Entry and the Efficiency of Venture Capital Markets with Endogenous Matching*

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June 22, 2011

Abstract

We develop an equilibrium model of the venture capital market with heterogenous entrepreneurs and venture capital firms (VCs). Each VC matches endogenously with an entrepreneur (two-sided matching), offering capital in exchange for an equity stake. We examine how barriers to entry affect (i) optimal VC contracts, (ii) the survival rate of new ventures, and (iii) market efficiency. We find that entry of VCs has a *ripple effect* throughout the entire market: All start-ups then receive more capital and become more likely to survive. However, the VC market will never converge to the efficient outcome, even when allowing for free entry.

Keywords: Entrepreneurship, Venture Capital, Barriers to Entry, Matching, Contracts, Efficiency.
JEL classifications: C78, D86, G24, L26, M13.

*An earlier version of this paper was circulated under the title "Matching and Contracting in the Venture Capital Market". We would like to thank Jean-Etienne de Bettignies, Ray DeGennaro, Claude Fluet, Thomas Hellmann, participants of the 2010 European Financial Management Symposium in Montreal, the 2010 Research Symposium on the Economics and Law of the Entrepreneur at Northwestern University, the 2010 Annual Congress of the European Economic Association in Glasgow, and the 2010 European Association for Research in Industrial Economics (EARIE) Conference in Istanbul, and workshop participants at Queen's University, University of Oklahoma, and Université du Québec à Montréal (UQAM) for valuable comments and suggestions.

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1 Introduction

The venture capital (VC) market has long been recognized as a fundamental source for entrepreneurs to finance their ventures, and thus to bring their innovative ideas to market (Kaplan, Sensoy, and Strömberg (2009)). A hallmark of the VC market is the process by which entrepreneurs are matched with VCs, which has important implications for the financing and ultimately the success of new ventures. The empirical literature has identified two forces at work: VCs with higher expertise select superior ventures to invest in (the "selection" effect); and VCs with higher management expertise provide greater value-added to their ventures (the "treatment" effect); see e.g. Hellman and Puri (2002), Sørensen (2007), and Nahata (2008). Clearly, then, the matching process matters greatly in the VC market.[1] The objective of this paper is to study the interplay between endogenous matching and barriers to entry and how they affect optimal VC contracts and market efficiency.

We develop an equilibrium model of the VC market with two-sided endogenous matching between a collection of VCs, that are heterogenous in terms of their management expertise, and a collection of entrepreneurs, that are heterogenous with respect to the quality (or market potential) of their business ideas. We show that, under reasonable conditions, the matching equilibrium is positive assortative (PAM): VCs with high expertise match with entrepreneurs that have business ideas with high market potentials. Once matched with an entrepreneur, each VC offers a contract which stipulates an allocation of equity and the VC's capital investment. Each entrepreneur then needs to exert (unobservable) effort which affects the success or failure of his venture.[2] Due to endogenous matching in our framework, each contract must not only satisfy the usual incentive compatibility constraint associated with the moral hazard problem, but also a condition that guarantees that an entrepreneur cannot be better off by contracting with an alternative VC. In this sense, VCs are competing for entrepreneurs with promising business ideas which—as we will show—plays a key role in the design of optimal VC contracts and the overall efficiency of the VC market.[3]

While the analysis of isolated VC-entrepreneur relationships in a contractual context provides important insights, a main implication of our study is that endogenous matching in the VC market significantly affects the structure and properties of VC contracts. Specifically, we show that each VC is forced to transfer more surplus to its matched entrepreneur through more attractive contracts than would occur in

[1] Matching is not only relevant in VC markets, but also matters in general principal-agent relationships (see, e.g., Ackerberg and Botticini (2002), Besley and Ghatak (2005), Serfes (2005, 2008) and Anderson and Smith (2010)).

[2] Thus, our framework exhibits a typical one-sided moral hazard problem with respect to the entrepreneur's effort, while the investment of the VC is contractible. For models with double-sided moral hazard in the context of the financing of new ventures, see e.g. Casamatta (2003), Repullo and Suarez (2004), and de Bettignies (2008).

[3] de Bettignies and Chemla (2008) also consider the effects of competition for business ideas, though in a different context. In their model, a firm wants to attract managers with high quality ideas for new ventures.

the absence of endogenous matching (i.e., as in an isolated principal-agent relationship). This in turn results in better capital endowments of new ventures, and hence, improves their prospects of success.

We then utilize our framework to examine how barriers to entry affect equilibrium contracts and the overall efficiency of the VC market.[4] Given the vertical structure of our equilibrium model, a lower entry cost will lead to entry at the bottom of the market (i.e., VCs with relatively low management expertise will enter the market). Interestingly, lower barriers to entry do not only benefit new entrepreneurs who then get venture capital, but also result in more favorable VC contracts for all other entrepreneurs in this market. They then receive more start-up capital which in turn improves their ventures' prospects of survival. We refer to this mechanism as the *ripple effect* of VC entry since it impacts all entrepreneur-VC pairs in the market. Although entry of low expertise VCs stimulates competition in the entire VC market, we find that entrepreneurs with less lucrative business ideas benefit relatively more from VC entry. The ripple effect thus subsides for ventures that are built on higher-quality entrepreneurial ideas.

There is a long-standing tradition in the economics literature to examine the relationship between entry and market efficiency in various market settings. As more firms enter the market, the equilibrium price can either (i) converge to marginal cost (as in Cournot, Bertrand, and Hotelling-type models), (ii) stay constant (as in models of monopolistic competition à la Dixit and Stiglitz (1977)), or (iii), can converge to a price strictly greater than marginal cost (as in discrete choice models, see e.g. Perloff and Salop (1985)). Moreover, barriers to entry have no effects on equilibrium prices in models with vertical differentiation due to the 'finiteness property'; see e.g. Shaked and Sutton (1982). In contrast, some oligopoly models predict excessive market entry because of the 'business stealing effect'; see e.g. Mankiw and Whinston (1986). In this paper, we propose a framework which facilitates a re-examination of the link between entry and market efficiency. Despite the existence of a continuum of VCs in our model, the market does not converge to the efficient outcome. Moreover, entry into the VC market is *insufficient* from a social welfare perspective, even in the absence of any barriers to entry. This is because market entry creates positive externalities for all ventures in the market, which are not internalized.

Our equilibrium model of the VC market is closely related to Terviö (2008) who devised an assignment model between heterogenous CEOs and firms to explain the diversity of CEO compensations. The model developed in this paper shares a number of features with Terviö (2008), namely two-sided matching between a continuum of heterogenous principals and agents. However, our model differs in two key aspects. First, we endogenize the contracts between VCs and entrepreneurs in a setting with moral hazard. Second, our model features non-transferable utility (NTU), implying that the outside options of

[4] For instance, strong networks between incumbent VCs constitute a typical barrier to entry in the VC market (e.g. Hochberg, Ljungqvist, and Lu (2010)).

entrepreneurs affect equilibrium contracts, and thus, the profitability of their ventures. The absence of non-distortionary side-payments is an important characteristic of many markets, including the VC market due to the limited wealth of entrepreneurs. These key differences to Terviö (2008) generate novel and important insights in how outside options of economic agents as well as barriers to entry affect equilibrium contracts and market efficiency.

Our paper is also close in spirit to the equilibrium models of the VC market as devised by Inderst and Müller (2004), Silviera and Wright (2010), and Dam (2007). Our paper differs in two important aspects from Inderst and Müller (2004) and Silviera and Wright (2010). First, both papers consider a *search* equilibrium, whereas we study *endogenous matching*. The search equilibria in their models are characterized by the ratio of the number of VCs to the number of entrepreneurs, commonly referred to as "market thickness" in the search theory literature. Using a matching framework, however, we show that it is not just the market thickness that matters; the qualities of entrepreneurial ideas and the management expertise of VCs are also critical for the optimal design of VC contracts, especially since these determine the (endogenous) outside options of entrepreneurs. Second, Inderst and Müller (2004) consider homogenous VCs and entrepreneurs, while Silviera and Wright (2010) introduce ex-post heterogeneity, i.e., the heterogeneity arises after a VC is matched with an entrepreneur. In contrast, we explicitly account for different match qualities by allowing entrepreneurs to be ex-ante heterogenous with respect to the market potential of their ideas, and VCs with respect to their management expertise. Dam (2007) also studies endogenous matching between venture capitalists and firms. However, his setting and predictions are substantially different from ours. He considers a market where venture capitalists are heterogeneous with respect to their monitoring ability, and firms are heterogeneous with respect to their levels of initial wealth. The matching equilibrium in his framework is negative assortative (NAM): venture capitalists with higher monitoring ability invest in firms with lower initial wealth.

The remainder of the paper is structured as follows. Section 2 introduces the features of our equilibrium model of the VC market. Section 3 characterizes the efficient (first-best) outcome as a benchmark for our analysis. Section 4 derives the optimal contract of an entrepreneur-VC pair in the absence of endogenous matching. Section 5 incorporates two-sided matching in the VC market and characterizes the equilibrium contracts. Section 6 examines the effects of barriers to entry on optimal VC contracts and the efficiency of the market equilibrium. Section 7 summarizes our key insights and concludes. All proofs are given in the Appendix.

2 The Model

Consider a continuum of risk-neutral entrepreneurs (ENs) and a continuum of risk-neutral venture capital firms (VCs), each of measure one. ENs are indexed by $i \in E = [0, \overline{E}]$, with $H(i)$ as the distribution of i and $h(i)$ as its density. VCs are indexed by $j \in V = [0, \overline{V}]$, with distribution $G(j)$ and density $g(j)$. All entrepreneurs are wealth constrained and their reservation utilities are normalized to zero. There are five dates:

1. ENs are endowed with ideas for new ventures.
2. VCs decide whether to enter the market.
3. Each VC matches with an entrepreneur and offers him a contract that consists of an equity share of the venture and an investment.
4. Entrepreneurs exert unobservable effort.
5. Profits of ventures are realized and payments are made.

At date 1, each entrepreneur i conceives an innovative business idea, denoted $\mu(i)$, where a higher value of $\mu(i)$ reflects an idea of superior quality (or market potential). Entrepreneurial ideas can be ranked according to their respective quality, where $\mu(i)$ is increasing and continuously differentiable in i (vertical ranking). To commercially exploit his idea, each entrepreneur relies on capital K as well as on management expertise, both provided by a VC.[5] Let $x(j)$ denote the management expertise of VC j, where a superior expertise is reflected by a higher $x(j)$, i.e., $x(j)$ is increasing and continuously differentiable in j (vertical ranking). Entrepreneurial ideas and VC expertise are common knowledge.[6]

At date 2, VCs decide whether to incur the sunk cost $F > 0$ to enter the market. The cost F reflects the extent of barriers to entry in the VC market. At date 3, each VC matches endogenously with one entrepreneur. VC j then makes its entrepreneur i a *take-it-or-leave-it* offer with a contract $\{\lambda_{ij}, K_{ij}\}$ that specifies an equity share λ_{ij} of the venture for the entrepreneur as well as the VC's investment K_{ij}. The cost of capital faced by each VC is exogenous and denoted $r > 0$. To simplify our notation,

[5] The dependency of entrepreneurs on VC expertise excludes debt as a viable form of financing their start-ups. However, if VC expertise was not crucial, debt financing could be preferred in some cases; see Ueda (2004) and Thiele and Tombak (2010). To account for debt financing as an alternative to VC financing, one could extend our framework by assuming a strictly positive outside option for entrepreneurs which reflects their expected utility when using debt to start their ventures. As will become clear from our analysis, however, this would not change our results.

[6] While a potential information asymmetry problem with respect to the qualities of entrepreneurial ideas is not the focus of this paper, one could extend our framework to account for such private information. As is well known from the standard adverse selection literature, entrepreneurs with high quality ideas would then extract an information rent. For matching models with asymmetric information or uncertainty about types see, e.g., Inderst (2005), Chakraborty, Citanna, and Ostrovsky (2010), and Anderson and Smith (2010).

we henceforth suppress the subscript ij when referring to an arbitrary EN-VC pair. The utility of an entrepreneur who remains unmatched is zero.

At date 4, after signing the contract $\{\lambda, K\}$, each entrepreneur exerts effort e to turn his idea into a marketable product. Implementing effort e imposes the cost $c(e) = e^2/2$. An entrepreneur's effort e determines the likelihood of whether his venture succeeds ($Y = 1$) or fails ($Y = 0$). Specifically, we assume that $\text{Prob}[Y = 1|e] = e$.[7] Finally, at date 5, the profit of each venture is realized and payments according to the respective contracts are made.

A key property of our framework is the reliance of entrepreneurs on the management expertise of VCs to turn ideas into marketable products. Intuitively, the value of this expertise is closely related to the quality of the entrepreneur's specific idea. To capture this notion, let $\Omega \equiv \Omega(\mu, x) > 0$ denote the *match quality* between an entrepreneur with idea μ and a VC with management expertise x. The match quality Ω is strictly increasing (and continuously differentiable) in both the entrepreneur's idea quality μ and the VC's expertise x, where ideas and expertise are complements (i.e., $\partial^2\Omega(\mu, x)/(\partial x \partial \mu) > 0$).

Let $\Pi \in \{0, \pi(K, \Omega)\}$ denote the gross profit of a venture, where $\Pi = \pi(K, \Omega) \geq 0$ if the venture succeeds (i.e., $Y = 1$). We assume that $\pi(K, \Omega)$ is increasing and concave in capital K and match quality Ω. We further assume that a higher match quality Ω makes every unit of capital K more productive, i.e., $\partial^2\pi(K, \Omega)/(\partial K \partial \Omega) \geq 0$. Finally, we make the following assumptions to ensure interior solutions:

$$\left.\frac{\partial \pi(K, \Omega)}{\partial K}\right|_{K=0} = \infty \text{ and } \left.\frac{\partial \pi(K, \Omega)}{\partial \Omega}\right|_{\Omega=0} = \infty.$$

3 The Efficient (First-Best) VC Contract

As a benchmark, we first derive the socially efficient (first-best) VC investment, denoted K^{fb}, and effort of the entrepreneur, denoted e^{fb}.[8] The problem of a social planner can be stated as follows:

$$\max_{\{e,K\}} S(e, K) = \pi(K, \Omega)e - e^2/2 - rK, \tag{1}$$

where $S(e, K)$ denotes the total surplus as a function of entrepreneurial effort e and VC investment K. The next Lemma characterizes the solution to this problem.

[7] To ensure interior solutions, we will assume that the venture's potential profit, denoted by π, is always smaller than one, so that even the first-best effort level e^{fb} guarantees that $\text{Prob}[Y = 1|e^{fb}] < 1$.

[8] Note that the entrepreneur's equity share λ determines only the allocation of surplus, and not the size of total surplus.

Lemma 1 (First-Best Outcome) *The socially efficient investment K^{fb} is characterized by*

$$\frac{\partial \pi(K^{fb}, \Omega)}{\partial K} = \frac{r}{\pi(K^{fb}, \Omega)}, \tag{2}$$

and the socially efficient effort level is $e^{fb} = \pi(K^{fb}, \Omega)$.

To ensure that $\text{Prob}[Y = 1|e^{fb}] = e^{fb} < 1$, we assume that $\pi(K^{fb}, \Omega) < 1$ for the socially optimal investment K^{fb} and any possible match quality Ω.[9] We will use the socially efficient investment K^{fb} and effort level e^{fb} to characterize the effect of endogenous matching and barriers to entry on the efficiency of VC contracts in a second-best environment.

4 Optimal VC Contracts in the Absence of Matching

We first ignore the entry decision of VCs as well as endogenous matching in the market, and derive the optimal contract for an arbitrary EN-VC pair. To do so, we proceed in two steps. First, we characterize the entrepreneur's effort choice for a *given* VC contract. Second, we derive the optimal VC contract by accounting for the entrepreneur's effort.

For a given match quality Ω, VC investment K, and equity share λ, the entrepreneur chooses effort e to maximize his expected utility:

$$\max_{\{e\}} U(e, \lambda, K, \Omega) = \lambda \pi(K, \Omega) e - e^2/2. \tag{3}$$

The entrepreneur's effort choice is thus given by

$$e^* = \lambda \pi(K, \Omega). \tag{4}$$

The entrepreneur's participation constraint is always satisfied (assuming an initial outside option of zero) because potential losses (up to K) are only incurred by the VC.

[9] Alternatively, one could incorporate a parameter $\zeta > 0$ in the entrepreneur's effort cost function so that $c(e) = \zeta e^2/2$. Assuming a sufficiently high cost parameter ζ would then also ensure that $\text{Prob}[Y = 1|e^{fb}] < 1$.

The VC has two instruments to indirectly control the entrepreneur's effort e^*: adjusting his equity share λ and the investment K. The optimal combination of λ and K maximizes the VC's expected profit as given by

$$\Pi(\lambda, K, e^*, \Omega) = (1-\lambda)\pi(K, \Omega)e^* - rK. \tag{5}$$

By accounting for e^* as defined by (4), the expected profit of the VC becomes

$$\Pi(\lambda, K, \Omega) = \lambda(1-\lambda)\pi^2(K, \Omega) - rK. \tag{6}$$

The optimal VC contract $\{\lambda^*, K^*\}$ is determined by two objectives: (i) to provide the entrepreneur with sufficient effort incentives to ensure the survival of his venture; and (ii), to equip the new venture with sufficient capital to generate an adequate return for the VC. The next Lemma characterizes the optimal VC contract for an arbitrary entrepreneur-VC pair in the absence of endogenous matching.

Lemma 2 (VC Contract without Endogenous Matching) *Consider an arbitrary EN-VC pair in the absence of endogenous matching. The optimal VC contract then comprises the equity share $\lambda^* = 1/2$ for the entrepreneur, and the VC investment K^* which is defined by*

$$\pi(K^*, \Omega)\frac{\partial \pi(K^*, \Omega)}{\partial K} = 2r. \tag{7}$$

In the absence of endogenous matching, it is optimal for the VCs to equally split the equity of new ventures.[10] We will show in the next section that the split of equity is *not* even when accounting for endogenous matching in the VC market. By comparing (7) with (2), it becomes clear that the optimal contract $\{\lambda^*, K^*\}$ entails under-provision of capital from an efficiency perspective, i.e., $K^* < K^{fb}$. A comparison of (4) with $e^{fb} = \pi(K^{fb}, \Omega)$ further reveals that the entrepreneur's effort is also inefficiently low, i.e., $e^* < e^{fb}$.

We can now derive the expected utility of the entrepreneur using the optimal contract $\{\lambda^*, K^*\}$ as stated in Lemma 2:

$$U^V(K^*, \Omega) = \pi^2(K^*, \Omega)/8. \tag{8}$$

[10]Note that this result is rooted in our specific assumptions about the entrepreneur's effort cost function $c(e)$, and how his effort affects the performance of the venture.

The superscript V indicates that the entire bargaining power rests with the VC. Thus, $U^V(K^*, \Omega)$ constitutes the lowest possible expected utility level for the entrepreneur in our matching framework, which will play a fundamental role in our analysis of the VC market equilibrium.

Finally, it is straightforward to show that both parties strictly benefit from a superior match quality Ω, rooted either in a more promising business idea (μ) or in greater VC expertise (x). This observation will have important implications for the properties of the matching process in the VC market, which we now turn to.

5 Market Equilibrium with Endogenous Matching

We now derive the optimal VC contracts in a two-sided market with heterogenous entrepreneurs and VCs. We proceed in two steps. In Section 5.1, we first identify the general properties of the matching equilibrium in the VC market. We then examine in Section 5.2 the optimal adjustment of VC contracts when taking endogenous matching into account.

5.1 Properties of the Matching Equilibrium

In a traditional principal-agent setting, an entrepreneur is forced to accept the offer of a specific VC (as long as the offer satisfies the individual rationality constraint). However, if entrepreneurs are free to choose the VC with the most attractive offer, optimal VC contracts must account for the best alternative available to an entrepreneur. The VC thus designs the contract $\{\lambda, K\}$ to maximize its expected profit subject to the entrepreneur receiving at least his outside value u (which we will characterize later). The constrained optimization problem of the VC is as follows:

$$\overline{\Pi}(u, \Omega) \equiv \max_{\{\lambda, K\}} (1-\lambda)\pi(K, \Omega)e^* - rK \tag{9}$$

s.t.

$$\lambda\pi(K, \Omega)e^* - (e^*)^2/2 \geq u, \tag{10}$$

where $e^* = \lambda\pi(K, \Omega)$. The maximized objective function $\overline{\Pi}(u, \Omega)$ defines the *bargaining frontier* between the VC and the entrepreneur. Whether the entrepreneur's individual rationality (IR) constraint (10) is binding clearly depends on his reservation utility u. We know that U^V, as defined by (8), constitutes the lower bound of u. This is the entrepreneur's expected utility when contracting with a VC in the absence of endogenous matching, assuming the entire bargaining power rests with the VC. The maximum value of u, denoted by U^E, constitutes the entrepreneur's expected utility in case he holds the entire bargaining power, with $U^E > U^V$. The next Lemma identifies an important property of the bargaining frontier $\overline{\Pi}(u, \Omega)$.

Lemma 3 (Bargaining Frontier) *The bargaining frontier* $\overline{\Pi}(u, \Omega)$ *is decreasing in the entrepreneur's reservation utility* u *for* $u \in [U^V, U^E]$.

We can now define the equilibrium of the VC market when each VC matches with one entrepreneur (*one-to-one matching*).

Definition 1 (Matching Equilibrium) *An equilibrium of the VC market consists of a one-to-one matching function* $m : E \to V$ *and payoff allocations* $\Pi^* : V \to \mathbb{R}_+$ *and* $u^* : E \to \mathbb{R}_+$ *that satisfy the following two conditions:*

(i) Feasibility of (Π^*, u^*) *with respect to* m*: For all* $i \in E$*,* $\{\Pi^*(m(i)), u^*(i)\}$ *is on the bargaining frontier* $\overline{\Pi}(u, \Omega(\mu(i), x(m(i))))$.

(ii) *Stability of* m *with respect to* $\{\Pi^*, u^*\}$*: There do not exist a pair* $(i, j) \in E \times V$*, where* $m(i) \neq j$*, and outside value* $u > u^*(i)$ *such that* $\overline{\Pi}(u, \Omega(\mu(i), x(j)) > \Pi^*(j)$.

These two conditions guarantee the existence of a stable matching equilibrium in the VC market. More precisely, the *feasibility* condition requires that the payoffs for VCs and entrepreneurs must be attainable, which is guaranteed whenever the payoffs of any $(i, m(i))$ pair are on the bargaining frontier $\overline{\Pi}(u, \Omega(\mu(i), x(m(i))))$. Moreover, the *stability* condition ensures that all matched VCs and entrepreneurs cannot become strictly better off by breaking their current partnership and matching with a new VC or entrepreneur.

We would obtain *positive assortative matching* (PAM) whenever entrepreneurs with high-quality ideas are matched with high-expertise VCs. The opposite occurs with *negative assortative matching* (NAM). The next definition formalizes the characteristics of positive versus negative assortative matching.

Definition 2 (Assortative Matching in the VC Market) *Consider two entrepreneurs i and i' with idea qualities $\mu(i) > \mu(i')$, and suppose that entrepreneur i is matched with VC $j = m(i)$ and entrepreneur i' is matched with VC $j' = m(i')$. The matching equilibrium is positive assortative (PAM) if the expertise of the VCs satisfy $x(j) > x(j')$; and negative assortative (NAM) if $x(j') > x(j)$.*

Applying the criteria derived by Legros and Newman (2007) to our framework, we can infer that the matching equilibrium is positive assortative if: (i) the cross-partial derivative of the bargaining frontier $\overline{\Pi}(u, \Omega)$ with respect to the entrepreneur's idea quality μ and the VC's management expertise x is positive, i.e., $\partial^2\overline{\Pi}/(\partial\mu\partial x) > 0$; and (ii), it is relatively easier for a high (versus low) expertise VC to transfer surplus to an entrepreneur, i.e., $\partial^2\overline{\Pi}/(\partial u\partial x) \geq 0$. The first condition is the standard complementarity condition that guarantees positive assortative matching in models with transferable utility (see Shapley and Shubik (1972) and Becker (1973)). However, as shown by Legros and Newman, this is not a sufficient condition to guarantee PAM whenever utility is non-transferable, as in our framework.[11]

Sørensen (2007) provides empirical evidence that the VC market is positive assortative (PAM): entrepreneurs with high-quality ideas receive start-up financing from more experienced VCs. The next Lemma derives sufficient conditions for PAM to arise in our model of the VC market.

Lemma 4 (PAM in the VC Market) *The matching between VCs and entrepreneurs is positive assortative (PAM) if the following two conditions are satisfied:*

(i) *The profit function $\pi(K, \Omega)$ is increasing and concave in the VC investment K and match quality Ω, with complementarity between the two, i.e. $\partial^2\pi(K, \Omega)/(\partial K\partial\Omega) \geq 0$.*

(ii) *The match quality $\Omega(\mu, x)$ is increasing in the entrepreneur's idea quality μ and the VC's expertise x, with complementarity between the two, i.e. $\partial^2\Omega(\mu, x)/(\partial\mu\partial x) \geq 0$.*

The properties of the venture's gross profit $\pi(K, \Omega)$ and the match quality $\Omega(\mu, x)$ are not only very intuitive, but also ensure that the matching equilibrium in our model of the VC market is positive assortative.

[11] In our framework, utility can be transferred via the investment K and equity share λ. These two instruments, however, transfer surplus imperfectly as they also affect the size of the surplus. Due to the zero wealth assumption for entrepreneurs, side payments from entrepreneurs to VCs are not feasible, which is an important characteristic of VC markets; see Sørensen (2007) for a discussion.

Positive assortative matching (PAM) implies that the matching function $m(i)$ is increasing in i. Note that the measure of ENs must be equal to the measure of VCs for the one-to-one matching equilibrium. Thus, it must hold that $H(i) = G(m(i))$ in order to ensure measure consistency. This implies that $m(i) = G^{-1}(H(i))$. Using this consistency condition, we can derive the slope of the matching function $m(i)$:

$$\frac{dm(i)}{di} = G^{-1'}(H(i))\,h(i) = \frac{h(i)}{G'(G^{-1}(H(i)))} = \frac{h(i)}{g(m(i))}. \tag{11}$$

The slope of the matching function $m(i)$ is equal to the ratio of the densities of EN and VC types, $h(i)$ and $g(m(i))$. Given that market entry is costly for VCs, there will be a cutoff expertise level below which VCs do not enter the market. We denote this cutoff by $\underline{j}(F) > 0$ as a function of the entry cost $F > 0$. The lowest-quality entrepreneur who still receives VC financing, denoted $\underline{i}$, is thus defined by $m(\underline{i}) = \underline{j}$. We henceforth use the index i when referring to the match between entrepreneur i and VC $m(i)$.

5.2 Endogenous Matching and Optimal VC Contracts

We can now characterize the equilibrium VC contracts $\{\lambda^M(i), K^M(i)\}$ under endogenous matching (indexed by the superscript M) for all $i > \underline{i}(F)$.

Proposition 1 (VC Contracts with Endogenous Matching) *The equilibrium VC contract between entrepreneur i and VC $m(i)$, with $i > \underline{i}(F)$, consists of the equity share*

$$\lambda^M(i) = \frac{\sqrt{2u^*(i)}}{\pi(K^M(i), \Omega)} \tag{12}$$

for entrepreneur i, and the VC investment $K^M(i)$ which is characterized by

$$\frac{\partial \pi(K^M(i), \Omega)}{\partial K(i)} = \frac{r}{\sqrt{2u^*(i)}}. \tag{13}$$

The equity share $\lambda^M(i)$ and investment $K^M(i)$ are both increasing in entrepreneur i's outside option $u^*(i)$ (i.e., $d\lambda^M(i)/du^*(i) > 0$ and $dK^M(i)/du^*(i) > 0$).

The equilibrium contract for entrepreneur i, $\{\lambda^M(i), K^M(i)\}$, accounts for his outside option $u^*(i)$. A better outside option $u^*(i)$ forces VC $m(i)$ to transfer more surplus to entrepreneur i by offering a higher equity stake $\lambda^M(i)$ and more capital $K^M(i)$. Furthermore, using (12) from Proposition 1 with (4) yields the equilibrium effort choice of entrepreneur i with $e^M(i) = \sqrt{2u^*(i)}$. Clearly, a better outside option $u^*(i)$ also results in a higher equilibrium level of effort, and thus improves the likelihood of the venture to succeed.

To fully establish the VC market equilibrium, we need to characterize the value of outside options $u^*(i)$ for all VC-backed entrepreneurs $i > \underline{i}(F)$. Note that the outside option $u^*(i)$ arises endogenously in our framework. Consider VC j which is matched with entrepreneur i (i.e., $m(i) = j$). Because the expected profit of a VC is increasing in the match quality Ω, all VCs j' with $j' < j$ would strictly prefer to match with entrepreneur i. However, it is VC j that has the highest willingness to pay for entrepreneur i, and thus to transfer the most utility to EN i. In contrast, it is not optimal for VC $m(i)$ to make a contract offer to a lower quality entrepreneur $i' < i$ because such a match would result in a lower expected profit. The equilibrium reservation utility $u^*(i)$ therefore ensures that no lower-expertise VC j', with $j' < j$, finds it profitable to outbid VC j and contract with entrepreneur i. The next Lemma provides a condition that characterizes the equilibrium outside options $u^*(i)$ for all entrepreneurs $i > \underline{i}(F)$.

Lemma 5 *The equilibrium outside option $u^*(i)$ for entrepreneur i, with $i > \underline{i}(F)$, is characterized by the ordinary differential equation*

$$\frac{du^*(i)}{di} = \frac{2u^*(i)\frac{\partial \pi(K^M(u^*(i)),\Omega)}{\partial \Omega}\frac{\partial \Omega}{\partial \mu}\frac{d\mu(i)}{di}}{2\sqrt{2u^*(i)} - \pi(K^M(u^*(i)),\Omega)} > 0, \tag{14}$$

with the initial condition $u^*(\underline{i}(F)) = \equiv U^V(\underline{i}(F))$. Both $\underline{i}(F)$ and $u^*(\underline{i}(F))$ are increasing in the entry cost F.

According to the Picard-Lindelöf Theorem, a unique solution $u^*(i)$ to (14) exists.[12] The unique solution $u^*(i)$ must then (implicitly) satisfy

$$u^*(i|F) = U^V(\underline{i}(F)) + \int_{\underline{i}(F)}^{i} \frac{du^*(s)}{ds} ds. \tag{15}$$

Among all entrepreneurs that receive VC financing, the lowest-quality EN $\underline{i}(F)$—who is matched in equilibrium with the zero-profit VC $m(\underline{i}(F))$—has the lowest possible utility $U^V(\underline{i}(F))$. The outside

[12]We need $du^*(i)/di$ to be Lipschitz continuous in u and continuous in i. Our assumptions ensure that $du^*(i)/di$ is continuous in i. If a differentiable function has a derivative that is bounded everywhere by a real number, then it is Lipschitz continuous. Hence, we need to assume that $\pi(K,\Omega)$ and $\partial\pi(\cdot)/\partial K$ are Lipschitz continuous with respect to K.

options of VC-backed ENs, $u^*(i|F)$, are increasing in i. The utility of EN i, with $i > \underline{i}$, thereby depends on the quality of all matches below in the ranking. How fast the outside option $u^*(i|F)$ increases in i depends, among other things, on the effect of entrepreneurial ideas on the match quality ($\partial\Omega/\partial\mu$), and on how fast the quality of ideas increases with the rank of the ENs ($d\mu(i)/di$). If, for example, all business ideas had the same quality (i.e., $d\mu(i)/di = 0$), then $du^*(i)/di = 0$ and all ENs would receive the same utility. We can view this case as a benchmark where endogenous matching does not have any effect on market outcomes. If ENs have business ideas of different qualities ($d\mu(i)/di > 0$), then endogenous matching clearly matters for the VC market equilibrium. The presence of diverse business ideas induces competition among VCs for lucrative investments, which results in improved outside options for entrepreneurs.

We can now shed light on how endogenous matching affects optimal VC contracts.

Proposition 2 (Endogenous Matching and VC Contracts) *The optimal contract $\{\lambda^M(i), K^M(i)\}$ between entrepreneur i and VC $m(i)$ with endogenous matching exhibits more equity and greater investment and effort levels compared to the benchmark contract $\{\lambda^*(i), K^*(i)\}$ without endogenous matching, i.e., $\lambda^M(i) > \lambda^*(i)$, $K^M(i) > K^*(i)$ and $e^M(i) > e^*(i)$. However, the equilibrium investment and effort levels, $K^M(i)$ and $e^M(i)$, are still below their socially efficient levels, i.e., $K^M(i) < K^{fb}(i)$ and $e^M(i) < e^{fb}(i)$.*

Endogenous matching in the VC market improves the outside options of all VC-backed entrepreneurs—except for the one with the lowest quality idea $\mu(\underline{i})$. According to Proposition 2, the competition for investment opportunities forces VCs to offer their entrepreneurs more equity $\lambda^M(i)$ and start-up capital $K^M(i)$. This enhances entrepreneurs' incentives to exert effort in order to turn their ideas into marketable products, and as a result, improves the survival rate of their ventures. However, despite the adjustments engendered by the matching process, the investment and effort levels (and thus the survival rate) remain sub-efficient from a social perspective.

6 Barriers to Entry and Market Efficiency: The Ripple Effect

We now use our matching framework to examine how barriers to entry, as captured by the entry cost F, affect the equilibrium of the VC market. We address the following two questions. First, how do barriers to entry affect investments in new ventures? Second, how sensitive is the survival rate of start-ups to entry barriers in the VC market?

We know from Lemma 5 that a smaller cost of entry F results in more lower-expertise VCs entering the market. These VCs match in equilibrium with lower-quality entrepreneurs who then receive capital for their ventures. More entrepreneurs receiving start-up financing is a direct effect of lower barriers to entry in the VC market. The next Proposition identifies an *indirect* effect of entry barriers on the VC market equilibrium.

Proposition 3 (Entry Barriers and Market Efficiency) *Lower barriers to entry result in (i) more equity and capital for all VC-backed entrepreneurs, and (ii), improved survival rates of their ventures (i.e., $d\lambda^M(i)/dF$, $dK^M(i)/dF$, $de^M(i)/dF < 0$ for all $i > \underline{i}(F)$).*

To briefly explain the rationale behind Proposition 3, consider entrepreneur $\underline{i}(F)$ who has the lowest idea quality $\mu(\underline{i}(F))$ among all VC-backed ENs. The reservation utility of entrepreneur $\underline{i}(F)$ is zero because only the lowest quality VC $m(\underline{i}(F))$ finds it optimal to make him a contract offer. Now suppose that the cost of entry decreases to F', with $F' < F$. VCs with lower expertise levels will then enter the market and match with the lower-quality entrepreneurs $i \in [\underline{i}(F'), \underline{i}(F))$ who would have remained unmatched for the entry cost F (as $d\underline{i}(F)/dF > 0$). The new VC $m(\underline{i}(F) - \varepsilon)$, with $\varepsilon \to 0$, is also willing to offer a contract to entrepreneur $\underline{i}(F)$, which in turn improves his outside option. Lemma 4 implies that entrepreneur $\underline{i}(F)$ is still matched in equilibrium with VC $m(\underline{i}(F))$. According to Proposition 1, however, VC $m(\underline{i}(F))$ must now offer its entrepreneur $\underline{i}(F)$ more equity and capital in order to account for his improved outside option. This motivates a higher effort level, and thus improves the venture's likelihood of survival. On the other hand, the overall profitability for VC $m(\underline{i}(F))$ is reduced, which in turn makes it optimal to offer the next best entrepreneur $\underline{i}(F) + \varepsilon$, with $\varepsilon \to 0$, a more attractive contract. The higher ranked VC $m(\underline{i}(F) + \varepsilon)$ is then forced to adjust the contract offer for its entrepreneur $\underline{i}(F) + \varepsilon$ in order to match his improved outside option, and so on.

Lower barriers to entry therefore benefit all entrepreneurs, and not just the ones that are financed by the new VCs. On the other hand, the "incumbent VCs" (i.e., the VCs that would have been in the market anyway) are then compelled to transfer more surplus to their entrepreneurs (through more attractive VC contracts). Nonetheless, the presence of more VCs improves the overall efficiency of the VC market: all start-ups are now equipped with more capital, which in turn improves their prospects of survival.[13] The

[13]This theoretical prediction is consistent with empirical evidence in Gompers and Lerner (2000) that the more intense the competition, the higher the amount of funding each venture receives.

key mechanism behind this implication is the interdependence of all ventures through the endogenous outside options of entrepreneurs. In this sense, reducing barriers to entry has a *ripple effect* throughout the entire VC market as it affects *all* entrepreneur-VC pairs.

The ripple effect of lower barriers to entry has another important implication: A VC entering the market exerts a positive externality on all VC-backed ventures, thus improving the efficiency of the entire VC market. However, this positive externality is not internalized. Entry is therefore always insufficient from a social welfare perspective, even in the absence of barriers to entry ($F = 0$).

The next Proposition identifies an interesting property of the ripple effect of lower entry barriers in the VC market (keep in mind that $dK^M(i)/dF$, $de^M(i)/dF < 0$).

Proposition 4 (The Ripple Effect) *There exits a threshold $\bar{i}$, with $\bar{i} > \underline{i}$, such that the ripple effect of lower barriers to entry decreases in i for $i < \bar{i}$ (i.e., $d^2K^M(i)/(dFdi)$, $d^2\lambda^M(i)/(dFdi)$, $d^2e^M(i)/(dFdi) > 0$ for $i \leq \bar{i}$).*

While all VC-backed entrepreneurs profit from more VCs competing for investment opportunities (the ripple effect), Proposition 4 implies that lower-quality entrepreneurs benefit relatively more from VC entry. In other words, the ripple effect of lower barriers to entry subsides for high-quality ventures as long as $i \leq \bar{i}$.[14] This also suggests that entrepreneurs with high quality ideas are more (but not completely) immune to less intense competition among VCs for investment opportunities.

6.1 A Numerical Example

To illustrate the subsiding ripple effect, consider an example with $\pi(K, \Omega) = K^\beta \Omega^\gamma$, $\Omega = \mu x$, $\mu = ai$, and $x = bj$. For simplicity, we assume that i and j are uniformly distributed on $[0, 0.45]$.[15] The uniform distribution implies a matching function with slope one. Figure 1 depicts the outside options $u^*(i|F)$ of all VC-backed entrepreneurs for $F = 0.05$ and $F = 0.07$, with parameter values $a = 2$, $b = 2$, $\beta = 1/3$, $\gamma = 1/3$, and $r = 1/10$ (tables showing the equilibrium outside options for selected entrepreneurs are provided in the Appendix).[16]

As can be seen from Figure 1, intensified competition among VCs stemming from lower barriers to entry ($F = 0.05$) improves the outside options of *all* VC-backed entrepreneurs in the market (except for the lowest-quality entrepreneur $\underline{i}(F)$). This ripple effect is clearly more pronounced for low-quality

[14] If $\bar{i} \notin E$, then the ripple effect subsides for all $i \in E$.

[15] The upper bound of the distribution ensures that $e^*(i) = \sqrt{2u^*(i)} < 1$ for all $i \in E$.

[16] We used Maple (a math software) to solve the differential equation (14) numerically.

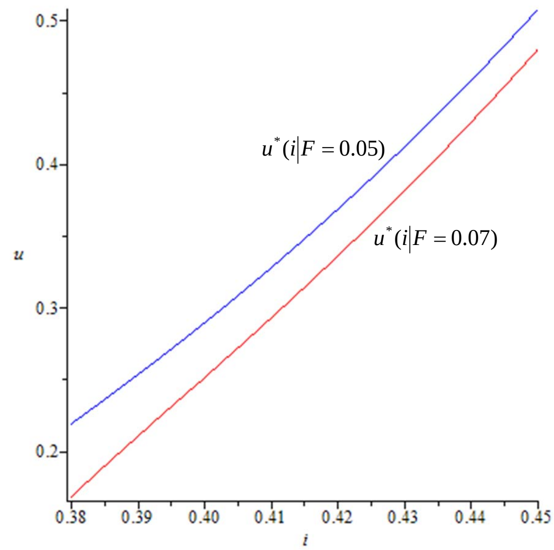

Figure 1: The ripple effect of market entry: A numerical example

entrepreneurs since the two curves move closer to each other as i increases. Entrepreneurs with high-quality ideas therefore benefit relatively less from market entry, which points to a subsiding ripple effect.

7 Conclusion

We develop an equilibrium model of the VC market with heterogenous entrepreneurs and heterogenous VCs. We examine how endogenous matching affects equilibrium contracts and the survival rate of new ventures. A key insight is that endogenous matching—which originates in the competition among VCs for entrepreneurial ideas—improves the outside options of entrepreneurs. This forces VCs to offer their entrepreneurs more equity and start-up capital, which in turn improves the survival rate of their ventures.

Our framework provides another important insight: Lower barriers to entry improves the efficiency of the entire VC market (the *ripple effect*). New ventures are then equipped with more capital, which in turn enhances their prospects of success. In this sense, lowering barriers to entry for VCs, e.g. by reducing capital requirements or administrative costs, would not only benefit entrepreneurs who will then receive VC financing. Also entrepreneurs contracting with incumbent VCs would benefit from VC entry because of better capital endowments of their ventures. This positive externality of entry on the entire market—which is not internalized—yields an equilibrium with an insufficient number of VCs, even in the absence of barriers to entry. Spurring entry into the VC market has therefore a multiplier effect on innovative activities, which are crucial for economic growth.

We consider the market for venture capital as the main application of our matching framework with non-transferable utility and endogenous entry. Our equilibrium model provides a series of novel and important insights with respect to the interrelationship between the VC market as a whole and the properties of individual VC contracts. However, our insights are much broader and apply in general to markets with vertically differentiated competition where surplus between any two parties in a contractual relationship cannot be transferred perfectly (NTU). Our model with endogenous matching implies that entry and exit in such vertically differentiated markets affect all contractual relationships, and not just the newly formed (entry) or dissolved (exit) matches. Even when allowing for free market entry, such markets do not converge to the efficient outcome.

Appendix

Proof of Lemma 1

The socially efficient effort level e^{fb} as well as the efficient investment K^{fb} are characterized by the following two first-order conditions:

$$\pi(K,\Omega) = e \tag{16}$$

$$\frac{\partial \pi(\cdot)}{\partial K} e = r. \tag{17}$$

Substituting (16) into (17) yields the lemma. □

Proof of Lemma 2

By accounting for the entrepreneur's effort choice e^* as given by (4), the optimal contract components λ^* and K^* are implicitly characterized by the following two first-order conditions:

$$2\lambda^*(1-\lambda^*)\pi(K^*,\Omega)\frac{\partial \pi(K^*,\Omega)}{\partial K} = r \tag{18}$$

$$(1-2\lambda^*)\pi^2(K^*,\Omega) = 0. \tag{19}$$

Solving (19) for λ^*, and substituting the resulting expression into (18) yields the lemma. □

Proof of Lemma 3

At the bargaining frontier, the constraint (10) must be binding. Using e^* as defined by (4), the binding constraint can be written as

$$\frac{1}{2}\lambda^2(\pi(K,\Omega))^2 = u.$$

Let λ^M denote the optimal equity share under endogenous matching. Thus, λ^M must satisfy

$$\lambda^M = \frac{\sqrt{2u}}{\pi(K,\Omega)}. \tag{20}$$

Substituting (20) and e^* (as defined by (4)) into (9) yields the unconstrained maximization problem for the VC:

$$\max_{\{K\}} \sqrt{2u}\pi(K,\Omega) - 2u - rK.$$

The optimal capital provision under endogenous matching, denoted $K^M(u,\Omega)$, is characterized by the first-order condition:

$$\sqrt{2u}\frac{\partial \pi(K,\Omega)}{\partial K} - r = 0. \tag{21}$$

Next, we can infer from (21) that

$$\frac{dK^M}{du} = -\frac{\frac{\partial \pi(\cdot)}{\partial K}}{2u\frac{\partial^2 \pi(\cdot)}{\partial K^2}} > 0 \text{ and } \frac{dK^M}{d\Omega} = -\frac{\frac{\partial^2 \pi(\cdot)}{\partial K \partial \Omega}}{\frac{\partial^2 \pi(\cdot)}{\partial K^2}} > 0. \tag{22}$$

Thus, the frontier contract $\{\lambda^M, K^M\}$ satisfies

$$\frac{\partial \pi(K^M,\Omega)}{\partial K} = \frac{r}{\sqrt{2u}} \text{ and } \lambda^M = \frac{\sqrt{2u}}{\pi(K^M,\Omega)}. \tag{23}$$

Substituting λ^M, as defined by (20), and e^*, as defined by (4), into (9) yields the bargaining frontier $\overline{\Pi}(\cdot)$:

$$\overline{\Pi}(u,\Omega) = \sqrt{2u}\pi(K^M(u,\Omega),\Omega) - 2u - rK^M(u,\Omega). \tag{24}$$

Differentiating the bargaining frontier $\overline{\Pi}(u,\Omega)$ with respect to u yields

$$\frac{d\overline{\Pi}(\cdot)}{du} = \frac{1}{\sqrt{2u}}\pi(\cdot) + \sqrt{2u}\frac{\partial \pi(\cdot)}{\partial K}\frac{dK^M}{du} - 2 - r\frac{dK^M}{du}, \tag{25}$$

which, by using (21), can be simplified to

$$\frac{d\overline{\Pi}(\cdot)}{du} = \frac{1}{\sqrt{2u}}\pi(\cdot) - 2. \tag{26}$$

Note that $d\overline{\Pi}(\cdot)/du = 0$ at $u = U^V$; see (8). Thus, $d\overline{\Pi}(\cdot)/du < 0$ for all $u > U^V$. Finally, it must hold that

$$\pi(\cdot) \geq -\frac{\left(\frac{\partial \pi(\cdot)}{\partial K}\right)^2}{\frac{\partial^2 \pi(\cdot)}{\partial K^2}} \tag{27}$$

so that $\overline{\Pi}(\cdot)$ is concave for all permissible values of u. □

Proof of Lemma 4

We verify whether the sufficient conditions for positive assortative matching (PAM) are satisfied. First, recall from Proof of Lemma 3 that the VC's unconstrained profit function $\overline{\Pi}(\cdot)$ is given by

$$\overline{\Pi}(u,\Omega) = \sqrt{2u}\pi(K^M,\Omega) - 2u - rK^M(u,\Omega).$$

Differentiating $\overline{\Pi}(\cdot)$ with respect to x yields

$$\frac{d\overline{\Pi}(\cdot)}{dx} = \sqrt{2u}\frac{\partial\pi(\cdot)}{\partial K}\frac{dK^M}{d\Omega}\frac{\partial\Omega}{\partial x} + \sqrt{2u}\frac{\partial\pi(\cdot)}{\partial\Omega}\frac{\partial\Omega}{dx} - r\frac{dK^M}{d\Omega}\frac{\partial\Omega}{\partial x}.$$

Using (21), we get the simplified expression

$$\frac{d\overline{\Pi}(\cdot)}{dx} = \sqrt{2u}\frac{\partial\pi(\cdot)}{\partial\Omega}\frac{\partial\Omega}{\partial x}. \tag{28}$$

Differentiating $d\overline{\Pi}(\cdot)/dx$ with respect to μ yields

$$\frac{d^2\overline{\Pi}(\cdot)}{dxd\mu} = \sqrt{2u}\left[\frac{\partial^2\pi(\cdot)}{\partial\Omega^2}\frac{\partial\Omega}{\partial\mu}\frac{\partial\Omega}{\partial x} + \frac{\partial^2\pi(\cdot)}{\partial\Omega\partial K}\frac{dK^M}{d\Omega}\frac{\partial\Omega}{\partial\mu}\frac{\partial\Omega}{\partial x} + \frac{\partial\pi(\cdot)}{\partial\Omega}\frac{\partial^2\Omega}{\partial x\partial\mu}\right]. \tag{29}$$

Using (22), we can rewrite (29) as

$$\frac{d^2\overline{\Pi}(\cdot)}{dxd\mu} = \sqrt{2u}\left[\frac{\partial^2\pi(\cdot)}{\partial\Omega^2}\frac{\partial\Omega}{\partial x}\frac{\partial\Omega}{\partial\mu} - \frac{\partial^2\pi(\cdot)}{\partial\Omega\partial K}\frac{\frac{\partial^2\pi(\cdot)}{\partial K\partial\Omega}}{\frac{\partial^2\pi(\cdot)}{\partial K^2}}\frac{\partial\Omega}{\partial\mu}\frac{\partial\Omega}{\partial x} + \frac{\partial\pi(\cdot)}{\partial\Omega}\frac{\partial^2\Omega}{\partial xj\partial\mu}\right]. \tag{30}$$

Since $\partial^2\Omega/\left(\partial x\partial\mu\right) \geq 0$, (30) is positive if

$$\sqrt{2u}\left[\frac{\partial^2\pi(\cdot)}{\partial\Omega^2} - \frac{\partial^2\pi(\cdot)}{\partial\Omega\partial K}\frac{\frac{\partial^2\pi(\cdot)}{\partial K\partial\Omega}}{\frac{\partial^2\pi(\cdot)}{\partial K^2}}\right]\frac{\partial\Omega}{\partial\mu}\frac{\partial\Omega}{\partial x} > 0. \tag{31}$$

Recall that $\pi(K,\Omega)$ is concave in both of its arguments. This implies that the Hessian determinant must be positive, i.e.,

$$\frac{\partial^2\pi(\cdot)}{\partial K^2}\frac{\partial^2\pi(\cdot)}{\partial\Omega^2}-\frac{\partial^2\pi(\cdot)}{\partial\Omega\partial K}\frac{\partial^2\pi(\cdot)}{\partial K\partial\Omega}\geq 0. \tag{32}$$

Thus, (31) is positive, which implies that $d^2\overline{\Pi}(\cdot)/(dxd\mu)\geq 0$. Differentiating $d\overline{\Pi}(\cdot)/dx$ (see (28)) with respect to u yields

$$\frac{d^2\overline{\Pi}(\cdot)}{dxdu}=\frac{1}{\sqrt{2u}}\frac{\partial\pi(\cdot)}{\partial\Omega}\frac{\partial\Omega}{\partial x}+\sqrt{2u}\frac{\partial^2\pi(\cdot)}{\partial\Omega\partial K}\frac{dK^M}{du}\frac{\partial\Omega}{\partial x}. \tag{33}$$

It is straightforward to show that $dK^M/du>0$. Thus, (33) is positive. □

Proof of Proposition 1

The optimal equity share $\lambda^M(i)$ and investment $K^M(i)$ follow directly from the derivations in the Proof of Lemma 3. Moreover, we know from Proof of Lemma 3 that $dK^M/du>0$. By using (20) and (22) one gets

$$\frac{d\lambda^M}{du}=\frac{\pi(\cdot)+\left(\frac{\partial\pi(\cdot)}{\partial K}\right)^2/\left(\frac{\partial^2\pi(\cdot)}{\partial K^2}\right)}{\sqrt{2u}\pi^2(\cdot)}. \tag{34}$$

The concavity condition for the bargaining frontier (27) implies $d\lambda^M/du>0$. □

Proof of Lemma 5

Consider the match between EN i and VC $j=m(i)$. The profit function of VC $m(i)$ is given by

$$\overline{\Pi}(i)=\sqrt{2u(i)}\pi(K^M(i),\Omega)-2u(i)-rK^M(i).$$

Holding $x(j)$ constant and applying the Envelope Theorem, we get

$$\frac{d\overline{\Pi}(i)}{di}=\frac{\pi}{\sqrt{2u(i)}}\frac{du(i)}{di}+\sqrt{2u(i)}\frac{\partial\pi}{\partial\Omega}\frac{\partial\Omega}{\partial\mu}\frac{d\mu(i)}{di}-2\frac{du(i)}{di}. \tag{35}$$

In the positive assortative matching equilibrium (PAM), it must hold that $d\overline{\Pi}(i)/di=0$ for all $j\in V$. If $d\overline{\Pi}(i)/di<0$, the matching equilibrium is not stable because VC j would be better off contracting with the marginally lower quality EN $i-\varepsilon$, with $\varepsilon\to 0$. If, on the other hand, $d\overline{\Pi}(i)/di>0$, then the matching equilibrium is not stable either. EN i would then be better off contracting with the marginally higher expertise VC $j+\varepsilon$, with $\varepsilon \to 0$. If a VC does not benefit from deviating locally, then it does not benefit from deviating globally.[17] By setting $d\overline{\Pi}(i)/di = 0$ and solving (35) for $du^*(i)/di$, we get the ordinary differential equation (ODE) which characterizes $u^*(i)$ for all $i > \underline{i}(F)$:

$$\frac{du^*(i)}{di} = \frac{2u^*(i)\frac{\partial \pi(K^M(u^*(i)),\Omega)}{\partial \Omega}\frac{\partial \Omega}{\partial \mu}\frac{d\mu(i)}{di}}{2\sqrt{2u^*(i)} - \pi(K^M(u^*(i)),\Omega)}. \tag{36}$$

We can now determine the initial condition for the ODE. To do so, we need to find an index $\underline{i}$ and an associated utility level $u(\underline{i})$ where VC $m(\underline{i})$ breaks even. Thus, $\underline{i}$ must satisfy the zero profit condition:

$$\overline{\Pi}(u(\underline{i}),\Omega) = \sqrt{2u(\underline{i})}\pi - 2u(\underline{i}) - rK^M(\underline{i}) - F = 0. \tag{37}$$

We now show that such a point exists and is unique. Totally differentiating (37) with respect to i and u yields

$$\frac{di}{du} = -\frac{\left(\frac{\pi}{\sqrt{2u}} - 2\right)}{\sqrt{2u}\frac{\partial \pi}{\partial \Omega}\left(\frac{\partial \Omega}{\partial \mu}\frac{\partial \mu(i)}{\partial i} + \frac{\partial \Omega}{\partial x}\frac{\partial x(j)}{\partial j}\frac{dm(i)}{di}\right)}.$$

Recall that $U^V = \pi^2/8$. It then follows that (i) $du(i)/di < 0$ for $u(i) < U^V$, (ii) $du(i)/di = 0$ at $u(i) = U^V$ and (iii) $du(i)/di > 0$ for $u(i) > U^V$. This implies that the zero-profit function (37) is U-shaped in the (i,u) space with its minimum at $u(i) = U^V = \pi^2/8$. Thus, the initial condition is $u^*(\underline{i}(F)) = U^V(\underline{i}(F))$.

Next, we examine how the entry cost F affects $\underline{i}(F)$ and $U^V(\underline{i}(F))$. Totally differentiating (37) with respect to i and F yields

$$\left[\left(\frac{\pi}{\sqrt{2u(i)}} - 2\right)\frac{du(i)}{di} + \sqrt{2u(i)}\frac{\partial \pi}{\partial \Omega}\left(\frac{\partial \Omega}{\partial \mu}\frac{\partial \mu(\underline{i})}{\partial i} + \frac{\partial \Omega}{\partial x}\frac{\partial x(j)}{\partial j}\frac{dm(\underline{i})}{di}\right)\right] di - dF = 0.$$

Using $u(i) = U^V = \pi^2/8$ we get

$$\frac{d\underline{i}(F)}{dF} = \frac{2}{\pi\frac{\partial \pi}{\partial \Omega}\left(\frac{\partial \Omega}{\partial \mu}\frac{\partial \mu(\underline{i})}{\partial i} + \frac{\partial \Omega}{\partial x}\frac{\partial x(j)}{\partial j}\frac{dm(\underline{i})}{di}\right)} > 0. \tag{38}$$

[17]This follows because under PAM the highest willingness to pay among VCs for an EN belongs to the VC who is currently matched with a marginally lower EN.

Differentiating U^V with respect to i gives

$$\frac{dU^V}{di} = \frac{\pi}{4}\left[\left(\frac{\partial \pi}{\partial K}\frac{dK^M}{d\Omega} + \frac{\partial \pi}{\partial \Omega}\right)\left(\frac{\partial \Omega}{\partial \mu}\frac{d\mu(\underline{i})}{di} + \frac{\partial \Omega}{\partial x}\frac{dx(\underline{j})}{dj}\frac{dm(\underline{i})}{di}\right) + \frac{\partial \pi}{\partial K}\frac{dK}{du}\frac{dU^V}{di}\right].$$

Using (22) and collecting dU^V/di on the LHS we get

$$\frac{dU^V}{di} = \frac{\frac{\pi}{4}\left[\left(\frac{\partial \pi}{\partial K}\frac{dK^M}{d\Omega} + \frac{\partial \pi}{\partial \Omega}\right)\left(\frac{\partial \Omega}{\partial \mu}\frac{d\mu(\underline{i})}{di} + \frac{\partial \Omega}{\partial x}\frac{dx(\underline{j})}{dj}\frac{dm(\underline{i})}{di}\right)\right]}{\left(1 + \frac{\left(\frac{\partial \pi}{\partial K}\right)^2}{\pi \frac{\partial^2 \pi}{\partial K^2}}\right)} > 0. \tag{39}$$

The sign follows from condition (27); see Proof of Lemma 3. Finally note that $dU^V/di > 0$ and $d\underline{i}(F)/dF > 0$ implies $dU^V(\underline{i}(F))/dF > 0$. □

Proof of Proposition 2

First, (22) implies $K^M(i) > K^*(i)$. Moreover, using (4) and Lemma 1 we get $e^M(i) = \sqrt{2u(i)}$. Thus, $de^M(i)/du(i) > 0$ and $e^M(i) > e^*(i)$. In the proof of Proposition 1 we proved $d\lambda^M/du > 0$. Next, the optimality condition for $K^M(i)$, (23), is only equivalent to the optimality condition for K^{fb}, (2), when $u^*(i) = (\pi(K, \Omega))^2/2$. However, on the bargaining frontier $\overline{\Pi}(\cdot)$ we have

$$u(i) = \frac{1}{2}\lambda^2\pi^2(K, \Omega) < \frac{1}{2}\pi^2(K, \Omega), \tag{40}$$

because $\lambda < 1$. Thus, $K^M(i) < K^{fb}(i)$ and $e^M(i) < e^{fb}(i)$. □

Proof of Proposition 3

Recall that $u^*(i|F)$ is defined by (15), where $u^*(\cdot)$ appears on both sides of (15). For any $i > \underline{i}(F)$, differentiating $u^*(i|F)$ with respect to F yields

$$\frac{du^*(i|F)}{dF} = \left(\frac{dU^V(\underline{i}(F))}{di} - \frac{du^*(\underline{i}(F))}{di}\right)\frac{d\underline{i}(F)}{dF} + \int_{\underline{i}(F)}^{i} \frac{d}{dF}\left(\frac{du^*(s)}{ds}\right) ds. \tag{41}$$

The first term of (41), $dU^V(\underline{i}(F))/di$, is positive and given by (39). The second term, $du^*(\underline{i}(F))/di$, is the differential equation (14) evaluated at the initial condition $i = \underline{i}$, and is positive. Note that

$du^*(\underline{i}(F))/di$ dominates $dU^V(\underline{i}(F))/di$ for $i \to \underline{i}$ because the denominator of $du^*(\underline{i}(F))/di$ then goes to infinity. Thus, $\left(dU^V(\underline{i}(F))/di - du^*(\underline{i}(F))/di\right) < 0$. Moreover, $d\underline{i}(F)/dF$ is positive and given by (38).

It remains to examine the term inside the integral. For parsimony, let $\pi \equiv \pi(K^M(u^*(i)), \Omega))$ and $u^* \equiv u^*(i)$. Differentiating (14), we obtain

$$\frac{d}{dF}\left(\frac{du^*(s)}{ds}\right) = \frac{\frac{du^*}{dF}\frac{\partial\Omega}{\partial\mu}\frac{d\mu(i)}{di}\left[2\frac{\partial\pi}{\partial\Omega}\left(\sqrt{2u^*}-\pi\right) + \left(2u^*\frac{\partial^2\pi}{\partial\Omega\partial K}\frac{dK}{du}\right)\left(2\sqrt{2u^*}-\pi\right) + 2u^*\frac{\partial\pi}{\partial K}\frac{\partial\pi}{\partial\Omega}\frac{dK}{du}\right]}{\left(2\sqrt{2u^*}-\pi\right)^2}.$$

Using (22), we get

$$\frac{d}{dF}\left(\frac{du^*(s)}{ds}\right) = \frac{\frac{du^*}{dF}\frac{\partial\Omega}{\partial\mu}\frac{d\mu(i)}{di}\overbrace{\left[2\frac{\partial\pi}{\partial\Omega}\left(\sqrt{2u^*}-\pi\right) - \left(\frac{\frac{\partial^2\pi}{\partial\Omega\partial K}\frac{\partial\pi}{\partial K}}{\frac{\partial^2\pi}{\partial K^2}}\right)\left(2\sqrt{2u^*}-\pi\right) - \frac{\frac{\partial\pi}{\partial\Omega}\left(\frac{\partial\pi}{\partial K}\right)^2}{\frac{\partial^2\pi}{\partial K^2}}\right]}^{\equiv\chi}}{\left(2\sqrt{2u^*}-\pi\right)^2}. \tag{42}$$

When evaluating the term inside the brackets in the numerator of (42) at the initial condition $u^* = \pi^2/8$, we obtain

$$\frac{d}{dF}\left(\frac{du^*(s)}{ds}\right) = \frac{du^*}{dF}\frac{\partial\Omega}{\partial\mu}\frac{d\mu(i)}{di}\frac{\partial\pi}{\partial\Omega}\underbrace{\left(-\pi - \frac{\left(\frac{\partial\pi}{\partial K}\right)^2}{\frac{\partial^2\pi}{\partial K^2}}\right)}_{\equiv\varphi}.$$

The concavity condition for the bargaining frontier, (27), implies $\varphi < 0$.

We can now identify the sign of $du^*(i|F)/dF$. As shown, (41) is negative for $i = \underline{i}$. For i sufficiently close to $\underline{i}$, (42) is also negative, excluding the du^*/dF term. This implies that $du^*(i|F)/dF < 0$ for $i \to \underline{i}$. We can now prove by contradiction that $du^*(i|F)/dF < 0$ must hold for all $i > \underline{i}$. Suppose for a moment that $du^*(i|F)/dF \geq 0$ for some i. Let $(i', m(i'))$ be the first VC-EN pair such that $du^*(i|F)/dF \geq 0$. This implies that all entrepreneurs i with $i < i'$ receive a lower utility in response to a higher fixed cost F. In contrast, VC $m(i')$ then transfers more surplus to its entrepreneur i' despite facing less competitive pressure from below. However, this contradicts Lemma 3. Thus, $du^*(i|F)/dF < 0$ for all i>i̲. Because $d\lambda^M(i)/du^*(i)$, $dK^M(i)/du^*(i)$, $de^M(i)/du^*(i) > 0$ (see Proposition 1), $du^*(i|F)/dF < 0$ implies $d\lambda^M(i)/dF$, $dK^M(i)/dF$, $de^M(i)/dF < 0$ for all $i > \underline{i}(F)$.
□

Proof of Proposition 4

Differentiating (41) from Proof of Proposition 3 with respect to i yields

$$\frac{d^2u^*(i|F)}{dFdi} = \frac{d}{dF}\left(\frac{du^*(i)}{di}\right).$$

Using $\sqrt{2u^*(i)} = e^* = \lambda\pi$, we can write χ from (42) as

$$\frac{2\pi\lambda\left(\pi_{KK}\pi_{\Omega} - \pi_{K\Omega}\pi_{K}\right) - \pi\left(2\pi_{KK}\pi_{\Omega} - \pi_{K\Omega}\pi_{K}\right) - \pi_{\Omega}(\pi_{K})^2}{\pi_{KK}}, \tag{43}$$

where the subscripts indicate partial derivatives. Using (27), it can easily be verified that (43) is negative for $\lambda = 1/2$. In contrast, (43) becomes positive for $\lambda = 1$. Moreover, we know from Proposition 1 that $d\lambda^M(i)/du^*(i) > 0$, and thus $d\lambda^M(i)/di > 0$. Hence, there exists a threshold $\bar{i}$ such that (43) is negative for $i < \bar{i}$, and positive otherwise. Depending on the functional forms, however, the threshold $\bar{i}$ is not necessarily in the relevant range (i.e., $\bar{i} \notin E$). Because $du^*(i|F)/dF < 0$ (see Proof of Proposition 3), we can infer from (42) that $d^2u^*(i|F)/(dFdi) > 0$ for $i < \bar{i}$, and $d^2u^*(i|F)/(dFdi) \leq 0$ otherwise. Thus, since $du^*(i|F)/dF < 0$, the ripple effect decreases in i as long as $i < \bar{i}$. This in turn implies that $d^2K^M(i)/(dFdi) > 0$, $d^2\lambda^M(i)/(dFdi) > 0$, and $d^2e^M(i)/(dFdi) > 0$ for $i < \bar{i}$. □

The Ripple Effect: A numerical example

When $F = 0.05$, the marginal VC is located at $\underline{i} = 0.34086$ and offers his assigned entrepreneur utility level $u^*(\underline{i}) = 0.075$. The next table presents the equilibrium outside options for selected i's when $F = 0.05$.

i	0.38	0.40	0.42	0.44
$u^*(i)$	0.2196	0.2899	0.3691	0.4588

When $F = 0.07$, the marginal VC-EN pair increases to $\underline{i} = 0.3707$, and the associated outside option to $u^*(\underline{i}) = 0.105$. The next table presents the equilibrium outside options for the same selected entrepreneurs.

i	0.38	0.40	0.42	0.44
$u^*(i)$	0.1684	0.2517	0.3365	0.4297

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