Common Agency with Caring Agents

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Abstract

This paper considers two extensions to the standard common agency model. First, the agent's objective need not be increasing in contributions. Second, the agent can reject contributions from principals. The concept of truthful equilibria is appropriately generalized and their key properties (including efficiency) are reaffirmed in the new context. Furthermore, an application of the theory discussing fiscal policy determination under lobbying is also presented. Economic intuition associated with such models is reexamined.

1. Introduction

Common agency or menu auction is an extension of the classic principal agent problem to a model where many principals share a common agent. Common agency has been used in most fields of economic analysis, but it has proven especially useful in the study of political influence (lobbying, corruption e.t.c.). It was introduced by Bernheim and Whinston (1986) and was extended by Dixit et. al. (1997) to general utility functions. Both these papers concentrate on a class of equilibria named "equilibria in truthful strategies" or "truthful equilibria". The equilibria in this class, are relatively easy to calculate and efficient. For these reasons, truthful equilibria have proven to be very popular amongst economists, over the years.

Yet common agency models come with a drawback. All fundamental results including efficiency of truthful equilibria are based on a key assumption. Namely, that the agent's objective is increasing in contributions. However in certain cases, especially in lobbying, such an assumption might prove restrictive.

This is so when the agent cares about the welfare of the principals. In particular, a typical specification of the agent's objective is to express it, as an increasing function of contributions and principals' utility. This specification is used very often in common agency lobbying models. Some well known examples are: Grossman and Helpman (1994), Dixit et. al. (1997), Persson (1998), Martimort and Stole (2003), Le Breton and Salanie (2003), Campante and Fereira (2007), e.t.c. The intuition in this case is that the agent's objective reflects the decision process of a politician who tries to balance the financial support he collects with the well being of the electorate. In such a case, the agent faces a tradeoff associated with contributions. On the one hand contributions have a direct positive effect on the agent's objective and on the other an indirect negative effect through the principals' utility. Under standard assumptions (i.e. Inada conditions), as contributions increase, at some point the negative effect might exceed the positive. Therefore, the agent's objective eventually might become decreasing in contributions.

The literature so far has avoided this problem in two ways. The first is by considering objectives and utilities that are quasi linear in contributions. In such a case, the marginal effect of contributions on the agent's objective is constant (typically positive) and thus, the objective is increasing. Probably the most known paper in this category is Grossman and Helpman (1994). Their paper on the determination of trade policy received empirical support by Goldberg and Maggi (1999) and was extended by Mitra (1999). Recent examples in this strand of literature are Damania and Fredriksson (2007) and Tomassi and Weinschelbaum (2007). However the problem with this approach is that it fails to incorporate income effects. The second way to get around the problem is by considering only the cases that combine income effects with an increasing objective, as in Dixit et al (1997). In this case, the range of applications is limited[1]. For example, Inada conditions for the principals' utility functions, will lead in many cases to a non monotonous objective. A typical case is when Inada conditions for utility are combined with an agent's objective that is linear in total contributions and principals' utility levels.

Although caring agents is my main motivation for considering non increasing objectives, one can easily come up with alternative ones. An example is the cost of secrecy. If contributions have to be kept secret (i.e. due to legal restrictions) the agent's objective might not be increasing in contributions. Assuming, for example, an increasing marginal cost of hiding money will typically suffice.

In order to surpass the limitations of the theory so far, one has to introduce non increasing objectives. This will broaden the range of possible applications and facilitate the development of flexible models that tackle distributional issues successfully.

Furthermore, as noted by Felli and Merlo (2006), there is an unrealistic aspect in the standard common agency model. In particular, the agent can't choose who lobbies him or in other words, he can't reject contributions[2]. Introducing the possibility of contribution rejections especially makes sense, when the agent's objective is not increasing. This is so, because in such a case, certain contributions can be "harmful" to the agent. As we will see later on, the agent need not actually reject contributions in equilibrium. Just the possibility for contribution rejection is sufficient to generate equilibria with the desired properties.

In view of the discussion above, this paper extends the standard common agency model by introducing two new features in Dixit et al. (1997) model. First, the agent can reject a contribution from a principal, either partially or in whole. Second, the agent's objective need not always be increasing in contributions.

These new features warrant some minor changes in the concept of truthfulness. Consequently, two new notions are defined in the paper: quasi truthful strategies and quasi truthful

[1] One of the few papers following Dixit et al (1997) is Campante and Fereira (2007).

[2] An issue related to contribution rejections is delegated common agency that is discussed by Martimort and Stole (2003) in an industrial organization context.

equilibria. These two notions are appropriately adopted versions of their counterparts in the standard model.

The main theoretical result of my paper is proposition 2, which states that quasi truthful equilibria are efficient. Along with proposition 1, which shows the relevance of quasi truthful schedules, proposition 2 offers support for the use of quasi truthful equilibria, in generalized common agency problems.

Furthermore, I develop an application of the general theory, from the field of lobbying. This application, discusses the determination of fiscal policy under lobbying, as in Dixit et al. (1997). In contrast to their work though, tax instruments are missing in my model (i.e. individual specific lump-sum taxes/subsidies are not available). Therefore, in the absence of lobbying, missing instruments imply an allocation that is not first best. Then, because of the efficiency of quasi truthful equilibria, the introduction of lobbying increases the efficiency of the resulting allocation in the economy. This result provides an explanation for the existence of lobbying. In other words, lack of tax instruments explains the emergence and persistence of lobbying. This argument reverses the standard intuition on the subject.

The rest of the paper is organized as follows. Section 2 discusses theory, section 3 considers the application, while section 4 concludes.

2. General theory

2.1 Basic Setting

This section discuses a variation of Dixit et al (1997) model, appropriately adopted to allow for contribution rejections and lack of monotonicity in the agent's objective. In particular, consider *n* principals and one agent.

Principals' utility, is given by a function $u_i : A \times R_+ \to R$ such that $u_i = u_i(a, c_i - r_i)$. Here, $a \in A$ is a policy vector chosen by the agent, $c_i \in R_+$ is a contribution paid to the agent by the principal *i* and $r_i \in [0, c_i]$ is a refund of contribution money. The non-negative difference $c_i - r_i$ will be called net contribution of *i*. Principals' utilities are differentiable and strictly decreasing, with respect to net contributions.

The objective of the agent is given by a function $G : A \times R_+^n \to R$ such that $G = G(a, c - r)$. Where $c = (c_1, c_2, \ldots c_n)$ and $r = (r_1, r_2, \ldots r_n)$. The agent's objective is continuous in net contributions, but unlike Dixit et al (1997) is not necessarily increasing in them.

Furthermore, for every policy choice by the agent, there exists a maximum contribution by each principal, $\overline{c_i(a)} < +\infty$. No restrictions are imposed on set A.

We say that a contribution $c_i \in R_+$ is feasible relative to a policy choice $a \in A$, if $0 \le c_i \le \overline{c_i(a)}$[3]. A refund $r_i \in R$ is called feasible, given a feasible contribution c_i, if $r_i \in [0, c_i]$. Finally, the vectors $c = (c_1, c_2, \ldots c_n)$ and $r = (r_1, r_2, \ldots r_n)$ are called feasible relative to a policy choice $a \in A$, if r_i and c_i respectively are feasible $\forall i \in \{1,2,\ldots,n\}$.

Principals and agent are part of a common agency game. Each principal uses contributions to affect the agent's policy choice in his favor. The agent on the other hand takes into account the contributions offered and decides on policy. Moreover, the agent can reject (refund) part or the whole contribution, if he finds it optimal to do so.

The sequence of play is as follows. First, the principals offer to the agent a feasible contribution schedule. Then, the agent decides on policy vector and refunds, in order to maximize his objective.

A feasible contribution schedule (function) for principal *i*, is a function $c_i : A \to R$, such that[4] $c_i(a) \in \left[0, \overline{c_i(a)}\right], \forall a \in A$.

We say that the triplet $(c(\cdot), a, r)$ is feasible, if $c(\cdot)$ is a vector of feasible contribution schedules, $a \in A$ and r is feasible for $c(a)$.

Now we can proceed to the definition of equilibrium.

Definition 1: **equilibrium**

A feasible triplet $(c^o(\cdot), a^o, r^o)$ is an equilibrium if:

a) $(a^o, r^o) \in \underset{\substack{a \in A \\ r\ feas.}}{\arg\max} \{G(a, c^o(a) - r)\}$

b) There does not exist a feasible triplet $(c^*(\cdot), a^*, r^*)$ and a principal i, such that $u_i(a^*, c_i^*(a^*) - r_i^*) > u_i(a^o, c_i^o(a^o) - r_i^o)$ and
$(a^*, r^*) \in \underset{\substack{a \in A \\ r\ feas.}}{\arg\max} \{G(a, c_i^*(a) - r_i, c_{-i}^o(a) - r_{-i})\}$,

where c_{-i}, r_{-i} stand for the vectors of contributions and refunds of all principals except principal i and the abbreviation *"r feas."* stands for r feasible for the respective vector of contributions[5].

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[3] $\overline{c_i(a)}$ is the maximum value a contribution can take, given policy choice a. It reflects the fact that contributions can't exceed disposable resources. Notice that the maximum contribution depends on policy choice, since the policy choice might affect disposable resources. i.e. a can be an income tax.

[4] The symbol $c_i(\cdot)$ refers to contribution functions/ schedules, c_i refers to contributions as real numbers, $c_i(a)$ refers to the value of $c_i(\cdot)$ at $a \in A$.

This is the standard definition of equilibrium in common agency, appropriately adopted to incorporate refunds. The maximization of the agent's objective reflects the fact that the agent decides both on policy a and refunds *r*. We can think of the agent as deciding first on policy and then on the amount of refunds. This is intuitive, since the refunds should not exceed contributions ($r_i \in [0, c_i(a)], \forall i \in \{1,2,\ldots,n\}$)[6]. The second maximization, with respect to refunds, is an addition to the standard equilibrium definition. It will allow us to generalize the properties commonly associated with truthfulness, in the current context.

Now we can introduce a suitable notion of truthfulness.

Definition 2: **Quasi-truthful schedules**

A payment function $c_i^T(\cdot; u_i^*)$, is a quasi truthful contribution schedule relative to the constant u_i^*, if $c_i^T(a; u_i^*) = \min\{\overline{c_i(a)}, \max[0, \phi_i(\alpha, u_i^*)]\}$ for all $a \in A$,

where $\phi_i(a, u_i^*)$ is implicitly defined as the solution of $u_i^* = u_i(a, \phi_i)$ with respect to ϕ_i.

This is the standard definition of truthful contribution schedules as it appears in the literature. Yet in the case considered here, unqualified use of the term truthful is not appropriate. Truthful payment schedules are named like this, because they exactly reflect, the principals' true preferences over the agent's possible actions. In the model considered here, the agent's actions include both policy choice and refunds. Thus, in order to be true to its name, a truthful schedule would have to depend on both policy choice and refunds. Since the contribution schedule described in the definition above is a function of policy choice alone, I have decided to use the term quasi truthful instead of plain truthful[7].

Now we can proceed to define quasi truthful equilibria.

Definition 3: **Quasi-truthful equilibrium**

A quasi truthful equilibrium is an equilibrium in which each equilibrium payment function is quasi truthful, relative to the equilibrium utility of the respective principal.

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[5] $\max\limits_{\substack{a \in A \\ r\ feas.}} \{G(a, c(a) - r\}$ means that r is restricted to be feasible for $c(a)$. Or $r_i \in [0, c_i(a)]\ \forall i$

[6] One could use the equivalent, but more complex notation: $\max\limits_{a \in A} \{\max\limits_{r\ feas.} \{G(a, c(a) - r)\}\}$

[7]Assuming that contribution schedules depend on refunds would be hard to justify, since casual observation of lobbying practices, is inconsistent with anything of the sort. Furthermore, proposition 3 in next section establishes that refunds need not appear in equilibrium. In this sense, contributions that depend on refunds would constitute a paradox.

This definition can be restated as follows:

Let $(c^o(\cdot),a^o,r^o)$ be an equilibrium and $u^o \equiv \{u_i^o\}_{i=1,2..L}$ be the vector of the respective equilibrium utility levels. Then, the equilibrium is quasi truthful if $c_i^o(a)=c_i^T(a;u_i^o)$ for all i and $a \in A$.

In the next subsection we will discuss the main results.

2.2 Main results

The point of this subsection is to show that quasi truthful equilibria constitute a suitable solution concept, for the generalized common agency framework discussed in this paper.

Proposition 1

The best response set of principal *i* to the contribution schedules $c_{-i}^o(\cdot)$ of the other principals, always contains a quasi truthful contribution schedule[8].

Proof:

See appendix.

Proposition 1 states that there is always a quasi truthful schedule in the best response set of principal *i*. In this sense, the principals have nothing to lose when using a quasi truthful strategy.

Proposition 2

Assume $(c^o(\cdot),a^o,r^o)$ is a quasi truthful equilibrium, then it is efficient. In other words, there does not exist a feasible pair (a^*,c^*) such that $u_i(a^*,c_i^*)\geq u_i(a^o,c_i^o(a^o)-r_i^o)\ \forall i$ and $G(\alpha^*,c^*)\geq G(a^o,c^o(a^o)-r^o)$ with at least one strict inequality.

Proof:

[8] The definition of the best response set is standard and is implicitly stated in the definition of equilibrium: A feasible contribution function $c_i(\cdot)$ belongs in the best response set of principal i to the feasible contribution functions of the other players $c_{-i}(\cdot)$, if there exists a feasible pair $(a',r')\in \arg\max_{\substack{a\in A\\ r\ feas.}}\{G(a,c_i(a)-r_i,c_{-i}(a)-r_{-i})\}$, such that for all feasible triplets $(c_i^*(\cdot),a^*,r^*)$ with $(a^*,r^*)\in \arg\max_{\substack{a\in A\\ r\ feas.}}\{G(a,c_i^*(a)-r_i,c_{-i}(a)-r_{-i})\}$,
$u_i(a',c_i(a')-r_i')\geq u_i(a^*,c_i^*(a^*)-r_i^*)$.

See appendix.

Efficiency of truthful equilibria is the most celebrated result in the relevant literature. It has been used to indicate the suitability of this equilibrium concept, as well as to derive results in economic applications.

Propositions 1, 2 above, generalize the main arguments offered in the literature, to support truthful equilibria[9]. Their purpose here is to establish that quasi truthful equilibria do not fall short of their counterparts in the standard model.

The intuition behind propositions 1 and 2 is straightforward. The fact, that the agent can reject a contribution, restores a form of monotonicity of his objective[10]. This point bridges the differences between this paper and the standard model. Therefore, the reasoning employed in the proof of the respective results in Dixit et al.(1997) can be used here as well.

Finally, proposition 3 below, indicates that if a quasi truthful equilibrium exists, then an equilibrium with zero refunds and an identical allocation of welfare also exists. Thus, refunds need not be observed in equilibrium.

Proposition 3

Assume $(c^o(\cdot), a^o, r^o)$ is an equilibrium of the game. Then, $(c^{\#}(\cdot), a^o, \underline{0})$ is also an equilibrium. Where $c^{\#}(a) = \begin{cases} c^o(a^o) - r^o & if\ a = a^o \\ c^o(a) & if\ a \neq a^o \end{cases}$

and the term $\underline{0}$ symbolizes a vector of zeros.

Proof:

See appendix.

Results 1-3 provide support for quasi truthful equilibria. The discussion that follows, places these results in the context of the relevant literature.

2.3 Discussion of general theory

The setting discussed so far is characterized by two differences in relation to the existing literature (especially Dixit et. al. (1997)). The first difference is that the agent can reject a part

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9 See propositions 2 and 4 in Dixit et al. (1997) and theorems 1 and 2 in Bernheim and Whinston (1986).

10 Consider a policy choice ***α*** and a vector of contributions ***c*** feasible for ***α***. Define $\equiv \max_{r\ feas.}\{G(a, c-r)\}$. Also, consider a vector of contributions c' that is also feasible for ***α***. Then one can easily show two things: i) if $c' \geq c$ then $G^*(a,c') \geq G^*(a,c')$ and ii) if $G^*(a,c') > G^*(a,c')$ then there exists at least one *i* such that $c_i' > c_i$. See also, the two fundamental results in the beginning of the appendix.

or the whole contribution offered by a principal. This assumption is consistent with intuition, since few would doubt that a policy maker can in principal reject contributions. Yet this assumption would be obsolete in the standard model, where the agent's objective is increasing in contributions. In such a case, the agent wouldn't care to reject any contributions. On the other hand, in the extension considered here, the agent's objective might be decreasing in contributions. Thus, the agent will be inclined to actually reject contribution money that is harmful to his welfare. Nevertheless, although politicians can in principal reject contributions, they aren't known for actually refunding contribution money. This issue is addressed by proposition 3. It states that refunds do not need to take place in equilibrium. In anticipation of a refund, the lobbies reduce their contributions accordingly. Thus, actual rejections need not occur. Just the possibility for contribution rejections is sufficient to generate efficient equilibria. This approach is consistent with the argument made by Feli and Merlo (2006) that certain lobbies remain inactive, because the policy makers do not wish to deal with them.

The second difference between this paper and the existing literature is that the agent's objective is not necessarily increasing in contributions. The blueprint for this assumption is an agent's objective that depends on the utility of the principals. In particular, assume that principals' utility, is given by $u_i = u_i(a, c_i)$, while the agent's objective by $G(a,c) = g(\sum_i c_i, u_1(a,c_1),.....,u_n(a,c_n))$. Then, Inada conditions for g and u_i imply that the objective of the agent is not monotonous[11]. As contributions from a principal increase, the value of the objective initially increases, but eventually it will stop doing so and start decreasing. This implies that standard common agency results can't be applied.

In order to avoid the issue, one can choose to consider only increasing objectives. In the general case, where income effects are preserved, this leads to limited results. This is because it is very hard to combine income effects with reasonable objectives that are increasing in contributions. To see this notice that even if the function g above is linear in its arguments, Inada conditions for u_i imply that G is not monotonous in general.

A less general approach is to combine a linear objective (i.e. $G = \sum_{i=1}^{n} (c_i + b_i u_i)$) with a quasi linear utility (u_i) for the principal (i.e. $u_i = x_i + f(a)$, where x_i is income net of contributions). Clearly, in such a case if $b_i < 1$ then G is increasing in contributions[12].

The problem with quasi-linearity is that it does not allow for income effects and thus "gives incomplete or implausible answers to distributional questions"[13].

[11] Depending on the context, Inada conditions just for principals' utilities might suffice.

[12] This is the case with Grossman and Helpman (1994) and a significant number of papers that follow them.

[13] Dixit et al (1997).

Except for an agent that cares for lobbies, non increasing objectives can come up in other cases as well. Examples are: the need to keep contributions secret (along with an increasing marginal cost of hiding money), agent's preferences over lobbies (the agent might not wish to receive money from lobbies he dislikes) e.t.c.

Thus, the main contribution of this paper is that it extends the range of applications of common agency and especially furthers the study of models that combine lobbying with income effects.

In accordance with the literature, the next section will indicate how the results derived can be used in lobbying applications.

3. Application

3.1 Model

Let us consider a typical, public good provision model.

Assume *n* principals (individuals) with utility functions $u_i = u_i(x_i, T)$. Here, T is a public good provided by the government and x_i is private good consumption by individual *i*. Function u_i is increasing, diferentiable and strictly quasi concave with respect to both its elements. The individuals' budget constraint is given by $c_i + t + x_i = e_i$. Where e_i is an exogenous endowment , t is a flat lump sum tax used to finance the public good and c_i is the net contribution paid to the agent to affect the choice of tax.

The agent's (government's) objective is $G = G(C, u_1, \ldots, u_n)$, where C is total net contributions received and G is continuous, increasing and quasi concave in all its elements. Total net contributions and public good satisfy: $C = \sum_{i=1}^{n} c_i$, $T = nt$ (balanced budget). The government decides on the tax level *t* and therefore, because of balanced budget on public good provision *T*. The symbols c_i, C are used to denote contributions net of refunds, in order to avoid excess use of notation[14].

Flat tax and non negative contributions, impose the following "institutional" restrictions:

1. $t \in A \equiv \lfloor o, e_j \rfloor$, where $e_j = \min\{e_1, \ldots, e_n\}$

2. $c_i \in C_i^t \equiv [o, e_i - t]$, $\forall i$

The above constitute an application of the general framework studied in this paper.

[14] One can notice that if we drop the adjective "net", associated with contributions, we can think of the model in the standard context provided by Dixit et al. (1997). This highlights the main point of this paper which is that we can apply standard results regardless of whether the agent's objective is increasing.

In particular, by substituting the private and public budget constraints into the utility functions we can express G, u_i as functions of c_i, t.

From now on, I will refer to the application as the game.

Assume that the game has an interior quasi truthful equilibrium. The term interior, means that the equilibrium tax t^o and all equilibrium net contributions c_i^o, belong in the interior of sets A and C_i^t respectively.

Then, the public and private budget constraints imply that equilibrium, public good provision and private consumption satisfy:

$T^o = nt^o > 0$ and $x_i^o = e_i - c_i^o - t^o \in (o, e_i), \forall i$ respectively.

Define $C^o = \sum_{i=1}^{n} c_i^o$.

Now, consider the equilibrium utility and objective levels:

$u_i^o = u_i(x_i^o, G^o)$ and $G^o = G(C^o, u_1(x_1^o, T^o), \ldots, u_n(x_n^o, T^o))$.

We can prove the following proposition:

Proposition 4

The equilibrium allocation $(C^o, \{x_i^o\}_{i=1,2,\ldots,n}, T^0)$ is first best.

In other words, there does not exist a non negative allocation $(C', \{x_i'\}_{i=1,2,\ldots,n}, T')$ with $u_i' = u_i(x_i', T')$, $G' = G(C', u_1(x_1', T'), \ldots, u_n(x_n', T'))$ and $\sum_{i=1}^{n} x_i' + T' + C' = \sum_{i=1}^{n} e_i$ such that $u_i' \geq u_i^o \; \forall i$ and $G' \geq G^o$ with at least one strict inequality.

Proof

See appendix.

3.2 Discussion of the application

Proposition 4 is not a direct application of the efficiency result in proposition 2. Proposition 2 is a constrained efficiency result. It states that any quasi truthful equilibrium achieves an allocation that cannot be Pareto improved by another institutionally feasible allocation. In the case above, this would mean, by any allocation that allows only for transfers from citizens to government and uses a flat lump sum tax to finance public good provision.

On the other hand, proposition 4 is an unconstrained efficiency result. It states that an interior quasi truthful equilibrium cannot be Pareto improved by any reallocation of total resources, whether institutionally feasible or not[15]. In this sense, proposition 4 makes a strong case for efficient lobbying.

Clearly, not all quasi truthful equilibria are interior, typically because some net contributions might be zero in equilibrium. In such a case, proposition 4 fails. Nevertheless, we can still support an efficient lobbying argument, through proposition 2, which holds for all quasi truthful equilibria, not just the interior ones. To see this, consider two changes in the game. Disallow lobbying and allow direct transfers from individuals to government through the tax (i.e. in the form of salary). In such a case, the only available instrument, to reallocate resources, is the flat lump sum tax. This tax enables transfers from citizens to government and finances public good provision. Then, proposition 2 states that introducing lobbying, in such an economy, leads to a weakly Pareto superior allocation of welfare. This is not strange, since contributions effectively increase the number of instruments that are available, for the reallocation of resources.

This argument along with proposition 4, have a straightforward implication. In particular, introducing lobbying in an economy that misses tax instruments, improves allocative efficiency. This argument can be thought as an explanation for the emergence and persistence of lobbying. Especially so, since missing tax instruments, characterize all actual economies[16].

Advocating that missing tax instruments cause the emergence of lobbying is the reverse of what Dixit et al (1997) argue, when discussing their application. Specifically, they claim that it is lobbying that causes economies to abstain from using lump sum taxes. Having a closer look at their argument can be of help. Dixit et al (1997), in their application, which is a variation of Diamond and Mirlees (1971), derive two fundamental results. First, they show that when given the choice, the government will not use distortive taxation. Second, they find, that lobbying groups or individuals do worse when efficient policy instruments are used. Thus, Dixit et al (1997) argue that if individuals can commit not to lobby for efficient policy instruments they will do so. Therefore, efficient fiscal instruments, (e.g. lump sum taxes) will not occur[17].

In summary, Dixit et al (1997), explain the observed lack of lump sum taxes with the help of lobbying. On the other hand, the application in 3.1 above, explains the emergence of lobbying via missing policy instruments. Discussing which of the two explanations is better, is far from

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[15] The only restriction is the economy's budget constraint $\sum_{i=1}^{n} x_i' + G' + C' = \sum_{i=1}^{n} e_i$

[16] Extensive discussion of this argument, along with explanation of differential incidence, can be found in Boultzis (2009). The analysis in that paper is based on the assumption that the agent can only reject a contribution as a whole.

[17] Many other explanations for the unwillingness of policy makers to use lump sum taxes are offered in the literature. Examples are: Hammond (1979), Coate & Morris (1995), Dixit and Londregan (1995).

the scope of this paper. Rather, I would like to point out that, by moving away from standard assumptions, this paper provided a new insight into lobbying. In this sense, the approach considered here, can increase our understanding of political influence.

4. Conclusion

This paper extends the standard common agency model in two directions. The first is allowing the rejection of agent's contributions and the second is considering non monotonous objectives for the agents. In this broader context, an appropriate variation of truthful equilibria is introduced. These equilibria preserve the key properties of truthful equilibria, including efficiency.

The new context is particularly useful when considering lobbying for government policy. In particular, it extends the application of common agency to models with non increasing objectives for the agents. One can think of many reasons for such an extension, but I think the most important is the need to consider income effects in the broadest possible context. So far common agency has been used by many economists to analyze political influence, in all fields of government policy. Yet this literature, in most cases, considers quasi linear objectives and utility functions and thus, fails to incorporate income effects. On the other hand, combining income effects with increasing objectives for the agents has limited applications. This is so, because even under very common assumptions (e.g. Inada conditions for principals' utility), it is very hard to achieve monotonicity for the objectives. Thus, the framework discussed in this paper, points to a new direction, by significantly widening the range of lobbying applications that allow the incorporation of income effects. Since in many government policy problems distribution is often the key issue, the ability to adequately model income effects can help us better understand lobbying and its effects.

An example of an application is given in section 3. There, I discuss a simple public good provision model with lobbying and derive a new insight, on the relation between fiscal policy and lobbying.

Appendix

Fundamental Results

Two fundamental results that "restore" monotonicity will be proven here. These results will be used in the proofs that follow.

Result 1

If $c_i \geq c_i'$ then

$$\max_{r\ feas.} G(a, c_i - r_i, c_{-i} - r_{-i}) \geq \max_{r\ feas.} G(a, c_i' - r_i, c_{-i} - r_{-i}) \geq \max_{r_{-i}\ feas.} G(a, c_i', c_{-i} - r_{-i})$$

Proof: Both inequalities hold, because if they didn't, the solution in the maximization problem in the RHS could also be achieved in the solution of the LHS maximization problem.

Result 2

If $\max_{r\ feas.} G(a, c - r) > \max_{r\ feas.} G(a, c' - r)$, then there exists an *i* such that $c_i > c_i'$.

Proof: If not the result of the maximization in the RHS would be feasible in the LHS.

Similarly, we can prove that:

If $G(a, c) > \max_{r\ feas.} G(a, c' - r)$, then there exists an *i* such that $c_i > c_i'$

Proof of proposition 1

Consider principal *i* and assume that $c_{-i}^o(\cdot)$ are the contribution schedules of the other principals. Also, assume that $c_i^o(\cdot)$ is a best response of principal *i* to $c_{-i}^o(\cdot)$ and $(a^o, r^o) \in \arg\max_{\substack{a \in A \\ r\ feas.}} \{G(a, c^o(a) - r)\}$ is the choice of the agent given $c^o(\cdot)$.

We will prove that the quasi truthful contribution schedule $c_i^T(\cdot)$ with respect to $\overline{u_i} \equiv u_i(a^o, c_i^o(a^o) - r_i^o)$ is also a best response to $c_{-i}^o(\cdot)$. In order to do that, we must prove that there exists $(a', r') \in \arg\max_{\substack{a \in A \\ r\ feas.}} \{G(a, c_i^T(a) - r_i, c_{-i}^o(a) - r_{-i})\}$ such that $u_i(a', c_i^T(a') - r') \geq \overline{u_i}$. Then, since $c_i^o(\cdot)$ is a best response, the last expression will hold as an equality.

Notice that $c_i^T(\cdot)$ is defined as the solution of $\overline{u_i} = u_i(a, c_i)$ with respect to c_i, provided this solution belongs in $\left[0, \overline{c_i(a)}\right]$.

Define as (a^T, r^T) the choice of the agent, given $(c_i^T(\cdot), c_{-i}^o(\cdot))$

(i.e. $(a^T, r^T) \in \arg\max_{\substack{a\in A \\ r\ feas.}} \{G(a, c_i^T(a) - r_i, c_{-i}^o(a) - r_{-i})\}$).

First, assume that for some r' feasible, $(a^o, r') \in \arg\max_{\substack{a\in A \\ r\ feas.}} \{G(a, c_i^T(a) - r_i, c_{-i}^o(a) - r_{-i})\}$.

Then, without loss of generality, we can set $a^T = a^o$. In this case, $c_i^T(a^T) = c_i^o(a^o) - r_i^o$. Furthermore, $r_i^T = 0$, since otherwise (a^o, r^o) wouldn't be the choice of the agent in the first place. Therefore, $u_i(a^T, c_i^T(a^T) - r_i^T) = u_i(a^T, c_i^T(a^T)) = u_i(a^o, c_i^o(a^o) - r_i^o) = \overline{u_i}$

Thus, in this case, $c_i^T(\cdot)$ is trivially a best response.

Second, consider the case where a^o is not chosen by the agent. (i.e. for all r' feasible, $(a^o, r') \notin \arg\max_{\substack{a\in A \\ r\ feas.}} \{G(a, c_i^T(a) - r_i, c_{-i}^o(a) - r_{-i})\}$).

In this case, we can show that $c_i^T(a^T) > c_i^o(a^T)$. To see this assume the contrary: $c_i^o(a^T) \geq c_i^T(a^T)$.

Then, the following chain of inequalities hold:

$$\max_{r\ feas.} \{G(a^T, c_i^o(a^T) - r_i, c_{-i}^o(a^T) - r_{-i})\} \geq \max_{r\ feas.} \{G(a^T, c_i^T(a^T) - r_i, c_{-i}^o(a^T) - r_{-i})\} >$$
$$> \max_{r\ feas.} \{G(a^o, c_i^T(a^o) - r_i, c_{-i}^o(a^o) - r_{-i})\} = G(a^o, c^o(a^o) - r^o)$$

Which is a contradiction, because a^T is not the policy the agent chooses given $c^o(\cdot)$.

The first inequality above is due to result 1 in the beginning of the appendix and $c_i^o(a^T) \geq c_i^T(a^T)$. The second holds because a^T is the choice of the agent given $(c_i^T(\cdot), c_{-i}^o(\cdot))$, while a^o is not. Finally, the equality holds, because $c_i^T(a^o) = c_i^o(a^o) - r_i^o$.

Thus, $c_i^T(a^T) > c_i^o(a^T) \geq 0$ or $c_i^T(a^T) > 0$. This last inequality implies that $u_i(a^T, c_i^T(a^T) - r_i^T) \geq u_i(a^T, c_i^T(a^T)) \geq \overline{u_i}$, which proves the proposition.

The last part in the chain of inequalities above is due to the definition of quasi truthful contribution schedules. Remember that $u_i(a, c_i^T(a)) < \overline{u_i}$ is possible only if $c_i^T(a) = 0$.

Proof of proposition 2

Assume the second inequality in the proposition is strict. Then, $G(a^*, c^*) > G(a^o, c^o(a^o) - r^o) \geq \max_{r\ feas.} \{G(a^*, c^o(a^*) - r\}$

The last inequality is due to the fact that (a^o, r^o) is part of an equilibrium and thus, maximizes $G(a, c^o(a) - r)$.

Since $G(a^*, c^*) > \max_{r\, feas.}\{G(a^*, c^o(a^*) - r)\}$, then by result 2 in the beginning of the appendix, there exists an *i* such that $c_i^* > c_i^o(a^*)$.

On the other hand, $u_i(a^*, c_i^*) \geq u_i(a^o, c_i^o(a^o) - r_i^o) \geq u_i(a^o, c_i^o(a^o)) = u_i(a^*, \phi_i(a^*; u_i))\ \forall\, i$. Thus, $c_i^* \leq \phi_i(a^*; u_i)$. Furthermore, since c^* is feasible given $a^*: 0 \leq c_i^* \leq \overline{c_i(a^*)}\ \forall i$. Therefore, $c_i^* \leq \max\left\{0, \min\left\{\phi_i(a^*; u_i), \overline{c_i(a^*)}\right\}\right\} = c_i^o(a^*)\ \forall i,$ which is a contradiction since there is an *i* for which $c_i^* > c_i^o(a^*)$.

Now, consider the case $G(a^*, c^*) = G(a^o, c^o(a^o) - r^o) \geq \max_{r\, feas.}\{G(a^*, c^o(a^*) - r)\}$. If the last inequality is strict, we can repeat the proof presented above. Thus, we only need to consider the case $G(a^*, c^*) = G(a^o, c^o(a^o) - r^o) = \max_{r\, feas.}\{G(a^*, c^o(a^*) - r)\}$.

In this case, by assumption, there exists a *j* such that: $u_j(a^*, c_j^*) > u_i(a^o, c_j^o(a^o) - r_j^o)$.

Also, following the argument presented above, we can show that $c_i^o(a^*) \geq c_i^*$ for all *i*.

Then, by applying result 1 repeatedly we get:

$$\max_{r\, feas.}\{G(a^*, c^o(a^*) - r\} \geq \max_{r_{-j}\, feas.}\{G(a^*, c_j^*, c_{-j}^o(a^*) - r_{-j})\} \geq G(a^*, c^*)$$

Using $G(a^*, c^*) = \max_{r\, feas.}\{G(a^*, c^o(a^*) - r)\} = G(a^o, c^o(a^o) - r^o)$ and the inequalities above:

$$\max_{r\, feas.} G(a^*, c_j^* - r_j, c_{-j}^o(a^*) - r_{-j}) \geq \max_{r_{-j}\, feas.}\{G(a^*, c_j^*, c_{-j}^o(a^*) - r_{-j})\} = G(a^o, c^o(a^o) - r^o) \geq$$
$$\geq G(a, c^o(a) - r)\ \forall a \in A \text{ and } r \text{ feasible for } a.$$

Now, we will show that for principal *j*, $c_j^o(\cdot)$ is not a best response to $c_{-j}^o(\cdot)$. In particular, consider:

$$\hat{c}_j(a) = \begin{cases} c_j^o(a)\ if\ a \neq a^* \\ c_j^*\ \ if\ a = a^* \end{cases}$$

Assume r' satisfies: $(a^*, r') \in \arg\max_{\substack{a \in A \\ r\, feas.}}\{G(a, \hat{c}_j(a) - r_j, c_{-i}^o(a) - r_{-j})\}$

Then, because $u_j(a^*,c_j^*-r_j') \geq u_j(a^*,c_j^*) > u_i(a^o,c_j^o(a^o)-r_j^o)$, it follows that $c_j^o(\cdot)$ is not a best response to $c_{-j}^o(\cdot)$, since principal j can do better by playing $\hat{c}_j(\cdot)$. This is a contradiction, since $c^o(\cdot)$ is part of an equilibrium.

Proof of proposition 3

It is straightforward that $c^{\#}(\cdot)$ is feasible and that a^o and the refunds $\underline{0}$ maximize the agent's objective.

To conclude the proof, we need to show that there does not exist a feasible triplet $(c_i'(\cdot),a',r')$ such that: $u_i(a',c_i'(a')-r_i') > u_i(a^o,c_i^o(a^o)-r_i^o)$ for some i and $(a',r') \in \arg\max\limits_{\substack{a\in A \\ r\ feas.}} \left\{G(a,c_i'(a)-r_i,c_{-i}{}^{\#}(a)-r_{-i})\right\}$.

Assume such a triplet exists.

Define: $A^{oi} = \left\{ a \in A : (a,r) \in \arg\max\limits_{\substack{a\in A \\ r\ feas.}} \left\{G(a,c_i'(a)-r_i,c_{-i}{}^{o}(a)-r_{-i})\right\} \right\}$

Notice that since $(c^o(\cdot),a^o,r^o)$ is an equilibrium, $a' \notin A^{oi}$

First, I will show that $a^o \in A^{oi}$. Using the definition of $c^{\#}(\cdot)$ and (a',r'):

$$G(a',c_i'(a')-r_i',c_{-i}{}^{\#}(a')-r_{-i}') \geq G(a,c_i'(a)-r_i,c_{-i}{}^{o}(a)-r_{-i}) \ \forall a \neq a^o \text{ and } r \text{ feasible}.$$

Then either $a' \in A^{oi}$ which is impossible, or

$$\max_{r\ feas.} G(a^o,c_i'(a^o)-r_i,c_{-i}{}^{o}(a^o)-r_{-i}) \geq G(a',c_i'(a')-r_i',c_{-i}{}^{\#}(a')-r_{-i}')$$

which proves that $a^o \in A^{oi}$.

Define r'^o implicitly in the following way:

$$\max_{\substack{a\in A \\ r\ feas.}} \left\{G(a,c_i'(a)-r_i,c_{-i}{}^{o}(a)-r_{-i})\right\} = G(a^o,c_i'(a^o)-r'^o{}_i,c_{-i}{}^{o}(a^o)-r_{-i}'^o)$$

Now, we can proceed with the rest of the proof.

Since $(c^o(\cdot),a^o,r^o)$ is an equilibrium and $a^o \in A^{oi}$ then:

$$u_i(a^o,c_i'(a^o)-r_i'^o) \leq u_i(a^o,c_i^o(a^o)-r_i^o) \Rightarrow c_i'(a^o)-r_i'^o \geq c_i^o(a^o)-r_i^o$$

Then we can generate the following chain of inequalities:

$$\max_{r\, feas.} G(a',c_i'(a')-r_i'-r_i,c_{-i}{}^{o}(a')-r_{-i})\geq$$
$$\geq G(a',c_i'(a')-r_i',c_{-i}{}^{o}(a')-r_{-i}')\geq G(a',c_i'(a')-r_i',c_{-i}{}^{\#}(a')-r_{-i}')=$$
$$=\max_{r\, feas.} G(a',c_i'(a')-r_i,c_{-i}{}^{\#}(a')-r_{-i})\geq \max_{r\, feas.} G(a^o,c_i'(a^o)-r_i,c_{-i}{}^{\#}(a^o)-r_{-i})\geq$$
$$\geq \max_{r_{-i} feas.} G(a^o,c_i'(a^o)-r'^o{}_i,c_{-i}{}^{\#}(a^o)-r_{-i})=\max_{r_{-i} feas.} G(a^o,c_i'(a^o)-r'^o{}_i,c_{-i}{}^{o}(a^o)-r_{-i}^o-r_{-i})\geq$$
$$\geq \max_{r_{-i} feas.} G(a^o,c_i^o(a^o)-r_i^o,c_{-i}{}^{o}(a^o)-r_{-i}^o-r_{-i})=G(a^o,c^o(a^o)-r^o)\geq$$
$$\geq G(a,c^o(a)-r)\forall a\in A \text{ and } r \text{ feasible for } a.$$

The first inequality is trivial. The second inequality holds because $c^{\#}(a)\leq c^o(a)\ \forall a\in A$ and result 1(applying it repeatedly). The first equality and the third inequality hold because of the definition of $(c_i'(\cdot),a',r')$. The fourth inequality holds because the outcome in the RHS is feasible in the maximization problem on the LHS. The second equality is because of the definition of $c^{\#}(\cdot)$, while the fifth inequality is because $c_i'(a^o)-r_i'^o\geq c_i^o(a^o)-r_i^o$ and result 1. The third equality is due to the definition of r_{-i}^o. Finally, the last inequality is due to the fact that $(c^o(\cdot),a^o,r^o)$ is an equilibrium.

Then, we arrive at a contradiction since $c_i^o(\cdot)$ is not a best response to $c_{-i}^o(\cdot)$.

Consider for example:

$$c_i=\begin{cases} c_i'(a')-r_i' \text{ if } a=a' \\ c_i^o(a) \text{ if } a\neq a' \end{cases}$$

Proof of proposition 4

Assume that the equilibrium allocation is not Pareto optimal. Then, there is another allocation $(C',\{x_i'\}_{i=1,2,\ldots,n},T')$ with $u_i'=u_i(x_i',T')$, $G'=G(C',u_1(x_1',T'),\ldots,u_n(x_n',T'))$, $\sum_{i=1}^{n}x_i'+T'+C'=\sum_{i=1}^{n}e_i$, $T'\geq 0$, $x_i'\geq 0\ \forall i$ and $C'\geq 0$ such that $u_i'\geq u_i\ \forall i$ and $G'\geq G^o$ with at least one strict inequality.

Because the functions u_i are quasi concave with respect to (x_i,T) and G is quasi concave with respect to $(C,\{u_i\}_{i=1,2,\ldots,n})$, then G is also quasi concave with respect to $(C,\{x_i\}_{i=1,2,\ldots,n},T)$. Consider the equilibrium utility and objective levels $u_i^o=u_i(x_i^o,T^o)$ and $G^o=G(C^o,u_1(x_1^o,T^o),\ldots,u_n(x_n^o,T^o))$. Then (x_i^o,T^o), (x_i',T') belong in the upper contour set of (x_i^o,T^o), and $(C^o,\{x_i^o\}_{i=1,2,\ldots,n},T^o)$, $(C',\{x_i'\}_{i=1,2,\ldots,n},T')$ in the upper contour set of $(C^o,\{x_i^o\}_{i=1,2,\ldots,n},T^o)$. Therefore, due to quasi concavity, all convex combinations of the respective pairs also belong in the respective upper contour sets.

Furthermore, since $(C', \{x'_i\}_{i=1,2,\ldots,n}, T')$ constitutes a strict Pareto improvement over the equilibrium, so do all the convex combinations described above.

Because the equilibrium is interior the following hold:

$$T^o > 0, x_i^o > 0 \ \forall i \text{ and } c_i^o = e_i - \frac{T^o}{n} - x_i^o > 0$$

Define: $c'_i = e_i - \frac{T'}{n} - x'_i$. Notice that c'_i can be negative, since the Pareto superior allocation does not need to satisfy any institutional restrictions.

Nevertheless, since $c_i^o > 0$, for all *i*, there exists $\lambda_i \in (0,1)$ such that $\lambda_i c'_i + (1-\lambda_i)c'_i > 0$. Define as λ the smaller of these λ_i and $c_i^* = \lambda c'_i + (1-\lambda)c_i^o$. Now consider the allocation $(C^*, \{x_i^*\}_{i=1,2,\ldots,n}, T^*)$ defined as: $C^* = \lambda C' + (1-\lambda)C^o$, $x_i^* = \lambda x'_i + (1-\lambda)x_i^o$ and $T^* = \lambda T' + (1-\lambda)T^o$.

This allocation clearly satisfies all feasibility and institutional restrictions imposed on our model: $T^* \geq 0$, $c_i^* \geq 0 \ \forall i$, $\sum_{i=1}^{n} c_i^* = C^* \geq 0$ and $x_i^* = e_i - \frac{T^*}{n} - c_i^* \geq 0$.

In such a case, we have a contradiction to proposition 2. This is so because $(C^*, \{x_i^*\}_{i=1,2,\ldots,n}, T^*)$ Pareto improves the equilibrium allocation and moreover, there is a flat tax and a set of feasible contributions that implement the same allocation.

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