Letter

Wireless Systems

Cell dimensioning in the CDMA uplink based on economic modelling†

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SUMMARY

We introduce a new approach, based on economic modelling, for determining the optimal cell coverage in the CDMA uplink. The approach takes into account resource usage in the uplink direction and user preferences, which are expressed through utility functions, and captures the tradeoff between having a large cell with many users versus having a smaller cell with fewer users, which however are able to transmit at a higher rate. Numerical investigations demonstrate how the optimal cell coverage is affected by the path propagation characteristics, the transmission power limit, the distribution of mobile users in a cell and the users’ preferences. Copyright © 2006 AEIT

1. INTRODUCTION

Due to their limited capacity, effective dimensioning of wireless networks in order to achieve efficient utilisation of scarce resources is important. One specific dimensioning problem is that of determining the cell coverage in a mobile cellular network. Moreover, code division multiple access (CDMA) systems, such as Wideband CDMA and IS-95, have the ability to dynamically adjust the cell coverage to achieve a balanced distribution of the load among neighbouring cells [1]; this procedure is also referred to as cell breathing. To achieve such load balancing, cells with heavy load could decrease their coverage, hence their load, whereas neighbouring cells that are less loaded could increase their coverage to accommodate mobile users that couldn’t be handled by the heavier loaded cell.

The objective of this paper is to propose and investigate a new approach for cell dimensioning based on maximising the economic efficiency, which is expressed by the aggregate utility of all mobile users in a CDMA cell. Maximising the aggregate utility results in network resources being used in a more efficient way and according to the actual user preferences; such efficient utilisation of resources results in more competitive services [2], which is important in today’s competitive telecommunications market. Other approaches to load balancing cannot achieve such efficiency, since they do not take into account user preferences. A user’s preferences are expressed through a utility function which is a relative measure for the satisfaction that a user obtains, or the performance that an application experiences, with a particular level of service, which in this paper is quantified by the average transmission rate. An important feature of our approach is that it captures, through a simple model, the tradeoff of having a large cell with many users, compared to having a small cell with fewer users, which however are able to transmit at a higher rate. Additionally, the

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The resource constraint can be approximated by \[ \sum_i r_i \gamma_i < W \] (1)

Hence, the amount of resources used by a mobile is given by the product of its transmission rate and target bit-energy-to-noise-density ratio (signal quality).

If each mobile \( i \) can transmit with maximum power \( p_i \), then the wireless resource constraint is \[ \sum_i r_i \gamma_i < W \] (2)

From the last equation, observe that if all mobiles have the same maximum power and the same resource usage, then the wireless resource constraint is determined by the mobile with the smallest channel gain \( g_i \).

The above resource usage model can be extended to take into account the interference from neighbouring cells, for example through the intercell interference coefficient [8]. Additionally, the model can be extended to the case of imperfect power control in the presence of shadow and Rayleigh fading, by introducing a maximum utilisation component in the right-hand side of the above equations to limit the load of the system. We do not discuss these extensions further, since they are not the focus of this paper and the resource constraint model expressed by Equation (2) suffices for our purposes, since it contains the dependence on the mobile transmission power limit, which we will further investigate within the economic model presented in the following section.

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3. ECONOMIC EFFICIENCY AND OPTIMAL CELL COVERAGE

In this section, we first formulate the problem of maximising the aggregate utility in the CDMA uplink for a given cell coverage. The model takes into account the corresponding resource constraint and the limited transmission power of mobile nodes, and will allow us to formulate the problem of optimally selecting the transmission rate and signal quality for mobiles in a cell with fixed coverage. Then, based on the aforementioned model, we will discuss the problem of finding the cell coverage that maximises the aggregate utility.

3.1. Economic efficiency

Consider the case of elastic (best-effort) traffic, where users value only the average throughput of successful data transmission. This throughput is given by the product $r_i P(\gamma_i)$ of the transmission rate and the probability of successful packet transmission, which depends on the target bit-energy-to-noise-density ratio $\gamma_i$. Thus, the utility for an elastic user $i$ that values only his average transmission rate has the form $U_i(r_i P(\gamma_i))$. For elastic traffic, the utility is typically concave, illustrating the law of diminishing return where as the average transmission increases the additional unit of service (transmission rate) yields less additional value.

The problem of maximising the aggregate utility (social welfare) of all mobile users $N$ in a CDMA cell is

$$\max \sum_{i} U_i(r_i P(\gamma_i))$$

subject to

$$\sum_{i} r_i \gamma_i \leq W - \frac{\eta}{\min \{\frac{\sum_{i} r_i \gamma_i}{\gamma_i}\}}(3)$$

where $i \in N$. Let $K$ be the set of mobiles whose transmission power constraints limit the cell coverage, that is the mobiles for which the constraint in Equation (3) is tight, hence

$$K = \{ k : k = \arg \min_{j} \left\{ \frac{\sum_{i} r_i \gamma_i}{\gamma_i} \right\} \}(4)$$

In the remainder of the paper, we will use the term 'border' mobile to refer to a mobile in set $K$. If the utilities $U_i(.)$ are differentiable and strictly concave, and the Lagrangian method for solving the above optimisation problem can be applied, then the Lagrangian is

$$L = \sum_{i} U_i(r_i P(\gamma_i)) + \lambda \left( W - \frac{\sum_{i} r_i \gamma_i}{\gamma_i} - \sum_{i} z_i \right)$$

where $\lambda$ is the shadow price (Lagrange multiplier) for the wireless resource constraint, which reflects the level of demand for the wireless resource. By setting $z_i = r_i \gamma_i$, and assuming $\gamma_i > 0$, the above Lagrangian can be written as

$$L = \sum_{i} U_i \left( z_i \frac{P(\gamma_i)}{\gamma_i} \right) + \lambda \left( W - \frac{u_{1k}}{u_{2k}} - \sum_{i} z_i \right)$$

Maximising the above Lagrangian is equivalent to maximising

$$\max \sum_{i} U_i \left( z_i \frac{P(\gamma_i)}{\gamma_i} \right) + \lambda \left( W - \frac{u_{1k}}{u_{2k}} - \sum_{i} z_i \right)$$

The last equation shows that the global problem of maximising the aggregate utility can be decomposed into two separate problems. The first gives the optimal signal quality $\gamma_i^*$ for each user

$$P_i(\gamma_i^*) = P_i(\gamma_i^*)$$

hence the optimal signal quality is independent of the user's utility and depends only on $P_i(\gamma_i^*)$. Moreover, note that the optimal signal quality is also independent of whether the mobile is located in the interior or on the border of the cell. A similar result, for the case where the mobile transmission power limits are not taken into account is shown in Reference [5].

The second optimisation problem gives the optimal transmission rate and is defined as follows

$$\max \sum_{i} U_i(r_i P(\gamma_i^*))$$

subject to

$$\sum_{i} r_i \gamma_i^* \leq W - \frac{\eta}{\min \{\frac{\sum_{i} r_i \gamma_i^*}{\gamma_i^*}\}}(7)$$

where $i \in N$ (set of all mobiles) and $k \in K$ (defined in Equation (4)).

From the first order conditions of the above optimisation problem, we find that the optimal transmission rates satisfy the following

$$U_i(r_i^* P(\gamma_i^*)) P(\gamma_i^*) = \lambda \gamma_i^*$$

if $i \not\in K$

$$U_i(r_i^* P(\gamma_i^*)) P(\gamma_i^*) = \lambda \left( \frac{1}{\gamma_i^*} + 1 \right) \gamma_i^*$$

if $i \in K$

where $\lambda$ is the shadow price for the constraint (7).
The optimal rates $r_i^*$ can be found in a distributed and decentralised manner, by sending to each mobile a 'price' signal that is proportional to the amount of resources they use. To achieve efficient resource utilisation, the shadow price can be iteratively adjusted based on a tatonnement process: the shadow price $\lambda$ is increased (decreased) if the aggregate demand that appears in the left-hand side of inequality (7) is greater (less) than the available wireless resource, which is given by the right-hand side of (7). Note that during the above iterative procedure for adapting the price $\lambda$, the set of border nodes $K$ can change. In each iteration, the set $K$ is determined by Equation (4).

The last two equations depict the contribution of each mobile user to the wireless load, expressed through a price, and can be used to estimate the optimal transmission rates for each mobile. In particular, this price for mobiles located at the cell border should be $\lambda \left( \frac{1}{2\pi d^2} + 1 \right)$, which is higher than the price $\lambda$ for mobiles located in the cell’s interior; this captures the fact that mobiles at the boundary influence the resource usage and the wireless resource constraint. Due to this, as we will illustrate in Section 4, the optimal transmission rate for mobiles at the cell border is lower than the optimal transmission rate for mobiles located in the interior of the cell.

In the case of multiple cells, the aggregate utility would involve the users of all cells. Additionally, each cell would have a resource constraint of the form (3). In this case, each cell would have its own shadow price which would reflect the level of demand in that cell.

3.2. Optimal cell coverage

The previous section discussed the optimal selection of the transmission rates and signal quality, for a fixed cell coverage. As expressed by the constraint (2), there is a tradeoff between the cell coverage and the achievable transmission rates. If we increase the coverage, the path gain for mobiles on the cell's boundary decreases, hence the right-hand side of (2) also decreases, thus reducing the aggregate rate that can be achieved by the mobiles. On the other hand, by decreasing the coverage, that is by not serving mobiles that are far from the base station, the remaining mobiles would be able to send at a higher rate, thus their utility could increase. Moreover, recall that the optimal signal quality is given by Equation (5), and is independent of the cell coverage, hence the coverage affects only the achievable transmission rates. In the case of multiple cells, the coverage of different cells is not independent, and one would seek to determine the aggregate utility for different placements of the boundary between neighbouring cells.

Under the above tradeoff, the optimal cell radius is the value that maximises the aggregate utility. As we will demonstrate in the numerical results section, for concave utility functions and the distribution of mobile users considered, the aggregate utility for a single cell is a unimodal function of the cell radius that is an increasing function of the cell radius until it obtains a maximum value, after which it is a decreasing function of the cell radius; this property allows us to find the optimal cell radius efficiently using an efficient search method, such as a golden section search [12].

4. NUMERICAL INVESTIGATIONS

In the numerical investigations, we consider the Okumura-Hata propagation model [11], according to which the path gain in an urban environment is

$$g_{urb}(d) = 1.82 \cdot 10^{-14} \times d^{-3.52}$$

where $d$ is the mobile’s distance from the base station in km. For a suburban environment, the path gain is

$$g_{sub}(d) = 1.15 \cdot 10^{-13} \times d^{-3.52}$$

Regarding the distribution of mobile users within a cell, we assume that the number of users within distance $d$ from the base station is given by

$$N(d) = \rho \pi d^2$$

In the numerical investigations, we consider the values $\rho = 20, 30$ and $v = 1, 2$, Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chip rate (W)</td>
<td>3.84 Mcps</td>
</tr>
<tr>
<td>Noise (W)</td>
<td>$10^{-1}$ Watt</td>
</tr>
<tr>
<td>Max. transmission power (P)</td>
<td>0.2, 0.6 Watt</td>
</tr>
<tr>
<td>Path gain ($g(d)$)</td>
<td>$kd^\gamma$, $\gamma = 3.52$, $k_{urb} = 1.82 \cdot 10^{-14}$, $k_{sub} = 1.15 \cdot 10^{-13}$</td>
</tr>
<tr>
<td># of mobiles (N(i))</td>
<td>$\mu i^\alpha$</td>
</tr>
<tr>
<td>BER (\gamma) (DPSK)</td>
<td>$\rho = 20, 30$, $v = 1, 2$, $0.5e^{-\gamma}$</td>
</tr>
<tr>
<td># of bits per packet (L)</td>
<td>60</td>
</tr>
<tr>
<td>Utility</td>
<td>$1 - e^{-\frac{b}{L}}$, $b = 0.1, 0.2$</td>
</tr>
</tbody>
</table>

$d$ is distance in km.

Table 1. Parameters for the numerical investigations.

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Next we describe the model for the packet success rate $P(\gamma)$, which we assume to be the same for all mobiles. For additive white Gaussian noise and a non-fading channel, the BER for differential phase shift keying (DPSK) modulation is [13]

$$\text{BER}(\gamma) = 0.5e^{-\gamma}$$

Although the above propagation model is simplified, the qualitative conclusions are the same with those that would be obtained for a more detailed model that includes shadowing and Rayleigh fading.

If there is no error correction, and bit errors are independent and are all detected, then the packet success probability is

$$P(\gamma) = (1 - \text{BER}(\gamma))^L$$

where $L$ is the number of bits in one packet. From this equation and Equation (5), we find that for $L = 60$, the optimal bit-energy-to-noise-density ratio is $\gamma^* = 5$. The optimal transmission quality can be derived for other modulation schemes and in the presence of forward error correction (FEC), in a similar manner [5]. The values of the other parameters used in the numerical investigations are shown in Table 1. For simplicity, we assume that all mobile users have the same utility and the same maximum transmission power.

Figure 1 shows the aggregate utility as a function of the cell radius (coverage), in an urban and suburban environment for different transmission power constraints. Observe that the dependence is concave, and the aggregate utility is maximised at some coverage. Furthermore, for a higher maximum transmission power, the aggregate utility increases and obtains its maximum at a larger cell radius. The reason for such behaviour is that an increasing power limit increases the right-hand side of the constraint in Equation (2). Indeed, a larger power limit enables mobiles to achieve a higher transmission rate, Figure 2. Also observe in Figure 2 that a mobile located at the cell border should transmit at a much lower rate in order to efficiently utilise the wireless resources. As discussed in the previous section, this is because such border nodes influence the wireless constraint, in addition to using wireless resources.

Figure 1 shows that for a suburban environment, where the path gain is higher than in an urban environment, both the aggregate utility and the coverage at which the aggregate utility is maximised are higher. Moreover, in a suburban environment the optimal transmission rate is higher.

Figure 3 shows that when the density of mobiles decreases with the distance from the base station (which is the case $v = 1$ in Table 1), the aggregate utility remains approximately the same as in the case where the density of mobile users is independent of the distance from the base station (which is the case $v = 2$). However, the coverage for which the aggregate utility is maximised is larger in the

Figure 1. Aggregate utility as a function of cell radius in an urban and suburban environment, for different mobile transmission power constraints $\beta$. The figure shows that the aggregate utility is higher, and is achieved at a larger coverage, in the case of a larger transmission power limit and a suburban environment.

Figure 2. Optimal transmission rate as a function of cell radius in an urban and suburban environment, for different mobile transmission power constraints $\beta$, and for internal and border nodes. The figure shows that the optimal transmission rate is smaller in the case of a smaller transmission power limit, an urban environment and mobiles at the cell border.

Figure 3 shows that when the density of mobiles decreases with the distance from the base station (which is the case $v = 1$ in Table 1), the aggregate utility remains approximately the same as in the case where the density of mobile users is independent of the distance from the base station (which is the case $v = 2$). However, the coverage for which the aggregate utility is maximised is larger in the

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Figure 3. Aggregate utility as a function of cell radius for different mobile distributions and densities. When the number of mobiles is \( N(d) \propto d^3 \), the density is uniform, whereas if it is \( N(d) \propto d \) the density decreases with the distance from the base station. The figure shows that the aggregate utility is independent of the distribution of mobiles, and is achieved at a higher coverage when the density does not decrease with the distance from the base station. On the other hand, the optimal cell coverage is independent of the density factor \( \rho \) in Table 1.

In the latter case (\( v = 2 \)). In other words, if by increasing the coverage, the number of mobiles that can be handled increases by a significant amount, then this can result in higher aggregate utility. Finally, Figure 3 shows that the distribution density factor \( \rho \) does not affect the optimal coverage, but does affect the value of the maximum aggregate utility.

Figure 4 shows that for a steeper utility, that is a utility with larger \( b \) (see Table 1) in which case mobile users have a higher utility for small values of the transmission rate, both the aggregate utility and the coverage at which the aggregate utility is maximised are higher. The reason for this behaviour is that if utility \( U_1 \) is steeper than \( U_2 \) we have \( U_1(x) > U_2(x) \), that is for a steeper utility, the same throughput \( x \) yields a higher user satisfaction; hence, increasing the coverage to accommodate more mobile users results in a higher aggregate utility compared to the case of a less steep utility, even if that means giving less resources to each user.

5. CONCLUSIONS

We have presented a new approach for determining the cell coverage in a CDMA wireless system, which takes into account the preferences of mobile users expressed through utility functions. This will be important in future wireless systems supporting data applications with different performance requirements, which should be taken into account for cell dimensioning. The approach is based on maximising the aggregate utility of a CDMA cell, and captures the tradeoff between having a large cell, hence accommodating a large number of mobile users, versus having a smaller cell, hence accommodating a smaller number of users, which however can achieve a higher transmission rate.

The analysis of our model shows the influence of mobile users located at the interior of a cell and on the border of the cell, on the resource usage and the wireless network constraint. When the mobile users value the average data throughput, we showed that the optimal signal-to-noise-density ratio depends solely on the packet success probability as a function of the signal quality, and is independent of the mobile user utility or whether he is located in the interior or on the boundary of the cell. On the other hand, the optimal transmission rate for a mobile at the boundary of a cell should be lower than for a mobile in the interior, since the former influences the wireless resource constraint.

The approach presented in this paper can be applied in practise by assuming a set of different classes, each class can refer to applications with the same performance requirements, hence the same utility function. The utility for each class can be estimated by measuring the quality or performance that applications corresponding to each class experience for different transmission rates, or by monitoring the choices made by actual users [14].

The focus of this paper was on the CDMA uplink. Unlike the uplink, the downlink is limited solely by the transmission power at the base station, and not the individual power.

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constraints at the mobile nodes; hence, the corresponding model for dimensioning a CDMA cell in the downlink direction would differ from the one presented in this paper. Another direction for further research is the investigation of the proposed model in the case of multiple cells, where interference from neighbouring cells must be taken into account.

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