Endogenous Spatial Differentiation with Vertical Contracting*

Frango Kourandi† Nikolaos Vettas‡

July 31, 2012

Abstract

We set-up a linear city model with duopoly upstream and downstream. There is a transportation cost when consumers buy from a retailer and when retailers buy from a wholesaler. All location and pricing decisions are endogenous. When the wholesalers choose their locations first, they locate closer to the center relative to the retailers, and relative to when they move simultaneously. Each wholesaler does this to strengthen the strategic position of his retailer by pulling him towards the market center resulting, in equilibrium, to a greater intensity of competition. All firms locate farther away from the market center and industry profits are higher under linear pricing relative to two-part tariffs and in turn relative to vertical integration.

Keywords: Spatial differentiation; Vertical contracting; Linear city; Strategic commitment.

JEL Codes: L13; R32.

*We thank, for their suggestions and discussions L. Linnemer, N. Matsushima, M. Peitz, P. Rey and participants in seminars and conferences where previous versions of this work have been presented.

†Department of Economics & Social Sciences, Telecom ParisTech, 46 Rue Barrault, Paris 75013, France; e-mail: kourandi@aueb.gr

‡Department of Economics, Athens University of Economics and Business, 76 Patision Str., Athens 10434, Greece and CEPR, UK; e-mail: nvettas@aueb.gr
1 Introduction

Motivation and main idea. Thanks to an extensive and important literature we understand many aspects of how oligopolistic firms choose their locations or horizontal differentiation before they compete in prices. Maximal, minimal or intermediate differentiation emerges depending on the balance between a direct demand effect that pulls firms towards the center of the market and a strategic price effect that pushes them away from one another. The literature, however, focuses on the direct market relation between the oligopolists and the final consumers. In contrast, the nature of vertical market relations is very important in many oligopolies. In other words, how upstream firms (wholesalers) trade with downstream firms (retailers) is interrelated with how downstream firms trade with final consumers. In this paper, we study the horizontal product differentiation and pricing decisions in vertically linked duopolies. Specifically, we endogenize the locations and pricing decisions of wholesalers and retailers on a Hotelling line.

Upstream-downstream trade and product differentiation are two key elements in many oligopolistic industries. Our analysis is particularly relevant for industries where both upstream and downstream oligopolists choose their locations. As usual, a location can be understood literally, in geographical space, and metaphorically, in the space of "horizontal" product characteristics. Consider, for example some agriculture or food industry where there may be oligopoly both upstream and downstream. Geographical location is a central consideration for all firms, since the transportation cost can be a very significant part of the total cost when the products are heavy to carry and perishable. But also in many other cases including concrete, steel, and many other commodities, upstream and downstream location choices are crucial. The "product variety" interpretation of locations is also interesting. Products can be differentiated at the wholesale level and retailers may have to pay a transformation cost so that they modify the features of the product and bring it closer to how they wish to position it in the retail market. For example, milk sold by wholesalers may differ with respect to its fat content and retailers have a cost when using that input to produce daily products, also differentiated with respect to their fat content. Thus, our analysis is applicable to any oligopolistic industry where transportation costs exist both upstream and downstream.

The model. We study a two-tier horizontally differentiated duopoly model. Each downstream firm is able to turn one unit it gets from its upstream firm into one unit in the final market. Upstream and downstream firms are differentiated with respect to their horizontal locations. In our basic model, firms choose any location on the real line while consumers are uniformly distributed within the unit interval. Choices are made sequentially. Upstream firms and then downstream firms choose their locations. Subsequently, upstream and then downstream firms choose their prices. Finally, consumers make their selection.

The questions that can be answered in a framework that combines vertical contracting and product differentiation elements refer both to the implications of upstream to downstream pricing for the location choices and to the role that upstream and/or downstream differentiation plays for
vertical trade and final prices. The strategic incentives are rich, as each firm’s choice affects the subsequent choices of both firms and of the final consumers. For instance, upstream locations affect downstream locations, then wholesale prices and then the final prices. The nature of competition downstream depends on the marginal cost of each retailer and, importantly, in our model this cost becomes endogenous since it depends on the location choices of the wholesalers and the retailers.

Main results. In one-level models of horizontal differentiation (that is, without upstream firms) there are two opposite forces, the "demand" effect, pushing firms close to each other, and the "price competition" effect, pushing them to the opposite direction. In our model there is a third force working through the wholesale prices and the transportation cost that the downstream firms pay when moving to the center of the line. The interaction of all these forces shapes the equilibrium. Somewhat surprisingly, given the significant complexity of the model, we are able to find a unique subgame perfect equilibrium which can be expressed in closed form in all of the cases we study. Under vertical separation, all firms locate outside the unit interval and when the wholesalers choose their locations first, they always locate closer to the city center than the retailers. One set of results refers to the comparison between various types of vertical trade: we first study vertical integration and then proceed to the case of vertical separation under linear pricing and under two-part tariffs. A second set of results studies how the timing of the location decisions matters.

Our main results can be summarized as follows. First, regardless of the timing of location choices, all firms locate farther away from the center under linear pricing, than under two-part tariffs and in turn than under vertical integration. Second, the social cost, including both the intermediate and the final transportation cost can be also ranked due to the modified location incentives. We find that prices are lower but total social cost is higher under linear pricing, then under two-part tariffs and then under vertical integration. We note that either with linear or two-part tariffs double marginalization emerges and final prices are higher than under vertical integration. Total equilibrium profits are also higher than under vertical integration.

Third, crucial insights are also obtained when we study alternative timing assumptions about the location choices. Regardless of the mode of vertical trade, when wholesalers choose their locations first, they choose to locate closer to the center relative to when location choices are simultaneous (or when retailers move first). They do this to credibly pull their retailers towards the center and offer them a stronger strategic position in the final market. Thus, locations are used as strategic commitment devices. In equilibrium, this strategic behavior leads to more intense competition and lower profits when wholesalers choose their locations first. The industry would, thus, not like the wholesalers to be able to choose their locations first. This result is reminiscent of the strategic delegation and contracting literature (as e.g. in Fershtman and Judd, 1987 or Bonanno and Vickers, 1988) but it operates through the location choices. Upstream players seek to gain unilaterally by offering a way for their downstream partners to credibly commit to more aggressive behavior: in our model more aggressive behavior means a location choice closer to the center. When upstream
locations are chosen first, each upstream firm strategically moves closer to the center so that it can offer (through the upstream-downstream transportation cost) a credible way for its downstream firm to also move towards the center. When both upstream firms do this, both downstream are drawn towards the center. In equilibrium, competition gets intensified, with industry profit being lower, than when this strategic effect is absent (e.g. when upstream locations do not proceed the downstream).

**Related literature.** Our paper brings together two distinct literatures, on product differentiation and on vertical contracting. Each contains a number of very influential papers: for some key results on product differentiation see e.g. Anderson *et al.* (1992), Gabszewicz and Thisse (1992) and for vertical relations see e.g. Rey and Tirole (2007) and Rey and Verge (2008). As each of these literatures is too large to survey here, we only discuss work that is more closely related to the specific setting of our model. The strand of the product differentiation literature we are building on, starts with the classic linear-city model of Hotelling (1929), modified by d’Aspremont *et al.* (1979) in a way that the strategic incentives can also be more easily characterized. Nevertheless, only very few papers have examined vertical chain interactions in a horizontal differentiation framework. This represents a significant gap in the literature since in reality many markets have an important vertical element. A key observation here is that in our set-up the marginal costs of the downstream firms are endogenously determined and that all location choices are affected by the terms of vertical trade.

Some papers have studied upstream monopoly with horizontally differentiated firms downstream. Gupta *et al.* (1994) assume that an upstream monopolist sets its wholesale price based on its observation of the locations chosen by the downstream firms and the downstream firms can price discriminate. Beladi *et al.* (2008) also study upstream monopoly and downstream duopoly, with two-part tariffs and the downstream firms not able to produce all varieties. Aiura and Sato (2008) examine an upstream monopolist at the center of the city and supplying two retailers. Retailers do not pay wholesale prices but only a transportation cost and choose their locations and final prices. In our model, there is oligopoly competition both upstream and downstream.

A location-price equilibrium is also analyzed by Brekke and Straume (2004) where each downstream firm has its own supplier, upstream firms bargain about the linear input prices with the downstream firms, but in contrast to our model, upstream firms are not product differentiated. In Allain (2002) and Laussel (2006) two upstream firms are exogenously brand differentiated and

---

1 Many variants of the linear city model have been studied. Among other, Anderson and Neven (1991) solve the location-pricing game when oligopolists compete in quantities. Ziss (1993) examines the d’Aspremont *et al.* model with heterogeneous production costs: if the marginal cost difference is sufficient small, a price and location equilibrium exists in which both firms enter and maximum differentiation emerges. Anderson and Engers (1994) study a price-taking equilibrium in the spatial setting. In Vettas (2003) and Vettas and Christou (2005), firms are horizontally as well as vertically differentiated. Tabuchi and Thisse (1995) and Lambertini (1997) allow firms to locate on the entire real line.

2 Dobson and Waterson (1996) study exclusivity in an exogenously non-horizontal differentiated successive duopoly at the upstream and downstream level, with consumers facing four varieties. Inderst and Shaffer (2007) analyze the impact of retail mergers on product variety in a non-Hotelling type differentiation model.
two downstream firms are exogenously spatially differentiated, thus, consumers face four different products. In our model, consumers care directly only about the final product locations and prices - we endogenize the location choices in both levels allowing input prices to be set by the wholesalers. Reisinger and Schnitzer (2010) study successive oligopolies with endogenous entry in the upstream and downstream level where firms are exogenously equidistantly located on a circle. Due to uncertainty in the realizations of the downstream firms’ locations, the downstream and upstream locations are not correlated with each other. In contrast, the main feature of our paper is that location choices are interrelated. The closest work to ours is probably by Matsushima (2004) who has also studied the two upstream and two downstream firm structure on the line. However, he studies simultaneous symmetric location choices of upstream and downstream firms within the unit interval, when the upstream firms price discriminate between the retailers and the transportation cost is paid by the wholesalers.\(^3\)

In our paper, by deriving in closed form the subgame perfect equilibrium of the general game with upstream and downstream duopoly when the location choices are either sequential or simultaneous, when all firms can locate anywhere on the real line and under alternative vertical structures, we obtain a rich set of results and insights about the interrelation between vertical trade and horizontal product differentiation.

**Roadmap.** The remainder of the paper is as follows. Section 2 sets up the model. Section 3 derives the equilibrium under vertical integration and studies its properties. Under vertical separation, the linear wholesale pricing game is examined in Section 4. Section 5 studies two-part tariffs. Section 6 concludes.

## 2 The model

We set up a model where upstream and downstream duopolies locate on a line as follows. Consumers are uniformly distributed on a [0,1] interval and have unit demands. There are two upstream firms, A and B, and two downstream firms, X and Y, each choosing a unique location on the entire real line. There is exclusive dealing and the downstream firms have a simple fixed-proportions technology: firm X turns each unit it purchases from firm A into one unit that it can sell to the final consumers; likewise, for firms Y and B.

The locations of firms A and B are denoted by \(a\) and \(1 - b\), respectively while the locations of firms X and Y are denoted by \(x\) and \(1 - y\), respectively (see Figure 1). Thus, for the A and X chain, \(a\) and \(x\) measure how much to the right of endpoint 0 each firm is located, while for the B and Y chain, \(b\) and \(y\) measure how much to the left of endpoint 1 each firm is located. Without loss of generality, we consider \(1 - a - b > 0\), so that A is located to the left of B. In the main body of the paper we will also focus on the case where \(1 - x - y > 0\) so that X is also to the left of Y; in

\(^3\)Matsushima (2009) studies the incentives for vertical mergers in a locations model.
Appendix A2 we also discuss the possibility that the downstream locations switch to the "wrong" side of the line relative to their upstream suppliers.

An important ingredient of our model that a transportation cost has to be paid both in the wholesale and retail market. So that our results are easily comparable to the literature, we follow the often used assumption that this cost is quadratic in distance. A consumer located at point \( z \) pays transportation cost \( t(x - z)^2 \) when purchasing a product from firm X and \( t(1 - y - z)^2 \) when purchasing a product from firm Y. In turn, firm X pays transportation cost \( \tau(x - a)^2 \) when purchasing a unit from firm A and firm Y pays transportation cost \( \tau(y - b)^2 \) when purchasing a unit from firm B. These costs, \( t > 0 \) and \( \tau > 0 \), may be understood as real transportation costs (for example, costs depending on the weight or the volume of the product) or as product characteristic transformation costs, necessary to convert one unit of the upstream firm’s input to one unit of final good.

We first consider linear pricing at the wholesale and retail level as our core scenario (but subsequently we also analyze the case when two-part tariffs are set at the wholesale level). A final consumer that purchases a unit from downstream firm X or Y pays the price set by this firm (\( p_X \) or \( p_Y \)) plus the transportation cost between the chosen firm’s location and his own location. We also assume that the basic reservation value of each consumer is high enough so that each consumer purchases one unit of the product. A downstream firm who purchases a unit from upstream firm A or B pays the price set by this firm (\( w_A \) or \( w_B \)) plus the transportation cost between the two trading firms. Apart from their location differences, the products are homogeneous. Production costs are assumed zero, for simplicity. Note that the final consumers care about the upstream locations and prices indirectly, that is, only to the extent that they affect the downstream locations and prices.

![Figure 1: Upstream and downstream locations (example)](image)

Each of the four firms maximizes its own profit and each consumer maximizes his own net surplus. We assume no information asymmetries and there is full observability from one stage of the game to the following. Our goal is to identify the subgame perfect equilibria of the game when all locations and prices are endogenous. We view locations as longer-run (and more difficult to change) variables than prices. Therefore, the main model we analyze is a five-stage game as follows:

1. Upstream firms A and B simultaneously choose their locations, \( a, b \).
2. Downstream firms X and Y simultaneously choose their locations, \(x, y\).

3. Upstream firms simultaneously set linear wholesale prices, \(w_A, w_B\): firm A charges \(w_A\) to firm X and firm B charges \(w_B\) to firm Y.

4. Downstream firms simultaneously set their (final) product prices, \(p_X, p_Y\).

5. Each consumer purchases one unit of the product from one of the downstream firms X and Y.

Naturally, this five-stage game will have to be analyzed by working backwards. The subgame perfect equilibrium that we will find reflects how the choices at each stage affect the choices at each of the subsequent stages. It should not be, thus, a surprise that the analysis becomes increasingly more complicated as we move from one step to the next and it is not always obvious how the various effects operate within the equilibrium expressions. Therefore, in order to obtain a better understanding of the problem, we first examine the vertical integration benchmark. Each vertical chain can choose different locations for its upstream and downstream divisions, however, since this is an integrated firm, the issue of pricing at the wholesale level becomes irrelevant (or, put more simply, stage 3 of our main model is eliminated). This modification of the problem allows us to isolate the study of the location incentives from that of upstream-downstream pricing and to obtain some first important insight into the problem (including that the order of location choices matters in a very important way). After the analysis of the vertical integration case, we turn to the case of our primary interest, vertical separation with linear pricing. We then see how the location incentives interact with the pricing incentives and how the equilibrium locations and the final prices are systematically different when we have vertical integration rather than separation. Finally, we modify our model to study the case of two-part tariffs at the wholesale level and we compare the different results.\(^4\) We find that the equilibrium outcome in this case can be viewed as in between the cases of vertical integration and of linear pricing.

### 3 Vertical integration

Under vertical integration (VI), each chain is a single firm and thus pricing issues do not arise at the wholesale level. As an initial observation, note that if each (integrated) vertical chain had to choose the same location for its upstream and its downstream division we would essentially return to the case of product differentiation on the line without a vertical structure. In a standard Hotelling model, there is no vertical structure (or, alternatively, if there are wholesalers these play a passive role). It is easy to see that VI with a single location for the upstream and the downstream

\(^4\)We also examine (in Appendix A) other extensions of the analysis: when firm locations cannot be on the entire real line but only within the unit interval; and when downstream locations can be at the opposite side on the line relative to their upstream suppliers.
division is equivalent to the absence of any vertical structure. In equilibrium the firms locate at distance \(-0.25\) from each endpoint and the prices are equal to \(1.5t\). This result directly follows from extending the d’Aspremont et al. (1979) model when firms are allowed to locate anywhere on the real line.\(^5\)

In this section, we study the case where the upstream and downstream divisions for each VI chain can be at different locations.\(^6\) We start by considering the case where these location choices are made simultaneously. Then we study the case where the upstream firms locations are chosen first. Of course, we are interested not only in the locations chosen but also in the final prices that result in equilibrium.

### 3.1 Simultaneous location choices

In the first stage of this vertical integration game, each of the two VI chains chooses the location of its upstream and downstream division simultaneously. In the second stage, the two VI chains compete for the consumers by setting the final prices. Finally, each consumer decides which retailer to buy from. We proceed backwards to solve for the subgame perfect equilibrium.\(^7\)

**Consumers’ choices and retail prices.** This part of the analysis corresponds to duopoly competition in the final (retail) market with fixed locations and potentially different unit costs.\(^8\) The reason that the costs may be different at the downstream level is that there is a transportation cost between each upstream and downstream division. The farther away a downstream division is located from its upstream supplier, the higher the cost it would have to pay for each unit it sells to the final consumers. For some locations and costs, both retailers sell in the market, while for locations and costs that significantly favor one of the retailers, we obtain a corner solution where the rival makes no sales. Given the firms’ costs, we calculate the demand functions for the downstream firms X and Y. Let \(z\) be the demand of firm X and \(1 - z\) be the demand of firm Y. In general, the firms’ profit functions are

\[
\Pi_X = (p_X - f_X)z, \quad \Pi_Y = (p_Y - f_Y)(1 - z),
\]

where \(f_X\) (respectively \(f_Y\)) is the aggregate marginal cost, that is, the wholesale price plus the transportation cost of firm X (respectively Y) with \(f_X = w_A + \tau(x - a)^2\) and \(f_Y = w_B + \tau(y - b)^2\).

Under VI, wholesale trade can be viewed as taking place at cost: \(w_A = w_B = 0\).\(^9\) When both

---

\(^5\)See e.g. Tabuchi and Thisse (1995) and Lambertini (1997).

\(^6\)Thus, one may think of each vertically integrated firm as having a factory and a point of sales possibly at different geographical locations.

\(^7\)The analysis is tedious at times and some of the formal details are omitted, or are presented in brevity to facilitate the continuity of the arguments and the intuition.

\(^8\)Ziss (1993) studies the linear city model with different unit costs. Our analysis in this part parallels his.

\(^9\)In contrast, we show in the next section that under vertical separation these prices are set at a positive level.
firms sell positive quantities, demand for each firm is characterized by the presence of an indifferent consumer located at \( z \), as follows:

\[
p_X + t(x - z)^2 = p_Y + t(1 - y - z)^2.
\]  

(2)

Of course, the location of the indifferent consumer depends on the downstream divisions’ locations, product prices and the transportation cost parameter \( t \), that is, \( z = z(p_X, p_Y, x, y; t) \). Solving (2) and taking into account the possibility that all consumers may choose to purchase from one firm, demand for firm X will be equal to

\[
z = \begin{cases} 
1 & \text{if } \frac{1 + x - y}{2} + \frac{p_Y - p_X}{2t(1 - x - y)} \geq 1 \\
\frac{1 + x - y}{2} + \frac{p_Y - p_X}{2t(1 - x - y)} & \text{if } 0 < \frac{1 + x - y}{2} + \frac{p_Y - p_X}{2t(1 - x - y)} < 1 \\
0 & \text{if } \frac{1 + x - y}{2} + \frac{p_Y - p_X}{2t(1 - x - y)} \leq 0.
\end{cases}
\]

(3)

Substituting for \( z \) into (1), we obtain the downstream profit functions. We note that the downstream demand and profit functions are continuous in both firms’ prices. Under VI, maximization of each chain’s total profit is equivalent to the maximization of the downstream profit. So we simply assume that prices are set by each downstream division to maximize its profits. By maximizing each downstream profit with respect to the corresponding retail price, we find:

**Lemma 1** The equilibrium price for firm X is:

\[
p_X = \begin{cases} 
f_Y - t(1 - x - y)(1 + y - x) & \text{if } f_X \leq \hat{f}_X \\
\frac{t(1 - x - y)(3 + x - y) + f_Y + 2f_X}{3} & \text{if } \hat{f}_X < f_X < \hat{f}_X \\
f_X & \text{if } f_X \geq \hat{f}_X,
\end{cases}
\]

(4)

where \( \hat{f}_X \equiv f_Y + t(1 - x - y)(3 - y + x) \), \( \hat{f}_X \equiv f_Y - t(1 - x - y)(3 - x + y) \). The equilibrium price for firm Y can be expressed in a symmetric way.\(^{11}\)

When firm X has low enough aggregate marginal cost (lower than the critical value \( \hat{f}_X \)), it is the only one that sells in the market, while Y has zero demand. The opposite is true when X has a high aggregate marginal cost (higher than \( \hat{f}_X \)). When the difference in the marginal costs of the two firms is not too large, \( \hat{f}_X < f_X < \hat{f}_X \), both downstream firms make sales in the final goods’ market.

Note that the equilibrium retail prices in Lemma 1 are valid for all location choices (either simultaneous or sequential timing in location choices) and for both the vertical integration and vertical separation cases (in the latter case, the aggregate marginal cost takes into account the wholesale prices as well). Note also three properties of the downstream market equilibrium in prices.

\(^{10}\)To simplify the exposition we suppress the arguments of this function in the following analysis.

\(^{11}\)For brevity we only provide throughout the paper the expressions for firm X and A, while the expressions for firms Y and B are suppressed since they can be written symmetrically.
First, the first derivative of the final prices and the profits with respect to the transportation cost parameter $t$ of the consumers is positive: the more differentiated the final goods are, the higher are the prices set by the retailers and the profits they obtain. Second, we have $dp_i/dt = 2/3 < 1$. Thus, following an increase in his aggregate marginal cost, a retailer passes part of this to the final consumers but absorbs the rest. Third, a change in the downstream locations affects the retail prices via two channels; the marginal costs and the transportation cost paid by the consumers.

**Location choices.** At this stage, the VI firms simultaneously choose their upstream and downstream locations anticipating equilibrium behavior at the retail level. Firms are allowed to locate anywhere on the real line, that is, also outside the unit interval where consumers are located. By inserting the final prices (4) into the joint profits of the VI firms (1) and by the fact that the aggregate marginal costs are simply the retail transportation costs, $f_X = \tau(x-a)^2$ and $f_Y = \tau(y-b)^2$, we obtain the profit functions depending on all four location choices. The profits of the VI chain A-X are:

$$
\Pi_X = \begin{cases} 
\tau((y-b)^2-(x-a)^2)-t(1-x-y)(1+y-x) & \text{if } \tau((y-b)^2-(x-a)^2) \geq t(1-x-y)(y-x+3) \\
\frac{\tau((x-a)^2-(y-b)^2) + t(y-x-3)(1-x-y))^2}{18t(1-x-y)} & \text{if } \tau((y-b)^2-(x-a)^2) < t(1-x-y)(y-x+3) \\
0 & \text{if } \tau((y-b)^2-(x-a)^2) \geq t(1-x-y)(y-x-3) \\
\text{if } \tau((y-b)^2-(x-a)^2) \leq t(1-x-y)(y-x-3).
\end{cases}
$$

(5)

Analogously, we can write the joint profits for the VI chain B-Y. The two VI firms choose their upstream and downstream locations simultaneously to maximize their joint profits. Clearly, no chain will choose asymmetric enough upstream and downstream locations to make their downstream division obtain zero demand and zero profits. Therefore, the profit function can be reduced to

$$
\Pi_X = \frac{\tau((x-a)^2-(y-b)^2) + t(y-x-3)(1-x-y))^2}{18t(1-x-y)}. 
$$

(6)

From the first order conditions for the upstream divisions (and the fact that the corresponding profit margins are positive), we find that each upstream division will be chosen to be at the same location as the corresponding downstream division, $a = x$ and $b = y$.\(^\text{12}\) Further, from the first order conditions for the downstream divisions, we obtain: $a = x = b = y = -0.25$. Thus, we have:

**Proposition 1** With simultaneous locations choices, the unique subgame perfect equilibrium outcome under vertical integration is:

$$
\hat{a}^{VI} = \hat{b}^{VI} = \hat{x}^{VI} = \hat{y}^{VI} = -0.25, \\
\hat{p}_X^{VI} = \hat{p}_Y^{VI} = 1.5t, \quad \hat{z}^{VI} = 0.5, \\
\hat{\Pi}_X^{VI} = \hat{\Pi}_Y^{VI} = 0.75t.
$$

\(^{12}\)The second order conditions hold and the first order conditions imply a unique maximum.
We note that all four firms locate outside the unit interval, at $-0.25$. Thus, the equilibrium locations when upstream and downstream locations are chosen simultaneously are the same as when these locations have to coincide: we obtain the same equilibrium outcome as the one where there is no vertical structure but two competing firms choose a location on the line and then compete in prices. This is not very surprising. As there is a transportation cost for upstream-downstream trade, and since all locations are chosen at the same time, and thus they can only affect retail prices, each chain chooses to minimize the distance between its upstream and its downstream division and, in equilibrium, this single location is the one we would also have in a model with only final market competition. The question is if anything changes when locations are not chosen simultaneously. It is easy to see that when the downstream locations are chosen before the upstream locations, the outcome does not change. The downstream locations are chosen first, as they would without a vertical structure and then the upstream divisions locate at the location of their downstream division in order to minimize their transportation cost.\footnote{For the details see Kourandi and Vettas, 2010.} If the downstream divisions are already located at $-0.25$, the upstream divisions have no better choice other than to also locate there to maximize their overall profits. However, importantly, in the next subsection we show that when the upstream locations are chosen before the downstream locations, the incentives for the firms get modified and they are willing to choose different locations for their upstream and downstream divisions even though this implies a positive transportation cost.

### 3.2 Upstream locations first

In the first stage of this vertical integration game, each of the two vertically integrated chains chooses the location of its upstream division. In the second stage, each chain chooses the location of its downstream division. In the third stage, the two VI chains compete for the consumers by setting the final prices and finally consumers decide which retailer to buy from. We proceed backwards to solve for the subgame perfect equilibrium.

**Consumers’ choices and retail prices.** The third stage of the game remains the same as in the simultaneous location model and the final prices for chain A-X are given by equation (4).

**Downstream locations.** In the second stage, taking as given the locations of the upstream divisions and assuming that the retail prices will be subsequently determined in equilibrium, the VI firms simultaneously choose the locations of their downstream divisions to maximize their joint profits. Like in the previous steps of our analysis, special care should be taken about the corner cases. Is it possible that for certain wholesale locations, a retailer cannot avoid obtaining zero demand no matter what location it would choose? We find that, if the upstream locations are asymmetric enough, we may obtain a corner solution in the continuation of the game, that is, one retailer finds it impossible to avoid making zero sales. Take, for example, division B (that supplies division Y) to be located far away from the unit interval and division A (that supplies X) to be
located at the center of the unit interval. Then, the transportation cost that retailer Y pays when supplied by division B is high compared to the rival’s transportation cost. Based on its cost advantage, division X maximizes its profits by choosing a location \( x \) that allows it to capture the whole market. The alternative, choosing a location far away from the unit interval would lead to zero demand or, in any event, to demand lower than unity and firm X’s profits would not be maximized.

We examine when such a corner solution will emerge. From the profit function (5), we have that for \( t(1-x-y)(y-x+3) - \tau((y-b)^2-(x-a)^2) \leq 0 \) firm X serves the whole demand. When firm B is located far away from the unit interval, its downstream division Y has to decide where to locate in order not to pay a high transportation cost but also to obtain some positive demand. If Y locates at its supplier’s location, it minimizes its own transportation cost but it is then located far away from the consumers. In contrast, if it locates in or near the unit interval it is close to the final consumers but faces a high transportation cost. In equilibrium, Y chooses a location \( y \) that maximizes the expression \( t(1-x-y)(y-x+3) - \tau((y-b)^2-(x-a)^2) \), that is, it minimizes the area where its demand is zero. By direct calculations it follows that \( y = (b\tau - t)/(\tau + t) \). Given this location \( y \), firm X maximizes its own profits. This argument holds symmetrically when firm A is located far away from the unit interval and firm X has a cost disadvantage. Of course, when neither firm has a high cost disadvantage in equilibrium the firms share the demand. From this analysis we obtain:

**Lemma 2** The equilibrium downstream location for firm X (as a function of the upstream locations) is

\[
x = \begin{cases} 
\frac{a\tau + t}{\tau + t} & \text{if } 9t - 4\tau(a-b-3)(1-a-b) \leq 0 \\
\frac{16\tau^2a(1-a-b)+4\tau(5a-a^2+b^2-1)-3t^2}{4(\tau+t)(4\tau(1-a-b)+3t)} & \text{if } 9t - 4\tau(a-b-3)(1-a-b) > 0 \\
\frac{a\tau - t}{\tau + t} & \text{if } 9t + 4\tau(a-b+3)(1-a-b) < 0.
\end{cases}
\]

The VI profits for chain A-X in this stage can be obtained by substituting the equilibrium downstream locations into the profits (5).

**Upstream locations.** In the first stage of the game, upstream locations will be chosen to maximize the joint profits of each chain. An upstream location will never be chosen so that the corresponding retailer obtains zero demand. Therefore, in equilibrium both VI chains avoid receiving zero demand and the relevant joint profit function for chain A-X reduces to:

\[
\Pi_X = \frac{t(3t + 2\tau(1-a-b))(9t + 4\tau(a-b+3)(1-a-b))^2}{36(\tau+t)(3t + 4\tau(1-a-b))^2}.
\]

Maximization of the joint profits and solving for the equilibrium yields: \( a = b = (3t + \tau - 3\sqrt{\tau^2 + t^2})/8\tau \).

**Proposition 2** With upstream locations chosen first, the unique subgame perfect equilibrium out-
come under vertical integration is:

\[ a^{VI} = b^{VI} = \frac{3t + \tau - 3\sqrt{\tau^2 + t^2}}{8\tau}, \quad x^{VI} = y^{VI} = \frac{\tau + t - 3\sqrt{\tau^2 + t^2}}{8(\tau + t)}, \]

\[ p_X^{VI} = p_Y^{VI} = \frac{3t(\tau^2(8\tau + 19t) + t^2(11\tau + 3t) + \sqrt{\tau^2 + t^2}(8\tau - 3t)(\tau + t))}{32\tau(\tau + t)^2}, \]

\[ z^{VI} = 0.5, \]

\[ \Pi_X^{VI} = \Pi_Y^{VI} = \frac{3t(\tau + t + \sqrt{\tau^2 + t^2})}{8(\tau + t)}. \]

We note that, in equilibrium, firms obtain positive profits for all parameter values and the downstream divisions are always located outside the unit interval (\( x^{VI} = y^{VI} < 0 \)). Further, upstream divisions are located closer to the center relative to the downstream firms (\( a^{VI} > x^{VI} \)) and upstream firms are located within the unit interval (\( a^{VI} > 0 \)) when \( t > 4\tau/3 \) and outside otherwise.

We discuss the equilibrium properties in detail in the next subsection.

### 3.3 Equilibrium properties and commitment incentives under VI

Let us now discuss the properties of the VI equilibrium we have derived under both timing assumptions. First, we study comparative statics. Second, we compare the equilibrium to the social optimum. Third, we discuss the different forces that shape the equilibrium in our model. Finally, we compare the two alternative VI cases.

**Comparative statics.** We study how equilibrium location, price and profits respond to changes of the two key parameters, \( t \) and \( \tau \).

**Corollary 1** With simultaneous locations choices i) as the downstream transportation cost parameter \( t \) increases, the retail prices and the VI profit levels increase while all four locations remain unchanged and ii) a change in the upstream transportation cost parameter \( \tau \) does not affect the equilibrium outcome. With upstream locations chosen first i) as the downstream transportation cost parameter \( t \) increases, the retail prices and the VI profits increase and the upstream divisions move closer to the center. However, as \( t \) increases, the downstream divisions move closer to the center when \( \tau > t \) and farther away from the center when \( \tau < t \). ii) As the upstream transportation cost parameter \( \tau \) increases, the retail prices increase when \( \tau > \tau^* \) and the VI profit levels increase when \( \tau > t \), and decrease otherwise. Also, as \( \tau \) increases, upstream divisions move farther away from the center, while the downstream firms move farther away from the center when \( \tau > t \) and move closer to the center when \( \tau < t \).

We obtain these results by directly differentiating the relevant expressions in Proposition 1 and 2. As the downstream transportation cost parameter \( t \) increases, retail prices increase (\( dp_X^{VI}/dt > 0 \) and \( dp_Y^{VI}/dt > 0 \)) in both the simultaneous and sequential location choices model and this also implies higher profit levels (\( d\Pi_X^{VI}/dt > 0 \) and \( d\Pi_Y^{VI}/dt > 0 \)), with symmetric relations holding for
firms B and Y. As should be expected, as final consumers view the products as more differentiated, competition becomes less intense. In the extreme case where \( t \) is zero, consumers buy from the cheapest retailer since there is no product differentiation at the downstream level, and this leads to zero retail prices and zero profits for all firms. With simultaneous locations choices, a change in \( t \) does not affect the equilibrium locations; a result similar to the standard d’Aspremont et al. (1979) model. However, when upstream locations are chosen first, an increase in \( t \) increases the upstream location (\( da^V / dt > 0 \)) for all values of \( \tau \), but downstream location increases (\( dx^V / dt > 0 \)) when \( \tau > t \). When final consumers view the products as more differentiated, upstream firms move closer to the center and retailers follow them when retail transportation costs are high. In the extreme case when \( t = 0 \), all four firms locate at \( a^V = b^V = x^V = y^V = 0.25 \).

A change in the upstream transportation cost parameter \( \tau \) does not affect the equilibrium outcome when locations are chosen simultaneously. However, when upstream locations are chosen first, an increase in \( \tau \) increases the profit levels (\( d\Pi^V_X / d\tau > 0 \)) when \( \tau > t \). Additionally, the retail prices increase with \( \tau \) when \( \tau \) is high enough.\(^{14}\) In the limit case where \( \tau \) approaches zero, we obtain: \( \lim_{\tau \to 0^+} p^V_X = 1.5t \), \( \lim_{\tau \to 0^+} \Pi^V_X = 0.75t \). We also obtain \( da^V / d\tau < 0 \) for all \( t \) and \( dx^V / d\tau < 0 \) for \( \tau > t \). In the limit case where \( \tau \) goes to zero, we obtain: \( \lim_{\tau \to 0^+} a^V = 0.125 \), \( \lim_{\tau \to 0^+} x^V = -0.25 \).

**Welfare and social optimum.** Before we discuss in more detail the forces that shape the equilibrium locations, it is useful to briefly turn to the welfare properties of the equilibrium under VI. Final consumers have unit demands, thus, the social cost (\( SC \)) can be simply expressed as the transportation cost paid by the retailers plus the transportation cost paid by the final consumers:

\[
SC = \int_0^2 t(k-x)^2 dk + \int_1^1 t((1-y-k)^2 dk + \tau(x-a)^2 z + \tau(y-b)^2 (1-z). \tag{8}
\]

By substituting the equilibrium locations from Proposition 1 and 2, we calculate that the total social cost in the VI equilibrium with simultaneous and sequential location choices, respectively, is:

\[
SC^V = \frac{13}{48} t \quad \text{and} \quad SC^{VI} = \frac{t\left(17t\tau + 17\tau^2 + 27t^2 - 9\sqrt{\tau^2 + t^2}(3t - \tau)\right)}{96t(t + \tau)}.
\]

A social planner, in contrast, would minimize the total transportation cost (8). Standard calculations imply that the socially optimal locations are \( a = b = x = y = 0.25 \). These locations would eliminate the upstream transportation cost and minimize the downstream transportation cost. The optimum (minimum) social cost obtained is then \( SC = t/48 \). Comparing this benchmark to the VI outcome we find that VI leads to a socially inefficient choice of locations (see Figure 2). In particular, with simultaneous location choices firms locate at \(-0.25\) which clearly leads to higher social cost than the 0.25 location. When upstream locations are chosen first, equilibrium locations are closer to the socially optimal ones but there is a positive transportation cost at the upstream

\(^{14}\)The retail prices increase with \( \tau \), when \( \tau > \tau^* \), where \( \tau^* \) solves \( dp^V_X / d\tau = 0 \).
level. Direct calculations show that for all parameter values \( \overline{SC}^{VI} \geq SC^{VI} \), that is, under VI, social cost is always lower when upstream locations are chosen first. We summarize

**Proposition 3** We have \( -0.25 < x^{VI} = y^{VI} < a^{VI} = b^{VI} < 0.25 \). VI leads to total transportation cost that is too high relative to the social optimum and this cost is higher when location choices are simultaneous.

![Figure 2: Equilibrium locations under VI](image)

Equilibrium forces. Now let us discuss the forces behind the equilibrium locations in more detail. In models of horizontal differentiation with duopolists selling directly to final consumers, there are two opposite forces, one pushing firms close to each other (to directly obtain higher demand) and one in the opposite direction (to reduce the intensity of price competition). In our model, these forces are modified and a third force also emerges. Given the locations of the other three firms, if a downstream division moves closer to the center of the unit interval this affects its aggregate marginal cost. This effect may be working to bring firms either closer to each other or apart, depending on the locations of the upstream firms.

Specifically, in a standard linear city duopoly model, as in d’Aspremont *et al.* (1979), the marginal production cost is exogenous. Denoting this cost by \( c \), profits are \( \Pi_X = (p_X - c)z \) for firm X. By the envelope theorem, we obtain:

\[
\frac{d\Pi_X}{dx} = (p_X - c)\left( \frac{dz}{dx} \bigg|_{\text{D.E.}} + \frac{dz}{dp_X} \frac{dp_X}{dx} \bigg|_{\text{P.C.E.}} \right),
\]

evaluated at the equilibrium prices.\(^{15}\) The direct Demand Effect (D.E.) pushes firms close to each other while the indirect Price Competition Effect (P.C.E.) pushes firms in the opposite direction. In our model the marginal production cost for the retailers is endogenous and equal to the transportation cost, \( f_X = \tau(a - x)^2 \).\(^{16}\) The profit function of firm X is \( \Pi_X = (p_X - f_X)z \) and by the


\(^{16}\)Under vertical separation discussed subsequently in the paper, the marginal production cost for the retailers would be equal to the wholesale price plus the transportation cost, \( f_X = w_A + \tau(a - x)^2 \).
envelope theorem we now obtain:  

\[
d\Pi_X \over dx = (p_X - f_X) \left( \frac{dz}{dy} + \frac{dz}{dy} \frac{dy}{dx} + \frac{dz}{dy} \frac{dy}{dx} \frac{df_X}{dx} + \frac{dz}{dy} \frac{dy}{dx} \frac{df_Y}{dx} \right) - \frac{df_X}{dx} z. \tag{9}
\]

In general, a change in the location \( x \) affects the aggregate marginal costs of both retailers and the final prices directly and indirectly via the change in the marginal costs.\(^{18}\) The direct Demand Effect (D.E.) still corresponds to \( dz/dx \) but the indirect Price Competition Effect (P.C.E.) now corresponds to a more complex expression as final prices are affected by the endogenous marginal production costs of the retailers. A change in \( x \), leads to a change in the retailers’ transportation cost, therefore, final prices change too. Finally, profits are also affected directly by the change in the Marginal Cost itself (M.C.E.).

**Corollary 2** In addition to the demand effect and the price competition effect, the aggregate marginal cost effect plays a key role in shaping the retailer location incentives. In the VI equilibrium when the locations of the upstream divisions are chosen first, the retailer’s incentives to move closer to the unit interval are affected positively by the demand effect and the aggregate marginal cost effect but negatively by the price competition effect.

As the profit margin \( (p_X - f_X) \) and the demand \( z \) are positive, the derivatives in (9) determine the effect of \( x \) on the profit \( \Pi_X \). After calculating these derivatives and calculating at the equilibrium locations, we find: \( \text{D.E.}|_{a=v_l, b=bv_l, x=xv_l, y=yv_l} > 0 \), \( \text{P.C.E.}|_{a=v_l, b=bv_l, x=xv_l, y=yv_l} < 0 \) and \( \text{M.C.E.}|_{a=v_l, b=bv_l, x=xv_l, y=yv_l} < 0 \) for every \( \tau \) and \( t \).\(^{19}\) Given that firms are located at the equilibrium locations (see Proposition 2), whenever retailer X moves closer to the unit interval it increases directly the demand it obtains and it decreases its aggregate marginal cost but the competition with the rival retailer is intensified. Therefore, compared to the standard linear city model (where firms are located at \( -0.25 \)), our vertical structure model introduces a third force that pushes retailers, in equilibrium, towards the unit interval (and closer to their suppliers) to decrease their marginal costs. Note also that the demand and the price competition effects in our model are modified and do not coincide with the respective effects in the standard linear city model. In equilibrium, the upstream divisions are located closer to the unit interval compared to the downstream divisions.

The forces pushing firms (X relative to Y and A relative to B) close to each other are dominated by

---

\(^{17}\)We apply the envelope theorem at the second stage of the game where \( \Pi_X \) is a function of all four locations.

\(^{18}\)Under VI, a change in \( x \) does not affect \( f_Y \), but under vertical separation we will show that \( f_Y \) is affected via a change in \( w_B \).

\(^{19}\)We have

\[
d\Pi_X \over dx = (p_X - f_X) \left( \frac{dz}{dy} + \frac{dz}{dy} \frac{dy}{dx} + \frac{dz}{dy} \frac{dy}{dx} \frac{df_X}{dx} + \frac{dz}{dy} \frac{dy}{dx} \frac{df_Y}{dx} \right) - \frac{df_X}{dx} z,
\]

with \( \text{D.E.}|_{a=v_l, b=bv_l, x=xv_l, y=yv_l} = \frac{1}{2} > 0 \), \( \text{P.C.E.}|_{a=v_l, b=bv_l, x=xv_l, y=yv_l} = -\frac{t+\tau}{t+\tau+\sqrt{t^2+\tau^2}} < 0 \) and

\[
\text{M.C.E.}|_{a=v_l, b=bv_l, x=xv_l, y=yv_l} = -\frac{3\tau(t+\tau-\sqrt{t^2+\tau^2})}{4(t+\tau)} < 0.
\]
the forces pushing firms to the opposite direction and this total effect is stronger for the downstream divisions.

Commitment incentives. To better understand the location incentives of the firms, we now compare the VI cases under the alternative timing assumptions about the order of the location choices. We have shown that, for all parameter values, $-0.25 < x^{VI} < a^{VI}$ and $-0.25 < y^{VI} < b^{VI}$. In the simultaneous locations game the VI firms move farther away from the city center relative to the case where upstream locations are chosen first. In addition, we find that the joint profits are higher compared to the sequential locations choice game. Why then the VI firms do not also choose their upstream locations at $-0.25$ in the sequential locations choice game, to obtain higher profits? Each VI firm has a unilateral incentive to deviate from $-0.25$ and to move its upstream division closer to the center given that its rival upstream division is at $-0.25$ and that downstream locations will follow equation (7). Each VI does this in an effort to obtain a strategic advantage: by moving its upstream division closer to the center, it makes it easier for the downstream division to be located closer to the center as well, thus creating a force that pushes the rival downstream division further away from the center than otherwise: $dx/db < 0$ by direct differentiation of (7). In such a way, a given chain unilaterally seeks to strengthen its position in the market. However, by acting like this, both chains become more competitive and both obtain lower equilibrium profits (something like a Prisoners’ Dilemma situation in locations when wholesalers choose locations first). Thus, we obtain the result that the vertically integrated firms obtain higher equilibrium profit when they set their locations simultaneously compared to the sequential game where upstream locations are chosen first.

**Corollary 3** When wholesalers choose their locations first, they locate closer to the center in an effort to also pull their retailers closer to the center, but they end up with lower equilibrium profits than in the simultaneous locations game.

Thus, we find that firms would prefer to be in a game where they set their locations simultaneously compared to the sequential game where upstream divisions move first. When upstream divisions set their locations first, they can unilaterally affect the downstream locations and this leads to a stronger competition between them. Likewise, if the location choices were reversed and downstream divisions chose their locations first, the equilibrium locations would coincide with the simultaneous equilibrium locations.\(^\text{20}\) If retailers are already located at $-0.25$, the suppliers have no other choice but to locate on their retailers’ locations to maximize their profits. In summary, all four firms prefer the downstream divisions’ locations to be chosen first (or that all locations are simultaneous) so that the upstream divisions locate at $-0.25$ and avoiding locations closer to the center.

This result is reminiscent of other results in the sequential choices and strategic commitment literature. In particular, Fershtman and Judd (1987), Bonanno and Vickers (1988), Brander and

\(^{20}\)For the details see Kourandi and Vettas, 2010.
Spencer (1985), and others have compared contracting and pricing incentives in structures with a vertical dimension. Owners reward managers for revenue or market shares, upstream firms choose low wholesale prices, governments choose export subsidies, all in an effort to unilaterally strengthen the strategic position of their downstream agents (technically, to shift their reaction functions to a more aggressive position). Here, we identify how this incentive for strategic commitment to aggressive behavior affects upstream and downstream equilibrium locations (and prices) along the real line.

4 Linear wholesale prices

Thus far we have examined the benchmark case of vertical integration. Now we turn to vertical separation (VS): all firms, upstream and downstream are independent and seek to maximize their own profit. The downstream firms pay linear wholesale prices to the upstream firms for each unit of input they purchase.\textsuperscript{21} Thus, the marginal production cost for retailer X is now equal to the wholesale price plus the transportation cost, \( f_X = w_A + \tau(a - x)^2 \). We proceed like before, by analyzing first the simultaneous location choices case and then the case where the upstream locations are chosen first. Finally, we discuss the equilibrium properties under vertical separation and compare to the vertical integration case.

4.1 Simultaneous location choices

In this game, all four firms make their location choices simultaneously in the first stage, while in the second stage the wholesale prices are determined by the upstream firms, followed by the final prices. Then, the final consumers decide which retailer to buy from. We proceed backwards to solve for the subgame perfect equilibrium.

Consumers' choices and retail prices. The last stage of this game is the same as in the VI model and the retail prices for firm X are given by equation (4) with the only difference that the aggregate marginal cost now also includes the wholesale prices.

Wholesale prices. In this stage, the two upstream firms compete by setting their wholesale prices. Each upstream firm exclusively supplies its retailer: firm A supplies firm X and firm B supplies firm Y. Even though each upstream firm cannot supply the rival retailer, competition takes place indirectly: the wholesale prices shape competition in the subsequent stage where retail prices and final demand is determined. If the wholesale price charged to retailer X by the upstream firm A is high, retailer X has a relative cost disadvantage compared with the rival retailer and, thus, the demand obtained by firm X (and, in turn, by firm A) is low.

Since a downstream firm turns one unit of the good it purchases in the wholesale market into one

\textsuperscript{21}Linear pricing is a natural starting point for our problem as it is widely used in practice and makes our results easily comparable to the standard one-level location models. We turn to two-part wholesale tariffs in Section 5 and compare the two cases.
unit it sells in the retail market, the demand function of firms A and B are \( D_A = z \) and \( D_B = 1 - z \) and their profit functions are \( \Pi_A = w_A z \) and \( \Pi_B = w_B (1 - z) \). Assuming equilibrium will follow when setting retail prices, the profit function for firm A can be expressed as:

\[
\Pi_A = \begin{cases} 
  w_A \left( \frac{\tau((y-b)^2-(x-a)^2)}{6t(1-x-y)} + 2(1-x-y)(y-x)+w_B-w_A \right) & \text{if } w_A \leq w_B + C \\
  0 & \text{if } w_B + C < w_A < w_B + D \\
  w_A \left( \frac{1+x-y}{2} \right) & \text{if } w_A \geq w_B + D, 
\end{cases}
\]  

(10)

where

\[
C = \tau((y-b)^2-(x-a)^2) + t(x-y-3)(1-x-y), \ D = \tau((y-b)^2-(x-a)^2) + t(x-y+3)(1-x-y).
\]

The profits are functions of the wholesale prices and all four locations. Note that corners may arise. For example, when the aggregate marginal cost \( w_A + \tau(a-x)^2 \) faced by firm X is low enough compared to its rival, firm A captures the whole demand via its retailer X (\( z = 1 \)). Here, we further stress the role that upstream and downstream locations play in our model since they crucially affect the marginal costs of the retailers. For some locations, even if an upstream firm charges a zero wholesale price to its retailer, it cannot obtain positive demand and the aggregate marginal cost of its retailer is always higher than the rival’s (this was also the case under VI for some parameter values and exogenous locations). This happens when the distance of one retailer from its supplier is very large compared to the rival’s distance from its own supplier. Clearly, these cases will tend to emerge when the transportation cost parameter \( \tau \) is relatively high.

Maximization of the upstream profit functions (see (10) and analogously for B) with respect to the wholesale prices gives:

**Lemma 3** The equilibrium wholesale price for firm A (as function of the upstream and downstream locations) is:

\[
w_A = \begin{cases} 
  \tau \left( \frac{(y-b)^2-(x-a)^2}{3} \right) + \frac{t(x-y+9)(1-x-y)}{3} & \text{if } 6t(1-x-y) - C \leq 0 \\
  \frac{\tau((y-b)^2-(x-a)^2) + t(x-y+9)(1-x-y)}{3} & \text{if } 6t(1-x-y) - C > 0 \\
  0 & \text{if } 6t(1-x-y) + D > 0 \\
\end{cases}
\]  

(11)

In equilibrium, when the upstream locations are asymmetric enough relative to the downstream locations, the disadvantaged upstream firm sets a zero price and there are no sales for itself (and its retailer), while the rival upstream firm sets the highest price that allows it to capture the whole demand. Otherwise, the market is shared (upstream and downstream). In either case, the

---

22 To illustrate, take for example \( t = \tau = 1, a = b = 0.2, x = 0.06, y = 0.9 \). Then even if \( w_B = 0 \) firm A captures the whole demand.

23 The profit functions are quasi-concave. When both firms have positive demand, from the first order conditions we obtain a unique critical point. At this point, the second order conditions are satisfied, thus, the local critical point is the total max.
equilibrium upstream profit for firm A can be obtained by substituting the equilibrium wholesale prices into expression (10).

Location choices. In the first stage, all four firms make their location choices simultaneously, each seeking to maximize its own profits. By substituting the wholesale and final prices that we have obtained in our proceeding analysis into the profits of the upstream and downstream firms, we obtain the profit functions depending on the four location choices. Since all firms avoid receiving zero demand, we find that:

\[
\begin{align*}
\Pi_X &= \frac{(\tau ((y - b)^2 - (x - a)^2) + t (x - y + 9) (1 - x - y))^2}{162t(1 - x - y)}, \\
\Pi_Y &= \frac{(\tau ((x - a)^2 - (y - b)^2) + t (y - x + 9) (1 - x - y))^2}{162t(1 - x - y)}, \\
\Pi_A &= 3\Pi_X, \quad \Pi_B = 3\Pi_Y.
\end{align*}
\]

From the first order conditions of the upstream firms, we find that each wholesaler chooses to have the same location as the corresponding retailer, \(a = x\) and \(b = y\). Further, from the first order conditions of the retailers we obtain: \(a = x = b = y = -1.75\). Thus, we have:

**Proposition 4** With simultaneous locations choices, the unique subgame perfect equilibrium outcome under vertical separation with linear wholesale prices is:

\[
\begin{align*}
\hat{a}^L &= \hat{b}^L = \hat{x}^L = \hat{y}^L = -1.75, \\
\hat{w}_A^L &= \hat{w}_B^L = 13.5t, \quad \hat{p}_X^L = \hat{p}_Y^L = 18t, \\
\hat{z}^L &= 0.5, \\
\hat{\Pi}_A^L &= \hat{\Pi}_B^L = 6.75t, \quad \hat{\Pi}_X^L = \hat{\Pi}_Y^L = 2.25t.
\end{align*}
\]

As in our VI model with simultaneous location choices, all firms are located outside the unit interval, however now at \(-1.75\). The equilibrium locations are independent of the parameter \(\tau\), as downstream firms are located at the same point of their suppliers, which means that they pay zero transportation costs in equilibrium. Equilibrium locations are also independent of the parameter \(t\); a result similar to the standard linear city model where firms are located in equilibrium at \(-0.25\) for all (quadratic) transportation cost parameters. Compared to that model, firms in our model are located farther away \((-1.75 < -0.25\)). The introduction of the wholesalers in the problem, which makes the production costs for the retailers endogenous, pushes the locations farther away. The forces that push the firms closer to each other are dominated by the forces that push them to the
opposite direction and the latter are stronger in a model with endogenous production costs.\textsuperscript{24,25}

In addition, by substituting the locations from Proposition 4 into expression (8), we calculate that in equilibrium the total social cost is $\overline{SC^L} = 193/48t$.

4.2 Upstream locations first

In this modification of the model, the upstream firms choose their locations first, followed by the downstream locations. As before, the wholesale prices are then chosen by the wholesalers followed by the retail prices and then finally the consumers decide which retailer to buy from.

Consumers’ choices, retail and wholesale prices. These stages of the game remain the same as in the simultaneous location model. The retail prices for firm X are given again by equation (4), while the wholesale prices for firm A are given by equation (11).

Downstream locations. Taking as given the locations of the upstream firms and assuming that the retail and wholesale prices will be subsequently chosen in equilibrium, the downstream firms simultaneously choose their locations to maximize their profits. The profit function for firm X is:

$$
\Pi_X = \begin{cases} 
2t(1 - x - y) & \text{if } 6t(1 - x - y) - C \leq 0 \\
\frac{\tau((y-b)^2-(x-a)^2)+t(y-x+9)(1-x-y))}{16t(1-x-y)} & \text{if } 6t(1 - x - y) - C > 0 \\
0 & \text{if } 6t(1 - x - y) + D > 0 \\
6t(1 - x - y) + D \leq 0.
\end{cases}
$$

Like in the previous steps of our analysis, special care should be taken about the corner cases. If upstream firms are located asymmetrically enough, this may favor a downstream firm and this chain may take the whole demand. From the profit function above, we calculate that for \( \tau \left( (x-a)^2-(y-b)^2 \right) + t \left( y-x+9 \right) \left( 1-x-y \right) \leq 0 \) firm X serves the whole demand. When firm B is located far away from the unit interval, its retailer Y has to decide where to locate in order not to pay a high transportation cost but also to have some positive demand. If Y locates exactly at its supplier’s location, it minimizes its own transportation cost but it is then located far away from the consumers. In contrast, when it locates in or near the unit interval it is close to the final consumers but faces a high transportation cost. In equilibrium, Y chooses a location \( y \) that maximizes the expression \( \tau \left( (x-a)^2-(y-b)^2 \right) + t \left( y-x+9 \right) \left( 1-x-y \right) \), that is, it minimizes the area where its demand is zero. By direct calculations it follows that \( y = (b\tau - 4t)/(\tau + t) \). Given this location \( y \), firm X maximizes its own profits. This argument holds symmetrically when firm A is located far

\textsuperscript{24}Brekke and Straume (2004) allow wholesale prices to be determined through bargaining between the upstream and downstream firms but they do not study the location choices of the upstream firms. In contrast, we assume that wholesale prices are set by the upstream firms and that their locations are endogenous. When the bargaining power of the upstream firms equals one, and there is no product differentiation at the upstream level \( (\tau = 0) \) the two models deliver the same results, firms locate at \(-1.75\).

\textsuperscript{25}Matsushima (2004) solves the simultaneous restricted locations choice model so that upstream firms price discriminate among downstream firms. He finds that for some parameters values downstream firms are located closer to the center of the unit interval compared to the upstream firms in an effort to reduce the wholesale prices they pay. In our model, there is no such incentive since each retailer has its own supplier and no price discrimination takes place.
away from the unit interval and firm X has a cost disadvantage. Of course, when neither firm has a
high cost disadvantage in equilibrium the firms share the demand. From this analysis we obtain:26

Lemma 4 The equilibrium downstream location for firm X (as function of the upstream locations)
is:

\[
x = \begin{cases} 
\frac{\alpha r + 5t + \sqrt{t(r(a-b)-9)(1-a-b)}}{r + t} & \text{if } 81t + 4r(b - a + 9)(1 - a - b) \leq 0 \\
\frac{16\alpha r^2 (1-a-b) + 4t (17a + 6b - a^2 - b^2 - 9r) - 63r^2}{4(r + t)(4r(1-a-b) + 9t)} & \text{if } 81t + 4r(b - a + 9)(1 - a - b) > 0 \\
\frac{\alpha r - 4t}{t + r} & \text{if } 81t - 4r(b - a - 9)(1 - a - b) \leq 0.
\end{cases}
\]  

(13)

The second order conditions of the downstream firms are satisfied at the equilibrium locations, since the profit
margins are positive.

The equilibrium downstream profits for firm X in this stage can be obtained by substituting the
equilibrium downstream locations into the profits (12).

Upstream locations. Now, upstream firms choose their locations each to maximize its profit. An
upstream firm will never choose a location that will make its own retailer face so high an aggregate
marginal cost that it would imply zero demand in the subsequent stage. Obviously, this is because
this would also lead to zero demand and profits for this upstream firm. Therefore, in equilibrium,
both upstream firms avoid receiving zero demand and they share the market. The relevant profit
function for firm A reduces to:

\[
\Pi_A = \frac{t (9t + 2\tau (1 - a - b))(81t + 4r(a - b + 9)(1 - a - b))^2}{108(t + t)(9t + 4r(1 - a - b))^2}.
\]

From the first order conditions, we obtain the equilibrium locations, \( a = b = (9t - 5\tau - 9\sqrt{\tau^2 + t^2})/8\tau \).
Note that \( a < 0 \) and \( b < 0 \), thus, upstream firms are located outside the unit interval for all values
of the parameters \( t \) and \( \tau \). We obtain:

Proposition 5 With upstream locations chosen first, the unique subgame perfect equilibrium out-
come under vertical separation with linear wholesale pricing is:

\[
\begin{align*}
&\alpha^L = \beta^L = \frac{9t - 5t - 9\sqrt{\tau^2 + t^2}}{8\tau}, \quad x^L = y^L = \frac{-(5\tau + t) + 9\sqrt{\tau^2 + t^2}}{8(t + t)}, \\
w_A^L = w_B^L = \frac{27t(t + \tau + \sqrt{\tau^2 + t^2})}{4(t + t)}, \\
p_X^L = p_Y^L = \frac{9t (t^2 (41\tau + 9t) + \tau^2 (32\tau + 73t) + \sqrt{\tau^2 + t^2} (32\tau - 9t)(\tau + t))}{32\tau (\tau + t)^2}, \\
z^L = 0.5, \\
\Pi_A^L = \Pi_B^L = \frac{27t(t + \tau + \sqrt{\tau^2 + t^2})}{8(t + t)}, \quad \Pi_X^L = \Pi_Y^L = \frac{9t (t + \tau + \sqrt{\tau^2 + t^2})}{8(t + t)}.
\end{align*}
\]

26The second order conditions of the downstream firms are satisfied at the equilibrium locations, since the profit margins are positive.
We note that, in equilibrium, the two downstream firms and the corresponding two upstream firms share the market equally, all four firms obtain positive profits and all locations are outside the unit interval. Further, upstream firms locate closer to each other (and to the market center) relative to the downstream firms ($a > x$). By substituting the locations from Proposition 5 into 8, we calculate that in equilibrium the total social cost is 

$$SC^L = \frac{18t(7t(\tau + t) + 9t^2 + \sqrt{t^2 + t^2(7(\tau - 9t))})}{64(\tau + t)} + \frac{t}{12} > 0.$$ 

4.3 Equilibrium properties and comparison to vertical integration

We discuss now the equilibrium properties under VS. We start by comparing our findings to the model without vertical structure. Second, we discuss the different forces that shape the equilibrium under VS. Third, we compare sequential to simultaneous location choices. Finally, we compare VI to VS.

*No vertical effects.* We contrast our results to the standard d’Aspremont *et al.* (1979) model, that is, the case where there is no vertical structure. As already discussed with only one stage of competition, in equilibrium, firms would locate at distance $-0.25$ from each endpoint and prices would be equal to $3t/2$. By direct comparison to Proposition 5, we find that:

**Corollary 4** In our vertical separation model with upstream locations chosen first, final prices and chain profits are higher than in the d’Aspremont *et al.* (1979) model. Downstream locations are farther away from the unit interval compared to the upstream locations and the latter are farther away compared to the d’Aspremont *et al.* (1979) model, $x^L = y^L < a^L = b^L < -0.25$.

The introduction of wholesalers in the model increases the final prices since each retailer’s marginal cost is increased by the wholesale price. Importantly, the aggregate chain profits are also higher in our vertical structure compared to the d’Aspremont *et al.* (1979) model. This result is related to other results under "vertical separation" such as Bonanno and Vickers (1988): upstream firms charge positive wholesale prices and in equilibrium overall profit increases. The distinguishing feature of our analysis is that we also have endogenous locations. As each vertical chain competes with the rival chain at both the upstream and downstream levels, competition is intensified and firms move farther away from the center compared to the case where there is only one level of competition.

**Equilibrium forces.** Similarly to the VI case, here there are again three forces that shape the location incentives: the demand effect, the price competition effect and the marginal cost effect. However, now the marginal cost also includes the wholesale prices, thus, a change in $x$, leads to a change in the wholesale prices and the retailers’ transportation cost, therefore, final prices change too.

**Corollary 5** In the vertical separation equilibrium with the upstream locations chosen first, the retailer’s incentives to move closer to the unit interval are affected positively by the demand effect and by the aggregate marginal cost effect but negatively by the price competition effect.
As in the VI case, the profit margin \((p_X - f_X)\) and the demand \(z\) are positive, thus, the derivatives in (9) determine the effect of \(x\) on the profit \(\Pi_X\). After calculating these derivatives and, then, by evaluating at the equilibrium locations, we find:

\[
\text{D.E.} = \left. \frac{\partial \Pi_X}{\partial x} \right|_{a=a^L, b=b^L, x=x^L, y=y^L} > 0, \\
\text{P.C.E.} = \left. \frac{\partial \Pi_X}{\partial z} \right|_{a=a^L, b=b^L, x=x^L, y=y^L} < 0 \quad \text{and} \\
\text{M.C.E.} = \left. \frac{\partial \Pi_X}{\partial t} \right|_{a=a^L, b=b^L, x=x^L, y=y^L} < 0 
\]

for every \(\tau\) and \(t\).

The forces pushing firms (X relative to Y and A relative to B) close to each other are dominated by the forces pushing firms to the opposite direction and this total effect is stronger for the downstream firms.

**Commitment incentives.** It is important to compare the equilibrium locations in our simultaneous locations game (Proposition 4) with the sequential locations game (Proposition 5). We find that \(x^L < a^L < 0\) and \(y^L < b^L < 0\), which means that all firms are located outside the unit interval with the upstream firms to be closer to the center. We can further calculate that for all parameter values \(-1.75 < x^L < a^L < 0\) and \(-1.75 < y^L < b^L < 0\). Thus, we find that in the simultaneous locations game firms move farther away from the city center than when upstream locations are chosen first (see Figure 3).

![Figure 3: Equilibrium locations under VS](image)

In addition, profits for both upstream and downstream firms and wholesale and final prices are higher compared to the sequential location choice game. The logic is the same as in the VI case: when wholesalers choose their locations first, they locate closer to the center in an effort to also pull their retailers closer to the center, but they end up with lower equilibrium profits than in the simultaneous locations game. Thus, we find again that firms would prefer to be in a game where they set their locations simultaneously compared to the sequential game where upstream firms move first.

**Vertical integration vs vertical separation.** It is important to compare the equilibrium outcome under vertical integration (Proposition 1 and 2) to vertical separation (Proposition 4 and 5). By

\[
\text{D.E.} \big|_{a=a^L, b=b^L, x=x^L, y=y^L} = \frac{1}{2} > 0, \\
\text{P.C.E.} \big|_{a=a^L, b=b^L, x=x^L, y=y^L} = -\frac{4(\tau+\tau t)}{3(\tau+\sqrt{\tau^2+t^2})} < 0 \\
\text{M.C.E.} \big|_{a=a^L, b=b^L, x=x^L, y=y^L} = -\frac{3(5(\tau+\tau t)-3\sqrt{\tau^2+t^2})}{4(\tau+\tau t)} < 0.
\]

In Appendix A we present two extensions of our basic model. First, we study the model where firms are not allowed to locate outside the unit interval and we find that firms are located at the two opposite endpoints (as firms locate always outside the unit interval under unrestricted locations). Second, we allow retailers to possibly locate at the opposite side of their wholesalers on the line. We find that for some upstream locations and parameter values it could also be an equilibrium in the second stage of the game that the retailers locate at the opposite side of the line relative to the corresponding wholesaler. We also show that the second stage equilibrium profit is lower in this arrangement. Still, we do not find an equilibrium of the entire game when all locations (upstream and downstream) are endogenous and are chosen so that wholesalers’ and retailers’ locations are chosen at the opposite side of one another.
direct calculations, we obtain:

**Proposition 6**

i) We have 
\[ 1.75 < x^L = y^L < a^L = b^L < -0.25 < x^{VI} = y^{VI} < a^{VI} = b^{VI} < 0.25. \]

ii) For any pair of symmetric locations, final prices under vertical separation are always higher than under vertical integration, 
\[ p_X^L (a = b, \ x = y) = p_Y^L (a = b, \ x = y) > p_X^{VI} (a = b, \ x = y) = p_Y^{VI} (a = b, \ x = y). \]

iii) The social cost under vertical separation is higher than under vertical integration, 
\[ SC^L > SC^{VI}. \]

![Figure 4: Locations under vertical separation and vertical integration](image)

Under vertical separation, downstream firms locate farther away relative to the upstream firms 
\( (x^L = y^L < a^L = b^L) \) and the latter are located farther away relative to the simultaneous locations choices under vertical integration \( (a^L = b^L < -0.25) \). Moreover, under vertical integration when upstream locations are chosen first, the downstream and upstream divisions are closer to the center relative to the corresponding vertical separation case \( (x^L = y^L < a^L = b^L < -0.25 < x^{VI} = y^{VI} < a^{VI} = b^{VI}) \) with the upstream firms located even closer to the center than the downstream firm. However, product differentiation under vertical integration is greater than the social optimum \( (x^{VI} = y^{VI} < a^{VI} = b^{VI} < 0.25) \).

Under vertical integration, a change in the location of firm X still affects the marginal cost \( (f_X) \) that it pays since it affects its transportation cost but, in contrast to our vertical separation model, there is no effect through the wholesale prices. The equilibrium locations under vertical integration are closer to the center, thus, price competition is less intense when there is no competition in wholesale prices.\(^{29}\)

Now, let us turn to the final prices. We find that, in equilibrium, the final prices under vertical separation are higher than under vertical integration \( (p_X^L = p_Y^L > p_X^{VI} = p_Y^{VI}) \). This result did not have to necessarily hold since in our model we have duopoly competition in each stage. We find that each upstream firm raises its price to a level that makes its retailer charge a higher final price than the one we would see in equilibrium under vertical integration. Only part of the negative effect of an increased wholesale price to its retailer is taken into account by the wholesaler, while the remainder of this effect emerges as an externality.

We can obtain some additional insights into the problem by comparing for any symmetric pair of upstream and downstream locations \( (a = b, \ x = y) \) the final prices under vertical separation and

\[^{29}\text{The negative effect of an increase in } x \text{ in the final demand via the Price Competition effect is less important, in equilibrium, under VI relative to the VS: } P.C.E.|_{a=b^L, b=b^L, x=x^L, y=y^L} > P.C.E.|_{a=a^L, b=b^L, x=x^L, y=y^L}.\]
under vertical integration. Let us fix the upstream and downstream locations at any arbitrary level (even when these are not equilibrium locations). We find that double marginalization emerges under vertical separation, with positive profit margins at both the upstream and downstream levels and higher chain profits compared to the vertical integration case. The idea here is similar to other models of vertical relations with price competition in the final market (see e.g. Bonanno and Vickers, 1988, and Rey and Stiglitz, 1995). When the retailers’ choices are strategic complements (as final prices are in our model) vertical separation tends to imply that wholesale prices will be set at levels that imply higher equilibrium final prices than under vertical integration. Of course, the important difference in our model is that product differentiation is endogenous and that location and pricing incentives are interrelated.

We can also compare the social costs and obtain that the social cost under vertical separation is higher than under vertical integration. Under vertical integration, the equilibrium locations are closer to the social optimum relative to the equilibrium locations under vertical separation, thus, social cost is higher under vertical separation in equilibrium.

5 Two-part tariffs

We now examine our problem under two-part tariffs. In addition to a per unit wholesale price that retailers pay when supplied by their suppliers, there can also be a fixed fee in the trade within each vertical chain ($F_A$ and $F_B$). Such pricing allows profit to be transferred upstream without affecting the marginal cost of the downstream firms through wholesale prices. We will show that this case is in between the linear pricing and the VI case, not only in terms of final prices and industry profits but also in terms of locations. However to analyze this case in a meaningful way we have to modify our model and relax the assumption that the upstream firms have all the pricing bargaining power. This is because then the upstream firms would always capture the entire chain profit leaving zero profits to the retailers and making them indifferent as to which location to choose earlier in the game. Therefore, we now assume that the upstream firm in each vertical chain bargains with the downstream firm over the two-part tariff terms in the third stage of the game. Specifically, we denote by $\theta$ the bargaining power of each upstream firm, with $\theta \in (0,1)$. As before, we start by considering the case where locations are chosen simultaneously. Then we study the case where upstream firms choose locations first, and finally we compare the equilibrium of the game to the linear pricing and to the vertical integration cases.

5.1 Simultaneous location choices

The timing of the game is the same as in our model with linear wholesale prices and simultaneous location choices with the only difference that firms in the second stage of the game bargain over the terms of the two-part tariff. We proceed backwards to find the subgame perfect equilibrium. The retail prices are given again by expression (4) but now the profits of the retailers are reduced
by the fixed fees, $\Pi_X - F_A$ and $\Pi_Y - F_B$ respectively (note that these fees may be negative).

In the second stage of the game, each vertical chain maximizes the Nash product with respect to the wholesale prices and the fixed fees $(w_i, F_i)$, $i = A, B$. The vertical chain $A-X$ solves:

$$\max_{w_A, F_A} (\Pi_A + F_A)^{\theta} (\Pi_X - F_A)^{1-\theta} = (w_A z + F_A)^{\theta} ((p_X - w_A - \tau(x-a)^2)z - F_A)^{1-\theta}.$$ 

Note that as the two chains bargain simultaneously, we are looking for a Nash equilibrium in Nash bargains. From the first order condition with respect to the fixed fee, we obtain $F_A = \theta (\Pi_A + \Pi_X) - \Pi_A$ and that firms in the vertical chain maximize their joint profits $\Pi_A + \Pi_X = (p_X - \tau(x-a)^2)z$ with respect to the wholesale price $w_A$ and share these profits via the fixed fees according to their bargaining power $\theta$ and $1 - \theta$.\(^{30}\) The equilibrium wholesale prices $w_A$ and $w_B$ are functions of all four locations and the profits are split according to the rule: $\Pi_A + F_A = \theta (\Pi_A + \Pi_X), \Pi_X - F_A = (1 - \theta) (\Pi_A + \Pi_X)$.

In the first stage, firms choose their upstream and downstream locations simultaneously. The four firms simultaneously seek to each maximize its profits with respect to its location. Since upstream and downstream firms share their joint profits according to their bargaining power, each chain maximizes its joint profits simultaneously with respect to the upstream and downstream locations. We find that each wholesaler chooses to have the same location as the corresponding retailer: $a = x$ and $b = y$ and further, from the first order conditions of the retailers we obtain: $a = x = b = y = -0.75$. Thus, we have:

**Proposition 7** With simultaneous locations choices, the unique subgame perfect equilibrium outcome under two-part tariffs is:

$$\hat{a}^T = \hat{b}^T = \hat{x}^T = \hat{y}^T = -0.75,$$

$$\hat{w}_A^T = \hat{w}_B^T = 2.5t, \hat{F}_A^T = \hat{F}_B^T = 1.25t (2\theta - 1),$$

$$\hat{p}_X^T = \hat{p}_Y^T = 5t, \hat{z}^T = 0.5,$$

$$\hat{\Pi}_A^T + \hat{\Pi}_X^T = \hat{\Pi}_B^T + \hat{\Pi}_Y^T = 2.5t.$$ 

All four firms locate outside the unit interval, now at $-0.75$. The fixed fees are positive when $\theta > 1/2$, otherwise for $\theta < 1/2$ the fixed fees become negative and the upstream firms should pay these fees to the retailers to carry their products. By substituting the equilibrium locations under two-part tariffs in the social cost function (8), we calculate that, in equilibrium, the total social cost is $\hat{SC}^T = 49/48t$.

\(^{30}\)For a proof see Milliou et al. (2003).
5.2 Upstream locations first

Now we allow the upstream firms to move first. The final prices and the two-part tariffs are determined as in the previous subsection. However, in the second stage, taking as given the locations of the upstream firms and assuming that the contract terms and the retail prices will be subsequently set in equilibrium, the downstream firms simultaneously choose their locations to maximize their profits, \( \Pi_X - F_A \) and \( \Pi_Y - F_B \). Following the same logic as under linear wholesale prices, we obtain the equilibrium downstream locations (as functions of the upstream locations). Proceeding to the first stage, upstream locations are chosen to maximize upstream profits. Of course, an upstream location will not be chosen so that the corresponding retailer obtains zero demand. Therefore, in equilibrium both vertical chains avoid receiving zero demand. We obtain:

**Proposition 8** With upstream locations chosen first, the unique subgame perfect equilibrium outcome under two-part tariffs is:

\[
\begin{align*}
a^T &= b^T = \frac{5t - \tau - 5\sqrt{\tau^2 + t^2}}{8\tau}, \quad x^T = y^T = \frac{-t + \tau + 5\sqrt{\tau^2 + t^2}}{8(\tau + t)}, \\
w_A^T &= w_B^T = \frac{5t \left( \tau + t + \sqrt{\tau^2 + t^2} \right)}{4(\tau + t)}, \quad F_A^T = F_B^T = \frac{(2\theta - 1)5t \left( \tau + t + \sqrt{\tau^2 + t^2} \right)}{8(\tau + t)}, \\
p_X^T &= p_Y^T = \frac{5t \left( \sqrt{\tau^2 + t^2} (16\tau - 5t)(\tau + t) + t^2 (21\tau + 5t) + \tau^2 (16\tau + 37t) \right)}{32\tau(\tau + t)^2}, \\
z^T &= 0.5, \quad \Pi^T_A + \Pi^T_X = \Pi^T_B + \Pi^T_Y = \frac{5t \left( \tau + t + \sqrt{\tau^2 + t^2} \right)}{4(\tau + t)}.
\end{align*}
\]

In equilibrium, the profits that are split according to the firms’ bargaining power. As in the sequential location model, the fixed fees are positive when \( \theta > 1/2 \) and otherwise negative. Further, all locations are outside the unit interval and upstream firms locate closer to each other (and to the market center) relative to the downstream firms (\( 0 > a^T > x^T \)). Note also that the wholesale prices are still positive, and in this sense, "double marginalization" again exists.

By substituting the equilibrium locations under two-part tariffs in the social cost function (8), we calculate that, in equilibrium, the total social cost is

\[
SC^T = \frac{t(53\tau(\tau + t) + 15(3\tau - 5t)\sqrt{\tau^2 + t^2})}{96\tau(\tau + t)}.
\]

5.3 Linear wholesale prices vs two-part tariffs

First, we compare the two-part tariff cases under our alternative timing assumptions about the order of the location choices. Then we compare the two-part tariff pricing case to the linear pricing case and to the vertical integration case. We calculate that, for all parameter values, \(-0.75 < x^T < a^T \) and \(-0.75 < y^T < b^T \). In the simultaneous locations game the firms move farther away from the city center relative to the case where upstream locations are chosen first. In addition, profits are higher compared to the sequential locations choice game, as well as the wholesale and retail prices. Again,
the argument is similar to that under VI and linear wholesale pricing concerning the incentives of
the upstream firms to offer a stronger strategic commitment to their downstream firms.

Now we compare the equilibrium outcome under two-part tariffs (Proposition 8) to the equilib-
rium outcome under linear pricing (Proposition 5) and to the vertical integration (Proposition 2)
case. By direct calculations, we obtain:

**Proposition 9.** We have i) \( a^L = b^L < a^T = b^T < a^{VI} = b^{VI} \), \( x^L = y^L < x^T = y^T < x^{VI} = y^{VI} \) with \( a^i = b^i > x^i = y^i \)
for \( i = L, T, VI \). ii) \( w^L_A = w^L_B > w^T_A = w^T_B > 0 \) and \( p^L_X = p^L_Y > p^T_X = p^T_Y > p^{VI}_X = p^{VI}_Y \) and iii) \( SC^L > SC^T > SC^{VI} \).

Under two-part tariffs, upstream and downstream firms locate between the locations chosen
under linear pricing and vertical integration. Competition under two-part tariffs is more intense
compared to the linear pricing but less intense compared to the vertical integration case. This is
also reflected on the wholesale and retail prices.

Under two-part tariffs, firms can also use fixed fees to capture profits. The presence of this
second instrument allows the wholesale price to play a stronger strategic role. As a result with
two-part tariffs price competition becomes more intense with lower wholesale and retail prices
compared to the linear pricing game. At the same time, these values are higher compared to the
vertical integration case. Considering also how equilibrium locations change, total chain profits
under linear pricing exceed the chain profits under two-part tariffs and the latter exceed the profits
under vertical integration: \( \Pi^L_A + \Pi^L_X > \Pi^T_A + \Pi^T_X > \Pi^T_B + \Pi^T_Y > \Pi^{VI}_X + \Pi^{VI}_Y \).

Finally, under vertical integration, the equilibrium locations are closer to the social optimum
relative to the equilibrium locations under vertical separation both under two-part tariffs and linear
pricing, thus, social cost is lower under vertical integration in equilibrium.

**6 Conclusion**

Our analysis contributes to two literatures, on horizontal differentiation and on vertical contracting.
We have studied a linear city model with duopoly upstream and downstream. Wholesalers and
then retailers choose their locations and then their prices, before consumers make their choices. We
are able to derive in closed form a unique subgame perfect equilibrium of this five stage game and
examine its properties. We find that wholesalers choose to become less differentiated (that is, locate
closer to the unit interval) than the retailers and that differentiation (in upstream and downstream
locations) is greater compared to the vertical integration benchmark, which in turn is greater than
in the social optimum. We also find positive profit margins both upstream and downstream and that
chain profits and final prices are higher than under vertical integration (“double marginalization”).
Thus, vertical separation in our duopoly implies both a higher social cost (of transportation) and
higher final prices for the consumers. Our study of two-part tariffs (with upstream-downstream
bargaining) shows that in this case industry profits, prices, and locations are in between the cases of
linear pricing and vertical integration.
Considering a number of extensions offers additional insights into the problem. We modify the order of location choices and find that when locations are chosen simultaneously by all four firms, the upstream and the downstream firms in each pair choose the same location. This location is also farther away from the city center relative to the case when upstream locations are chosen first: in that case, the wholesalers locate closer to the city center with the goal to also pull their retailers closer to the center, so that they can each strategically strengthen their retailer’s position in the downstream market. Still, since this happens for both vertical chains, the end result is a more competitive market (than when all locations are chosen simultaneously) and profits are lower.

While the results presented here take a relatively simple and clear form, the equilibrium calculations are quite involved, as one should expect from a five stage game, especially when care has to be taken for possible corner solutions. Yet, a number of extensions and modifications appear promising. In companion work we consider the role of price discrimination at the wholesale level (upstream competition is intensified as firms compete for the demand of each retailer separately) and non exclusive vertical relations (due to the discontinuity of the profit functions under some parameter values the upstream firms always undercut their rival and no equilibrium in pure strategies exist in the wholesale prices). Additional modifications of the product differentiation structure, pricing and contracting and the timing of the game may offer additional important insights, and so would models where consumers may directly care also about the upstream choices (and not only indirectly, as in our model). Of course, empirical work that studies the interplay of product differentiation and vertical contracting will also be of great interest and hopefully our theoretical study of provides some foundations in this direction.

References


Appendix A  We explore here extensions of our basic model. First, we study the model where firms are not allowed to locate outside the unit interval. Second, we allow retailers to possibly locate to the opposite side of their wholesalers on the line.

Appendix A1: Locations in the unit interval  We modify the linear wholesale pricing model so that firms are restricted to locate within the \([0,1]\), an assumption often made in the literature. Our analysis of the game is as in the linear wholesale pricing model with upstream locations first, however now \(a, b, x\) and \(y\) cannot take negative values. Thus, we have to solve for the restricted location choices in the first and second stage of the game with the other stages remaining the same.

In the second stage, the downstream firms set their locations and their profit functions are given by expression (12). Can one of the two retailers in equilibrium serve the whole market? Firm X takes the whole demand if \(\tau ((x-a)^2 - (y-b)^2) + t (y - x + 9) (1 - x - y) \leq 0\). Given that all firms are located in the unit interval, this expression can be negative only when \(\tau (y - b)^2\) is higher than \(\tau (x-a)^2 + t (y - x + 9) (1 - x - y)\). Thus, firm Y can avoid receiving zero demand by setting \(y = b\), that is, locate at the same point as its supplier. Firm Y has some local monopoly power and always serves some consumers located close to that firm. Likewise, firm X can avoid receiving zero
demand by setting $x = a$. Therefore, in equilibrium no retailer can serve the whole demand, each retailer can assure at least some sales. The profit functions, thus, reduce to:

\[
\Pi_X = \frac{\left( \tau \left( (y-b)^2 - (x-a)^2 \right) + t\left( x - y + 9 \right) (1 - x - y) \right)^2}{162t\left( 1 - x - y \right)},
\]

\[
\Pi_Y = \frac{\left( \tau \left( (x-a)^2 - (y-b)^2 \right) + t\left( y - x + 9 \right) (1 - x - y) \right)^2}{162t\left( 1 - x - y \right)}.
\]

From the first order conditions, we obtain:\(^{31}\)

\[
x = \frac{16a\tau^2 (1 - a - b) + 4t\tau (17a + 6b - a^2 + b^2 - 7) - 63t^2}{4 (t + \tau) (4\tau (1 - a - b) + 9t)},
\]

\[
y = \frac{16b\tau^2 (1 - a - b) + 4t\tau (17b + 6a - b^2 + a^2 - 7) - 63t^2}{4 (t + \tau) (4\tau (1 - a - b) + 9t)}.
\]

However, the pair of locations $(x, y)$ is positive only if the numerators of $x$ and $y$ are positive (also note that $x, y < 1$). Otherwise, due to the concavity of the profit functions, the equilibrium locations become zero. We get four cases, depending on the values of $a, b$ and the parameter values $t, \tau$. In the second stage of the game, both retailers may be located within $[0,1]$, both retailers may be located at the two opposite endpoints or one retailer within $[0,1]$ and the rival at the endpoint.

Nevertheless, in the first stage of the game, the wholesalers choose symmetric locations ($a = b$), thus, we cannot obtain, as an equilibrium outcome, asymmetric locations downstream. Further, from the analysis of the unrestricted locations model, we find that all four firms are located outside the unit interval. Thus, in the restricted locations model, we cannot obtain an equilibrium with all four firms located in $[0,1]$, since there does not exist such an equilibrium in the unrestricted locations model.

In the unrestricted locations model, the forces that push retailers away from the center of the unit interval are stronger relative to the wholesalers. Thus, there are two possible equilibrium outcomes in the restricted locations model. Retailers locate at the two opposite endpoints and the wholesalers either locate within the unit interval or at the endpoints as their corresponding retailers. We prove that when retailers are located at the two opposite endpoints ($x = y = 0$), the wholesalers maximize their profits, in the restricted locations model, by locating at the two opposite endpoints as their retailers ($a = b = 0$).

The following results summarize the equilibrium outcome and the social cost in equilibrium.\(^{32}\)

\(^{31}\)The profit margin for firm X is positive when $\tau \left( (y-b)^2 - (x-a)^2 \right) + t\left( x - y + 9 \right) (1 - x - y) > 0$ and for firm Y when $\tau \left( (x-a)^2 - (y-b)^2 \right) + t\left( y - x + 9 \right) (1 - x - y) > 0$. The second order conditions are satisfied in equilibrium.

\(^{32}\)In equilibrium, the inequality $f_X < f_X < \bar{f}_X$ is satisfied.
**Proposition A1** The equilibrium outcome with locations restricted in the unit interval is:

\[ a^r = b^r = x^r = y^r = 0, \]
\[ w^r_A = w^r_B = 3t, \quad p^r_X = p^r_Y = 4t, \]
\[ z^r = 0.5, \]
\[ \Pi^r_A = \Pi^r_B = 1.5t, \quad \Pi^r_X = \Pi^r_Y = 0.5t. \]

The social cost is simply equal to the transportation cost of the consumers since this is a unit final demand model (prices do not affect welfare) and the downstream firms are located at the same point as their suppliers, \( SC^r = t/12. \)

As the transportation cost parameter \( t \) increases, the social cost increases too. As \( t \) increases, the products become more differentiated and the profits of the upstream and downstream firms increase. We observe that the cost parameter \( \tau \) that reflects the transportation cost paid by the downstream firms when supplied by the upstream firms does not affect the equilibrium prices and profits. This is because in equilibrium the retailers are located at the same point as their suppliers. The wholesale and product prices in the restricted locations model are lower than in our basic model where firms can locate anywhere on the real line. Again, there are three forces that affect the location choices. The demand effect, the price competition effect and the aggregate marginal cost effect. The forces that push firms farther away from each other dominate the forces that push them towards each other and this lead to maximum differentiation. This result was expected since in the unrestricted locations model we have obtained that all four firms locate outside the unit interval (and since the profit functions are quasi-concave).

We should also note that if we modify the restricted locations model to have all four firms simultaneously choose their locations, we obtain that again upstream firms locate at the same point as their retailers and that maximum differentiation occurs \( (a = x = 0 \text{ and } b = y = 0) \) as opposed to the unrestricted locations model where all firms locate at \( -1.75. \) So, in the simultaneous locations choice model with locations within \([0,1]\) we obtain the same equilibrium outcome compared to the sequential location choices with restricted locations.

**Appendix A2: Retailers locating at the opposite side from the wholesalers** In our analysis we have assumed that upstream firm A locates to the left of B: this is simply a matter of labeling and without loss of generality. We have also proceeded to the equilibrium derivation assuming that each downstream firm locates at the same side of the line as the corresponding upstream \((X \text{ with } A \text{ and } Y \text{ with } B).\) Doing so has allowed us to simplify the exposition in the first two stages of the game and focus on the core arguments. Here, for completeness, we investigate the possibility that, while A is to the left of B, X locates to the right of Y. We find that for some upstream locations and parameter values it could also be an equilibrium in the second stage of the game that the retailers locate at the opposite side of the line relative to the corresponding
wholesaler. We also show that second stage equilibrium profit is lower in this arrangement. Still, we do not find an equilibrium of the entire game when all locations (upstream and downstream) are endogenous and are chosen so that wholesalers’ and retailers’ locations are chosen at the opposite side of one another.

We proceed with the analysis. Let us fix the upstream locations (with $1 - a - b > 0$). Thus far we have assumed that $1 - x - y > 0$. We calculate the profit functions of the downstream firms when firm X is located at the same point of Y (with $1 - x - y = 0$) or to the right of firm Y (with $1 - x - y < 0$). The overall profit function of firm X is then:

$$
\Pi_X = \begin{cases} 
\Pi^L_X & \text{if } 1 - x - y > 0 \\
0 & \text{if } 1 - x - y = 0 \\
\Pi^R_X & \text{if } 1 - x - y < 0,
\end{cases}
$$

where $\Pi^L_X$ is the profit for X when X is to the left (L) of Y (expression (12)) and $\Pi^R_X$ is the profit for X when X is to the right (R) of Y with:

$$
\Pi^R_X = \begin{cases} 
\frac{2t(x + y - 1)}{162t(x+y-1)} & \text{if } 6t(x + y - 1) - D \leq 0 \\
\frac{(\tau((x-b)^2-(x-a)^2)+t(y-x+9)(x+y-1))^2}{162t(x+y-1)} & \text{if } 6t(x + y - 1) - D > 0 \\
0 & \text{if } 6t(x + y - 1) + C \leq 0
\end{cases}
$$

where

$$
C = \tau((y-b)^2-(x-a)^2) + t(x-y-3)(1-x-y), \quad D = \tau((y-b)^2-(x-a)^2) + t(x-y+3)(1-x-y).
$$

If there are equilibrium locations, say $(\bar{x}, \bar{y})$, where X is located to the right of Y these would be given by the maximization of the downstream firms’ profits $\Pi^R_X$ and $\Pi^R_Y$ respectively. From the first order conditions, we have:

$$
\bar{x} = \frac{16a\tau^2(a + b - 1) + 4t\tau(19a + 12b + a^2 - b^2 - 11) + 99t^2}{4(\tau + \tau)(4\tau(a + b - 1) + 9t)},
$$

$$
\bar{y} = \frac{16b\tau^2(a + b - 1) + 4t\tau(19b + 12a + b^2 - a^2 - 11) + 99t^2}{4(\tau + \tau)(4\tau(a + b - 1) + 9t)}
$$

if $9t + 2\tau(a + b - 1) > 0$, $\frac{81t + 4\tau(a-b-9)(1-a-b)}{9\tau + 4\tau(a+b-1)} > 0$ and $\frac{81t + 4\tau(a-b-9)(a+b-1)}{9\tau + 4\tau(a+b-1)} > 0$. The first constraint is necessary to obtain positive equilibrium profits and the other two to obtain positive profit margins (non-negative quantities). Thus, if such an equilibrium $(\bar{x}, \bar{y})$ exists, it will satisfy these
Figure A2 illustrates the nature of the problem. When the upstream firms are located at the two opposite endpoints and the transportation cost parameters are equal to one, we obtain $\hat{y} = 1.375$. In Figure A2, we plot the profit function of firm X allowing X to be located either to the left or to the right of firm Y. When firm X is located to the left of firm Y and far enough from the unit interval and its own supplier, it gets zero demand and profits. There is an interval where firm X is located to the left of firm Y and both firms sell to the final consumers. However, when X is located close to Y, either to the left or to the right, X captures the whole demand. Firm X has a cost advantage compared to firm Y and reduces its retail price to capture the whole demand. This cost advantage is greater when firm X is located to the right of firm Y, since firm X is closer to its supplier, firm A. Thus, the profits of firm X when it captures the whole demand are higher when it chooses the side that is closer to its supplier. There is again an interval where the two retailers share the market, but now firm X is to the right of firm Y. Finally, if firm X is located to the right of firm Y and away from the unit interval, it obtains zero demand since it has a significant cost disadvantage and is located away from [0,1] where consumers live.

We prove that for symmetric upstream locations ($a = b$) and some parameter values, given $y = \hat{y} = (11t + 4a\tau) / 4(t + \tau)$, firm X obtains higher profits when it locates to the right of firm Y and shares the market with Y compared to the profits obtained when i) it locates to the left of Y and shares the market, ii) it locates to the left of Y and captures the whole demand or iii) it locates to the right of Y and captures the whole demand. Figure A2 presents an example. Employing symmetry, we prove that, for certain parameter values and given $\hat{x}$, the best response of firm Y is $\hat{y}$.

**Proposition A2** For fixed and symmetric upstream locations and for $t \geq 4\tau (1 - 2a) / 9$, the pair of locations $\hat{x} = \hat{y} = (11t + 4a\tau) / 4(t + \tau)$ constitutes an equilibrium in the second stage.

If the transportation cost parameter of the consumers $t$ is high enough (compared to $\tau$, or equivalently $\tau$ is low enough), it can also be an equilibrium in the second stage that retailers locate at the opposite side of the corresponding wholesaler. In the extreme case where $t$ is positive and $\tau$ is
zero, retailers pay zero transportation costs when supplied by their wholesalers. This is equivalent to locating at the optimum points on the real line either to the same or the opposite side of their wholesalers. In Figure A2, we assume \( a = b = 0 \) and \( \tau = 1 \), thus, we need \( t \geq 4/9 \), which is satisfied.

Therefore, in the second stage for \( a = b \) and \( t \geq 4\tau(1-2a)/9 \), retailers either locate at the same side (see \( (x^*, y^*) \), from equation (13) for \( a = b \)) or at the opposite side of their wholesalers \((\tilde{x}, \tilde{y})\):

\[
x^* = y^* = \frac{-7t + 4a\tau}{4(t + \tau)}, \quad \Pi_X^L (x^*, y^*) = \Pi_Y^L (x^*, y^*) = \frac{t(9t + 2\tau(1-2a))}{4(t + \tau)}
\]

and
\[
\tilde{x} = \tilde{y} = \frac{11t + 4a\tau}{4(t + \tau)}, \quad \Pi_X^R(\tilde{x}, \tilde{y}) = \Pi_Y^R(\tilde{x}, \tilde{y}) = \frac{t(9t - 2\tau(1-2a))}{4(t + \tau)}.
\]

We now compare the profits in the two equilibria and find that equilibrium \((x^*, y^*)\) Pareto dominates \((\tilde{x}, \tilde{y})\) since \( \Pi_X^L (x^*, y^*) > \Pi_Y^R(\tilde{x}, \tilde{y}) \).\(^{33}\) Downstream equilibrium profits are higher when retailers are located closer to their suppliers.

Can this behavior be part of a subgame perfect equilibrium in the whole game? If we make the upstream firms locate at the points indicated in Proposition 1 \((a^* = b^* = (9t - 5\tau - 9\sqrt{\tau^2 + t^2})/8\tau)\), we calculate that we can have this additional equilibrium \(\tilde{x} = \tilde{y} = \left(31t - 5\tau - 9\sqrt{\tau^2 + t^2}\right)/8(t + \tau)\) in the second stage of the game when \( t \geq 4\tau/3 \). Retailers locate either at \((x^*, y^*)\) or at \((\tilde{x}, \tilde{y})\) in the second stage for these fixed upstream locations. However, we have proved numerically that the pair \((a^*, b^*)\) is not an equilibrium pair of locations in the first stage, when it is anticipated that the retailers, in the equilibrium of the subsequent stage, locate at \((\tilde{x}, \tilde{y})\). Each wholesaler then has an incentive to deviate from \((a^*, b^*)\).

**Appendix B (For referees: NOT for publication)**

**Appendix B1: Profit function of firm X for locations x at the left or the right of y**

Thus far, we have derived the profits for firm X when it is located to the left of firm Y \((1-x-y > 0)\) as a function of all four locations. Now, we derive the profits for X when it is located at the same point \((1-x-y = 0)\) or to the right of firm Y \((1-x-y < 0)\). When X is located to the right of Y, it is still supplied by A which is located to the left of B. Thus, when X is located to the right of Y is not completely symmetric to when Y is located to the right of X, since they have different suppliers.

When \(1-x-y = 0\), there is no product differentiation downstream, thus, consumers buy one unit of the product from the cheapest retailer. The demand for firm X, becomes:

\[
z = D_X = D_A = \begin{cases} 
1 & \text{if } p_X < p_Y \\
\frac{1}{2} & \text{if } p_X = p_Y \\
0 & \text{if } p_X > p_Y.
\end{cases}
\]

\(^{33}\)For symmetric upstream locations, we have \(a = b < 1/2\).
Downstream price competition is very intense and pushes final prices to the marginal cost. Firm X has aggregate marginal cost \( f_X = w_A + \tau(x-a)^2 \) and firm Y has \( f_Y = w_B + \tau(y-b)^2 \). Thus, final price is set equal to \( \max\{f_X, f_Y\} \) and the firm with the lowest aggregate marginal cost captures the whole demand and enjoys positive profits.

Since firm X is supplied by firm A, the profit function for firm A is:

\[
\Pi_A = \begin{cases} 
  w_A & \text{if } w_A < w_B + \tau \left( (y-b)^2 - (x-a)^2 \right) \\
  0 & \text{if } w_A \geq w_B + \tau \left( (y-b)^2 - (x-a)^2 \right).
\end{cases}
\]

Downstream competition is transferred at the upstream level and the wholesale prices are reduced to the difference in the transportation costs of the two retailers. If firm B charges a wholesale price higher than that level, firm A can undercut \( w_B \) and capture the whole demand via its retailer X.

Thus:

\[
w_A = \begin{cases} 
  \tau \left( (1-x-b)^2 - (x-a)^2 \right) & \text{if } (1-x-b)^2 - (x-a)^2 > 0 \\
  0 & \text{otherwise}.
\end{cases}
\]

Nevertheless, these wholesale prices lead to zero downstream profits, \( \Pi_X = \Pi_Y = 0 \).

When \( 1-x-y < 0 \), firm X is located to the right of firm Y. The indifferent consumer is given by:

\[
p_Y + t(z-(1-y))^2 = p_X + t(x-z)^2
\]

and the demand for firm Y, now \( z \), is:

\[
z = D_Y = \begin{cases} 
  1 & \text{if } \frac{1+x-y}{2} + \frac{p_X-p_Y}{2(x+y-1)} \geq 1 \\
  \frac{1+x-y}{2} + \frac{p_X-p_Y}{2(x+y-1)} & \text{if } 0 < \frac{1+x-y}{2} + \frac{p_X-p_Y}{2(x+y-1)} < 1 \\
  0 & \text{if } \frac{1+x-y}{2} + \frac{p_X-p_Y}{2(x+y-1)} \leq 0.
\end{cases}
\]

The profit functions for firm X and Y, respectively, are: \( \Pi_X^R = (p_X - f_X)(1-z) \) and \( \Pi_Y^R = (p_Y - f_Y)z \). The superscript \( R \) refers to the case where firm X is to the right of firm Y.

From the first order conditions, we obtain the equilibrium final price for firm X (likewise, for firm Y):

\[
p_X^R = \begin{cases} 
  f_Y - t(x+y-1)(1+x-y) & \text{if } f_X \leq \frac{f_Y^R}{x} \\
  \frac{t(x+y-1)(3+y-x)+f_Y+2f_X}{3} & \text{if } \frac{f_Y^R}{x} < f_X < \frac{f_Y^R}{x} \\
  f_X & \text{if } f_X \geq \frac{f_Y^R}{x},
\end{cases}
\]

where \( \frac{f_Y^R}{x} \equiv f_Y + t(3-x+y)(x+y-1) \) and \( \frac{f_X^R}{x} \equiv f_Y - t(3-y+x)(x+y-1) \).

In the third stage, upstream firms seek to maximize their profits \( \Pi_A^R = w_A(1-z) \), \( \Pi_B^R = w_Bz \) with respect to their wholesale prices. Assuming equilibrium in the subsequent stage, the profit
function for firm A becomes:

$$\Pi^R_A = \begin{cases} 
0 & \text{if } w_A \geq w_B + C \\
(1 - \frac{2t(y-x)(x+y-1) + w_A - w_B + \tau((x-a)^2-(y-b)^2)}{6t(x+y-1)} + \frac{1+x-y}{2}) w_A & \text{if } D + w_B + < w_A < w_B + C \\
 & \text{if } w_A \leq w_B + D,
\end{cases}$$

where

$$C \equiv \tau((y-b)^2-(x-a)^2) + t(x-y-3)(1-x-y), \ D \equiv \tau((y-b)^2-(x-a)^2) + t(x-y+3)(1-x-y).$$

Similarly to the basic analysis, we calculate the equilibrium wholesale prices for firm A (analogously, for firm B):

$$w^R_A = \begin{cases} 
0 & \text{if } 6t(x + y - 1) + C \leq 0 \\
\frac{\tau((y-b)^2-(x-a)^2) + t(y-x+9)(x+y-1)}{3} & \text{if } 6t(x + y - 1) - D > 0 \\
\tau \left((y-b)^2-(x-a)^2\right) - t(x-y+3)(x+y-1) & \text{if } 6t(x + y - 1) - D \leq 0,
\end{cases}$$

and the corresponding profits for firm X are:

$$\Pi^R_X = \begin{cases} 
0 & \text{if } 6t(x + y - 1) + C \leq 0 \\
\frac{\tau((y-b)^2-(x-a)^2) + t(y-x+9)(x+y-1))^2}{16t(x+y-1)} & \text{if } 6t(x + y - 1) - D > 0 \\
2t(x+y-1) & \text{if } 6t(x + y - 1) - D \leq 0.
\end{cases}$$

Expression 12 presents the profits for firm X when located to the left of firm Y ($\Pi^L_X$) and this appendix derived the profits for firm X when X is located at the same point or to the right of firm Y ($\Pi^R_X$). Thus,

$$\Pi_X = \begin{cases} 
\Pi^L_X & \text{if } 1 - x - y > 0 \\
0 & \text{if } 1 - x - y = 0 \\
\Pi^R_X & \text{if } 1 - x - y < 0.
\end{cases}$$

**Appendix B2: Derivation of the downstream equilibrium locations** In our basic analysis of linear wholesale prices with upstream location choices first, we proved that, in the second stage, there exist an equilibrium $x^* = \frac{16a^2(1-a-b)+4t(17a+6b-a^2+b^2-7)-63b^2}{4(1+t)(1-a-b)+9t}$, $y^* = \frac{16a^2(1-a-b)+4t(17b+6a-b^2+a^2-7)-63b^2}{4(1+t)(1-a-b)+9t}$ when firm X is located to the left of firm Y. Here, we prove that given $y = y^*$, firm X prefers to locate to the left of firm Y at point $x^*$, compared to the right of firm Y. For simplicity, we provide the proof for symmetric upstream locations ($a = b$) since this will be the equilibrium outcome.

Initially, we prove that given $y = y^* = \frac{-7t+4a}{4t+7}$, firm X cannot capture the whole demand either located to the left or to the right of firm Y. Firm X would serve the whole market when located to the left of firm Y if: $\tau \left((x-a)^2-(y-b)^2\right) + t(y-x+9)(1-x-y) \leq 0$ or if $x \in (x_1, x_2)$ with
\[
\begin{align*}
x_1 &= \frac{20t + 4ar - 3(9t - 16r + 32ar)}{4(t + \tau)}, \quad x_2 = \frac{20t + 4ar + 3\sqrt{t(9t - 16r + 32ar)}}{4(t + \tau)}. \quad \text{Since } 1 - y^* < x_1 < x_2, \text{ the inequality } \\
\tau \left( (x - a)^2 - (y - b)^2 \right) + t(y - x + 9)(1 - x - y) &\leq 0 \text{ is never satisfied. Firm X would serve the whole market when located to the right of firm Y if: } \\
\tau \left( (x - a)^2 - (y - b)^2 \right) + t(x - y + 9)(x + y - 1) &\leq 0 \text{ or if } x \in (x_4, x_3) \text{ with } x_3 = \frac{-16t + 4ar + 3\sqrt{t(81t + 16r - 32ar)}}{4(t + \tau)}, \quad x_4 = \frac{-16t + 4ar - 3\sqrt{t(81t + 16r - 32ar)}}{4(t + \tau)}. \quad \text{Since } 1 - y^* > x_3 > x_4, \\
\text{the inequality } \tau \left( (x - a)^2 - (y - b)^2 \right) + t(x - y + 9)(x + y - 1) &\leq 0 \text{ is never satisfied.}
\end{align*}
\]

Thus, we have to prove that firm X prefers to share the market with firm Y and locate to the left of Y than to the right. Firm X maximizes its profits located to the left of Y at \( x^* = \frac{-7t + 4ar}{4(t + \tau)} \) with \( \Pi_X^L(x^*, y^*) = \frac{t(9t - 2r(2a - 1))}{4(t + \tau)} \) and to the right of Y at \( x^+ = \frac{5t + 16r - 20ar}{12(t + \tau)} \) with \( \Pi_X^R(x^+, y^*) = \frac{(9t - 2r(2a - 1))(9t - 16r + 32ar)^2}{576(t + \tau)} \) for \( 9t + 16r(2a - 1) > 0 \). The latter constraint is necessary to ensure that both firms have positive demand when X is located to the right of Y. If \( a \) is low enough, that is, upstream firms are located away from the unit interval, firm X can never obtain positive demand when located to the right of firm Y since its supplier A is far away. By direct comparison we obtain: \( \Pi_X^L(x^*, y^*) > \Pi_X^R(x^+, y^*) \).

**Appendix B3: Derivation of the second equilibrium in the second stage for fixed and symmetric upstream locations**

For symmetric upstream locations (\( a = b \)) and \( y = \tilde{y} = \frac{11t + 4ar}{4(t + \tau)} \), the profits of firm X when it locates to the right of firm Y and firms share the market are maximized at \( x = \frac{11t + 4ar}{4(t + \tau)} \) with \( \Pi_X^R(\tilde{x}, \tilde{y}) \).\(^{34}\) On the contrast, when firm X locates to the left of firm Y and firms share the market profits are maximized at \( x^- = \frac{-15t + 2r(a - 2) + 4\sqrt{(9t - 8r + 16ar)^2}}{6(t + \tau)} \) with \( \Pi_X^L(\tilde{x}^-, \tilde{y}) \).

When firm X locates to the left of firm Y and captures the whole demand, profits are maximized at \( x_m^L = \frac{20t + 4ar - 3\sqrt{t(81t + 16r(2a - 1))}}{4(t + \tau)} \) with \( \Pi_X^L(x_m^L, \tilde{y}) \). Finally, firm X captures the whole demand located to the right of firm Y by maximizing its profits at \( x_m^R = \frac{-16t + 4ar + 3\sqrt{t(9t + 16r(1 - 2a))}}{4(t + \tau)} \) with \( \Pi_X^R(x_m^R, \tilde{y}) \).

The \((\tilde{x}, \tilde{y})\) pair of locations constitutes an equilibrium in the second stage of the game when:
\[
\Pi_X^L(\tilde{x}, \tilde{y}) \geq \Pi_X^L(x^-, \tilde{y}), \quad \Pi_X^L(\tilde{x}, \tilde{y}) \geq \Pi_X^L(x_m^L, \tilde{y}) \quad \text{and } \Pi_X^R(\tilde{x}, \tilde{y}) \geq \Pi_X^R(x_m^R, \tilde{y}). \quad \text{By direct calculations we find that when } \Pi_X^L(\tilde{x}, \tilde{y}) \geq \Pi_X^L(x^-, \tilde{y}), \text{ then } \Pi_X^L(\tilde{x}, \tilde{y}) \geq \Pi_X^L(x_m^L, \tilde{y}) \quad \text{and } \Pi_X^R(\tilde{x}, \tilde{y}) \geq \Pi_X^R(x_m^R, \tilde{y}) \quad \text{hold true. Inequality } \Pi_X^L(\tilde{x}, \tilde{y}) \geq \Pi_X^L(x^-, \tilde{y}) \text{ is satisfied when } t \geq 4r(1 - 2a) \div 9.\]

\(^{34}\)The profit functions \( \Pi_X^L \) and \( \Pi_X^R \) for firm X are presented at expression (12) and Appendix A2, respectively.