

Aggregative Games with Entry¹

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Abstract

For aggregative games we use a cumulative reaction function approach to determine the long-run effects of an exogenous change (e.g., cost changes, cooperation, leadership) affecting some players. Even though the affected players' equilibrium actions and payoffs, and the number of active players change, the aggregator stays the same: free entry completely undoes the short-run impact. Unaffected players' equilibrium payoffs are unchanged, whether or not actions are strategic complements or strategic substitutes. When the payoffs of agents outside the game are solely a function of the aggregator, the welfare (total payoffs) change from an exogenous change are measured simply as the change in payoffs to the directly affected player(s). An important case covers Logit and CES models in Bertrand differentiated products oligopoly: we show that the IIA property of demands implies that consumer surplus depends on the aggregator alone, and conversely, and the Bertrand pricing game is aggregative. We show how these findings are affected when entrants are heterogeneous. The results are applied to several contexts, including Cournot oligopoly, differentiated products Bertrand competition, mergers, privatization, rent-seeking, research joint ventures, and congestion.

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1 Introduction

Consider a mixed oligopoly industry of a public firm and private firms producing differentiated products with demand characterized by the CES model and a free entry zero profit condition. The public firm is then privatized, and there is entry or exit so that private firms again make zero profit. At the new equilibrium, total surplus is higher if and only if the privatized firm was making a loss before the change, and the size of the loss is an exact measure of the welfare gain moving to the new equilibrium. Or consider an industry with logit demands and suppose there is a merger, with a zero profit condition for marginal firms both before and after. Even though the merged firm's price rises and there is more variety through new entry, consumer surplus is unaltered. Or consider an R&D race with free entry. Some firms participate in the race as members of a cooperative R&D arrangement. R&D cooperation has no impact on the aggregate rate of innovation in the industry (hence need not be encouraged or discouraged) even though the number of participants in the race increases with R&D cooperation. Or indeed compare a homogenous Cournot industry with free entry with one where a firm acts as Stackelberg leader. The size of fringe firms is the same in these industrial structures, although there are fewer of them. Moving outside of IO, consider a rent-seeking (lobbying) game (Tullock) where efforts determine success probabilities via a contest success function. Equilibrium efforts of unaffected firms are independent of efficiency gains (to lobbying) to infra-marginal firms.

What ties together all these examples is that they are all aggregative games, and moreover, consumer surplus is a monotonic function of the aggregator. Many noncooperative games studied in the economics literature have the structure of an aggregative game, where each player's payoff depends on his/her own action and an aggregate of all players' actions. The goal of this paper is to determine the long-run effects of an exogenous change in such models.

We are interested in situations where the exogenous change affects some of the players in the market. The exogenous change can be a cost shock, a change in the objective function of the affected firms (such as a merger) or a change in the timing of the moves (such as leadership). We develop a unifying framework to analyze the impact of the change on the aggregate variable, producer surplus, and consumer surplus when the equilibrium

is characterized by free entry. The analysis deploys the cumulative best reply function introduced by Selten (1970), for which we derive the corresponding maximal profit function as a key tool to characterize the equilibrium.

In Section 2, we explain the framework and provide the basic definitions. Section 3 contains the core results. We show the following:

- The aggregator stays the same in the long run even though the affected players' equilibrium actions and payoffs, and the number of active players change.
- The unaffected players' equilibrium payoffs are unchanged.
- When the payoffs of agents outside the game (e.g., consumers) are solely a function of the aggregator, the welfare (total payoffs) change from an exogenous change are measured simply as the change in payoffs to the directly affected player(s).

In many of the problems considered in the literature, the main focus is on the aggregator (e.g., the total supply in the market, the probability of success in R&D). The first result stated above implies that free entry completely undoes the short-run impact of the change on the aggregator. In Section 5, we show this to be the case even if some players in the game choose their actions cooperatively or one of the players acts as a Stackelberg leader. The second result holds whether or not actions are strategic complements or strategic substitutes. The third result implies that in those cases when the aggregator is a sufficient statistic for consumer surplus, the change will have no impact on consumer surplus in the long run. An important case covers Logit and CES models in Bertrand differentiated products oligopoly. We show in Section 4 that the IIA property of demands implies that consumer surplus depends on the aggregator alone, and conversely, and the Bertrand pricing game is aggregative.

Our framework reveal the key assumptions behind many of the existing results in the literature. In Section 6, we present several examples from the literature, applying our results in the context of Cournot oligopoly, differentiated products Bertrand competition, mergers, privatization, rent-seeking, research joint ventures, and congestion. We demonstrate that the unifying theme for all these results in the literature is that they are based on aggregative games. We show that exploiting the aggregative structure of the game yields more

general results which can be obtained more directly. Our discussion includes new results and insights.

An important assumption behind all of our results is that the type of the marginal firm remains the same before and after the change. In Section 7, we show how the results are affected when entrants are heterogeneous. We then take the integer constraint into account in Section 8, and conclude in Section 9 by discussing further applications.

Two related papers in the literature are Corchon (1994) and Acemoglu and Jensen (2010). Both papers provide comparative statics results for aggregative games in the short run. Our contribution is to consider entry. In addition, we present consumer welfare and total welfare results.

2 Preliminaries

2.1 Game

We consider two-stage games where in the first stage, firms simultaneously make entry decisions. Entry involves a sunk cost. In the second stage, after observing how many firms have sunk the entry cost, the firms simultaneously choose their actions. We will also consider sequential choices of actions in the applications.

We consider a class of aggregative games whereby payoffs can be written as the sum of the actions (or strategic variables) of all agents as well as one's own action. These strategic variables may be monotonic transformations of the "direct" choices: in some central examples of Bertrand games with differentiated products, we will write the strategic variables as monotonically decreasing in prices. For what follows, it may be easiest for the reader to think of Cournot games, where the aggregator is simply the sum of all firms' output choices. Other applications will be brought in where pertinent.

For (homogeneous product) Cournot games, payoffs are written simply as a function of own output, q_i , and the sum of all other firms' outputs, Q_{-i} . That is, $\pi_i = p(Q_{-i} + q_i) q_i - C_i(q_i)$, or indeed $\pi_i = p(Q) q_i - C_i(q_i)$, where Q is the value of the aggregator, here total output. Note that here the consumer surplus depends only on the price, $p(Q)$, and so tracking what happens to the aggregator is a sufficient statistic for tracking what happens to consumer welfare. The benchmark case to which we shall sometimes refer is the Cournot

model with log-concave demand, $p(Q)$, and constant marginal cost, $C_i(q_i) = c_i q_i + K$, where K is the sunk entry cost.

Another example of an aggregator is in the Bertrand differentiated products model with logit demands. Here we have $\pi_i = (p_i - c_i) \frac{\exp[(s_i - p_i)/\mu]}{\sum_{j=0}^n \exp[(s_j - p_j)/\mu]}$ where the s_j represent vertical “quality” parameters and $\mu > 0$ represents the degree of preference heterogeneity across products. The “outside” option has price 0. The denominator of this expression is the aggregator. It can be written as the sum of individual choices by defining $q_j = \exp[(s_j - p_j)/\mu]$ so that we can think of firms as choosing the values q_j . Then we write $\pi_i = (s_i - \mu \ln q_i - c_i) \frac{q_i}{Q}$. Note here that q_j varies inversely with price, p_j , so that the strategy space $p_j \in [c_j, \infty]$ under the transformation becomes $q_j \in [0, \exp[(s_j - c_j)/\mu]]$. Strategic complementarity of prices implies strategic complementarity of the q 's.

2.2 Payoffs

Consider the sub-games in the second (post-entry) stage of the game.

Let A be the set of active firms: an active firm is one which has sunk the entry cost. Let Q be the value of the aggregator (total output in the Cournot case), and let $q_i \geq 0$ be each active firm's contribution, so $Q = \sum_{i \in A} q_i$. Assume furthermore that each firm's strategy set is compact and convex.¹ We write the profit function as $\pi_i(Q, q_i)$, which encapsulates the property of the aggregative game that i 's profits depend only on the aggregator and player i 's own action. We shall use the notation $\pi_{i,1}(\cdot)$ and $\pi_{i,2}(\cdot)$ to refer to the partial derivatives with respect to the first and second arguments of this function.

Let $Q_{-i} = Q - q_i$ be the aggregate choices of all firms in A other than i . Then we can write i 's profit function as $\pi_i(Q_{-i} + q_i, q_i)$. For given Q_{-i} , let $\pi_i^*(Q_{-i})$ be i 's maximized profit function, so $\pi_i^*(Q_{-i}) = \pi_i(Q_{-i} + r_i(Q_{-i}), r_i(Q_{-i}))$ where $r_i(Q_{-i})$ denotes the best-reply (or reaction) function, i.e., $r_i(Q_{-i}) = \arg \max_{q_i} \pi_i(Q_{-i} + q_i, q_i)$.

Assumption 1 (Competitive Games) $\pi_i(Q_{-i} + q_i, q_i)$ is strictly decreasing in Q_{-i} for $q_i >$

¹Even if actions are not a priori bounded, we can often render them so by ruling out outcomes with negative payoffs. For example, in the benchmark Cournot game, we can define the maximum value of q_i as the solution to $p(q_i) = c$ and rule out outputs where price necessarily would be below the marginal cost. For the logit differentiated products demand we define $q_i = \exp\left(\frac{s_i - p_i}{\mu}\right)$, and note that the firm will not price below marginal cost, c_i , so that the strategy set can be restricted to $q_i \in \left[0, \exp\left(\frac{s_i - c_i}{\mu}\right)\right]$.

0.

This competitiveness assumption means that agents are hurt by larger rival actions. Note that the aggregator we use for Bertrand differentiated product games will vary inversely with price, so competitiveness applies in that context too. Assumption 1 implies that $\pi_i^*(Q_{-i})$ is strictly decreasing for $Q_{-i} < \bar{Q}_{-i}$, where \bar{Q}_{-i} is defined as the smallest value of Q_{-i} such that $r_i(\bar{Q}_{-i}) = 0$.² This will imply below that there is a unique free entry equilibrium.

Competitiveness means that agents impose negative externalities upon each other. The assumption therefore rules out aggregative games where agents impose positive externalities upon each other. Important examples studied in the literature include the public goods contribution game (see, e.g., Cornes and Hartley, 2007). However, in such games it is not always relevant to have a free-entry condition closing the model.³

Assumption 2 (Payoff functions)

- a) For any given $Q_{-i} \in \mathbb{R}^+$, $\pi_i(Q_{-i} + q_i, q_i)$ is continuous, twice differentiable, and strictly quasi-concave in q_i for $Q_{-i} < \bar{Q}_{-i}$, with a strictly negative second derivative with respect to q_i at the maximum.
- b) For any given $Q \in \mathbb{R}^+$, $\pi_i(Q, q_i)$ is continuous, twice differentiable, and strictly quasi-concave in q_i for $Q < \bar{Q}$, with a strictly negative second derivative with respect to q_i at the maximum.

Part (a) takes as given the actions of all other players, while part (b) takes as given the aggregator.⁴ As we see below, 2(a) is pertinent when players make their action decisions after entry decisions have been made, and so recognize how their actions affect the aggregator. 2(b) is relevant when one player commits to its action before the others enter, and applies to the case of leadership. To see that there is a difference in the two assumptions, consider the case of Cournot competition, with $\pi_i = p(Q)q_i - C(q_i)$, and consider the stronger assumption of profit concavity in q_i . Then 2(a) implies that the derivative

²Note that \bar{Q}_{-i} is allowed to differ across firms. For example, when marginal cost is constant, in the central case of homogenous products Cournot oligopoly with constant marginal costs, this means $p(\bar{Q}_{-i}) = c_i$.

³Note too that if there are positive externalities, then the solution to an entry game would have either all agents entering, or none at all.

⁴Both conditions apply for the main models we consider below.

$p''(Q)q_i + 2p'(Q) - C'''(q_i) \leq 0$, while 2(b) implies simply that $C'''(q_i) \geq 0$. Neither condition implies the other.

Assumption 2a implies a continuous and differentiable best response function, $r_i(Q_{-i})$, $i \in A$. Given the strategy spaces are convex and compact, equilibrium existence follows from Brouwer's fixed point theorem in the standard manner (see e.g., Theorem 2.1 in Vives, 1999). The best response functions are characterized by $\frac{d\pi_i(Q_{-i}+q_i, q_i)}{dq_i} = \pi_{i,1}(Q_{-i} + q_i, q_i) + \pi_{i,2}(Q_{-i} + q_i, q_i) = 0$. We will concentrate on two cases for these reaction functions.

The case of strict strategic substitutes arises for $\frac{d^2\pi_i}{dq_i dQ_{-i}} < 0$. This means that the derivative $\frac{d\pi_i(Q_{-i}+q_i, q_i)}{dq_i}$ decreases in Q_{-i} so that the value of q_i where this is zero, which characterizes the best response, is smaller when Q_{-i} is larger. Then, $r_i(Q_{-i})$ is a strictly decreasing function for $Q_{-i} < \bar{Q}_{-i}$ and is equal to zero otherwise. This holds in the benchmark Cournot model if demand is log-concave.

Conversely, the case of strict strategic complements applies when $\frac{d^2\pi_i}{dq_i dQ_{-i}} > 0$. Then $r_i(Q_{-i})$ is strictly increasing because marginal profits rise with rivals' strategic choices.

The next assumption ensures that the value of the aggregator does not fall when rivals increase their actions.

Assumption 3 (Reaction function slope) $\frac{d^2\pi_i}{dq_i^2} < \frac{d^2\pi_i}{dq_i dQ_{-i}}$.

The condition is readily verified in the benchmark Cournot model, as well as the logit and CES models.⁵ The next result shows that this condition implies there will be no over-reaction: if all others have collectively increased their actions, the reaction of i should not cause the aggregator to fall lower than where it started.⁶

Lemma 1 *Under Assumption 3, $r'_i(Q_{-i}) > -1$ and the relation $Q_{-i} + r_i(Q_{-i})$ is monotonically increasing in Q_{-i} .*

Proof. Because $r_i(Q_{-i})$ is defined by the first-order condition $\frac{d\pi_i(Q_{-i}+q_i, q_i)}{dq_i} = 0$, then $r'_i(Q_{-i}) = \frac{-\frac{d^2\pi_i}{dq_i dQ_{-i}}}{\frac{d^2\pi_i}{dq_i^2}}$. Because the denominator on the LHS is negative by the second-

⁵Consider the Cournot profit function $p(Q)q_i - C_i(q_i)$, with first derivative $p'(Q)q_i + p(Q) - C'_i(q_i)$. The stability condition in Assumption 3 is $p''(Q)q_i + 2p'(Q) - C'''_i(q_i) < p''(Q)q_i + p'(Q)$ or $p'(Q) < C'''_i(q_i)$, which readily holds for $C'''_i(q_i) \geq 0$.

⁶See also Vives (1999, p. 42).

order condition (see Assumption 2), then Assumption 3 implies that $r'_i(Q_{-i}) > -1$. Finally, note that $Q_{-i} + r_i(Q_{-i})$ is necessarily increasing in Q_{-i} if $r'_i(Q_{-i}) < -1$. ■

Given the monotonicity relation established in Lemma 1, then we can specify pertinent relations as functions of Q instead of Q_{-i} , and this will be used extensively in the analysis that follows. In particular, since $Q = Q_{-i} + r_i(Q_{-i})$, we can invert this relation (given that the RHS is monotonic, by Lemma 1) to write $Q_{-i} = f_i(Q)$. This is used in the next sub-section. The construction of $Q_{-i} = f_i(Q)$ is illustrated in Figure 1 for the case of strategic substitutes and Figure 2 for strategic complements. In each case, we construct Q for given Q_{-i} , and $f_i(Q)$ is then given by flipping the axes (inverting the relation).

2.3 Cumulative best reply function

The cumulative best reply of firm i gives the optimal action of firm i which is consistent with a given value of the aggregator, Q . The concept was first introduced by Selten (1970).⁷

Let $\tilde{r}_i(Q)$ stand for this cumulative best reply function, and below we will work with $\tilde{r}_i(Q)$ instead of $r_i(Q_{-i})$. Hence, $\tilde{r}_i(Q)$ stands for the portion of Q optimally produced by firm i (i.e., $Q - Q_{-i} = r_i(Q_{-i}) = \tilde{r}_i(Q)$). A continuous and differentiable $r_i(Q_{-i})$ gives us a continuous and differentiable $\tilde{r}_i(Q)$ function (by construction).

Geometrically, $\tilde{r}_i(Q)$ can be constructed in the following way. For strategic substitutes, Assumption 3 gives us $q_i = r_i(Q_{-i})$ decreasing as a function of Q_{-i} , with slope greater than -1 . At any point on the reaction function, draw down an isoquant (slope -1) to reach the Q_{-i} axis, which it attains before the reaction function reaches the axis. The x -intercept is the Q corresponding to the Q_{-i} once augmented by i 's reaction function contribution. This also relates $q_i = \tilde{r}_i(Q)$. Clearly, Q and q_i are negatively related. This construction is illustrated in Figure 3.

The case of strategic complements gives Q and q_i positively related. Hence, strategic substitutability or complementarity is preserved in the cumulative best replies: for strategic substitutes a higher total means a lower own contribution, and for strategic complements a higher total means a higher own contribution. This construction is illustrated in Figure 4.

⁷Cornes and Hartley (2009) call it the replacement function.

Lemma 2 *The slope of the cumulative reaction function is*

$$\frac{d\tilde{r}_i}{dQ} = \frac{r'_i}{1+r'_i} < 1.$$

If Assumption 3 holds, $\tilde{r}_i(Q)$ is strictly decreasing for $Q < \bar{Q}_{-i}$ for strict strategic substitutes. For strict strategic complements, $\tilde{r}_i(Q)$ is strictly increasing.

Proof. By definition, $\tilde{r}_i(Q) = r_i(f_i(Q))$. Differentiating yields

$$\frac{d\tilde{r}_i(Q)}{dQ} = \frac{dr_i(Q_{-i})}{dQ_{-i}} \frac{df_i(Q)}{dQ}.$$

Because $Q_{-i} = f_i(Q)$ from the relation $Q = Q_{-i} + r_i(Q_{-i})$, applying the implicit function theorem gives us

$$\frac{df_i}{dQ} = \frac{1}{1+r'_i}$$

and hence $\frac{d\tilde{r}_i}{dQ} = \frac{r'_i}{1+r'_i}$. Note that $\tilde{r}'_i \rightarrow 0$ as $r'_i \rightarrow 0$ and $\tilde{r}'_i \rightarrow -\infty$ as $r'_i \rightarrow -1$. For the case of strategic substitutes, because $-1 < r'_i < 0$ by Lemma 1, then $\tilde{r}'_i < 0$. For the case of strategic complements, $0 < \tilde{r}'_i < 1$. ■

Therefore, the slope of the cumulative best reply has the same sign as the slope of the best reply itself. Lemma 2 implies that $\sum_i \tilde{r}_i(Q)$ will be decreasing for strategic substitutes. This is useful because the cumulative best reply function can be used to establish the existence of an equilibrium: this is why the concept emerged in the literature (Selten, 1970) in the first place. An equilibrium exists if and only if $\sum_i \tilde{r}_i(Q)$ has a fixed point.⁸ Equilibrium uniqueness follows from Assumption 3 (Vives, 1999, p. 43) for the strategic substitutes case; for strategic complements, it suffices to assume that $\sum \tilde{r}'_i(Q) < 1$, where the sum is over all active firms.

Define now $\tilde{\pi}_i^*(Q)$ as the value of profit given a total output of Q and given that firm i is maximizing its profit given the output of the others, and doing so renders the total as Q . Because $\pi_i^*(Q_{-i}) = \pi_i(Q_{-i} + r_i(Q_{-i}), r_i(Q_{-i}))$ and because Q_{-i} can be written in terms of Q as $Q_{-i} = f_i(Q)$, then $\pi_i^*(Q_{-i}) = \pi_i(Q, r_i(f_i(Q))) = \pi_i(Q, \tilde{r}_i(Q)) = \tilde{\pi}_i^*(Q)$. This

⁸Note that flat parts in $\sum_i \tilde{r}_i$ would not cause a problem. Hence, we do not need $\sum_i \tilde{r}_i$ to be strictly decreasing (at least for the present purpose). If flat portions are coming from flat portions in $r_i(Q_{-i})$, then the strict strategic substitutes assumption is violated. Note that if $r_i(Q_{-i})$ has flat parts, then $\tilde{r}_i(Q)$ will have flat parts.

shows how the maximized value written in terms of the aggregator is related to the standard maximum value function written in terms of the sum of actions of all other players.

Lemma 3 *Under Assumptions 1-3, the function $\tilde{\pi}_i^*(Q)$ is strictly decreasing for $Q < \bar{Q}_{-i}$ and is constant (zero) otherwise.*

Proof. Recall that $\pi_i^*(Q_{-i})$ is strictly decreasing in Q_{-i} for $Q_{-i} < \bar{Q}_{-i}$ and is constant otherwise. Hence, since $Q_{-i} = f_i(Q)$ is a strictly increasing function for $Q_{-i} < \bar{Q}_{-i}$ (by Lemma 1), then the function $\tilde{\pi}_i^*(Q)$ defined by $\tilde{\pi}_i^*(Q) = \pi_i^*(f_i(Q))$ is strictly decreasing in Q for $Q < \bar{Q}_{-i}$ and is constant otherwise. ■

An insightful alternative proof is as follows. Profits are maximized at $q_i = \tilde{r}_i(Q)$ with associated profit level $\tilde{\pi}_i^*(Q) = \pi_i(Q, \tilde{r}_i(Q))$. Hence $\frac{d\tilde{\pi}_i^*(Q)}{dQ} = \pi_{i,1}(Q, \tilde{r}_i(Q)) + \pi_{i,2}(Q, \tilde{r}_i(Q)) \frac{d\tilde{r}_i(Q)}{dQ}$. Recalling that $\pi_{i,1}(Q, \tilde{r}_i(Q)) + \pi_{i,2}(Q, \tilde{r}_i(Q)) = 0$ by the first-order condition defining optimal choice of q_i and hence the cumulative best reply function, the desired derivative becomes $\frac{d\tilde{\pi}_i^*(Q)}{dQ} = \left(1 - \frac{d\tilde{r}_i(Q)}{dQ}\right) \pi_{i,1}(Q, \tilde{r}_i(Q))$. This is negative by Assumption 1 and Lemma 2.

2.4 Free Entry Equilibrium (FEE)

We are now in a position to define an equilibrium with entry. First define by Q_A the equilibrium value of the aggregator when the set A of agents has sunk the entry cost. That is, agents enter and the sub-game ensues with aggregator outcome Q_A . We already know from above that Q_A is unique given Lemma 3. Notice there could be some zero actions in there: this depends on whether $\tilde{r}_i(Q)$ is symmetric across firms. If it is, then we will have a symmetric equilibrium. Of course, following an exogenous change, which is what we are interested in, $\tilde{r}_i(Q)$ will no longer necessarily be symmetric.

Definition 1 *A Free Entry Equilibrium (FEE) is defined by a set of entrants, A , such that*

$$\tilde{\pi}_i^*(Q_A) \geq K \quad \text{for all } i \in A$$

and

$$\tilde{\pi}_i^*(Q_{\{A+i\}}) < K \quad \text{for all } i \notin A.$$

The first of these conditions means the firms which are in the market do not prefer to exit (and earn zero), while the second condition means the converse, where the notation $\{A + i\}$ denotes the set of agents A plus i . Generally, an equilibrium set of agents will not be unique. This is especially true when there are many homogenous agents.

We are interested in comparing equilibria before and after some exogenous change, under the stipulation that the marginal entrant type is the same both before and after the exogenous change. That is, we assume that there are some homogenous agents, which are all endowed with the same profit function, and that the marginal entrant type is one of these, both before and after. Let E denote the set of firms with the same profit function.⁹ Let the set of other firms be A_I . They are in A both before and after the change, so we are interested in equilibria where $A_I \subset A$. The exogenous change affects some (or all) of the other firms in the industry. There may also be other firms, not directly affected by the change, which differ from the marginal entrant type. The assumption of symmetry of the marginal entrants is a crucial assumption. We relax it in Section 7. The firms which are directly affected by the change may get some advantage (like a lower cost of production, superior product quality) or indeed they may be governed by different behavior, like Stackelberg leadership, or get a different objective function, as with a merger or nationalization.

A substantial part of the literature on free entry equilibrium assumes that entry occurs until the marginal entrant firm makes zero profit (and effectively ignores any possible integer issue). To the end of illuminating the role of the zero profit condition in the equilibrium configuration, we introduce the following definition.

Definition 2 *At a Zero Profit Homogenous Entrants Equilibrium (ZPHEE):*

$$\begin{aligned}\tilde{\pi}_i^*(Q_A) &\geq K && \text{for all } i \in A_I \subset A \\ \tilde{\pi}_i^*(Q_A) &= K && \text{for all } i \in E \cap A,\end{aligned}$$

where $E \cap A$ is non-empty.

The ZPHEE is used by many papers in the literature, but it can be criticized because it does not account for integer constraints. In a later section, we specifically account for

⁹We sometimes refer to these firms as the fringe firms, keeping in mind that they always choose their actions strategically.

integers, using a bounds approach.

Suppose now that there is some change (affecting the firms $i \in A_I$) that shifts the equilibrium set of agents from A to A' , both of which contain A_I , and both A and A' constitute Zero Profit Homogenous Entrants Equilibria. We wish to characterize the positive and normative characteristics of the comparison of these equilibria.

3 Core propositions

We are now in a position to prove the central positive and normative results.

3.1 Aggregator

Proposition 1 *Suppose that there is some change affecting the firms $i \in A_I$, which shifts the ZPHEE set of agents from A to A' , both of which contain A_I and at least one Fringe firm. Then $Q_A = Q_{A'}$.*

Proof. By Lemma 3, $\tilde{\pi}_i^*(Q)$ is strictly decreasing in Q , which implies that there is a unique solution for the aggregator at any ZPHEE. As long as $\tilde{\pi}_i^*(0) > K$, it is given by $Q = \tilde{\pi}_i^{*-1}(K)$, for $i \in E$. Hence, we must have $Q_A = Q_{A'} = \tilde{\pi}_i^{*-1}(K)$. ■

Proposition 1, while simple, is quite a powerful result. The composition of Q_A and $Q_{A'}$ may be quite different from each other due to the change experienced by the infra-marginal firms. For example, in the context of privatization with the CES model (Anderson, de Palma, and Thisse, 1997), there can be more or fewer firms present in the market. Moreover, the result applies irrespective of whether the actions of the firms are strategic substitutes or complements. In most models without entry, which analyze firm behavior in the short run, whether the actions of the firms are strategic substitutes or complements has a significant impact on the equilibrium predictions.

The crucial assumption behind Proposition 1 is that the marginal entrant has the same type before and after the change. We discuss in Section 7 how the results are modified if this is not the case.

3.2 Individual actions of fringe and unaffected firms

Proposition 1 implies that the equilibrium actions of the fringe firms will be the same before and after the change.

Proposition 2 *Suppose that there is some change directly affecting the firms $i \in A_I$, which shifts the ZPHEE set of agents from A to A' , both of which contain A_I . Then $q_{i,A} = q_{i,A'}$ for all $i \in E$, and $q_{i,A} = q_{i,A'}$ for all unaffected firms $i \in A_I$. Any change making the affected firms more aggressive in aggregate (in the sense of raising the sum of their $\tilde{r}(Q)$'s) will decrease the number of fringe firms.*

Proof. Since $Q_A = Q_{A'}$ and the cumulative best reply function $\tilde{r}_i(Q)$ is the same for all $i \in E$, we have $\tilde{r}_i(Q_{A'}) = \tilde{r}_i(Q_A)$. Similarly, for an unaffected firm (that is, one whose profit function remains unchanged) we have $\tilde{r}_i(Q_{A'}) = \tilde{r}_i(Q_A)$. Finally, because $Q_A = Q_{A'}$, if insiders become more aggressive, then there must be fewer fringe firms because $q_{i,A} = q_{i,A'}$ for all $i \in E$. ■

For the Cournot model, this means that *per firm* fringe output stays the same at a long-run equilibrium (although the number of fringe firms will typically change) following any change to some infra-marginal firms. Likewise, any infra-marginal firms not directly affected will also see no change in their output. For the privatization model noted above (Anderson, de Palma, and Thisse, 1997), the equilibrium prices of private firms must be the same independent of the presence of a public firm.

Although the equilibrium action of each active fringe firm does not change, the number of fringe firms may change substantially due to exit or entry after the change. If insiders get more aggressive (we explore below conditions under which this holds true), then there is less "room" for the fringe firms.

3.3 Producer surplus

We now turn to the question of total producer surplus following a change to some (or all) of the non-fringe active firms.

Proposition 3 *Suppose some firms' profit functions change. Then all other firms' rents remain unchanged at a ZPHEE, so the change in producer surplus equals the change in rents to the affected firms.*

Proof. This follows directly from Propositions 1 and 2. The aggregator remains the same, the best replies remain the same, and, since the profit functions are the same, the rents remain the same for the unaffected firms. Hence, the total change to producer surplus from the change is measured as the change in the affected firms' rents. ■

Thus, all insiders not directly affected will not change their equilibrium behavior, and nor do their rents change. Nor do the rents to fringe firms change, even though their numbers may change, since all $i \in E$ make zero profit whether in the industry or not. Changes in total producer surplus therefore follow what happens to the affected insiders.

3.4 Consumer surplus

In oligopoly examples, the other agents who can be affected by changes are consumers. Since it is the consumers who generate the demand system, their behavior is integral to the firms' problem. Their welfare is an important or even decisive (under a consumer welfare standard) criterion for evaluating the desirability of changes in the industry.

In the benchmark case of Cournot competition with homogeneous goods, the aggregator Q is total output, and consumer welfare depends directly on the aggregator via the market price, $p(Q)$. Therefore, an increase in the aggregator is a sufficient statistic for consumer welfare to rise. This is the case whenever the consumer welfare depends just on the aggregator.

Proposition 4 *Suppose that consumer surplus depends solely on the value of the aggregator. Suppose that there is some change affecting the firms $i \in A_I$, which shifts the ZPHEE set of agents from A to A' , both of which contain A_I and at least one Fringe firm. Then consumer surplus remains unchanged.*

Proof. By Proposition 1, $Q_A = Q_{A'} = \tilde{\pi}^{*-1}(K)$ at any ZPHEE, and the result follows immediately. ■

There are important other cases when consumer welfare depends solely on the value of the aggregator (and not its composition). These are discussed further in Section 4 below. Although the result stated in Proposition 4 follows immediately from Proposition 1, as it will become clearer in the applications we discuss in Section 6, that the change would have no impact on consumer surplus in the long run is not obvious.

The result does not hold if the composition of Q matters to consumers. This may be the case for example when there is an externality, like pollution, which varies across firms. Then a shift in output composition to less polluting firms raises welfare.

3.5 Total welfare

The results in Sections 3.3 and 3.4 give us total welfare results.

Proposition 5 *Suppose that consumer surplus depends solely on the value of the aggregator. Suppose that there is some change affecting the firms $i \in A_I$, which shifts the ZPHEE set of agents from A to A' , both of which contain A_I and at least one Fringe firm. Then the change in welfare is measured solely by the change in the affected firms' rents.*

Proof. This result follows immediately from Propositions 3 and 4. ■

That is, since the aggregator remains unchanged at any ZPHEE, then consumer welfare is unaffected and so is the rent earned by all infra-marginal firms not directly affected by the change.

4 Consumer welfare and Bertrand differentiated product games

So far, we have noted that the normative properties of the model hold if consumer surplus is only a function of the aggregator. Otherwise, if composition of the aggregator matters to consumers (examples are provided below¹⁰), then consumer welfare may be affected following exogenous changes even if the game is aggregative so the aggregator does not change.

Consider Bertrand (pricing) games, with differentiated products, and suppose the profit function takes the form $\pi_i = (p_i - c_i) D_i(\vec{p})$ where \vec{p} is the vector of prices set by firms (and

¹⁰Include the linear demand examples, aggregative games in SR/LR (Cournot but not Bertrand), but CS depends on composition.

prices are infinite for inactive firms, reflecting the unavailability of their goods) and $D_i(\vec{p})$ is firm i 's demand function. We are interested in the conditions – which are therefore the properties of the demand function, $D_i(\vec{p})$ – under which the game is an aggregative one and in which also the consumer welfare depends only on the value of the aggregator.

To pursue these questions, consider a quasi-linear consumer welfare (indirect utility) function, $V(\vec{p}, Y) = \phi(\vec{p}) + Y$. Demands are generated from the price derivatives of $\phi(\cdot)$. Suppose first that we can write $\phi(\vec{p})$ as an increasing function of the sum of decreasing functions of p_i , so $\phi(\vec{p}) = \tilde{\phi}\left(\sum_i g_i(p_i)\right)$ where $\tilde{\phi}' > 0$ and $g_i'(p_i) < 0$. Then $D_i(\vec{p}) = -\tilde{\phi}'\left(\sum_i g_i(p_i)\right) g_i'(p_i) > 0$, which therefore depends only on the summation and the derivative of $g_i(\cdot)$. Assume further that $D_i(\vec{p})$ is decreasing in own price ($\frac{dD_i(\vec{p})}{dp_i} = -\tilde{\phi}''\left(\sum_i g_i(p_i)\right) [g_i'(p_i)]^2 - \tilde{\phi}'\left(\sum_i g_i(p_i)\right) g_i''(p_i) < 0$).¹¹ Since $g_i(p_i)$ is decreasing, its value uniquely determines p_i and hence the term $g_i'(p_i)$ in the demand expression. Therefore, demand can be written as a function solely of the summation and $g_i(p_i)$. This means that the game is aggregative, by choosing $g_i(p_i) = q_i$ and $Q = \sum_i q_i$. Hence too the consumer welfare depends only on Q (and not its composition). This structure has another important property, namely that the demand functions satisfy the IIA property: the ratio of any two demands depends only on their own prices and is independent of the price of any other option in the choice set. That is, $\frac{D_i(\vec{p})}{D_j(\vec{p})} = \frac{g_i'(p_i)}{g_j'(p_j)}$. In summary:

Proposition 6 *Let $\pi_i = (p_i - c_i) D_i(\vec{p})$ and $D_i(\vec{p})$ be generated by an indirect utility function $V(\vec{p}, Y) = \tilde{\phi}\left(\sum_i g_i(p_i)\right) + Y$ where $\tilde{\phi}$ is twice differentiable, strictly convex in p_i , and $g_i(p_i)$ is twice differentiable and decreasing. Then demands exhibit the IIA property, the Bertrand pricing game is aggregative, and consumer welfare depends only on the aggregator, $Q = \sum_i q_i$.*

Important examples include the logit and CES models. For the logit model, $V = \mu \ln\left(\sum_i \exp[(s_i - p_i)/\mu]\right) + Y$, and the action variables are $q_i = \exp[(s_i - p_i)/\mu]$.

For the CES model, it is more common to write the direct utility function first, which in quasi-linear form is $U = \frac{1}{\rho} \ln\left(\sum_i x_i^\rho\right) + X_0$, where X_0 denotes numeraire consumption

¹¹For the logsum formula which generates the logit model, we have $g_i(p_i) = \exp[(s_i - p_i)/\mu]$ and so $g_i''(p_i) > 0$. However, $\tilde{\phi}$ is concave in its argument, the sum.

and x_i is consumption of the numeraire commodity. This implies that demands are $x_i = \frac{p_i^{-\lambda-1}}{\sum_j p_j^{-\lambda}}$ where $\lambda = \frac{\rho}{1-\rho}$. Then $X_0 = Y - 1$ and $V = \frac{1}{\rho} \ln \left(\sum_i \left(\frac{p_i^{-\lambda-1}}{\sum_j p_j^{-\lambda}} \right)^\rho \right) + Y - 1$, or

$$V = \frac{1}{\lambda} \ln \left(\sum_j p_j^{-\lambda} \right) + Y - 1.^{12}$$

There is also a converse to Proposition 6. Suppose that demands exhibit the IIA property. Then, following Theorem 1 (p. 389) in Goldman and Uzawa (1964), and imposing quasi-linearity, V must have the form $\tilde{\phi} \left(\sum_i g_i(p_i) \right) + Y$ where for them $\tilde{\phi}(\cdot)$ is increasing and $g_i(p_i)$ is any function of p_i . If we further stipulate that demands must be differentiable, then the differentiability assumptions made in Proposition 6 must hold, and assuming that demands are strictly downward sloping implies that $\tilde{\phi} \left(\sum_i g_i(p_i) \right)$ must be strictly convex in p_i . In summary:

Proposition 7 *Let $\pi_i = (p_i - c_i) D_i(\vec{p})$ and $D_i(\vec{p})$ be twice continuously differentiable and strictly decreasing in own price; furthermore suppose that the demand functions satisfy the IIA property. Then the demands $D_i(\vec{p})$ can be generated by an indirect utility function $V(\vec{p}, Y) = \tilde{\phi} \left(\sum_i g_i(p_i) \right) + Y$ where $\tilde{\phi}$ is twice differentiable, strictly convex in p_i , and $g_i(p_i)$ is twice differentiable and decreasing. Then the Bertrand game is aggregative, and consumer welfare depends only on the aggregator, $Q = \sum_i q_i$.*

However, the fact that a game is aggregative does not imply the IIA property on $V(Q)$. For example, the linear differentiated products demand system of Ottaviano and Thisse (2004) is an aggregative game for a fixed number of firms (i.e., in the short-run) with Bertrand competition since demand can be written as a sum of all prices and own price. However, it does not satisfy the IIA property.¹³

5 Examples of exogenous changes

The exogenous change we refer to in the results above can take many forms. In this section, we consider two examples. First, we determine the effects of cooperation (hence a change in

¹² A useful task is to find a general V which nests these two cases.

¹³ As shown in Erkal and Piccinin (2010), the same demand system is an aggregative game both in the short run and the long run under Cournot competition, but again it does not satisfy the IIA property.

the objective function of some of the firms). Second, we look at leadership (hence a change in the timing of moves). A third sub-section puts these into a common perspective.

5.1 Cooperation

Suppose that two players cooperate by maximizing the sum of their payoffs. An obvious application is a merger, though collusion or a research joint venture also fit the scenario. We deal with the short-run analysis explicitly below in the merger section; here we concentrate on the long-run consequences.

We first consider the impact of the cooperation on the actions of the cooperating players, j and k . Before the pact, their cumulative reaction functions are given by $\tilde{r}_j(Q)$ and $\tilde{r}_k(Q)$. The cumulative best reply is defined by the solution $q_j = \tilde{r}_j(Q)$ to the first order condition

$$\pi_{j,1}(Q, q_j) + \pi_{j,2}(Q, q_j) = 0,$$

and similarly for $\tilde{r}_k(Q)$. Under cooperation the players jointly solve

$$\max_{q_j, q_k} \pi_j(Q, q_j) + \pi_k(Q, q_k).$$

There are two first order conditions which determine q_j and q_k . The first of these is

$$\pi_{j,1}(Q, q_j) + \pi_{j,2}(Q, q_j) + \pi_{k,1}(Q, q_k) = 0, \quad (1)$$

which differs from the independent player case by the last term, which is the internalization of the effect of the aggregator on the sibling payoff. From this first order condition, we can solve for optimal choice of q_j as a function of Q and q_k . Similarly, the second first order condition defines the optimal choice of q_k as a function of Q and q_j . These two first order conditions can be solved simultaneously to determine the optimal choices of q_j and q_k as functions of the aggregator Q , and the solution yields $\tilde{r}_j^c(Q)$ and $\tilde{r}_k^c(Q)$ as the individual cumulative best reply functions for the cooperative pact. Adding them up gives the cumulative best reply for the pact, $\tilde{R}^c(Q)$.

The following lemma states that the merger causes the cumulative best reply function of the firms which are part of the pact to shift down (an illustration is in the Cournot competition section).

Lemma 4 *Let A be the set of active players in the market. Suppose that two of the players in A , j and k , cooperate. Then, $\tilde{r}_j^c(Q) \leq \tilde{r}_j(Q)$, $\tilde{r}_k^c(Q) \leq \tilde{r}_k(Q)$, and $\tilde{R}^c(Q) < \tilde{r}_j(Q) + \tilde{r}_k(Q)$.*

Proof. First consider the case when both j and k are active in production after the merger. Assumption 1 states that $\pi_k(Q_{-k} + q_k, q_k)$ is strictly decreasing in Q_{-k} . Writing $\pi_k(Q_{-k} + q_k, q_k)$ directly as $\tilde{\pi}_k(Q, q_k)$, this implies that $\tilde{\pi}_k(\cdot)$ is decreasing in its first argument. Hence, the third term in (1) is negative by Assumption 1 so that, for any q_k , the choice of q_j must be lower at any given Q . That is, firms j and k will choose lower actions (lower quantity in the Cournot benchmark, higher price in the differentiated products Bertrand oligopoly) as a result because they internalize the negative effect of their action on each other. The same logic applies for the first order condition for q_k . This implies that $\tilde{R}^c(Q) < \tilde{r}_j(Q) + \tilde{r}_k(Q)$.

Note that in the special case where the firms produce homogeneous products at constant but heterogeneous marginal costs, the lower-cost firm, call it k , will do all of the production after the merger. In this case, we have $0 = \tilde{r}_j^c(Q) < \tilde{r}_j(Q)$, $\tilde{r}_k^c(Q) = \tilde{r}_k(Q)$, and $\tilde{R}^c(Q) < \tilde{r}_j(Q) + \tilde{r}_k(Q)$. ■

In the context of mergers, that the combined entity would decrease its total production after the members internalize the externalities they impose on each other was stressed by Salant et al. (1983) in the case of Cournot competition with homogeneous products. In Lemma 4, we illustrate this result more generally (for any aggregative game) using the concept of the cumulative best reply function for the pact.

The following proposition summarizes the results in the long run case.

Proposition 8 *Let A be the set of active firms in the market. Suppose a subset C of the firms in A cooperate and a ZPHEE prevails after the cooperation as well as before. Then:*

- (i) *The value of the aggregator and, as a result, the non-cooperating firms' profits and consumer welfare remain unchanged.*
- (ii) *The payoffs to cooperating players all weakly decrease.*

Proof. (i) By Proposition 1, the aggregator does not change if the fringe is homogeneous. That the non-merged firms' profits and consumer welfare remain unchanged follows from

Lemma 3 and Proposition 4.

(ii) Recall that $\tilde{\pi}_j^*(Q) = \pi_j(Q, \tilde{r}_j(Q))$ stands for the maximized profit of firm j before the cooperation. By definition, $\tilde{r}_j(Q)$ stands for the portion of Q that is optimally produced by firm j . We know from Lemma 4 that each cooperating firm produces weakly less after the cooperation: $\tilde{r}_j^c(Q) \leq \tilde{r}_j(Q)$. Hence, it must be the case that $\tilde{\pi}_j^*(Q) \geq \tilde{\pi}_j^c(Q)$. ■

Proposition 8 applies generally in the case of symmetric or asymmetric firms as long as the type of the marginal firm does not change. If the firms are symmetric to start with, and making zero profits, then, after the cooperation (and following entry), the unaffected firms continue to make zero profits while the affected firms make negative profits (if the cooperation results in no additional benefits, such as fixed cost savings).

5.2 Leaders and followers

Etro (2006, 2007, 2008) considers the effect of introducing a Stackelberg leader into the free-entry model. His main results can be derived succinctly and extended using the framework above. The cumulative best reply approach provides a simple and direct proof of the result that the leader's profits are higher than when it acts simultaneously with the other firms, regardless of whether actions are strategic complements or substitutes.

The leader chooses q_l and in doing so rationally anticipates the subsequent entry level and follower firm action levels.

A first result on welfare is quite immediate:

Proposition 9 *Assume that one firm acts before the others (Stackelberg leader), and the subsequent equilibrium is a ZPHEE. Assume too that consumer surplus depends only on the value of the aggregator, Q . Then welfare is higher than under the Nash equilibrium, although consumer surplus stays the same.*

Proof. The consumer surplus result follows because Q is the same, given the outcome is a ZPHEE. Welfare is higher because the leader's rents must rise: it always has the option of choosing the Nash action level, and can generally do strictly better. ■

As discussed in Section 4, examples of cases where this results hold include markets with homogenous goods Cournot competition and Bertrand competition with demands satisfying the IIA property (such as logit/CES).

We now determine how the leader's action differs from the action it would choose in a simultaneous-move game. The cumulative reaction function $\tilde{r}_i(Q)$ (relevant for simultaneous moves) is implicitly defined by

$$\pi_{i,1}(Q, \tilde{r}_i(Q)) + \pi_{i,2}(Q, \tilde{r}_i(Q)) = 0$$

By Assumption 1, $\pi_{i,1}(Q, \tilde{r}_i(Q)) < 0$, and hence the second term must be positive at the solution. A Stackelberg leader, however, rationally anticipates that Q is unchanged by its own actions (Proposition 1), and so its optimal choice of action is determined by

$$\pi_{i,2}(Q, q_L) = 0$$

Hence, by Assumption 2b, the action taken by the leader in the long run must be larger than the action it takes in a simultaneous-move game. This result does not depend on whether we have a game with strategic complements or strategic substitutes.

Proposition 10 (*Replacement effect*) *Assume that one firm acts before the others (Stackelberg leader), and the subsequent equilibrium is a ZPHEE. Then its action level is higher and there are fewer fringe firms, although each fringe firm retains the same action level.*

We term the above result the Replacement Effect because, with a fixed Q , the leader would rather do more of it itself, knowing that it crowds out one-for-one the following fringe. In some cases, the leader wants to fully crowd out the fringe. For example, in the Cournot model, with $\pi_i(Q, q_i) = p(Q)q_i - C_i(q_i)$ we have $\frac{\partial \pi_i(Q, q_i)}{\partial q_i} = p(Q) - C'_i(q_i)$, so the leader will fully crowd out the fringe if $p(Q) \geq C'_i(Q)$. This condition necessarily holds if marginal cost is constant (since $p(Q) > c$ at a ZPHEE).

Finally, we compare with the short-run analysis when the number of firms is fixed. The equilibrium behavior of a first-mover with a fixed number of firms depends on whether actions are strategic substitutes or complements. If one of the firms has the privilege to choose its action before the others, it will take into account the impact of its action on the behavior of the followers. This implies that if actions are strategic complements, $dQ/dq_i > 1$. Since in a simultaneous-move Nash equilibrium $dQ/dq_i = 1$, the leader acts less aggressively than it would in a simultaneous-move game. On the other hand, if actions

are strategic substitutes (i.e., $dQ/dq_i < 1$), the leader acts more aggressively than it would in a simultaneous-move game. The leader's actions are determined by the relation

$$\pi_{i,1}(Q, q_L) \frac{dQ}{dq_L} + \pi_{i,2}(Q, q_L) = 0. \quad (2)$$

The comparison of short-run and long-run equilibria is quite striking. The larger leader result is familiar from the classic strategic substitutes case of Cournot competition, though there the followers react by reducing output, and the leader endogenizes this effect. In the canonical strategic complements model of differentiated products, the leader sets a higher price, inducing a higher price from the followers (and this induced higher price is the objective since the action level Q is reduced).¹⁴ At the ZPHEE, by contrast, the leader sets a lower price (higher q_i) and all fringe firms have the same price, regardless of the presence of a leader.

5.3 A common theme

The results above can be tied together with a simple common graphical theme. First, Assumption 2b implies that profit as a function of q_i for given Q is quasi-concave. The corresponding marginal profit, $\pi_{i,2}(Q, q_i)$ is graphed in Figure Since Q is determined at a ZPHEE independently of the actions of firm i , then firm i 's profit is represented as the area under this derivative. The various solutions above and the corresponding profit levels can now be depicted. First, the leadership point is the value of q_i where $\pi_{i,2}(Q, q_i) = 0$. Clearly it gives the highest profit of any solution. In comparison, the solution where i plays simultaneously with the other firms after entry involves $\pi_{i,1}(Q, q_i) + \pi_{i,2}(Q, q_i) = 0$. As argued above, because by Assumption 1 the first term is negative, the second is positive at the solution. Hence, the action level is lower, and the corresponding profit level is lower. The lower profit is represented as the triangle in Figure

Now consider cooperation (merger). As argued in the cooperation section above, each party to the merger now chooses an even lower action level (because now $\pi_{i,1}(Q, q_i) + \pi_{i,2}(Q, q_i) + \pi_{k,1}(Q, q_k) = 0$, and a further negative term has been added by Assumption 1), and hence each party to the cooperation now nets an even lower payoff. This loss per party (plant or product, depending on the interpretation or context in the IO setting) is

¹⁴These results can be quite readily derived within our framework.

represented in the figure as the trapezoid, and shows the loss compared to simultaneous and uncoordinated action.

6 Applications

In this section, we consider several implications of the results stated above. It is readily confirmed in these examples that Assumption 1 which states that $\pi_i(Q, q_i)$ is decreasing in Q holds quite generally.

6.1 Cost changes and producer surplus (rents) of affected firms

We start by analyzing cost or quality improvements. For simplicity, we analyze the effect if a single insider is affected, supposing that it is the only one affected.

We are first interested in how the change affects the affected firm's rents, and we draw out the crucial distinction between the total profit and the marginal profit effect of the change. We then consider how the change affects the number of the fringe firms.

In what follows, we denote firm i 's type parameter by θ_i , and we are interested in how an improvement to θ_i translates to changes in producer surplus. We therefore introduce parameters θ_i into the profit functions of insiders.

Proposition 11 *Suppose $\frac{d\pi_i(Q_{-i}+q_i, q_i; \theta_i)}{d\theta_i} > 0$ for some $i \in A_I$. Then higher θ_i raises firm i 's equilibrium rents at a ZPHEE if $\frac{\partial^2 \pi_i(Q_{-i}+q_i, q_i; \theta_i)}{\partial \theta_i \partial q_i} \geq 0$.*

Proof. Proposition 1 implies that at a ZPHEE with homogeneous fringe firms, Q remains unchanged as θ_i rises. Hence, we wish to show that $\frac{d\tilde{\pi}_i^*(Q; \theta_i)}{d\theta_i} > 0$ with Q fixed.

Let $f_i(Q, \theta_i)$ denote the Q_{-i} locally defined by $Q - Q_{-i} - r_i(Q_{-i}, \theta_i) = 0$. Then, by the implicit function theorem,

$$\frac{df_i(Q, \theta_i)}{d\theta_i} = \frac{-\partial r_i / \partial \theta_i}{1 + \partial r_i / \partial Q_{-i}}.$$

Invoking Lemma 1, the denominator is positive. Hence, this expression is negative if $\partial r_i / \partial \theta_i$ is positive. Applying the implicit function theorem to the reaction function, which is given in implicit form as $\frac{d\pi_i(Q_{-i}+q_i, q_i; \theta_i)}{dq_i} = 0$, shows that $\partial r_i / \partial \theta_i \geq 0$ if and only if $\frac{\partial^2 \pi_i(Q_{-i}+q_i, q_i; \theta_i)}{\partial \theta_i \partial q_i} \geq 0$. Hence, $\frac{df_i(Q, \theta_i)}{d\theta_i} \leq 0$ under this condition.

Now, since by definition $\tilde{\pi}_i^*(Q; \theta_i) = \pi_i^*(f_i(Q, \theta_i); \theta_i)$, we have

$$\frac{d\tilde{\pi}_i^*(Q; \theta_i)}{d\theta_i} = \frac{\partial \pi_i^*(Q_{-i}; \theta_i)}{\partial Q_{-i}} \frac{df_i(Q, \theta_i)}{d\theta_i} + \frac{\partial \pi_i^*(Q_{-i}; \theta_i)}{\partial \theta_i} \quad (3)$$

The term $\frac{\partial \pi_i^*(Q_{-i}; \theta_i)}{\partial Q_{-i}}$ on the RHS is negative by Assumption 1. Hence, $\frac{\partial \pi_i^*(Q_{-i}; \theta_i)}{\partial Q_{-i}} \frac{df_i(Q, \theta_i)}{d\theta_i}$ is positive under the condition stated in the Proposition.

Consider now the last term on the RHS (3). Since $\pi_i(Q_{-i} + q_i, q_i; \theta_i)$ increases with θ_i by supposition, so does the maximized value represented by $\pi_i^*(Q_{-i}; \theta_i)$: hence, $\frac{\partial \pi_i^*(Q_{-i}; \theta_i)}{\partial \theta_i} > 0$.

Hence, $\frac{d\tilde{\pi}_i^*(Q; \theta_i)}{d\theta_i} > \frac{\partial \pi_i^*(Q_{-i}; \theta_i)}{\partial \theta_i} > 0$, and i 's rents necessarily rise at a ZPHEE. ■

The qualification $\frac{\partial^2 \pi_i(Q_{-i} + q_i, q_i; \theta_i)}{\partial \theta_i \partial q_i} \geq 0$ in Proposition 11 represents an increasing *marginal* profitability. If marginal profits instead fell with θ_i , then there is a tension between the direct effect of the improvement to i 's situation, and the induced effect through a lower action. This tension is illustrated in the following example, where we show that a cost improvement that has a “direct” effect of raising profits may nonetheless end up decreasing them once we duly account for the new free entry equilibrium reaction.

Example 1 Consider a Cournot model with demand $p(Q) = 1 - Q$. Suppose then that firm costs are $C(q) = cq + K$ for all Fringe firms, and $C_1(q) = (c + \theta)q_1 - \beta\theta + K$ for firm 1. The Fringe firms' first order conditions are $1 - Q - c = q$, and their zero profit condition is $q = \sqrt{K}$. For firm 1, the first order conditions are $1 - Q - c - \theta = q_1$, where we see the impact of higher marginal cost reducing output. Hence, $q_1 = q - \theta = \sqrt{K} - \theta$. Since profit is $q_1^2 + \beta\theta - K$, then, using the zero profit condition of the fringe and the output relation just derived this profit is $(\sqrt{K} - \theta)^2 + \beta\theta - K$ at the zero-profit free entry equilibrium solution. The derivative of this profit expression with respect to θ is $-2(\sqrt{K} - \theta) + \beta$, which we can write as $-2q_1 + \beta$. Now notice that the “direct” effect of a marginal change in θ is $-q_1 + \beta$, meaning that this is the change in profit if all outputs were held constant (of course, firm 1's output would adjust, but, by the envelope theorem, this expression measures the profit change for given output of rivals). Clearly, depending on the size of β , a positive direct effect can nonetheless mean a negative final effect, once we factor in the output restriction of the affected firm, and the entry response of the fringe.

The example shows that the direct profit effect and the marginal profit effect can work in different directions. There are, of course, other examples in the literature where the response

of rivals can overwhelm the direct effect.¹⁵ Bulow, Geanakoplos, and Klemperer (1985) provide several examples where a purported benefit turns into a liability once reactions are factored in, in the context of multi-market contact. The original Cournot Merger Paradox of Salant, Switzer, and Reynolds (1983) is a case where merging firms end up worse off after coordinating their output. This latter example can easily be tailored to fit the case at hand, with free entry. Note first that two merging firms in an industry of symmetric firms earn zero profits both before and after merging, so there is no long-run benefit (or cost) to them merging. Suppose instead then that two firms in a (Cournot) industry had lower marginal production costs than their rivals, and the industry is in free entry equilibrium. Then when these two low-cost firms merge, they will restrict output and more firms will enter. The final state will have them worse off than before merging, at least if there are no synergies from the merger.¹⁶

The next result says how a change to a single firm affects its equilibrium action.

Proposition 12 *Suppose $\frac{d^2\pi_i(Q_{-i}+q_i;\theta_i)}{d\theta_i dq_i} > 0$. Then, a higher θ_i for firm i increases its equilibrium output at a ZPHEE.*

Proof. We know from Proposition 1 that Q is unchanged when θ_i changes for an infra-marginal firm i . Hence we wish to show that $\frac{d\tilde{r}_i(Q;\theta_i)}{d\theta_i} > 0$: which means that firm i produces more as a result of the change in θ_i .

As we argued in Proposition 11, $\frac{d^2\pi_i(Q_{-i}+q_i;\theta_i)}{d\theta_i dq_i} > 0$ if and only if $\frac{dr_i(Q_{-i},\theta_i)}{d\theta_i} > 0$.

Now, by definition $\tilde{r}_i(Q;\theta_i) = r_i(f_i(Q,\theta_i);\theta_i)$, where we recall that $f_i(\cdot)$ denotes the Q_{-i} locally defined by the relation $Q - Q_{-i} - r_i(Q_{-i};\theta_i) = 0$. Hence,

$$\frac{d\tilde{r}_i(Q;\theta_i)}{d\theta_i} = \frac{\partial r_i(Q_{-i};\theta_i)}{\partial Q_{-i}} \frac{df_i(Q)}{d\theta_i} + \frac{\partial r_i(Q_{-i};\theta_i)}{\partial \theta_i}.$$

As we showed in Proposition 11,

$$\frac{df_i(Q)}{d\theta} = \frac{-\partial r_i/\partial \theta_i}{1 + \partial r_i/\partial Q_{-i}}.$$

Hence,

$$\frac{d\tilde{r}_i(Q;\theta_i)}{d\theta_i} = \frac{\partial r_i/\partial \theta_i}{1 + \partial r_i/\partial Q_{-i}}, \tag{4}$$

¹⁵ Although we do not know of examples using free entry as the mechanism?

¹⁶ Move this discussion to the merger section.

which is positive since the denominator is positive for strategic substitutes or strategic complements under Assumption 3 (see Lemma 1). ■

The proof relies on showing how the individual cumulative reaction function changes. If there are several affected firms, and they all change the same way (in the sense that $\frac{d\pi_i(Q_{-i}+q_i;\theta_i)}{dq_i}$ is moved in the same direction by the individual changes), then the number of fringe firms goes down if the individual marginal profitabilities move up, etc. (see Proposition 11 above).

The result at the end of Proposition 12 (equation (4)) can be illustrated graphically. This is done in Figure 5 for strategic substitutes, and Figure 6 for strategic complements. In both cases, the cumulative reaction functions shift up when the reaction functions do. Indeed, given $Q = Q_{-i} + r_i(Q_{-i};\theta)$, a higher θ leads to a higher $r_i(Q_{-i})$ and hence to a higher Q . Hence, inverting, there is a higher Q for a given Q_{-i} and so, since Q_{-i} is increasing in Q , we have now a *lower* Q_{-i} for a given Q , which means a *higher* $\tilde{r}(Q)$. That is, q_i is now a larger part of any total; as indeed i reacts with a higher action to any Q_{-i} .

The construction used in Figures 3 and 4 also neatly illustrates how a higher θ raises $\tilde{r}_i(Q)$. This is done for the two cases in Figures 7 and 8, respectively. Since a higher θ raises the reaction function, the same Q line needs to get to a higher q . For the case of strategic complements the reaction function slopes up; Assumption 3 means it is flatter than the diagonal. From the reaction function, draw a line of slope -1 back to the Q axis to give the Q_{-i} associated to Q and q ; Q_{-i} is increasing in both. Hence \tilde{r} increases when r increases. Raising θ raises the reaction function. The same Q_{-i} (extending the 45-degree line) associates to a higher q and Q . Alternatively, hold Q constant by going vertically above the reaction function to the new one, and observe that the associated q rises while Q_{-i} falls. This indicates the property that if higher a θ means a higher \tilde{r} , then it means a higher r , and conversely.

As an example, suppose that firm i is subsidized in the context of a Cournot world. This raises its received marginal profit to $\pi_{i,2}(Q, q_i) + s$, as shown in Figure ..., where s is the subsidy per unit paid. The subsidy makes firm i more aggressive, as per our cost change analysis above, and therefore induces a higher action level, and a greater fraction of the fixed total. As shown in Figure ..., the firm's profit rises by the area ... which is greater

than the cost of the subsidy, which is area This insight underpins the analysis of ... in the international trade context.

6.2 Cournot competition

A classic example of an aggregative game is the Cournot oligopoly model.¹⁷ For illustration purposes, consider the derivation of $\tilde{r}_j^m(Q)$ and $\tilde{r}_k^m(Q)$ in the next example, where there is a merger between two producers of homogeneous products in a market with Cournot competition. Note that since the merging firms have different cost functions in this example, it allows for cost savings.

Example 2 Consider a merger between firms j and k in a market where firms engage in Cournot competition and demand is $P = 1 - Q$. The cost function of firm j is $C_j(q_j) = q_j^2$ and the cost function of firm k is $C_k(q_k) = q_k^2/2$. The merger allows the firms to shift production between themselves so that each unit is produced by the firm with the lower marginal cost for that unit. In equilibrium, the last unit produced at both firms should have the same marginal cost.

The firms maximize $(1 - Q)q_j - q_j^2 + (1 - Q)q_k - q_k^2/2$. The FOCs are

$$1 - Q - q_j - 2q_j - 2q_j - q_k = 1 - Q - q_j - k - 4q_j - 2q_k = 1 - Q - 3q_j - q_k = 0$$

$$1 - Q - k - 2q_k - q_k - q_j = 1 - Q - j - k - 3q_k - 2q_j = 1 - Q - 2q_k - q_j = 0$$

The first expression gives us q_j as a function of Q and q_k . Similarly for the second expression. We can solve these two expressions simultaneously to get the optimal q_j and q_k as functions of Q . We get

$$\tilde{r}_j^m(Q) = \frac{1 - Q}{5} \text{ and } \tilde{r}_k^m(Q) = \frac{2(1 - Q)}{5}$$

where we use the notation $\tilde{r}_j^m(Q)$ and $\tilde{r}_k^m(Q)$ to denote the quantities produced by firms j and k within the merged entity, respectively.

¹⁷Material on subsidies here (see the exogenous changes section)? Or, a leader (drive out the rivals if c is constant) example? Common property resources? International trade example?

6.3 Mergers

A merger implies that the merged firms can maximize joint profits. After the pact, we can treat the merged entity as a multi-plant (in case of homogeneous products) rationalization of production across plants owned by formerly disparate owners; or a multi-product firm (in case of differentiated products) pricing different variants. The results of the cooperation section apply directly to mergers in the long run. Here we derive the short run results and compare with those for the long-run equilibrium effects of a merger. As illustrated here, this objective is made simple with the device of the cumulative reaction function.

A merger can result in cost savings and/or synergies. As explained in Farrell and Shapiro (1990), synergies allow the merged entity to improve their joint production capabilities (through learning, for example) while cost savings result from the opportunity to better allocate production among the different facilities (rationalization). We focus on mergers without synergies although the analysis can be extended to include mergers with synergies. Clearly, if there are synergies created by the merger, Lemma 4 may no longer hold.

A merger may result in fixed cost savings (perhaps due to economies of scope) or a more optimal allocation of production within the merged entity by shifting production from high-cost facilities to low-cost ones. Note that the second kind of cost saving is more likely to happen if the firms are producing homogeneous products. If the firms are producing differentiated products, then it may not be feasible to shift production between firms.

Consider the impact of the merger on the value of the aggregator in the short run. Recall that the equilibrium condition for the aggregator, for a given set of firms, A , is

$$\sum_{i \in A} \tilde{r}_i(Q) = Q.$$

In an aggregative game, a merger does not affect the reaction functions or the cumulative reaction functions of the firms which are not part of the merger. Therefore, we only need to keep track of what happens to the cumulative reaction functions of those merging, which we have just argued goes down. Hence, the sum $\sum_{i \in A} \tilde{r}_i(Q)$ (i.e., $\sum_{i \neq j,k} \tilde{r}_i(Q) + \tilde{R}^m(Q)$) goes down once there is a merger. Since it will intersect the 45° line at a lower Q , the equilibrium value of Q necessarily falls in the short run for the strategic substitutes case and, invoking the slope condition, also for the strategic complements case.

The new equilibrium values for the remaining firms follow directly from the reaction function slope ($\tilde{r}'_i(Q)$, which has the same sign as $r'_i(Q_{-i})$). They therefore rise under strategic substitutability; for example, in the Cournot case, other firms expand output, and hence the merged firm's total output must contract by a higher amount to deliver the lower total Q . For the strategic complements case, others' actions fall (higher prices under Bertrand competition), and the merged firm's actions fall for the twin reasons of the direct lowering of the reaction functions and their positive slope. In both cases though, since Q has fallen, profits of non-merging firms have risen (since $\tilde{\pi}_i^*(Q)$ is decreasing by Lemma 3). Also, consumer welfare decreases as long as it decreases in the value of the aggregator. In summary:

Proposition 13 *Let A be the set of active firms in the market. Suppose a subset M of the firms in A merge. If there are no merger-specific synergies, the value of the aggregator decreases in the short run. Hence, the non-merged firms' profits go up, and consumer welfare goes down*

In the short run, in the absence of cost savings or synergies, the merged firms may or may not benefit from the merger. If actions are strategic substitutes, since the non-merged firms' response does not reinforce the merged firms' actions, mergers are not profitable unless they include a sufficiently large percentage of the firms in the market. This effect is exemplified in the Cournot merger paradox of Salant, Switzer, and Reynolds (1983). Without synergies, non-merged firms benefit from a merger while merging firms can lose. If actions are strategic complements (as in the case of Bertrand competition), Deneckere and Davidson (1985) show that mergers are always profitable. However, non-merged firms still benefit more from a merger than the merged firms. This is because each merged firm cannot choose the action that maximizes its individual profits while each non-merged firm does.

In the long run, Part (i) of Proposition 8 states that with entry, whether actions are strategic substitutes or strategic complements, the value of the aggregator returns back to its pre-merger level. It is important to comment on how general this result is. It (and hence its implications) holds for all games which can be classified as aggregative games in the long run. Davidson and Mukherjee (2007) shows this result in the case of homogeneous

goods Cournot competition. Our contribution is to extend it to multi-product firms and to firms participating in differentiated goods markets with Bertrand (CES, and logit demand systems) and Cournot competition (Ottaviano and Thisse, 2004). This result has been shown in Davidson and Mukherjee (2007) for the case of homogeneous goods Cournot competition.

Part (i) of Proposition 8 also implies that entry in the long run counteracts the negative impact of mergers on consumer welfare in the short run. Given our results in Section 4, this holds for any demand system where consumer welfare can be written as a function of the aggregator only. As we show in Section 4, this includes demand systems which satisfy the IIA property.

In terms of welfare, Proposition 8 implies that mergers are socially desirable from a total welfare standpoint if and only if they are profitable. Put another way, the models we consider here build in that property. The result is tempered by integer issues, by a rising fringe supply, and if the long-run takes a long time to attain. Most important, perhaps, is the global competition property built into the demand functions satisfying the IIA property.

Propositions 13 and 8 together imply that while the non-merged firms benefit from the merger in the short run, their profits go back to the pre-merger level (zero) in the long run. Importantly, part (ii) of Proposition 8 implies that in the absence of merger-specific synergies and/or fixed cost savings, the merger affects the profits of its member firms adversely.¹⁸ Hence, the profitability of mergers under Bertrand competition no longer holds in the long run. Merger-specific synergies and/or fixed cost savings are required for the profitability of mergers in the long run.

¹⁸In the special case where the firms produce homogeneous products at constant but heterogeneous marginal costs, it is profitable for the lower-cost firm to do all of the production after the merger. In this case, the lower-cost firm's profits will not be affected by the merger while the higher-cost firm will make strictly less as a result of the merger since it will shut down. In such cases, there may be side payments between the firms, which are not taken into account in the statement of Proposition 8. If there are side payments between the firms, their post-merger payoffs will clearly be different from what they individually make as a result of the merger.

6.4 Contests

Another example of an aggregative game can be found in the literature on contests, where players spend effort to win a prize.¹⁹ One specific context in which contests take place is in research and development (R&D).

6.4.1 Competition and cooperation in R&D environments

Starting with Loury (1979) and Lee and Wilde (1980), the standard approach to R&D competition assumes that the size of the innovation is exogenously given, but its timing stochastically depends on the R&D investments chosen by the firms through a Poisson process (see Reinganum (1989) for a survey). This mainstream model of R&D competition has been used to analyze issues ranging from the impact of market structure on innovation to the optimal design of patent policy.

Each firm which participates in the R&D race operates an independent research facility. Time is continuous, and the firms share a common discount rate r . Firms choose an investment x at the beginning of the race, which provides a stochastic time of success that is exponentially distributed with hazard rate $h(x)$. A higher value of $h(x)$ corresponds to a shorter expected time to discovery. Suppose that $h'(x) > 0$, $h''(x) < 0$, and $h(0) = 0$. $\lim_{x \rightarrow 0} h'(x)$ is sufficiently large to guarantee an interior equilibrium and $\lim_{x \rightarrow \infty} h'(x) = 0$.

The main difference between Loury (1979) and Lee and Wilde (1980) is in the formulation of the investment costs. Loury (1979) assumes that a firm which chooses an investment x pays x at $t = 0$. Lee and Wilde (1980) assumes that the same firm pays a fixed cost F at $t = 0$ and a flow cost x as long as it stays active in the R&D race.

Following Lee and Wilde (1980), we can express the payoff function of participant i in the R&D race as

$$\frac{h(x_i)V - x_i}{r + h(x_i) + \alpha_i} - F,$$

where V stands for the value of the innovation and $\alpha_i = \sum_{j \neq i} h(x_j)$ stands for the aggregate hazard rate of the rival firms. Using our notation from above, we can think of each firm as

¹⁹Starting with the seminal contributions of Tullock (1967 and 1989), Krueger (1974), Posner (1975) and Bhagwati (1982), a large literature has emerged to analyze the various aspects of contests seen in social, political and economic life.

choosing $q_i = h(x_i)$. Hence, letting $Q_{-i} = \sum_{j \neq i} h(x_j)$ and $Q = \sum_i h(x_i)$, we can the firm's payoff function as

$$\frac{q_i V - h^{-1}(q_i)}{r + Q} - F.$$

Clearly, we have an aggregative game, where each firm's payoff depends on its own action and the aggregator, Q .

Let $r(Q_{-i}, V, r)$ and $\tilde{r}(Q, V, r)$ stand for the reaction function and the cumulative reaction function of firm i , respectively. Using the implicit function theorem, it is straightforward to show that both functions are upward sloping. Hence, investment choices are strategic complements.²⁰

Although the discussion above treats the number of firms as fixed, several papers have assumed that the number of participants in the R&D race is determined by free entry.²¹ Using the envelope theorem, we can verify that Lemma 3 holds within this framework.²² This powerful result allows one to pin down the aggregate investment level in a long-run equilibrium.

One obvious exogenous change to consider in such a framework is the impact of R&D cooperation on market structure and performance. Erkal and Piccinin (2010) study the impact of R&D cooperation in a long run equilibrium. They compare free entry equilibria with R&D competition to free entry equilibria with R&D cooperation, where an exogenously given number of firms participate in the R&D race as members of a cooperative R&D arrangement. Following Kamien, Muller and Zang (1992), two common ways in which cooperative R&D has been defined in the literature are R&D cartels and RJV cartels. In an R&D cartel, the partner firms choose their investment levels to maximize their joint profit function, but they do not share the outcomes of their research efforts. In an RJV cartel, the partner firms both choose their investment levels to maximize their joint profit function and share the outcomes of their research efforts. In both cases, the firms compete

²⁰Following Loury's (1979) formulation of the investment costs yields a payoff function of the form $\frac{q_i V}{r+Q} - h^{-1}(q_i)$. In this case, $\frac{\partial r}{\partial Q_{-i}} \geq 0$ as $q_i \geq \frac{Q+r}{2} = Q_{-i} + r$. Hence, actions are strategic substitutes if $q_i < Q_{-i} + r$ and strategic complements if $q_i > Q_{-i} + r$. However, in a symmetric equilibrium with $N \geq 2$, actions are necessarily strategic substitutes since $Q^* = Nh(x^*)$.

²¹Loury (1979) and Lee and Wilde (1980) show that with free entry, there will be too much entry into the R&D race. For some more recent contributions, see Denicolo (1999 and 2000), Etro (2004), Erkal (2005) and Erkal and Piccinin (2010).

²²See Lemma 1 in Erkal and Piccinin (2010).

in the product market following the R&D stage. Hence, if the firms form an R&D cartel and if one of the cartel members is the first to develop the innovation, there is only one firm with the new technology in the product market whereas if the firms form an RJV cartel and if one of the cartel members is the first to develop the innovation, all of the cartel members compete in the product market using the new technology.

Proposition 1 above implies that regardless of the type of cooperation, as long as there are participants in the R&D race which are not members of the cooperative R&D arrangement, the aggregate rate of innovation ($Q = \sum_i h(x_i)$) remains the same with and without a cooperative R&D arrangement. This is despite the fact that the number of participants in the R&D race is different under R&D competition and R&D cooperation. This surprising indifference result implies that any welfare gain from R&D cooperation cannot be driven by its impact on the aggregate rate of innovation.²³

Regarding the impact of R&D cooperation on firm profitability, Erkal and Piccinin (2010) show that R&D cartels are unprofitable in the long run. The logic behind this result is similar to the one presented in Lemma 4 and Proposition 8 in Section 6.3. That is, the members of the R&D cartel, because they maximize joint profits, reduce their investment choices. This causes them to make less profits than they would make in an industry without an R&D cartel. If firms are symmetric, then it implies that they would be making negative profits instead of zero profits.

6.4.2 R&D subsidies

Another relevant policy question which can be explored within our framework is the impact of R&D subsidies in a long run equilibrium. R&D subsidies are used in many countries throughout the world. Depending on the program, the goal may be to support specific types of industries, technological problems, or early vs. late stage research.

Consider a subsidy program that affects only a subset of the firms in an industry.²⁴ Suppose that, as in the Lee and Wilde (1980), investment in R&D entails the payment of a fixed cost F at $t = 0$ and a flow cost, and that the subsidy decreases the recipient's marginal

²³For the short-run effects of RJVs in a Poisson environment, see Miyagiwa and Ohno (2002).

²⁴Alternatively, we can think about R&D subsidies in an international setting, where the firms in a single country or a group of countries can benefit from R&D subsidies.

cost of R&D. Based on our results in section 3, we can state the following.

Proposition 14 *Let A be the set of active firms in the market. Suppose the government adopts an R&D subsidy policy which affects the firms $i \in A_I \subset A$. Then:*

- (i) The rate of innovation in the short run is higher with the policy than without.*
- (ii) The long run rate of innovation is unaffected by the policy.*
- (iii) The number of participants in the R&D race is lower with the policy than without.*
- (iv) With the policy, the investment choices of the affected firms increase and the investment choices of the unaffected firms do not change in the long run.*
- (v) With the policy, the expected profits of the affected firms go up and the expected profits of the fringe firms remain unchanged in the long run.*

Proof. (i) The R&D subsidy causes the recipients' reaction functions to shift up. Since actions are strategic complements in Lee and Wilde (1980), this causes the rate of innovation in the short run, Q , to increase.

(ii) Follows from Proposition 1.

(iii) Follows from Proposition 12.

(iv) Follows from Propositions 12 and 2.

(v) That the affected firms earn higher with the policy follows from Proposition 11. The fringe firms make zero by definition in a long run equilibrium. ■

The important result in Proposition 14 is that although the government can increase the rate of innovation in the short run by adopting an R&D subsidy policy, it cannot affect the rate of innovation in the long run with this kind of a policy.

6.5 Privatization of public firms

Anderson, de Palma and Thisse (1997) use a CES model to compare free-entry equilibria (ignoring integer constraints) in two scenarios. The first is when some firms are run as public companies, with the objective of maximizing their contribution to social surplus. There remains a competitive fringe. Note first that the public firms do not price at marginal cost, they price above it. This is because they are assumed to care about producer surplus of other private firms – raising price just above marginal cost has a second-order effect on

the sum of public firm profit plus consumer surplus, but has a first-order effect of increasing private firms' profits. They also price below private firms because they care about consumers too. Paradoxically perhaps, (paradoxically because the private firms are the assumed profit maximizers) it can also happen that the public firms make a profit at a ZPHEE, even though the private firms (with the same cost and demand structure) do not. This is because public firms price lower and produce more.²⁵

Since the CES model has the IIA property, Proposition 7 applies: the game is an aggregative one, and consumer surplus depends only on the value of the aggregator. Moreover, then our core propositions apply. Namely from Proposition 5, the affect of privatization is to change total welfare by the change in the rents earned by the nationalized firms. Hence, if they earned profits as nationalized firms, total surplus falls by the size of the profits now no longer earned. In this case, consumers experience a price rise, but this is exactly offset by the change in product variety as new entrants are attracted to the industry by reduced price competition from the erstwhile public firms.²⁶ Note that this suggests that profitable public firms ought not be privatized if there is free-entry, and if the market demands are well characterized by the IIA property.

6.6 Leadership and credible commitment

We assumed above the leader firm is able to credibly commit to its action. However, this may not be feasible without some commitment device, and so the leader may be constrained in the extent to which it can crowd out the fringe. Nevertheless, the tenor of the above results goes through: the leader will be more aggressive in order to credibly commit to crowd out the fringe, by presenting a tougher reaction function.

Following Brander and Spencer (1983) and Spencer and Brander(1983), suppose that

²⁵Recall that price leaders in strategic complements Bertrand differentiated products games will price above the Nash equilibrium level in order to induce a favorable price rise from the price followers. All earn more than at the Nash price equilibrium, but the leader earns less than the follower(s). The nationalized firm's incentives run the other way: it prices below the private firms, and this brings down their prices. This per se lowers profits for all, but can lower them more for the private firms. Of course, if the public firm prices close enough to marginal cost, its profits must be lower than the private firms'.

²⁶Lemma 3 in Anderson, de Palma and Thisse (1997) - that all private firms charge the same price in a long-run equilibrium - follows from Proposition 2 above. Proposition 2 in Anderson et al. (1997) -that the social gain (loss) from privatization equals the loss (profit) made by the public firms before they are privatized - is Proposition 5 above.

the leader can irrevocably choose its cost technology, and by doing so present a more or less aggressive stance to the subsequent Fringe movers.

More formally, let us append a cost function $\Gamma(\theta_i)$ to the leader's profit function, with the idea that higher θ_i shifts the net profit function, which we wrote before as $\pi_i(Q_{-i} + q_i, q_i; \theta_i)$, and so we now write the firm's profit as $\pi_i(Q_{-i} + q_i, q_i; \theta_i) - \Gamma(\theta_i)$. Naturally then, $\Gamma'(\theta_i) > 0$. For example, a higher capital investment translates into a lower marginal cost of production.

We first describe the standard ZPHEE when the firm in question acts in standard Nash fashion, but we add to its choice the level of θ_i . We then compare to the case in which the firm acts as leader.

As we know from Section 3.3, the reaction function for given $\theta_i, r_i(Q_{-i}, \theta_i)$, is determined by

$$\frac{d\pi_i(Q_{-i} + q_i, q_i; \theta_i)}{dq_i} = 0.$$

As we noted above (see (2)), this can be written as

$$\frac{\partial\pi_i(Q, q_i; \theta_i)}{\partial Q} + \frac{\partial\pi_i(Q, q_i; \theta_i)}{\partial q_i} = 0, \quad (5)$$

where the first term is negative by Assumption 1 and so the second term must be positive at the solution. We now also have a new first order condition:

$$\frac{d\pi_i(Q_{-i} + q_i, q_i; \theta_i)}{d\theta_i} = \Gamma'(\theta_i), \quad (6)$$

which equalizes marginal benefits and marginal costs.

Consider now the case of the leader firm, which chooses θ_l in advance of the fringe's actions. Regardless of what it does, it knows that the total market action, Q_{ZPHEE} , is independent of its own actions. It also rationally anticipates that it will choose $q_l = \tilde{r}_l(Q_{ZPHEE}, \theta_l)$ in the sub-game in which actions and entry decisions are determined. Therefore, its choice of θ_l is determined by the solution to the problem

$$\max_{\theta_l} \pi_l(Q_{ZPHEE}, q_l; \theta_l) - \Gamma(\theta_l)$$

where $q_l = \tilde{r}_l(Q_{ZPHEE}, \theta_l)$, and the leader treats Q_{ZPHEE} as fixed. The derivative with respect to θ_l is given by

$$\frac{\partial\pi_l(Q_{ZPHEE}, q_l; \theta_l)}{\partial q_l} \frac{d\tilde{r}_l(Q_{ZPHEE}; \theta_l)}{d\theta_l} + \frac{\partial\pi_l(Q_{ZPHEE}, q_l; \theta_l)}{\partial\theta_l} - \Gamma'(\theta_l) \quad (7)$$

where $\frac{d\tilde{r}_i(Q_{ZPHEE};\theta_l)}{d\theta_l}$ has the sign of $\frac{\partial^2 \pi_i(Q_{-i}+q_i, q_i; \theta_i)}{\partial \theta_i \partial q_i}$ whether or not actions are strategic substitutes or strategic complements.²⁷

Now, since Q_{ZPHEE} is the same whether or not the firm acts as a leader, the first term $\frac{\partial \pi_i(Q_{ZPHEE}, q_i; \theta_l)}{\partial q_i}$ in (7) is positive when evaluated where θ_l is given by the non-strategic (simultaneous move) choice, i.e., (6), at which point the last two terms in (7) are also zero. Therefore the derivative (7) at that choice has the sign of $\frac{\partial^2 \pi_i(Q_{-i}+q_i, q_i; \theta_i)}{\partial \theta_i \partial q_i}$. This means that the choice of θ_l is then higher under pre-commitment (assuming the problem is sufficiently well-behaved that it is quasi-concave) if marginal profit is increasing in θ_i and it is lower if marginal profit is decreasing. In either case, $\tilde{r}_i(Q)$ increases from the simultaneous game level: this is the Aggressive Leaders result of Etro (2006).²⁸

Short-run incentives with a fixed number of firms can also readily be compared. The leader wants to do at the margin whatever causes the other firms to cut back, so this is clearly to be more aggressive if actions are strategic substitutes (see Figure 11), and to be less aggressive if actions are strategic complements (Figure 12).²⁹

7 Heterogeneous fringe

We have assumed until now that the Fringe consists of firms with the same profit functions. The simplest generalization is when firms differ in a manner that can be unambiguously ranked, such as costs, qualities, or entry costs.

7.1 Heterogeneous entry costs

We first analyze the case of differing entry costs, by supposing that all fringe firms have the same profit functions, except that they differ by idiosyncratic K . Note that Lemmas 1, 2 and 3 of Section 2 of the paper are still pertinent to the current case since they apply to the post-entry sub-games.

Let $K(n)$ denote the entry cost of the n th lowest cost fringe entrant (rather similar to a supply curve: they are ranked in order of increasing costs). Assume the marginal firm

²⁷ Recall from (4) that $\frac{d\tilde{r}_i(Q, \theta_i)}{d\theta_i} = \frac{\partial r_i / \partial \theta_i}{1 + \partial r_i / \partial Q_{-i}}$.

²⁸ Results for the case of heterogeneous followers can be readily derived.

²⁹ With a lower leader action, rivals reply with lower actions, which is the response the leader wants to induce under Assumption 1.

earns zero profit at a zero profit equilibrium. Then the equilibrium solution for any set of active firms, A , is given by

$$\sum_{i \in A} \tilde{r}_i(Q) = Q, \quad (8)$$

which is a fixed point. We assume the LHS has slope less than 1 (see Figures 11 and 12 below). This is guaranteed for strategic substitutes since the LHS is decreasing because all its components are. For strategic complements, matters are more delicate. Assuming a slope less than 1 at any fixed point guarantees a unique equilibrium. Alternatively, following a device frequently deployed in analyzing comparative statics with strategic complements, we can describe the comparative static properties of the largest and smallest equilibria. The slope condition must hold for both.

Suppose now that one of the insiders' marginal profit rises (for example, due to a cost reduction), and this causes the equilibrium set of firms to move from A to A' . From Proposition 12, we know that if all fringe firms have the same entry cost, such a change increases the affected firm's action while leaving the value of the aggregator unchanged. In the following proposition, we state how the equilibrium changes in the case of heterogeneous fringe firms.

Proposition 15 *Let entry costs be strictly increasing across fringe firms. Suppose one of the insiders ($j \in A$) experiences a change that increases its marginal profit. Let A and A' stand for the set of firms in the two ZPHEE before and after the change, respectively. Then: (i) $Q_A < Q_{A'}$, (ii) the change causes some fringe firms to exit (i.e., the number of fringe firms is higher at A than at A'), (iii) each fringe firm chooses a higher (lower) action if and only if actions are strategic substitutes (complements), and (iv) insider firm j (the firm which has been exposed to the change) chooses a higher action.*

Proof. (i) To show that $Q_A < Q_{A'}$, we can break down the fixed-point condition (8) above to write

$$n\tilde{r}_f(Q) + \sum_{i \in A_f} \tilde{r}_i(Q) = Q. \quad (9)$$

The marginal fringe entrant makes zero profit, or

$$\tilde{\pi}_f(Q) = K(n). \quad (10)$$

Together, these two equations determine the equilibrium outcome.

Totally differentiating (9) gives

$$\tilde{r}_f(Q) dn + \left(n\tilde{r}'_f(Q) + \sum_{i \in A_I} \tilde{r}'_i(Q) \right) dQ + d\tilde{r}_j(Q) = dQ \quad (11)$$

where we take into account the fact that insider firm j experiences a change that increases its marginal profit (a positive $d\tilde{r}_j(Q)$ means it is more aggressive). Totally differentiating (10), solving for dn , and substituting in (11) gives

$$\left(\tilde{r}_f(Q) \frac{\partial \tilde{\pi}_f(Q) / \partial Q}{K'(n)} + \left(n\tilde{r}'_f(Q) + \sum_{i \in A_I} \tilde{r}'_i(Q) - 1 \right) \right) dQ = -d\tilde{r}_j(Q) \quad (12)$$

Note from (10) that Q and n vary inversely, since $\tilde{\pi}_f(Q)$ is a decreasing function (for both strategic substitutes and complements). Hence, the first term on the LHS of (12) is always negative. The second term, in parentheses, is negative immediately for strategic substitutes, and by invoking the slope condition noted in Assumption 3 for strategic complements. Hence, if insider firm j experiences a change that renders it more aggressive, it must be the case that $Q_A < Q_{A'}$.

(ii) From Lemma 3, recall that $\tilde{\pi}_i^*(Q)$ is strictly decreasing. Hence, if $Q_A < Q_{A'}$, then $\tilde{\pi}_i^*(Q_A) > \tilde{\pi}_i^*(Q_{A'})$. The result then follows from (10), which implies that there must be fewer fringe firms in A' than in A since the marginal firm has a lower gross profit and hence a lower entry cost, too.

(iii) Whether fringe firms' actions are larger or smaller depends on the sign of $\tilde{r}'_f(Q)$. Lemma 2 implies that if $Q_A < Q_{A'}$, the fringe firms in A' choose a lower (higher) action iff actions are strategic substitutes (complements).

(iv) A change that increases the marginal profit of insider firm j causes its reaction function to shift out. Since $Q_A < Q_{A'}$, the firm clearly chooses a higher action if actions are strategic complements. We prove by contradiction that the same result holds in the case of strategic substitutes. Suppose not and suppose that the insider firm j chooses a lower action. But then the value of the aggregator must be larger in order to overturn the marginal profit effect of increasing the action level. From (ii) and (iii), there must be fewer fringe firms and each fringe firm must choose a lower action under strategic substitutes.

Since every action level is smaller, this is inconsistent with the purported higher level of the aggregator. Hence, the insider firm j must produce more. ■

Proposition 15 states that a change which increases the marginal profit of one of the insiders increases the equilibrium value of the aggregator irrespective of whether the actions of the firms are strategic substitutes or complements. For example, for the Cournot model, this would imply a higher total output level and for the Logit model, this would imply a lower price level (which implies a higher total output level).³⁰ In cases where the aggregator value increases consumer welfare, consumers must be better off in both cases.

Proposition 15 also states that such a change will always result in exit by the fringe firms, which implies that the fringe firms which remain active in the market must be earning lower rents after the change. (This also follows from the fact that the value of the aggregator is smaller in the new equilibrium and $\tilde{\pi}_i^*(Q)$ is strictly decreasing by Lemma 3.) Although the firm which experiences the change reacts positively to it by increasing its own action, whether the fringe firms' actions increase or decrease depends on the sign of $\tilde{r}'_f(Q)$. The equilibrium with only fringe firms is illustrated in Figure 9, while Figure 10 shows the case where other firms are also present.

7.2 The Logit model with differentiated quality-costs

The analysis above readily adapts to the case of a fringe with different quality-costs and the same entry cost, K . Anderson and de Palma (2001) considered this model, but did not look at the comparative static properties of the equilibrium (instead, they considered the pricing, profit, and output comparison of firms within an equilibrium: higher quality cost firms have higher mark-ups and sell more, while entry is excessive.)

Suppose then that $\pi_i = (p_i - c_i) \frac{\exp(s_i - p_i)/\mu}{\sum_{j=0, \dots, n} \exp(s_j - p_j)/\mu}$ where the s_j represent vertical "quality" parameters and $\mu > 0$ represents the degree of preference heterogeneity across products. The "outside" option has price 0 and "quality" s_0 . The denominator of this expression is the aggregator, $Q = \sum_{j=0, \dots, n} \exp(s_j - p_j)/\mu$. This can be written as the sum of individual choices by defining $q_j = \exp(s_j - p_j)/\mu$ so that we can think of firms as choosing the values q_j . Then we write $\pi_i = (s_i - \mu \ln q_i - c_i) \frac{q_i}{Q}$. Note here that q_j varies inversely

³⁰The Cournot case concurs well with intuition for a dominant firm-fringe scenario.

with price, p_j , so that the strategy space $p_j \in [c_j, \infty]$ under the transformation becomes $q_j \in [0, \exp(s_j - c_j)/\mu]$. Strategic complementarity of prices implies strategic complementarity of the q 's.

Label firms by decreasing quality cost, so that $s_1 - c_1 \geq s_2 - c_2 \geq \dots \geq s_n - c_n$, etc. The free-entry equilibrium is described by zero profits for the marginal firm, firm n , so that

$$\tilde{\pi}(Q; n) = K.$$

where $\tilde{\pi}(Q; n)$ denotes the profit value of the n th firm when the aggregator has value Q , and is decreasing in Q and n . Let A be the set of active firms, i.e., the first n firms. The fixed point condition becomes

$$\sum_{i \in A} \tilde{r}(Q; i) = Q.$$

Then total differentiation of the latter condition gives

$$\tilde{r}(Q; n) dn + \left(\sum_{i \in A} \tilde{r}'(Q; i) - 1 \right) dQ = -d\tilde{r}_j(Q; j)$$

where we assume that insider firm j experiences a change that causes its reaction function to change. Since the zero profit condition implies $(\partial \tilde{\pi}(Q; n) / \partial Q) dQ + (\partial \tilde{\pi}(Q; n) / \partial n) dn = 0$, we can write

$$\left(\tilde{r}(Q; n) \frac{-\partial \tilde{\pi}(Q; n) / \partial Q}{\partial \tilde{\pi}(Q; n) / \partial n} + \left(\sum_{i \in A} \tilde{r}'(Q; i) - 1 \right) \right) dQ = -d\tilde{r}_j(Q; j).$$

Since $\frac{\partial \tilde{\pi}(Q; n) / \partial Q}{\partial \tilde{\pi}(Q; n) / \partial n} > 0$ and $\sum_{i \in A} \tilde{r}'(Q; i) < 1$ by the slope condition, the LHS bracketed term is negative, and so a higher aggregator results from a change that makes firm j more aggressive. Then fewer firms are active at the new equilibrium, and each unaffected one has a higher q_i , meaning a lower mark-up. Consumers are better off because the aggregator has risen.

8 Integer constraints

So far, we have considered changes that lead to long-run profits of zero for fringe firms both before and after any changes. In this section, we will take the integer constraint into

account, and thus determine bounds on Q_A and $Q_{A'}$ - equilibrium output levels before and after the change - and the associated welfare consequences.

8.1 Strategic substitutes

The argument will be phrased in terms of Q_A , but the same arguments apply to $Q_{A'}$.

Let Q_L and Q_U stand for the lower and the upper bound on the equilibrium aggregate output level with a discrete number of firms and at least one fringe entrant. That is, we are looking for the lowest value of Q_A such that there is no entry. If $Q_A < Q_L$, there will be entry. Similarly, for the upper bound, we are looking for the highest amount of Q_A such that there is no exit: $\tilde{\pi}_i^*(Q_U) \geq K$ for all $i \in A$. If $Q_A > Q_U$, there will be exit.

The first thing to note is that the aggregate output level cannot be larger than the equilibrium output level in the case when the number of fringe firms is treated as a continuous variable. Hence, Q_U is defined by $\pi_i^*(Q_U) = K$.

As Q_A decreases, the potential entrants outside of the market have increasing incentives to enter the market: however, they need to rationally anticipate that the post-entry equilibrium, having observed the pre-entry statistic, will not exceed Q_U . Hence, we can determine the lower bound by considering their incentives to enter. If the potential entrant expects the aggregate reaction of the rivals to be $Q_{-i} < Q_U - \tilde{r}_i(Q_U)$, it will enter the market. This is because by definition $\pi_i^*(Q_U) = K$ and so for all $Q_{-i} < Q_U - \tilde{r}_i(Q_U)$, the potential entrant will expect to make positive profits upon entry. If it expects $Q_{-i} \geq Q_U - \tilde{r}_i(Q_U)$, it will not enter the market.

Hence, we can treat $Q_U - \tilde{r}_i(Q_U)$ as a lower bound on Q_A . Actually, under strategic substitutability, Q_A will be higher than this because the rival firms will choose higher actions in the absence of entry: entry tends to be accommodated in the sense that incumbents reduce their equilibrium.

We can now show the following result.

Proposition 16 *Assume actions are strategic substitutes. Any free-entry equilibrium with at least one fringe entrant must have $Q_A \in [Q_L, Q_U]$, where Q_U is defined by $\pi_f^*(Q_U) = K$ and $Q_L = Q_U - \tilde{r}_f(Q_U)$.*

Proof. First, note that $Q_U = \pi_f^{*-1}(K)$ is the upper bound on the aggregator with at least one fringe firm active, since $\pi_i^*(Q)$ is decreasing in Q by Lemma 3 and Q_U is the largest aggregator value at which an active fringe firm can make a non-negative profit.

Second, we wish to show that any $Q_A < Q_L$ cannot be an equilibrium because it must attract profitable entry by a fringe firm. This is equivalent to showing that if a single fringe entrant joins the set A of active firms, then the ensuing equilibrium aggregator value will not be above $Q_U = \pi_f^{*-1}(K)$. Suppose this is not the case and suppose $Q_{\{A+i\}} > Q_U$ with $i \in E$. Since, by Lemma 2, $\tilde{r}_i(Q)$ is decreasing under strategic substitutes, each firm in A must be choosing a lower action than before (i.e., at the equilibrium with the fringe firm excluded). This means their actions sum to less than Q_A . Likewise, for the incremental fringe entrant, $\tilde{r}_f(Q_{\{A+i\}}) < \tilde{r}_f(Q_U) = Q_U - Q_L$ under strategic substitutability (also by Lemma 2). Hence, its action must be less than its action at Q_U . But then the sum of the actions cannot exceed Q_U , a contradiction. ■

Hence, before and after any change, the equilibrium value of the aggregator lies in $[Q_L, Q_U]$. The maximum consumer welfare increase from the change from A to A' is given by $CW(Q_U) - CW(Q_L)$. The maximum consumer welfare decrease from the change is given by $CW(Q_L) - CW(Q_U)$.³¹

Example 3 Consider Cournot competition with symmetric firms and linear demand. Suppose firms are symmetric after the change. Suppose demand is given by $P = 1 - Q$, and marginal costs are zero. Then, $Q_U = 1 - \sqrt{K}$, because each firm would just make zero profit by producing its equilibrium output of $q_i = \sqrt{K}$. The best response function is given by $r_i(Q_{-i}) = \frac{1-Q}{2}$ and the cumulative best response function is given by $\tilde{r}_i(Q) = 1 - Q$. Hence, the lower bound is given as $Q_U - \tilde{r}_i(Q_U) = Q_L = 1 - 2\sqrt{K}$. Note that there are $n = \frac{1-\sqrt{K}}{\sqrt{K}}$ firms at Q_U if this is an integer. Suppose then we took out one (indifferent) firm, and check what the new equilibrium total quantity is. For this benchmark Cournot model, $n - 1$ firms produce a total output of $\frac{n-1}{n}$. Substituting the value of n given above, we have a total output of $\frac{1-2\sqrt{K}}{1-\sqrt{K}} > Q_L$, which represents the actual lower bound in this case. The ratio of Q_L to the actual one gets small as K gets small, as indeed does the ratio $\frac{Q_L}{Q_U}$.

³¹ Graphing q_i against Q_{-i} and Q reveals that as $Q_{-i} = Q_L$ increases (which happens when K decreases), q_i decreases.

8.2 Strategic complements

Dealing with the bounds for the strategic complements case is more tricky because $\sum \tilde{r}'_i(Q)$ can be arbitrarily close to 1. The equilibrium output with one less firm may be very far away from Q_U . At the other extreme, if $\sum \tilde{r}'_i(Q)$ is close to zero, then Q_L is close to the bound found above for the case of strategic substitutes.

9 Further applications and comments

Aggregative games are common in other applications, such as public finance, where a selectively-applied exogenous tax or subsidy may affect the marginal or fixed costs of firms (see, e.g., Anderson et al., 2001a and 2001b; Besley, 1989), and in public good games (see Cornes and Hartley, 2007). They are also extensively applied in international trade models, along with the free entry condition, in order to analyze unilateral changes in trade policy (see, e.g., Etro, forthcoming; Markusen and Stahler, ... (NBER); Markusen and Venables, ...; Venables, 1985; Horstmann and Markusen, 1986 and 1992). Most recently, CES models are at the heart of the empirical renaissance in trade (see Melitz, ...; Melitz and Ottaviano, ...; and Helpman and Itzhoki, ...; 2007 survey article in JEP). Many of the papers in international trade which use CES models assume monopolistic competition for reasons of tractability. Our paper shows that this may not be a necessary assumption. Using the aggregative nature of the game may allow one to obtain results in a framework with strategic interaction.

Further applications of CES based models arise in the new economic geography. Common property resource problems and congestion problems (including consumer information overload) have a strong aggregative game presence also.

One of the shortcomings of the applications (and results) we have considered so far is that they are all based on models of global competition. Some special cases of localized competition which are aggregative games include the Hotelling model with two firms and the circular city model with three firms. The short-run results presented above would apply in these special cases.

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Figure 1: Construction of $\tilde{r}_i(Q)$, Strategic Substitutes Case

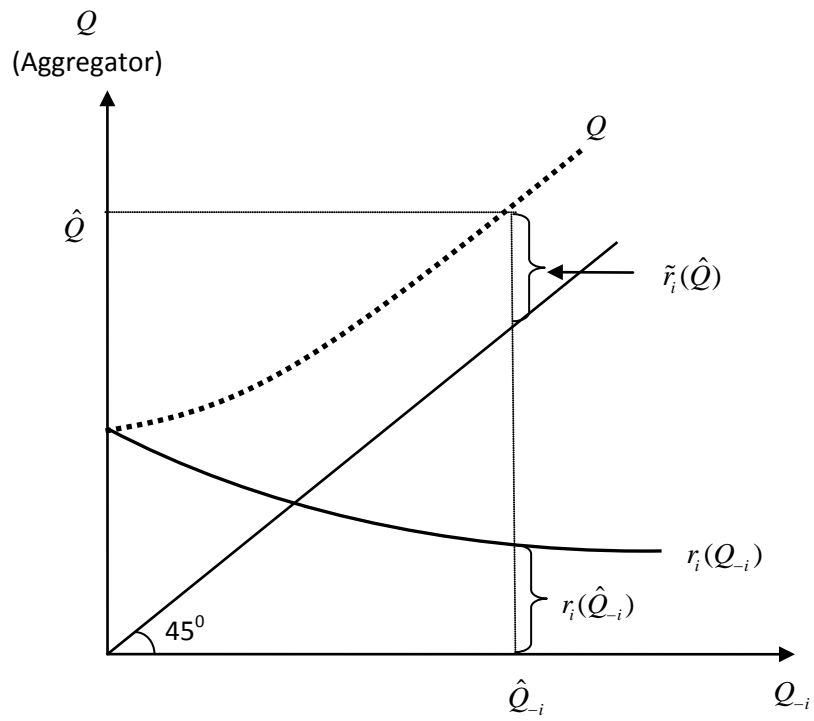


Figure 2: Construction of $\tilde{r}_i(Q)$, Strategic Complements Case

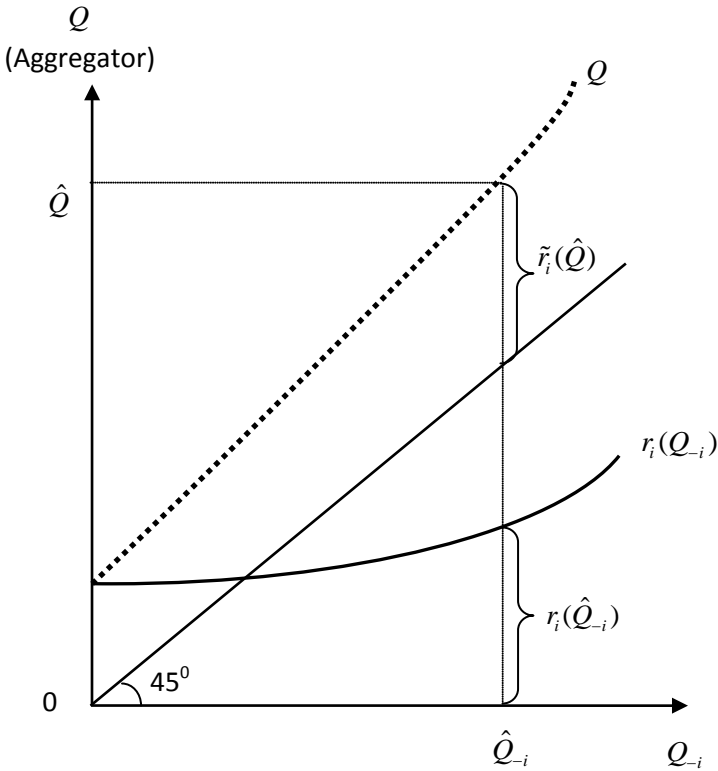


Figure 3: Construction of $\tilde{r}_i(Q)$, Strategic Substitutes Case

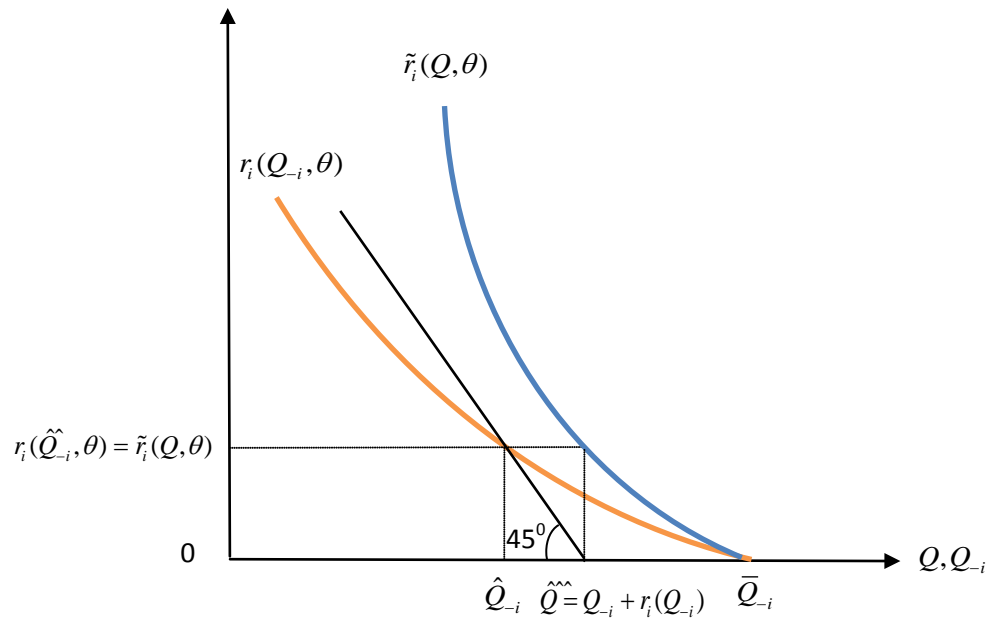


Figure 4: Construction of $\tilde{r}_i(Q)$, Strategic Complements Case

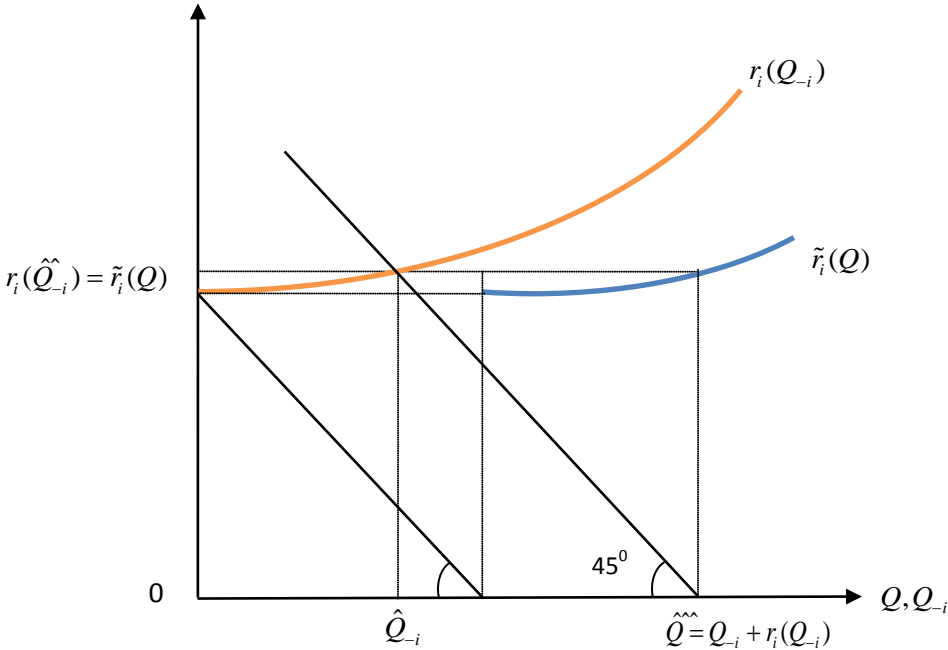


Figure 5: Effect of Change in θ on $r_i(Q_{-i})$ and $\tilde{r}_i(Q)$, Strategic Substitutes Case

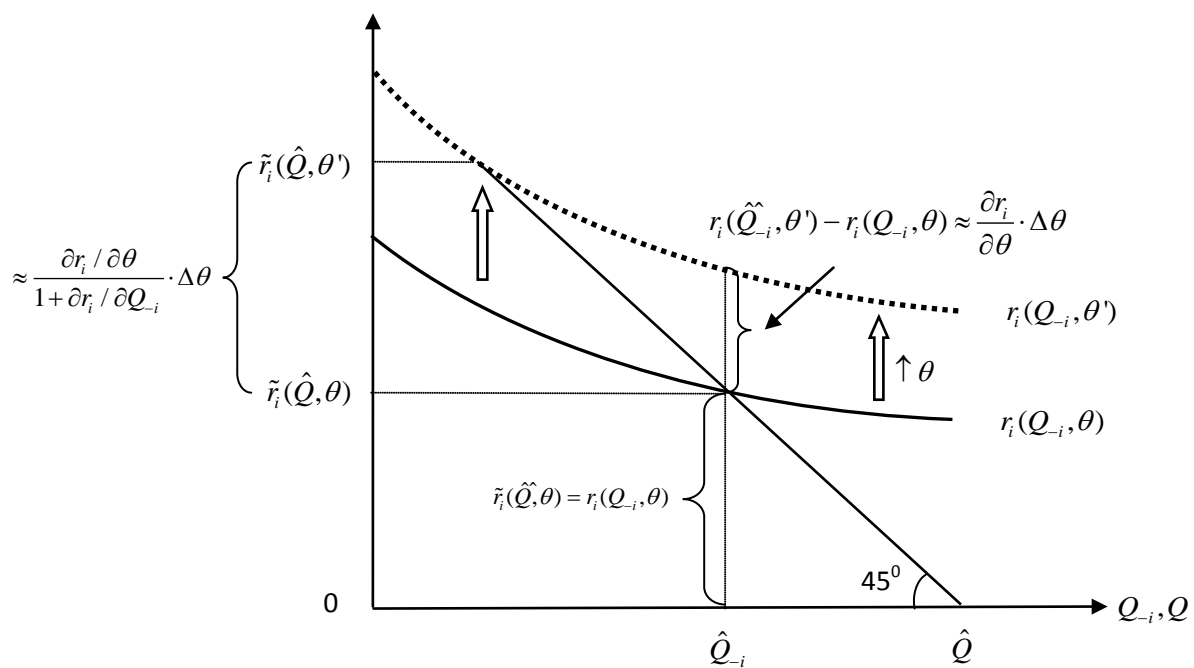


Figure 6: Effect of Change in θ on $r_i(Q_{-i})$ and $\tilde{r}_i(Q)$, Strategic Complements Case

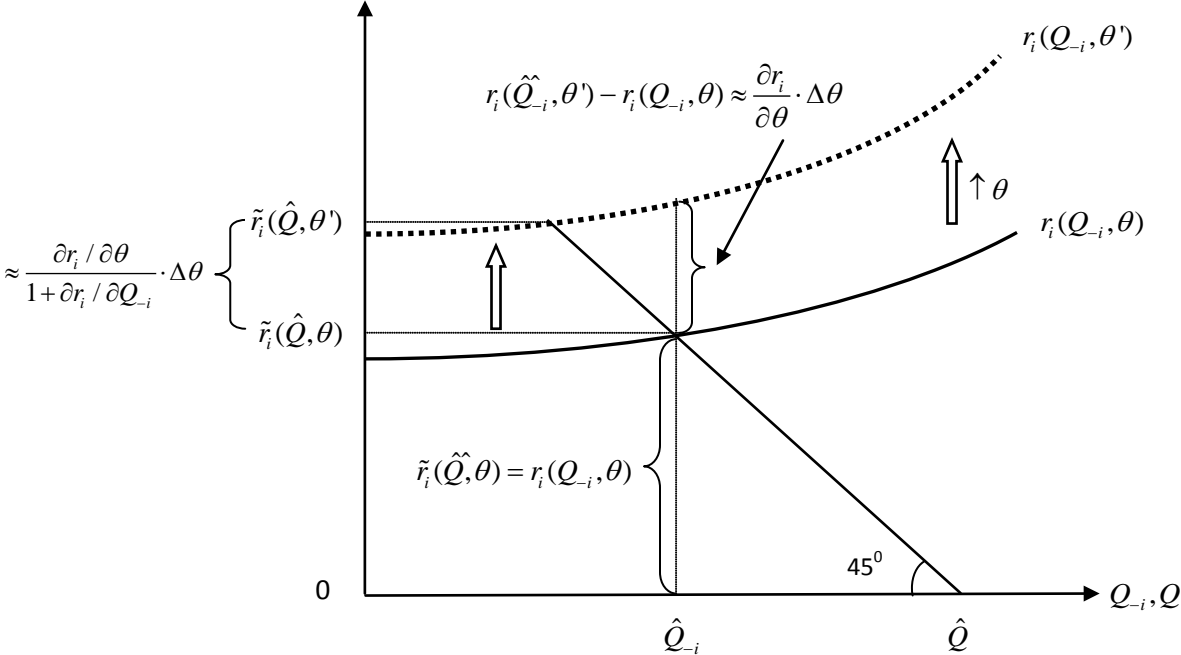


Figure 7: More Aggressive Reaction Shifts up $r_i(Q_{-i})$ and $\tilde{r}_i(Q)$, Strategic Substitutes Case

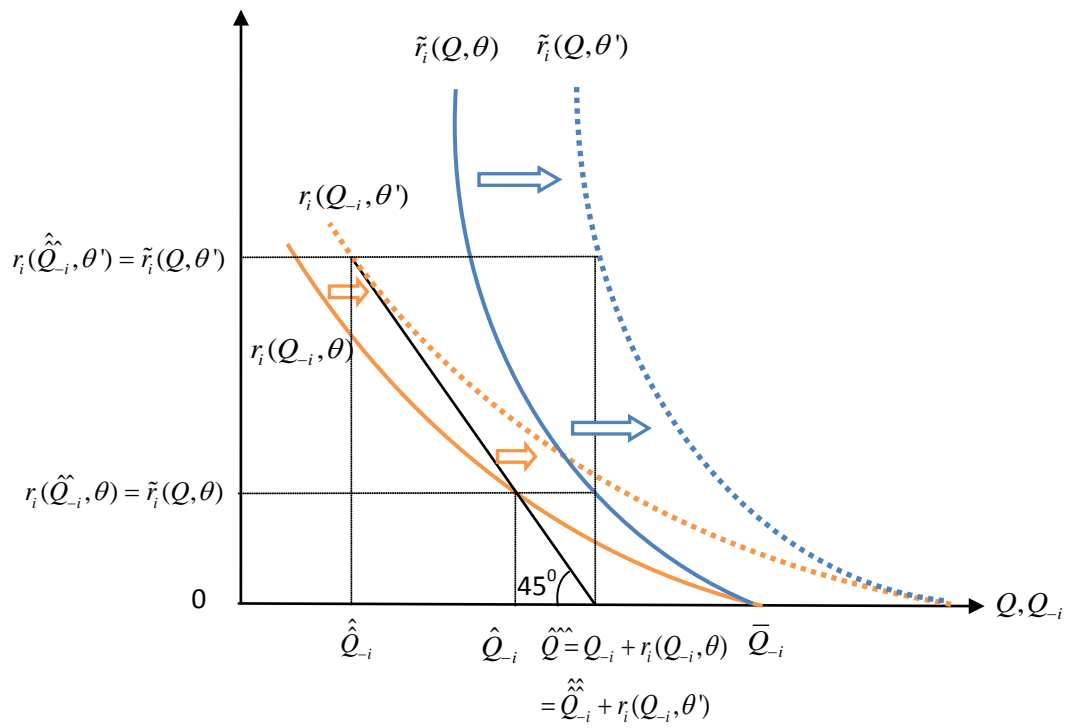


Figure 8: More Aggressive Reaction Shifts up $r_i(Q_{-i})$ and $\tilde{r}_i(Q)$, Strategic Complements Case

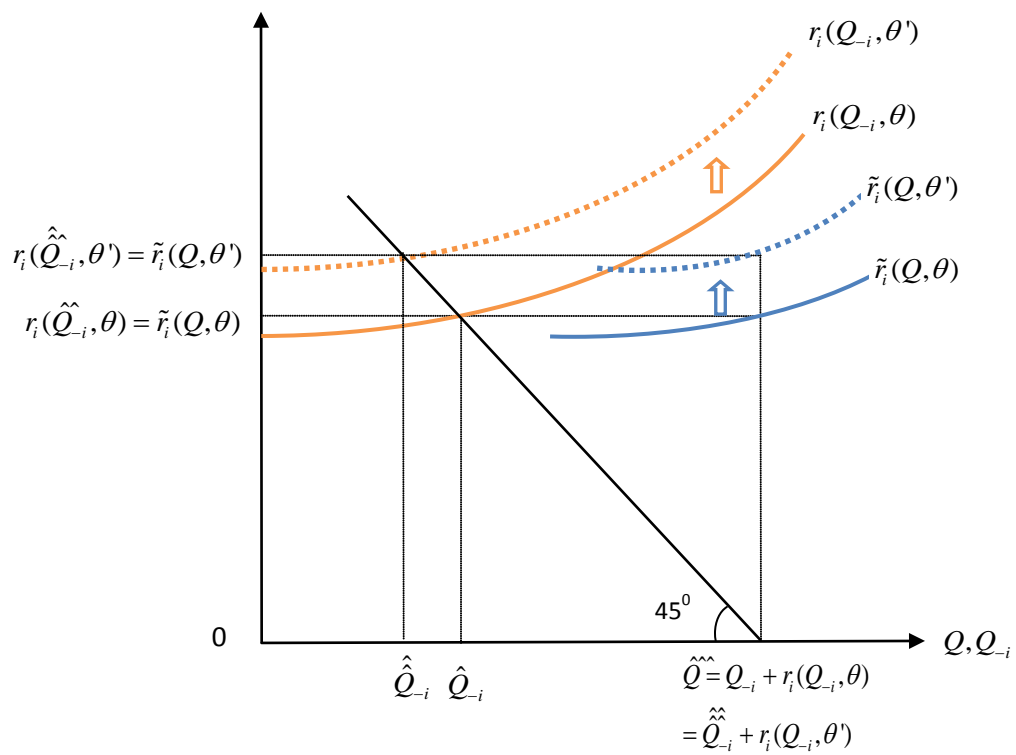


Figure 9: Increasing entry cost, equilibrium with fringe firms only

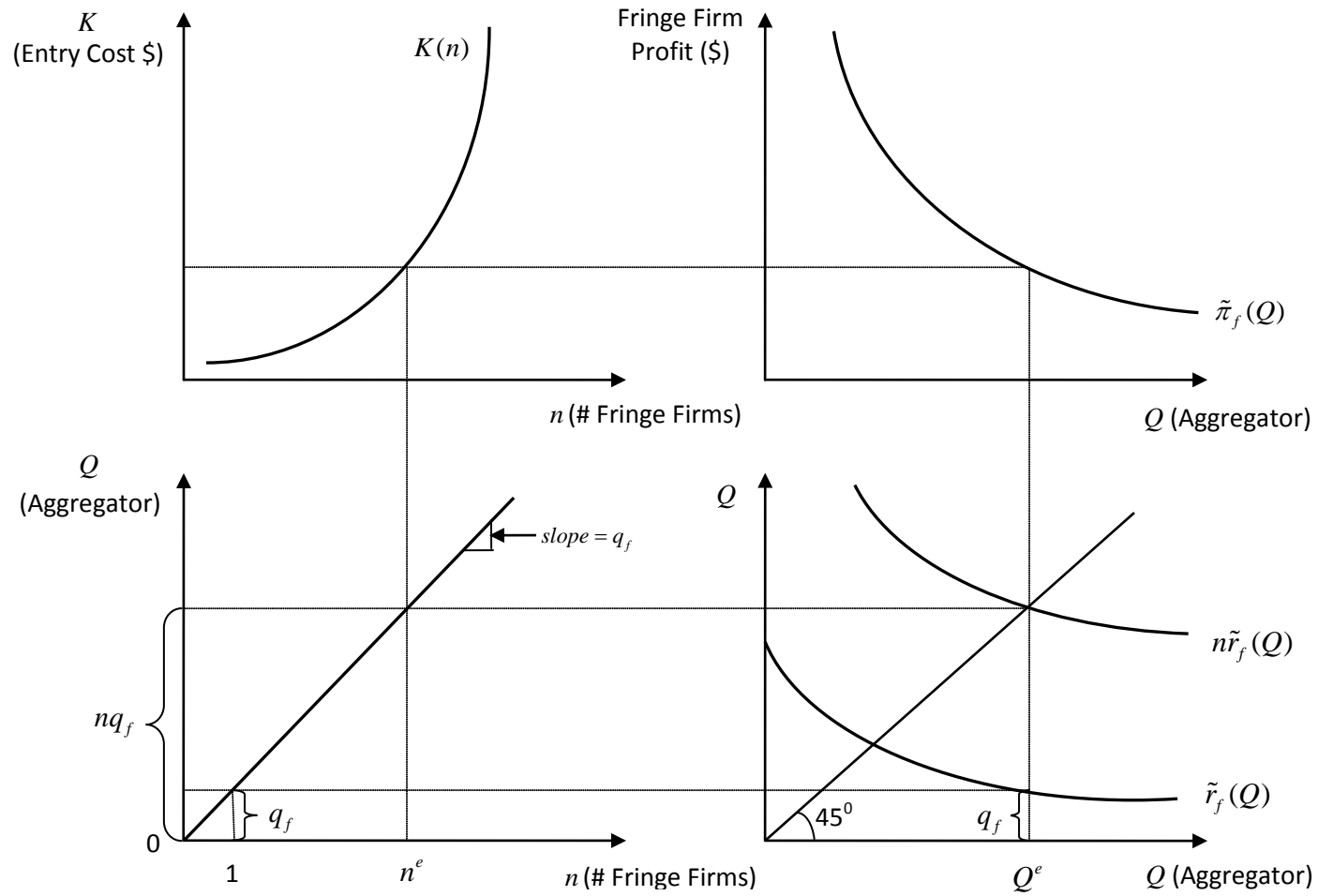


Figure 10: Increasing entry cost, equilibrium with fringe firms and incumbents

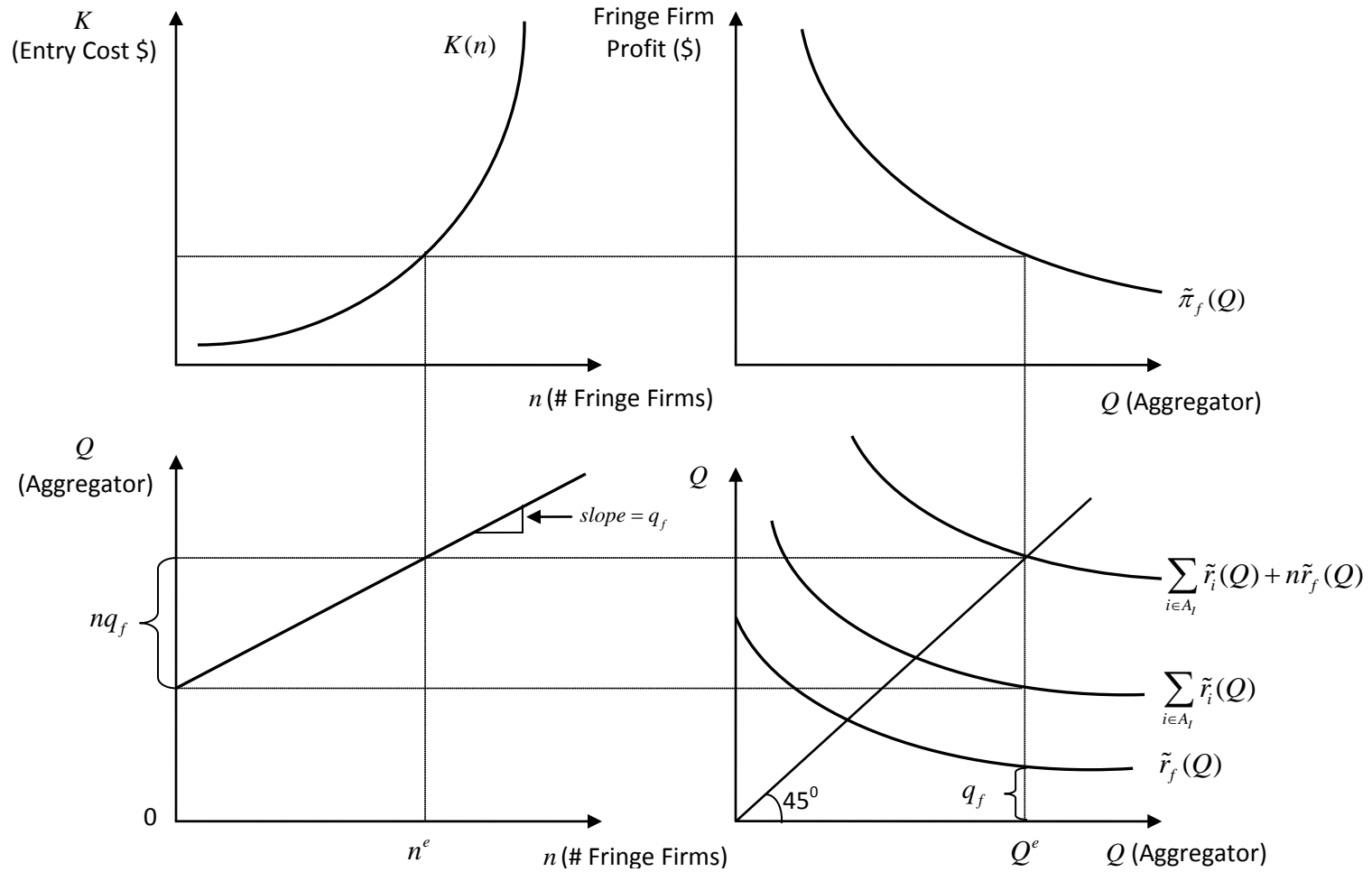


Figure 11: Fixed Point, Short Run. More Aggressive Firms Reaction, Strategic Substitutes Case

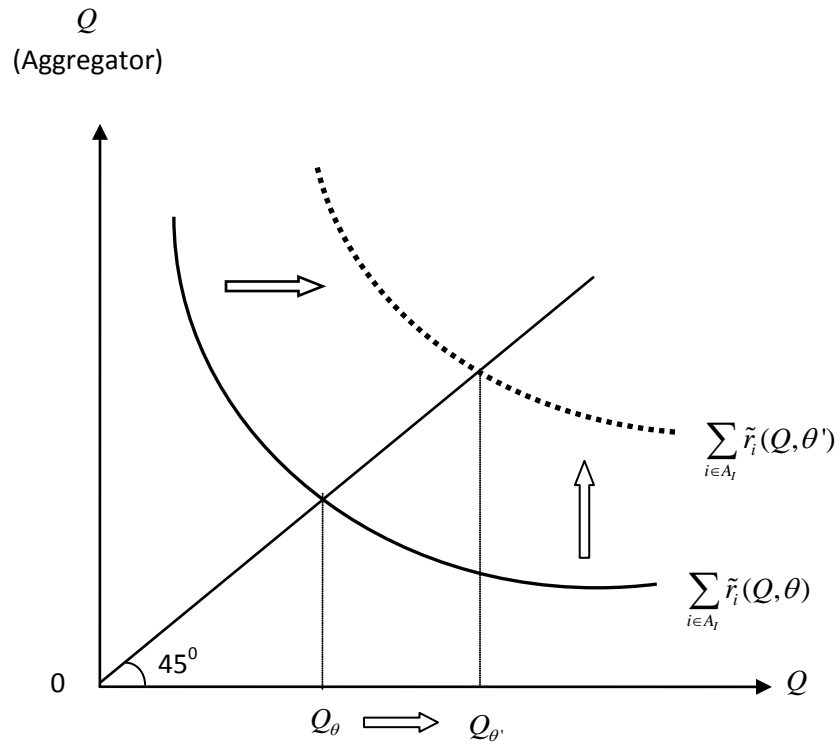
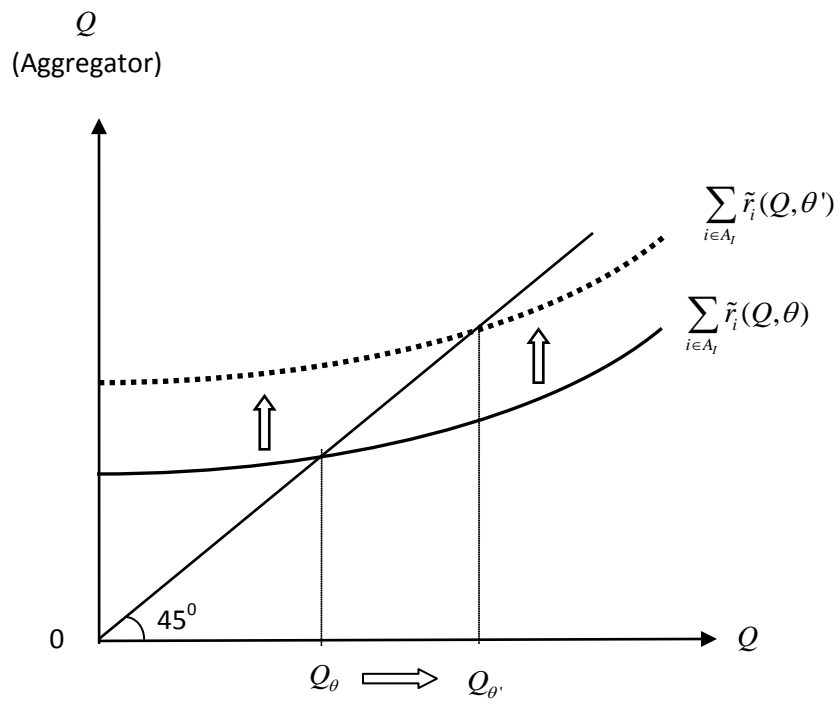
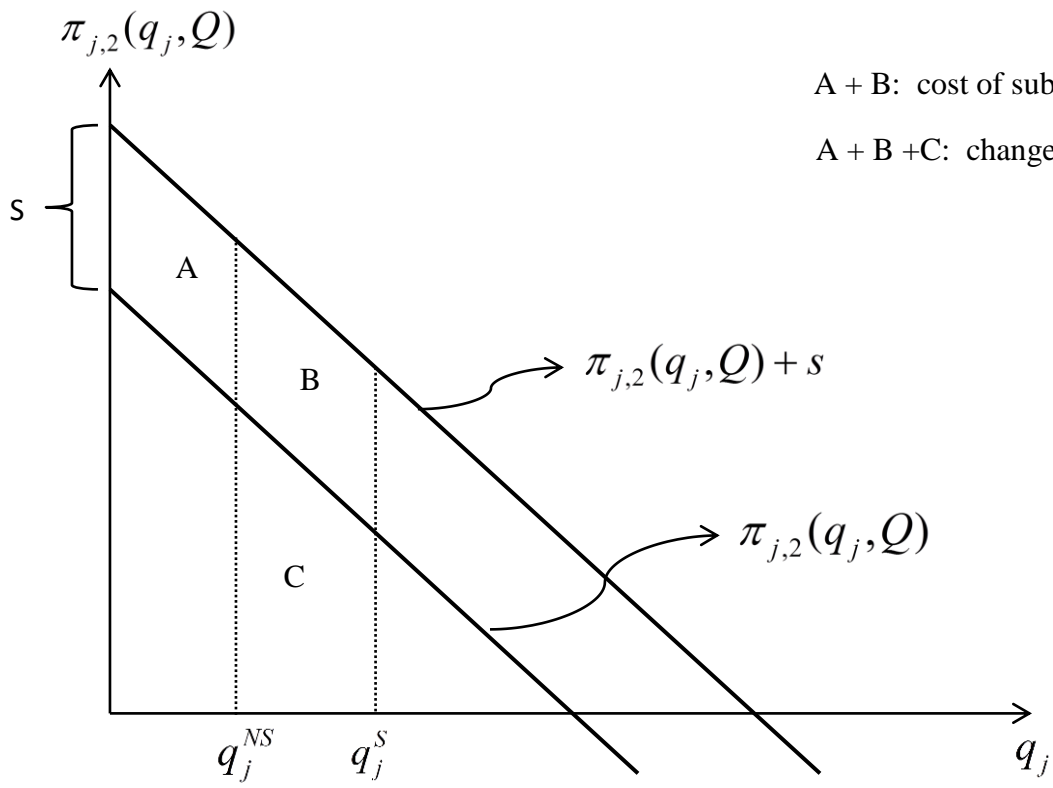


Figure 12: Fixed Point, Short Run. More Aggressive Firms Reaction, Strategic Complements Case





A + B: cost of subsidy

A + B + C: change in profit

