On Vertical Contracting under Incomplete Information and the Role of Observable Contracts

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Abstract

This paper deals with the pricing decision of an upstream firm, vertically linked with downstream competing firms, when it is less informed regarding the marginal cost of them. In this framework the upstream firm (principal) has rather two options, either offering a menu of contracts (designing a direct truthful mechanism) or offering a single contract. The former choice under contract choice observability leads to a "separating" case, full information disclosure, while the latter one (when trade occurs with both types) to a "pooling" case, without information disclosure. Both cases may form an equilibrium depending on the extent of differentiation between the two potential types with respect to efficiency. Thus, it is no longer optimal for the upstream firm to price using a menu (when this is a viable option), as it is when the choice of the contract remains secret.

Keywords: vertical relations; screening; asymmetric information; two part tariffs

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1 Introduction

Recent industrial organization literature has given considerable attention on vertically related markets since most real market structures fall in this category. From a theoretical perspective there is a growing literature that combines vertical relations and asymmetric information and main intention is to add new features in this kind of models by making them more realistic. Vertical relations refer to trading between upstream (input producers or final good manufacturers) and downstream firms (wholesalers or retailers). This kind of vertical linkages are frequent in real world markets and in many occasions informational asymmetries occur affecting them. The upstream firms use sophisticated contracts and make take-it-or-leave-it offers having two things in mind to maximize as much as possible the total rents and to appropriate the highest possible share of them. Thus, their pricing decision determines both the total rents as well as their distribution among the participating firms. A common approach, when the upstream parties suffer from less information, is to use a menu of contracts so as the potential types of the informed parties to be appropriately screened and thus truthfully report their type. The screening activity and type revelation does not come without cost though and thus it leaves information rents to the more efficient types, the so called distributional distortion. Besides that information asymmetries in this setting bring about a second distortion known as the excessive pricing distortion. The upstream firm/principal has to increase wholesale prices offered to the less efficient type to prevent mimicking.

In this paper we present a detailed analysis of an upstream firm’s pricing decision. We built on the realistic scenario where an upstream firm reaches final consumers through two downstream intermediaries which are better informed regarding their marginal cost of production. We examine in detail how the upstream firm designs the optimal pricing schemes by underlining two distinct cases regarding the observability of the contract choice when a menu pricing is used. Downstream firms’ choices from a menu of contracts can be either known to the rival (competing) firm or unknown/secret with important implications on how pricing is formed in equilibrium and on the way the profits are being shared.

In general, an upstream (principal) that deals with better informed downstream intermediaries has rather two options, either offering a menu of contracts and thus screen retailers’ types or
offering a single contract to all types. When the contract choices are not observable by the rival firm in the downstream market no information disclosure occurs and downstream rivals compete in an incomplete information framework. On the other hand when the contract choice of the downstream rivals is observable the former choice leads to a "separating" case, with full information disclosure, while the latter one (when trade occurs with both types) to a "pooling" case, without information disclosure. We demonstrate that both cases may form an equilibrium depending on the extent of differentiation between the two potential types with respect to the production efficiency. Thus, when we incorporate observability on the contract choice it is no longer optimal for the upstream firm to price using a menu (when this is a viable option), as it is when the choice of the contract remains secret. We present an example where under certain conditions the separation of the informed parties’ types is not optimal due to the consequences on the dispersion of information among the participants.

Our paper is clearly related to the literature that combines vertical relations with asymmetric information. Most of the existing literature uses a bilateral monopoly set up and focuses on the main distortions introduced due to the asymmetry of information as well as on the comparison of commonly used contract types/vertical restraints, Gal-Or (1991), Blair and Lewis (1994), Crocker (1983), Martimort and Piccolo (2007), Accconcia Martina and Piccolo (2008), Corbett Zhou and Tang (2004). According to our knowledge the only papers that use more than one agent is the seminal paper by Rey and Tirole (1986), Herweg and Müller (2011) and Hansen and Motta (2012) but only that of Hansen and Motta uses competing agents in an incomplete information framework. In addition we depart from the existing literature by making the chosen contract (from a menu) observable to competing firms. In Hansen and Motta the menu that is offered is public but the choice of each downstream firm is not observed and in Rey and Tirole there is uncertainty regarding the demand and the production cost, both are unknown at the contracting stage and being known only to the downstreams before they make their pricing decisions. In addition, we perform the analysis under a two-part tariff scheme while most of the papers use a quantity forcing contract.

The fact that the contract choice from a menu is observable leads to a separation of the types and thus to information disclosure since each downstream firm is forced to accept a contract that is
targeted to its specific type. Designing the optimal contract the upstream principal imposes the standard incentive compatibility (IC) and participation constraints (PC). Incentive constraints make possible the separation of the potential types and thus under certain conditions it may be optimal for the upstream firm to switch to a single contract pricing strategy avoiding in this way information disclosure. In this paper we explicitly show that under the observability assumption both the menu and the single contract pricing can prevail under certain circumstances while with unobservable contract choice menu pricing is without doubt the best pricing strategy when this is a viable choice.

The rest of the paper is as follows. In Section 2, we introduce our model. In Section 3, we perform the equilibrium analysis both under contract choice observability and unobservability as well as under a single contract. In Section 4, we analyze the optimal pricing by the upstream firm and we discuss some variations and Section 5 concludes.

2 The model

We use a vertically related industry which consists of risk-neutral firms in a triangular market formation. We consider a framework in which the information is asymmetrically dispersed and thus the analysis rests upon the principal-agent relationship. There is one upstream firm $U$ (the principal) which sells through two intermediaries, downstream firms $D_1$ and $D_2$ (the agents). $U$ is less informed regarding the marginal cost of production of the downstream firms but it has the bargaining power and makes take-it-or-leave-it offers which take the form of either a menu of two-part tariff contracts or a single two-part tariff contract. We also assume that: (i) The upstream supplier is able to offer different contract terms to the downstream agents (no arbitrage), (ii) Contract offers are being observed by the downstream firms (no commitment problem), (iii) Demand is known to everyone (common knowledge), (iv) Inverse demand is a linear one $p(Q) = a - Q$.

Retailer’s unit cost $c_i$ takes two possible values high or low $c_i \in \{c_L, c_H\}$, this is determined

\footnote{Without loss of generality and for expositional ease we assume that $c_L = 0$. Note that if we had $c_L > 0$ then the analysis should be qualitatively the same as the one presented in the next sections. This holds because what determines the optimal pricing by $U$ is the difference $c_H - c_L$.}
by nature and it is common knowledge that $\text{prob}(c_i = c_L) = \theta$ with the residual probability equal to $\text{prob}(c_i = c_H) = 1 - \theta$. Since the downstream firms enjoy an information advantage the upstream firm has to decide whether it will proceed in a screening activity by offering as many contracts as the number of types, designing a direct truthful mechanism with a menu of contracts, recalling the revelation principle (Myerson 1979) or it will offer just a single contract. The retailers accept or reject the offer by considering their reservation utility without having the option for any counteroffers. Reservation utility is assumed equal to zero\textsuperscript{2}. Crucial assumption and novel insight in our model is the observability of the contract choice. Specifically, the upstream firm makes its offer and if this is a menu of contracts then the choice that each downstream firm makes, through its reporting, is being observed by the rival firm. The game unfolds as follows:

- At time $t = 1$, $c_i$ is being realized and only retailer $i$ observes it.
- At time $t = 2$, the upstream firm makes take-it-or-leave-it public offers to the retailers.
- At time $t = 3$, the retailers accept or refuse the offer they receive and this is known to the rival firm.
- At time $t = 4$, if they accept they must trade according to the contract selected and place their quantities and if they refuse they get their outside option which is normalized to zero.

Our notational convention will be as follows. We will use interchangeably $H$ and $L$ as superscripts and subscripts to denote respectively the choice of the high and low cost firm and superscripts $OB$ for the observable case and $UN$ for the unobservable one. We will use $SH$, when trading is conducted only with the efficient type following a shutdown policy. Finally, we use $SC$, when a single two-part tariff contract is used by the upstream firm that at the same time ensures participation of both types.

3 Characterization of the equilibrium

We proceed in the analysis of the menu pricing as well as of the single contract pricing scheme. For completeness we present both menu pricing cases, i.e. when the contract choice is observable

\textsuperscript{2}See Julien (2000) for a detailed analysis of the role of the participation constraints and Acconcia et al. (2008) for a type dependent reservation utility model.
and unobservable. Our main intention is to explain how the equilibrium is formed in each case and the way it is being affected by the degree of differentiation of the two types. The main question at hand is whether we obtain an equilibrium that separates the informed parties’ types (menu pricing) or one that is not informative, with a single two-part tariff contract. It is important to note that the exact contract terms of a single contract may make both downstream types to accept the same (unique) contract offer or to keep alive only the low-cost firm. In what follows each case is being analyzed separately.

3.1 Menu pricing (unobservable contract choice)

The upstream firm is less informed compared to the downstreams and thus it offers a menu \( T_i = [(w_{iL}, f_{iL}), (w_{iH}, f_{iH})] \) and in this case the choice each downstream firm makes is not observable. We need to assume that \( c_H < M \equiv \frac{a(1-\theta)}{3+5\theta} \) to ensure that equilibrium quantities are positive\(^3\).

Since the choice is not observable there is no information transmission to the downstream through the pricing decision of the upstream firms. Thus, the downstream firms set their quantities in an environment with incomplete information and a Bayesian (B.N.E) equilibrium is formed that specifies an action for each possible type. In terms of the wholesale prices we obtain:

\[
q_i^L(w_i, w_j) = \frac{1}{3} [a + w_j - 2w_i + \frac{(1-\theta)c_H}{2}] \tag{1}
\]

\[
q_i^H(w_i, w_j) = \frac{1}{3} [a + w_j - 2w_i - c_H - \frac{\theta c_H}{2}], i \neq j \tag{2}
\]

The equilibrium quantities depend on both \( w_i \) and \( w_j \) and each retailer reports honestly and reveals its type given that its rival is expected to behave in the same way. The notation we use in order to present in a proper way the constraints is \( q_i^L(w_{iL}, w_{jK}) \) whenever firm i is a low cost firm (and chooses \( w_{iL} \)) and firm j can be either high or low cost and \( q_i^H(w_{iH}, w_{jK}) \) whenever firm i is a high cost firm and chooses \( w_{iH} \) from the menu. The second entry always refers to the rival firm’s choice given that it behaves honestly \( k \in \{L, H\}^4 \).

\(^3\)Note that a corner solution leads to the use of a single contract and that happens when \( c_H > M \).

\(^4\)Expected profits of \( D_i \):

- if \( c_i = c_H \rightarrow \theta(a - q_i^H(w_{iH}, w_{iL}) - q_j^L(w_{jL}, w_{jH}) - w_{iH} - c_H)q_i^H(w_{iH}, w_{iL}) + (1-\theta)(a - q_i^H(w_{iH}, w_{iH}) - \).
In its maximization program $U$ takes into account both the participation and the incentive compatibility constraints so as to ensure participation by the downstream retailers and to avoid transferability of demand. Anticipating the retailers’ behavior $U$ maximizes its expected profits subject to the relevant-binding constraints:\footnote{The relevant constraints in cases like this are: (i) the participation constraint of the inefficient type (the high cost) (ii) the incentive compatibility constraint of the efficient type (the low cost).}

\[
\max \{w_{1L}w_{1H},w_{2L},w_{2H},f_{1L},f_{1H},f_{2L},f_{2H}\} \quad E\Pi^U = w_{1H}[(1 - \theta)^2 q_1^H(w_{1H}, w_{2H}) + (1 - \theta)^2 q_1^H(w_{1H}, w_{2L})] + w_{1L}[\theta^2 q_1^L(w_{1L}, w_{2L}) + \theta(1 - \theta)q_1^L(w_{1L}, w_{2H})] + w_{2H}[(1 - \theta)^2 q_2^H(w_{2H}, w_{1H}) + (1 - \theta)^2 q_2^H(w_{2H}, w_{1L})] + w_{2L}[\theta^2 q_2^L(w_{2L}, w_{1L}) + \theta(1 - \theta)q_2^L(w_{2L}, w_{1H})] + (1 - \theta)[f_{1H} + f_{2H}] + \theta[f_{1L} + f_{2L}]
\]

s.t.

(i) $E\Pi_D(w_{iH})|_{c_i=c_H} \geq f_{iH}$ \hspace{1cm} (P.C.)

(ii) $E\Pi_D(w_{iL})|_{c_i=c_L} \geq f_{iL}$

(iii) $E\Pi_D(w_{iL})|_{c_i=c_L} - f_{iL} \geq E\Pi_D(w_{iH})|_{c_i=c_L} - f_{iH}$ \hspace{1cm} (I.C.)

(iv) $E\Pi_D(w_{iH})|_{c_i=c_H} - f_{iH} \geq E\Pi_D(w_{iL})|_{c_i=c_H} - f_{iL}$

The participation constraints ensure that each possible type should at least cover the status quo utility assuming for simplicity that it is equal to zero. On the other hand the incentive compatibility constraints induce a particular type to select the right contract. Hence, under the selected contract the expected profits are higher than with any other choice given the behavior of the rival firm.

After substituting $f_{iL}, f_{iH}$ into the profit function of the upstream firm using the binding constraints ((i) and (iii)) and maximizing with respect to $w_{iL}, w_{iH}$ we obtain the equilibrium

\[
q_1^H(w_{iH}, w_{iH}) - w_{iH} - c_H)q_1^H(w_{iH}, w_{iH}) - f_{iH} - q_1^L(w_{iL}, w_{iL}) - w_{iL} - c_L)q_1^L(w_{iL}, w_{iL}) + (1 - \theta)(a - q_1^L(w_{iL}, w_{iH}) - q_1^H(w_{iH}, w_{iL}) - w_{iL} - c_L)q_1^H(w_{iL}, w_{iH}) - f_{iL}
\]

• if $c_i = c_L \rightarrow \theta(a - q_1^L(w_{iL}, w_{iL}) - q_1^L(w_{iL}, w_{iL}) - w_{iL} - c_L)q_1^L(w_{iL}, w_{iL}) + (1 - \theta)(a - q_1^L(w_{iL}, w_{iH}) - q_1^H(w_{iH}, w_{iL}) - w_{iL} - c_L)q_1^H(w_{iL}, w_{iH}) - f_{iL}$
contract terms:

\[ w_{1L}^{UN} = w_{2L}^{UN} = \frac{1}{4} [a - c_H(7 + 2\theta)], \quad w_{1H}^{UN} = w_{2H}^{UN} = \frac{a(1 - \theta) - c_H - \theta c_H(11 + 2\theta)}{4(1 - \theta)}; \]

\[ f_{1L}^{UN} = f_{2L}^{UN} = \frac{1}{16} \left[ a^2 + 2ac_H(3 + 2\theta) - \frac{c_H^2(4\theta^3 - 27\theta - 25)}{1 - \theta} \right]; \]

\[ f_{1H}^{UN} = f_{2H}^{UN} = \frac{a^2(1 - \theta)^2 - 2ac_H(1 - \theta)(1 + \theta(5 + 2\theta)) + \frac{c_H^2(1 + \theta(18 + \theta(29 + 40(3 + \theta)))))}{16(1 - \theta)^2}. \]

It can be easily confirmed that the wholesale price offered to the high cost firm is always higher than the one offered to the low cost, \( w_{iL}^{UN} < w_{iH}^{UN} \) and the opposite holds for the fixed fees where the upstream firm requires a higher fee from the low cost firm, \( f_{iL}^{UN} > f_{iH}^{UN} \). The contract terms are designed in this way since the upstream firm wants to subtract as much as possible of the total rents using the fees. It tries to separate the types, makes the downstreams to report truthfully, and at the same time leads the more efficient ones more to produce more. The upstream firm’s equilibrium profits are:

\[ \Pi^U = \frac{(a - c_H)^2 - [a^2 - c_H^2 - 2ac_H]\theta + 6c_H^2\theta^2}{4(1 - \theta)}. \]

The required (expected) information rents for truthful reporting can be expressed as the difference between the expected profits of the low cost firm and the relevant fee and it is positive as expected:

\[ E\Pi_{D_i}(w_{iL}^{UN})|_{c_i=c_L} - f_{iL}^{UN} = \frac{c_H(a(1 - \theta) - 2c_H(3 + \theta))}{4(1 - \theta)} > 0. \]

### 3.2 Menu pricing (observable contract choice)

As we have already explained the upstream firm offers a menu of contracts in order to screen the types of the downstream retailers. The relevant restriction for positive equilibrium quantities is \( c_H < W \equiv \frac{3a(1 - \theta)}{15 + 14\theta + 3\theta^2} \). Following the revelation principle the number of contracts used are as many as the number of types. A higher number of contracts is redundant and thus the contract offers have the following form, \( T_i = [(w_i^L, f_i^L), (w_i^H, f_i^H)] \). Focusing then on direct truthful mechanisms, it is important in designing the optimal contract to impose both the participation (P.C.) and incentive compatibility (I.C.) constraints as in the previous case. In particular, downstream
firms, by accepting a contract offer, reveal their true type (marginal cost) since they are forced
to do that by the incentive compatibility constraints. Clearly, this feature of the model gives a
signaling characteristic on the offered contracts, that is information is transmitted to the down-
streams. Therefore, downstream competition is one with complete information that leads to the
standard Cournot outcomes with the following equilibrium quantities:

\[
q_{RQ}^i(w_i, w_j) = \frac{1}{3}\{a - 2w_i - 2c_i + w_j + c_j\}, \quad R, Q = \{L, H\}
\]

where \(c_i, c_j\) denote the exogenously given (determined by nature) marginal cost, \(w_i, w_j\) the
wholesale prices that each downstream firm pays and \(R\) refers to the cost level of firm \(i\) and \(Q\) to
that of firm \(j\).

The upstream firm maximizes its expected profits subject to the relevant (binding) constraints:

\[
\begin{align*}
\max_{\{f_i^L, f_i^H, f_j^L, f_j^H\}} E\Pi^{US} = & \quad w_1^H[(1 - \theta)q_1^{HH}(w_1^H, w_2^H) + (1 - \theta)\theta q_1^{HL}(w_1^H, w_1^L)] \\
& + w_1^L[\theta^2 q_1^{LL}(w_1^L, w_2^L) + \theta(1 - \theta)q_1^{LH}(w_1^L, w_2^H)] \\
& + w_2^H[(1 - \theta)^2 q_2^{HH}(w_2^H, w_1^H) + (1 - \theta)\theta q_2^{HL}(w_2^H, w_2^L)] \\
& + w_2^L[\theta^2 q_2^{LL}(w_2^L, w_1^L) + \theta(1 - \theta)q_2^{LH}(w_2^L, w_2^H)] \\
& + (1 - \theta)[f_1^H + f_2^H] + \theta[f_1^L + f_2^L]
\end{align*}
\]

s.t.

(i) \(E\Pi_{Di}(w_i^H)\bigg|_{c_i=H} \geq f_i^H\) \quad (P.C.)

(ii) \(E\Pi_{Di}(w_i^L)\bigg|_{c_i=L} \geq f_i^L\)

(iii) \(E\Pi_{Di}(w_i^L)\bigg|_{c_i=L} - f_i^L \geq E\Pi_{Di}(w_i^H)\bigg|_{c_i=L} - f_i^H\) \quad (I.C.)

(iv) \(E\Pi_{Di}(w_i^H)\bigg|_{c_i=H} - f_i^H \geq E\Pi_{Di}(w_i^L)\bigg|_{c_i=H} - f_i^L\)

where \(E\Pi_{Di}(w_i)\bigg|_{c_i}\) represent the expected profits of each downstream firm given its cost type
(high or low) and the wholesale price it pays to the upstream firm. The binding constraints in
adverse selection models is (i) and (iii) and thus there is no rent at the bottom (no profits for the
high cost firm) but information rents have to be given to the efficient (low cost) firm for a truthful
reporting. Making the appropriate substitutions and maximizing with respect to the wholesale
price we obtain the optimal contract terms:

\[ w_{1}^{LOB} = w_{2}^{LOB} = \frac{1}{4}[a - 3(3 + \theta)c_H], \quad w_{1}^{HOB} = w_{2}^{HOB} = \frac{1}{4(1 - \theta)}[a(1 - \theta) + \theta(14 + 3\theta)c_H] - c_H; \]

\[ f_{1}^{LOB} = f_{2}^{LOB} = \frac{1}{144}[9a^2 + 78ac_H - 343c_H^2 + \frac{768c_H^2}{1 - \theta} + 14c_H(3a + 5c_H) + 33c_H^2\theta^2]; \]

\[ f_{1}^{HOB} = f_{2}^{HOB} = \frac{9a^2(1 - \theta)^2 - 6ac_H(1 - \theta)(3 + \theta)(1 + 7\theta) + c_H^2(3 + \theta)^2(3\theta^2 + 30\theta + 1)}{144(1 - \theta)^2}. \]

As in the previous case the equilibrium wholesale price is higher for the high cost, \( w_{i}^{LOB} < w_{i}^{HOB} \) and the fixed fee for the low cost, \( f_{i}^{HOB} < f_{i}^{LOB} \). The upstream firm’s equilibrium profits are:

\[ \Pi^{US} = \frac{1}{36(1 - \theta)}[9a^2(1 - \theta) - 6ac_H(1 - \theta)(3 + \theta) + (9 + \theta(37 + \theta(5 + 7\theta)))c_H^2] \]

The (expected) information rents are:

\[ E\Pi_{D_i}(w_{i}^{LOB})|_{c_i=c_L} - f_{i}^{LOB} = \frac{c_H(3a(1 - \theta) + c_H(1 - \theta(26 + 7\theta)))}{9(1 - \theta)} > 0 \]

The following lemma summarizes the comparison between the equilibrium wholesale prices for the two cases analyzed above.

**Lemma 1** Under menu pricing:

(i) the equilibrium wholesale price offered to the high cost firm is always higher when the contract choice of the downstream rivals is observable, \( w_{i}^{HOB} > w_{i}^{UN} \).

(ii) the equilibrium wholesale price offered to the low cost firm is always higher when the contract choice of the downstream rivals is unobservable, \( w_{i}^{LOB} < w_{i}^{UN} \).

Lemma 1 informs us about the comparison of the respective equilibrium wholesale prices when menu pricing is used by the upstream firm. It demonstrates that when the contract choice of the downstream firm is observable by its rival and thus information disclosure occurs the upstream firm pricing decision intensifies asymmetries between the downstreams’ two potential types. In particular it offers a higher wholesale price to the high cost firm and a lower one to the low cost firm relative to the case where the choice of the menu is unobservable. This is the case because the upstream firm faces stronger constraints (participation and incentive constraints) when the choice
is observable since when each of downstream firm truthfully reports its type this becomes known not only to the upstream firm but to its downstream competitor as well. Thus, the upstream firm is forced to offer much more differentiated (distorted) wholesale prices in this case in order to be accepted by the downstremes and prevent mimicking from the low cost ones. We should note that under certain conditions the equilibrium wholesale prices offered to the low cost firms $w_{iL}^{LOB}, w_{iL}^{UN}$ as part of the menu can be negative. This means that the upstream firm in order to separate the types as well as to control the downstream competition may be forced to subsidize downstream production of the low cost firms\(^6\).

A straightforward implication of Lemma 1 is that under observable contract choice the fixed fee for the low cost type is higher than under unobservable contract choice but the fee for the high type is lower. This is in line with the above comparison since with a lower wholesale price the downstream firms become more aggressive (due to strategic complementarity) and thus the upstream firm enjoys higher profits due to higher fixed fees.

### 3.3 Single contract pricing

#### 3.3.1 Trade with both types

Instead of offering a menu of contracts the upstream firm could also stick on a single contract offer. This has important implications on the equilibrium formation of this game. When the upstream firm offers a single contract both downstream firms’ types participate taking as given the contract offer and thus no information disclosure occurs. In this case the relevant required restriction is $c_H < V \equiv \frac{\alpha}{1+2\theta}$. Downstream firms then compete and set their quantities under incomplete information since they learn nothing from the contract offers and the Bayesian equilibrium quantities are given in (1) and (2) above.

\(^6\)Note that in a similar setting with one upstream firm but with symmetric information subsidization occurs when the contract offers are secret and the upstream firm faces the well-known commitment problem, see e.g. McAfee and Schwartz (1995).
The upstream firm maximizes the following expression:

\[
\max_{w_1, w_2, f_1, f_2} \ E\Pi^{UP} = w_1[(1-\theta)q_1^H(w_1, w_2)+\theta q_1^L(w_1, w_2)]+w_2[(1-\theta)q_2^H(w_2, w_1)+\theta q_2^L(w_2, w_1)]+f_1+f_2
\]

s.t. \( E\Pi_{D_i}(w_i)\big|_{c_i=c_H} \geq f_i, i = 1, 2 \)

In this case there is no need for incentive compatibility constraints since there is only a single contract offered by the upstream firm. In order to ensure that both possible types participate the maximum value of the fixed fee should be equal to the expected profits of the high cost firm. Otherwise, either the downstreams will not participate or the upstream will not obtain the maximum rents. Clearly, the respective participation constraints are binding and the optimal contract terms are the following:

\[
w_1^{SC} = w_2^{SC} = \frac{1}{4}[a - (1 - 4\theta)c_H], \quad f_1^{SC} = f_2^{SC} = \frac{1}{16}(a - c_H(1 + 2\theta))^2.
\]

The upstream firm’s equilibrium profits are:

\[
\Pi^{UP^*} = \frac{1}{4}[(a - c_H)^2 + 2c_H^2\theta^2]
\]

and the required cost for this pricing method, that is the downstream (low cost) firms’ expected profits are:

\[
E\Pi_{D_i}(w_i^{SC})\big|_{c_i=c_L} = \frac{1}{4}(a - 2c_H\theta)c_H
\]

### 3.3.2 Trade with the low-cost only (shutdown)

Another possible scenario is the one in which the upstream firm offers an appropriate contract to sell only to the low cost firms keeping the less efficient downstreams out of the market, following a shutdown policy. To achieve this it has to offer a contract that is accepted only by firms producing with a low cost. Since all the participating firms are risk neutral, the fixed fee offered by the upstream firm should combine the profits that a low cost firm should get either facing a
low cost rival (duopoly) or being a monopoly in the market:

\[ f^{SH} = \theta \Pi^D(c_L) + (1 - \theta)\Pi^M(c_L) \]

where \( \Pi^D(c_L) \) and \( \Pi^M(c_L) \) represent the duopoly and monopoly profits when \( c = c_L \) respectively. The upstream firm maximizes the following expression:

\[
\max_{w^{SH}, f^{SH}} E\Pi^{USH} = w^{SH}[\theta^2(2q^{LL}_i) + 2\theta(1 - \theta)q^M_i] + [\theta^2 + 2\theta(1 - \theta)]f^{SH}
\]

The optimal wholesale price, the fixed fee and upstream firm’s profits are equal to:

\[
w^{SH} = \frac{a[9 - \theta(12 - 5\theta)]}{18 - 5(3 - \theta)\theta}, \quad f^{SH} = \frac{a^2(3 - \theta)^2(9 - 5\theta)}{4(9 - \theta)^2}, \quad \Pi^{USH} = \frac{a^2(3 - \theta)^2\theta}{2(9 - \theta)}
\]

Clearly, the downstream low cost firm(s) get zero expected profit while high cost ones do not participate.

**Lemma 2** Under single contract pricing, the equilibrium wholesale price offered when \( U \) trades with both potential types is higher than the one under shutdown, \( w^{SC}_i > w^{SH}_i \).

As expected the equilibrium wholesale price when trading takes place with both types is higher than the one with shutdown policy. This is true since in the latter case a high cost firm does not participate and the upstream firm offers a lower price to the respective (low cost) downstreams in order to make them produce as much as possible without distorting that much the equilibrium wholesale price. By doing this it keeps their cost at low levels and thus it is able to appropriate higher profits through the fixed fees, \( f^{SC}_i < f^{SH}_i \).

### 4 Optimal pricing

In this section we present the optimal way that \( U \) decides to price its products. The upstream firm makes take-it-or-leave-it offers to the downstream ones, set the prices, and thus is looking for the best pricing strategy. For completeness and for comparison reasons we present below the optimal choice of the upstream firm in both the unobservable and the observable contract choice.
cases. The analysis that follows demonstrates that in any case the chosen pricing scheme depends on the degree of differentiation between the two potential types.

4.1 Unobservable contract choice

When the contract choice each downstream makes is unobservable then a menu of contracts cannot be used for information transmission. The Proposition below presents the optimal choice of the upstream firm.

Proposition 1 According to the value of the marginal cost of the high type, $c_H$, we obtain the following three cases:

(i) When $0 < c_H < M$, a menu of two-part tariff contracts is preferred.

(ii) (a) If $0 < \theta < 0.803$ then $B \in (M, V)$ and when $M < c_H < B$ a single two-part tariff (for both types) is preferred and when $B < c_H < V$ a single contract for the low-cost type (shutdown) is preferred.

(b) If $0.803 < \theta < 1$ then $B \notin (M, V)$, with $B < M$ and when $M < c_H < V$ a single contract for the low-cost type (shutdown) is preferred.

(iii) When $c_H > V$, a single two-part tariff for the low-cost type (shutdown) is the only viable option.

Proposition 1 describes in detail how the upstream firm prefers to price its products (Figure 1). Clearly, when the menu pricing is a viable option, $c_H \in (0, M)$, then it is the best pricing scheme. This is the case because by offering a menu of contracts the upstream firm is able to screen the downstream firms’ potential types and to force each of them to select the appropriate contract. In this way it balances the trade-off between rent extraction and efficiency in the sense that it maximizes allocative efficiency by taking into consideration the fact that it is being constrained since information is asymmetrically distributed. In order to make the downstream firms to perfectly reveal their type it has to sacrifice a part of the total profits the so called information rents. Furthermore, we should note two things. First, the use of either a menu of contracts or a single contract does not provide extra information to the downstreams and the competition among them takes place in an incomplete information framework. Hence, the use of
each of the pricing schemes does not alter the conditions regarding information dispersion under which downstream firms compete something that it does not hold when the choice of the menu can be viewed by the rival firm. Second, it can be easily verified that the expected information rents required for the separation of the types under menu pricing is always lower than the rents that the low cost firms can secure under a single contract. Hence, the latter pricing method is more costly for the upstream firm and this is the reason why screening of the potential types is preferred.

When \( c_H \) takes intermediate values, \( c_H \in (M, V) \), a single contract is preferred and in general when the probability for the low type is high enough then shutdown is preferred. Finally, when \( c_H \) is high enough, \( c_H > V \), the upstream firm trades only with the efficient type since by keeping the inefficient one alive is too costly. Obviously, when \( c_H \) is very high then the two potential types become very differentiated in terms of efficiency and thus after some point it is not optimal for the upstream principal to keep trading with the inefficient type since the required cost becomes prohibitively high and thus the high cost does not produce at all. The same holds when \( \theta \) is very high as in Proposition 1 (ii) (b) since then it is very likely that the downstream intermediary is of low cost and thus shutdown policy seems a reasonable choice.

![Figure 1: Optimal pricing under unobservable contract choice.](image)

**4.2 Observable contract choice**

When the contract choice each downstream makes is observable as it has been explained in the text above full information disclosure occurs and thus any informational advantages disappear.
Comparing the respective profit expressions we find that both pricing strategies, menu or single contract pricing, form an equilibrium depending on the parameter values and in particular on the difference between the marginal cost of the two types. The main insights are described in the following proposition with a graphical representation in Figure 2 below:

**Proposition 2** According to the value of the marginal cost of the high type, $c_H$, we obtain the following four cases:

(i) When $0 < c_H < G$, a single two-part tariff (for both types) is preferred.

(ii) When $G < c_H < W$, a menu of two-part tariff contracts is preferred.

(iii) (a) If $0 < \theta < 0.904$ then $B \in (W, V)$ and when $W < c_H < B$, a single two-part tariff (for both types) is preferred and when $B < c_H < V$ a single contract for the low-cost type (shutdown) is preferred.

(b) If $0.904 < \theta < 1$ then $B \notin (W, V)$, with $B < W$ and when $W < c_H < V$ a single contract for the low-cost type (shutdown) is preferred.

(iv) When $c_H > V$, a single contract for the low-cost type (shutdown) is the only viable option.

Proposition 2 informs us about the best pricing strategy of the upstream firm when the contract choice is observable under a menu pricing scheme (Figure 2). In particular, when $c_H$ is not high enough and thus there is no important cost difference between the efficient and the inefficient type then a single contract is preferred by $U$. This seems a quite reasonable decision since the more similar the two potential types the less eager is the upstream firm to separate them through its pricing decision, avoiding in this way the required information rents. We should note that in most of the cases in this area ($0 < c_H < G$) the information rents required for separation are higher than the rents obtained by the downstreams under a single contract. Thus, it seems that it is more costly for the upstream firm to screen the potential types when they are not that differentiated.

Also, in this case the wholesale prices offered differ more than in the previous (non observability) one, Lemma 1, due to the fact that secret information becomes common knowledge and this heavily affects downstream competition. Both expected allocative efficiency as well as expected information rents are being affected by $U$’s pricing decision. With two-part tariff contracts the upstream firm has two instruments available and it can use the wholesale prices so as to control
downstream competition and the fees to transfer upstream the highest possible proportion of the total rents. We can easily verify that when menu pricing is preferred ($G < c_H < W$) then the part of the profits that $U$ obtains from the fees is higher than under a single contract. On the other hand, when $0 < c_H < G$ under certain conditions the opposite holds. Therefore, as Proposition 2 (i) states $U$ prefers a single contract that keep both types alive when $c_H$ is not high enough and this comes in contrast to the previously mentioned result (Proposition 1) where no information is revealed.

![Figure 2: Optimal pricing under observable contract choice.](image-url)

When instead $c_H$ gets higher and above the $G$ threshold value then considerable differences occur (regarding the production efficiency) and thus $U$ prefers to offer a menu of contracts by trading off the pros and cons of each pricing scheme. In this way it separates the potential downstream types since incentive compatibility constraints must hold and thus information disclosure occurs. Why is that? On the one hand a single pricing contract scheme avoids sacrificing information rents and this is the reason why it is preferred when efficiency differences are not very significant. On the other hand a menu pricing scheme achieves much higher levels of allocative efficiency and therefore it is preferred as long as the types are more differentiated (up to point $W$ since only when $c_H \in (0, W)$ menu pricing takes place). When $c_H$ get higher than $W$ qualitatively the results are the same as when the contract choice is unobservable (Proposition 1 or Figure 1). In particular when $c_H > V$ is high enough then shutdown is the only attractive pricing strategy and when $c_H \in (W, V)$ that holds only when $\theta$ is high enough (Proposition 2 (iii) (b)).
It is important to note that in the absence of the upstream sector - in a one-tier Cournot duopoly - the total expected profits are higher in a complete information framework in which each firm is perfectly informed about its rival’s cost relative to the one with incomplete information. Thus, one would expect that it is to the best interest of the upstream firm to lead through its pricing decision in a separation in the downstream sector and thus to strongly prefer the menu pricing strategy. In the observable contract choice case where type separation is possible this is not necessarily true when the two types are lowly differentiated as Proposition 2 informs us. This is the case mainly because the asymmetry of information requires the upstream firm to leave information rents to the downstream firm and thus this separation becomes costly enough.

4.3 Discussion

(i) Quantity Forcing contracts: In the current literature on vertical relations under asymmetric information it is commonly used quantity forcing contracts instead of two-part tariffs, e.g. Gal-Or (1991), Martimort and Piccolo (2007), Hansen and Motta (2012). These kind of contracts specify the (fixed) tariff paid by the downstream party as well as the quantity transferred downstream, \{q_i, T_i\}. Using this type of contracts whether the rival firm observes the selection from the menu is irrelevant because after the acceptance there is no other choice made by the downstream firms. Thus, the mechanism that we describe above under two-part tariff contracts and the separation of types due to information disclosure does not occur. For this reason and given the proposed contract menus the downstream competition always takes place under incomplete information and each firm makes its choice based on the cost type. Using menu pricing the upstream firm’s equilibrium profits are:

\[
\Pi_{UF^*} = \frac{a^2(1 - \theta) - 2ac_H(1 - \theta) + c_H^2(1 + \theta)}{4(1 - \theta)}
\]

By comparing \(\Pi\)'s profits under this kind of contracts and under two-part tariffs used in subsection 3.1 we realize that it obtains higher profits in the latter case. Thus, the two-part tariffs are more desirable from the upstream firm’s viewpoint since they bring higher profits. This happens because the two-part tariffs are more flexible in the sense that production is delegated
to the downstreams. $U$ makes decisions about the wholesale prices, which affect the downstream choices, and the fixed fees and then downstream choices form the equilibrium in quantities.

(ii) **Downstream Exclusion:** One question that naturally arises is whether the upstream firm, although it suffers from less information, prefers to trade with both downstream firms or it is better to exclude one of them and use one intermediary. For simplicity we focus only on the cases where menu pricing is one of the options, i.e. $c_H \in (0, W)$. In order to identify whether downstream exclusion is optimal we should compare $U$’s profits in each case with the one when the downstream market is monopolized. When the contract choice is unobservable then $U$ never excludes one of the downstreams. When instead the contract choice is observable with the above illustrated consequences on the equilibrium the following results hold. We should note that when $U$ trades with one downstream firm offering a menu of two-part tariff contracts is always the best pricing strategy\(^7\) and its equilibrium profits are:

$$
\Pi_{UM}^{*} = \frac{1}{4(1-\theta)}[(1-\theta)(a-c_H)^2+\theta^2 c_H^2]
$$

When $0 < c_H < G$ and thus a single two-part tariff is used the following holds:

$$(i) \Pi_{UM}^{*} - \Pi_{UP}^{*} > 0 \text{ when } \frac{1}{2} < \theta < 1$$
$$(ii) \Pi_{UM}^{*} - \Pi_{UP}^{*} < 0 \text{ when } 0 < \theta < \frac{1}{2}.$$

For the range of the parameters in which a single contract is the equilibrium pricing strategy the upstream firm prefers to trade with one intermediary and thus to exclude one of the downstream firms when $\theta$ is high enough.

In case of a menu pricing (i.e. $G < c_H < W$), then by setting $\Pi_{UM}^{*} - \Pi_{US}^{*} = 0$ and solving for $c_H$ we obtain a threshold point $K = \frac{6a(1-\theta)}{46+66\theta+7\theta^2}$. Upstream profits under a downstream monopolist are higher ($\Pi_{UM}^{*} - \Pi_{US}^{*} > 0$) when $0 < c_H < K$ but $K > G$ only when $\frac{1}{2} < \theta < 1$. Thus only for this parameter values exclusion may occur. Our analysis reveals that in the presence of asymmetric information and contract choice observability downstream exclusion may be optimal

\(^7\)Shutdown occurs only for very high values of $\theta$. 

even with risk neutral downstreams. This comes in contrast with Hansen and Motta (2012) in which they show that exclusion never arises with risk neutral firms. The explanation for this divergence comes entirely from the observability of the contract choice that we assume and the implied information disclosure.

5 Conclusion

We have investigated the pricing decision of an upstream firm, vertically linked with downstream competing firms, when it is less informed regarding the marginal cost of them. Within the framework described above the upstream firm has to decide whether to offer a menu of contracts, designing a direct truthful mechanism or to offer a single contract. The former choice under contract choice observability leads to full information disclosure, while the latter one, when trade occurs with both types, to no information disclosure and as we comment above this has important implications. We demonstrate that the pricing schemes used by the upstream firm are heavily depend on whether downstreams’ contract choices from a menu are observed or not by the rival firm and by the extent of differentiation between the two potential types since this determines how costly is each pricing scheme. Both cases may form an equilibrium and that depends on the extent of differentiation since under unobservable contract choice and low differentiation a menu pricing is preferred while if the contract choice is observable then for low enough differentiation a single contract with both types prevails. Thus, it is no longer optimal for the upstream firm to price using a menu (when this is a viable option), as it is when the choice of the contract remains secret.

6 Appendix

Relevant threshold values:

(i) In order for the menu pricing and contract choice unobservability to be viable (to avoid corner solution) we should ensure that the quantities specified as Bayes-Nash equilibrium are positive for each possible combination of the wholesale prices. It can be easily confirmed that the strongest constraint comes from the case in which a firm is high cost, it selects the relevant
wholesale price from the menu, \( w_{iH}^{UN} \), and its rival is low cost. Setting \( q_i^H(w_{iH}^{UN}, w_{jL}^{UN}) = 0 \) and solving for \( c_H \) we obtain the relevant threshold, \( M \equiv \frac{a(1-\theta)}{3+5\theta} \).

(ii) In order for the menu pricing and contract choice observability to be viable (to avoid corner solution) we should ensure that the quantities specified as a Cournot equilibrium are positive for each possible combination of the wholesale prices. It can be easily confirmed that the strongest constraint comes from the case in which a firm is high cost, it selects the relevant wholesale price from the menu, \( w_i^{HOB} \), and its rival is low cost. Setting \( q_i^{HL}(w_i^{HOB}, w_j^{LOB}) = 0 \) and solving for \( c_H \) we obtain the relevant threshold, \( W \equiv \frac{3a(1-\theta)}{15+14\theta+3\theta^2} \).

(iii) In order for the single contract pricing with both types to be a viable choice \( q_i^H(i^{SC}, w_j^{SC}) \) must be positive. Setting it equal to zero and solving for \( c_H \) we obtain the relevant threshold, \( V \equiv \frac{a}{1+2\theta} \).

(iv) Note that in the shutdown case there is no restriction since the high cost firm(s) do not participate.

**Proof of Proposition 1:**

(i) When \( 0 < c_H < M \), we find that \( \Pi^{U*} - \Pi^{UP*} > 0, \Pi^{U*} - \Pi^{USH*} > 0 \).

(ii) Setting \( \Pi^{UP*} - \Pi^{USH*} = 0 \) and solving for \( c_H \) we obtain the relevant threshold, \( B \equiv \frac{a(9-\theta)-\sqrt{a^2(9-\theta)(9-\theta(15-2\theta)(10-(6-\theta)\theta))}}{(9-\theta)(1+2\theta)} \). We can easily confirm that \( B \in (M, V) \) if \( 0 < \theta < 0.803 \) and then when \( M < c_H < B \), we find that \( \Pi^{UP*} - \Pi^{USH*} > 0 \) and when \( B < c_H < V \) we find that \( \Pi^{UP*} - \Pi^{USH*} < 0 \).

If \( 0.803 < \theta < 1, B \notin (M, V) \) with \( B < M \) and we find that \( \Pi^{UP*} - \Pi^{USH*} < 0 \).

(iii) When \( c_H > V \), shutdown is the only viable option.

**Proof of Proposition 2:**

(i) Setting \( \Pi^{UP*} - \Pi^{US*} = 0 \) and solving for \( c_H \) we obtain the relevant threshold, \( G \equiv \frac{6a(1-\theta)}{46+57\theta+25\theta^2} \). When \( 0 < c_H < G \) we find that \( \Pi^{UP*} - \Pi^{US*} > 0, \Pi^{UP*} - \Pi^{USH*} > 0 \).

(ii) When \( G < c_H < W \) we find that \( \Pi^{UP*} - \Pi^{US*} < 0, \Pi^{US*} - \Pi^{USH*} > 0 \).

(iii) Setting \( \Pi^{UP*} - \Pi^{USH*} = 0 \) and solving for \( c_H \) we obtain the relevant threshold, \( B \). We can easily confirm that \( B \in (W, V) \) if \( 0 < \theta < 0.904 \) and then when \( W < c_H < B \), we find that \( \Pi^{UP*} - \Pi^{USH*} > 0 \) and when \( B < c_H < V \) we find that \( \Pi^{UP*} - \Pi^{USH*} < 0 \).

If \( 0.904 < \theta < 1, B \notin (W, V) \) with \( B < W \) and we find that \( \Pi^{UP*} - \Pi^{USH*} < 0 \).
(iv) When $c_H > V$, shutdown is the only viable option.

Note that $0 < G < W < V$ and $0 < M < V$.

7 References


Martimort, D. (1996), "Exclusive dealing, common agency and multiprincipals incentive the-


