Price Discrimination in Two-Sided Markets*

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Abstract

We examine the profitability and the welfare implications of price discrimination in two-sided markets. Platforms have information about the preferences of the agents that allows them to price discriminate within each group. The conventional wisdom from one-sided horizontally differentiated markets is that price discrimination hurts the firms and benefits consumers, *prisoners' dilemma*. Moreover, it is well-known that the presence of indirect externalities in two-sided markets intensifies the competition. Despite all these, we show that the possibility of price discrimination, in a two-sided market, may actually *soften* the competition. Therefore, the implications of price discrimination assuming that the market is one-sided may not carry over to two-sided markets. This is the case regardless of whether prices are public or private, although private prices boost profits. Our analysis also sheds light on the welfare properties of price discrimination in intermediate goods markets, such as Business-to-Business (B2B) markets.

Keywords: Price discrimination; Two-sided markets; Indirect network externalities; Market segmentation.

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1 Introduction

Two-sided markets have recently received significant attention in the industrial organization literature [e.g., Armstrong (2007), Caillaud and Jullien (2003), Rochet and Tirole (2003) and Rochet and Tirole (2005)]. Two-sided (or multiple-sided) markets are markets that are organized around intermediaries or "platforms" with two (or multiple) sides who should join a platform in order for successful exchanges (trade) to take place. Indirect network externalities play an important role in these markets. For example, videogame platforms (e.g., Nintendo, Sony, Microsoft) need to attract both gamers and game developers. TV networks need to attract advertisers and viewers.[1] Credit cards need merchants and users.[2] More formally, a two-sided market is defined as a one where the volume of transactions between end-users depends on the structure of the fees and not only on the overall level of fees charged by platforms [Rochet and Tirole (2005)].

The development of the Internet and the rapid growth of sophisticated software tools have enabled firms to collect large amounts of information about consumer preferences, characteristics and purchasing history. Firms can use such information to segment consumers into distinct groups and target each group with different prices and products of different qualities and attributes.[3] The purpose of this paper is to study the issue of price discrimination in two-sided markets. There exists a relatively large literature on oligopolistic price discrimination in "one-sided" markets,[4] but this paper is among the first ones that examine this problem in the context of a two-sided market.[5] We assume that there are two platforms that are horizontally differentiated. The literature on price discrimination in one-sided markets where products are horizontally differentiated suggests–under the standard Hotelling-type assumptions–that price discrimination leads to lower prices and profits for the firms (prisoners' dilemma).[6] The reason is best response asymmetry [Corts (1998)].[7] Under horizontal differentiation, one firm's strong market is the other firm's weak market and vice versa. When price discrimination is feasible, a firm can charge a low price to the loyal customers of the

[1] In general, most media and advertising markets are two-sided markets, e.g., Anderson and Coate (2005) and Anderson and Gabszewicz (2006).

[2] Other examples of two-sided markets include, newspapers, scholarly journals, magazines, shopping malls, dating services and Business-to-Business (B2B) markets. See also the papers cited above for more detailed discussions and examples.

[3] Dell, for example, follows this practice. According to the June 8, 2001 *Wall Street Journal*: "One day recently, the Dell Latitude L400 ultralight laptop was listed at $2,307 on the company's Web page catering to small businesses. On the Web page for sales to health-care companies, the same machine was listed at $2,228, or 3% less. For state and local governments, it was priced at $2,072.04, or 10% less than the price for small businesses." As another example, credit cards target credit card holders of rival issuers with rates that are typically lower than the rates they charge to their own customers.

[4] By "one-sided" markets we simply mean markets with no externalities.

[5] For a survey of the literature on oligopolistic price discrimination in one-sided markets we refer the reader to Armstrong (2006) and Stole (2003).

[6] See, for example, Thisse and Vives (1988), Shaffer and Zhang (1995), Bester and Petrakis (1996), Chen (1997) and Fudenberg and Tirole (2000).

[7] Best response asymmetry is *only* a necessary condition for prices and profits to go down when firms price discriminate.

rival firm, while at the same time it can keep its price high to its own loyal customers. The problem is that the other firm can follow the same strategy, resulting in a very intense competition among the firms for consumers.

Therefore, the conventional wisdom is that, in markets with roughly symmetric firms and products that are horizontally differentiated, price discrimination is beneficial for the consumers (at least on average).[8] The advice then given to policymakers and antitrust authorities is that they should not worry much about firms acquiring and using consumer information with the intention to customize prices, because after all firm competition for each consumer dissipates profits and transfers most of the surplus to consumers.

Furthermore, it is well-known that the presence of indirect externalities in two-sided markets can intensify the competition, Armstrong (2007). Therefore, one would expect that price discrimination in a two-sided market will generate a very competitive environment with low prices and profits. Nevertheless, we show that the possibility of price discrimination may actually *soften* the competition, even in a market with symmetric and horizontally differentiated platforms. The game need not be a prisoners' dilemma. In particular, price discrimination in two-sided markets is possible to increase prices for (almost) all consumers relative to uniform prices. Our result has important theoretical and policy implications because it demonstrates that price discrimination is more likely to be anti-competitive in two-sided markets than it is in one-sided markets. More fundamentally, it suggests that two-sided markets can be very different from one-sided markets. An interesting implication is that firms in two-sided markets may seek to acquire customer information (in order to facilitate price discrimination) more aggressively than in one-sided markets.

Our analysis can also apply to intermediate goods markets where price discrimination is more likely to raise antitrust concerns than in final goods markets. Indeed, in the Unites States price discrimination is illegal in intermediate goods markets under the Robinson-Patman act. Each platform in our model can be viewed as a Business-to-Business (B2B) website which matches input suppliers with producers [e.g., Caillaud and Jullien (2003)]. The Internet facilitates the collection and application of information about the users' preferences and characteristics and price cuts can only be observed by targeted agents.[9] An interesting question which arises then is whether platforms should be restricted to charge uniform prices. We will return to this interpretation of our model later.

The model we develop consists of two platforms that are horizontally differentiated. There are two groups of agents and each agent is assumed to join only one platform (single-homing). Agents from one group that contemplate joining a given platform care about the number of agents from

[8] When some kind of firm asymmetry is introduced the game may no longer be a prisoners' dilemma, e.g., Shaffer and Zhang (2002) and Liu and Serfes (2005a). Price discrimination benefits the firm with the larger market share.

[9] See the FTC report on "Competition policy in the world of B2B electronic market places." Whether all prices are observable by all agents is crucial in the presence of network externalities. As we show the equilibrium depends on the price observability assumption.

the other group that will join the same platform.[10] This (indirect) externality is captured by the cross-group externality parameters. Each platform charges lump-sum prices. Under a uniform pricing rule each member of a group that joins a platform pays the same price (across groups the prices of a platform are allowed to differ). Under price discrimination each agent pays a different price (perfect price discrimination). We show that if the cross-group externality parameters are high enough price discrimination increases platform profits and hurts consumer welfare.

The intuition for this result is as follows. A uniform equilibrium price charged by a platform to (say) group 1 agents balances optimally the following three effects: i) loss (gain) of inframarginal rents, ii) gain (loss) of marginal agents from group 1 and iii) gain (loss) of agents from group 2. The third effect is called a *feedback* effect and it has to do with the two-sidedness of the market. A price reduction to group 1 first increases the number of group 1 agents who join the platform. This induces more group 2 agents to join the platform, which in turn allows the platform to extract more revenue from group 1 agents. This feedback effect is responsible for lowering the equilibrium (uniform) prices because platforms compete more aggressively to sign up agents. The cross-group externality parameters affect the magnitude of the feedback effect. The higher the values of these parameters the lower the uniform equilibrium prices and profits.

Under perfect price discrimination the feedback effect is *absent*.[11] Let's explain why. We focus on group 1 agents. Each agent in equilibrium joins the platform that is closest to his "ideal" platform. When prices can be customized competition is for each agent individually. Each platform charges to the agents that are located in the rival platform's territory marginal cost prices and to its own agents a platform can charge a premium over marginal cost. This premium is equal to the transportation cost difference a particular agent will incur by joining instead the rival platform and the difference in the memberships of group 2 agents across the two platforms. The latter difference is zero in a symmetric equilibrium. If a platform now lowers its price to a particular agent (or a group of agents) the platform will loose rents, but it will not sign up more agents. Due to the "limit price" nature of the problem under perfect price discrimination the feedback effect disappears in equilibrium. Equilibrium discriminatory prices are free of the cross-group externality parameters. Furthermore, price discrimination, as it is well-known from one-sided models, intensifies the competition. This effect is also present in our model. But because the cross-group externalities are absent under perfect price discrimination and present when prices are uniform, we can conclude that when these externalities are strong perfect price discrimination leads to higher (average) prices and profits.

Perfect price discrimination helps us to derive a clean prediction and also to extract a clear

[10] Examples include newspapers and scholarly journals. Advertisers care about the number of people who read a particular newspaper and readers care about the number of advertisements in a newspaper. In the market of scholarly journals, authors care about the number of readers and readers care about the number of authors (research papers), McCabe and Snyder (2006).

[11] This result depends crucially on the assumption that prices cannot become negative. We offer a discussion on this in the main body of the paper.

intuition. But how much of our result is due to the assumption of perfect price discrimination? To answer this important question, we extend our model to allow for imperfect price discrimination. Platforms can segment the agents into groups but this segmentation is not perfect. In particular, we employ the segmentation approach that was developed in Liu and Serfes (2004).[12] Platforms lack the necessary information needed to identify the preferences of each agent with perfect accuracy. Nevertheless, platforms possess some information which can be used to segment agents into groups. Each agent segment pays a different price. We allow the number of segments to vary exogenously in an attempt to capture the quality of information. A higher number of available segments implies that platforms can identify the preferences of each agent with higher precision (higher quality of consumer information). In the limit, we recover the perfect price discrimination paradigm. Imperfect price discrimination is interesting because it combines features from both the uniform pricing and the perfect discrimination cases. Further, it allows us to draw a more comprehensive picture of equilibrium profits and welfare under varying levels of quality of consumer information.

We examine two different cases depending on whether all prices are observed by all agents before they join a platform. In the first case, we assume that prices are public. This assumption may not be very realistic when platforms discriminate via many prices, but it can serve as a benchmark case. Also, it may not be a bad assumption if the number of segments (and hence prices) is small. In the second case, we assume that agents observe each platform's two regular prices (one for each group), but they do not observe the targeted discounts off the regular prices. Each agent only observes the discount that is offered to him (private prices). This distinction matters in the presence of network externalities.[13]

We show that equilibrium profits can be non-monotonic (U-shaped) with respect to the quality of agent information, regardless of whether prices are public or private. Competition is more intense when prices are public. A price cut to a specific agent segment that is observed by everyone also attracts agents from other segments to the platform. So, targeted price cuts are more lucrative when they are publicly observed than when they are not. This suggests that firms have incentives to make prices less transparent. The U-shape pattern of the equilibrium profits with respect to the degree of segmentation (quality of information) implies that price discrimination hurts the platform profits (relative to the uniform price profits) when the quality of information is low. When the quality of information is high–*and unlike the case in one-sided markets*– price discrimination can benefit the platforms.

When the market is a B2B market, then our result implies that price discrimination will lead to higher input prices if and only if platforms have detailed information about the preferences of the participants. To arrive at this result, we assume that each firm is seeking to buy only one unit of the input and each input seller sells only one unit. The platforms facilitate the matching process

[12] Liu and Serfes (2005a & 2005b) and Eruysal and Ouksel (2006) have employed the segmentation approach as well to study the profitability of price discrimination in other contexts.

[13] As we will argue later, it does not matter when firms price discriminate perfectly.

between the two sides [as in Caillaud and Jullien (2003)]. Let's assume that platforms have very good information about the agents. If platforms are allowed to customize their prices then firms end up paying higher prices for the right to trade a unit of the input. Now if we assume that the prices the participants pay to join a platform do not affect the bargaining process between an input supplier and a firm that will ensue once a matching takes place, then a higher price charged by a platform will lead to a higher overall price a firm will have to pay in order to acquire its input. If firms can pass part of this extra cost on to consumers, then price discrimination is anti-competitive. However, the reverse may be true if platforms do not possess very detailed information about the participants. In this case the cost of acquiring the input is reduced due to price discrimination.

Caillaud and Jullien (2003) and Armstrong (2007) also allow for price discrimination. In Caillaud and Jullien agents in each group are homogeneous and therefore price discrimination means different prices charged to each group of agents, while within each group the price is constant. This is also the meaning of price discrimination in Armstrong (2007), although he allows for heterogeneous populations of agents. In contrast, we allow the prices within each group to vary. Price discrimination in Armstrong's model can lead to higher or lower prices and profits, but the condition that determines the profitability of price discrimination is qualitatively different from the condition (and intuition) we derive in this paper. In Armstrong the *differences* between: i) the degrees of platform differentiation and ii) the cross-group externalities across groups play an important role. In contrast, in our paper the *levels* matter. As a consequence, if we assume complete symmetry (i.e., same degrees of platform differentiation and same cross-group externalities across groups) then price discrimination always yields the same prices and profits with uniform prices in Armstrong's paper. In our paper, however, this is not the case.

The rest of the paper is organized as follows. In section 2 we present the model. In section 3 we perform the analysis assuming that platforms can target each agent perfectly. We relax the perfect price discrimination assumption is section 4. We conclude we section 5. The appendix contains the proofs of propositions.

2 The description of the benchmark model

The benchmark model we develop is similar to Armstrong's (2007) modeling framework. There are two groups of agents $\ell = 1, 2$ and two horizontally differentiated platforms $k = A, B$. We will denote the "other" group of agents by m. We capture platform differentiation as follows. There is a continuum of agents of group ℓ that is distributed on the $[0, 1]$ interval according to the distribution function $F_\ell(\cdot)$ with density f_ℓ. The distributions are independent across the two groups of agents and symmetric about $\frac{1}{2}$, i.e., $F_\ell\left(\frac{1}{2}\right) = \frac{1}{2}$ and $f_\ell(x) = f_\ell(1-x)$. The two platforms are located at the two end points of each interval, with platform A located at 0 and platform B located at 1, see figure 1. The common per-unit transportation cost of both groups is denoted by $t > 0$. We assume that each agent joins only one platform (single-homing). Each member of a group who joins a given

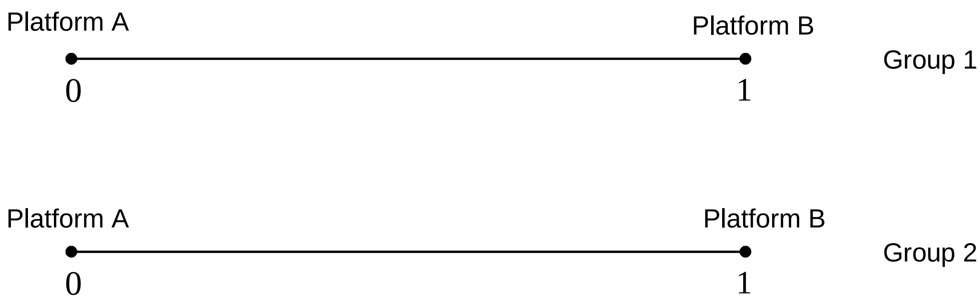

Figure 1: Graphical represenation of the benchmark model

platform cares about the number of members from the other group who join the same platform. Denote by $n_{\ell k}$ the number of participants from group ℓ that platform k attracts. The maximum willingness to pay for a member of group ℓ if he joins platform k is given by $V + \alpha_\ell n_{mk}$, where V is a standalone benefit each agent receives independent of the number of participants from the other group on platform k. The parameter $\alpha_\ell > 0$ measures the cross-group externality for group ℓ participants. The indirect utility of an agent from group ℓ who is located at point $x \in [0, 1]$ is given by,

$$U_\ell = \begin{cases} V + \alpha_\ell n^e_{mA} - tx - p_{\ell A}(x), & \text{if he joins platform } A \\ V + \alpha_\ell n^e_{mB} - t(1-x) - p_{\ell B}(x), & \text{if he joins platform } B \end{cases} \tag{1}$$

where $p_{\ell k}(x)$ is platform k's lump-sum charge to a group ℓ participant who is located at point x and n^e_{mk} denotes the expectations agents from group ℓ have about how many agents from group m will join platform k. Under a uniform pricing rule prices are constant across all agents in the same group (prices are allowed to vary across groups), while under discriminatory pricing the price each agent pays depends on his preferences (location). We assume that V is high enough which ensures that the market is covered. The costs are zero.

The timing of the game is as follows. In stage 1, the two platforms make, simultaneously, their pricing decisions. In stage 2, the agents decide which platform to join.

3 Analysis

First, we solve for a symmetric equilibrium assuming that each platform charges uniform prices to the agents of each group. Second, we assume that each platform can price discriminate perfectly the agents of each group. We assume that agents have rational expectations about how many agents will join each platform.

3.1 No price discrimination

Each agent observes all prices before he decides which platform to join, [e.g., Caillaud and Jullien (2003) and Armstrong (2007)]. The location of the marginal agent from group ℓ, who is indifferent between A and B, is given by,

$$\begin{aligned} V+\alpha_\ell n_{mA}^e - tx - p_{\ell A} &= V+\alpha_\ell n_{mB}^e - t(1-x) - p_{\ell B} \Rightarrow \\ x_\ell &= \frac{\alpha_\ell\left(n_{mA}^e - n_{mB}^e\right) - p_{\ell A} + p_{\ell B} + t}{2t}. \end{aligned} \tag{2}$$

where $n_{mA}^e = F_m\left(x_\ell^e\right)$ and $n_{mB}^e = 1-F\left(x_\ell^e\right)$. Therefore, the implicit functions for the market shares are given by,

$$x_1 = \frac{\alpha_1\left[2F_2\left(x_2^e\right)-1\right]-\left(p_{1A}-p_{1B}\right)+t}{2t};\; x_2 = \frac{\alpha_2\left[2F_1\left(x_1^e\right)-1\right]-\left(p_{2A}-p_{2B}\right)+t}{2t}.$$

Since expectations are rational we must have $x_\ell = x_\ell^e$, or $n_{mk}^e = n_{mk}$. By invoking the implicit function theorem we can derive the effect of prices on the market shares,

$$\begin{aligned} \frac{\partial x_1}{\partial p_{1A}} &= \frac{\partial x_2}{\partial p_{2A}} = -\frac{t}{2\left[t^2-\alpha_1\alpha_2 f_1\left(x_1\right) f_2\left(x_2\right)\right]}, \frac{\partial x_1}{\partial p_{2A}} = -\frac{\alpha_1 f_2}{2\left[t^2-\alpha_1\alpha_2 f_1\left(x_1\right) f_2\left(x_2\right)\right]} \\ \text{and } \frac{\partial x_2}{\partial p_{1A}} &= -\frac{\alpha_2 f_1}{2\left[t^2-\alpha_1\alpha_2 f_1\left(x_1\right) f_2\left(x_2\right)\right]}. \end{aligned} \tag{3}$$

For the Jacobian of the system of the implicit functions to have a non-zero determinant it must be that $t^2-\alpha_1\alpha_2 f_1\left(x_1\right) f_2\left(x_2\right) \neq 0$, for all x_1 and x_2. We further assume that $t^2-\alpha_1\alpha_2 f_1\left(x_1\right) f_2\left(x_2\right) > 0$, for all x_1 and x_2.

The platforms' profit functions are given by,

$$\begin{aligned} \pi_A &= p_{1A}n_{1A}+p_{2A}n_{2A} = p_{1A}F_1\left(x_1\right)+p_{2A}F_2\left(x_2\right) \text{ and} \\ \pi_B &= p_{1B}n_{1B}+p_{2B}n_{2B} = p_{1B}\left[1-F_1\left(x_1\right)\right]+p_{2B}\left[1-F_2\left(x_2\right)\right]. \end{aligned}$$

The first order conditions of platform A are given by,

$$\begin{aligned} \frac{\partial \pi_A}{\partial p_{1A}} &= F_1\left(x_1\right)+p_{1A}f_1\left(x_1\right)\frac{\partial x_1}{\partial p_{1A}}+p_{2A}f_2\left(x_2\right)\frac{\partial x_2}{\partial p_{1A}} = 0, \\ \frac{\partial \pi_A}{\partial p_{2A}} &= F_2\left(x_2\right)+p_{2A}f_2\left(x_2\right)\frac{\partial x_2}{\partial p_{2A}}+p_{1A}f_1\left(x_1\right)\frac{\partial x_1}{\partial p_{2A}} = 0. \end{aligned}$$

Each first order condition has three terms. Suppose platform A lowers its price for group ℓ. The first two terms in each first order condition capture the reduction in inframarginal rents and the increase in marginal consumers respectively. Since more agents from group ℓ join platform k, platform k becomes more attractive to the members of group m. The third term represents the additional revenue from the increase in the number of agents from group m that join platform k. (This third effect will be absent when platforms price discriminate perfectly).

We look for a symmetric equilibrium where platforms charge the same prices to each group. We assume that regularity conditions hold so that a symmetric sharing equilibrium exists.[14] Using (3), the symmetric solution to the system of first order conditions is given by,

$$p_{1A}^{*} = p_{1B}^{*} = \frac{t - \alpha_2 f_1\left(\frac{1}{2}\right)}{f_1\left(\frac{1}{2}\right)} \text{ and } p_{2A}^{*} = p_{2B}^{*} = \frac{t - \alpha_1 f_2\left(\frac{1}{2}\right)}{f_2\left(\frac{1}{2}\right)}. \tag{4}$$

Each platform serves one half of the members of each group. The equilibrium prices depend positively on the differentiation parameter t, negatively on the strength of the cross-group externality α_ℓ and negatively on the number of marginal agents $f_\ell\left(\frac{1}{2}\right)$. When the externality for group ℓ is stronger firms offer lower prices to the members of group m. Potentially, the price can be negative, but we do not allow for this possibility (so we assume that $t > \max\left\{\alpha_1 f_2\left(\frac{1}{2}\right), \alpha_2 f_1\left(\frac{1}{2}\right)\right\}$). The implication of this assumption is that differentiation is more important than cross-group externality.

The equilibrium profits are,

$$\pi_A = \pi_B = \frac{t - \alpha_2 f_1\left(\frac{1}{2}\right)}{2 f_1\left(\frac{1}{2}\right)} + \frac{t - \alpha_1 f_2\left(\frac{1}{2}\right)}{2 f_2\left(\frac{1}{2}\right)}. \tag{5}$$

3.1.1 Uniform distribution

If we assume that the distribution is uniform, then the equilibrium prices and profits are,

$$p_{1A}^{*} = p_{1B}^{*} = t - \alpha_2 \text{ and } p_{2A}^{*} = p_{2B}^{*} = t - \alpha_1 \tag{6}$$

$$t - \frac{(\alpha_1 + \alpha_2)}{2}. \tag{7}$$

3.2 Perfect price discrimination

3.2.1 Prices are public

Now we assume that platforms can discriminate perfectly and prices cannot be negative. (Below we discuss how the equilibrium changes when prices are allowed to be negative). Agent utility is given by (1) and platforms compete on an agent by agent basis. Each agent receives a targeted offer. Arbitrage is not feasible. We assume that agents observe all prices before they decide which platform to join. We make this assumption in order to be consistent with the uniform price case. We discuss about how our results might change if we relax the assumption that all prices are public in the next section.

[14] We were not able to come up with clean conditions on the distribution functions that would ensure the strict concavity (or quasi-concavity) of the objective functions. For instance, the monotone hazard rate property is not enough. When the distribution is uniform, then the profit functions are strictly concave provided that $2t > (\alpha_1 + \alpha_2)$. When this condition holds, then a symmetric sharing equilibrium exists. Otherwise, one platform may corner the entire market.

The symmetric equilibrium is,

$$\begin{aligned} p_{\ell A} &= t\,(1-2x) \text{ and } p_{\ell B}=0, \text{ for } x \leq \frac{1}{2} \text{ and} \\ p_{\ell A} &= 0 \text{ and } p_{\ell B}=t\,(2x-1)\,, \text{ for } x \geq \frac{1}{2}. \end{aligned} \tag{8}$$

Each agent is indifferent between the two platforms, and we assume that he joins the one that is closest to his location. It is easy to see that no platform has an incentive to deviate from (8). The price platform A (say) charges to agents located in its own turf is a limit price. It is a price that prevents the rival platform from making any sales to these agents. Hence, an equilibrium discriminatory price does not balance the same three effects a uniform price does. Rather, the primary purpose of a price is to drive the rival platform out of the market for each agent. The premium a platform can charge is equal to the transportation difference an agent will incur if he instead joins the rival platform. In addition, a platform can enjoy an extra premium if it has acquired more agents, but in a symmetric equilibrium this premium vanishes. Therefore, cross-group externalities do not affect the equilibrium prices when firms have the ability to price discriminate perfectly. In this respect, the equilibrium prices are the same with those from a one-sided market.[15]

The equilibrium profits are given by,

$$\pi_A = t - 2t\int_0^{1/2} x\left[f_1(x)+f_2(x)\right]dx \text{ and } \pi_B = 2t\int_{1/2}^{1} x\left[f_1(x)+f_2(x)\right]dx - t.$$

If the distribution is uniform, then the equilibrium profits are equal to $\frac{t}{2}$.

Remark (allowing for negative prices). If we allow for negative prices, then (8) is no longer an equilibrium. Each platform has an incentive to lower its price below zero by ε in order to attract the rival platform's agents and then raise its prices to its own agents. Such a deviation is profitable. The symmetric equilibrium in this case is,[16]

$$\begin{aligned} p_{\ell A} &= t\,(1-2x)-\alpha_m \text{ and } p_{\ell B}=-\alpha_m, \text{ for } x \leq \frac{1}{2} \text{ and} \\ p_{\ell A} &= -\alpha_m \text{ and } p_{\ell B}=t\,(2x-1)-\alpha_m, \text{ for } x \geq \frac{1}{2}. \end{aligned} \tag{9}$$

As one can immediately see prices are affected by the cross-group externality parameter. However, in many cases negative prices are unrealistic [see also Armstrong (2007) for a discussion on this issue]. Moreover, the above equilibrium is not very plausible. Here, we assume that agents in

[15] We have implicitly assumed that coordination among agents is not feasible. If agents could coordinate, then they would all become better off by agreeing to join the same platform, after the firms have announced their prices. This follows from the fact that, in equilibrium, each agent is indifferent between the two platforms. Hence, coordination would increase the membership of one platform and would make all agents better off. Firms, of course, would anticipate that and they would change their prices. In the rest of the paper coordination is assumed away.

[16] The proof is standard and it is omitted.

$\left[0, \frac{1}{2}\right]$, for example, who are also indifferent between the two platforms join platform A. But what if we allow agents to make small mistakes (trembles) and instead join platform B? Is B ready to honor its commitment and pay these agents α_1 or α_2 (or sell the product at prices below marginal cost)? This does not seem very likely since platform B will loose money on these agents without being able to recoup its loses by raising its prices to its own agents. Although (9) is an equilibrium, it is not a very "plausible" one. Finally, when prices are private (which may be a more realistic assumption in the presence of so many prices) (8) is an equilibrium, even when prices can become negative. In other words, the credible signal that the negative price sends to the whole market disappears once prices cannot be observed publicly.

For all these reasons, in the remaining of the paper we assume that prices cannot become negative.

3.2.2 Prices are private

So far we have assumed that prices are public. We recognize that this assumption may not be very realistic in the context of perfect price discrimination. Alternatively, we can assume that platforms target each consumer with *private coupons* that represent discounts off the regular prices.[17] Given the cross-group externalities, beliefs are important in this case. What is an agent's belief about the offers made to other agents if he receives an out-of equilibrium offer? We could then assume that beliefs are passive [e.g., McAfee and Schwartz (1994)]. If price offers were secret, and beliefs were passive, then (8) would continue to constitute an equilibrium. To see this, suppose that a platform raises unilaterally its prices to a group of agents in its territory. Each agent, however, continues to believe that market shares will not change and *given* that agents are indifferent, in equilibrium, between the two platforms they will *all* switch to the rival platform. Hence, such a deviation is unprofitable. Price cuts would also be unprofitable because a reduction in price to an agent (or a group of agents) will not lead to higher market share. The distinction between public and private prices does make a difference in the imperfect price discrimination case that we examine later.

3.3 Price comparison

We compare the equilibrium uniform prices given by (6) with the discriminatory prices given by (8). We will exploit the fact that uniform prices depend on the cross-group externality parameters, while discriminatory price do not. Discriminatory prices, as it is the case in one-sided markets that are characterized by horizontal differentiation, are decreasing in the degree of consumer loyalty to

[17]We note that the analysis of the previous section (i.e., the uniform prices case) is not affected because we can reasonably assume that agents observe each platform's regular prices. In general, when prices are private platforms face a problem similar to the commitment problem in bilateral contracting contexts with externalities, e.g., Segal and Whinston (2003). Platforms would have an incentive to raise their prices under passive beliefs. This is because when price changes are secret a price increase is not very costly due to the absence of the feedback effect. The next section confirms this intuition.

a platform. Agents located very close to one or the other platform pay higher prices than those located in the middle. The highest price is t and the lowest is 0. If we compare these prices with the no discriminatory prices, $\frac{t-\alpha_\ell f_m\left(\frac{1}{2}\right)}{2f_m\left(\frac{1}{2}\right)}$, we will see that it is possible that nearly all agents pay higher prices under price discrimination if α_ℓ is arbitrarily close to $\frac{t}{f_m\left(\frac{1}{2}\right)}$. It then becomes obvious that there exists a threshold for the cross-group externality parameters above which perfect price discrimination hurts the platforms' profits.

When the distribution is uniform, equilibrium profits increase with price discrimination if and only if $\frac{t}{2} > t - \frac{(\alpha_1+\alpha_2)}{2}$. This is the case if and only if,

$$t < (\alpha_1 + \alpha_2)\,. \tag{10}$$

Furthermore, the necessary and sufficient condition for a market sharing equilibrium to exist is $2t > (\alpha_1 + \alpha_2)$. Therefore, there exists a range of parameters such that uniform equilibrium prices are given by (6) and price discrimination leads to higher profits.

Armstrong (2007) compares price discrimination with uniform prices. In his model price discrimination is defined as the uniform pricing rule in our model, i.e., when platforms charge each group a different price. A uniform pricing rule in Armstrong's model is when a platform charges both groups the same price. Armstrong shows that price discrimination is profitable if and only if,

$$(t_1 - t_2)^2 > (\alpha_1 - \alpha_2)^2\,. \tag{11}$$

Our condition (10) for a profitable price discrimination is very different from Armstrong's condition (11). In our case the levels matter, whereas in Armstrong's case the differences matter more. If the transportation parameters are the same across groups ($t_1 = t_2$), as it is the case in our model, then price discrimination is never profitable in Armstrong's model, while it may be in our model.

Finally, as it is well-known [e.g., Thisse and Vives (1988)], price discrimination in one-sided markets when preferences are uniformly distributed and firms are symmetric is *always* a prisoners' dilemma. The profits under perfect price discrimination are $\frac{t}{4}$, while under a uniform pricing rule they are $\frac{t}{2}$. In contrast, in two-sided markets, when (10) is satisfied the game is not a prisoners' dilemma. Firms in one-sided markets have "more incentives" to eliminate the practice of price discrimination than in two-sided markets. Because information about consumer tastes facilitates price discrimination one would expect that firms in two-sided markets will seek to develop and acquire customer databases more aggressively than their counterparts in one-sided markets.

4 Imperfect price discrimination

Now we relax the assumption of perfect price discrimination. Rather, we assume that platforms can segment the agents into groups, but this segmentation is not perfect. Platforms can develop or acquire information which helps them to segment the agents into distinct groups based upon

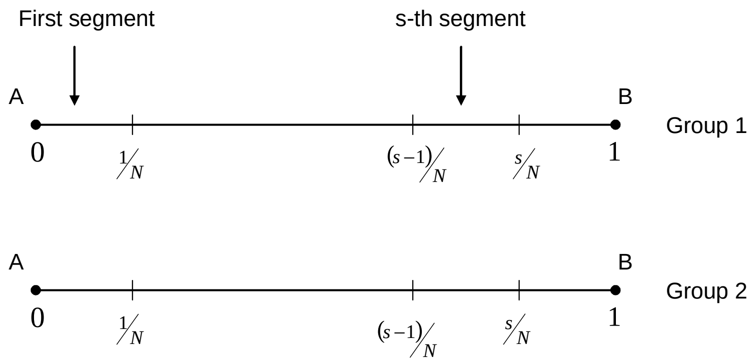

Figure 2: Market segmentation

each agent's relative degree of platform loyalty. We assume that the information partitions each $[0,1]$ interval into N sub-intervals (indexed by s, $s = 1, ..., N$) of equal length, see figure 2. A platform can charge different prices ($p_{\ell ks}$, $\ell = 1, 2$, $k = A, B$ and $s = 1, ..., N$) to different groups of consumers [see Liu and Serfes (2004)]. To make the analysis tractable, we assume that the distribution of preferences is uniform and $\alpha_1 = \alpha_2 = \alpha$.

We further assume that $N = 2^i$, where i takes on all integer values, $i = 0, 1, 2, 3, 4....$ Hence, N will parameterize the precision of agent information, with higher N's being associated with higher information precision. Moreover, $N = 1$ corresponds to no price discrimination and $N = \infty$ to perfect price discrimination. Note that N, for simplicity, does *not* take on all integer values, but rather $N = 1, 2, 4, 8, 16...$(i.e., information refinement). We assume that information of precision N is available to both platforms and that the current state of technology dictates N which the firms take as exogenously given. Hence, our approach is static and the effect of information improvements on the equilibrium is captured by comparative statics analysis.

To be consistent with the previous analysis, first we assume that consumers observe all the prices before they choose which platform to join. This assumption seems reasonable when the number of segments is small. An alternative assumption is that agents receive targeted discounts and therefore each agent observes only his own price (and the regular (highest) prices). In the previous section we showed that this distinction does not matter as long as beliefs are passive. However, under imperfect price discrimination, the analysis will change. So, we also derive the equilibrium by assuming that prices are private.

4.1 Prices are public

We denote by $n_{\ell ks}$ the number of agents from group ℓ in segment s that join platform k. Then, the total number of agents from group ℓ that join platform k is given by $n_{\ell k} = \sum_{s=1}^{N} n_{\ell ks}$. The marginal agent from group ℓ in segment s is given by,

$$x_{\ell s} = \frac{\alpha\left(n^e_{mA} - n^e_{mB}\right) - p_{\ell As} + p_{\ell Bs} + t}{2t}. \tag{12}$$

From (12) the implicit expressions for the segment s shares are given by,

$$n_{1As} = \frac{\alpha\left(n^e_{2A} - n^e_{2B}\right) - p_{1As} + p_{1Bs} + t}{2t} - \frac{s-1}{N};\ n_{2As} = \frac{\alpha\left(n^e_{1A} - n^e_{1B}\right) - p_{2As} + p_{2Bs} + t}{2t} - \frac{s-1}{N}, \tag{13}$$

and

$$n_{1Bs} = \frac{s}{N} - \frac{\alpha\left(n^e_{2A} - n^e_{2B}\right) - p_{1As} + p_{1Bs} + t}{2t}; n_{2Bs} = \frac{s}{N} - \frac{\alpha\left(n^e_{1A} - n^e_{1B}\right) - p_{2As} + p_{2Bs} + t}{2t}, \tag{14}$$

for $s = 1, ..., N$.

Now we sum up over all segments. This yields,

$$\begin{aligned} \sum_{s=1}^{N} n_{1As} &= \frac{\alpha\left(n^e_{2A} - n^e_{2B}\right) N - \sum_s p_{1As} + \sum_s p_{1Bs} + Nt}{2t} - \sum_{s=1}^{N} \frac{s-1}{N}, \\ \sum_{s=1}^{N} n_{2As} &= \frac{\alpha\left(n^e_{1A} - n^e_{1B}\right) N - \sum_s p_{2As} + \sum_s p_{2Bs} + Nt}{2t} - \sum_{s=1}^{N} \frac{s-1}{N} \end{aligned} \tag{15}$$

and

$$\begin{aligned} \sum_{s=1}^{N} n_{1Bs} &= \sum_{s=1}^{N} \frac{s}{N} - \frac{\alpha\left(n^e_{2A} - n^e_{2B}\right) N - \sum_s p_{1As} + \sum_s p_{1Bs} + Nt}{2t}, \\ \sum_{s=1}^{N} n_{2Bs} &= \sum_{s=1}^{N} \frac{s}{N} - \frac{\alpha\left(n^e_{1A} - n^e_{1B}\right) - \sum_s p_{2As} + \sum_s p_{2Bs} + Nt}{2t}. \end{aligned} \tag{16}$$

Since expectations are rational we must have $n_{\ell k} = \sum_{s=1}^{N} n_{\ell ks} = n^e_{\ell k}$. Solving (15) and (16) with respect to $n_{\ell k}$, $\ell = 1, 2$ and $k = A, B$, we obtain the aggregate market shares explicitly as a function of prices,

$$\begin{aligned} n_{1A} &= \frac{\alpha N\left(\sum_s p_{2Bs} - \sum_s p_{2As}\right) - t\left(\sum_s p_{1As} - \sum_s p_{1Bs}\right) + t^2 - \alpha^2 N^2}{2\left(t^2 - \alpha^2 N^2\right)}, \\ n_{2A} &= \frac{\alpha N\left(\sum_s p_{1Bs} - \sum_s p_{1A}\right) - t\left(\sum_s p_{2As} - \sum_s p_{2Bs}\right) + t^2 - \alpha^2 N^2}{2\left(t^2 - \alpha^2 N^2\right)}, \end{aligned} \tag{17}$$

and

$$\begin{aligned} n_{1B} &= \frac{\alpha N\left(\sum_s p_{2As} - \sum_s p_{2Bs}\right) - t\left(\sum_s p_{1Bs} - \sum_s p_{1As}\right) + t^2 - \alpha^2 N^2}{2\left(t^2 - \alpha^2 N^2\right)}, \\ n_{2B} &= \frac{\alpha N\left(\sum_s p_{1As} - \sum_s p_{1Bs}\right) - t\left(\sum_s p_{2Bs} - \sum_s p_{2As}\right) + t^2 - \alpha^2 N^2}{2\left(t^2 - \alpha^2 N^2\right)}. \end{aligned} \tag{18}$$

Next, we substitute (17) and (18) into (13) and (14) to obtain the segment shares $n_{\ell ks}$, $\ell = 1, 2$, $k = A, B$ and $s = 1, ..., N$. Segment s demand depends on all $4N$ prices.

We denote by $\mathbf{p}_{\ell k}^{N}=(p_{\ell k1},p_{\ell k2},...,p_{\ell kN})$ the n-dimensional vector of prices charged by platform k to the agents of group ℓ. The platforms' profit functions are then given by,

$$\begin{aligned}\pi_A\left(\mathbf{p}_{1A}^{N},\mathbf{p}_{2A}^{N},\mathbf{p}_{1B}^{N},\mathbf{p}_{2B}^{N}\right) &= \sum_{s=1}^{N} p_{1As}n_{1As}+\sum_{s=1}^{N} p_{2As}n_{2As} \text{ and}\\ \pi_B\left(\mathbf{p}_{1A}^{N},\mathbf{p}_{2A}^{N},\mathbf{p}_{1B}^{N},\mathbf{p}_{2B}^{N}\right) &= \sum_{s=1}^{N} p_{1Bs}n_{1Bs}+\sum_{s=1}^{N} p_{2Bs}n_{2Bs}.\end{aligned}$$

Each platform chooses $\mathbf{p}_{1k}^{N}$ and $\mathbf{p}_{2k}^{N}$ to maximize profits, given the two vectors of the rival platform. The presence of the externalities complicates the problem. This is due to the fact that we cannot treat each segment separately from the other segments, as it would be the case if the externalities were absent [see Liu and Serfes (2004, proposition 1)]. In two-sided markets a price a platform charges to agents in a particular segment affects the number of agents from that segment who join the platform which, in turn, affects the number of agents from the other group that join this *and* other segments of the platform. Nevertheless, we built on the results in Liu and Serfes (2004) and we were able to characterize the symmetric equilibrium as the following proposition demonstrates. We maintain the assumption that brand differentiation is more important than externalities, i.e., $t>\alpha$.

Proposition 1 (Prices are public). *The symmetric equilibrium is described as follows:*

- *Suppose that platforms can segment each group of agents into two segments, i.e., $N=2$. Assuming that $t>2\alpha$ a symmetric pure strategy equilibrium is described as follows:*[18]

  1. *If $t>3\alpha$ the equilibrium profits are*
  $$\pi_k=\frac{5t}{9}-\alpha. \tag{19}$$
  Platforms share each segment demand and charge positive prices in all agent segments.
  2. *If $t\in(2\alpha,3\alpha]$ the equilibrium profits are*
  $$\pi_k=\frac{t\,(t-2\alpha)\,(t-\alpha)}{(3\alpha-2t)^2}. \tag{20}$$
  Platforms charge zero prices in the rival platform's agent segment and positive prices in their own segment. Platforms do not make any sales in the segments that are in the rival platform's own territory.

- *Suppose that platforms can segment each group into more than two segments, i.e., $N\geq 4$. If $t\leq\frac{\alpha N}{2}$, then a symmetric pure strategy equilibrium exists. The equilibrium profits are*
$$\pi_k=\frac{t}{2}-\frac{t}{N}. \tag{21}$$

[18] If $t\leq 2\alpha$, then one platform may corner the market.

Platforms do not share any segment demand, which implies that each agent joins the platform that is closest to his location.

Proof. See appendix. ■

Proposition 1 says the following. First let's assume that $N \geq 4$. Fix t and α. Equilibrium profits do not depend on α when $t \leq \frac{\alpha N}{2}$. (Note that the latter condition is not very restrictive especially when N is large). This is because platforms do not share any segment demand and the price is a limit price. On the other hand, the uniform price equilibrium profits, from (7), are equal to $t - \alpha$. It can be easily seen that $\frac{t}{2} - \frac{t}{N} > t - \alpha$, provided that $N > \tilde{N} \equiv \frac{2t}{2\alpha - t}$ (and assuming that $t < 2\alpha$, i.e., platforms are not too differentiated), see figure 3.[19] In the remaining of this section we maintain the assumption that $t < 2\alpha$. Therefore, if $N > \tilde{N}$, (and platforms are not too differentiated) then price discrimination with symmetric platforms leads to higher profits than those under uniform prices. For example, if $t = 1$ and $\alpha = .6$, then $\tilde{N} = 10$. This implies that price discrimination is profitable for the platforms when they have the ability to segment each group of agents into at least ten segments (i.e., $N \geq 10$). (Recall that our definition of segmentation allows the number of segments to go from 2 to 4 to 8 and so forth). Thus, we see that the result we derived by assuming perfect price discrimination continues to hold even when firms discriminate imperfectly, provided that the quality of information is high enough.

Now let's assume that $N = 2$. The equilibrium in this case requires that $t > 2\alpha$ which contradicts the assumption, $t < 2\alpha$, that we made above (and we maintain in the rest of the paper). We need this condition because otherwise prices become zero and the equilibrium may involve tipping. We do not pursue the possibility of tipping further. If, for a moment, we assume that $t > 2\alpha$, then proposition 1 holds (for $N = 2$) and we can easily observe that equilibrium profits are lower than the uniform price profits, i.e., $\frac{t(t-2\alpha)(t-\alpha)}{(3\alpha-2t)^2} \leq \frac{5t}{9} - \alpha < t - \alpha$. Overall, the relationship between equilibrium profits and the quality of information N is U-shaped, see figure 3. In drawing figure 3 we assumed, for convenience, that N is a continuous variable and that $t < 2\alpha$. Moreover, we reasonably assumed that the profits for the platforms when $N = 2$ cannot be greater than the profits in proposition 1 (where $t > 2\alpha$ is assumed).

4.2 Prices are private

We assume that agents observe each platform's regular (highest) prices, but they do not observe all the targeted discounts off the regular prices. Each agent only observes the price offers made to him. Agents form conjectures about the membership of each platform that must be confirmed in equilibrium. An unexpected deviation by a platform is not going to be observed by anyone, except the segment of agents which the price deviation targets. We assume that beliefs are passive. Hence, a deviation will only have an effect on the demand in the targeted agent segment and nowhere else.

[19] We need this condition because, as we proved in a previous section, if $t > 2\alpha$ then even perfect price discrimination is not profitable relative to uniform prices.

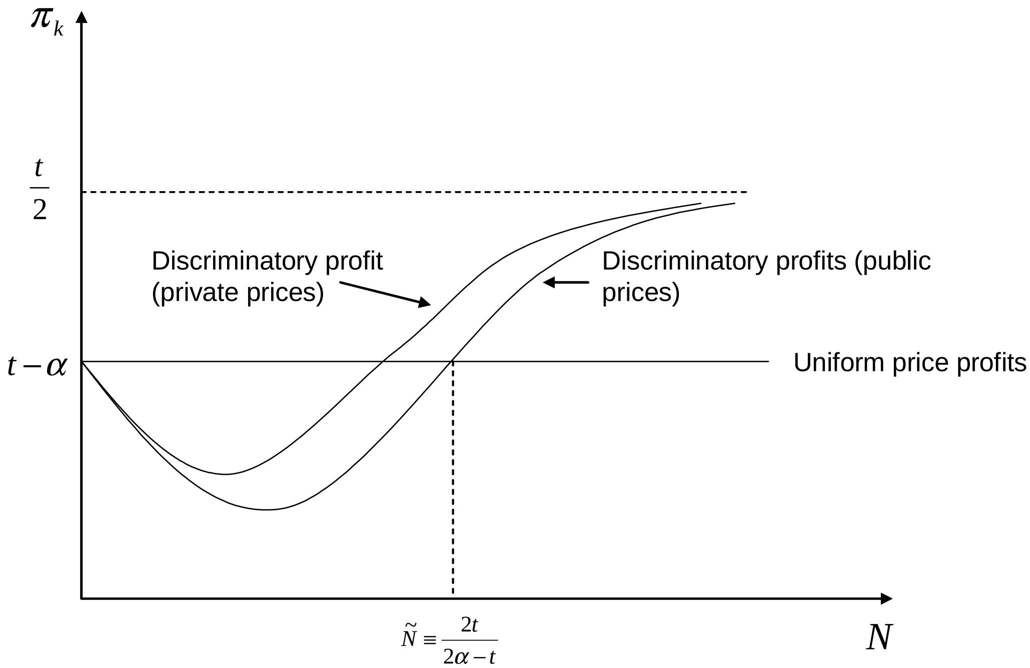

Figure 3: Profit comparison (in drawing the figures, for convenience, we assumed that N is a continuous variable)

Proposition 2 (Prices are private). *Assume that $N \geq 2$. Platforms share the segment demand only in the middle two segments. The equilibrium profits are given by,*

$$\pi_k = \frac{t}{2} - \frac{t}{N} + \frac{20t}{9N^2}. \tag{22}$$

The equilibrium profit exhibits a U-shape as a function of N.

Proof. The proof is the same as the proof of proposition 1 in Liu and Serfes (2004) where there are no externalities. ■

We know from one-sided markets that price discrimination gives rise to two effects: an *intensified competition* effect and a *surplus extraction* effect, Liu and Serfes (2004). When the segmentation is coarse the first effect is more dominant. As platforms move to finer partitions the second effect becomes increasingly stronger. Overall, the relationship between equilibrium profits and the number of available consumer segments (quality of information) is U-shaped. Moreover, the equilibrium profits are always below the uniform price profits. In two-sided markets the aforementioned two effects are also present. An additional effect is due to the cross-group externalities. Network externalities affect the equilibrium prices when a segment demand is shared and prices are public. Otherwise, they do not influence the equilibrium prices. Nevertheless, externalities affect the equilibrium indirectly in the following sense. When prices are public price cuts are more lucrative precisely because of the externalities. This forces the platforms to compete more vigorously and leads to an equilibrium where each platform drives the rival platform out of all of the segments in its own turf. Equilibrium prices are independent of the externalities since sharing of segment demand does not take place (prices are limit prices). This is the case when $N \geq 4$ as proposition 1 documents.

4.3 Price and profit comparison

Equilibrium profits when prices are private are higher than the profits when prices are public, see figure 3. This comparison is straightforward when $N \geq 4$ based on (21) and (22). Both profit functions converge to $\frac{t}{2}$ (the profit under perfect price discrimination) as N goes to infinity. When $N = 2$ the profits when prices are private are (from (22)) $\pi_k = \frac{5t}{9}$. This profit is greater than the profit ((19) and (20)) when prices are public. Essentially, the externalities push the uniform profits and the discriminatory profits with public prices down. They do not affect, however, the profits when prices are private. Moreover, the discriminatory profits with public prices are affected less than the uniform profits

When prices are public and $N \geq 4$ (and $t \leq \frac{\alpha N}{2}$) the outcome is efficient because each agent joins the platform that is closest to his location. This is not true when prices are private where some agents (those in the middle two segments) do not join the closest platform.

5 Conclusion

We examine the issue of price discrimination in two-sided markets. We assume that there are two symmetric horizontally differentiated platforms and two groups of agents. Agents from both groups must join a platform for successful trades to take place. Platforms possess information about the agents' brand preferences which can be used to customize prices. Our first result indicates that when the externality effect is strong perfect price discrimination yields higher profits to the platforms relative to the profits under uniform prices. This result is in sharp contrast with the prisoners' dilemma prediction in oligopolistic one-sided price discrimination models.

Then, we allow for imperfect price discrimination. We analyze two different cases depending on whether agents observe all prices before they decide which platform to join: i) prices are private and ii) prices are public. This distinction is relevant for two reasons. First, when platforms charge many prices it seems reasonable (at least in some cases) to assume that not all prices are observed by all agents. Second, observability matters due to the presence of network externalities. We demonstrate that in either case the equilibrium profits exhibit a U-shape pattern with respect to the quality of information platforms possess about the preferences of the agents. In the limit, i.e., as the quality of information tends to become perfect, we recover the perfect price discrimination paradigm. This suggests that when the information is not very accurate price discrimination makes the platforms worse off, while with accurate information price discrimination may be profitable.

Moreover, profits, when prices are public, are always lower compared to the profits when prices are private. An implication of this result is that platforms may have incentives to make their prices less transparent. Our analysis can also be used in intermediate goods markets, such as B2B markets. In this case, price discrimination will lead to lower final prices if and only if market segmentation is coarse.

A Proof of proposition 1

The proof is divided into two parts. In the first part we solve for the equilibrium assuming that the number of segments is two, $N = 2$. In the second part we assume that $N \geq 2$. We do this because there is a qualitative difference in the solution between the two cases. In the first case firms may share the segment demand, whereas in the second case segment demand is not shared (under some conditions). The incumbent platform is able to drive the rival out of all segments in its own territory. Platform A's own territory is the $\left[0, \frac{1}{2}\right]$ part of the intervals and platform B's own territory is the $\left[\frac{1}{2}, 1\right]$ part. When we say "incumbent platform" we refer to the platform that is operating in its own territory.

Case 1: $N = 2$. So each platform can charge two prices to a given group of agents. This implies that each platform competes with four prices.

Let $p_{\ell ks}$ denote platform k's price in segment s for group ℓ. It can be showed (details are omitted) that when $t > 2\alpha$, each firm's profit function is concave in its own prices. The implication of concavity is that in equilibrium neither firm will corner the market.

- Prices in all segments are positive.

In this case first order conditions that are satisfied with equality are necessary and sufficient.

We calculate the first order conditions,[20] and we assume symmetry in prices across firms

$$(p_{1B1}, p_{1B2}, p_{2B1}, p_{2B2}) = (p_{1A2}, p_{1A1}, p_{2A2}, p_{2A1}).$$

Then, we solve the first order conditions to obtain

$$p_{\ell A1} = \frac{2t}{3} - \alpha \text{ and } p_{\ell A2} = \frac{t}{3} - \alpha,\ \ell = 1, 2.$$

When $t > 3\alpha$, both prices are positive. By substituting the above prices back into the profit function, we can obtain the equilibrium profits,

$$\pi_k = \frac{5t}{9} - \alpha.$$

No unilateral deviation is profitable because the objective functions are strictly concave.

- One price is positive and the other price is zero.

When $t \leq 3\alpha$, one price becomes zero. This is the price that each platform charges to the agents in the rival platform's own territory, i.e., $p_{\ell A2} = p_{\ell B1} = 0$, for $\ell = 1, 2$. By following similar steps

[20] Expression for the profit function is very lengthy to report.

as above and assuming that platform A obtains zero profit in segment 2 and positive profit only from segment 1 we can obtain the prices for platform A that satisfy the first order conditions (from symmetry we can derive the prices for platform B)

$$p_{\ell A1} = \frac{t(-2\alpha + t)}{-3\alpha + 2t}, \text{ for } \ell = 1, 2.$$

When $t > 2\alpha$, each platform has positive price in its own turf for each group. Next, we consider a deviation. Due to symmetry, consider platform A only. There is no profitable deviation in $p_{\ell A1}$, $\ell = 1, 2$, due to the strict concavity of the profit function. The only possible deviation is for platform A to increase $p_{\ell A2}$, $\ell = 1, 2$ above zero. We show that this is not profitable (details are omitted).

Case 2: $N \geq 4$. In this case each each platform competes with $2N$ prices. Given our assumption that $N = 2^i$, where $i = 0, 1, 2, 3, 4, ...$, there are always two middle segments in each $[0, 1]$ interval: one is to the left of $\frac{1}{2}$ and the other is to the right of $\frac{1}{2}$.

Claim 1: Firms may share the demand only in the middle two segments. In the remaining segments the incumbent platform drives the rival out of the market.

Let $m_1 = \frac{N}{2} - 1$ and $m_2 = \frac{N}{2} + 2$ denote the adjacent two segments (left and right respectively) to the two middle segments in each $[0, 1]$ interval. Then, the two middle segments are denoted by $M_1 = m_1 + 1$ and $M_2 = m_2 - 1$ (same for both groups of agents). (For example, if $N = 16$, $m_1 = 7$ and $m_2 = 10$. The middle two segments are $M_1 = 8$ and $M_2 = 9$). In Liu and Serfes (2004, proposition 1), where market segmentation is modeled the same way as in the present paper, we showed that firms share the segment demand only in the middle two segments, while in the other segments the incumbent firm drives the rival firm out of the market. This is true for any $N \geq 4$. The difference between that paper and the present paper is that the model in Liu and Serfes (2004) is one-sided. This implies that the feedback effect is absent. The feedback effect intensifies the competition and leads to lower prices, implying that the incumbent platform has more incentives to drive the rival out of the market. Hence, platforms in the present paper cannot share more than the middle two segments.

Claim 2: In any pure strategy equilibrium prices in the middle two segments must be zero.

Let $p_{\ell AM_1}$ and $p_{\ell AM_2}$ denote platform A's prices in the two middle segments in both groups, $\ell = 1, 2$. So, platform B's prices are $p_{\ell BM_1}$ and $p_{\ell BM_2}$, for $\ell = 1, 2$.

We first prove that $p_{\ell kM_1} > 0$ and $p_{\ell kM_2} > 0$, $\ell = 1, 2$ and $k = A, B$ cannot be an equilibrium. Let's focus on platform A. Assume an infinitesimal change of the prices, say $dp_{\ell AM_1}$ and $dp_{\ell AM_2}$, $\ell = 1, 2$, such that platform A would lose market share, $-dn_{\ell A}$, $\ell = 1, 2$, in the two middle segments. Then a price change of $-dp_{\ell AM_1}$ and $-dp_{\ell AM_2}$, $\ell = 1, 2$ would lead to a gain of market share of $-dn_{\ell A}$, $\ell = 1, 2$.

Let $d\pi_L$ denote the aggregate (i.e., both groups combined) change in platform A's profit in segments $s = 1, ..., m_1$, i.e., left segments, $d\pi_M$ in the two middle segments, $s = M_1, M_2$, and $d\pi_R$ the change in the right segments, $s = m_2, ..., N$. Obviously $d\pi_R = 0$. For platform A not to find the $dp_{\ell AM_1}$ and $dp_{\ell AM_2}$, $\ell = 1, 2$ change profitable, it must be that the resulting profit goes down, i.e,

$$d\pi_L + d\pi_M + 0 \leq 0.$$

After platform A loses share in the middle segments, we can show that it will not find it in its best interest to drive platform B out of the market in segment M_1, and possibly not in some other left segments as well. But suppose that platform A, *suboptimaly*, still drives platform B out of the market in all left segments $s = 1, ..., m_1$, and let $d\pi'_L$ be the corresponding profit change. Then we have,

$$d\pi_L \geq d\pi'_L \Rightarrow d\pi'_L + d\pi_M + 0 \leq 0. \tag{23}$$

Now consider a price decrease of $-dp_{\ell AM_1}$ and $-dp_{\ell AM_2}$, $\ell = 1, 2$. The corresponding profit change is

$$-d\pi'_L - d\pi_M + d\pi_R.$$

$d\pi_R > 0$ since platform A will gain some share in each right segment $s = m_2, ..., N$. This is because platform A lowered its prices.

For platform A not to find the $-dp_{\ell AM_1}$ and $-dp_{\ell AM_2}$, $\ell = 1, 2$ change profitable, it must be that the resulting profit goes down, i.e,

$$-d\pi'_L - d\pi_M + d\pi_R \leq 0.$$

But this is impossible, since from (23) $-d\pi'_L - d\pi_M \geq 0$ and $d\pi_R > 0$. Therefore, if platform A has no incentive to increase its prices, it will certainly have incentives to lower them, which implies that strictly positive prices in the middle two segments cannot be an equilibrium.

Next we prove that $p_{\ell AM_1} > 0$, $p_{\ell AM_2} = 0$, $p_{\ell BM_1} = 0$ and $p_{\ell BM_2} > 0$, for $\ell = 1, 2$ cannot be an equilibrium either.

The logic is the same as above, except that when $p_{\ell AM_1}$ decreases, there is not only gain in the right segments (as above), but also gain in the second middle segment, which will strengthen our argument.

Claim 3: When $t \leq \frac{\alpha N}{2}$, zero prices in the middle two segments constitute an equilibrium. The prices in the remaining segments are limit prices that drive the rival platform out of the incumbent platform's territory. Hence, $p_{\ell As} = 0$ for all $s = M_1, ..., N$ and $p_{\ell Bs} = 0$ for all $s = 1, ..., M_2$. Moreover, $p_{\ell As} = t - \frac{2ts}{N}$, for $s = 1, ..., m_1$ and $p_{\ell Bs} = t - \frac{2ts}{N}$, for $s = m_2, ..., N$, $\ell = 1, 2$. By summing up all segments we can derive the candidate equilibrium profits

$$\pi_k = \frac{t}{2} - \frac{t}{N}.$$

To sum up, our symmetric candidate equilibrium involves zero prices in the middle two segments and limit prices in all the remaining segments where the incumbent platform drives the rival out of the markets in its own turf. In the absence of indirect externalities this is not an equilibrium. A platform would have incentives to unilaterally raise its middle prices above zero [as in Liu and Serfes (2004)]. What may prevent a platform from doing so is the loss of market share in the middle segments and the resulting loss of profits in all the remaining segments due to the cross-group externalities. To prove that such a deviation is indeed unprofitable we take a shortcut. When a platform deviates in the middle two segments we allow the indirect externality to affect only the adjacent segments. Below, we explain how we check for such a deviation.

Due to symmetry, consider only platform A's deviation. Fix platform B's prices. Let $p^{dev}_{\ell AM_1}$ and $p^{dev}_{\ell AM_2}$, $\ell = 1, 2$, denote platform A's deviating prices in the two middle segments. Platform A does not have incentives to change its prices in the left or right segments. In the left segments it is capturing all the segment demand, while in the right segments its prices and market share are zero.

Note that platform A can only increase its prices in the first middle segments by charging $p^{dev}_{\ell AM_1} > 0$, $\ell = 1, 2$. In the second middle segments a price increase will clearly be unprofitable (since the segment is in platform B's own territory). When platform A increases its prices in the first middle segments, it will loose market share in those segments. It may also loose market share in other segments as well. We assume that platform A deviates in the first middle two segments and we also consider the feedback effect in the adjacent two segments m_1, i.e., in the segments that are adjacent to the left of the first middle segments. We ignore the feedback effect in the remaining segments. The resulting profits are denoted by $\hat{\pi}^{dev}_A$, while the "true" deviation profits are denoted by π^{dev}_A. Because the feedback effect can only hurt the deviating platform's profits we have $\hat{\pi}^{dev}_A \geq \pi^{dev}_A$. Then we show that $\hat{\pi}^{dev}_A$ is strictly concave in $p^{dev}_{\ell AM_1}$, $\ell = 1, 2$. We calculate firm A's deviating profit $\hat{\pi}_{A,dev}$ as a function of $p^{dev}_{\ell AM_1}$ and then calculate $\frac{d\hat{\pi}^{dev}_A}{dp_{\ell AM_1}}$. We find that

$$\left.\frac{d\hat{\pi}^{dev}_A}{dp_{\ell AM_1}}\right|_{p_{\ell AM_1}=0} \leq 0,\ \ell = 1, 2, \text{ when } t \leq \frac{\alpha N}{2}.$$

The above condition is sufficient (but not necessary) for a local deviation to be unprofitable. Since $\hat{\pi}^{dev}_A = \pi^{dev}_A$ when $p^{dev}_{\ell AM_1} = 0$, $\ell = 1, 2$ we can identify the above threshold. Since $\hat{\pi}^{dev}_A$ is concave no global deviation is profitable either. Finally, the fact that $\hat{\pi}^{dev}_A \geq \pi^{dev}_A$ ensures that no deviation is profitable when we consider the true deviation profits. Hence, our candidate equilibrium is indeed an equilibrium.

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