# Output Taxes on Duopoly with Brand-Name and Quality Differentiation: Designing policies to provide incentives for efficient investment in abatement

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**Abstract**

The present paper examines whether and the extent to which an environmental policy based on a tax per unit of output, can provide firms with adequate incentives to invest in abatement. We focus on output taxes since, despite their obvious disadvantages relative to emissions taxes, are widely used in reality. We examine two policy designs and we compare their effectiveness in inducing abatement investment and their social welfare consequences. According to the first policy option the policymaker moves first committing to a tax rate, while in the second it allows firms to move first and choose abatement investment, knowing that in the next stage the regulator will impose an output tax. In both cases, firms compete in prices in the last stage. We find that the second policy option yields higher levels of abatement, lower tax rates and higher production. This is because even though firms move first, the anticipation of the upcoming intervention induces them to choose higher abatement level. It is as if the policymaker moves at a stage prior to the first, announcing a tax schedule on which firms respond by choosing higher abatement investment. Although the effect of the second policy option on consumer surplus is always positive (especially when they are environmentally aware), the combination of high abatement and high production can result in higher or lower environmental damages depending on the specification of emissions as a function of abatement. We find that when emissions are a convex function of abatement, the second policy option yields lower environmental damages and higher social welfare when the cost of abatement is quite high and consumers are not very environmentally aware.

**JEL classifications:** D4, L1, Q55, Q58

# 1 Introduction

Despite the obvious efficiency advantages of a tax per unit of pollution, there are many instances in which environmental regulation is based on taxes per unit of output. While a tax per unit of pollution provides incentives to reduce both production and pollution per unit of output, an output tax produces only the former incentives. However, most of the actual environmental taxes target output instead of pollution. There are many reasons for that, such that emissions may be difficult or impossible to measure, abatement technology improves faster than policymakers can adjust to, or that a pollution per unit of output is hard to design and very complex to administer. Examples of output taxes include some of the most important environmental taxes, such as the gasoline tax and industrial effluent taxes which are calculated using an industry-wide rate of effluent per unit output.[1] An output tax coincides perfectly with a pollution tax only if pollution per unit of output is fixed. The literature has noted the difference and a number of papers compare the effects of emissions and output taxes in a variety of settings.[2]

The present paper analyzes a situation in which the regulator can use only an output tax to control pollution and we examine whether the timing of the policy intervention could have an effect on firms' choice of abatement per unit of output. In order to make the model more realistic we examine an oligopolistic output market in which firms compete in prices. Furthermore, we assume that reducing abatement per unit of output improves the quality of the product as perceived by consumers. This is consistent with consumers' increasing environmental awareness which leads to a variety of firms' voluntary environmental investments such as the implementation of environmental management systems and ecolabelling. Thus, in our setting, an investment in abatement reduces pollution per unit of output and in the same time induces an increase in demand. We consider two orders of moves. In the first game, the policymaker moves first, setting a particular tax per unit of output, taking into account its effect both on firms' choice of prices (short-run effect) and on firms' choice

[1] For an extensive discussion of environmental taxes per unit of output, including the presentation of a number of examples, see Fullerton et al. (2001).

[2] For example, Schmutzler and Goulder (1997) compare emissions and output taxes in a partial equilibrium framework and in the presence of imperfect monitoring of emissions, while Fullerton et al. (2001) in a general equilibrium framework. Goulder et al. (1997) examine the interactions with pre-existing distortionary taxes.

of emissions per unit of output (long-run effect). In the second game, the policymaker moves second, after firms have chosen their abatement per unit of output investment.

One might expect that a per unit of output tax will not affect firms' choice of abatement per unit of output and thus, the timing of the intervention will not affect abatement investment choices. Contrary to this layman's presumption we find that the effect of the two games on abatement technology differ considerably. Even more surprisingly we find that when the policymaker moves second induces firms to choose higher investment in abatement. Higher environmental quality increases demand, since consumers are environmentally aware, and induces a lower tax rate which, given strategic complementarity, reinforces the production of higher level of output. Thus, despite the fact that the policy maker does not have a first mover advantage in the second game, its tax choice yields higher abatement investment and output.

The intuition of these results might be better understood if we think that the first game corresponding to a policy option in which the regulator sets a specific tax rate which, since it is given at the second stage, does not affect firms' choice of abatement investment, while the second game corresponds to a policy option in which the policymaker offers a tax schedule. It is as if in the second game there is a stage prior to the first one, in which the policymaker offers a tax schedule, which is time consistent, given that the regulator actually chooses the tax in the second stage. The fact that in the second policy option firms when making their abatement investment decisions are aware of the fact that the regulator will impose a tax at the second stage, induces them to choose a higher investment in abatement in order to manipulate the tax rate. Thus, the first policy option corresponds to a typical per unit of output tax that does not affect abatement investment, while the second policy option indirectly offers incentives to reduce per unit of emissions.

Although the second policy option shares characteristics with an emission tax, we should not forget that it relies on a per unit of output tax and thus, it is not optimal in the sense of Pigouvian efficiency. Actually we find that, if the cost of abatement investment is low enough, the second policy option leads to overshooting in abatement investment. This in turn yields higher output, which although increases consumer surplus, it could increase pollution and environmental damages as well. This, combined with high cost of abatement could lead to lower social welfare relative to the first policy option. However, when investment in abatement technology is expensive, the second policy option is welfare superior assuming that total emissions are convex in abatement.

We also find that, ceteris paribus, the more environmentally aware consumers are the more likely is that the first policy option dominates. This is because consumers' environmental awareness provides firms with incentives to invest in abatement even in the first policy option in which the output tax fails to do so. In the second policy option though, leads to overinvestment in abatement and thus to higher abatement costs and environmental damages. Furthermore, the degree of competition in the product market plays some role as well. As expected, fiercer competition reduces welfare, due to reduced investment in abatement in both policy options. Overall, we show that the choice of the policy option has important consequences and thus, the timing of intervention should be considered seriously by the environmental authorities.

Whether and the extent to which environmental policy can provide incentives for investment in abatement is an important question both from a theoretical and a policy point of view. Although there is no conclusive evidence that environmental policy does indeed lead to innovation and the adoption of new abatement technologies (see for example OECD (2010)), an extensive theoretical literature has developed examining various aspects of this issue. An important part of the literature compares the effectiveness of different policy instruments, showing that economic instruments and especially environmental taxation perform better than command and control. Most of this literature, reviewed comprehensively by Requate (2005), assumes perfect competition and examines also uncertainty over environmental damage, abatement costs and technological progress. In the case of imperfect competition, Montero (2002) finds that tradable permits might not always provide stronger incentives for innovation. Innes and Bial (2002) examine environmental policies that combine taxes and standards in order to provide incentives for innovation in a Bertrand setting.

Another part of the literature examines the effect of the environmental policy's stringency on abatement investment. It has been shown that, when the output market is not taken into consideration, a stricter policy induces higher level of investment (see for example Milliman and Prince (1989) and Jung et al. (1996)). When the effect of environmental policy on output is taken into account, it has been shown that the effect of higher environmental taxes on abatement investment is ambiguous, since it could respond to higher taxes by either investing in abatement and/or reducing output (see Ulph (1997) and Katsoulacos and Xepapadeas (1996)). More recently, Dijkstra and Gil-Molto (2011) examine the effect of increasing environmental taxation on output's environmental quality (emissions intensity of output). They study both cases of ex ante and ex post commitment and they find that the rate of emissions per unit of output could be increasing after a certain point as the policy becomes stricter.

Closer to our work, Petrakis and Xepapadeas (1999) compare the same two policy options in a monopoly setting but assuming that the regulator uses a tax on emissions. Similar to our results, they find that in the second policy option, that is, when the monopolist moves first, it chooses a higher level of abatement investment inducing thus a relatively lower tax rate. However, they find that the second policy option is always welfare inferior to the first policy option. The difference in the results is due to the fact that Petrakis and Xepapadeas use an emissions tax and they do not consider environmentally aware consumers. It is clear that first mover advantage allows policymaker, when she is able to design and administer an emission tax, to derive the optimal Pigouvian tax (given the market distortion) and thus, the first policy option yields higher welfare. However, in the case of an output tax considered in the present paper, first mover advantage does not ensure optimal choice of abatement and thus, welfare maximization.

There is an extensive literature examining the effect of output rather than emission taxes. An important segment of the literature considers output taxes assuming that consumers show higher willingness to pay for products that exhibit higher environmental quality. Most of this literature is using a vertically differentiated product model such as Arora and Gangopadhyay (1995), Cremer and Thisse (1999), Constantatos and Sartzetakis (1999) and Moraga-Gonzalez and Padron-Fumero (2002). More recently Bansal and Gangopadhyay (2003) consider a model with output tax and examine the effect of these policies on abatement levels.

The remaining of the paper is organized as follows. Section 2 describes the model and the two policy options. Sub-Sections 2.1 to 2.3 solve for the first and second policy option respectively and Section 3 presents the comparison of the two policy options. Section 4 concludes the paper.

## 2 The model

We consider a product offered by two firms, each producing a distinct variety. Brand-name differentiation between the two product types is exogenous. Production and consumption of both product types generate pollution resulting in environmental damages $D$, which are assumed linear in pollution, i.e. $D = dE$, where $d > 0$. We further assume that emissions are increasing with output at a rate $v_i$, denoting both firms' abatement efforts and the environmental quality of the product's type as perceived by consumers. The rate of pollution per unit of output can be reduced if firms choose a higher investment in abatement leading to more environmentally-friendly quality. Thus, firm $i$'s level of pollution is $e_i(v_i, q_i)$, where $q_i$ is firm $i$'s output, $E = \sum_{i=1}^{2} e_i$ , $i = 1, 2$. We assume that emissions function is increasing at a constant rate in $q_i$ and decreasing and convex in $v_i$, that is, $\frac{\partial e_i}{\partial q_i} > 0$, $\frac{\partial^2 e_i}{(\partial q_i)^2} = 0$, $\frac{\partial e_i}{\partial v_i} < 0$, and $\frac{\partial^2 e_i}{(\partial v_i)^2} \leq 0$.

Higher environmental qualities may also yield consumption benefits, either because consumers care about the environment and/or because greener products are associated with superior quality attributes (e.g., reduced probability of health damages, increased nutrition content, etc.). The representative consumer's utility is $u = U(q_i, q_j) + m$, where $m$ is consumption of a numeraire good and,[3]

$$U = (a + \theta v_1)q_1 + (a + \theta v_2)q_2 - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) \ , \tag{1}$$

where $\gamma \in (0, 1)$ measures brand-name differentiation, $\theta \in [0, 1]$ is an index relating environmental quality to consumption and $a > 0$. The utility function in (1) exhibits a taste for brand name variety in that the representative consumer prefers to consume both firms' product. As $\gamma \to 0$ substitutability between brand-names disappears, and so does competition between the two firms; as $\gamma \to 1$ the two product types become almost homogeneous. When $\theta = 0$, consumers are indifferent about the environmental quality of a product they consume; in all other cases, we can call $v_i$,

[3] A similar type of utility function has been introduced by Dixit (1979) and used in many works such as Singh and Vives (1984). For a comprehensive and complete presentation, see Martin (2002). The particular utility specifiaction has been employed, among others, by Sartzetakis et al. (2012). There is a number of papers, see for example Bansal and Gangopadhyay (2001), taking into account the fact that environmentally aware consumers are willing to pay a higher price for higher environmental quality.

"the environmental quality of product $i$", While there is need for an upper bound in $\theta$, there is no theoretical reason for this bound to equal 1; we adopt this value only for convenience.

Production requires fixed and variable costs. We assume the latter to be linear in quantity and independent of the level of $v$, they can therefore be normalized to zero. Fixed costs are an increasing convex function of abatement effort $v_i$. That is, $v_i$ denotes both firms' abatement efforts and their product's environmental quality as perceived by perfectly informed and environmentally aware consumers. Each firm's cost is specified as

$$C_i = \frac{k}{2} v_i^2 \ , \tag{2}$$

where $k > 0$. The two firms compete in prices.

The government intervenes to endogenize environmental externalities. We consider the case that the regulator uses a per unit of output tax, as a proxy to a tax on pollution. Social welfare $SW$ is,

$$SW = U(v_i, q_i) - D(v_i, q_i) - \sum\nolimits_{i=1}^{2} C_i(v_i) \ . \tag{3}$$

Since the timing of government's intervention plays an important role, in what follows we examine and compare two possibilities: (A) the government chooses the tax before firms choose quality, (B) firms' choose quality first and then government chooses the tax. Both games have three stages with price competition at the last stage. In game $A$, the government chooses an output tax rate, then firms choose their quality and then they compete in prices. In game $B$ the order of the first two moves is reversed: at the first stage firms choose quality, at the second the government imposes a tax rate and firms compete in price at the third stage. In both games, consumers' environmental awareness induces abatement investment. In game $A$ although the regulator has a first mover advantage, it does not have the appropriate policy instrument to affect firms' choice of abatement. In model $B$ firms anticipate the impact of their abatement decision on the tax rate that the regulator chooses at the second stage. By appropriately choosing their investment in abatement, they can obtain a more favorable treatment by the regulator. That is, in model $B$, firms choose their qualities based on a tax schedule that the government has provided to them and which is time consistent, or renegotiable-proof, in the sense that the government will not depart from it at the second stage.

## 2.1 The pricing stage

Working backwards, the determination of the last stage equilibrium is common in both games. Maximization of (1) yields the demand functions for the two product types,

$$q_i = \frac{a(1-\gamma) + \theta v_i - \gamma\theta v_j - p_i + \gamma p_j}{1-\gamma^2}\ , \tag{4}$$

where $i, j = 1, 2$ and $i \neq j$.

Firm $i$'s profit function is,

$$\Pi_i = (p_i - t_i) q_i(p_i) - \frac{k}{2} v_i^2\ , \tag{5}$$

where $t_i$ is the tax rate per unit of output. Maximizing each of the above functions with respect to its corresponding price yields the reaction functions,

$$p_i = \frac{a(1-\gamma) + \gamma p_j + t_i + v_i - \gamma v_j}{2}\ . \tag{6}$$

Solving the above system yields equilibrium prices,

$$p_i = \frac{a(2-\gamma-\gamma^2) + 2t_i + \gamma t_j + (2-\gamma^2)\,\theta v_i - \gamma\theta v_j}{4-\gamma^2}\ . \tag{7}$$

Substituting the prices from (7) back into the demand (4) and profit (5) functions yields quantities and profits as function of qualities and taxes,

$$q_i\left(v_i, v_j, t_i, t_j, a, \gamma, \theta\right) = \frac{a(2-\gamma-\gamma^2) - (2-\gamma^2)(t_i - \theta v_i) + \gamma(t_j - \theta v_j)}{(4-\gamma^2)\,(1-\gamma^2)}\ , \tag{8}$$

$$\Pi_i\left(v_i, v_j, t_i, t_j, a, \gamma, \theta, k\right)\ . \tag{9}$$

Note that, as expected, a higher tax rate yield lower production and higher prices, $\frac{\partial q_i}{\partial t_i} < 0$, and $\frac{\partial p_i}{\partial t_i} > 0$. Furthermore, the choice of higher abatement increases firm's output, $\frac{\partial q_i}{\partial v_i} > 0$.

In order to solve the next two stages of the two games ($A$ and $B$) we use two specifications of the pollution function $e_i(v_i, q_i)$. For simplicity, in both specifications, we assume that pollution is a linear function of production. The first specification assumes that without any investment in abatement, pollution per unit of output is $\overline{v}$ and that as environmental investment in abatement increases, pollution per unit of output decreases linearly, that is, $e_i = (\overline{v} - v_i)\, q_i$. This linear specification allow us to derive full analytical solution for both games, which we present in the Appendices.[4] We also employ a more realistic pollution function, in which pollution per unit of output is a decreasing, convex function of environmental quality and in particular we assume, $e_i = \left(1 - \frac{v_i}{1+v_i}\right) q_i$. Although this specification also yields closed form solutions, we choose not to present the results because of their complexity. We illustrate the results of this specification only graphically.

## 2.2 Model A (t-v-p)

In game A the government moves first and chooses tax rates per unit of output. At the second stage both firms maximize their profit functions (9) in order to choose $v_i$, their investment in abatement (which defines their product's environmental quality). Assuming that second order conditions hold, profit maximization yields firms' reaction functions in the $(v_1, v_2)$ space, from which firm $i$'s equilibrium abatement as function of taxes and the model's parameters is obtained,[5]

$$v_i \left(t_i, t_j, a, \gamma, \theta, k\right) \ . \tag{10}$$

As shown in Appendix 1, $\frac{\partial v_i}{\partial t_i} < 0$ and $\frac{\partial v_i}{\partial t_j} > 0$. An increase in the tax rate levied on firm $i$'s product, reduces its choice of abatement, while the opposite is true for an increase in the tax rate on firm $j$'s product.

Substituting firms' quality choices from (10) into (8) we derive quantities as functions of taxes and the model's parameters, $q_i = q_i(t_i, t_j, a, \gamma, \theta, k)$. Notice that since we assume symmetric firms, both abatement and quantity choices are the same for both firms. We then substitute qualities and quantities into (3) to obtain social welfare as function of taxes and the model's parameters, $SW = SW(t_i, t_j, a, \gamma, \theta, k, d)$.

Assuming second order conditions hold, the system $\frac{\partial SW}{\partial t_i} = 0$, and $\frac{\partial SW}{\partial t_i} = 0$ yields a unique real symmetric solution,[6]

$$t_A^* \left(a, \gamma, \theta, k, d\right) \ , \tag{11}$$

[4]he linear specification of emissions is similar to the one used in Petrakis and Xepapadeas (1999) and yields similar results.

[5]The analytical solution, for the case that emission per unit of output is a linear function of quality, is provided in Appendix 1.

[6]Since firms are completely symmetric, both taxes are the same. The subscript $A$ denotes equilibrium values in the first model.

Depending on the relative strenght of competition to environmental damages, this could actually be a tax or a subsidy. Substituting the optimal tax from (11) into the expression (10) we obtain the equilibrium quality, $v_A^* (a, \gamma, \theta, k, d)$. We can also derive the equilibrium values of quantity, $q_A^* (a, \gamma, \theta, k, d)$ , utility $U_A^* (a, \gamma, \theta, k, d)$, environmental damage $D_A^* (a, \gamma, \theta, k, d)$ ,and social welfare $SW_A^* (a, \gamma, \theta, k, d)$.

## 2.3 Model B (v-t-p)

In game $B$ the government considers investment in abatement as given when choosing the tax rate. However, firms anticipate the upcoming taxation, and choose their abatement in accordance. It might seem like the regulator acts myopically, taking into account only the short-run effects on prices and ignoring the effects of taxation on abatement choice. However, this is not the case, since the regulator does not intervene unexpectedly, taking free-market abatement investment as given. Instead, firms know that the regulator will intervene in the next stage and take this information into account. It is like the regulator offers to the firms a tax schedule and firms choose their investment in abatement based on this schedule which is time consistent.

At the second stage the government chooses the tax rate given firms' abatement choice. We substitute optimal quantities from (8) into the social welfare function (3), to obtain social welfare as function of qualities, taxes and the model's parameters, $SW = SW(v_i, v_j, t_i, t_j, a, \gamma, \theta, d)$. Maximization of the social welfare function with respect to $t_i$ yields unique, real tax rates as function of qualities,[7]

$$t_i (v_i, v_j, a, \gamma, \theta, d) \ . \tag{12}$$

Equation (12) gives the tax schedule which is known to firms when choosing quality at the first stage. As shown in Appendix 2, the higher is the firm's choice of abatement, the lower is the tax levied on its product, that is $\frac{\partial t_i}{\partial v_i} < 0$. The higher is the abatement chosen by the firm, the lower is the environmental damage and thus, the environmental tax is lower. Furthermore, $\frac{\partial t_i}{\partial v_j} > 0$, because of strategic complementarity between the firms' choice variables.

[7]As in game $A$, if the environmental externality is not very prominent relative to the market distortion, the government will subsidize production. The analytical solution is relegated to Appendix 2.

At the first stage, firms, taking into account the tax schedule in (12), choose abatement in order to maximize their profits. Substituting taxes from (12) into the profit functions (9) yields profits as functions of qualities only, that is, $\Pi_i = \Pi_i(v_i, v_j, a, \gamma, \theta, k, d)$. The two firms maximize their profit function and the system of the first order conditions yields the optimal abatement effort,[8]

$$v_B^* (a, \gamma, \theta, k, d) \ . \tag{13}$$

Because of the symmetry assumption, both firms choose the same abatement effort. Substituting optimal abatement from (13) into equation (12) yields the equilibrium value of tax,

$$t_B^* (a, \gamma, \theta, k, d) \ . \tag{14}$$

Because of the assumed symmetry between firms, the government levies the same tax, or gives the same subsidy to both of them. By substituting the optimal abatement and taxes from (13) and (14) into (8), we obtain the equilibrium values of quantity, $q_B^* (a, \gamma, \theta, k, d)$ , and then utility $U_B^* (a, \gamma, \theta, k, d)$, environmental damage $D_B^* (a, \gamma, \theta, k, d)$ ,and social welfare $SW_B^* (a, \gamma, \theta, k, d)$.

# 3 Comparison of the two models

As it was mentioned above, the specification of emission as $e_i = (\overline{v} - v_i) q_i$ allows for manageable analytical solutions in both games, which are presented in Appendices 1 and 2. For the same emissions specification we obtain the following results.[9]

**Proposition 1** *Assuming a linear specification of emissions, $e_i = (\overline{v} - v_i) q_i$, that second order conditions for the maximization of profits and social welfare hold and positive values for firms' choice of abatement and output,*

*i. Firms choose higher abatement in game B relative to game A, $v_B^* > v_A^*$*
*ii. Firms produce higher output in game B relative to game A, $q_B^* > q_A^*$*
*iii. The regulator imposes a higher tax in game A relative to game B, $t_A^* > t_B^*$*
*iv. Social welfare is higher in game A relative to game B, $SW_A^* > SW_B^*$*

[8] The subscript $B$ denotes equilibrium values in model $B$.
[9] The proof of Proposition 1 is relegated to Appendix 3.

When firms choose abatement first, anticipating the regulator's intervention, they choose higher abatement effort relative to the case in which they choose their abatement after the announcement of a particular tax. Firms choice of high abatement effort is an attempt to avoid high tax rates. Since higher abatement effort corresponds to higher environmental quality as perceived by consumers, it induces higher demand and thus production is also higher when firms move first. As a result utility unambiguously increases, but environmental damage could also increase despite the improvement in quality, since output increases as well. Taking into account the cost of quality, which increases in an increasing rate, firms' overinvestment in abatement in game $B$ leads to lower social welfare relative to game $A$.

However, the social welfare comparison depends on the particular specification of the emissions function. Up to now we have assumed that emissions per unit of output decrease linearly as firms invest in higher abatement effort. However, it is more realistic to assume that abatement technology is such that investment in abatement decreases emissions at a faster rate. Thus, we employ an emissions function which is decreasing and convex in environmental quality, $e_i = \left(1 - \frac{v_i}{1+v_i}\right) q_i$. As we already mentioned, although closed form solutions can be obtained using this functional form, they are extremely complicated and therefore we choose not to present them. We will present the results with simulations and we will compare the two emissions specifications.

Since the cost of abatement is a critical parameter in the model, we will present all simulations with respect to the cost parameter $k$.[10] At the end of the Section we will examine the effect of the other parameters of the model. In all Figures the blue line illustrates the results of game $B$, while the red line the results of model $A$. All Figures associated with the model using the emission specification $e_i = (\overline{v} - v_i)\, q_i$, present results for values of $k > k_{\min}$ for which we obtain positive values of quality and quantity.

Figure 1 illustrates abatement choice as function of the cost parameter $k$ for both emission specifications. Since the tax rat is always lower when the government takes the qualities as given (model B, blue line), it is clear that firms' abatement choice will

[10] For the simulations supporting the illustrations in this Section we use the following values for the parameters: $a = 60$, $\theta = 0.8$ and $\gamma = 0.5$. For the linear specification of emissions $\overline{v} = 20$ and $d = 1.2$. For the convex specification of emissions $d = 50$.

always be higher when the government moves second. Firms choose higher abatement (environmental quality) when they move first, anticipating the imposition of a tax at a later stage. There are two reasons for which firms invest in abatement. First, higher abatement results in higher environmental quality increasing consumers' demand and second it affects the tax rate when firms move first. In the case that firms face a given tax rate when they choose abatement, they do not have any incentive to choose higher abatement effort, since the tax is levied on a per unit of output basis. Therefore, in game $B$ firms have an additional incentive relative to game $A$ which drives them to choose higher abatement effort.

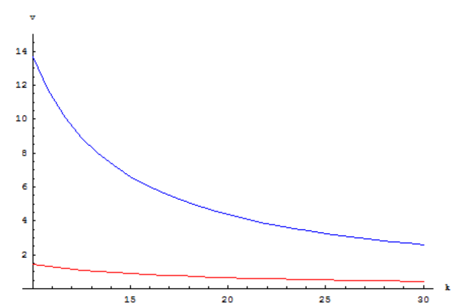

Fig. 1a. $v(k)$ with $e_i = (\overline{v} - v_i)\, q_i$

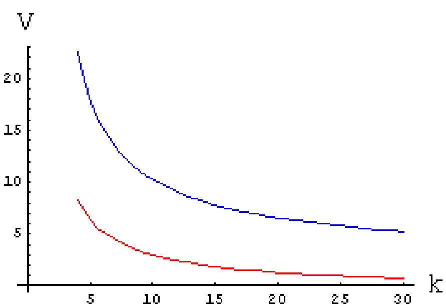


Fig. 1b. $v(k)$ with $e_i = \left(1 - \frac{v_i}{1+v_i}\right) q_i$

In response to the higher choice of abatement in game $B$, the regulator imposes a lower tax. This is illustrated in Figure 2.

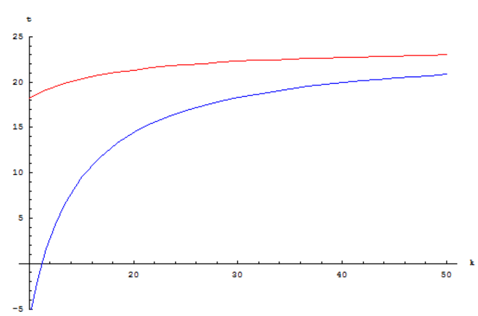

Fig. 2a. $t(k)$ with $e_i = (\overline{v} - v_i)\, q_i$

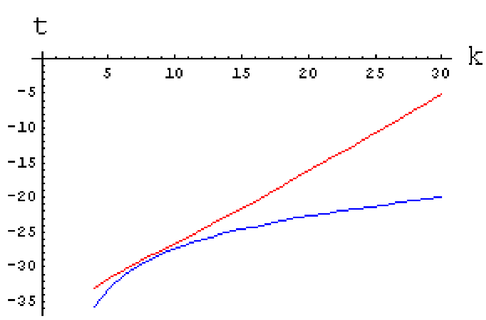


Fig. 2b. $t(k)$ with $e_i = \left(1 - \frac{v_i}{1+v_i}\right) q_i$

Figure 3 illustrates output as function of $k$. Output is always higher in game $B$, since quality is higher – and thus consumers' demand– and the tax rate is lower, which implies lower marginal cost of production, intensified competition which in turn leads to lower prices because of strategic complementarity.

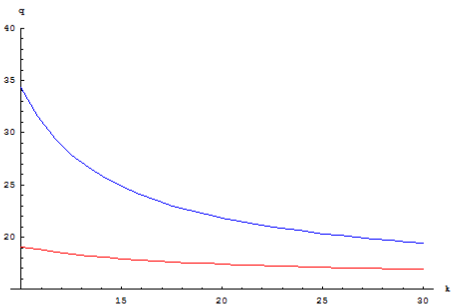


Fig. 3a. $q(k)$ with $e_i = (\overline{v} - v_i)\, q_i$

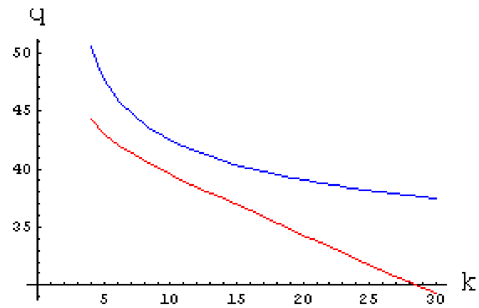


Fig. 3b. $q(k)$ with $e_i = \left(1 - \frac{v_i}{1+v_i}\right) q_i$

Therefore, in game $B$, firms choose higher abatement effort so as to force the regulator to levy a lower tax, which it actually does since the tax schedule known to the firm is renegotiation proof. Higher abatement and lower tax, which leads to lower prices, yields higher output and therefore higher utility as shown in Figure 4.

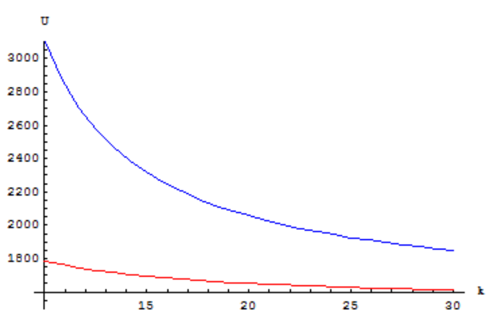


Fig. 4a. $U(k)$ with $e_i = (\overline{v} - v_i)\, q_i$

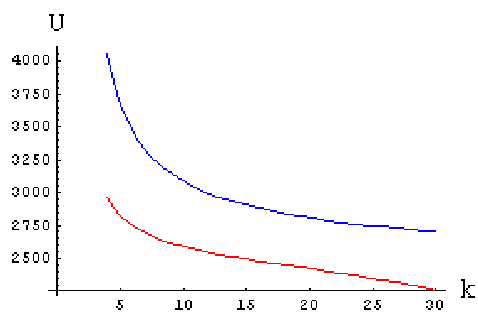


Fig. 4b. $U(k)$ with $e_i = \left(1 - \frac{v_i}{1+v_i}\right) q_i$

The effect of higher output outweighs the effect of increased abatement / environmental quality leading to higher environmental damages as shown in Figure 5a.

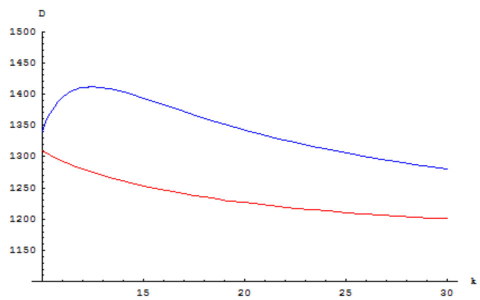


Fig. 5a. $D(k)$ with $e_i = (\overline{v} - v_i)\, q_i$

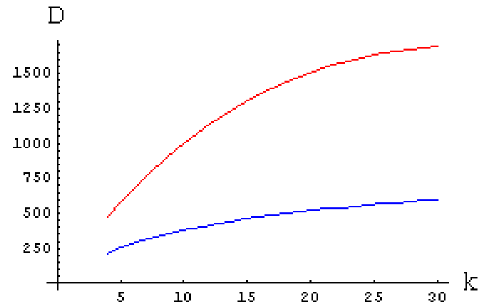


Fig. 5b. $D(k)$ with $e_i = \left(1 - \frac{v_i}{1+v_i}\right) q_i$

The combination of higher environmental damages with higher cost of quality, outweighs the benefits of increased utility, yielding always lower social welfare, in the case of the linear emissions specification, as shown in Figure 6a.

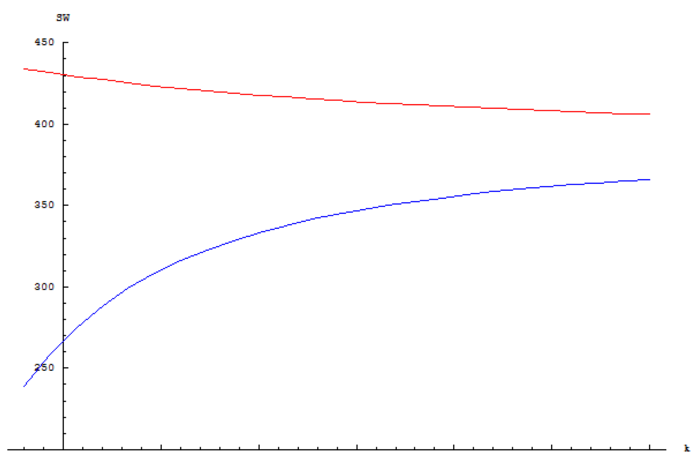


Fig. 6a. $SW(k)$ with $e_i = (\overline{v} - v_i)\, q_i$

In their attempt to avoid increased taxation, firms overinvest in abatement when the cost of abatement is relative low. As the cost of abatement, $k$, increases, the equilibrium of the two games converges, since the choice of abatement in both games tends to a minimum value supported by consumers' environmental awareness (qualities tend to zero as $\theta \to 0$).

Using the definition of social welfare from equation (3) we can analyze the three venues through which a change in $k$ affects social welfare

$$\frac{dSW}{dk} = \frac{\partial U(v_i, q_i)}{\partial k}dk - \frac{\partial D(v_i, q_i)}{\partial k}dk - \frac{\partial \sum_{i=1}^{2} C_i(v_i)}{\partial k}dk$$

An increase in $k$ affects abatement choice and this in turn affects the price choice at the last game and thus output. As we showed above, $\frac{\partial q_i}{\partial v_i} > 0$. Thus, we can define the sign of the three effects on social welfare,

i. $\frac{\partial U(v_i,q_i)}{\partial k} = \left[\underset{+}{\frac{\partial U}{\partial v_i}} + \underset{+}{\frac{\partial U}{\partial q_i}} \underset{+}{\frac{\partial q_i}{\partial v_i}}\right] \underset{-}{\frac{\partial v_i}{\partial k}} < 0$, an increase in abatement cost unambiguously decreases utility.

ii. $\frac{\partial D(v_i,q_i)}{\partial k} = \left[\underset{-}{\frac{\partial D}{\partial v_i}} + \underset{+}{\frac{\partial D}{\partial q_i}} \underset{+}{\frac{\partial q_i}{\partial v_i}}\right] \underset{-}{\frac{\partial v_i}{\partial k}} \gtreqless 0$, the sign of the effect on environmental damage depends on the relative strength of the direct and the indirect effect. In Figure 5a, for low values of $k$, the direct effect dominates in the case of game $B$ and environmental damages increase as k increases.

iii. $\frac{\partial \sum_{i=1}^{2} C_i(v_i)}{\partial k} = \underset{+}{\sum_{i=1}^{2} \frac{v_i^2}{2}} + k \sum_{i=1}^{2} v_i \underset{-}{\frac{\partial v_i}{\partial k}}$, which in the case of symmetry reduces to $v\left(v + 2k\underset{-}{\frac{\partial v}{\partial k}}\right) < 0$, since the indirect effect dominates for the admissible values of $k > k_{\min}$.

Therefore, as $k$ increases, utility decreases, total abatement costs decrease, and environmental damage could increase or decrease. Since in model $B$ firms choose higher abatement effort, total cost of abatement decreases faster as $k$ increases. This explains why social welfare is increasing in model $B$, despite the fact that environmental damage is increasing for small $k$s.

In the case that emissions are decreasing and convex in environmental quality, $e_i = \left(1 - \frac{v_i}{1+v_i}\right) q_i$, environmental damage is increasing in $k$, as shown in Figure 5b, since the direct effect dominates, that is $\underset{-}{\frac{\partial D}{\partial v_i}} + \underset{+}{\frac{\partial D}{\partial q_i}} \underset{+}{\frac{\partial q_i}{\partial v_i}} < 0$ for all values of $k$ in both model $A$ and $B$. Since, for this specification emissions are decreasing faster as quality improves, environmental damages are lower in model $B$. Therefore, for low $k$s, for which quality costs drop sharply in model $B$, social welfare is increasing in $k$ and then is dropping at a slower rate than in model $A$. Thus, although for low values of

$k$, firms in model $B$ overshoot quality and welfare is lower relative to model $A$, there is a critical value of k after which social welfare in model $B$ exceeds that in model $A$. The above analysis is illustrated in Figure 6b and summarized in Proposition 2,

**Proposition 2** *Assuming a convex specification of emissions,* $e_i = \left(1 - \frac{v_i}{1+v_i}\right) q_i$, *that second order conditions for the maximization of profits and social welfare hold and positive values for firms' choice of abatement and output,*

*i.* *Firms choose higher abatement in game* $B$ *relative to game* $A$, $v_B^* > v_A^*$
*ii.* *Firms produce higher output in game* $B$ *relative to game* $A$, $q_B^* > q_A^*$
*iii.* *The regulator imposes a higher tax in game* $A$ *relative to game* $B$, $t_A^* > t_B^*$
*iv.* *Social welfare is higher in game* $A$ *relative to game* $B$, $SW_A^* > SW_B^*$, *when cost of abatement is low, but* $SW_B^* > SW_A^*$, *when cost of abatement is high.*

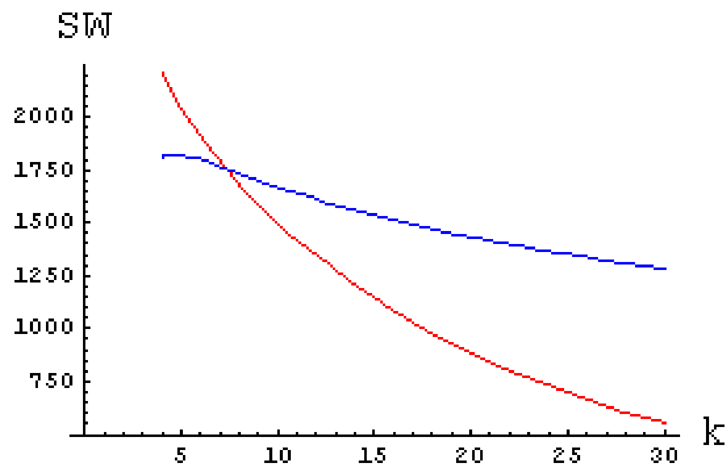


Fig. 6b. $SW(k)$ with $e_i = \left(1 - \frac{v_i}{1+v_i}\right) q_i$

The above result has important policy implications. If the cost of environmental quality is high, it might be welfare superior inducing firms to invest in higher abatement effort by providing them with a renegotiation-proof tax schedule instead of choosing what seems to be the social optimal tax rate. The intuition explaining why firms choose a higher levels of abatement when moving second is as follows. When the regulator moves first, firms take the tax as given, choosing abatement and then prices. However, since the tax is a per unit of output tax, it does not affect abatement choices which depend solely on consumers' willingness to pay for environmental quality. The tax affects only the price choice. In model $B$, although the regulator moves second, the fact that firms anticipate its intervention makes the game as if there is a stage zero in which the regulator determines a tax schedule and then in stage one firms choose abatement based on this time-consistent tax schedule. Therefore, in model $B$ taxation influences the abatement choice as well.

The overinvestment in abatement that model $B$ yields, is socially beneficial when high abatement costs restrain firms from choosing the socially desirable quality. However, if abatement costs are low, then overinvestment in abatement could decrease welfare since it leads to higher than the socially optimal abatement effort.

We can now examine the important effects of the degree of consumers' environmental awareness, denoted in our model by the parameter $\theta$. Firms' choice of abatement are increasing in $\theta$, with the increase being faster in model $B$. Abatement is higher in model $B$ for any value of $\theta$. The tax rate is smaller in model $B$ and is decreasing as $\theta$ increases. Production is higher in model $B$ and is increasing in both models with $\theta$. For the linear specification of emissions, environmental damages are increasing in both models as $\theta$ increases and are higher in model $B$ for any value of $\theta$. On the contrary, for the convex specification of emissions, environmental damages are decreasing in both models as $\theta$ increases and are lower in model $B$ for any value of $\theta$. For the linear specification of emissions, social welfare is higher in model $A$ for any value of $\theta$. For the convex specification of emissions, social welfare is higher in model $B$ for low values of $\theta$ but as $\theta$ increases social welfare in model $A$ becomes higher. This happens because the increased incentives for investing in abatement that model $B$ offers, leads to higher social welfare when abatement is not reinforced by consumers behavior. However, when $\theta$ is high, the combination of incentives leads to overinvestment in abatement and thus to lower social welfare. Therefore, the second policy option is welfare superior for high cost of abatement and low environmental awareness, cases in which the model $A$ cannot provide adequate incentives for investment in abatement.

Quite important is the role of the degree of competition in the output market. When $\gamma$ decreases, the two types of the product become more distinct and firms' market power increases and so does the market distortion. Therefore, for smaller values of $\gamma$ the product tax focuses more on the market distortion and it can become negative in both models. Because of this, abatement (environmental quality), output and profits are increased, while prices are decreased in both models. Fiercer competition, higher $\gamma$, reduces welfare in both models due to reduced quality. Note that without intervention, less brand- name differentiation (hence, stiffer price-competition for any choice of qualities) may result in higher environmental damage and higher profits. This is due to the fact that relaxation of price competition due to brand-name differentiation intensifies competition at the previous stage (quality choice).

# 4 Conclusions

Within a differentiated duopoly model we examine output taxes when firms can choose abatement investment. We assumed that higher abatement effort yields both higher private consumption benefits and lower emissions per unit of output. We examine two policy options differing with respect to the timing of policymakers' intervention. In model $A$ the policymaker moves first and chooses a tax rate per unit of output, then firms choose abatement and finally they compete in prices. This model corresponds to the ideal constrained welfare maximization (second best) exercise. In model $B$, firms choose abatement first and then, the policymaker chooses the tax rate; at the last stage firms again compete in prices. Model $B$ corresponds to a situation where firms adjust their abatement decision in anticipation of the forthcoming output tax. It is as if the regulator announces and commits to a tax schedule that depends on firms' abatement choice, prior to the firms' choice of abatement. Therefore, although firms move first, they adjust their abatement choice in order to manipulate the tax rate.

We analyze two specifications of emissions. In the first one, abatement reduces emissions in a constant rate, while in the second emissions are a convex function of abatement. We show that for both emissions specifications, model $B$ yields higher abatement level, higher output, lower tax rates and prices. he combination of higher abatement (lower emissions per unit of output) and higher output could yield higher or lower environmental damages. We show that for the linear specification of emissions, environmental damages are higher in model $B$, while for the convex specification environmental damages are higher in model $A$. This result combined with higher abatement costs in model $B$, yields lower social welfare in model $B$ for the linear emissions specification. However, for the convex specification of emissions, model $B$ yields higher social welfare when the cost of abatement is relatively high. Therefore, a policy option in which the policymaker is able to commit to a tax schedule rather than to a specific tax rate could be welfare superior when the regulator does not have the choice of an emission tax, limiting itself to a tax per unit of output.

Given the preliminary stage of our analysis, there is a number of issues to be resolved and extensions to be considered. We are currently attempting to provide a full analytical solution of Proposition 2 that is currently supported only by simulations. We are also examining the robustness of our results in the case of strategic substitutability. Our analysis could also be extended to consider the effect of the different policy options on domestic firms' international competitiveness.

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# 6 Appendix 1: Analytical solution of Model A

At the second stage, firms maximize profits from which we obtain firms' reaction functions in the $(v_1, v_2)$ space. Solving the system yields qualities as functions of taxes and the model's parameters,

$$v_i = \frac{2(2-\gamma^2)\left[\Psi a - (2-\gamma^2)\left[(4-\gamma^2)k - 2\right]t_i + \gamma(4-\gamma^2)kt_j\right]}{4(2-\gamma^2)^2 - 4(4-\gamma^2)(2-\gamma^2)^2 + (1-\gamma^2)(4-\gamma^2)^3k^2}\ , \tag{15}$$

where, $\Psi = [(2-\gamma)(1-\gamma)(2+\gamma)^2k - 2(2-\gamma^2)] > 0$. The second order conditions of the profit maximization ensure that the denominator of (15) is positive, and therefore the following is true: $\frac{\partial v_i}{\partial t_i} < 0$ and $\frac{\partial v_i}{\partial t_j} > 0$. An increase in the tax rate reduces firm $i$'s choice of quality, while a tax increase on firm $j$'s product incrases firm $i$'s choice of quality. This is because $t$ is a per unit of output tax and given when firms decide their quality, which means that the regulator does not provide firms with any incentive to increase environmental quality and, at the same time, a higher tax reduces marginal revenue. Substituting the $v$'s from (15) back into (8) we derive quantities as functions of taxes only, $q_i = q_i(a, \gamma, \theta, k, t_i, t_j)$, which we then substitute into (3) to derive social welfare as function of taxes, $SW = SW(a, \gamma, \theta, k, d, \overline{v}, t_i, t_j)$.

At the first stage, the regulator maximizes $SW$ to choose the tax rates $t_i$. Since firms are symmetric, both taxes are the same,[11]

$$t_A^* = \frac{\Phi n\overline{v} - Xa}{D} \ , \tag{16}$$

where, $\Phi = (4-\gamma^2)\left[(2-\gamma)^2(1+\gamma)(2+\gamma)k - 2(2-\gamma^2)\right] > 0$, $X = [16(k+2d) - 4(6k+6d-1)\gamma^2 + (9k+4d-2)\gamma^4 - k\gamma^6] > 0$, and $D = 16\left[(1+\gamma)k - 2d - 1\right] + \gamma^2\left[8 + 4(6-\gamma^2)d - (1+\gamma)(8-\gamma^2)k\right] > 0$. Depending on the values of the parameters, the government could either subsidize or tax the two firms. When the environmental externality is not prominent, that is, $d\overline{v}$ is relatively small, the government will subsidize production to decrease the existing market distortion.

We then substitute taxes from (16) into the expression (15) in order to obtain equilibrium qualities,

$$v_A^* = \frac{2(4-\gamma^2)(2-\gamma^2)(a - n\overline{v})}{D} \ . \tag{17}$$

Because of the symmetry assumption, both firms choose the same quality at the equilibrium. The higher is the environmental externality, the lower is the quality that firms choose to offer, since the tax is increasing on the level of environmental externality. By substituting the optimal qualities from (17) into $q_i = q_i(a, \gamma, k, t_i, t_j)$ and $SW = SW(a, \gamma, n, \overline{v}, k, t_h, t_g)$, we obtain the equilibrium values $q_A^* = \frac{k(4-\gamma^2)^2(a-d\overline{v})}{D}$, and $SW_A^* = \frac{k(4-\gamma^2)^2(a-d\overline{v})^2}{D} = (a - d\overline{v})q_A^*$.

# 7 Appendix 2: Analytical solution of Model B

In model $B$ the regulator attempts to correct the environmental externality given that the firms have chosen qualities at an earlier stage, anticipating government's intervention. At the second stage the government chooses the tax level given the firms' quality choice. We substitute optimal quantities from (8) into the social welfare function (3), to obtain social welfare as function of qualities and taxes, $SW = SW(a, \gamma, \theta, k, d, \overline{v}, v_i, v_j, t_i, t_j)$. Maximization of this social welfare function yields the optimal tax or subsidy as function of the qualities chosen by the firms,

$$t_i = (2-\gamma)d\overline{v} - (1-\gamma)a - (1+2d)v_i + \gamma(1+d)v_j \ . \tag{18}$$

[11] The subscript $A$ denotes equilibrium values in the first model.

As in the previous case, if the environmental externality is not very prominent the government will subsidize production. It is also clear from (12) that the higher is the firm's choice of quality, the lower is the tax levied on its product, that is $\frac{\partial t_i}{\partial v_i} < 0$. The higher is the quality chosen by the firm, the lower is the environmental damage and thus, the environmental tax is lower. Furthermore, $\frac{\partial t_i}{\partial v_j} > 0$, because of strategic complementarity between the firms' choice variables.

At the first stage, the firms take taxes and chose qualities in order to maximize their profits. Substituting taxes from (18) into the profit functions (9) yields profits as functions of qualities only, that is, $\Pi_i = \Pi_i(a, \gamma, \theta, k, d, \overline{v}, v_i, v_j)$. The two firms maximize their profit function and the system of the first order conditions yields optimal qualities,[12]

$$v_B^* = \frac{2(1+n)(a - d\overline{v})}{(1+\gamma)k - 2(1+d)^2} \; . \tag{19}$$

Because of the symmetry assumption, both firms choose the same quality. Substituting optimal qualities from (19) into equation (18) yields the equilibrium value of tax,

$$t_B^* = \frac{\left[(2+\gamma-\gamma^2)k - 2(1+d)\right] d\overline{v} - \left[(1-\gamma^2)k + 2(1+d)\right] a}{(1+\gamma)k - 2(1+d)^2} \; . \tag{20}$$

Because of the assumed symmetry between firms, the government levies the same tax, or gives the same subsidy to both of them. By substituting the optimal qualities from (19) and the optimal taxes from (20) into (8) and $SW = SW(a, \gamma, \theta, k, d, \overline{v}, v_h, v_g, t_h, t_g)$, we obtain the equilibrium values $q_B^* = \frac{k(a-d\overline{v})}{(1+\gamma)k-2(1+d)^2}$, and $SW_B^* = \frac{\left[((1+\gamma)k-4(1+d)^2\right]k(a-d\overline{v})^2}{[(1+\gamma)k-2(1+d)^2]^2} = \frac{(1+\gamma)k-4(1+n)^2}{k}\left(q_B^*\right)^2$.

# 8 Appendix 3: Proof of Proposition 1

i. We start by comparing the optimal qualities that the firms choose under the two models. From (17) and (19) we obtain,

$$v_B^* - v_A^* = \frac{2\Xi_1(a - d\overline{v})}{\left[(1+\gamma)k - 2(1+d)^2\right] D} \; ,$$

where $\Xi_1 = 8\left[(1+2d)k - (1+d)2d\right] + 8(1+2d)k\gamma - 2(4kd+k+2-n-6d^2)\gamma^2 - 2(1+4d)k\gamma^3 + (2+kd-2d^2)\gamma^4 + dk\gamma^5$. This expression is positive for the range of values of

[12] The subscript $B$ denotes equilibrium values in the second model.

the parameters that satisfy second order conditions and positivity constraints. Given that the denominator is also positive, we have that $v_B^* > v_A^*$ for all admisible values of the parameters.

ii. We then compare optimal taxes that the two different models yield. >From (16) and (20) we obtain,

$$t_A^* - t_B^* = \frac{2\Xi_2(a - d\overline{v})}{\left[(1+\gamma)k - 2(1+d)^2\right] D} \ ,$$

where $\Xi_2 = 8(k + 2d) + (4 - \gamma)\left[4 + (2-\gamma)^2(1+\gamma)(2+\gamma)k + 4 - 2\gamma^2\right] d^2 +$ $\left[(2-\gamma)^2(1+\gamma)(2+\gamma)k - 8\gamma^2\right] d - \left[14k - 4 - \left[\left[(3-\gamma)(2+\gamma)\gamma - 2\right] k - 2\gamma\right] \gamma\right] \gamma^2 > 0$. Given that the denominator is also positive we have that $t_A^* > t_B^*$ for all admisible values of the parameters.

iii. We move to the comparison of quantities next. The difference between $q_B^*$ and $q_A^*$ is,

$$q_B^* - q_A^* = \frac{2k\Xi_3(a - d\overline{v})}{\left[(1+\gamma)k - 2(1+d)^2\right] D} \ ,$$

where, $\Xi_3 = 8 - 4\gamma^2 + \gamma^4 + 4(4-\gamma^2)n + (4-\gamma^2)n^2 > 0$. Given that the denominator is also positive we have that $q_B^* > q_A^*$ for all admisible values of the parameters.

iv. The difference between $SW_B^*$ and $SW_A^*$ is,

$$SW_A^* - SW_B^* = \frac{4k\Xi_4(a - d\overline{v})^2}{\left[(1+\gamma)k - 2(1+d)^2\right]^2 D} \ ,$$

where, $\Xi_4 = 4k + 8[k + 2n(1+n)^2]n + 4k(1+2n)\gamma - 2[k(1+3n) - 4(1-n)(1+n)^2 n]\gamma^2 - 2k(1+3n)\gamma^3 + [1 + n(k - 2n + n^3)]\gamma^4 + kn\gamma^5 > 0$.Given that the denominator is also positive we have that $SW_A^* > SW_B^*$ for all admisible values of the parameters.