), if \( c > 0.7 \).

The possibility of a merger at the beginning of each period leads to higher investment levels compared to the case where this possibility is present only at the beginning of the pricing game. The incentive to overinvest is driven by the expectation of merging in the future. Because the option to merge is present also at later periods, firms must reinforce their competitive position in case their rival denies the proposition for a merger in the first and second period of the pricing subgame. It turns out that each firm’s competitive position is positively related to its capacity, which translates to firms’ choosing relatively higher levels of initial capacity. Note that, from the point of view of the merged firm, this is inefficient, since it can sell at most three units. Also note that, even for positive cost levels, though sufficiently low, each firm invests in three units.

5 Welfare implications

In this Section we examine the welfare implications of a merger between the two firms leading to a monopoly situation. First, we compute and compare the total expected surplus under duopoly and under monopoly, gross of investment costs. In our model the monopolist always extracts the whole surplus by setting a price equal to the buyers’ valuation. Therefore, in case of a merger, total expected surplus (gross of investment costs) equals the expected monopoly profit. Note that this

28 See Technical Appendix for a proof.
surplus is the same in the setting where a merger is possible either only before the pricing game or at the beginning of each period. This is because in both settings, in equilibrium, a merger occurs just after the initial investment and before firms compete in prices. Hence, for given capacities, the profit is equal to the expected monopoly profit. However, total expected surplus net of investment costs is different in the two settings, because equilibrium capacities (and capacity installment costs) are different (see Propositions 3 and 4).

Regarding the duopoly case, note that, if a unit is sold in a period $t$, the created surplus equals the realized value $v_t$. The price at which the unit is sold determines only how the total surplus is shared between the seller and the buyer. The expected total surplus (gross of investment costs) at the beginning of period $t$, before firms learn $v_t$, can be calculated as

$$TS_t(k^A, k^B) = \int_{v_t \in R_A} \left(v_t + TS_{t+1}(k^A-1, k^B)\right) dv_t + \int_{v_t \in R_B} \left(v_t + TS_{t+1}(k^A, k^B-1)\right) dv_t +$$

$$+ \int_{v_t \in R_C} \left(\Pr_t^A((k^A, k^B), v_t) \left(1 - \Pr_t^B((k^A, k^B), v_t)\right) (v_t + TS_{t+1}(k^A-1, k^B)) + \right.$$

$$+(1 - \Pr_t^A((k^A, k^B), v_t)) \Pr_t^B((k^A, k^B), v_t) (v_t + TS_{t+1}(k^A, k^B-1))) +$$

$$+ \int_{v_t \in R_D} TS_{t+1}(k^A, k^B) dv_t.$$

Regions $R_A$, $R_B$, $R_C$ and $R_D$ designate the subsets of the interval $[0, 1]$ as $v_t$ varies, in which firm $A$ sells in period $t$ with certainty ($R_A$), firm $B$ sells with certainty ($R_B$), firms play mixed strategies ($R_C$), and none of the firms sells ($R_D$). What determines the difference between the trade surpluses in the monopoly and the duopoly case is the existence and the areas of $R_C$ and $R_D$, where it is possible that none of the firms sells. It turns out that the probability with which a monopolist with capacity $(k^A + k^B)$ sells is higher than the probability with which a sale is realized under duopoly with capacities $(k^A, k^B)$.

**Remark 2** For some capacity levels, total expected surplus (gross of investment cost) is lower under duopoly than under monopoly (resulting from a merger). Otherwise, the two surpluses are equal.

We have shown thus far, that when faced with demand uncertainty, the merged firm can manage the sale of total capacity over time better than a duopoly. However, we have also established that the possibility of a merger may lead to higher capacity than under duopoly without a merger. The net welfare created by the merger reflects the tension between these two effects: increased trade surplus and increased cost due to (possibly) excess investment. We examine this trade-off by computing total welfare in the settings with and without a merger.

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29 Note that the regions indicated depend on the capacity levels and on time.
First, regarding the case where a merger can be realized only at the beginning of the pricing game, in Table 3 we present the equilibrium capacity pairs that arise in the merger and the duopoly case for different values of \( c \) (see Propositions 2 and 3). The case that implies the higher welfare between the two settings is marked with an asterisk.\(^{30}\)

<table>
<thead>
<tr>
<th>Cost</th>
<th>Equilibrium investment levels without a merger</th>
<th>Equilibrium investment levels with a merger</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c \in (0.03, 0.18] )</td>
<td>((2, 1)_1 ) or ((1, 2)_1 ) *</td>
<td>((2, 2)_1 )</td>
</tr>
<tr>
<td>( c \in (0.18, 0.33] )</td>
<td>((2, 1)_1 ) or ((1, 2)_1 )</td>
<td>((2, 1)_1 ) or ((1, 2)_1 ) *</td>
</tr>
<tr>
<td>( c \in (0.33, 0.52] )</td>
<td>((1, 1)_1 )</td>
<td>((1, 1)_1 ) *</td>
</tr>
<tr>
<td>( c \in (0.52, 0.6] )</td>
<td>((0, 1)_1 ) or ((1, 0)_1 ) *</td>
<td>((1, 1)_1 )</td>
</tr>
<tr>
<td>( c \in (0.6, 0.7] )</td>
<td>((0, 1)_1 ) or ((1, 0)_1 )</td>
<td>((0, 1)_1 ) or ((1, 0)_1 )</td>
</tr>
<tr>
<td>( c &gt; 0.7 )</td>
<td>((0, 0)_1 )</td>
<td>((0, 0)_1 )</td>
</tr>
</tbody>
</table>

Table 3: Equilibrium capacities without and with a merger, when the latter can be realized only at the beginning of the pricing game.

We observe that, when \( c \in (0.03, 0.18] \cup (0.52, 0.6] \), the merger performs worse than the duopoly since any benefit of having one more unit of capacity (in the case of a merger) is outweighed by its cost. Of course, total welfare is higher with a merger for cost levels such that the (prospect of a) merger does not alter the equilibrium investment levels, which occurs when \( c \in (0.18, 0.52] \). If \( c > 0.6 \) the net surplus is the same in both settings since the market equilibrium outcome is in fact a monopoly (only one firm has positive capacity).

In Table 4 we present the equilibrium capacity levels, for different values of \( c \), when a merger is not possible and when it can be realized at the beginning of each period and, again, compare total welfare. The difference with the case where a merger can be realized only at the beginning of the pricing subgame, is that now firms have a greater incentive to invest in capacity (compare Propositions 3 and 4), which results in a higher investment cost more often than before. Once again we observe that the cost of excess investment outweighs the expected benefit from higher trade surplus, which happens for \( c \in (0.03, 0.02] \cup (0.33, 0.35] \cup (0.52, 0.6] \). The merger is now optimal, if \( c \in (0.2, 0.33] \cup (0.35, 0.52] \), where invested capacities are the same in the two settings.

We summarize in the following Proposition.

**Proposition 5** For values of the capacity installment cost such that total invested capacity is the same in the merger and the no-merger case, a merger between the two firms is welfare improving. For cost levels that, in the prospect of a merger, result in excess capacity, total welfare is higher under

\(^{30}\) For details see Technical Appendix.
Thus, it is possible that welfare increases with a merger, although it leads to a monopoly situation and despite the absence of gains in terms of direct cost reductions and other synergies, which have been established as the main sources of welfare benefits generated by mergers. The increase of total surplus in our case stems from the opportunity that a merger offers to the two firms to better manage total capacity, given demand uncertainty. The opportunity cost of capacity may be greater when firms act noncooperatively compared to the case they merge (their capacities), which reduces the incentive of each firm to sell when current demand is relatively low (in expectation of higher future demand). Although in our case there is no ex ante cost asymmetry between the firms and no post-merger reduction of direct production costs, if we consider a more “general” cost measure that includes the opportunity cost of capacity, the effect of a merger may also be interpreted in terms of a cost “efficiency” created by the merger.

### 6 Conclusion

In this paper, we have examined how selling constraints across a number of time periods influence the strategic behavior of firms under demand uncertainty. We have considered a simple duopolistic setting where, first, firms choose capacities and then they compete in prices over three periods. Demand (represented by a single buyer valuation) is realized in each period, and the firms set prices simultaneously, knowing the valuation of the current-period buyer but being uncertain about the valuation of future buyers. We have characterized how capacities evolve endogenously, according to the sellers’ pricing strategies and the demand realizations. In addition, we have analyzed how
mergers affect the intertemporal behavior of the sellers and their investment decisions. Finally, we have evaluated how mergers may affect welfare.

We obtain a number of results and insights. The main consequence of the introduction of future demand uncertainty in the dynamic setting is that it creates an opportunity cost for the available capacity. Depending on the capacity levels of both firms and on the current, as well as expected demand, a firm may wish to avoid selling in a given period. Importantly, the opportunity cost of a unit of capacity for each firm depends on its expectation that its rival will make a sale or not, in case it does not make a sale itself. In particular, over a region of intermediate demand realizations, a firm may strictly prefer that it does not sell a unit and that its rival sells a unit. Overall, in contrast to the nonstochastic demand case, a low capacity firm may have an incentive to forego a sale, if current-period demand is low, and in fact may not sell at all. In general, the analysis of price competition here creates two insights, both driven by the level of opportunity costs in equilibrium: that the allocation of capacity by a dynamic duopoly may be inefficient (implying outcomes with unused capacity and unsatisfied demand) and that the pattern of pricing and sales (in particular, whether the high or the low capacity firm makes a sale) in each period depends on the demand realizations.

The characterization of equilibrium capacity levels shows that the firms do not invest in excess capacity (except for the case where the unit capacity cost is close to zero). This does not hold, if they expect they can merge in the future. When firms invest, under the prospect of a future merger, their incentives to overinvest are higher than if no prospect for a merger exists. Mergers in general may affect welfare in many ways. In our analysis, the following trade-off emerges. On the one hand, for given capacities, a merger between the two firms typically increases total surplus from sales. This happens because the opportunity cost of a unit of capacity differs when total capacity is owned by a single firm compared to when it is divided between the two sellers. By merging, the two firms can more efficiently manage total capacity utilization over time and would not forego a sale that is efficient to take place. On the other hand, for some levels of the capacity installment cost, the prospect of a merger leads to higher (and inefficient) investment levels. Combining both effects, we find that, for certain cost ranges, the net impact of a merger on welfare is positive.

Our model has been kept simple enough to allow a relatively clean analysis and intuition. There are several directions in which the analysis could be generalized in future work. Each is expected to enrich the results in certain dimensions but at the cost of nontrivial modeling complications. First, we have considered pricing over three-periods. This allows enough horizon for the main results to come out, but obviously is not fully general. Increasing the number of periods in our setting would come at the significant cost of expending the state space (the possible levels of capacities) and make an analytic solution intractable and complicated. Still, some infinite horizon variation of the general problem may be tractable if it is simplified along some other dimensions (e.g. investment...
at predetermined times, with capacity that lasts only for a small number of periods, or a smaller number of possible demand levels). Second, we have assumed here unit demand within each period. Any inefficiency comes from a failure to sell a unit to a buyer when a unit is available with an opportunity cost below the valuation. More general demand structures (and the associated standard pricing distortions and deadweight losses) would naturally add a new dimension to the comparison between duopoly and monopoly, overall making a merger less socially desirable than our model claims it would be. Third, demand uncertainty here comes through the future periods and firms set prices depending on the current level of demand. Our preliminary results, in a companion paper, indicate that if firms had to set prices before even seeing the current demand, the problem would be somewhat modified: while the dependence of prices and sales on the current period would be lost, multiplicity of equilibria would emerge in many cases. Of course, the opportunity costs of capacity would still be present as a driving force. Fourth, we have modelled demand realizations as independent across periods. If, for whatever reasons, we had some demand correlations (positive or negative) across periods, the results would have to be modified accordingly (with the case of perfect positive correlation bringing us back to the certainty benchmark). Of course, explicit calculations in the case of correlated demand would be substantially more complicated, as price distributions for future periods would have to be updated conditional on current demand realizations. And finally, empirical work that relates the evolution of firms’ capacities with their pricing strategies appears very promising.31

References


