Labor Immobility and the Transmission Mechanism of Monetary Policy in a Monetary Union*

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April 10, 2013

Abstract

It is believed that a common monetary policy in a monetary union will have identical effects on different countries as long as these countries have identical fundamentals. We show that, when there is specialization in production, the terms of trade react to the shock. The transmission mechanism of a monetary shock has in this case an additional channel, the terms of trade. This is the case even if state

*We thank the editor, the associate editor, an anynomous referee and participants in the seminars at the Bank of Portugal, at ISEG, at FCEE Católica-Lisbon, at the SED Meetings, and at the Annual Congress of the European Economic Association for helpful comments. Bernardino Adão and Isabel Correia gratefully acknowledge financial support of FCT. The opinions are solely those of the authors and do not necessarily represent those of the Banco de Portugal.

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contingent assets can be traded across countries. For a reasonable parametrization, the differential on the transmission across countries is quantitatively significant when compared with the effect on the union's aggregates. Monetary shocks create cycles with higher volatility in "poor" countries than in "richer" ones.

Key words: Monetary Union; Transmission Mechanism of Monetary Policy; Labor Immobility; Idiosyncratic Effects.

JEL: E31; E41; E58; E62

1 Introduction

The widespread view in the profession is that homogeneous shocks in a set of countries or regions have no idiosyncratic effects if these countries or regions are identical. The standard hypotheses in the international macroeconomic literature imply that aggregate shocks do not affect the terms of trade when countries are identical. Therefore, relative consumptions, relative incomes and the current accounts do not change in response to such a shock. Instead, in this paper we want to stress the effects of common shocks in identical countries, by not closing the potential role of the terms of trade and of the current account in the transmission of common shocks, and therefore allowing for different outcomes across similar countries.

Countries in this paper are similar in the sense that they have identical preferences and technologies, but they are specialized in the production of aggregate tradable goods with different income elasticities. We do not explain formally how identical countries specialize in goods with different income elasticities to keep the model simple. But it is not difficult to devise set ups that could originate such an outcome. A simple paradigm of such a situation would be equal production technologies for the goods, with increasing returns to scale, or with constant returns to scale and an initial sunk entry cost for each firm. Each country specializes in the production of a set of the goods, and the aggregate tradable goods, in which the countries specialize, will have in general different income elasticities. The number of firms in each of the countries, and the pattern of trade and specialization across countries would be indetermined as in Krugman (1980).

The main result of this paper conveys more importance to monetary policy because - by impacting on the terms of trade and on relative allocations - it is more powerful than in the traditional view where a common monetary policy cannot affect similar countries differently. In this sense, this paper represents a step forward in trying to understand the transmission mechanism of monetary policy in a monetary union. We evaluate numerically the idiosyncratic effects of a monetary policy shock, in a simple model where countries have an identical nominal rigidity, and conclude that the asymmetric effects have the potential to be quantitatively significant. *We focus on a monetary shock in a monetary union, but the conclusions extend to any other common shock.*

Market completeness is an important assumption in the open macroeconomics literature. The effects of idiosyncratic shocks may change substan-

tially when this hypothesis is dropped, and therefore the non-existence of markets has non-trivial implications. The empirical plausibility of this assumption, is associated with the importance of changes in the current account in the transmission of shocks across countries. Although this assumption is clearly identified as determinant in the analysis of idiosyncratic shocks, or different exogenous transmission mechanisms, its importance for common shocks and identical transmissions mechanisms has not yet been explored in the literature.[1]

Typically to avoid the indeterminacy of the aggregates in the steady-state when markets are incomplete, and the associated non-stationarity in the dynamics, it is necessary to introduce a modification to the standard models to induce stationarity (see Schmitt-Grohë and Uribe, 2003, and Correia et al (1995) for further details). In this paper, even with incomplete markets, we maintain simplicity and tractability, because we consider a model that has well defined aggregates and is stationary at the union level, while it is non-stationary at the country level. That is, we have a particular form of aggregation even when asset markets are incomplete[2]. The crucial market incompleteness is the labor immobility across countries. The existence of state-contingent asset markets across countries is qualitatively unimportant for our results.

It is interesting that here the so much publicized role of the terms of trade as an insurance mechanism is reversed. It is exactly the endogenous response of the terms of trade to the common shock that leads to the asymmetric responses of the various economic variables across countries to the common shocks. It is well documented that, in response to an idiosyncratic productivity shock, the country whose productivity increased the most will produce relatively more but the relative price of the bundle of goods it produces will decrease also. Thus, the terms of trade reaction in response to idiosyncratic shocks will determine a smaller dispersion of the relative income of the countries. In the context of a simple model, Cole and Obstfeld (1991), demonstrated that the gains from completing the markets can be modest, as the terms of trade are a good insurance scheme for countries without state contingent asset markets. Even though, they provide full in-

[1]The literature includes, among many others, Benigno (2004), Benigno and Benigno (2003, 2006), Benigno and Thoenissen (2006), Carlstrom, Fuerst, Ghironi and Hernandez (2006), Cole and Obstfeld (1991), Corsetti and Pesenti (2001, 2005), Corsetti, Luca and Leduc (2008), Dotsey and Duarte (2008) and Ghironi (2006).

[2]See Adão and Correia (2009).

surance only for a very small set of parameters, for a larger set of other realistic parameters they provide almost full insurance. Therefore, Cole and Obstfeld (1991) conclude that "the terms of trade may play an important role by automatically pooling national economic risks". More recently Ghironi (2006) showed these results may not be robust. The terms of trade can be a poor substitute for a full insurance scheme since, in more complex models, the transmission of idiosyncratic shocks has effects in an incomplete market framework that can be quantitatively fairly different from the ones obtained in a complete market set up. Our analysis is just on aggregate shocks, and in an environment where the insurance provided by the terms of trade would not be enough to replicate complete markets. We show that terms of trade react to the common shock and therefore the outcomes of the shock differ across countries.

To develop the intuition for the change in the terms of trade, and to introduce the aggregation results, we consider first an economy where firms have no restrictions on the way they choose prices. For the monetary shock to have real effects in the flexible price economy we assume that money has a role in transactions. Latter, to get an idea of the quantitative importance of this effect, we consider an alternative environment where firms set prices according to a Calvo mechanism, Calvo (1983). In this environment the model is solved numerically with log-linearization of the equilibrium equations. Unlike what happens in most closed economy models, where with the first order approximation the behavior of relative prices is lost, here there is a change in the terms of trade. This occurs because we impose labor immobility across countries and non-homothetic preferences. These are our crucial assumptions. If instead we had assumed homothetic preferences for the households, the same result could be obtained if government expenditures were introduced, identical across countries but whose composition across goods did not coincide with the one of the households. In this way total demand (private and public) would be again non-homothetic, and the result would be preserved.

The assumption of identical income elasticity across goods is clearly rejected by empirical micro studies. For example, Blundell, Pashardes and Weber (1993) states that "In our sample of UK survey data for 15 years we find strong evidence of (...) the presence of nonlinearity in the micro-level Engel curves". Another example, Banks, Blundell and Lewbel (1997) establish that Engel curves differ across goods, and that they depend on higher order income terms through coefficients which are price dependents.

These empirical facts are not relevant for macro models if the assumption

of homogeneity/nonhomotheticity is not central for the issues under study. Therefore, for convenience and tractability most macro models use homogeneous momentary preferences. However, there is a large literature that shows that this assumption is not harmless for the understanding of some facts, which under homogeneity turn into puzzles. For instance in growth models, when markets are open to international capital, it is very difficult to explain both the direction of capital flows, as well as the cross section growth rates, when preferences are homogeneous. In Rebelo (1992) it is shown that the introduction of nonhomothetic preferences can solve both questions. Another example in growth is when aggregate consumption has to be disaggregated due to the question under scrutiny. To study the sectorial structural transformation associated with development, the different income elasticity across agriculture, manufacture and service goods play a fundamental role. This is shown in Kongsamut, Rebelo and Xie (2001). In trade, the general equilibrium models used have necessarily consumption goods disaggregation as a main modeling assumption. Again here nonhomothetic preferences are fundamental. Examples are studies that explain trade determinants (e.g. Markusen (1986) and Hunter (1991)) or the factor content of trade, e.g. Davis and Weinstein (2003), Trefler (1995) and Chung (2005).

Thus, in models with a positive degree of disaggregation, like those used to understand the transmission of monetary policy through countries that trade, the abstraction of homogeneity should be used cautiously. If the individual goods that compose an aggregate traded good have different income elasticities then it should be expected that different aggregate traded goods would also have different income elasticities. This possibility is typically ignored as this literature of monetary policy transmission assumes homogeneity of preferences. Our paper explores this concern.

How is an aggregate shock transmitted across countries? In a closed economy an aggregate shock does not affect the relative price of goods. However, this result does not extend to economies with more than one country. Aggregate shocks with asymmetric subsistence levels will have identical effects in all countries if and only if labor is perfectly mobile across countries. In this paper we want to stress that when there is less than perfect mobility of labor across countries then aggregate shocks will have asymmetric effects if countries are specialized in goods with different subsistence levels. It is the interaction of labor immobility and the specialization of countries in goods with different income elasticities that originates the asymmetric effects of aggregate shocks.

The intuition behind the main result of the paper is straightforward. Since goods have different income elasticities, an aggregate shock, which can be a monetary shock or any other aggregate shock, changes the relative demands for the goods. Because countries are specialized in goods with different income elasticities some countries become relatively better off or worse off, depending on the sign of the shock and the goods in which they specialize. The countries that specialize in the goods with higher income elasticity benefit relatively more with a positive aggregate shock than the countries that specialize in goods with lower income elasticity. The opposite occurs with a negative aggregate shock; all countries are worse off but the countries that specialize in the goods with lower income elasticity are relatively better off than the countries that specialize in goods with higher income elasticity.

The paper proceeds as follows. In Section 2 we describe the basic two-country monetary union when firms have no price setting restrictions. In Section 3 we show how the equilibrium can be computed analytically. The solution for the terms of trade is explored to identify how the common shock can create heterogeneous outcomes. In Section 4 we use an environment with price-setting frictions a la Calvo, where the degree of stickiness is the same for every firm independently of the country, and we show numerically that the idiosyncratic effects are significative when compared with the union's wide effects of the monetary shock. Section 5 contains concluding remarks.

2 The Model with Flexible Prices

The model considered is a standard international macroeconomic model. The monetary union has two countries with identical tastes, technologies and initial assets. We denote the home country with H and the foreign country with F. The union is populated by a continuum of households in the interval $[0, 1]$. The households in the segment $[0, \theta]$ live in country H and the households in the segment $(\theta, 1]$ live in country F. Each firm produces a distinct good and each good is identified with the firm that produces it. Firms use technologies that are linear in labor, and productivity is identical across goods and across countries.[3] The goods produced in the union are normalized to the unit in-

[3]We do not model the trade pattern, instead consider it as exogenous. In general the endogenous pattern of international trade in a world with identical countries is not unique. There are many set ups that could originate the pattern of trade we take: one for which the subsistence level of the exported goods is different from the one of the imported goods.

terval $[0,1]$. The goods in the interval $[0,\theta]$ are produced in country H and the goods in the interval $(\theta,1]$ are produced in country F.[4] The relative sizes of the countries can be different, but as we will see later, that is irrelevant for the results. As it is usual in the literature, it is assumed that there is no firm entry dynamics in response to monetary shocks.[5]

The monetary authority of the monetary union issues the common currency, that is distributed endogenously across countries in order to satisfy demand. Monetary policy is conducted by an interest rate rule, which is the instrument of monetary policy. It is assumed that seigniorage is transferred through lump sum transfers equitatively across countries.

There are union-wide markets for the goods but the market for labor is segmented across countries. Labor is homogeneous and perfectly mobile inside each country but immobile across countries.

The history of events up to period t, $(s_0, s_1, ..., s_t)$ is $s^t \in S^t$ and the initial realization s_0 is given. The aggregate productivity and nominal interest rate are the random variables indexed to these histories. Notice that we are not allowing idiosyncratic shocks across countries and firms. There is a state contingent nominal asset traded inside each country, and there is a non-state contingent nominal asset traded across countries.

The results of this paper would still hold if instead we had assumed that the nominal state contingent asset was traded across countries[6]. On the other hand, labor immobility across countries is a crucial assumption when associated with nonhomothetic preferences. If we had either perfect mobility of labor or homothetic preferences the terms of trade channel of the monetary transmission mechanism would be closed. Labor immobility across countries is the natural assumption in trade and international macro.

For instance, with increasing returns, or alternatively with constant returns but industry entry costs as in Krugman (1980), there will be specialization and trade across countries to collect the gains from the economies of scale. Since typically the production locations are indeterminate, as in Krugman (1980), without loss of generality we can take the trade patterns as historically determined.

[4]The number of goods produced in each country does not have to coincide with its size, however that simplifies the analysis and the notation.

[5]Few are the papers that allow for endogenous firm entry over the business cycle. See for instance Bilbiie et al (2007).

[6]In the appendix 2 we consider the case when the contingent nominal bond can be traded across countries. Notice that the existence of state contingent markets across countries when labor is immobile is in general not necessary nor sufficient for the existence of a representative household for the union.

Nonhomotheticity of preferences across disaggregated consumption goods is validated by a vast literature, and we suppose this property holds for aggregate tradable goods too. In other words the pattern of trade, which we take as history determined, extends the nonhomoteticity for individual goods to nonhomoteticity for traded aggregates. This nonhomoteticity implies that the traded goods will have different income elasticities for unitary terms of trade.

We consider the monetary transmission mechanism in two environments differentiated by the type of firms' price setting. A simple one to gain intuition on the transmission mechanism and another, more complex, to evaluate the magnitude of the transmission mechanism. In this section, we assume that firms set prices contemporaneously and solve analytically for prices and for the aggregate allocation. Since in a flexible price economy there must be a friction for the monetary policy to have real effects, we assume that households have to satisfy a cash in advance constraint. This friction was introduced by Clower (1967) and became one of the most popular frictions in monetary theory after Lucas (1980). In section 5 we assume that firms set prices à la Calvo and derive numerically the transmission mechanism.

2.1 Households

Given the described set-up, there are two representative households, one for each country. Preferences are of the Stone-Geary type and identical across these two consumers. The preferences of the representative consumer in country H are

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t\right), \ 0 < \beta < 1 \tag{1}$$

where E_0 is the expectation conditional on the information available at time 0, β is a discount factor, N_t is hours of labor of the representative household of country H and C_t is the composite consumption in excess of subsistence levels of the representative household of country H. The C_t is defined as:

$$C_t = \left[\int_0^{\theta} \widetilde{c}_{h,t}(j)^{\frac{\sigma-1}{\sigma}} dj + \int_{\theta}^{1} \widetilde{c}_{f,t}(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

with $\widetilde{c}_{h,t}(j) = c_{h,t}(j) - \overline{c}_{h,t}(j)$, for the goods produced in country H, $j \in [0, \theta]$, and $\widetilde{c}_{f,t}(i) = c_{f,t}(i) - \overline{c}_{f,t}(i)$, for the goods produced in country F, $i \in (1, \theta]$,

where $\overline{c}_{h,t}(j) \geq 0$ and $\overline{c}_{f,t}(i) \geq 0$, which we interpret as subsistence levels, and where the consumptions of the representative household of country H are $c_{h,t}(j)$ of good j produced in country H, $j \in [0,\theta]$, $c_{f,t}(i)$ of good i produced in country F, $i \in (1,\theta]$, and $\sigma > 1$ is the elasticity of substitution between the various goods.

Notice that we allow for different subsistence levels across goods. To simplify the notation we take that all goods produced in each country have the same subsistence level, but what is crucial is that the average subsistence level of the goods produced in each country H be different from the one in country F. The economy could be simplified further and all the arguments would go through, if instead, we had considered just two goods, with different subsistence levels. However, since in section 3 we have Calvo price setting firms and would need to have many individual goods, we decided to have many individual goods from the start.

The variables concerning country F are indexed with a star. Thus, the composite consumption in excess of the subsistence levels of the representative household of country F, C_t^* is defined as:

$$C_t^* = \left[\int_0^{\theta} \widetilde{c}_{h,t}^*(j)^{\frac{\sigma-1}{\sigma}} dj + \int_{\theta}^{1} \widetilde{c}_{f,t}^*(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

with $\widetilde{c}_{h,t}^*(j) = c_{h,t}^*(j) - \overline{c}_{h,t}(j)$, and $\widetilde{c}_{f,t}^*(i) = c_{f,t}^*(i) - \overline{c}_{f,t}(i)$, where $c_{h,t}^*(j)$ denotes the consumption by the representative household of country F of good j produced in country H, and $c_{f,t}^*(i)$ denotes consumption of good i produced in country F. The subsistence levels across goods for the representative household of country F are equal to the ones for the representative household of country H.

It is convenient to rewrite the composite consumption in excess of the subsistence levels for the representative consumer of country H as

$$C_t = \left[\theta C_{h,t}^{\frac{\sigma-1}{\sigma}} + (1-\theta) C_{f,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \ 0 < \theta < 1, \text{ for all } t,$$

where the aggregate $C_{h,t}$ corresponds to the composite consumption in excess of the subsistence levels of the continuum of goods produced in country H, and $C_{f,t}$ corresponds to the composite consumption in excess of the subsistence levels of the continuum of goods produced in country F. These bundles are defined as

$$C_{h,t} = \left[\frac{1}{\theta}\int_0^{\theta} \widetilde{c}_{h,t}(j)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}, \text{ for all } t$$

and

$$C_{f,t} = \left[\frac{1}{1-\theta} \int_{\theta}^{1} \widetilde{c}_{f,t}(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \text{ for all } t.$$

Similar variables can be defined for the representative consumer of country F. Thus, $C^*_{h,t}$ is the composite consumption in excess of the subsistence levels of the continuum of goods produced in country H, and $C^*_{f,t}$ is the composite consumption in excess of the subsistence levels of the continuum of goods produced in country F.

Let $p_{h,t}(j)$, for $j \in [0,\theta]$ be the price of good j produced in country H, and let $p_{f,t}(i)$, for $i \in (1,\theta]$ be the price of good i produced in country F. By standard procedures three things can be shown, that the price of $C_{h,t}$ or $C^*_{h,t}$ is[7] $P_{h,t} = \left[\left(\frac{1}{\theta}\right)^{\sigma} \int_0^{\theta} p_{h,t}(j)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$, that the price of $C_{f,t}$ or $C^*_{f,t}$ is $P_{f,t} = \left[\left(\frac{1}{1-\theta}\right)^{\sigma} \int_{\theta}^{1} p_{f,t}(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$, and that the price of C_t or C^*_t is

$$P_t = \left[\theta^{\sigma} P_{h,t}^{1-\sigma} + (1-\theta)^{\sigma} P_{f,t}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

In this environment domestic consumers do not have a higher share of domestic produced goods in their consumption bundle. There is no consumption home bias in this set up. The goods produced in country H enter in the consumption basket of the representative household of country H, C_t, in the same manner as in the consumption basket of the representative household of country F, C^*_t. The assumption that every good is tradable and the assumption of identical preferences, without home bias, implies an identical consumer price index across countries in a monetary union.

Consumers choose the consumptions of the individual goods that minimize the expenditure to achieve any given composite consumption. The demands for the individual goods are given by conditions (4) and (5):

$$\frac{\widetilde{c}_{h,t}(j)}{C_{h,t}} = \left(\frac{1}{\theta}\right)^{\sigma} \left(\frac{p_{h,t}(j)}{P_{h,t}}\right)^{-\sigma} = \frac{\widetilde{c}^*_{h,t}(j)}{C^*_{h,t}}, \text{ for } j \in [0,\theta], \text{ and} \tag{4}$$

$$\frac{\widetilde{c}_{f,t}(i)}{C_{f,t}} = \left(\frac{1}{1-\theta}\right)^{\sigma} \left(\frac{p_{f,t}(i)}{P_{f,t}}\right)^{-\sigma} = \frac{\widetilde{c}^*_{f,t}(i)}{C^*_{f,t}}, \text{ for } i \in (1,\theta]. \tag{5}$$

[7] The $P_{h,t}$ is the minimum expenditure necessary to buy one more unit of $C_{h,t}$. Formally $P_{h,t} \equiv \min \int_0^{\theta} p_t(j)\widetilde{c}_{h,t}(j)dj$ s.t. $\left[\frac{1}{\theta}\int_0^{\theta} \widetilde{c}_{h,t}(j)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}} = 1$.

During each period households make a sequence of choices in the various markets according with the Lucas timing. In each period the assets markets open first and close before the goods markets open. Thus, in the beginning of period t, households of country H and country F enter the financial markets and allocate the wealth they brought from the previous period plus the transfer made to them by the central bank, X_t and X_t^*, respectively, between state-contingent bonds, non-state contingent bonds, B_t and B_t^*, respectively, - remunerated at a gross interest rate R_t - and cash balances, M_t and M_t^*, respectively. After leaving the financial markets, the households enter in the goods and labor markets. They supply labor, demand goods produced in both countries and face a cash-in-advance constraint, stating that all nominal consumption must be purchased with their cash-balances. At the end of the period, households of country H and country F receive wages W_t and W_t^*, respectively.

Households in every country can trade state contingent assets, but cannot trade these assets with households of the other country. For this reason, in equilibrium, the net supplies, in each of the countries, of these assets are zero. We use this result, by not including these assets, in the budget constraints of the households.

Households of country H maximize utility (1) subject to cash-in-advance constraints and budget constraints. The cash-in-advance constraints are

$$P_{h,t}C_{h,t} + P_{f,t}C_{f,t} + E_t \leq M_t, \text{ for all } t,$$

where $E_t \equiv \int_0^\theta p_{h,t}(j)\overline{c}_{h,t}(j)dj + \int_\theta^1 p_{f,t}(i)\overline{c}_{f,t}(i)di$ and the budget constraints are

$$\begin{aligned} &M_{t+1} + B_{t+1} - X_{t+1} = M_t + B_t R_t + W_t N_t \\ &-P_{h,t}C_{h,t} - P_{f,t}C_{f,t} - E_t, \text{ for all } t. \end{aligned}$$

Households of country F have a similar problem. They maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t^*, N_t^*\right),$$

subject to the cash-in-advance constraints

$$P_{h,t}C_{h,t}^* + P_{f,t}C_{f,t}^* + E_t = M_t^*, \text{ for all } t$$

and budget constraints

$$M^*_{t+1} + B^*_{t+1} - X^*_{t+1} = M^*_t + B^*_t R_t + W^*_t N^*_t$$
$$-P_{h,t}C^*_{h,t} - P_{f,t}C^*_{f,t} - E_t, \text{ for all } t.$$

The first-order conditions of the households can be summarized in the following equations, which hold for every state and all t:

$$\frac{\theta}{1-\theta}\left(\frac{C_{f,t}}{C_{h,t}}\right)^{\frac{1}{\sigma}} = \frac{P_{h,t}}{P_{f,t}} = \frac{\theta}{1-\theta}\left(\frac{C^*_{f,t}}{C^*_{h,t}}\right)^{\frac{1}{\sigma}} \tag{6}$$

$$\frac{-u_{N_t}}{u_{C_{h,t}}} = \frac{W_t}{R_t P_{h,t}} \tag{7}$$

$$\frac{-u_{N^*_t}}{u_{C^*_{f,t}}} = \frac{W^*_t}{R_t P_{f,t}} \tag{8}$$

$$\frac{1}{\beta R_t} = E_t\left(\frac{u_{C_{i,t+1}}P_{i,t}}{u_{C_{i,t}}P_{i,t+1}}\right) = E_t\left(\frac{u_{C^*_{i,t+1}}P_{i,t}}{u_{C^*_{i,t}}P_{i,t+1}}\right), \text{for } i = h, f \tag{9}$$

Conditions (6) state that the relative consumptions (net of subsistence levels) of the aggregate goods produced in each country, are inversely proportional to the relative price of the goods. Conditions (7) and (8) state that in each country the intratemporal marginal rate of substitution between leisure and consumption is equal to the relevant real wage times the inverse of the gross interest rate. In the terminology of Lucas and Stokey (1987), the interest rate introduces a wedge between the marginal rate of substitution and the relevant real wage paid by firms because leisure is a credit good and consumption is a cash good. Conditions (9) are the households' intertemporal conditions. They are an implication of the standard non-arbitrage opportunity condition: the marginal utility at date t of one unit of money must be equal to the expected marginal utility at date $t+1$ of the proceeds that result from buying bonds at time t in the amount of one unit of money.

2.2 Firms

The production functions are identical across goods and use labor as its unique input. If good j is produced in the home country it has the following production technology,

$$y_t(j) = A_t n_t(j) \text{ , with } j \in [0, \theta], \tag{10}$$

where $y_t(j)$ is the production of good j, $n_t(j)$ is labor employed by the firm producing good j, and A_t is the technology level. Similarly for any good produced in the foreign country,

$$y_t^*(i) = A_t n_t^*(i) \text{ , with } i \in (\theta, 1], \tag{11}$$

where $y_t^*(i)$ is the production of good i and $n_t^*(i)$ is labor employed by the firm producing good i.

In each country, the goods and labor markets are competitive. However, there is no labor mobility between countries. Firms in each economy hire labor at a certain wage rate, W_t at home and W_t^* in the foreign country.

For all t and for all $j \in [0, \theta]$, firm j in country H chooses the production level that maximizes its profits subject to its production function, taking all prices and wages as given. The first-order condition of this problem implies that firms at home set their prices equal to the marginal cost

$$\frac{P_{h,t}}{\theta} = p_{h,t}(j) = \frac{W_t}{A_t}, \text{ for } j \in [0, \theta] \text{ and for all } t. \tag{12}$$

The price-setting behavior of the firms in the foreign country is symmetric and therefore,

$$\frac{P_{f,t}}{1-\theta} = p_{f,t}(i) = \frac{W_t^*}{A_t} \text{ for } i \in (\theta, 1] \text{ and for all } t. \tag{13}$$

Given (12) and (13), then all firms in the same country set the same price, $p_{h,t}(j) = p_{h,t}$, for $j \in [0, \theta]$, and $p_{f,t}(i) = p_{f,t}$ for $i \in (\theta, 1]$ and for all t. Equal prices and conditions (4) and (5) imply $\widetilde{c}_{h,t}(j) = C_{h,t}$ and $\widetilde{c}_{h,t}^*(j) = C_{h,t}^*$, for $j \in [0, \theta]$ and $\widetilde{c}_{f,t}(i) = C_{f,t}$ and $\widetilde{c}_{f,t}^*(i) = C_{f,t}^*$, for $i \in (\theta, 1]$ and for all t.

2.3 Monetary Authority

The monetary union authority does two things: sets the interest rate, R_t, and injects money in the economy, through lump-sum transfers: X_t to the representative home household and X_t^* to the representative foreign household, so that money demand is satisfied. The money supply in the monetary union evolves according to $M_t^S = M_{t-1}^S + X_t + X_t^*$, where M_t^S is the total money supply in the union in period t.

2.4 Clearing Conditions

In equilibrium, all markets clear. Since there are no government bonds, the stock of bonds held by every representative household coincides with the external assets held by the country[8]. The bond market clearing condition is therefore:

$$B_t + B_t^* = 0. \tag{14}$$

The labor markets clearing conditions are:

$$\theta N_t = \int_0^\theta n_t(j)dj, \tag{15}$$

and

$$(1-\theta)\, N_t^* = \int_\theta^1 n_t^*(i)di. \tag{16}$$

The clearing of the goods markets means that production of goods equals the respective consumption. Production of good j is $A_t n_t(j)$, for $j \in [0, \theta]$. Total consumption of good j is equal to the sum across consumers in the union of their consumption of good j. Each household of country H consumes $\widetilde{c}_{h,t}(j)+\overline{c}_{h,t}(j)$ of good j, which is equal to $C_{h,t}+\overline{c}_{h,t}(j)$. Since there is a mass θ of identical consumers in country H, the total consumption by the households in country H of good j is $\theta\left[C_{h,t} + \overline{c}_{h,t}(j)\right]$. Using a similar argument it can be shown that the total consumption by the households in country F of good j is $(1-\theta)\left[C_{h,t}^* + \overline{c}_{h,t}(j)\right]$. Total consumption of good j in the union is $\theta C_{h,t} + (1-\theta)C_{h,t}^* + \overline{c}_{h,t}(j)$. Thus, clearing in the goods markets implies:

$$\theta C_{h,t} + (1-\theta)C_{h,t}^* + \overline{c}_{h,t}(j) = A_t n_t(j), \text{ for } j \in [0,\theta] \text{ and for all } t, \tag{17}$$

and

$$\theta C_{f,t} + (1-\theta)C_{f,t}^* + \overline{c}_{f,t}(i) = A_t n_t^*(i), \text{ for } i \in (\theta,1] \text{ and for all } t. \tag{18}$$

2.5 The Equilibrium with Labor Mobility

A competitive equilibrium is a sequence for each country of policies, allocations and prices such that the private agents (firms and households) solve their problems given the sequences of policies and prices and markets clear.

[8] As said before, state-contingent assets market clearing in every country was already assumed to save on notation.

When labor is mobile the two country monetary union is similar to a closed economy, with labor mobility. In this case $W = W^*$. Let $\mathfrak{p}_t \equiv \frac{p_{f,t}}{p_{h,t}}$ denote the terms of trade. Using the firms pricing conditions, (12) and (13), it is immediate that $p_{h,t} = p_{f,t}$, or that $\mathfrak{p}_t = 1$, for all t and all states. The non-existence of idiosyncratic shocks implies that markets are complete, even without state contingent bonds. Given identical fundamentals, including identical initial net external asset positions and money holdings in each country, the equilibrium is identical in both countries. The relative size of the country, $\frac{\theta}{1-\theta}$, is irrelevant and the per capita allocations are identical across countries. Thus, a monetary shock or a change in the interest rate would have an identical effect, namely on per capita aggregate consumption and hours of work, of the two countries.

Labor mobility across countries is crucial for those identical outcomes, since in general the aggregate labor supply in a particular country does not coincide with the aggregate labor demand in that country. For $\mathfrak{p}_t = 1$, relative demands (net of the subsistence levels) in every country $\frac{\widetilde{c}_{f,t}}{\widetilde{c}_{h,t}}$ and $\frac{\widetilde{c}^*_{f,t}}{\widetilde{c}^*_{h,t}}$ are equal to one. However, the relative goods demands, $\frac{c_{f,t}}{c_{h,t}}$ and $\frac{c^*_{f,t}}{c^*_{h,t}}$, are not equal to one and more labor is allocated to the firms (and the respective country) with the relatively higher demands.

With homotheticity of preferences we would have also $\mathfrak{p}_t = 1$, for all t and all states. In this case the per capita allocations would be identical across countries. Thus, even if labor is immobile across countries, the effects of aggregate shocks would be similar across countries. This is the standard result in the literature referred in the introduction. In what follows we consider the case of non-homothetic preferences.

2.6 The Equilibrium without Labor Mobility

Typically, the non existence of a global labor market for the union, implies incomplete markets. In this case equilibrium prices and aggregate allocations for the union cannot be computed independently of the allocations of each country. This loss of aggregation implies a more complex problem than the one of a closed economy with labor mobility. Even if there was a market for contingent assets, in general it would not be possible to compute the equilibrium without keeping track of the country variables over time, (see Appendix 2).

However, there is a class of preferences, even with labor market segmentation, that allows the computation of the equilibrium prices and aggregate allocations independently of the distribution of the allocations across countries. That class of preferences is the GHH class proposed by Greenwood, Hercowitz and Huffman (1988). When preferences belong to the GHH class the equilibrium has an aggregation property. Even without complete markets we can solve for the aggregate quantities and prices without having to keep track on the distribution of the allocations across countries. This aggregation result comes from the fact that labor supply is independent of the wealth distribution.

We show that the path for the equilibrium terms of trade can be determined uniquely as a function of the nominal interest rate and aggregate productivity. The equilibrium terms of trade determines the aggregate labor supply in each country and production (and consumption) of every good in the union, for every date and state. Later, in a second stage, using the income level in each country, the path of the equilibrium terms of trade and the interest rate, we compute the sequence of aggregate consumption in each country.

The instantaneous utility function of the representative home consumer is

$$u\left(C_t, N_t\right) = \frac{1}{1-\phi}\left(C_t - \epsilon\frac{\left(N_t\right)^{1+\chi}}{1+\chi}\right)^{1-\phi}, \phi > 0, \chi > 0.$$

In this case the intratemporal decisions, (6), (7) and (8), can be used to obtain the supplies of labor,

$$N_t = \left\{\frac{\left[\theta + (1-\theta)\left(\frac{1}{\mathfrak{p}_t}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}}{\epsilon R_t}\frac{W_t}{p_{h,t}}\right\}^{\frac{1}{\chi}}. \tag{19}$$

Using the fact that the representative household for the foreign country has identical preferences we get:

$$N_t^* = \left\{\frac{\left[\theta\mathfrak{p}_t^{\sigma-1} + (1-\theta)\right]^{\frac{1}{\sigma-1}}}{\epsilon R_t}\frac{W_t^*}{p_{f,t}}\right\}^{\frac{1}{\chi}}. \tag{20}$$

Observe, as noted before, that there is no income or wealth effect on the supply of labor. The supply of labor in each country is a function of the interest rate, the terms of trade and the real wage in the production.

2.6.1 The Aggregate Equilibrium

In each country, there is no heterogeneity across firms. Although they produce different goods, they have the same linear technology and face the same wage. Therefore, the equilibrium relative price across goods produced inside each country is always one. However, as we show below, in general the relative price across goods produced in different countries, i.e. the terms of trade, is different from one.

We proceed by showing first that the terms of trade, $\mathfrak{p}_t$, in every state and date, is uniquely determined, and independent of the distribution of aggregate consumptions across countries. Given this relative price, the relative consumption of the goods, hours per capita and real wages across countries, $\left\{\frac{C_{h,t}}{C_{f,t}}, \frac{C^*_{h,t}}{C^*_{f,t}}, N_t, N^*_t, \frac{W_t}{P_{h,t}}, \frac{W^*_t}{P_{f,t}}\right\}$, are also determined in every state and date, independently of distributional considerations.

The equilibrium vector $\left\{\frac{C_{h,t}}{C_{f,t}}, \frac{C^*_{h,t}}{C^*_{f,t}}, N_t, N^*_t, \frac{W_t}{P_{h,t}}, \frac{W^*_t}{P_{f,t}}\right\}$ satisfies a set of static equations for each date and state, and this lack of dynamics in the aggregate economy enables us to obtain a closed form solution for it. If that was not the case, for instance if capital was an input in production or prices were sticky, as in the next section, we would have to solve numerically for this equilibrium vector, but the level of complexity of such procedure would be similar to the one in a standard closed economy model, with a representative household.

For every date and state the equilibrium vector $\left\{\frac{C_{h,t}}{C_{f,t}}, \frac{C^*_{h,t}}{C^*_{f,t}}, N_t, N^*_t, \frac{W_t}{P_{h,t}}, \frac{W^*_t}{P_{f,t}}\right\}$ depends on the level of technology and the interest rate, at that state and date[9]. To compute the distribution of consumptions across countries, we use the intertemporal budget constraints and intertemporal conditions for households in each country together with the realized values of this equilibrium

[9]It is well known in the literature, that setting an exogenous path for the interest rate does not, in the flexible price environment, determine uniquely the path of prices, or the path of inflation in a stochastic environment with monetary shocks. The indeterminacy is reflected in the initial price level, which given identical economies with zero stock of initial external assets, does not affect the real equilibrium.

vector, for every date and state.[10] Once we have that, we can compute the productions of each individual good as well as the aggregate production of each country.

The equilibrium conditions described above, (6), (12), (13), (19) and (20), imply,

$$\frac{C_{h,t}}{C_{f,t}} = \frac{C^*_{h,t}}{C^*_{f,t}} = \mathfrak{p}_t^{\sigma}, \tag{21}$$

$$\frac{W_t}{p_{h,t}} = \frac{W^*_t}{p_{f,t}} = A, \tag{22}$$

$$N_t = \left\{ \frac{A}{\epsilon R_t} \left[\theta + (1-\theta) \left(\frac{1}{\mathfrak{p}_t} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} \right\}^{\frac{1}{\chi}}, \tag{23}$$

and

$$N^*_t = \left\{ \frac{A}{\epsilon R_t} \left[\theta \mathfrak{p}_t^{\sigma-1} + (1-\theta) \right]^{\frac{1}{\sigma-1}} \right\}^{\frac{1}{\chi}}. \tag{24}$$

Conditions (21) indicate that the relative net consumption, net of the subsistence level, of the good produced in country H depends negatively on the relative price of that good. Conditions (22) say that producer wages, measured in terms of the national goods, are a positive function of the common technology level. Via (23) and (24) we know that each labor supply depends negatively on the interest rate and positively on the technology level and the relative price of the good that uses that labor as input.

Using (15) and (17) we get

$$\theta C_{h,t} + (1-\theta) C^*_{h,t} + \overline{C}_{h,t} = A_t \frac{1}{\theta} \int_0^{\theta} n_t(j) dj = A_t N_t \equiv Y_{h,t} \tag{25}$$

where $\overline{C}_{h,t} = \frac{1}{\theta} \int_0^{\theta} \overline{c}_{h,t}(j) dj$. The parameter $\overline{C}_{h,t}$ can be interpreted as the average of the subsistence levels for the goods produced in country H. Equation (25), (23) and the production functions (10) imply

$$\begin{aligned} & A \left\{ \frac{A}{\epsilon R_t} \left[\theta + (1-\theta) \left(\frac{1}{\mathfrak{p}_t} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} \right\}^{\frac{1}{\chi}} - \overline{C}_{h,t} \qquad (26) \\ = \;& \theta C_{h,t} + (1-\theta) C^*_{h,t}. \end{aligned}$$

[10] We followed a straightforward procedure that is described in the Appendix 1.

We have a similar market clearing condition for the goods produced in country F:

$$\begin{aligned} & A\left\{\frac{A}{\epsilon R_t}\left[\theta\left(\mathfrak{p}_t\right)^{\sigma-1}+(1-\theta)\right]^{\frac{1}{\sigma-1}}\right\}^{\frac{1}{\chi}}-\overline{C}_{f,t} \\ = \quad & \theta C_{f,t}+(1-\theta)\, C^*_{f,t}. \end{aligned} \tag{27}$$

where $\overline{C}_{f,t}=\frac{1}{1-\theta}\int_\theta^1 \overline{c}_{f,t}(i)di$. We obtain, making use of (21), that the ratio of these two clearing conditions, (26) and (27), is given by:

$$RNS_t \equiv \frac{\Upsilon_t\left[\theta+(1-\theta)\left(\frac{1}{\mathfrak{p}_t}\right)^{\sigma-1}\right]^{\frac{1}{\chi(\sigma-1)}}-\overline{C}_{h,t}}{\Upsilon_t\left[\theta(\mathfrak{p}_t)^{\sigma-1}+(1-\theta)\right]^{\frac{1}{\chi(\sigma-1)}}-\overline{C}_{f,t}}=(\mathfrak{p}_t)^{\sigma}\equiv RND_t \tag{28}$$

where $\Upsilon_t \equiv A\left(\frac{A}{\epsilon R_t}\right)^{\frac{1}{\chi}}$.

The left hand side of (28) is the relative net supply, net of the subsistence level, (RNS_t) of the home good. The RNS_t depends negatively on the relative price of the foreign good. The right hand side of (28), RND_t, is the relative net demand, net of the subsistence level, of the home good. This ratio depends positively on the relative price of the foreign good. Market clearing, $RNS_t = RND_t$, guarantees a unique $\mathfrak{p}_t$.

Therefore, given the interest rate path and the technology level, we can use (28) to compute the equilibrium path for the terms of trade, use (21) to compute the relative consumptions, and use (23) and (24) to compute the labor supplies. Real wages are given by (22).

When $\overline{C}_{h,t}=\overline{C}_{f,t}$, it is immediate to see that the equilibrium relative price is one, $\mathfrak{p}_t = 1$, as it satisfies (28). In this particular case labor immobility is irrelevant, since in this case $W_t = W^*_t$.

If $\overline{C}_{h,t} > \overline{C}_{f,t}$, it is easy to verify, using (28), that for a relative price equal to one, the RNS_t will be less than one, but the RND_t will be equal to one. As RNS_t is a negative function of $\mathfrak{p}_t$ and RND_t a positive function of $\mathfrak{p}_t$, the equilibrium relative price $\mathfrak{p}_t$ will have to be smaller than one. Thus, the quantity produced, and consumed of the home good will be larger than the quantity produced of the foreign good. We state this result as a Proposition.

Proposition 1: If the average subsistence level of the goods produced in country H, $\overline{C}_{h,t}$, is larger than the average subsistence level of the goods produced in country F, $\overline{C}_{f,t}$, then the equilibrium terms of trade are smaller

than one, $\mathfrak{p}_t < 1$. *Conversely, if* $\overline{C}_{h,t} < \overline{C}_{f,t}$ *then* $\mathfrak{p}_t > 1$. *Finally, if* $\overline{C}_{h,t} = \overline{C}_{f,t}$ *then* $\mathfrak{p}_t = 1$.

We have shown that if $\overline{C}_{h,t} > \overline{C}_{f,t}$, in equilibrium the supply of the home good is higher than the supply of the foreign good. The per capita output will be higher in the home country than in the foreign country. Moreover, the per capita hours of work will be relatively higher in the home country because productivity is identical in both economies. Thus, in a stationary equilibrium, the per capita total consumption is also relatively higher in the home country.

Corollary: If $\overline{C}_{h,t} > (<)\overline{C}_{f,t}$ *then, for a constant* R_t, *the home country will have higher (lower) consumption and production than the foreign country.*

The main result of this section goes against the intuition that two regions identical in per capita fundamentals should have identical per capita equilibrium allocations. As we saw above, this intuition would be correct if there was a global labor market for the whole monetary union. In this case allowing for trade across countries of contingent nominal assets would be irrelevant. However the inverse is not true. The presence of a market of state contingent nominal assets, when the labor market is segmented, will result in different per capita allocations among countries[11]. Thus, this main result is crucially driven by the labor immobility assumption.

Even though countries have identical preferences, production functions and initial wealth the average subsistence level of the goods in which they specialize may be different across countries. We do not explain the trade pattern. Specialization is completely exogenous in the model, as we wanted to have a simple model. However, as referred in section 2, in the presence of increasing returns, or alternatively with constant returns but industry entry costs as in Krugman (1980), there will be specialization and trade across countries to reap the economies of scale.

The bias that the terms of trade different from one imposes on each specific country's equilibrium implies, as we describe in the next section, that a common shock will have asymmetric outcomes across countries.

2.7 The Effects of the Monetary Shock

As we described, the differences across countries are related with the equilibrium terms of trade being different from one. We first determine how the

[11] We show this in the Appendix 2.

terms of trade react to the monetary policy shock and later we discuss the effect of this change in the individual countries.

2.7.1 The Response of the Terms of Trade to the Monetary Shock

Understanding how does the terms of trade react to a common shock is therefore key to understand how that shock can lead to different outcomes across countries. Later we will describe quantitatively these effects in a model with a price stickiness. Right now we explore the intuition behind the effects of the common shock on the terms of trade in the model with flexible prices.

We saw that if $\overline{C}_{h,t} = \overline{C}_{f,t}$ the equilibrium relative price would be one. In this case a decline in the interest rate makes leisure relatively more expensive in both countries and as a result households supply more labor and productions increase. But, for $\mathfrak{p}_t = 1$, the relative production remains unchanged and the same happens with the relative demand. Thus, the equilibrium relative price does not change. In this particular case the monetary shock has identical effects across countries. However in general, we have $\overline{C}_{h,t} \neq \overline{C}_{f,t}$, and an aggregate monetary shock, a revision of the interest rate, changes the terms of trade. This happens because the change in the interest rate, for the initial equilibrium terms of trade, leads to a discrepancy between the relative demand and the relative supply. To show this we make use of Figure 1. Figure 1 shows how the relative price of the home good is affected by a decline of the interest rate. The relative net demand, RND_t, is a negative function of the relative price of the home good $(1/\mathfrak{p}_t)$, and is not a function of the interest rate. The relative net supply, RNS_t, is a positive function of the relative price of the home good, and also a function of the interest rate. When the interest rate decreases the the curve RNS_t moves to the right if $\overline{C}_{h,t} > \overline{C}_{f,t}$ and to the left if $\overline{C}_{h,t} < \overline{C}_{f,t}$. We prove this now. Let RNS_t be rewritten as $\frac{k-\frac{\overline{C}_{h,t}}{Y_{f,t}}}{1-\frac{\overline{C}_{f,t}}{Y_{f,t}}}$, where $k \equiv \frac{Y_{h,t}}{Y_{f,t}}$, then

$$\frac{\partial RNS_t}{\partial R_t} = \frac{\partial RNS_t}{\partial Y_{f,t}} \cdot \frac{\partial Y_{f,t}}{\partial R_t},$$

where

$$\frac{\partial RNS_t}{\partial Y_{f,t}} = \frac{\frac{\overline{C}_{h,t}}{Y_{f,t}^2}\left(1-\frac{\overline{C}_{f,t}}{Y_{f,t}}\right)-\frac{\overline{C}_{f,t}}{Y_{f,t}^2}\left(k-\frac{\overline{C}_{f,t}}{Y_{f,t}}\right)}{\left(1-\frac{\overline{C}_{f,t}}{Y_{f,t}}\right)^2}.$$

Notice that k is not a function of the interest rate. Since $Y_{f,t}$ decreases when R_t increases, i.e. $\frac{\partial Y_{f,t}}{\partial R_t} < 0$, the sign of the change in RNS_t due to a decrease in R_t is the sign of $\frac{\partial RNS_t}{\partial Y_{f,t}}$,

$$-\text{sign}\frac{\partial RNS_t}{\partial R_t} = \text{sign}\frac{\partial RNS_t}{\partial Y_{f,t}} = \text{sign}\left\{\overline{C}_{h,t} - k\overline{C}_{f,t}\right\}.$$

If $\overline{C}_{h,t} > \overline{C}_{f,t}$, then from (28) we get $\mathfrak{p}_t < 1$, $k > 1$, and

$$\frac{1 - \frac{\overline{C}_{h,t}}{Y_{h,t}}}{1 - \frac{\overline{C}_{f,t}}{Y_{f,t}}} = (\mathfrak{p}_t)^{\sigma} \frac{1}{k} < 1.$$

Using the inequality $(\mathfrak{p}_t)^{\sigma} < k$ and (28) we obtain

$$k - \frac{\overline{C}_{h,t}}{Y_{f,t}} < k\left(1 - \frac{\overline{C}_{f,t}}{Y_{f,t}}\right).$$

Thus, $\overline{C}_{h,t} - k\overline{C}_{f,t} > 0$ and $\frac{\partial RNS_t}{\partial Y_{f,t}} > 0$. This proves that $\frac{\partial RNS_t}{\partial Y_{f,t}} > 0$ (< 0) for any $\overline{C}_{h,t} > \overline{C}_{f,t}$ $(\overline{C}_{h,t} < \overline{C}_{f,t})$. Thus, after a decline in the interest rate the curve RNS_t moves to the right if $\overline{C}_{h,t} > \overline{C}_{f,t}$ and to the left if $\overline{C}_{h,t} < \overline{C}_{f,t}$. Using Figure 1 it is straightforward to see that the relative price of the home good, $\frac{1}{\mathfrak{p}_t}$, decreases (increases) for $\overline{C}_{h,t} >(<) \overline{C}_{f,t}$. This result is stated as a proposition:

Proposition 2: In general a positive monetary shock (a decrease in R_t) in a monetary union with two identical countries leads to an increase (decline) of the terms of trade, $\mathfrak{p}_t$, when $\overline{C}_{h,t} >(<) \overline{C}_{f,t}$.

Proposition 1 and 2 allow us to say that:

Corollary: Monetary shocks in a monetary union create fluctuations characterized by lower volatility of output for the country with higher average output. The mechanism responsible for both the different average and cycle is the path of the terms of trade.

The Corollary states that the country with higher output, in per capita terms, is also the one that is going to experience lower volatility of production and hours of work. When the shock is positive it is the richer country that benefits the less, while when the shock is negative it is the richer country that is harmed the less. There are two channels through which a change in

the interest rate affects every national economy, in this flexible price model. In the cash-in-advance economy, that we consider here, the interest rate is a wedge between the marginal rate of substitution and the marginal rate of transformation. A reduction in the interest rate has a direct effect over production, increases the production of both goods, due to the decrease in the wedge. And has an indirect effect through its consequence on the terms of trade. Our emphasis on this channel comes from the fact that it is this indirect effect of the monetary shock that creates the asymmetric responses across countries. As we have seen in Proposition 1 there is a one to one relationship between the interest rate and the terms of trade. The decline of the interest rate, through its indirect effect on the terms of trade, will have a negative effect over the production in one country and a positive effect over the production of the other. This indirect effect affects with opposite signs the households' incomes of each country.

2.7.2 The Effects of the Monetary Shock in the Individual Countries

To determine the effect on aggregate consumption in every country it is necessary to take a position on the assets markets across countries. When the only asset traded across countries is the state non-contingent bond, the equilibrium consumption for each country can be computed after the determination of the aggregate equilibrium for the union. As we just described we can compute the aggregate allocations and terms of trade with no information about the distribution of consumption across countries. Given these equilibrium values, the consumption path of each country is pinned down using the remaining equilibrium conditions: the households' budget constraints and intertemporal equations and the non Ponzi game conditions. The construction of the intertemporal constraints for each country is straightforward but cumbersome. The Appendix 1 describes this construction as well as the determination of the consumption of each country.

Once understood that monetary shocks affect the terms of trade, it is immediate to see that monetary shocks can affect differently every national economy. For temporary shocks, and given the chosen GHH preferences, those different effects will be temporary for those aggregates which are stationary like labor and output. However, consumption of either good or aggregate consumption in each country will be affected permanently. These permanent effects on consumption are associated with permanent effects on

the position of each economy in external assets holdings. Given an initial position of zero net foreign debt, and conditional to the temporary monetary shock that we have been analyzing, the country that produces the good whose price is temporarily higher will have a consumption higher forever and a permanent balance of trade deficit, that will be financed by the assets accumulated during the periods when, given the higher terms of trade, the economy had a trade balance surplus with the rest of the union. The more pronounced the cumulative effect on the terms of trade, the higher will be the permanent effect on consumption and on the net asset position of each national economy.[12]

The distribution of the inflation tax revenue is another way through which monetary policy could have asymmetric effects on the countries. We assumed that this seigniorage distribution is equitable to highlight the terms of trade channel.

Thus, we can conjecture, and quantify later, that the welfare level of the rich country is higher not only because it has a higher stationary level of consumption and production, but also because it has less volatile consumption and hours of work.

3 The Model with Calvo Prices

It remains to see whether the asymmetric effects coming from the terms of trade reaction to the monetary policy shocks are quantitatively significant. Since most of the recent literature stresses nominal rigidities as the main transmission mechanism of monetary shocks, we analyze whether the sort of reasoning developed in the previous section can be extended to that type of environments, and quantify the potential difference of outcomes across countries when there is nominal rigidities. Therefore, in this section we consider an extremely simplified model with the most used nominal rigidity in the literature, namely we impose that firms set prices according to Calvo (1983). We study the effects of a monetary shock in this environment and, as before, we investigate its transmission to the terms of trade over time. We are interested in determining whether the response of this variable to a monetary

[12]The results hold even with state contingent markets. Since the preferences used in this paper are not separable, the lower volatility of hours in the rich country will be transmitted in lower volatility of consumption when marginal utilities of aggregate consumption across countries are smoothed across states.

shock is significantly different from zero. If so, then the effects of this terms of trade path on the asymmetric transmission of the shock are similar to the ones described in the previous section. The transmission of the shock to the prices is different, but the transmission of the terms of trade to the other equilibrium variables is the same, since we maintain the same environment for the households. We evaluate the magnitude of this asymmetric shock for a basic calibration of the model, and compare quantitatively the idiosyncratic effects of the monetary shock with its union wide effect.

We begin by describing the changes introduced in the model described in section 2 to accommodate the sticky price friction. The behavior of households is the same as in the flexible prices economy. In contrast, firms behave differently. Firms behave as monopolisitc competitors and set prices as in Calvo (1983).[13]

To take into account the possibility of heterogeneous price behavior by firms, we follow Calvo (1983) and assume that in each period only a fraction $(1-\xi_p)$ of firms is able to change prices optimally. Those firms that cannot re-optimize update their prices according to the lagged inflation in the continuum of goods produced in their country. We maintain the identical countries assumption by using the same probability of revising prices across firms and across countries.

When a specific firm can re-optimize it chooses the price that maximizes expected discounted future profits subject to the total demand for good j produced in country H. The log-linearization of the first order condition of this problem, around the steady state, and the aggregation of the log-linearized equations, for both optimizing and non-optimizing firms, yields the following equation for the aggregate inflation of the goods produced in the home country, $\pi_{h,t}$, defined as the growth rate of $P_{h,t}$,

$$\widehat{\pi}_{h,t} - \tfrac{\beta}{1+\beta} E_t \widehat{\pi}_{h,t+1} - \tfrac{1}{1+\beta} \widehat{\pi}_{h,t-1} - \tfrac{\left(1-\xi_p\right)\left(1-\xi_p\beta\right)}{\xi_p(1+\beta)} \left(\widehat{W}_t - \widehat{P}_{h,t} - \widehat{A}_t \right) = 0, \quad (29)$$

where the variables with hat denote deviations from their steady state values. In this framework, inflation of the goods produced in the home country

[13] *Ravn et al (2008) show, in a closed economy, that when preferences display non-homotheticity and firms are monopolistic competitors then mark-ups move with aggregate shocks. In order to isolate our results from varying mark ups we assume that governments tax profits from firms and give contingent subsidies to production, in the spirit of Correia, Teles and Nicolini (2008), so that the mark ups are invariant with aggregate shocks.*

depends on lagged inflation, future inflation and current marginal costs of the goods produced in the home country.

The problem of each foreign firm that can choose the price is similar to the problem of the domestic firm that can choose the price. Similarly, those foreign firms that cannot re-optimize update their prices with the lagged inflation in the continuum of goods produced in their country. The equation for the inflation of goods produced in the foreign country, $\pi_{f,t}$, defined as the growth rate of $P_{f,t}$, is

$$\widehat{\pi}_{f,t} - \tfrac{\beta}{1+\beta} E_t \widehat{\pi}_{f,t+1} - \tfrac{1}{1+\beta} \widehat{\pi}_{f,t-1} - \tfrac{\left(1-\xi_p\right)\left(1-\xi_p\beta\right)}{\xi_p(1+\beta)} \left(\widehat{W}_t^* - \widehat{P}_{f,t} - \widehat{A}_t \right) = 0, \quad (30)$$

is completely analogous to (29), with the variables $\widehat{\pi}_{f,t}$, $\widehat{W}_t^*$, and $\widehat{P}_{f,t}$ replacing $\widehat{\pi}_{h,t}$, $\widehat{W}_t$ and $\widehat{P}_{h,t}$, respectively.

As is standard in the literature the central bank conducts monetary policy through an interest rate rule that guarantees local determinacy. In its loglinearized form the simple rule followed by the central bank is

$$\widehat{R}_t = \rho_0 \cdot \widehat{R}_{t-1} + \rho_1 \cdot \widehat{\Pi}_t + \widehat{\varepsilon}_t \quad (31)$$

where Π_t is the inflation of the union, the growth level of P_t, $\widehat{\varepsilon}_t$ is a random shock to the monetary policy and ρ_0 and ρ_1 coefficients.[14] What we want to study is the transmission mechanism of a monetary policy shock, i.e. to determine the effects on the main variables of an innovation in $\widehat{\varepsilon}_t$.

The economy with flexible prices is identical to the economy with sticky prices, except that the firms and the central bank behave differently. Thus, the system of equations that determines the equilibrium in the flexible prices economy differs from the system of equations that determines the equilibrium in the sticky prices economy, only on those equations associated with the behavior of the firms and the central bank. More specifically, the first order conditions (12) and (13) are replaced with the first order conditions of the firms' problems, described in this section, (29) and (30). The central bank now, instead of choosing the interest rate, follows the simple rule, (31).

[14] In its nonlinearized version the interest rate is $\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_0} \left(\frac{\Pi_t}{\Pi}\right)^{\rho_1} \varepsilon_t$, where R is the steady state interest rate and Π the gross steady state inflation rate of the union, *which we take to be one*. We do not discuss the optimality of this rule, as we do not assume that monetary policy aims at minimizing a specific loss function. Instead we assume that the interest rate rule is a good representation of the behavior of the monetary policy maker.

preferences,	$\beta = 0.993$	$\chi = 0.5$	$v = 1.2$
	$\phi = 1$	$\theta = 0.5$	$\epsilon = 1$
technology		$A = 1$	
levels of subsistence	$\overline{C}_h = 0.2$	$\overline{C}_f = 0.1$	
price-setting frictions		$\xi_p = \xi_p^* = 0.67$	

Table 1: The benchmark calibration

3.1 The Effects of a Monetary Shock

3.1.1 Calibration

The calibration follows the literature, so we will not justify it in detail (see, for example, Christiano, Eichenbaum and Evans (2005)). Table 1 presents the calibration of all parameters. In the Calvo price setting environment we assume that firms change prices on average every 3 quarters. The firms' steady state mark-up is calibrated to be 1.2 and the inverse of the elasticity of labor supply to be 0.5. The countries are of equal size. Preferences are such that the consumption subsistence level is higher for goods produced in country H.

As said before the central bank follows an interest rate rule and the monetary shock is identified as a disturbance $\widehat{\varepsilon}_t$ in that interest rate policy rule. The specific interest rate rule considered was

$$\widehat{R}_t = 0.95 \cdot \widehat{R}_{t-1} + 1.5 \cdot \widehat{\Pi}_t + \widehat{\varepsilon}_t.$$

This rule satisfies fundamental requisites: the equilibrium interest rate obtained from it has a high degree of persistence as in the data, and has parameters that guarantee local determinacy of the equilibrium.

We assume that all revenue raised by the central bank from the inflation tax is redistributed back to each fiscal authority. To maintain neutrality we take that each government receives an identical per capita payment from the central bank.

3.1.2 Results

Figure 2 shows the shock and the persistent path of the monetary instrument, the nominal interest rate. All variables are measured in deviations from the steady state. For the chosen parameters, on impact the annualized interest

rate declines 40 basis points. The magnitudes of the aggregate effects are roughly in line with the ones find in the literature, even though our model misses many details necessary to replicate the exact qualitative and quantitative characteristics of the monetary shock on the aggregate equilibrium. As usual the expansionary shock increases production of all goods, consumption and inflation. The dynamics are somewhat different from the ones found in more sophisticated models, namely the inverted U shape is missing since we do not have any real friction in this model.

Qualitatively the terms of trade under sticky prices appear to behave as they do under flexible prices, i.e. according with Proposition 2. An expansionary monetary shock implies, under sticky prices, a decline in the terms of trade for the country that has them higher in the steady-state.[15]

We want to use this numerical exercise to take a position on the magnitude of the asymmetrical effects on the aggregate consumption in every country. Therefore we construct, as described in the Appendix 1, the country level path of private consumption given the path of the nominal interest rate, the path of inflation, the path of the terms of trade and the path of income in each country. Figure 3 shows the paths in levels of the country variables. We compute the percentage deviations of aggregate consumption in the home and in the foreign country and represent them in Figure 4. As we expected, the country variables response is not identical after the monetary shock. The monetary shock has two effects. It has a direct effect, as in the one good closed economy model, an identical increase in the consumption of each country. However, there is an additional indirect way of transmitting the monetary shock - through the terms of trade. This channel has opposite effects in each country and therefore the sum of both the direct and the indirect effect creates an asymmetry in the response of consumption in each country. For the calibration that is proposed in this example the asymmetry implies that on impact the consumption in the home good increases by 1.09 pp while in the foreign country it increases by 1.25 pp. This means that the differential is 0.16 pp. To take a position on whether this is a small or large number we use as metric the response, on impact to the shock, of the union aggregate consumption (Fig 2). It increases on impact by 1.17 pp. Therefore the differential across countries is 14% of this aggregate effect. According to this number the asymmetry is significative. In addition, as stated in

[15]This response of the terms of trade to a monetary shock is robust to changes in the parameters.

Proposition 2, the effect in the output is lower for the "richer" country.

The Corollary to Proposition 2 is confirmed in this environment too. When there are monetary shocks, the lower volatility of the momentary utility and consumption, for the home country reinforces the higher stationary value of the utility and consumption in this country.

In a monetary union, a monetary policy shock affects differently countries with the same fundamentals. The shock is amplified in the poorer country and is restrained in the richer one. The effect of a contractionary monetary policy shock, which is negative on the aggregate, is moderated in the richer country and is augmented in the poorer country. Thus, although positive monetary shocks tend to make countries more similar in per capita terms, the opposite occurs with negative shocks and therefore they reinforce the welfare asymmetry that characterizes these countries in a stationary environment.

4 Concluding Remarks

The conventional wisdom is that common shocks should be transmitted identically in a set of countries connected by trade and with no differences in fundamentals. Therefore, a monetary shock in a monetary union composed by identical countries would not have idiosyncratic effects.

In this paper we show that this conventional wisdom is not a general result. The crucial assumptions are segmented labor markets and non unitary income elasticities of total demand for the goods. The first is the standard one in international economics. The second is not the usual one in macroeconomics when the focus is on one aggregate consumption per date and state. It is a well documented fact that different goods have different income elasticities and is reasonable to assume that the aggregate traded consumption goods inherited this property.

We described the effects of a common monetary shock but it is trivial to verify that these results can be extended to common technology shocks. Since monetary policy should be used primarily for stabilization policy, i.e. in response to technological or other shocks, it is crucial to understand not only the monetary transmission but also the transmission of these other shocks.

Controlling for country-specific shocks, our result is that richer countries exhibit less volatile cycles, which is not in conflict with the empirical evidence. During most of the last 50 years, less developed countries tend to have higher output volatility. The plot of the standard deviation of annual (per capita)

growth rates against the level of real GDP per capita, for a large cross section of countries, has a quite pronounced negative slope (see Koren and Tenreyro (2007) and Lucas (1988)). There are some explanations in the literature for this result, our paper proposes a different one.

This result can be regarded as complementary to the well known result that changes in the terms of trade, originated from country specific shocks, can replicate, partially or totally, state complete markets. This automatic insurance mechanism of the terms of trade, that occurs with idiosyncratic shocks, is reversed when shocks are common. As we showed the change in the terms of trade is the mechanism that makes the common shock have asymmetric country specific outcomes.

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5 Appendix 1: Aggregate Consumption Across Countries

Here we show how the equilibrium consumption path of each country is determined. In section 2.6 we showed how the equilibrium path of the variables $\left\{\frac{C_{h,t}}{C_{f,t}}, \frac{C^*_{h,t}}{C^*_{f,t}}, N_t, N^*_t, \frac{W_t}{P_{h,t}}, \frac{W^*_t}{P_{f,t}}, \mathfrak{p}_t\right\}$ is determined once the path of the interest rate is given. The consumption path of each country is computed using this vector of variables together with the intertemporal budget constraints and intertemporal conditions for each country.

Conditions (9) imply

$$\left(\left[\theta+(1-\theta)\frac{C_{f,t}^{\frac{\sigma-1}{\sigma}}}{C_{h,t}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}C_{h,t}-\epsilon\frac{(N_t)^{1+\chi}}{1+\chi}\right)\left[\theta+(1-\theta)\frac{C_{f,t}^{\frac{\sigma-1}{\sigma}}}{C_{h,t}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$

$$=\left(\beta R_t\frac{P_{h,t}}{P_{h,t+1}}\right)^{-\frac{1}{\phi}}\left[\theta+(1-\theta)\frac{C_{f,t+1}^{\frac{\sigma-1}{\sigma}}}{C_{h,t+1}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$

$$\left(\left[\theta+(1-\theta)\frac{C_{f,t+1}^{\frac{\sigma-1}{\sigma}}}{C_{h,t+1}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}C_{h,t+1}-\epsilon\frac{(N_{t+1})^{1+\chi}}{1+\chi}\right),\ t=0,1,... \tag{32}$$

There is a similar condition for the foreign country,

$$\left(\left[\theta+(1-\theta)\frac{C_{f,t}^{\frac{\sigma-1}{\sigma}}}{C_{h,t}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}C_{h,t}^{*}-\epsilon\frac{(N_t^*)^{1+\chi}}{1+\chi}\right)\left[\theta+(1-\theta)\frac{C_{f,t}^{\frac{\sigma-1}{\sigma}}}{C_{h,t}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$

$$=\left(\beta R_t\frac{p_{h,t}}{p_{h,t+1}}\right)^{-\frac{1}{\phi}}\left[\theta+(1-\theta)\frac{C_{f,t+1}^{\frac{\sigma-1}{\sigma}}}{C_{h,t+1}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$

$$\left(\left[\theta+(1-\theta)\frac{C_{f,t+1}^{\frac{\sigma-1}{\sigma}}}{C_{h,t+1}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}C_{h,t+1}^{*}-\epsilon\frac{\left(N_{t+1}^*\right)^{1+\chi}}{1+\chi}\right),\ t=0,1,...$$

If we add up the home constraints, after multiplying them by θ, and the foreign constraints, after multiplying them by $1-\theta$, and use the resource constraints we get

$$\left(\left[\theta+(1-\theta)\frac{C_{f,t}^{\frac{\sigma-1}{\sigma}}}{C_{h,t}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}\left(A_tN_t-\overline{C}_h\right)-\epsilon\frac{\theta(N_t)^{1+\chi}+(1-\theta)(N_t^*)^{1+\chi}}{1+\chi}\right)\left[\theta+(1-\theta)\frac{C_{f,t}^{\frac{\sigma-1}{\sigma}}}{C_{h,t}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$

$$=\left(\beta R_t\frac{P_{h,t}}{P_{h,t+1}}\right)^{-\frac{1}{\phi}}\left[\theta+(1-\theta)\frac{C_{f,t+1}^{\frac{\sigma-1}{\sigma}}}{C_{h,t+1}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$

$$\left(\left[\theta+(1-\theta)\frac{C_{f,t+1}^{\frac{\sigma-1}{\sigma}}}{C_{h,t+1}^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}\left(A_{t+1}N_{t+1}-\overline{C}_h\right)-\epsilon\frac{\theta(N_{t+1})^{1+\chi}+(1-\theta)\left(N_{t+1}^*\right)^{1+\chi}}{1+\chi}\right),\ t=0,1,... \tag{33}$$

Equations (33) determine $\left\{\frac{P_{h,t}}{P_{h,t+1}}\right\}_{t=0}^{\infty}$, as all the other variables in these equations are already known.

Given $\left\{\frac{P_{h,t}}{P_{h,t+1}}\right\}_{t=0}^{\infty}$ from the intertemporal conditions (32) we obtain $\{C_{h,t}\}_{t=1}^{\infty}$ as a function of $C_{h,0}$. Given $\left\{\frac{P_{f,t}}{P_{h,t}}\right\}_{t=0}^{\infty}$, we get $\{C_{f,t}\}_{t=0}^{\infty}$ as a function of $C_{h,0}$, as well. The intertemporal budget constraint of the representative consumer in the home country is

$$\sum_{t=0}^{\infty} Q_t \left(P_{h,t}\left(C_{h,t}+\overline{C}_h\right)+P_{f,t}\left(C_{f,t}+\overline{C}_f\right)-P_{h,t}Y_t\right)=\mathbb{W}_0+\sum_{t=0}^{\infty} Q_t X_t \quad (34)$$

where $Q_t=\frac{\beta^t u_{C_{h,t}} P_{h,0}}{u_{C_{h,0}} P_{h,t}}$, is the value at 0 of a monetary unit at t, $Q_0=1$, and $\mathbb{W}_0$ is the initial nominal wealth of the representative household of the home country. Once we rewrite $\{C_{h,t}\}_{t=0}^{\infty}$ and $\{C_{f,t}\}_{t=0}^{\infty}$ as functions of $C_{h,0}$, condition (34) determines the value of $C_{h,0}$. Given the value of $C_{h,0}$ we can compute the whole path $\{C_{h,t}, C_{f,t}\}_{t=0}^{\infty}$. Using equation (2) we obtain the equilibrium path of the home country aggregate consumption. The foreign consumptions $\left\{C^*_{h,t}, C^*_{f,t}, C^*_t\right\}_{t=0}^{\infty}$ can be obtained in a similar manner, or instead by using the resource constraints.

6 Appendix 2: Complete Markets and Labor Mobility

As claimed in the text when labor is mobile the two country economy is similar to a typical closed economy.

Proposition 5: Independently of the households' preferences if countries have zero initial wealth and labor is mobile across countries then the per capita consumption of every good and the supply of labor are equal across households. Thus, state contingent markets are redundant.

Proof: If labor is mobile implies equal nominal wages across countries, $W_t=W^*_t$, the price-setting behavior of firms, (12) and (13), implies, $p_t(j)=p_t(i)=p_t$, for all t, $j\in[0,\theta]$ and $i\in(\theta,1]$. Therefore the terms of trade are one, i.e. $\mathfrak{p}_t=1$, $N_t=N^*_t$, $C_{h,t}=C_{f,t}=C_t$ and $C^*_{h,t}=C^*_{f,t}=C^*_t$. In this case the period t intertemporal budget constraints for the representative

households are

$$\sum\nolimits_{s=t}^{\infty} E_t Q_{t,s+1} \left[P_{h,s} \left(C_{h,s} + \overline{C}_h \right) + P_{f,s} \left(C_{f,s} + \overline{C}_f \right) - W_s N_s \right] = \mathbb{W}_t + \sum_{t=0}^{\infty} Q_t X_t$$
for all states and t,

and

$$\sum\nolimits_{s=t}^{\infty} E_t Q^*_{t,s+1} \left[P_{h,s} \left(C^*_{h,s} + \overline{C}_h \right) + P_{f,s} \left(C^*_{f,s} + \overline{C}_f \right) - W_s N^*_s \right] = \mathbb{W}^*_t + \sum_{t=0}^{\infty} Q_t X_t,$$
for all states and t,

where $\mathbb{W}_t$ is the nominal wealth of the home representative household in period t, and $Q_{t,t+1}$ is the price, in the home state-contingent market, at date t of one monetary unit at a particular state at date $t+1$. Thus $Q_{t-1,t} \frac{u_{C_{h,t-1}}}{P_{t-1}} = \frac{\beta u_{C_{h,t}}}{P_t}$, for all dates and states. $Q_{t,s} = Q_{t,t+1} ... Q_{s-1,s}$, $t \geq 0$, $s \geq t+1$, and $Q_{t,t} = 1$. $\mathbb{W}^*_t$ and $Q^*_{t,s+1}$ are defined similarly. Clearly if $\mathbb{W}_0 = \mathbb{W}^*_0 = 0$ then $C_t = C^*_t$, for all dates and all states, satisfies all intertemporal budget constraints.■

Thus, in equilibrium the per capita aggregate consumption and the supply of labor is independent of the country of residence and the existence of a market for a nominal state contingent bond across countries is redundant.

However, if labor is immobile a single nominal state contingent market for the union is not enough to avoid changes in the terms of trade, and asymmetric responses of output and consumption across countries as a result of common shocks in the union.

Proposition 6: A monetary shock in a monetary union environment with labor immobility across countries and a nominal state global contingent bond market has asymmetric effects across similar countries.

In this environment the terms of trade, hours per capita, productions and real wages across countries, $\left\{ \frac{C_{h,t}}{C_{f,t}}, \frac{C^*_{h,t}}{C^*_{f,t}}, N_t, N^*_t, \frac{W_t}{P_{h,t}}, \frac{W^*_t}{P_{f,t}}, \mathfrak{p}_t \right\}$, continue to be determined in every state and date, by the same equations. Thus, these variables behave in the same way in the two different environments. The terms of trade and the differences in hours across countries change with a monetary shock and the country with higher average output will experience lower volatility of output. It remains to see, in this context, how each country's consumption reacts to the aggregate shock. The existence of a global

state contingent asset implies that the ratios of the marginal consumptions of dates $t+1$ and t must be equal across countries,

$$\frac{u_{C_{t+1}}}{u_{C_t}} = \frac{u_{C^*_{t+1}}}{u_{C^*_t}}, \text{ for all dates and states.} \tag{35}$$

Condition (35) entails that there is a constant $\alpha > 0$, such that

$$u_{C_t} = \alpha \cdot u_{C^*_t}, \text{ for all dates and states.} \tag{36}$$

For the particular instantaneous utility function used (36) implies:

$$\left[C_t - \epsilon \frac{(N_t)^{1+\chi}}{1+\chi}\right] = \alpha' \cdot \left[C^*_t - \epsilon \frac{(N^*_t)^{1+\chi}}{1+\chi}\right], \text{ with } \alpha' = \alpha^{-\frac{1}{\phi}}. \tag{37}$$

It is clear, from (37), that the differences in hours across countries, for all dates and states, will be reflected in differences in the aggregate consumption across countries.■

We have shown that even if there is a state contingent asset, tradable across countries, a common shock will have idiosyncratic effects across similar countries. There will be transactions of the state contingent asset to smooth out marginal utilities of consumption and leisure across countries, but nevertheless a common shock will affect the terms of trade and lead to differences across countries in state contingent hours, output and aggregate consumption.

7 Appendix 3 (not for publication): Determination of the Labor Supply

The labor supply of the home country is implied by the following set of equalities

$$
\begin{aligned}
\frac{\epsilon\left(N_t\right)^{\chi}}{\theta^{\frac{1}{\sigma}}}\left(\frac{C_{h,t}}{C_t}\right)^{\frac{1}{\sigma}} &= \frac{\epsilon\left(N_t\right)^{\chi}}{\theta^{\frac{1}{\sigma}}\left[\theta^{\frac{1}{\sigma}}+(1-\theta)^{\frac{1}{\sigma}}\left(\frac{C_{f,t}}{C_{h,t}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}} \\
&= \frac{\epsilon\left(N_t\right)^{\chi}}{\left[\theta^{\frac{1}{\sigma}+\frac{\sigma-1}{\sigma}}+(1-\theta)^{\frac{1}{\sigma}+\frac{\sigma-1}{\sigma}}\left(\frac{\theta}{1-\theta}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{C_{f,t}}{C_{h,t}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}} \\
&= \frac{\epsilon\left(N_t\right)^{\chi}}{\left[\theta+(1-\theta)\left(\frac{\theta}{1-\theta}\frac{C_{f,t}}{C_{h,t}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}} \\
&= \frac{\epsilon\left(N_t\right)^{\chi}}{\left[\theta+(1-\theta)\left(\frac{1}{\mathfrak{p}_t}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}} \\
&= \frac{W_t}{R_t P_{h,t}}.
\end{aligned}
$$

Similarly for the foreign country.